An overview of our accomplishments during the final term of this grant (1 September 2008 – 30 June 2012) is given below. These fall mainly into three categories: fast algorithms for linear eigenvalue problems; solution algorithms and modeling methods for PDEs with uncertain coefficients; and preconditioning methods and solvers for models of computational fluid dynamics (CFD). Publications referenced are listed immediately below.

Main Accomplishments

**Fast eigenvalue solvers** (Publications P1, P6, P9, P10, P12, P15, P17). Results include detailed analysis of the performance of Rayleigh quotient iteration for finding the smallest eigenvalues of positive-definite matrices when required matrix solves are replaced by approximate (inexact) computations (P1); development of robust preconditioning strategies for use in combination with subspace iteration (P6) and Arnoldi iteration (P9) for eigenvalue problems; and demonstration of the efficient use of new methods for computing eigenvalues associated with Hopf bifurcation in stability analysis of dynamical systems (P10, P12, P17).

**Computational algorithms for PDEs with uncertain coefficients** (P3, P5, P8, P11, P13, P16). Progress in this area includes several studies of the diffusion equation where the diffusion coefficient is a random field. In particular, we have developed an efficient preconditioner for the first-order (Darcy) formulation of the problem derived from a multigrid method designed to handle $H(\text{div})$ operators weighted by random diffusion coefficients; analysis and experimental results establish textbook multigrid convergence (P3). We have performed a careful study (P5) comparing the performance of the stochastic Galerkin and stochastic collocation methods for solving the stochastic diffusion equation, showing that if the dependence on stochastic parameters is linear, then the Galerkin method can be significantly more effective when a good (multigrid) preconditioner is used for the coupled Galerkin system. We have also developed an efficient method for handling such problems when the coefficient is of log-normal form (a particular example of a nonlinear function of the parameters); in this case, we take advantage of the fact that the diffusion problem can be transformed to an equivalent convection-diffusion problem depending linearly on the coefficients (P8). Finally, we have shown that the performance of collocation methods can be improved using adaptive strategies, and that methods of kernel density estimation can be used to estimate statistical quantities in cases when joint densities are not available (P11).

**Preconditioners and solvers for models of CFD** (P2, P4, P7). We have made a careful determination of the effects of boundary conditions on the performance of preconditioners for the
incompressible Navier-Stokes equations, leading to significant improvements in performance (P2). In addition, we have shown that the preconditioning methods developed through this project are robust and easily adapted to sophisticated models, including one concerned with buoyancy driven flow that includes thermal effects in the model (P7), and another simulating microfluidic flows driven by an electrokinetic process (P4).

Publications


**Presentations of Results**


ICIAM 2011, Vancouver, CA (two minisymposium presentations), July 2011.
Copper Mountain Conference on Iterative Methods, Copper Mountain, CO, April 2012.
Conference in honor of Tony Chan’s 60th birthday, UCLA, Los Angeles, CA, June 2012.

Purdue University, Mechanical Engineering Seminar, Feb. 2010.
University of Delaware, Analysis and PDE Seminar, April 1010.
NIST, Mathematical and Computational Sciences Division Seminar, June 1010.
Computational Science and Engineering Symposium, University of Illinois, April 2011.
Virginia Polytechnic Institute, Mathematics Colloquium, Nov. 2011.
University of Freiberg, Germany, Numerical Analysis Seminar, Jan. 2012.
Numerical Analysis Seminar, remote presentation to consortium of UK universities (Oxford, Bath, Bristol, Imperial and Warwick), May 2012.

**Personnel**

The grant supported Howard Elman (PI) and four graduate students in Maryland’s Applied Mathematics and Scientific Computing Program: Fei Xue (Ph.D. 2009), Christopher Miller (Ph.D. 2012), Minghao Wu (Ph.D. 2012), and Edward Phillips. It also supported two post-doctoral appointments, Elisabeth Ullmann (April - September, 2009) and Qifeng Liao (February 2010 - June 2012). In addition, it supported two visits from visiting professor Alastair Spence from the University of Bath, England, in January 2009 and June 2010.