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***A Heavy Ion Fireball freeze-out Dipion Cocktail for
Au-Au Collisions at root $s = 200\text{GeV}$
(Part 1).***

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A Heavy Ion Fireball freeze-out Dipion Cocktail for Au-Au Collisions at $\sqrt{s_{NN}}=200$ GeV (Part 1).

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Abstract

In this paper we develop all the ingredients that come into play in the freeze-out of the heavy ion fireball for Au-Au collisions at $\sqrt{s_{NN}}=200$ GeV into dipions. The resonance production of particles that decay into dipions plus minijets that also decay into dipions are explored. The final state re-scattering of the pions from minijets play an important role in the mass spectrum of the cocktail. Mass shifts due to minijet interference which depend on the volume of the re-scattering pions is presented. The effective mass balance function is explained. The dipion mass spectrum within a p_t range(intermediate p_t) is fitted using thermal and minijet amplitudes.

1 Introduction

The ultra-relativistic heavy ion collision starts out as a state of high density nuclear matter called the Quark Gluon Plasma(QGP) and expands rapidly to freeze-out. During the freeze-out phase quarks and gluons form a system of strongly interacting hadrons. These hadrons continue to expand in a thermal manner until no further scattering is possible because the system becomes too dilute. However this transition from quarks and gluons(partons) into hadrons is not a smooth affair. The expansion is very rapid and some faster or hard scattered partons fragment directly into hadron through a minijet[1] process. Thus we have thermal and minijet hadrons present in the last scattering of the hadrons. This mixture of sources is considered in this paper and applied to the dipion mass spectrum of the heavy ion fireball formed in Au-Au collisions at $\sqrt{s_{NN}}=200$.

The paper is organized in the following manner:

Sec. 1 is the introduction to the cross sections for $\pi \pi$ scattering. Sec. 2 develops a two component model with direct production and decay plus a background $\pi \pi$ component which must re-scatter in order to satisfy unitarity. Sec. 3 consider two channel unitary scattering $\pi \pi$ and $K \bar{K}$ for $J^{PC} = 0^{++}$ and shows what re-scattering would look like in the $\pi \pi$ channel. Sec. 4 introduces the balance function for dipion data and applies it to the two component model. Sec. 5 applies the two component model to dipion data within a p_t range. Sec. 6 presents the summary and discussion. Finally there are two appendices. Appendix A show the mathematical details of how the two component model is worked out. Appendix B determines the maximum value of the α parameter using photo-production data reported in Ref.[2].

1.1 $\pi \pi$ scattering cross section

For the first part of this story we will define what a scattering cross section is. We will first only consider elastic scattering of pions. Two pions can scatter at a certain energy which we will call $M_{\pi\pi}$. The differential cross section σ at a given $M_{\pi\pi}$ is

$$\frac{d\sigma}{d\phi d\theta} = \frac{1}{K^2} \left| \sum_{\ell} (2\ell + 1) T_{\ell} P_{\ell}(\cos\theta) \right|^2 \quad (1)$$

where ϕ and θ are the azimuthal and scattering angles, respectively. T_{ℓ} is a complex scattering amplitude and ℓ is the angular momentum. P_{ℓ} is the Legendre polynomial, which is a function of $\cos\theta$. K is the flux factor equal to the pion momentum in the center of mass. The T_{ℓ} elastic scattering amplitudes are complex amplitudes described by one real number which is in units of angles. The form of the amplitude is

$$T_{\ell} = e^{i\delta_{\ell}} \sin\delta_{\ell} \quad (2)$$

We note that δ depends on the value of ℓ and $M_{\pi\pi}$. We will use the phase shifts given in Ref[3].

2 The two component model

Let us consider two pions scattering in the final state of the heavy ion collision. The scattering will be in some ℓ partial wave. The $M_{\pi\pi}$ of the scattering dipion system will depend on the probability of the phase space of the overlapping pions. The pions emerge from a close encounter in a defined quantum state with a random phase. We will call this amplitude A and note that the absolute value squared of the amplitude is proportional to the phase space overlap. The emerging pions can re-scatter through the quantum state of the pions, which is a partial wave or a phase shift. We have amplitude A plus A times the re-scattering of pions through the phase shift consistent quantum state of A . The correct unitary way to describe this process is given by Ref[4] equation(4.5)

$$T = \frac{V_1 U_1}{D_1} + \frac{\left(V_2 + \frac{D_{12} V_1}{D_1} \right) \left(U_2 + \frac{D_{12} U_1}{D_1} \right)}{D_2 - \frac{D_{12}^2}{D_1}} \quad (3)$$

In the above equation we have two terms, 1 and 2. The first term denoted by 1 is the $\pi\pi$ scattering through ℓ -wave which will become the amplitude A mentioned above, where V is the incoming and U is the outgoing $\pi\pi$ system. The second term denoted by 2 is the direct production of the $\pi\pi$ system in the ℓ -wave with V being the production, the propagation being D and the decay being U . We see that there are terms D_{12} which involves a loop of pions between scattering pions and the formation of a resonance ($\ell=1$ would be a ρ) by the pions.

2.1 Final equation for the two component model

The complete derivation is in Appendix A. From the appendix we get two terms, one being the direct production of the resonance or $\pi\pi$ ℓ -wave phase shift and the second being the resonance from re-scattering. The final equation 6 has two important factors, one is two-body phase space and the other is a coefficient α . This coefficient is related to the real part of the $\pi\pi$ re-scattering loop and is given by equation 4. When the pions re-scatter or interact at a close distance or a point the real part of α has its maximum value of α_0 . While if the pions re-scatter or interact at a distance determined by the diffractive limit the value of α is zero.

In the equation 6 $|T|^2$ is the cross section for ℓ partial wave produced, where D is the direct production amplitude and A is the amplitude introduced above for the re-scattering pions into the ℓ -wave with δ_ℓ the $\pi\pi$ phase shift[3]. The q is the $\pi\pi$ center of mass momentum. At a given p_t and y bin, D will have a thermal factor as a function of $M_{\pi\pi}$. The α which is the real part of the re-scattering factor has a simple form given by

$$\alpha = (1.0 - \frac{r^2}{r_0^2})\alpha_0 \quad (4)$$

where r is the radius of re-scattering in fm's and r_0 is 1.0 fm or the limiting range of the strong interaction ranging to $r = 0.0$ for point like interactions.

The dependence of A is calculated by the phase space overlap of dipions added as four vectors and corrected for proper time, with the sum having the correct p_t , y and phase space weighting for ℓ^{th} partial wave for a given $M_{\pi\pi}$.

Finally we must use the correct two body phase space. For a two body system of pions, phase space goes to an constant as $M_{\pi\pi}$ goes to infinity. Let us choose this constant to be unity. Phase space which is denoted by PS is equal to

$$PS = \frac{2qB_\ell(q/q_s)}{M_{\pi\pi}} \quad (5)$$

where B_ℓ is a Blatt-Weisskopf-barrier factor[5] for ℓ angular momentum quantum number. The q_s is the momentum related to the range of interaction of the $\pi\pi$ scattering. 1 fm is the usual interaction distance which implies that q_s is .200 GeV/c. For the ρ meson $\ell=1$ the barrier factor is $B_1 = \frac{(q/q_s)^2}{(1+(q/q_s)^2)}$. The phase space factor PS as a function of q near the $\pi\pi$ threshold is given by $q^{2\ell+1}$. Thus in the appendix we use $q^{2\ell+1}$ for the factor PS except for equation 6 which is the final equation.

$$|T|^2 = |D|^2 \frac{\sin^2 \delta_\ell}{PS} + \frac{|A|^2}{PS} |\alpha \sin \delta_\ell + PS \cos \delta_\ell|^2 \quad (6)$$

3 Re-scattering through the Swave $\pi\pi$ is a two channel problem

In Sec. 2 we derived equation 6 considering only elastic scattering of the $\pi\pi$ system. If we consider the Dwave it couples to the $f_2(1270)$ ($J^{PC} = 2^{++}$) with 85% of the cross section in the $\pi\pi$ channel. The Pwave couples to the $\rho(770)$ ($J^{PC} = 1^{--}$) where 100% is in the $\pi\pi$ channel. The Swave $\pi\pi$ ($J^{PC} = 0^{++}$) couples to two resonances the σ and the $f_0(980)$. The σ is purely elastic while the $f_0(980)$ is split between the $\pi\pi$ and $K\bar{K}$ channels. These two channels plus two resonances gives an additional complexity to the re-scattering problem.

In order to handle the $\pi\pi$ and $K\bar{K}$ channels we will use the K-matrix approach. When we are below the $K\bar{K}$ threshold the system is only a one channel problem and the K-matrix is only a single term of the matrix

$$K_{11} = \tan\delta_0. \quad (7)$$

We see that when $\delta_0 = 90^\circ$ that there will be a pole in the K-matrix. It is standard to expand the K-matrix as a sum of poles.

$$K_{11} = \sum_i \frac{\frac{2\gamma_i^2 q_{\pi\pi}}{M_{\pi\pi}}}{(M_i^2 - M_{\pi\pi}^2)} \quad (8)$$

Where γ_i is the coupling of the pole(i^{th}) to the $\pi\pi$ channel, $q_{\pi\pi}$ is the center of mass momentum of the $\pi\pi$ channel and M_i is the mass of the pole(i^{th}). The T-matrix is given by

$$T_{11} = e^{i\delta_0} \sin\delta_0 = \frac{K_{11}}{(1 - iK_{11})} = (1 - iK)^{-1}K \quad (9)$$

When both channels are open $j = 1 \pi\pi$ and $j = 2 K\bar{K}$, the K-matrix is given by

$$K_{11} = \sum_i \frac{\frac{2\gamma_{i1}^2 q_1}{M_1}}{(M_i^2 - M_1^2)}, K_{21} = \sum_i \frac{\frac{2\gamma_{i2}\sqrt{q_2}\gamma_{i1}\sqrt{q_1}}{M_1}}{(M_i^2 - M_1^2)}, K_{12} = \sum_i \frac{\frac{2\gamma_{i1}\sqrt{q_1}\gamma_{i2}\sqrt{q_2}}{M_1}}{(M_i^2 - M_1^2)}, K_{22} = \sum_i \frac{\frac{2\gamma_{i2}^2 q_2}{M_1}}{(M_i^2 - M_1^2)}. \quad (10)$$

The T-matrix is given by

$$T = (\delta - iK)^{-1}K. \quad (11)$$

We fit the Swave ($J^{PC} = 0^{++}$) $\pi\pi$ of Ref[3] using three poles for the σ , f_0 and some background from higher mass poles. The T_{11} amplitude is shown in Figure 1.

Figure 1: The T_{11} amplitude for the Swave ($J^{PC} = 0^{++}$) $\pi\pi$ comes from a fit to of Ref[3] using three K-matrix poles for the σ , f_0 and some background from higher mass poles.

Two pions scattering in the final state of the heavy ion collision in a Swave will be our amplitude A , where the emerging pions can re-scatter through the Swave. We have amplitude A plus A times the re-scattering of pions through the Swave phase shift.

The term of equation 6 $PS e^{i\delta_0} \cos \delta_0$ is equal to $PS(1 + i e^{i\delta_0} \sin \delta_0)$. T_{11} which is equal to $e^{i\delta_0} \sin \delta_0$ for the one channel case becomes $T_{11} = \eta e^{i\delta_0} \sin \delta_0$ for the two channel case. Thus the re-scattering term becomes $PS(1 + i \eta e^{i\delta_0} \sin \delta_0)$ or $PS(1 + iT_{11})$. Using our K-matrix fit to the Swave one obtain the re-scattering term plotted in Figure 2. In the lower mass we see a shift of the spectrum to a lower mass. In the next section will see the same effect for $\rho(770)$ resonance that re-scattering will shift its mass to lower values. This shift will be caused by a direct production plus the re-scattering adding together creating a shifted $\rho(770)$ [6]. We see that the f_0 is a narrow resonance. The f_0 resonates at the $K \bar{K}$ threshold. Direct production of the f_0 gives a bump at the $K \bar{K}$ threshold and the re-scattering of $\pi\pi$ also gives such a bump at the $K \bar{K}$ threshold(Figure 2). Therefore we will only consider the f_0 as a resonance being directly produced and decaying into $\pi\pi$ near the $K \bar{K}$ threshold.

4 The balance function for dipion effective mass

Up to this point in the paper we did not specify the charge of the pions considered. With the idea of the balance function we look at the creation of pairs of opposite charge pions. The QGP fireball begins mostly neutral without a large excess of charge. Pairs of quarks and anti-quarks are created finally forming hadrons mainly pions. Thus for every π^+ there is a π^- which balance out the charge. The balance function measures the kinematic variable between the balancing charge[7]. In our case we want to measure effective mass of the $\pi^+\pi^-$ pairs. The dipion effective mass balance function is defined by

$$B(M_{\pi\pi}) = \frac{1}{N} \sum_{events} \sum_{pairs} \frac{1}{2} \left\{ \frac{(M_{\pi^+\pi^-} - M_{\pi^+\pi^+})}{N_+} + \frac{(M_{\pi^+\pi^-} - M_{\pi^-\pi^-})}{N_-} \right\}, \quad (12)$$

where N is the number of events, N_+ is the number of positive pions per event, and N_- is the number of negative pions per event.

Let us consider the effective mass dipion balance function for p-p collisions $\sqrt{s_{NN}}=200$ GeV[8]. The effective mass shows a large bump at the K^0 mass from $K_S^0 \rightarrow \pi^+\pi^-$. Above this bump there is another mass bump which could be the $\rho(770)$. At this mass Pwave should be the most important partial wave so let us assume this is the case. The balance function drops off with mass so we assume an exponential fall off with $M_{\pi^+\pi^-}$. The $\rho(770)$ should be directly produced and decay through the Pwave phase shift. We will use equation 6 to fit the p-p balance function. Figure 3 shows the fit to the STAR data, where the solid line is the fit and dashed line is the $\rho(770)$. The exponential is amplitude A and the dotted line is A plus A times the re-scattering of pions through the Pwave phase shift. The value of α is 0.37 which means the radius of re-scattering of the pions is 0.9 fm using equation 4 with the value of α_0 equal to 2. The solid line which is a good fit to the data peaks at .73 GeV/c. We see that the shift of this peak is caused by the dotted line added to the dashed line.

Figure 2: We plot for the S-wave ($J^{PC} = 0^{++}$) $\pi\pi$ re-scattering term $|PS(1 + iT_{11})|^2$. The T_{11} amplitude comes from a fit to of Ref[3] using three K-matrix poles for the σ , f_0 and some background from higher mass poles.

Figure 3: The fit to the p-p balance function STAR data[8], where the solid line is the fit and dashed line is the $\rho(770)$. The exponential is amplitude A and the dotted line is A plus A times the re-scattering of pions through the Pwave phase shift.

We turn to the central Au-Au balance function measured by STAR[8]. Again using the same functions that we used above we fit the data. The solid line is the fit and dashed line is the $\rho(770)$. The exponential function is amplitude A and the dotted line is A plus A times the re-scattering of pions through the Pwave phase shift. The value of α is 0.44 which means the radius of re-scattering of the pions is 0.88 fm using equation 4 with the value of α_0 equal to 2. We see it is a little smaller volume where pions can re-scatter.

5 The dipion effective mass cocktail over a p_t range in Au-Au

In order to form this cocktail we need to consider three important ingredients that come into play. First is the thermal production of resonances that decay into $\pi^+\pi^-$ as a function of dipion p_t . Second is the minijet production of $\pi^+\pi^-$ that is not through a resonance decay and determine its dipion effective mass spectrum as a function of dipion p_t . Also we need the break up of the spectrum into partial waves. Third is rewriting equation 6 in a form that uses resonance or Breit-Wigner parameters (mass, widths) instead of phase shifts plus modify the equation to use the derived minijet amplitudes.

5.1 Thermal production of resonances

Thermal resonance production will have a Boltzmann weighting of the dipion effective mass spectrum. Since we are projecting in p_t this weighting will be an exponential function of the transverse mass divided by the temperature[6,9-13].

$$Weight(M_{\pi\pi}) = \frac{M_{\pi\pi}}{\sqrt{M_{\pi\pi}^2 + p_t^2}} \exp \frac{-\sqrt{M_{\pi\pi}^2 + p_t^2}}{T} \quad (13)$$

This weight times the Breit-Wigner line shape is the thermal production of the resonance which decays into the dipion system. The Breit-Wigner line shape is given by

$$BW(M_{\pi\pi}) = \frac{M_{\pi\pi} M_0 \Gamma}{(M_0^2 - M_{\pi\pi}^2)^2 + M_0^2 \Gamma^2}. \quad (14)$$

Where Γ is the $M_{\pi\pi}$ dependent total width

$$\Gamma = \Gamma_0 \frac{\frac{q B_\ell(q/q_s)}{M_{\pi\pi}}}{\frac{q_0 B_\ell(q_0/q_s)}{M_0}} \quad (15)$$

with Γ_0 being the total width at resonance, B_ℓ is the Blatt-Weisskopf-barrier factor[5] for the ℓ of the resonance, q is the $\pi\pi$ center mass momentum, q_0 is q at resonance, M_0 is the mass of the resonance, and q_s is center mass momentum related to the size (1.0 fm is used $q_s = .200$ GeV/c).

Figure 4: The fit to the Au-Au balance function STAR data[8], where the solid line is the fit and dashed line is the $\rho(770)$. The exponential is amplitude A and the dotted line is A plus A times the re-scattering of pions through the Pwave phase shift.

5.2 Minijet production of dipions

Partons that under go a hard scattering fragment into hadrons[1, 14]. These hadrons become part of the outward flow of hadrons with the thermal hadrons. Hadrons that have long life times will decay outside the freeze-out volume. For these long lived resonances the dipion spectrum will be given by the Breit-Wigner line shape of the last subsection. Thus the source of these resonances either thermal or minijet fragmentation will not be apparent. Resonances like the $\eta(c\tau = 154000 \text{ fm})$, the $\omega(c\tau = 24 \text{ fm})$, the $\eta'(c\tau = 100 \text{ fm})$, the $K^*(c\tau = 4 \text{ fm})$, and the $\phi(c\tau = 50 \text{ fm})$ are decaying outside the freeze-out volume. All other resonances decay inside this volume like the $\sigma(c\tau = 1/3 \text{ fm})$, the $\rho(770)(c\tau = 1.3 \text{ fm})$, and the $f_2(1270)(c\tau = 1 \text{ fm})$. Pions from these decays become a source for re-scattering with pion directly produced or the ones that arise from decays.

We will use the minijet fragmentation code of PYTHIA[14] in order to estimate the dipion effective mass spectrum. We set up correct kinetics by using minijets that are predicted by the program HIJING[15] for Au-Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$. We cycle through the minijets produced by HIJING forming dipion pair($\pi^+\pi^-$) combinations within a given minijet fragmentation. We remove the long lived resonance listed above because they can be handled through the direct thermal production. All pairs that come directly from the second class short lived resonances are also not considered. They also can be handled through the direct thermal production. However combinations of their decay pions with other pions are considered as a source of minijet dipions. We see from Figure 5 that the hadrons that fragment from the minijets are moving in the same direction and will interact with each other.

The dipions from this minijet source can be selected for its dipion p_t range and decomposed into each partial wave $\ell(\ell\text{wave})$ obtaining an amplitude A for equation 6. We can separate out this dipion spectrum by using statistical and kinematic weighting. At the highest dipion mass the S, P, D, and F waves have a $2\ell + 1$ statistical weighting. S is 1, P is 3, D is 5, and F is 7 making 16 units of probability. The kinematic weighting is given by the Blatt-Weisskopf-barrier factors. At each dipion mass we have a dipion spectrum from the minijets with a selection on dipion p_t which we call $jet(M_{\pi^+\pi^-})$. Let us define Z as the ratio of the center mass dipion momentum q divided q_s which is .200 GeV/c(size of 1.0 fm) all squared

$$Z = \frac{q^2}{q_s^2}. \quad (16)$$

The Fwave minijet dipion weight is given by

$$F = \frac{7}{16} \frac{Z^3}{(Z^3 + 6Z^2 + 45Z + 225)}. \quad (17)$$

The Dwave minijet dipion weight is given by

$$D = \frac{5}{9} (1.0 - F) \frac{Z^2}{(Z^2 + 3Z + 9)}. \quad (18)$$

Minijet

Parton Shower

Figure 5: A minijet parton shower.

The Pwave minijet dipion weight is given by

$$P = \frac{3}{4}(1.0 - D - F) \frac{Z}{(Z + 1)}. \quad (19)$$

The Swave minijet dipion weight is given by

$$S = (1.0 - P - D - F). \quad (20)$$

Thus $F(M_{\pi^+\pi^-}) = Fjet(M_{\pi^+\pi^-})$, $D(M_{\pi^+\pi^-}) = Djet(M_{\pi^+\pi^-})$, $P(M_{\pi^+\pi^-}) = Pjet(M_{\pi^+\pi^-})$, and $S(M_{\pi^+\pi^-}) = Sjet(M_{\pi^+\pi^-})$.

Let us choose a dipion p_t range and plot the above minijet spectrum. We choose $1.6 \text{ GeV}/c < p_t < 1.8 \text{ GeV}/c$ and plot in Figure 6 the Swave, Pwave, Dwave, and Fwave from minijets coming from HIJING for Au-Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$.

5.3 Equation 6 for Breit-Wigner parameters

In this subsection we will alter equation 6 so it can use Breit-Wigner parameters (mass, width) instead of phase shifts. We will also need to modify the re-scattering part of the equation in order to have the correct threshold behavior we have just introduced into the minijet partial waves above. The phase shift can be written for the ℓ^{th} wave as

$$\cot\delta_\ell = \frac{(M_\ell^2 - M_{\pi\pi}^2)}{M_\ell\Gamma_\ell}, \quad (21)$$

where M_ℓ is the mass of the resonance in the ℓ wave and Γ_ℓ is its total width.

$$\Gamma_\ell = \Gamma_{0\ell} \frac{\frac{qB_\ell(q/q_s)}{M_{\pi\pi}}}{\frac{q_\ell B_\ell(q_\ell/q_s)}{M_\ell}} \quad (22)$$

with $\Gamma_{0\ell}$ the total width at resonance, B_ℓ is the Blatt-Weisskopf-barrier factor for the ℓ of the resonance, q is the $\pi\pi$ center mass momentum, q_ℓ is q at resonance, M_ℓ is the mass of the resonance, and q_s is center mass momentum related to the size (1.0 fm is used $q_s = .200 \text{ GeV}/c$).

Using equation 21 we rewrite equation 6 as

$$|T_\ell|^2 = |D_\ell|^2 \frac{\sin^2\delta_\ell}{PS_\ell} + \frac{|A_\ell|^2 \sin^2\delta_\ell}{PS_\ell} |\alpha + PS_\ell \cot\delta_\ell|^2 \quad (23)$$

The D_ℓ is the thermal production term and is constant except for the Boltzmann weight. The expected threshold behavior $q^{2\ell+1}$ comes from the $\sin\delta_\ell$ term. Since there is $\sin^2\delta_\ell$ one of the $q^{2\ell+1}$ is killed off by dividing by PS_ℓ . In Figure 6 we have put into our minijet A_ℓ the correct threshold $q^{2\ell+1}$ so we need kill off the $q^{2\ell+1}$ of the other $\sin\delta_\ell$ term. Therefore equation 6 for our minijet A_ℓ we will use

$$|T_\ell|^2 = |D_\ell|^2 \frac{\sin^2\delta_\ell}{PS_\ell} + \frac{|A_\ell|^2 \sin^2\delta_\ell}{PS_\ell^2} |\alpha + PS_\ell \cot\delta_\ell|^2 \quad (24)$$

Figure 6: The dipion p_t range $1.6 \text{ GeV}/c < p_t < 1.8 \text{ GeV}/c$ showing the S-wave, P-wave, D-wave, and F-wave from minijets coming from HIJING for Au-Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$.

Rewriting equation 6 for each partial wave with Breit-Wigner parameters the first term becomes

$$|T_\ell|_1^2 = |D_\ell|^2 \frac{M_{\pi\pi}^2}{\sqrt{M_{\pi\pi}^2 + p_t^2}} \exp \frac{-\sqrt{M_{\pi\pi}^2 + p_t^2}}{T} \frac{M_\ell \Gamma_\ell}{(M_\ell^2 - M_{\pi\pi}^2)^2 + M_\ell^2 \Gamma_\ell^2}, \quad (25)$$

while the second term

$$|T_\ell|_2^2 = |A_\ell|^2 \frac{M_\ell^2 \Gamma_\ell^2}{(M_\ell^2 - M_{\pi\pi}^2)^2 + M_\ell^2 \Gamma_\ell^2} \left| \alpha + \frac{2q B_\ell(\frac{q}{q_s})(M_\ell^2 - M_{\pi\pi}^2)}{M_{\pi\pi} M_\ell \Gamma_\ell} \right|^2 \left(\frac{M_{\pi\pi}^2}{4q^2 B_\ell^2(\frac{q}{q_s})} \right). \quad (26)$$

$$|T|^2 = \sum_\ell |T_\ell|^2 \quad (27)$$

where

$$|T_\ell|^2 = |T_\ell|_1^2 + |T_\ell|_2^2 \quad (28)$$

and $|A_0|^2 = S(M_{\pi^+\pi^-})$, $|A_1|^2 = P(M_{\pi^+\pi^-})$, $|A_2|^2 = D(M_{\pi^+\pi^-})$, and $|A_3|^2 = F(M_{\pi^+\pi^-})$.

5.4 STAR data dipion p_t range (1.6 GeV/c < p_t < 1.8 GeV/c)

We have fitted a dipion p_t range using equation 27 above for the STAR data Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV 40% to 80% centrality. We included minijets up to $\ell = 3$ and resonances σ $\ell = 0$, $\rho(770)$ $\ell = 1$, and $f_2(1270)$ $\ell = 2$. Using the arguments of Sec. 3 we added the f_0 as a direct thermal term ($|T_0|_1^2$) and only the σ interfered with $\ell = 0$ minijet background. Two other thermal terms are present in the cocktail, the K_S^0 and the ω_0 .

Finally the threshold effective mass region .280 GeV to .430 GeV is dominated by the Swave and receives contributions from minijet fragmentation, $\pi\pi$ Swave phase shift, η decay, HBT adding to the like sign $\pi\pi$ distribution that has been subtracted away from the unlike sign $\pi\pi$ and the coulomb correction between the charged pions. The minijet fragmentation is the least known of the effects since we relied on PYTHIA, however there are large uncertainty in all the other effects. So for this fit we let the minijet fragmentation be free to fit the data and let the Breit-Wigner parameters for the σ determine the Swave phase shifts plus leaving out all other effects. The results of this fit is shown in Figure 7. Table I shows the Breit-Wigner parameters used in the fit.

Table I. The Bret-Wigner Parameters of the fit.

Table I		
resonance	mass(GeV)	width(GeV)
σ	1.011	1.015
f_0	0.973	0.041
ρ	0.748	0.147
f_2	1.275	0.185

Figure 7: Fit to STAR dipion effective mass distribution ($1.6 \text{ GeV}/c < p_t < 1.8 \text{ GeV}/c$) for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV 40% to 80% centrality using equation 27. See text for complete information.

The value of α for this p_t range is .806. For coherent photo-production of the $\rho(770)$ from STAR data[2] the value of α is 2.0 or α_0 because the two pions emerge from a point source(see Appendix B). Thus the radius of the size associated with re-scattering is .773 fm for this p_t range from equation 4.

6 Summary and Discussion

In this article we start with the basic definition of elastic $\pi\pi$ scattering. Next we show how re-scattering of pions depends on the unitary condition that interactions present in the phase shift of an orbital state must interact all the time. The process of parton fragmentation into dipion states through unitarity leads to a equation of production and re-scattering in a given orbital quantum number. This equation (equation 6) has two components in each orbital state: one being the thermal production of resonances in a dipion orbital state, the other is the re-scattering of dipions coming from parton or minijet fragmentation into the dipion orbital state which do not come directly from the resonance. Unitarity requires that there must be re-scatter through the resonance of the phase shift. This equation is used over and over again with the details presented in the appendix.

Equation 6 considers only elastic scattering of the $\pi\pi$ system. We considered the Dwave which couples to the $f_2(1270)$ ($J^{PC} = 2^{++}$) with 85% of the cross section in the $\pi\pi$ channel. The Pwave which coupled to the $\rho(770)$ ($J^{PC} = 1^{--}$) where 100% is in the $\pi\pi$ channel. The Swave $\pi\pi$ ($J^{PC} = 0^{++}$) couples to two resonances the σ and the $f_0(980)$. The σ is purely elastic while the $f_0(980)$ is split between the $\pi\pi$ and $K\bar{K}$ channels. We saw that the f_0 was a narrow resonance. The f_0 resonates at the $K\bar{K}$ threshold. Direct production of the f_0 gives a bump at the $K\bar{K}$ threshold and the re-scattering of $\pi\pi$ also gives such the same bump at the $K\bar{K}$ threshold Sec. 3 (Figure 2). Therefore we considered the f_0 as a resonance being directly produced and decaying into $\pi\pi$ near the $K\bar{K}$ threshold.

We used equation 6 to fit the dipion effective mass balance function. We assumed the Pwave was the most important partial wave in the dipion effective mass range of the fit. The balance function drops off with mass so we assumed an exponential fall off with $M_{\pi^+\pi^-}$. The two parts of equation 6 where the first part being the $\rho(770)$ directly produced and decaying through the Pwave phase shift with the second part being the exponential function plus re-scattering through the $\rho(770)$. With this simple model we were able to fit the p-p balance (Figure 3) and the central Au-Au balance(Figure 4). The observed mass shifts were due to the re-scattering effect, being ~ 40 MeV.

We chose a dipion p_t range to do a cocktail fit to the effective dipion mass spectrum. We needed three important ingredients in order to do this fit. First is the thermal production of resonances that decay into $\pi^+\pi^-$ as a function of dipion p_t . Second is the dipion effective mass spectrum as a function of dipion p_t coming from minijet production not through resonance decay. Third we needed to rewrite equation 6 in a form that uses resonance or Breit-Wigner parameters (mass, widths) instead of phase shifts. Once these three ingredients were developed we were successful in doing a cocktail fit (Figure 7).

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A Appendix

Starting with equation (4.5) from Ref[4]

$$T = \frac{V_1 U'_1}{D_1} + \frac{\left(V_2 + \frac{D_{12} V_1}{D_1}\right) \left(U'_2 + \frac{D_{12} U'_1}{D_1}\right)}{D_2 - \frac{D_{12}^2}{D_1}} \quad (29)$$

In order to have the correct threshold kinematics, we define

$$U'_1 = U_1 \sqrt{q^{2\ell+1}} \quad (30)$$

$$U'_2 = U_2 \sqrt{q^{2\ell+1}} \quad (31)$$

where q is the $\pi\pi$ center of mass momentum and ℓ is the value of the angular momentum. The amplitude A of the text is given by

$$\frac{V_1 U_1}{D_1} = A \quad (32)$$

Thus we have

$$\frac{V_1 U'_1}{D_1} = A \sqrt{q^{2\ell+1}} \quad (33)$$

The phase shift for the ℓ^{th} partial wave will be given by δ_ℓ , where

$$\frac{U'_2 U'_1}{D_2} = e^{i\delta_\ell} \sin \delta_\ell \quad (34)$$

The above equality is true if the D_1 mode plays no role in the $\pi\pi$ scattering in the ℓ^{th} partial wave. But in the initial state there is a large production of D_1 . The U 's are the basic coupling of the D 's to the $\pi\pi$ system. In order to decouple D_1 from the $\pi\pi$ system U_1 must go to zero. We can maintain a finite production of D_1 if we define

$$V_1 = \frac{1}{U_1} \quad (35)$$

Thus the first term in the equation becomes

$$\frac{V_1 U'_1}{D_1} = \frac{\frac{1}{U_1} U_1 \sqrt{q^{2\ell+1}}}{D_1} = \frac{\sqrt{q^{2\ell+1}}}{D_1} = A \sqrt{q^{2\ell+1}} \quad (36)$$

The form of D_{12} is given by

$$D_{12} = \alpha U_1 U_2 + i q^{2\ell+1} U_1 U_2 \quad (37)$$

D_{12} is the real and imaginary part of the two pion loop from state 1 to state 2. The U 's are the $\pi\pi$ couplings and the imaginary part goes to zero at the $\pi\pi$ threshold. The α factor is α_0 for re-scattering coming from a point source, but goes to zero when re-scattering is diffractive. A simple form for α is given by

$$\alpha = (1.0 - \frac{r^2}{r_0^2}) \alpha_0 \quad (38)$$

where r is the radius of re-scattering in fm's and r_0 is 1.0 fm or the limiting range of the strong interaction. The second term of the first equation is

$$\frac{\left(V_2 + \frac{D_{12}V_1}{D_1}\right) \left(U_2' + \frac{D_{12}U_1'}{D_1}\right)}{D_2 - \frac{D_{12}^2}{D_1}} \quad (39)$$

Rewriting

$$\frac{\left(V_2 + \frac{\alpha U_1 U_2 V_1}{D_1} + i q^{2\ell+1} \frac{U_1 U_2 V_1}{D_1}\right) \left(U_2' + \frac{D_{12}U_1'}{D_1}\right)}{D_2 - \frac{D_{12}^2}{D_1}} \quad (40)$$

Let us make substitutions

$$V_1 = \frac{1}{U_1}, U_2 = \frac{U_2'}{\sqrt{q^{2\ell+1}}}, \frac{1}{D_1} = A, D_{12} = 0 \quad (41)$$

The second term becomes

$$\frac{\left(V_2 + \frac{A\alpha U_2'}{\sqrt{q^{2\ell+1}}} + i\sqrt{q^{2\ell+1}} A U_2'\right) U_2'}{D_2} \quad (42)$$

The first term is

$$\frac{V_1 U_1'}{D_1} = A \sqrt{q^{2\ell+1}} \quad (43)$$

Adding the first and the second terms and substituting the phase shift,

$$T = \frac{V_2}{U_2} \frac{e^{i\delta_\ell} \sin\delta_\ell}{\sqrt{q^{2\ell+1}}} + A \left(\frac{e^{i\delta_\ell} \alpha \sin\delta_\ell}{\sqrt{q^{2\ell+1}}} + \sqrt{q^{2\ell+1}} e^{i\delta_\ell} \cos\delta_\ell \right) \quad (44)$$

The term with the factor $\frac{V_2}{U_2}$ is the direct production of the dipion system. We shall call this amplitude D . The re-scattered amplitude is A and is modified by the dipion phase shift. These two amplitudes have some random phase and are not coherent. Thus the cross section is

$$|T|^2 = |D|^2 \frac{\sin^2\delta_\ell}{PS} + \frac{|A|^2}{PS} |\alpha \sin\delta_\ell + PS \cos\delta_\ell|^2 \quad (45)$$

B Appendix

In this appendix we determine the value α coming from a point source (thus α_0 see equation 4 and 38) using photo-production data reported in Ref.[2]. When a photon interacts with a strong field a ρ or a $\pi^+ \pi^-$ pair is formed in a Pwave, both states are coherent with each other and must be added together (see equation 3 of Ref.[2]). We write equation 3 using the phase shift of Pwave $\pi^+ \pi^-$ scattering and the width (Γ_0) of the ρ resonance. Threshold behavior is added using the phase space factor (PS) for Pwave production (see equation 5 of Sec. 2.1).

$$\frac{dN}{dM_{\pi\pi}} = \left| A_\rho \frac{\sin\delta e^{i\delta}}{\Gamma_0^{1/2} (\frac{PS}{PS_0})^{1/2}} + (\frac{PS}{PS_0})^{1/2} B_{\pi\pi} \right|^2 \quad (46)$$

The Pwave phase factor is

$$PS = \frac{\frac{2q(q/q_s)^2}{(1+(q/q_s)^2)}}{M_{\pi\pi}}, \quad (47)$$

where q is the center mass momentum of the $\pi \pi$ and q_s is related to the range of interaction of the $\pi\pi$ scattering. 1 fm is the usual interaction distance which implies that q_s is .200 GeV/c. $M_{\pi\pi}$ is the effective mass of the system. The Pwave phase factor at the ρ resonance is

$$PS_0 = \frac{\frac{2q_0(q_0/q_s)^2}{(1+(q_0/q_s)^2)}}{M_\rho}, \quad (48)$$

where q_0 is the center mass momentum of the $\pi \pi$ at the ρ mass.

From equation 46 we pull out a common phase space ratio from both terms,

$$\frac{dN}{dM_{\pi\pi}} = \frac{1}{(\frac{PS}{PS_0})} \left| A_\rho \frac{\sin\delta e^{i\delta}}{\Gamma_0^{1/2}} + (\frac{PS}{PS_0}) B_{\pi\pi} \right|^2. \quad (49)$$

The ratio of the $\pi \pi$ production amplitude over the ρ production amplitude as averaged over world data is

$$\frac{B_{\pi\pi}}{A_\rho} = 0.85. \quad (50)$$

Using equation 50 and substituting this relation for A_ρ into equation 49 we obtain

$$\frac{dN}{dM_{\pi\pi}} = \frac{B_{\pi\pi}^2}{(\frac{PS}{PS_0})} \left| \frac{\sin\delta e^{i\delta}}{0.85\Gamma_0^{1/2}} + (\frac{PS}{PS_0}) \right|^2. \quad (51)$$

The second term of equation 51 can be expanded because $1 = e^{-i\delta}e^{i\delta}$ becoming

$$\frac{dN}{dM_{\pi\pi}} = \frac{B_{\pi\pi}^2}{(\frac{PS}{PS_0})} \left| \frac{\sin\delta e^{i\delta}}{0.85\Gamma_0^{1/2}} + (\frac{PS}{PS_0})\cos\delta e^{i\delta} - i(\frac{PS}{PS_0})\sin\delta e^{i\delta} \right|^2. \quad (52)$$

There is a common phase factor($e^{i\delta}$) on all three terms which has a magnitude of 1. The magnitude for all three terms can be written as

$$\frac{dN}{dM_{\pi\pi}} = (\frac{PS}{PS_0})B_{\pi\pi}^2\sin^2\delta + \frac{B_{\pi\pi}^2}{PS(PS_0)} \left| \frac{PS_0\sin\delta}{0.85\Gamma_0^{1/2}} + PS\cos\delta \right|^2. \quad (53)$$

The second term has an amplitude of the form of equation 6 with

$$\alpha\sin\delta + PS\cos\delta. \quad (54)$$

This implies that the maximum value of α is α_0 and given by

$$\alpha_0 = \frac{PS_0}{0.85\Gamma_0^{1/2}} \simeq 2.0. \quad (55)$$

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