Radiation Field Simulation and Estimation Algorithms for a Mobile Sensor and a Stationary Unknown Source

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Auspices and Disclaimer

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Radiation Field Simulation and Estimation Algorithms for a Mobile Sensor and a Stationary Unknown Source: Initial Results

May 13, 2009

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This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344
We Have an Interdisciplinary Team

• Simon Labov (LLNL/GHS) Principal Investigator
• Tom Edmunds (LLNL/NSED): Systems Engineering
• Yiming Yao (LLNL/NSED): Simulations
• Larry Hiller (LLNL, Physics): Simulations, Algorithms, systems
• Maya Gokhale (LLNL/ CS): Networks
• Gardar Johannesson (LLNL/NSED): Algorithms
• Dale Sloan (LLNL): Physics
• Richard Wheeler (LLNL): Physics
• Karl E. Nelson (LLNL/NSED): Algorithms, physics
• Grace Clark (LLNL/NSED: Estimation/Detection Algorithms

• Garrett Jernigan (UCB): Algorithms
• Adel Ganem (Zontrak Inc., San Ramon, CA): Networks
• K. Mani Chandy (Caltech): Algorithms
• Annie Liu (Caltech): Algorithms
• Ryan McLean (Caltech): Algorithms
• Matt Wu (Caltech): Algorithms
Agenda

• Introduction
• Algorithm R&D Plans
• Technical Approach
• Current Results
• Discussion and Plans
Algorithm R&D Plans in Priority Order

- Derive and Implement the Background and Source Simulation Algorithms
- Document the Background and Source Simulation Algorithms
- Derive and Document the Proposed Backpropagation Algorithm

- Implement the Proposed Backpropagation Algorithm and possibly some others

- R&D for a new full inversion algorithm with proximity and energy constraints
  - For a single block of measurements
  - For multiple blocks of measurements

- Future Work:
  - Fold in attenuation away from the source: occlusions, shielding, etc.
  - Fold in asymmetric sources: occlusion near the source
Problem Definition
Problem Definition

Given:

• A Simple two-dimensional (planar) radiation field (no buildings, etc.)
  - A grid of radiation samples

• A single constant radiation point source

• A single mobile radiation sensor traveling along a planar trajectory one meter above the plane measuring counts per second at each grid point

• Measurements of the sensor position (GPS) along the trajectory

Goals:

• Estimate the radiation (counts/sec) at each point on the 2D grid based only upon the sensor measurements acquired along the sensor trajectory.

• Detect the source, estimate its location and estimate its radiation strength.
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Derivation of the Algorithms for Simulating Background and Source Radiation

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Simulations Algorithms Derivation p.1

Problem Description: The Spatial Grid

\[ \mathbf{p}(k) = \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix} \]

(From GPS)

\[ \mathbf{x}(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix} = \begin{bmatrix} i \Delta \\ j \Delta \end{bmatrix} \]

= Location Vector of a Point on the Spatial Grid

\[ k = \text{time or position index on the sensor trajectory} \]

\[ \Delta = \text{spatial sample interval (m)} \]

(Assume \( \Delta_i = \Delta_j = \Delta \))

\[ m = \text{position index on the spatial grid} \]

Injected Source Location

\[ q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} i \Delta \\ j \Delta \end{bmatrix} \]

Notation: The spatial grid has been converted to a vector (see next page)
NOTATION FOR POINTS ON THE SPATIAL GRID

LET

\[ \mathbf{X} = \begin{bmatrix}
X_1 \\
\vdots \\
X_M
\end{bmatrix} = \begin{bmatrix}
X_1(i,j) \\
\vdots \\
X_M(i,j)
\end{bmatrix} \]

TREAT THE GRID AS A 2D ARRAY, THEN CONVERT IT TO A VECTOR

DO A LExicographic ORDERING AS IN MATLAB \( \mathbf{X}(:) \) 

\( \Rightarrow \) A VECTOR FORMED BY RASTER-SCANNING BY COLUMNS

\[ \mathbf{X}(:) = \begin{bmatrix}
X_C(1) \\
X_C(2) \\
\vdots \\
X_C(M)
\end{bmatrix} \]

\[ \mathbf{X}(m) = \begin{bmatrix}
X_1(m) \\
X_2(m) \\
\vdots \\
X_M(m)
\end{bmatrix} \]

\( X_1(m) = \text{Row Coord.} \) 
\( X_2(m) = \text{Column Coord.} \)

\[ m = IJ \]
Let $X(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix}$ for $m = 1, 2, \ldots, M$.

- $x_1(m)$ = ROW INDEX
- $x_2(m)$ = COLUMN INDEX

$X^T(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix}$

$X = \begin{bmatrix} X^T(1) \\ X^T(2) \\ \vdots \\ X^T(M) \end{bmatrix}$

$X^T = \begin{bmatrix} x_1(1) & x_2(1) \\ x_1(2) & x_2(2) \\ \vdots & \vdots \\ x_1(M) & x_2(M) \end{bmatrix}$
DEFINITIONS

\[ \Delta = \text{SPATIAL SAMPLING INTERVAL (METERS)} \]

\[ i = 1, 2, \ldots, \quad I = \text{RAY INDEX ON THE SPATIAL GRID (SOUTH)} \]

\[ j = 1, 2, \ldots, \quad J = \text{COLUMN INDEX (EAST)} \]

\[ c_{s}(i, j) = \text{RADIATION (CARTIES) ON THE SPATIAL GRID = TRUTH (SIMULATED)} \]

\[ \Omega = \begin{bmatrix} \omega(i,j) \end{bmatrix} = I \times J \text{ MATRIX OF RADIATION VALUES ON THE SPATIAL GRID} \]

\[ \varphi(k) = \begin{bmatrix} \varphi_1(k) \\ \varphi_2(k) \end{bmatrix} = 2 \times 1 \text{ VECTOR OF LOCATION COORDINATES FOR THE SENSOR AT A GIVEN TIME AND POSITION ON ITS TRAJECTORY (FROM GPS) } \\
\text{AND POSITION FOR MEASUREMENT yi ON TRAJECTORY } \]

\[ \varphi_1(k) = \text{SCALAR COORDINATE FOR SENSOR ALONG A ROW ON THE GRID} \]

\[ \varphi_2(k) = \text{COLUMN } \]

\[ \varphi_1(k) = i \Delta, \quad i = \text{ROW INDEX ON THE GRID} \]

\[ \varphi_2(k) = j \Delta, \quad j = \text{COLUMN} \]
\[
\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 2 \times 1 \text{ vector of the location of the point of interest on the spatial grid at a given position. We wish to estimate the radiation at this point of interest.}
\]

\[
q = \text{the position of a true point source (e.g., injected)}
\]

\[
k = \text{position index for a vector position on the sensor trajectory}
\]

\[
n = \text{time index} = 1, 2, \ldots, N \quad (k \text{ or } \ell \text{ used } "i")
\]

\[
t_n = nt = \text{time (seconds) at time index } n
\]

\[
t = \text{time sample interval (seconds)}
\]
\[ R[\mathbf{p}(k), \mathbf{x}(m)] = \text{SCALAR EUCLIDEAN DISTANCE BETWEEN A POINT AT } \mathbf{p}(k) \text{ AT TIME OR POSITION } k \text{ AND A POINT } \mathbf{x}(m) \text{ ON THE SPATIAL GRID.} \]

\[ \mathbf{p}(k) = \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix}, \quad \mathbf{x}(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix} \]

\[ R[\mathbf{p}(k), \mathbf{x}(m)] = \lVert \mathbf{p}(k) - \mathbf{x}(m) \rVert \]

\[ = \sqrt{\mathbf{e}(k) - \mathbf{x}(m) \mathbf{e}(k) - \mathbf{x}(m)} \]

\[ = \sqrt{\begin{bmatrix} p_1(k) - x_1(m) \\ p_2(k) - x_2(m) \end{bmatrix} \begin{bmatrix} p_1(k) - x_1(m) \\ p_2(k) - x_2(m) \end{bmatrix}} \]

\[ \text{for } \mathbf{e} = [e_1, e_2]^T \]

\[ \lVert \mathbf{e} \rVert = \langle e, e \rangle^{\frac{1}{2}} \]

\[ = (e^T e)^{\frac{1}{2}} \]

\[ = \sqrt{e^2 + e_2^2} \]

\[ \text{Simulations Algorithms Derivation p.6} \]
Define

\[ R \left[ \mathbf{x}(k) , \mathbf{z} \right] = 11 \left| \mathbf{p}(k) - \mathbf{z} \right| \]

\[ R \left[ \mathbf{p}(k) , \mathbf{z} \right] = \frac{1}{2} \left( \mathbf{p}(k) - \mathbf{z} \right)^T \Sigma_{p}^{-1} \left( \mathbf{p}(k) - \mathbf{z} \right) \]

\[ S(m) = \text{RADIATION VALUE (COUNTS/SEC) AT ONE METER ABOVE THE GRID PLANE} \]

\[ = \frac{\text{COUNTS}}{\text{SEC}} \]

\[ = \text{THIS IS WHAT WE WANT TO ESTIMATE} \]
Grace A. Clark, Ph.D.

Simulations Algorithms Derivation p.8

GACS Notes on How to Simulate the Mean of the Poisson Process to Make the Measurements $y(h)$

$y(h) = \text{Sensor measurement at position } h$

$\lambda(h) = \text{Poisson } \mu[\lambda(h)] = \text{A Poisson draw at position } h$

$\mu[\lambda(h)] = \sum_{m=1}^{M} \frac{\omega(m)}{R[\lambda(h), x(m)]^2 + 1} + \frac{\sigma(m)}{R[\lambda(h) - \bar{q}]^2 + 1}$

$\omega(m) = \text{Background radiation (cts/sec) at position } m$

$R[\lambda(h), x(m)] = \| \lambda(h) - x(m) \|$

$\sigma(m) = \text{Distance between the sensor position } \lambda(h) \text{ and the spatial position } x(m)$

$\sigma(m) = \text{Radiation (cts/sec) of the source at position } m$

$\frac{\sigma(m)}{\bar{q}} = \text{Position of the source}$
We wish to simulate the radiation as follows:

\[
s(\mathbf{r}) = \begin{cases} 
0, & \text{no source} \\
S, & \text{injected constant source}
\end{cases}
\]

- \( s(\mathbf{r}) \geq 0 \) (always positive)

- A realistic range of values for simulation:
  - \( \mu \approx 40 \text{ counts/sec} \) for background mean
  - Range of \( \theta \) = \((+20, +80)\) over the map
How to simulate the BG RAD:

1. **RNG**
2. **BLURRING FILTER** with $1/r^2$ kernel
3. **W\&N**
   - $W \sim N[\mu, \sigma^2]$

$W(M) = \text{TRUE RAD (cm^3/sec)}$

Kernel width $\approx 5-10$ meters
Simulations of Background Radiation, Source Radiation and Sensor Measurements

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First Step in the Background Radiation Simulation:  
*Construct a 2D Gaussian Distributed Array*

\( W = \text{Gaussian Distributed Radiation Field to be Lowpass Filtered in the Next Step with a } 1/R^2 \text{ Filter Kernel to Form the "True" Radiation Field.}\)

This represents one realization of a draw from the Gaussian RNG.
Simulated Background Radiation Field: Gaussian Array
Convolved with a “1/R^2” 2D Filter Kernel:
Background Radiation Field + Injected Poisson Point Source Radiation

\[ Y_{\text{inj}} = \text{BACKGROUND RADIATION FIELD } + \text{ SOURCE RADIATION (counts/sec)} \]

Y_{\text{inj}} = What the truth would look like once the radiation is transported to all the grid points.

This represents one realization of a draw from the Gaussian RNG.
Background Radiation + Point Source Location + Sensor Trajectory
\( U = \text{Mean of the Poisson BG Measurements and} \)
\( U_{\text{inj}} = \text{Mean of the Source Measurements Along the Sensor Trajectory} \)
Y = Poisson BG Measurements and
Yinj = Poisson Source Measurements Along the Sensor Trajectory
Conclusions and Plans

- Derive and Implement the Background and Source Simulation Algorithms
- Document the Background and Source Simulation Algorithms
- Derive and Document the Proposed Backpropagation Algorithm

  - Implement the Proposed Backpropagation Algorithm and possibly some others

  - R&D for a new full inversion algorithm with proximity and energy constraints
    - For a single block of measurements
    - For multiple blocks of measurements

- Future Work:
  - Fold in attenuation away from the source: occlusions, shielding, etc.
  - Fold in asymmetric sources: occlusion near the source
Clark_DNDA_Results_2
Preliminary Radiation Field Estimation Results
Using the Back Propagation Algorithm
June 5, 2009
Preliminary Results for the Backpropagation algorithm with Simulated Data

• This is the first radiation map result estimated using the Back Propagation algorithm
• I have not yet had time to validate the results
• The program was cancelled today, so I am documenting the results I have to date
Experiment E2: Backpropagation Algorithms to Estimate the Radiation Field

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Background Plus Sensor Trajectory and Source

Y bkg = Simulated Background Field Overlaid with
The Line = the Location Vector = p of the Sensor Platform Trajectory
The Star Depicts the Location of the Injected Source
This represents one realization of a draw from the Gaussian RNG

Source Location is Denoted by the Star
Histogram of the Simulated Background
Histogram of the Simulated Injected Source

Histogram of $\gamma_{inj}$ = the Simulated Injected Array

Theoretically Optimal Number of bins = $\log_2(N_{samples})+1 = 18$
**Mean of the Simulated Poisson Measurements**

- *U = Mean of the Poisson Measurements and U injected = Mean of the Injected Source Along the Sensor Trajectory (two vectors of means vs. distance)*
- *This represents one realization of a draw from the Gaussian RNG*
Simulated Measurements Along the Trajectory

- $Y = \text{Poisson Draw Measurement}$ and $Y_{\text{injected}} = \text{Poisson Draw Injected Source}$
- Along the Sensor Trajectory (two vectors of means vs. distance)
- This represents one realization of a draw from the Gaussian RNG to Make $U$
- and one draw each from a Poisson Process to Make $Y$ and $Y_{\text{inj}}$

Graph:
- Y (Measured Radiation)
- Y Injected (Injected Radiation)

Counts/sec vs. Distance (meters) or Time
Histogram of the Sensor Measurement (w / Source)
$A(m)$, $B(m)$ and $S(m)$, where $S = A./B$

$S(m)$ denotes the Estimated Radiation at Grid Position $m$
Estimated Radiation Map Using Back Propagation

- Red = High
- Blue = Low
Histogram of the Estimated Radiation Map

Histogram of SS = the Estimated Rad Field Array SS
\[ y = Y = \text{Measurements for Background Only (No Source)} \]

Theoretically Optimal Number of bins = \( \log_2(N_{\text{samples}}) + 1 = 16 \)
Conclusions and Plans

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