DOE Grant ER64439: Final Report, October 2011

"Collaborative Research: Robust Climate Projections and Stochastic Stability of Dynamical Systems"

Michael Ghil, P.I., Dept. of Atmospheric & Oceanic Sciences and IGPP, UCLA
James C. McWilliams, Co-P.I., Dept. of Atmospheric & Oceanic Sciences and IGPP, UCLA
David Neelin, Co-P.I., Dept. of Atmospheric & Oceanic Sciences and IGPP, UCLA
Ilya Zaliapin, Co-P.I., Mathematics and Statistics Dept., Univ. Nevada, Reno
M. D. Chekroun, UCLA; D. Kondrashov, UCLA; and E. Simonnet, INLN, collaborators

1 Introduction

The project was completed along the lines of the original proposal, with additional elements arising as new results were obtained. The originally proposed three thrusts were expanded to include an additional, fourth one. (i) The effects of stochastic perturbations on climate models have been examined at the fundamental level by using the theory of deterministic and random dynamical systems, in both finite and infinite dimensions. (ii) The theoretical results have been implemented first on a delay-differential equation (DDE) model of the El-Niño/Southern-Oscillation (ENSO) phenomenon. (iii) More detailed, physical aspects of model robustness have been considered, as proposed, within the stripped-down ICTP-AGCM (formerly SPEEDY) climate model. This aspect of the research has been complemented by both observational and intermediate-model aspects of mid-latitude and tropical climate. (iv) An additional thrust of the research relied on new and unexpected results of (i) and involved reduced-modeling strategies and associated prediction aspects have been tested within the team's empirical model reduction (EMR) framework. Finally, more detailed, physical aspects have been considered within the stripped-down SPEEDY climate model.

The results of each of these four complementary efforts are presented in the next four sections, organized by topic and by the team members concentrating on the topic under discussion.

2 Random dynamical systems and stochastic parametrizations

Team members involved: M. D. Chekroun, M. Ghil and E. Simonnet

Atmospheric, oceanic and coupled climate dynamics is one of the main areas of application of dynamical systems theory. The stability — linear, nonlinear and structural — of climate model simulations has been an important area of investigation since the work of J. G. Charney, E. N. Lorenz, H. Stommel, P. D. Thompson, G. Veronis, and others. So far, these investigations have been mainly confined to deterministic models.

The crucial field of modeling subgrid-scale phenomena has been increasingly moving to stochastic parametrizations. It seems necessary, therefore, to address stability issues not only in the context of deterministically nonlinear models, but also in the presence of random perturbations. This topic has been widely studied in the mathematical and physical literature. During the past two decades, the mathematical theory of random dynamical systems (RDS; L. Arnold, *Springer-Verlag*, 1998) has made substantial progress.

We present here our theoretical results concerning the effect of noise in topological, as well as statistical terms. These results are illustrated with several "toy" models, and their physical relevance to more detailed and realistic models is discussed. We illustrate this discussion with some striking numerical results [Ghil *et al.* (2008a), Chekroun *et al.* (2011a)]. Most of our arguments are illustrated using a noisy version of the Arnol'd tongues [Ghil *et al.* (2008a)], and the paradigmatic Lorenz system, perturbed by multiplicative noise [Chekroun *et al.* (2011a)]. The former is a well-known metaphor for the Devil's staircase phenomena present in ENSO models of considerable realism, as well as in observations of interannual climate variability in the Tropical Pacific and beyond.

2.1 Noise effects and model statistics: A change of paradigm [**Chekroun** *et al.* **(2011a)**]

The first issue that arises in the field of stochastically perturbed dynamical systems concerns the effect of noise on the phase portrait of a deterministic system. To address this issue, we need to clarify first the appropriate framework to consider. When noise is added to a (chaotic) deterministic system, several ways exist to assess its effects on the "dynamics". At this stage we need the appropriate tools to dissect the dynamics.

(a) **The classical approach**. To analyze the effect of the noise on the *measure* supported by the attractor — roughly speaking on the probability of the flow visiting more or less often a certain region of the attractor — one relies on the *stochastic analysis approach.* Mathematically, it consists of solving the *Fokker-Planck equation*, whose solution yields the probability density function (PDF) on the system's phase space. An important difference with respect to the deterministic picture is that this PDF is not supported by an "attractor" of the stochastic system, but merely by a neighborhood of the attractor of the deterministic, unperturbed system.

(b) **The random dynamical system (RDS) approach**. This approach adopts a *pathwise analysis*, rather than the preceding one, based on an *ensemble of realizations.* At first glance, this approach may appear more laborious than the previous one. In fact, it provides a much greater degree of detail, and the detailed information it provides also yields the PDF.

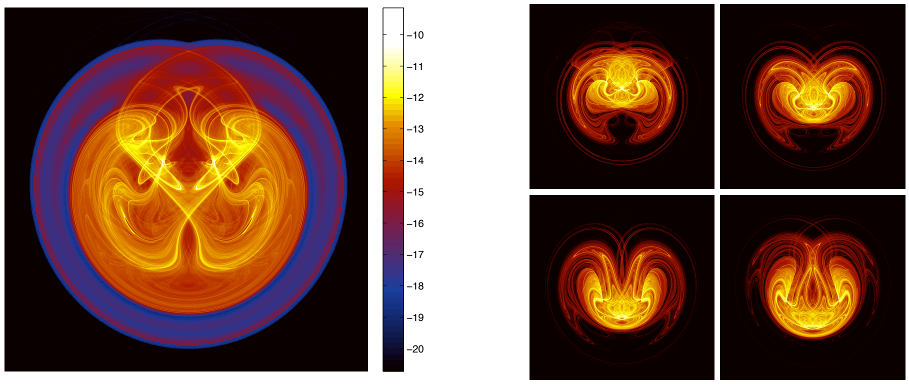
To understand this relatively novel approach, we first define the concept of a *pullback attractor*, which replaces the by now familiar concept of attractor in the case of nonautonomous systems, with time-dependent forcing. This concept allows one to answer the following question:

Q: For a fixed realization ω, and at a fixed time t — the time at which the system is observed — what are the most probable regions of the phase space in which the "flow" governed by the dynamical system ends up, from a set of initial data taken in a remote past s, *i.e.* $s << t$?

We see right away that the question involves a two-time description of the motion, in contrast to the one-parameter (semi-)group description that suffices to describe the long-time asymptotics of the flow for an autonomous systems, with no forcing at all or with forcing that does not depend on time.This two-parameter description of the motion gives rise to a family $\mathcal{A}(t)$ of objects called a *pullback attractor*; this family gives the asymptotic regime at time t, where the system is initialized in a remote past s and exhibits good dissipative properties. When the time-dependent forcing is random, the pullback attractor becomes a *random attractor.* This concept is subtler than its "time-independent cousin"; see [Ghil *et al.* (2008a)]. Roughly speaking, for each realization ω and each fixed time t, the "pullback asymptotic regime" — starting from a remote past s and stopping the dynamics at time t — is determined by the random attractor $\mathcal{A}(\omega; t)$. The random attractor $\mathcal{A}(\omega)$ supports a measure μ_ω; this measure, together with $\mathcal{A}(\omega)$, provides the full answer to question **Q**.

Let us now clarify the differences between the two approaches on a paradigmatic dynamic system, such as the stochastically perturbed Lorenz model. In approach (a), we observe, for small enough noise intensity, that the resulting PDF looks very much like that of the unperturbed system. This phenomenon is well known by physicists and is called *stochastic stability* by mathematicians.

On the other hand, in approach (b) — valid even in the case of large noise intensity — the statistics given by μ_ω are in general very different from one realization to another, as illustrated in Figure 1(b) below. More importantly, two systems may have very similar PDFs but very different random attractors [Chekroun *et al.* (2011a)]. The implication is that, for the same realization ω of a noise process perturbing two different climate models, the asymptotic regimes at time t of either system — *i.e.* the random attractor $\mathcal{A}(\omega)$ and its measure μ_ω — can be very different, whereas their PDFs



(a) (b)

Figure 1: The stochastic Lorenz model's random attractor and the sample measure it supports. **(a)** A snapshot of the Lorenz 63 model's random attractor $\mathcal{A}(t,\omega)$ and of the corresponding sample measure $\mu_{t,\omega}$, for a given t and a fixed realization ω. Each of the 3 model equations of the Lorenz system are perturbed by multiplicative vector-valued white noise, $\sigma[x,y,z]^T \mathrm{d}W_t$, where W_t is a real-valued Wiener process and $\sigma > 0$ is the noise intensity. The figure corresponds to projection onto the (y,z) plane. The Lorenz model's deterministic parameter values are the classical ones, *i.e.* $r = 28$, $Pr = 10$, and $b = 8/3$, while $\sigma = 0.3$. The color bar is on a log-scale and quantifies the probability to end up in a particular region of phase space. **(b)** Four snapshots of the sample measure supported by the random attractor in panel **(a)**. These snapshots illustrate the stochastically perturbed system's recurrent motion, which is driven by the deterministic system's unstable limit cycles and is thus related to the latter's well-known lobe dynamics. Random SRB measures are therefore able to translate the nonlinear deterministic recurent motion's into probabilistic terms [Chekroun *et al.* (2011a)].

look almost identical. Roughly speaking, one might say that the stochastic analysis approach provides only the asymptotic expectation, whereas the RDS approach does much more: the former provides a "hammer" and the latter provides a "scalpel." This scalpel, however, also has a blunt end, to provide the overall statistics when they are useful.

In [Chekroun *et al.* (2011a)], a stochastic version of the Timermann and Jin model (STJ herefater)— roughly speaking, a Galerkin version of the Cane-Zebiak model — has been examined by using these new concepts and tools. The RDS analysis of this model has revealed fundamental differences with respect to the one of the stochastic Lorenz model (SLM). The low frequency variability (LFV) exhibited by the STJ model is associated here with a random attractor much more singular than in the case of the SLM, with only one peak of the random measure μ_ω which evolves according to the LFV of the model. This low frequency motion of such a peak — giving for a fixed realization ω the most probable state w.r.t. a uniform distribution of initial data — offers completely new perspectives for prediction, from a pathwise point of view rather than the ensemble approach classically adopted by the climate community. A pathwise approach to prediction has been already introduced in the case of ENSO [Chekroun *et al.* (2011b)], by exloiting the well-known LFV of ENSO, and by relying on the Empirical Model Reduction (EMR) method (Kravtsov et al. 2005; Kondrashov et al. 2005) which builds nonlinear stochastic inverse ENSO models from the data, where the path on which lives the EMR model is estimated.

Since a climatic system, on any spatial or temporal scale, is never closed, taking into account time-dependent forcing is always necessary. We have introduced the appropriate concept of pullback attractor to replace the earlier, and by now well understood, concept of "strange attractor," which is useful only for time-independent forcing. In addition, we have shown how to apply the extension of pullback attractors to random forcing, namely random attractors (RAs), to a paradigmatic example of climatic toy model, the stochastically driven Lorenz model. Such RAs render much more precise and useful information than the vague notions associated with the concept of "fuzzy attractors," often encountered in the applied literature concerning stochastic systems.

2.2 Noise effects on structural stability [Ghil *et al.* (2008a)]

In the purely deterministic context, most dynamical systems are structurally unstable and thus we expect highly nonlinear and complex climate models to be so. If true, this would bode ill for future climate-change simulations and the entire IPCC enterprise: it might imply that the current range of uncertainties is intrinsically irreducible (McWilliams, *PNAS*, 2007). A key question that motivated therefore the theoretical part of our project initially was: Can noise "smooth" a system's phase-parameter portrait? In particular, can it convert the system from structurally unstable to stable and, if so, in which sense?

The RDS approach does provide the appropriate framework to answer this question; see [Ghil *et al.* (2008a)]). Roughly speaking, this approach shows that we can define an appropriate extension of the concept of flows valid in the context of stochastic dynamics. This extension is called a cocycle and is based on a parametrization of the noise via an ergodic bijective transformation of the probability space.

This concept of a *cocycle* allows us to classify RDS systems and to study the PDFs supported by the RA, as well as topological robustness issues.

Ghil et al. [Ghil *et al.* (2008a)] have shown, at least for certain discrete maps, that noise can stabilize the system, in the topological sense. In fact, we have rigorously proved, in the case of *V. I. Arnol'd's circle maps*, that when noise is added to the system, the perturbed system exhibits a finite number of topological classes, independent of the noise realization, whereas the unperturbed, purely deterministic system exhibits an infinite number of topological classes.

What is the consequence of such rigorous results for the interpretation of climate model simulations and observations? The presence of certain manifestations of a Devil's staircase has been documented across the full hierarchy of ENSO models, as well as in certain observational data sets. Interestingly, both general circulation models (GCMs) and observations only exhibit a few broad steps of the staircase, such as $4:1=4$ yr, $4:2=2$ yr, and $4:3\cong 16$ months.

Our numerical and mathematical results [Ghil *et al.* (2008a)] are therefore consistent with these spectral results obtained for detailed numerical models of major climatic phenomena such as ENSO. Indeed, whether explicitly modeling subgrid scales stochastically or not, small-scale processes can best be described as random. Thus it is reasonable to think of some form of "noise" being present in both GCMs and observations. The disappearance of the finer peaks — which correspond to high-denominator rationals among the Arnol'd "tongues" — in our RDS results, as the noise level increases is thus entirely consistent with the above-mentioned GCM and observational results.

Note that, for stochastic flows like those of the noise-driven Lorenz model, we do not have yet the type of topologically rigorous results obtained for the Arnol'd maps. We have shown, however, numerically that the statistics captured by the invariant measures μ_ω are robust with respect to variations in the noise intensity. These numerical results are in agreement with other results obtained subsequently in [Chekroun *et al.* (2011a)] where it has been shown that such invariant measures may vary linearly with the noise intensity in the case of the stochastic Lorenz system. This picture is however far to be the rule as it will be further demonstrated in [Chekroun *et al.* (2011c)] for ENSO related models; see Section 3.1 below for a short account on these last results.

3 A delay-differential model of ENSO variability [Ghil *et al.* (2008b)]

Team members involved: M. D. Chekroun, M. Ghil and I. Zaliapin

We have considered a toy model for ENSO variability that is governed by a delay-differential equation (DDE) with a single, fixed delay and additive periodic forcing:

$$\frac{dh(t)}{dt} = -\tanh[h(t-\tau)] + b\cos(2\pi\omega t); \tag{1}$$

here $t \geq 0$, and the parameters a, κ, τ, and ω are real and positive. Work by Suarez and Schopf (*JAS*, 1988), Battisti and Hirst (*JAS*, 1989), and Tziperman et al. (*Science*, 1994) has shown that DDE

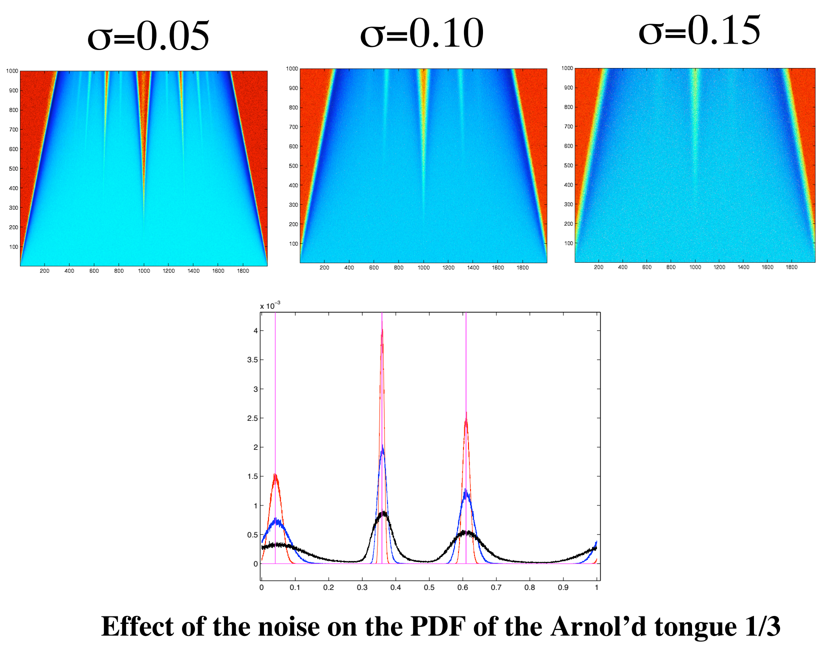


Figure 2: Arnol'd tongues in the presence of additive noise with different noise amplitudes σ. Upper panels: Arnol'd tongues for $\sigma = 0.05, 0.10$ and 0.15; lower panel: PDF for $\epsilon = 0.9$ (i.e. for highly nonlinear effects, see [Ghil *et al.* (2008a)]) and the three σ-values in the upper panels: $\sigma = 0.05$ (red curve), $\sigma = 0.10$ (blue curve), and $\sigma = 0.15$ (black curve). The observed disappearance of Arnol'd tongues as noise intensity increases, is consistent with the smoothing of the power spectrum observed in detailed stochastic models of ENSO as well as in observations. From [Ghil *et al.* (2008a)].

models can effectively capture complex phenomena found in much more detailed ENSO models, as well as in observational data sets. DDE models are very simple and, at the same time, exhibit rich and complex behavior. Stability and bifurcation analysis for such models can be carried out analytically only to some extent, but numerical methods are being actively developed (Baker, *JCAM*, 2000; Baker et al., 1995; Engelborghs et al., 2001), and we have not yet taken full advantage here of either approach.

Our toy model necessarily ignores a multitude of actual physical mechanisms and processes that might affect ENSO dynamics. For example, an alternative to the additive forcing, currently used in Eq. 1, could be a seasonal modulation of the air-sea coupling coefficient κ, which seems to be better justified by the physics of the problem. It would be equally interesting to see how the inclusion of a positive feedback, associated with Kelvin wave propagation, might affect model dynamics. These and other modifications of the current basic Eq. 1 deserve to be carefully explored and we plan to do so in the future.

To initiate the stability and bifurcation analysis of ENSO related DDE models, we started here with a descriptive numerical exploration of Eq. 1 over a wide range of physically relevant parameter values. We studied parameter dependence of various trajectory statistics, and report the existence of a large domain in parameter space where the statistics maps are strikingly discontinuous. The local continuous dependence theorem (Proposition 1 in [Ghil *et al.* (2008b)]) suggests, at least, that the reported discontinuities in global solutions point to the existence of unstable solutions of Eq. 1; the complex discontinuity patterns (see Fig. 3) lead us to suspect the presence of a rich family of unstable solutions that underlie a complicated attractor. It would be interesting to study in greater detail the unstable solutions and understand their role in shaping the system dynamics.

Summarizing the model results in terms of their relevance for ENSO dynamics, we emphasize the following observations. A simple DDE model Eq. 1 with a single delay reproduces the Devils staircase scenario documented in other ENSO models, including ICMs and GCMs, as well as in observations (Jin et al., 1994, 1996; Tziperman et al., 1994, 1995; Ghil and Robertson, 2000). The model illustrates, in simplest possible terms, the role of the distinct parameters: strength of seasonal forcing b vs. ocean

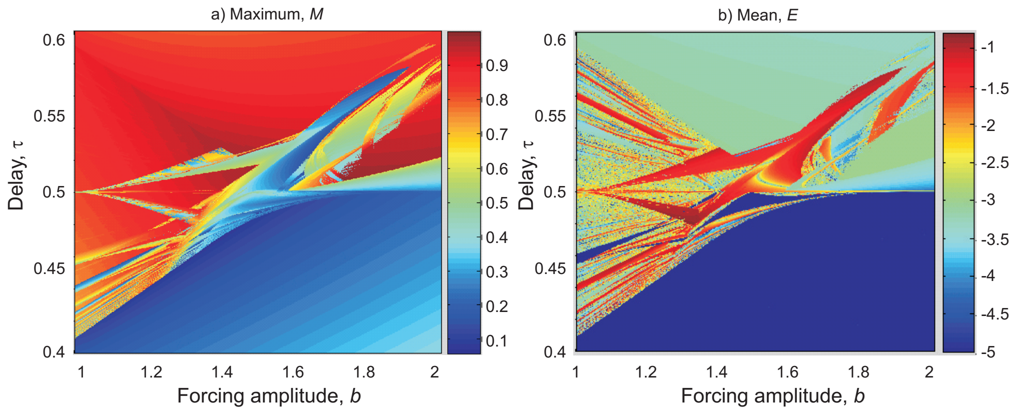


Figure 3: A sharp neutral curve separates the unstable, parameter-sensitive domain (warm colors) from the stable, parameter-insensitive one (cold colors). This curve becomes fractal as the ocean-atmosphere coupling κ increases. The system exhibits strong parametric instabilities. From [Ghil *et al.* (2008b)].

atmosphere coupling κ and transit time τ of oceanic waves across the Tropical Pacific.

We find in [Ghil *et al.* (2008b)] spontaneous transitions in mean thermocline depth, and hence in sea surface temperature (SST), as well as in extreme annual values; these transitions occur for purely periodic, seasonal forcing. The model generates intraseasonal oscillations of various periods and amplitudes, as well as interdecadal variability. The latter result suggests that interdecadal variability in the extratropical, thermohaline circulation (Dijkstra, 2005; Dijkstra and Ghil, 2005) might interfere constructively with ENSOs intrinsic variability on this time scale.

The former result might likewise suggest that Madden- Julian oscillations (Madden and Julian, 1971, 1972, 1994) and westerly wind bursts (Gebbie et al., 2007; Harrison and Giese, 1988; Verbickas, 1998; Delcroix et al., 1993) in the Tropical Pacific are affected by ENSOs interannual modes at least as much as they affect them in turn. Boulanger et al. (2004) and Lengaigne et al. (2004), among others, provide a comprehensive discussion of how weather noise could be responsible for the complex dynamics of ENSO, and, in particular, whether wind bursts could trigger El Niño events. Arguments about a "stochastic paradigm" for ENSO, with linear or only mildly nonlinear dynamics being affected decisively by weather noise, vs. a "deterministically chaotic paradigm," with decisively nonlinear dynamics, have also been discussed by Ghil and Robertson (2000).

A sharp neutral curve in the $(b - \tau)$ plane separates smooth parameter dependence in the solutions map of "metrics" (Taylor, 2001; Fuglestvedt et al., 2003) from "rough" behavior. We expect such separation between regions of smooth and rough dependence of solution metrics on parameters in much more detailed and realistic models, where it is harder to describe its causes as completely.

Finally, it appears that, even in as simple a model as our DDE, the mean, extrema and periodicity of solutions can change (a) spontaneously, without any change in the external forcing; and (b) one of these characteristics can change considerably, while others change but very little. Furthermore, certain parts of parameter space involve only small and smooth changes, while others involve large and sudden ones. It is quite conceivable that such behavior might arise in intermediate climate models (Jin et al., 1996; Neelin et al., 1994, 1998) and GCMs (Murphy et al., 2004; Stainforth et al., 2005).

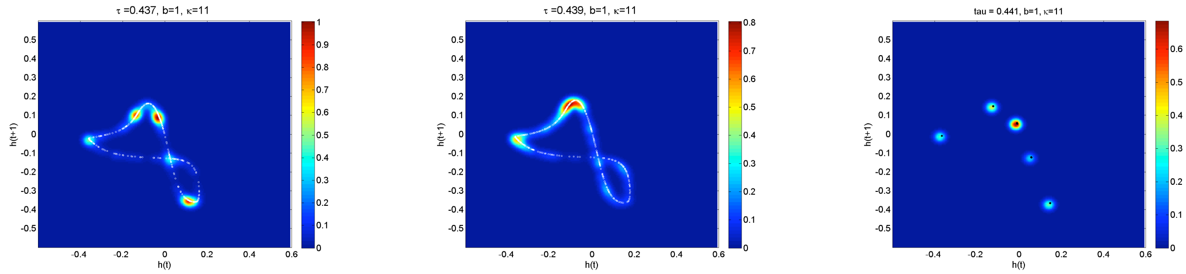
3.1 High sensitivity in a DDE-ENSO model via its pullback attractor [Zaliapin and Ghil (2010b), Chekroun *et al.* (2011c)]

ENSO is the dominant LFV mode of global climate on seasonal-to-interannual time scales. The physical growth mechanism of ENSO is quite well understood: it is due to the positive atmospheric feedbacks on equatorial SST anomalies via the surface wind stress. Still, ENSO's unstable, quasi-periodic behavior prevents its robust prediction, even at subannual lead times. Conceptual numerical modeling plays a prominent role in understanding ENSO variability and developing prediction methods for it.

To understand, simulate and predict such complex LFV phenomena, our project will explore a full hierarchy of models Initiated in the 1980s, the study of conceptual ENSO models has significantly contributed to shedding new light on many of its aspects, including quasi-periodic behavior, onset of instabilities, phase locking, power spectrum, and interdecadal variability. This part of the project focuses on the theoretical and numerical exploration of a conceptual model of ENSO dynamics; it will help us achieve a rather comprehensive understanding of ENSO's underlying mechanisms and their interplay, as well as advance our fundamental understanding of the mechanisms of climate sensitivity across the full modeling hierarchy.

During the currently funded DOE project, we considered a nonlinear DDE model of ENSO with additive, periodic forcing [Ghil *et al.* (2008b), Zaliapin and Ghil (2010b)]. The model is a simplified, one-delay version of the two-delay model of Tziperman *et al.* (Science, 1994), and it includes two mechanisms essential for ENSO variability: a delayed, negative feedback and the seasonal forcing; see the caption of Fig. 4. Before this project, DDE model studies of ENSO have been limited to linear stability analysis of fairly unrealistic steady-state solutions; case studies of particular trajectories; or one-dimensional (1-D) scenarios of transition to chaos, where one varies a single parameter while the others are kept fixed.

In this work, we explored the DDE model's behavior within broad parameter ranges. This detailed numerical exploration [Ghil *et al.* (2008b), Zaliapin and Ghil (2010b)] has revealed the existence of (i) several scenarios relevant to ENSO dynamics, including the intermittent, 2–7-year occurrence of El Niño and La Niña events, interdecadal variability, and even subseasonal variability reminiscent of Madden-Julian oscillations or westerly wind bursts; (ii) phase locking of solutions to the seasonal cycle, with temperature maxima and minima tending to occur at the same season, as observed; and (iii) parametric instabilities in the location of the extrema. Such instabilities were found for many other important, long-term statistics of the model solutions — like the overall mean and the period — and regions of high sensitivity in parameter space have been localized with great precision [Ghil *et al.* (2008b), Zaliapin and Ghil (2010b)].



(a) (b) (c)

Figure 4: The pullback attractor (PBA, bright) for the ENSO-DDE model $h'(t) = -\tanh\left[\kappa\, h(t-\tau)\right] + b\,\cos(2\pi\, t)$, of [Ghil *et al.* (2008b), Zaliapin and Ghil (2010b)]; the parameter values are $b = 1$ and $\kappa = 11$, while (a) $\tau = 0.437$, (b) $\tau = 0.439$, and (c) $\tau = 0.441$, and the invariant sample measure supported by the PBA is represented in grey scale. The parameters τ, κ and b are all positive; they denote respectively wave delay, the strength of ocean-atmosphere coupling, and the amplitude of the seasonal forcing. The function $h(t)$ represents the thermocline depth deviations from the annual mean in the eastern Tropical Pacific. The model is quasi-periodic in panels (a) and (b), while it has period 5 in panel (c), where the PBA consists of 5 points. The panels illustrate a change of about 1 % in the parameters from (a) to (b) and (b) to (c), whereas the corresponding changes in the invariant measures are respectively about 60 % and 97 %, in the L_1 norm (*i.e.*, in mean absolute value of difference); from [Chekroun *et al.* (2011c)].

More importantly for the scope of the proposed work, the novel tools of pullback attractors (PBAs) and of time-dependent invariant measures, as discussed in Section 2, have demonstrated their utility to understand the model's sensitivity to its parameters. We have shown, in effect, with mathematical rigor that the sharp variations of the model's statistics reported in [Ghil *et al.* (2008b), Zaliapin and Ghil (2010b)] can be explained by their dependence on the sample measures μ_t that are supported by the PBA $\mathcal{A}(t)$; see [Chekroun *et al.* (2011c)].

For instance, let $S(t,s)$ denote the two-time flow associated with our ENSO model (1) in some appropriate phase space X, see e.g. [Chekroun *et al.* (2011c)]. Then the time average $(t-s)^{-1}\int_s^t \phi(S(u,s)h_0)\mathrm{d}u$, where ϕ is some metric of the system, converges as $s \to -\infty$ to the ensemble average with respect to the sample measure μ_t at the frozen time t. Sensitive dependence in model statistics is thus equivalent to sensitive dependence in the sample measure μ_t. Figure 4 shows that a change of about 1% in parameters can lead to significant changes in these sample measures, thus leading to a high sensitivity of the model solutions' mean and other metrics [Ghil *et al.* (2008b), Zaliapin and Ghil (2010b)].

Having applied to our model the PBA approach, cf. [Ghil *et al.* (2008a)] and [Chekroun *et al.* (2011a)], we have thus shown that its dynamics — whether periodic or quasi-periodic — occurs on a 2-D torus, which is driven by the time-periodic forcing; see Fig. 4. This behavior reflects the competition between ENSO's two oscillatory mechanisms: the seasonal forcing and the self-sustained oscillation due to the delayed feedbacks. Such an interpretation is much harder to obtain from the complex, parameter-sensitive dynamics of the model using more traditional approaches.

We propose to analyze to which extent this type of sensitivity is generic, by considering more realistic ENSO models that will include additional delayed feedbacks, both positive and negative. We will further explore the quasi-biennial and quasi-quadrennial modes of variability associated with ENSO [Chekroun *et al.* (2011b)]. This funded DOE project has established the existence of "Devil's bleachers" in the DDE-ENSO model's periods as a function of parameters. In particular, we found multidimensional "tongues" of constant, low-frequency periods of 2, 3, 5 and 7 years. Within the proposed PBA framework, we have further studied the dependence of the dominant LFV modes on parameters and explore the dynamics when these parameters change slowly in time, as expected in the context of global warming. These tools will be used to study ENSO's LFV modes in the intermediate model of F.-F. Jin and J. D. Neelin, and will be confronted with methods recently developed by the team to assess parameter sensitivity in high-end models [Neelin *et al.* (2010)].

In preparation for the study of ENSO's impact on river networks, [Zaliapin *et al.* (2010)] proposed a novel, robust description of envirodynamics on such networks. This dynamical description on transport encompasses fluxes of water, sediment and biota.

4 Parameter dependence and sensitivity of climate models

Team members involved: A. Bracco, M. Ghil, J. C. McWilliams and J. D. Neelin

4.1 AGCM studies [Neelin *et al.* (2010)]

A number of basic issues in the dependence of climate model simulations on model parameters are poorly understood [Ghil *et al.* (2008a), Hillerbrand and Ghil (2008)]. To investigate the source of uncertainties in climate projections, one has to address the question of whether a reduction in parametric uncertainties in GCMs would lead to better convergence in their predictions. In current GCM modeling practice, every time a model change is made—typically in a parameterization, let's say for definiteness just changing a given parameter value— other aspects of the model have to be retested and retuned. This exercise is carried out so far primarily by trial-and-error, doing a run, checking maps of favorite fields, deciding whether the new fields are acceptable and, if not, changing some other aspect and repeating. Frustrating aspects of this process involve unexpectedly large changes in previously satisfactory fields, as well as persistent errors in certain fields, even when making changes that should correct these errors.

To assess the sensitivity systematically in multiple parameters and multiple objective functions or metrics as well as to guide optimization of the CESM model with respect to regionally dependent physical parameterization, a metamodeling approach has been introduced in Neelin et al (2010). This approach adapts a set of methods from the computer science and engineering literature, specifically for the type of climate modeling problem addressed here, and has been successfully applied to another atmospheric general circulation model (GCM) for related goals. In Neelin et al (2010) analytical regionally dependent models have been derived from GCM spatio-temporal fields — *the metamodels* — in order to approximate computationally- intensive objective functions at a much reduced computational cost compared to brute-force approaches, while keeping a satisfactory level of accuracy. This in turn permits economic application of tools from multi-objective optimization and insight into regional sensitivity.

We tested this procedure for several parameters of the ICTP-AGCM that are known to impact the model solution, including the subgrid-scale wind gustiness $v*$, which controls the minimum wind speed in the bulk formula for surface fluxes; the relative humidity RH from the deep convective parameterization, which controls the moisture towards which the convection adjust the column; the cloud albedo; and a viscosity parameter. For each of these parameters, we assign an admissible range based on the properties of the parameterization and the model numerics.

Figure 5 shows results from ensembles of ten 25-year simulations, forced by observed SSTs for each parameter end-point, for all combinations of two simultaneously varying end-points (*i.e.* min–min, min–max, max–min, and max-max for any two parameters), and for a sequence of values through the interior of the parameter range. The results suggest that for many climatic variables of interest, the curvature of common objective functions, such as global RMS error, varies fairly smoothly throughout the parameter range explored. Low-order polynomial fits to the model output as a function of parameter — quadratic in model field and 4th-order in cost function — based on end-point values and a few interior ones thus provide a very successful strategy for many quantities.

The smoothness holds even when the nonlinearity in the model response is strong enough to produce negative curvature of the objective function for the RH parameter (Figure 5a). Optima frequently occur at the end of the feasible parameter range (Figs. 5a,b), a result that (i) requires us to treat this as a constrained optimization problem; and (b) suggests a way of identifying aspects of the parameterization problem that need careful scrutiny of the physics by the climate modeler.

Figures 5c,d show the spatial fields associated with the (c) linear and (d) quadratic contributions to the sensitivity of precipitation to the gustiness parameter $v*$. These fields are shown as the contributions to the difference between the maximum gustiness and the standard value. Since these two contributions have quite different properties, in terms of model sensitivity and optimization, visualizations such as these spatial maps aid insight into regional aspects. Approximate analytic solutions for the optimization problem provide additional visualization possibilities.

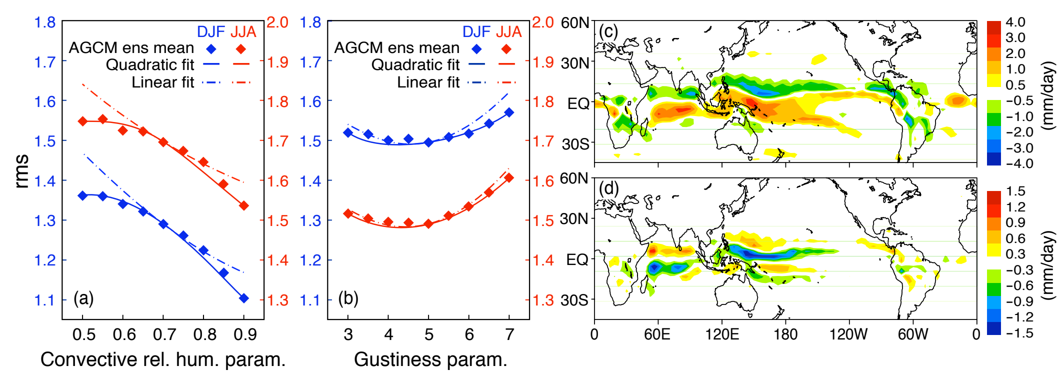


Figure 5: (a) Root-mean-square (RMS) error of the ensemble mean ICTP-AGCM precipitation (diamonds) in boreal winter (DJF, blue) and boreal summer (JJA, red) relative to NCEP reanalysis, as a function of the relative humidity parameter RH. Also shown are curves of linear (dashed) and quadratic (solid) fits to the model output's spatial fields as a function of the parameter. (b) As in (a) but as a function of the gustiness parameter $v*$. (c,d) The linear (c) and quadratic (d) contributions to the change in JJA precipitation at the maximum feasible value of $v*$ relative to its standard values. These maps allow one to visualize important aspects of the sensitivity and optimization problem.

The usefulness of low-order fitting procedures opens the possibility of developing standard optimization procedures that follow existing strategies for high-dimensional design problems with computationally expensive black-box functions.One can thus formulate a constrained optimization problem that allows fast, repeated optimizations using commercially available optimization packages (e.g. KNITRO). This problem also permits a multi-objective optimization approach, in which one can consider separate objective functions for each important climate variable, as opposed to considering a single cost function with pre-determined, arbitrary weights — on which different users might disagree — for several climate variables.

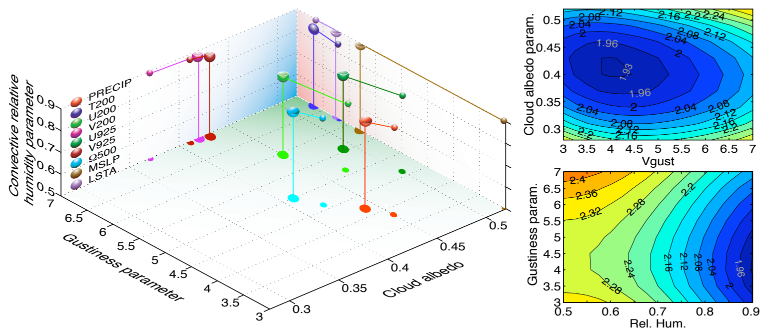


Figure 6: (a) Multi-objective optima calculated using an order-N (small sphere) or order-N^2 (large sphere) algorithm for calculating the RMS-error-based objective functions that correspond to several model variables: PRECIP is precipitation, T is upper-air temperature, U, V are wind components, Ω is vertical velocity, MSLP is mean sea level pressure and LSTA is land surface temperature; levels are in hPa. (b,c) The RMS error "basin" surrounding the optimum in precipitation, shown in the $RH - v*$ plane and the gustiness-cloud albedo plane.

Finally, this fitting procedure allows the user to quantify the maximum improvement that can be achieved in a given variable, and the extent to which an associated degradation in other variables might occur. Figure 6a exemplifies the contradiction among individual objective functions, chosen here as RMS errors for several key variables and shown as the location of the optima in parameter space for different climate variables: the spread of the optima in parameter space is substantial. A multi-objective procedure that yields a strict partial ordering for these optima, *i.e.* yields information about the trade-offs in optimizing for different variables, clearly provides more information for climate modelers than blind optimization for a weighted sum.

Also shown in Fig. 6a is the extent to which a fit that requires $\mathcal{O}(N)$ model field evaluations, where N is the number of parameters, can provide a good approximation versus one requiring $\mathcal{O}(N^2)$ evaluations. For most variables an $\mathcal{O}(N)$ algorithm gives a reasonable first approximation. A brute-force algorithm that simply samples the parameter space with a number of points M in each parameter direction would require $\mathcal{O}(M^N)$ evaluations, so it is very promising that a pragmatic, easy to implement approach has the potential to bring the computational problem to below $\mathcal{O}(N^2)$.

4.2 Observational and intermediate-model studies [**Bordi** *et al.* **(2009)**], [**Deremble** *et al.* **(2009, 2011a,b), Feliks** *et al.* **(2010, 2011), [Groth and Ghil (2011)]** [**Moron** *et al.* **(2011), Taricco** *et al.* **(2009)**]

Complementing the ICTP-AGCM studies above, we undertook several methodolgical, observational and intermediate-model studies, in order to bridge the gap (a) between the simple models of Sections 2 and 3, on the one hand; and (b) between the theoretical and modeling work and the observed climate, on the other. These studies were mainly conducted by the P.I., M. Ghil, in collaboration with researchers not supported by this grant.

Much of the oscillatory response of the climate system to natural and anthropogenic forcing involves the synchronization of climate signals across large distances. The spatial aspects of these responses are familiar by now as "teleconnections"; their combined, spatio-temporal aspects, however, are less well known, but essential to our understanding of the interaction between natural climate variability and anthropogenic forcing.

[Deremble *et al.* (2009, 2011a,b)] studied various aspects of natural variability of the atmosphere's mid-latitude LFV. [Deremble *et al.* (2009)] considered fundamental dynamical aspects of weather regimes and their predictability. Following up on the P.I.'s earlier DOE-supported work on the mid-latitude oceans' impact on atmospheric LFV, [Deremble *et al.* (2011a)] extended the study of the impact of an oceanic SST front on an intermediate, quasi-geostrophic (QG), three-level model (QG3). The dry version of the model is due to Marshall and Molteni (1993) but [Deremble *et al.* (2011a)] included, for the first time, humidity effects, while [Deremble *et al.* (2011b)] investigated the bifurcation sequence of a barotropic QG atmosphere in a coupled ocean-atmosphere model. This work led to the Ph. D. thesis of B. Deremble, defended at the Université Pierre-et-Marie Curie (P-6) in Paris; he is currently post-doc'ing with Prof. W. K. Dewar at Florida State University and is planning to pursue a research career in the United States.

[Bordi *et al.* (2009)] studied the bimodality of zonal-flow regimes in an atmospheric GCM and in a simple QG model, emphasizing the role of stratospheric dynamics. This work complements the previously mentioned studies of SST front impacts. [Feliks *et al.* (2010)] investigated oscillatory climate modes in century-long, land-based records of Eastern Mediterranean climate and the synchronization of these modes with the North Atlantic Oscillation (NAO). [Feliks *et al.* (2011)] used SODA reanalysis data for the North Atlantic ocean to drive a QG model and showed that the Gulf Stream's 7-8-year variability can be a cause of the similar periodicity observed in the NAO, thus supporting an earlier conjecture of Kondrashov et al. (*GRL*, 2005) that such a synchronization might have occurred between the NAO and the Nile River floods over more than a millennium. [Taricco *et al.* (2009)] studied Mediterranean climate over two millennia in marine, rather than continental records.

Given the importance of these teleconnective synchronizations, [Groth and Ghil (2011)] improved substantially the methodology of describing and explaining phase synchronization in chaotic oscillators by applying the project team's Multi-channel Singular Spectrum Analysis (M-SSA: Ghil et al. 2002) to the signals under study. [Moron *et al.* (2011)] also used M-SSA to investigate the effects of the modulated annual cycle and of intraseasonal oscillation on daily-to-interannual rainfall variability across monsoonal India.

The studies of [Roques and Chekroun (2010)] and [Roques and Chekroun (2011)] are also related to fundamental, methodological questions in the modeling of phenomena governed by reaction-diffusion equations, like the energy balance models of theoretical climate dynamics (Ghil, 1976; Held and Suarez, 1974; North, 1975). In particular, [Roques and Chekroun (2010)] considers the interaction of spatially inhomogeneous and nonlinear effects on the bimodality of solutions to such models.

5 Empirical model reduction (EMR) and past noise forecasting (PNF) [Kravtsov *et al.* (2009), Chekroun *et al.* (2011b), Kondrashov *et al.* (2011)]

Team members involved: M. Ghil, M. D. Chekroun and D. Kondrashov

Our major statistical analysis tools included the empirical model reduction (EMR) method, cf. [Kravtsov et al. (2005); Kondrashov et al. (*J. Clim.*, 2005) and [Kravtsov *et al.* (2009)], for fitting a nonlinear stochastic dynamical model to various climatic data sets, as well as the multivariate signal-detection techniques such as Empirical Orthogonal Function (EOF) analysis (Preisendorfer 1988) and M-SSA (Ghil et al. 2002). Our work contributed in two inter-related directions: (1) EMR as a major tool for identification and analysis of nonlinear regimes of atmospheric circulation – anomalously persistent patterns that potentially result in the predictable modes: [Kondrashov *et al.* (2011)] – application to an idealized atmospheric model; and (2) EMR-related ENSO modeling and "Past Noise Forecasting" (PNF) prediction: [Chekroun *et al.* (2011b)].

Signatures of nonlinear dynamics were analyzed by [Kondrashov *et al.* (2011)] by studying the phase-space tendencies of the dry QG3 model with topography. Nonlinear, stochastic, low-order prototypes of the full QG3 model were constructed in the phase space of this models empirical orthogonal functions using the empirical model reduction (EMR) approach. The phase-space tendencies of the EMR models closely match the full QG3 models tendencies. The component of these tendencies that is not linearly

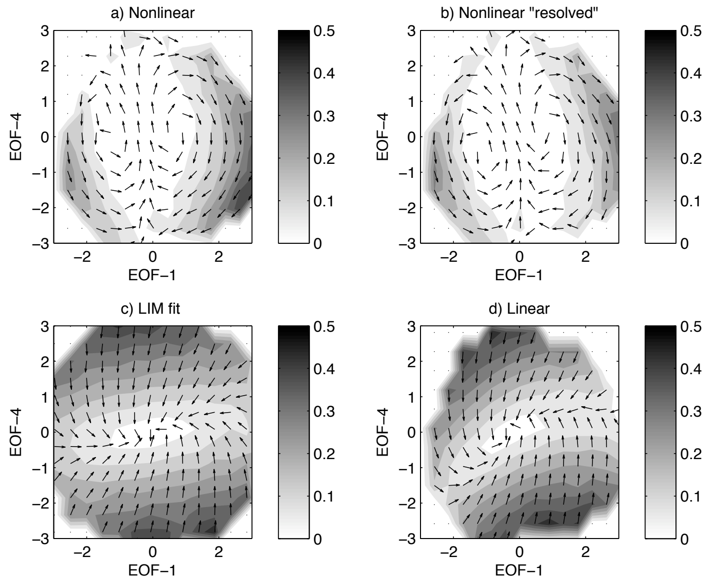


Figure 7: Detailed budget of mean phase-space tendencies for the plane spanned by EOFs 1 and 4: (a) tendencies due to the EMR model's full nonlinear operator; (b) nonlinear resolved contribution of the EMR model; c) LIM fit; and (d) linear tendencies of the nonlinear EMR.

parameterizable is shown to be dominated by the interactions between resolved modes rather than by multiplicative noise associated with unresolved modes (see Fig. 7). The method of defining the leading resolved modes and the interactions between them plays a key role in understanding the nature of the QG3 models dynamics, whether linear or nonlinear, deterministic or stochastic.

Forecasts of the EMR-ENSO model of Kondrashov et al. (2005) are included in the plume of ENSO forecasts compiled monthly by the International Research Institute for Climate and Society (IRI), and its real-time prediction skill is highly competitive against the 24 state-of-the-art dynamical and statistical models included in the IRI plume. Long-range ENSO forecasts, however, are typically unreliable for most models — whether statistical or fully coupled GCMs — beyond 7–8 months. Interannual and interdecadal predictions are in fact major challenges for climate dynamics. In the ENSO-forecast context, the PNF method exhibits significantly better skill at 6–16 months ahead, when compared with currently used dynamic and statistic prediction methods for ENSO indices and the global SST field [Chekroun *et al.* (2011b)]. This methodology is not limited to ENSO prediction and would be useful for a wide class of ocean-atmosphere processes that exhibit low-frequency variability modes (LFMs); see *Supporting Information* in [Chekroun *et al.* (2011b)].

In the ENSO case, a nonlinear EMR model is built using past observations and the path of the "weather" noise that drove this model over previous finite-time windows is estimated. The PNF method is then applied in two steps. First, noise samples — obtained as copies of this estimated path — are selected from past time intervals that resemble the LFM phase prior to a particular forecast. Then, these noise samples — having the same length, say T, as the "copy window" — are used to drive the system into the future over a lead time equal to T. The LFMs are characterized by the reconstructed components given by MSSA (Ghil et al. 2002). This conditioning of the noise on the initial LFM phase improves forecast skill. Linear response theory — familiar from non-equilibrium statistical physics and dynamical systems theory — helps explain the PNF method's success under suitable circumstances, which are numerically verifiable in the case of ENSO modeling ([Chekroun *et al.* (2011b)], [Chekroun *et al.* (2011a)]).

ENSO forecasting by PNF indicates significantly improved skill in prediction of Tropical Pacific SSTs beyond 6 months, when compared with the already competitive standard EMR scheme driven into the future by a large number of noise realizations; see Figs. 8(a), 8(b). This improvement is due,

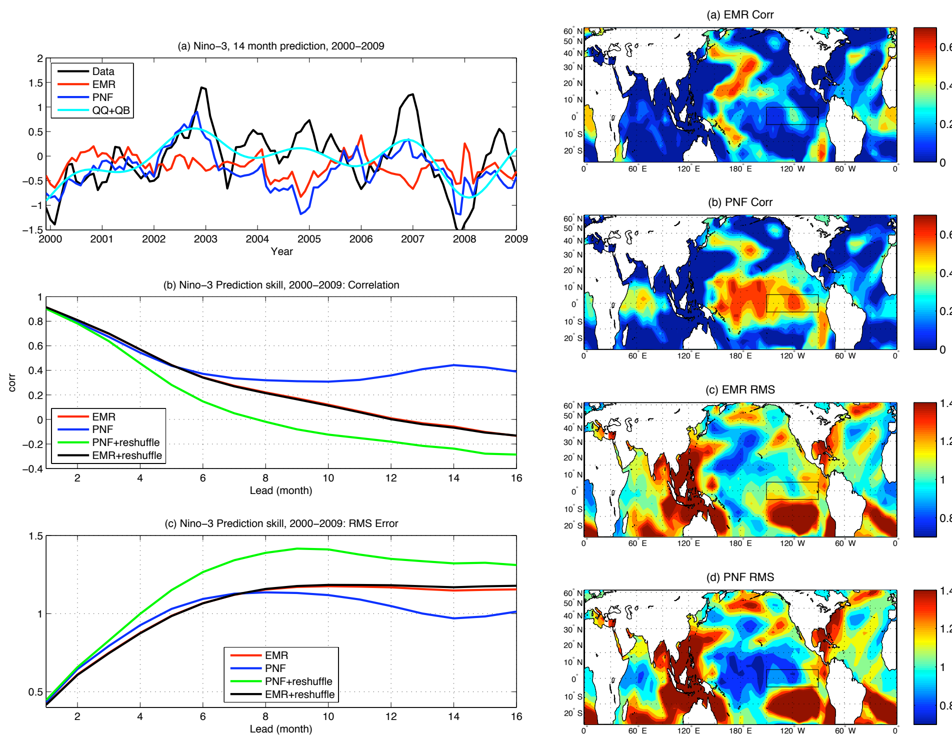


Figure 8: **(a)**: Average prediction skill of the EMR and PNF methods. The two lower panels show the respective Niño-3 forecasting skill (in anomaly correlation and root-mean-square) obtained by computing the mean of the EMR-ENSO when driven by a large set of noise realizations (red curves), and when driven by a smaller subset of noise realizations selected by the PNF method (blue curves). These skill scores are computed for 114 forecasts issued each month — from October 1998 to March 2008 (upper panel) — using only data prior to that month, i. e., no "look-ahead" whatsoever is involved; the scores are represented for leads of 1–16 months ahead. From a 6-month lead forecast on, the PNF method avoids the drop in skill exhibited by EMR. These results are further compared with a reshuffled version of the estimated noise by EMR and PNF (magenta and green curves, respectively).
(b): Local prediction skill, as given by maps of anomaly correlation (`Corr`) and normalized rms errors (`RMS`) for 14-month lead forecasts by PNF and EMR models, trained on SST data for the 1950–2000 interval and validated on the 2000–2009 interval. PNF skill is uniformly better — i. e., higher `Corr` and lower `RMS` — in the equatorial Tropical Pacific and Indian Ocean, where ENSO is the most active.

at least in part, to the method's capturing the energetic phases of the ENSO LFMs.

List of publications acknowledging this grant

[Ghil *et al.* (2008a)] Ghil, M., M.D. Chekroun, and E. Simonnet, 2008: Climate Dynamics and fluid mechanics: Natural variability and related uncertainties, *Physica D*, **237**, 2111–2126.

[Ghil *et al.* (2008b)] M. Ghil, I. Zaliapin, and S. Thompson (2008b), A delay differential model of ENSO variability: Parametric instability and the distribution of extremes. *Nonlin. Proc. Geophys.*, **15**, 417–433.

[Hillerbrand and Ghil (2008)] Hillerbrand, R. and M. Ghil, 2008: Anthropogenic climate change: Scientific uncertainties and moral dilemmas, *Physica D*, **237** (14-17), 2132–2138.

[Bordi *et al.* (2009)] Bordi, I., K. Fraedrich, M. Ghil, and A. Sutera, 2009: Zonal-flow regime changes in a GCM and in a simple quasi-geostrophic model: The role of stratospheric dynamics, *J. Atmos. Sci.*, **66**, 1366–1383.

[Deremble *et al.* (2009)] Deremble, B., F. D'Andrea, and M. Ghil, 2009: Fixed points, stable manifolds, weather regimes and their predictability, *Chaos*, **19** (4), 043109.

[Kravtsov *et al.* (2009)] Kravtsov, S., D. Kondrashov, and M. Ghil, 2009: Empirical model reduction and the modeling hierarchy in climate dynamics, in *Stochastic Physics and Climate Modelling*, Eds. T. N. Palmer and P. Williams, Cambridge Univ. Press, pp. 35–72.

[Taricco *et al.* (2009)] Taricco, C., M. Ghil, S. Alessio, and G. Vivaldo, 2009: Two millennia of climate variability in the Central Mediterranean, *Clim. Past*, **5**, 171181, 2009.

[Feliks *et al.* (2010)] Feliks, Y., M. Ghil, and A. W. Robertson, 2010: Oscillatory climate modes in the Eastern Mediterranean and their synchronization with the North Atlantic Oscillation, *J. Clim.*, **23**, 40604079.

[Zaliapin and Ghil (2010a)] I. Zaliapin, and M. Ghil, 2010: Another look at climate sensitivity, *Nonlin. Processes Geophys.*, **17**, 113–122.

[Zaliapin and Ghil (2010b)] I. Zaliapin and M. Ghil, 2010: A delay differential model of ENSO variability, Part 2: Phase locking, multiple solutions, and dynamics of extrema, *Nonlin. Processes Geophys.*, **17**, 123–135.

[Zaliapin *et al.* (2010)] Zaliapin, I., E. Foufoula-Georgiou, and M. Ghil, 2010: Transport on river networks: A dynamical approach, *J. Geophys. Res.Earth Surface*, **115**, F00A15.

[Brachet *et al.* (2011)] Brachet, S., F. Codron, Y. Feliks, M. Ghil, H. Le Treut, and E. Simonnet, 2011: Atmospheric circulations induced by a mid-latitude SST front: A GCM study, *J. Clim.*, accepted.

[Chekroun *et al.* (2011a)] Chekroun, M. D., E. Simonnet, M. Ghil, 2011: Stochastic climate dynamics: Random attractors and time-dependent invariant measures, *Physica D.*, **240** (21), 1685–1700.

[Chekroun *et al.* (2011b)] Chekroun, M. D., D. Kondrashov and M. Ghil, 2011: Predicting stochastic systems by noise sampling, and application to the El Nio-Southern Oscillation, *Proc. Natl. Acad. Sci. USA*, **108** (29), 11766–11771.

[Chekroun *et al.* (2011c)] Chekroun, M.D., M. Ghil and I. Zaliapin, 2011: The role of invariant measures and pullback attractors in conceptual El Niño models with delays, in preparation.

[Deremble *et al.* (2011a)] Deremble, B., G. Lapeyre, and M. Ghil, 2011: Atmospheric dynamics triggered by an oceanic SST front in a moist quasi-geostrophic model, *J. Atmos. Sci., sub judice.*

[Deremble *et al.* (2011b)] Deremble, B., E. Simonnet, and M. Ghil, 2011: Multiple atmospheric equilibria and oscillatory modes in a mid-latitude ocean-atmosphere model, *J. Clim., sub judice.*

[Feliks *et al.* (2011)] Feliks, Y., M. Ghil, and A. W. Robertson, 2011: The atmospheric circulation over the North Atlantic as induced by the SST field, *J. Clim.*, **24**(2), 522–542.

[Groth and Ghil (2011)] Groth, A., and M. Ghil, 2011: Multivariate singular spectrum analysis and the road to phase synchronization, Phys. Rev. E, 84, 036206 (10 pp.).

[Kondrashov *et al.* (2011)] Kondrashov, D., S. Kravtsov and M. Ghil, 2011: Signatures of nonlinear dynamics in an idealized atmospheric model, *J. Atmos. Sci.*, **68**,1–3.

[Moron *et al.* (2011)] Moron, V., A. W. Robertson, and M. Ghil, 2011: Impact of the modulated annual cycle and intraseasonal oscillation on daily-to-interannual rainfall variability across monsoonal India, *Clim. Dyn., sub judice.*

[Roques and Chekroun (2010)] Roques, L., and M. D. Chekroun, 2010: Does reaction-diffusion support the duality of fragmentation effect?, *Ecological Complexity*, **7** (1), 100–106.

[Roques and Chekroun (2011)] Roques, L., and M. D. Chekroun, 2011: Probing chaos and biodiversity in a simple competition model, *Ecological Complexity*, **8** (1), 98–104.

[Neelin *et al.* (2010)] Neelin, J. D., A. Bracco, H. Luo, J. C. McWilliams, and J. E. Meyerson, 2010: Considerations for parameter optimization and sensitivity in climate models. *Proc. Natl. Acad. Sci. USA*, **107** (47).