Theoretical study of proton beam preparation for the $g-2$ experiment

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Abstract

Beam preparation for the $g-2$ experiment is studied. Because of limited 2.5-MHz rf voltage and the 66-ms limited time duration allowance for the whole process, the Booster batch must be first captured to a lower 2.5-MHz rf voltage followed by a rotation in the 2.5-MHz bucket in order to achieve the required narrow half bunch width of 50 ns. The appearance of bunch tails can be reduced by first rotation by a linear barrier voltage before the adiabatic capture.

1 Introduction

Collaborators at BNL would like to move their $g - 2$ experiment to Fermilab. After the shutdown of the Tevatron, $\bar{p}$ accumulation will no longer be necessary. The Recycler Ring can be used to accumulate protons directly from the Booster while the Main Injector (MI) is in acceleration with a probable cycle time ~ 1.4 s. During each of these MI acceleration cycles, the Recycler can accumulate 20 Booster batches (at 15 cycles per sec). 12 batches will be used for neutrino experiments, leaving behind 8 batches. The $g - 2$ experiment would like to use 6 of these left-behind batches. Each of these 6 batches will come at a 66 ms interval. The followings are what we need to do with each of these 6 Booster batches:

1. Transferred from Booster, each batch ($1.59 \mu\text{s}$) consists of 84 52-MHz bunches.
2. Need to debunch first
3. Then capture into 4 mega-bunches (2.5 MHz), each of total length $\lesssim 100$ ns
4. These 4 mega bunches will be extracted one at a time to the $g - 2$ ring.

All the above have to be performed within 66 ms before the 2nd batch arrives. The real problem is they wish to perform everything with present existing equipment, without as little additional purchases as possible.

In this note, we analyze some possibility of achieving the required proton beam. Experiment on the beam preparation will be carried out in the MI, because there is no 53-MHz and 25-MHz rf's in the Recycler at this moment. Thus this analysis will be performed using the properties of the MI, some of which are given in Table I.

2 Longitudinal Kinematics

The mega bunch will not be small inside the 25-MHz rf bucket. Therefore we need some more exact expressions relating the bunch length, energy spread, and

Table I: Some properties of the Main Injector at injection.

Energy E	8.9383 GeV
Period T_0	1.11339 μ s
Slip factor η	-8.9150×10^{-3}
25-MHz rf harmonic h	28

bunch area. From the equations of motion

$$\begin{aligned}\frac{d\tau}{dt} &= -\frac{\partial H}{\partial \Delta E} = \frac{\eta \Delta E}{\beta^2 E}, \\ \frac{d\Delta E}{dt} &= -\frac{\partial H}{\partial \tau} = -\frac{eV}{T_0} \sin h\omega_0\tau,\end{aligned}\tag{2.1}$$

the Hamiltonian is

$$H = -\frac{\eta \Delta E^2}{2\beta^2 E} + \frac{eV}{2\pi h} [1 - \cos h\omega_0\tau],\tag{2.2}$$

where τ is the time advance and ΔE is the energy offset referenced to some synchronous particle, V is the rf voltage, β is the relativistic parameter, and $T_0 = 2\pi/\omega_0$ is the revolution period. Consider a bunch matched to the rf with half width $\hat{\tau}$. The energy offset is given by

$$\Delta E = \sqrt{\frac{2eV\beta^2 E}{\pi h|\eta|}} \sqrt{\sin^2 \frac{h\omega_0\hat{\tau}}{2} - \sin^2 \frac{h\omega_0\tau}{2}}.\tag{2.3}$$

The area of the bunch is

$$A = 4 \int_0^{\hat{\tau}} \Delta E d\tau = \sqrt{\frac{128eV\beta^2 E}{\pi h^3 \omega_0^2 |\eta|}} I,\tag{2.4}$$

where $a = h\omega_0\hat{\tau}/2$ and

$$I = \int_0^a \sqrt{\sin^2 a - \sin^2 x} dx.\tag{2.5}$$

To evaluate the integral, let $\sin x = \sin a \sin \theta$. Then

$$\begin{aligned}I &= \int_0^{\pi/2} \frac{\sin^2 a \cos^2 \theta}{\sqrt{1 - \sin^2 a \sin^2 \theta}} d\theta = \int_0^{\pi/2} \left[\sqrt{1 - \sin^2 a \sin^2 \theta} - \frac{1 - \sin^2 a}{\sqrt{1 - \sin^2 a \sin^2 \theta}} \right] d\theta \\ &= E(m) - (1 - m)K(m),\end{aligned}\tag{2.6}$$

with $m = \sin^2 a$. The bunch area is therefore

$$A = \sqrt{\frac{128eV\beta^2 E}{\pi h^3 \omega_0^2 |\eta|}} \left[E(m) - (1-m)K(m) \right]. \quad (2.7)$$

When the bunch is small compared with the bucket, $m = \sin^2 a \ll 1$ and $E(m) - (1-m)K(m) \approx \pi m/4$. The bunch area reduces to

$$A \approx \sqrt{\frac{128eV\beta^2 E}{\pi h^3 \omega_0^2 |\eta|}} \frac{\pi}{4} \frac{h^2 \omega_0^2 \hat{\tau}^2}{4} = \sqrt{\frac{\pi eV\beta^2 E h}{2|\eta|}} \omega_0 \hat{\tau}^2. \quad (2.8)$$

When the bunch is small or when the rf is exactly linear, actually from the first equation of motion

$$\widehat{\Delta E} = \frac{\beta^2 E}{|\eta|} \omega_s \hat{\tau}. \quad (2.9)$$

The the bunch area is

$$A = \pi \widehat{\Delta E} \hat{\tau} = \frac{\pi \beta^2 E}{|\eta|} \sqrt{\frac{|\eta| h e V}{2\pi \beta^2 E}} \omega_0 \hat{\tau}^2 = \sqrt{\frac{\pi eV\beta^2 E h}{2|\eta|}} \omega_0 \hat{\tau}^2, \quad (2.10)$$

agreeing with the result of the elliptical-function formula.

3 Method

We now analyze the capture of a totally debunched beam into four MI 2.5-MHz rf bucket. We assume non-adiabatic debunching. At the present, the Booster bunch as energy offset $\Delta E = 5.2557$ MeV. For one-quarter of a Booster batch, the bunch area $A = 4.18$ eVs. From Eq. (2.7), substituting half bunch length $\hat{\tau} = 50$ ns, the required 2.5-MHz rf voltage is $V = 215$ kV, which is the voltage to capture of the mega bunch assuming no dilution at all. Unfortunately, the maximum 2.5-MHz rf voltage is only 75 kV, and for reasons of safety, usually it can be fired up to 60 kV only. Inversely, with only 60 kV of rf to capture a mega bunch of half width $\hat{\tau} = 50$ ns, an initial bunch area must be less than 2.21 eVs. Such a small bunch area for a quarter of a batch is not possible from the Booster even if an adiabatic debunching is performed. Actually, for a bunch of 4.18 eVs

matched to the 2.5-MHz bucket, the half bunch length will be 69.9 ns at 60 kV, and 65.9 ns at 75 kV.

Because the 2.5-MHz rf voltage is far too low to match to the mega bunch of half width 50 ns, the only way to produce such a narrow bunch is to adiabatically capture the Booster batch first into four 2.5-MHz buckets with a low rf voltage. After that the rf voltage is raised suddenly to the maximum to rotate the captured mega bunches into narrow bunches.

In order to obtain a final bunch of half width $\hat{\tau} = 50$ ns at $V = 60$ kV, before the 90 degree rotation, the energy offset is, with the aid of Eq. (2.3), $\widehat{\Delta E}_c = 14.15$ MeV. Assume a perfect capture before the rotation with capture voltage V_c . The original rectangular bunch of $A = 4.18$ eVs is now matched to the rf voltage V_c with maximum energy offset $\widehat{\Delta E}_c = 14.15$ MeV and half width $\hat{\tau}_c$. To obtain the latter, we eliminate V from Eqs (2.3) and (2.7), resulting in

$$\frac{h\omega_0 A}{8\widehat{\Delta E}_c} = \frac{E(m) - (1-m)K(m)}{\sqrt{m}}, \quad (3.11)$$

with $m = \sin^2 h\omega_0 \hat{\tau}_c / 2$. The half bunch length before rotation is $\hat{\tau} = 98.14$ ns. Substituting back into Eq. (2.7) gives the necessary capture voltage of $V_c = 17.61$ kV. Simulation of such a process is performed, starting from a uniformly distributed rectangular bunch with 4000 macro-particles, captured with rf voltage from $V_{c1} = 0.1$ kV to $V_c = 17.61$ kV, according to the voltage ramp curve [1]

$$V(t) = \frac{V_0}{\left[1 - \frac{\omega_s t}{n_{ad}}\right]^2}, \quad (3.12)$$

which is derived from the criterion that the relative rate of change in energy offset is much slower than the spontaneous synchrotron frequency. Here, the adiabatic ratio n_{ad} should be set to much larger than unity for adiabaticity. With the first choice of $n_{ad} = 200$, the synchrotron period at V_{c1} is $T_{s1} = 2\pi/\omega_{s1} = 524.2$ ms. The capture time is 15.46 s or 1388337 turns. The captured bunch is shown in the top plot of Fig. 1 in black dots. The finally rotated bunch at $V_r = 60$ kV by 90° in ~ 5.5 ms or 493 turns is shown as red dots. We see that the captured half

Figure 1: Top: Simulation showing adiabatic capture with rf voltage from $V_{c1} = 0.1$ kV to $V_c = 17.61$ kV in 15.46 s (adiabatic ratio $n_{ad} = 200$). After that a rotation with rf voltage $V_r = 60$ kV in 5.5 ms. Bottom: Same simulation but with initial rf voltage $V_{c1} = 1$ kV and capture time 40.27 ms ($n_{ad} = 2$).

bunch length is just below 100 ns and half height just below 15 MeV and the final rotated bunch has a half width roughly 50 ns, in agreement of what we expect from the above computation. A very low starting voltage V_{c1} and a very large adiabatic ratio n_{ad} have been used in the simulation just for the purpose of having smooth bunch shapes with minimal tails so that comparison with computation can be made with ease.

4 Realistic Capture

In reality, a larger starting voltage ~ 1 kV and a smaller adiabatic ratio $n_{ad} \sim 2$ will be used so as to limit the capture time. We also show in the lower half of Fig. 1 a similar simulation with the starting rf voltage $V_{c1} = 1$ kV and adiabatic ratio $n_{ad} = 2$. The peak capture voltage V_c and the rotation voltage V_r are unchanged. The capture time is 40.27 ms. After that rotation is another 5.5 ms. A clear difference is the appearance of long tails after the capture and rotation.

The first mega bunch is now be extracted. The rest of the bunches will be extracted after successive half-synchrotron periods, as illustrated in Fig. 2. The total time from the beginning of the capture to the extraction of the 4th mega bunch is 80 ms. Although this duration is still larger than the required 66 ms, it can easily be shortened by decreasing the capture time. The price to pay is obviously longer and thicker bunch tails.

Since the severity of tail appearance depends on the capture time, we try to perform simulation with various capture times and compare the results. Figure 3 the capture from $V_{c1} = 1$ kV to $V_r = 60$ kV followed by rotation with capture time 40, 100, 201, 302, and 403 ms, corresponding to adiabatic ratio $n_{ad} = 2, 5, 10, 15, 20$. We see that to get rid of most tails, we have to employ $n_{ad} \geq 10$ or a capture time larger than 200 ms. Of course, this capture time is too long to be acceptable.

Figure 2: Fast capture is performed with adiabatic factor $n_{\text{ad}} = 2$ from the 2.5 MHz rf voltage 1 kV to 17.61 kV, and then rotation at 60 kV for 90° . Second harmonic rf is turned off. Capture time is 40.27 ms (3616 turns). Then there is a wait of 5.0 ms (449 turns) for extraction (top-left) when the bunch becomes shortest. The second extraction (top-right) is at 16.3 ms (1463 turns) after capture. The third extraction (bottom-left) is at 27.8 ms (2496 turns) after capture. The fourth extraction (bottom-right) is at 39.7 ms (3565 turns) after capture. Total time for the whole procedure is 80.0 ms (7181 turns) excluding debunching.

Figure 3: Capture is performed with various adiabatic factors n_{ad} from the 2.5 MHz rf voltage 1 kV to 17.61 kV and then rotation at 60 kV for 90° . Second harmonic rf is turned off. From left to right and top to bottom: $n_{\text{ad}} = 2$ with capture time 40 ms, $n_{\text{ad}} = 5$ with capture time 100 ms, $n_{\text{ad}} = 10$ with capture time 201 ms, $n_{\text{ad}} = 15$ with capture time 302 ms, $n_{\text{ad}} = 20$ with capture time 403 ms.

5 Using Barriers

The capture time can be reduced by using linear barrier voltage. It is possible to use the linear barrier to rotate the debunched beam by 90° to become a bunch of narrow width. If we start from a debunched beam of $\Delta E = \pm 5.2557$ MeV and wish to obtain a bunch of half-width $\hat{\tau} = 50$ ns, the linear barrier voltage at the edge of the 2.5 MHz bucket is given through Eq. (2.9) by

$$V_b = \frac{|\eta|}{2\beta^2 E h} \left[\frac{\widehat{\Delta E T_0}}{\hat{\tau}} \right]^2 = 24.7 \text{ kV}, \quad (5.13)$$

where we have used the fact that V_b is equivalent to having the sinusoidal peak rf voltage πV_{rf} . Unfortunately, the present Finemat cavity in the MI has a maximum allowable barrier voltage of only 16 kV. We can, however, perform a combination of barrier rotation and 2.5-MHz rf capture instead. The debunched Booster beam is first rotated partially by the barrier voltage and then captured with the 2.5-MHz rf. A negative 20% of second harmonic is added to help the linearization of the sinusoidal rf. What we did is first turn on the linear barrier with $V_b = 16$ kV at the bucket edge, turn it off after for 7 ms and then turn on the sinusoidal rf adiabatically up to 10 kV (-25% second harmonic) to capture the beam with adiabatic ratio $n_{\text{ad}} = 2$. Finally, the bunch is rotated to its narrowest width by firing the rf at 60 kV. The 7-ms barrier rotation, the initial 10-kV capture voltage, and the adiabatic ratio of 2 are adjusted so that the final bunch is at the narrowest position having $\hat{\tau} \approx 50$ ns. The extraction times of the second, third, and fourth bunches are at an additional 13.5, 27.0, and 40.5 ms of rotation by the sinusoidal rf. The bunch shapes are depicted in Fig. 4. In this simulation, the total time for the process is only 57.4 ms, within the required 66 ms. We see that the tails of the first extracted bunch are minimal, but increase gradually for the second, third, and fourth bunches, due to synchrotron rotation. This tail effect can be further decrease if the barrier voltage can be made larger.

Unfortunately, such a barrier-assisted method could be implemented in the MI. This is because the Finemat barrier rf at the MI is an inductive barrier. It is

not possible to program the barrier voltage in a saw-tooth profile across Booster batch so that the rf voltage across each mega bunch becomes linear. Such a saw-tooth profile can only be programmed with the resistive barrier rf, for example the one in the Recycler ring. However, this resistive barrier rf in the Recycler can only maintain a maximum voltage of 2 kV. Since resistive barrier rf is expensive, we cannot foresee a 16-kV resistive barrier rf to be built for the $g-2$ experiment.

References

- [1] K.Y. Ng, *Continuous multiple injections at the Main Injector*, Phys. Rev. ST Accel. Beams 5, 061002 (2002); Fermilab Physics Note FN-715, 2002.

Figure 4: Debunched beam is first rotated by a linear barrier with voltage $V_b = \pm 16$ kV at the two edges of the 2.5 MHz bucket. The barrier voltage is then turned off and a 2.5 MHz rf voltage of $V_{rf} = 10$ kV (with -25% of second harmonic) to capture the beam with adiabatic ratio $n_{ad} = 2$. The first bunch can now be extracted. Second, third, and fourth bunches are extracted after another 13.5, 27.0, 40.5 ms of rotation by the sinusoidal rf. Total time for the whole procedure is 57.4 ms after debunching.