Liquid Argon
Maximum Convective Heat Flux
vs Liquid Depth

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D0 EN 237

DEPTH IN LIQUID ARGON VERSUS MAXIMUM CONVECTIVE HEAT FLUX

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(report on work done in February 1988)

Introduction.

In order to help answer questions about the magnitude of heat flux to the liquid argon in a liquid argon calorimeter which could cause boiling (bubbles), calculations estimating the heat flux which can be removed by free convection were made in February, 1988. These calculations are intended to be an estimate of the heat flux above which boiling would occur. No formal writeup was made of these calculations, although the graph dated 3 Feb 88 and revised (adding low-velocity forced convection lines) 19 Feb 88 was presented in several meetings and widely distributed. With this description of the calculations, copies of the original graph and calculations are being added to the D0 Engineering Note files.

Assumptions.

The liquid argon surface is in equilibrium with argon vapor at a pressure of 1.3 bar, so the surface is at 89.70 K. The liquid is entirely at this surface temperature throughout the bulk of the volume, except locally where it is warmed by a solid surface at a higher temperature than the bulk liquid. This surface temperature is taken to be the boiling temperature of argon at the pressure corresponding to 1.3 bar plus the liquid head; hence it is a function of depth below the surface. The free and forced convection correlations used are from Kreith, "Heat Transfer", for heated flat plates in a large (i.e., no other objects nearby enough to disturb the flow) uniform volume of fluid. Heat flux is a function of plate size, really length along the flow path (since a boundary layer increases in thickness starting from the leading edge of the plate), and orientation (i.e., vertical or horizontal).

Method.

A table (on page 1) was made using ΔT above the surface temperature of 89.70 K as the independent variable. From this the saturation (boiling) pressure corresponding to 89.70 K plus ΔT and depth below the surface were found. It can be seen that the

liquid density at a constant (surface) temperature is practically constant with depth, and this density was used to calculate depth for a given pressure. A density at the elevated temperature (saturation temperature) also has to be tabulated since it is the difference between this and the bulk density that drives the free convection. Other fluid properties (thermal conductivity, viscosity, and Prandtl number) are found for the pressure at the depth and an average of the saturation (surface) and bulk temperatures. Grashof number includes a factor L^3 (L is plate length), so Grashof divided by L^3 is calculated first.

The tables on pages 2 and 3 contain Grashof number, Nusselt number, and convection coefficients for the various lengths and orientations, tabulated as a function of head. The product of ΔT and the convection coefficient gives the heat flux, tabulated on page 4.

Convection coefficients are also calculated for low-velocity forced convection for comparison.

The resulting heat fluxes were then plotted as a function of distance below the liquid surface. Note that depth is on the vertical axis with zero at the top, and heat flux is on the horizontal axis.

Results.

The maximum heat flux which can be carried away by free convection (i.e., the heat flux above which boiling occurs) is .001 W/sq.cm. at 4 inches below the surface and 0.1 to 0.2 W/sq.cm. 15 feet below the surface. Forced convection over a 1 cm plate with a fluid velocity of 1 cm/sec, or a 10 cm plate at 10 cm/sec, is about like free convection. The line for much higher heat flux is 10 cm/sec flow over a 1 cm plate.

Discussion.

The two key assumptions here are that the bulk of the liquid is at the surface temperature, and that the threshold of boiling is when the solid surface is at the saturation temperature of the liquid. It is possible that experiments would give free convection heat fluxes much lower or much higher than these results if these assumptions are in error.

Much higher heat fluxes via free convection, especially near the surface where I used very small ΔT 's, might be possible if nucleation of boiling occurs at some surface temperature significantly (like a degree) above the saturation temperature of the liquid. Nucleation would not occur at a lower ΔT than was used here, since it was just the ΔT to the saturation temperature.

So this calculation took the most pessimistic possible assumption (in terms of avoiding boiling) regarding the onset of nucleation.

Conversely, the limits of free convection might be much lower than calculated here, again especially near the surface, since the mechanism of heat dissipation from the bulk involves warming of the bulk liquid above the surface temperature and transfer of heat through a boundary layer to the surface where evaporation takes away the heat (Atkinson, et. al., "Heat and Evaporative Mass Transfer Correlation at the Liquid-vapour Interface of Cryogenic Liquids", ICEC 10, 1984). Based on that paper, the predicted heat flux to EC, and the surface area for evaporation in EC, I calculate that the liquid would be superheated to a depth of about 3 feet. This free convection calculation would result in no allowable heat flux to that depth since the liquid is already at or above saturation temperature.

A third factor which could cause reality to differ from these calculations is that the geometry is not a small heated flat plate suspended in fluid. Corners, edges and irregularities will disrupt the boundary layer and enhance free convection heat transfer. But a hot spot on the inner vessel wall will have no leading edge, so the velocity profile will be different from what is assumed here. It may be best approximated by the 10 cm plate in these calculations since that is mostly covered by a thicker boundary layer than the 1 cm plate, but the total heat added is small enough that flow is still laminar. The vertically oriented 100 cm plate has a higher heat flux to the liquid than the 10 cm plate since for the 100 cm plate flow is turbulent rather than laminar.

Conclusions.

As I have indicated in the above discussion, there is considerable uncertainty in these results. Near the surface I can imagine a difference from these calculations of two orders of magnitude in either direction. At 15 feet deep I have much more confidence in these predictions; I would expect them to be within a factor of two of experimental results for the onset of boiling for heated objects in the argon.

SUBJECT DEPTH IN LIQUID ARGON VERSUS
MAXIMUM CONVECTIVE HEAT FLUX

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TJP
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Map out Heat Flux vs Depth
for Various Plate Sizes, Laminar
and Turbulent Flow

Under a 1.3 bar (89.70 K) surface

k_{avg} wall (sat) to 5 at depth W/cmK	ΔT (K)	ρ_{sat} at $T + \Delta T$ (g/cm ³)	ρ_{∞} at T_{surf}, P_{depth} (g/cm ³)	μ_{avg} (wall to 5 at depth) (g/cm s)	Gr/L^3 (cm ⁻³)	P_{depth} (P_{sat} at T_{surf}) (bar)	head (ft) (using ρ_{∞})	Pr_{avg} wall (sat) to at depth
1.241×10^{-3}	.10	1.3773	1.3780	2.419×10^{-3}	1.616×10^5	1.314	0.34	2.40
1.242×10^{-3}	.20	1.3767	1.3780	2.415×10^{-3}	3.009×10^5	1.327	0.66	2.40
1.242×10^{-3}	.30	1.3761	1.3780	2.412×10^{-3}	4.407×10^5	1.340	0.97	2.39
1.243×10^{-3}	.40	1.3755	1.3780	2.408×10^{-3}	5.816×10^5	1.353	1.29	2.39
1.244×10^{-3}	.50	1.3749	1.3780	2.404×10^{-3}	7.232×10^5	1.367	1.63	2.38
1.245×10^{-3}	.60	1.3742	1.3780	2.401×10^{-3}	8.883×10^5	1.380	1.94	2.38
1.246×10^{-3}	.70	1.3736	1.3780	2.397×10^{-3}	1.032×10^6	1.393	2.26	2.37
1.247×10^{-3}	.80	1.3730	1.3780	2.394×10^{-3}	1.175×10^6	1.407	2.60	2.36
1.247×10^{-3}	.90	1.3724	1.3780	2.390×10^{-3}	1.319×10^6	1.420	2.91	2.36
1.248×10^{-3}	1.00	1.3717	1.3780	2.386×10^{-3}	1.489×10^6	1.434	3.25	2.35
1.253×10^{-3}	1.50	1.3686	1.3780	2.370×10^{-3}	2.246×10^6	1.505	4.98	2.34
1.257×10^{-3}	2.00	1.3654	1.3781	2.349×10^{-3}	3.082×10^6	1.580	6.80	2.32
1.261×10^{-3}	2.50	1.3623	1.3781	2.336×10^{-3}	3.868×10^6	1.657	8.67	2.31
1.266×10^{-3}	3.00	1.3591	1.3781	2.322×10^{-3}	4.697×10^6	1.736	10.59	2.30
1.270×10^{-3}	3.50	1.3559	1.3782	2.309×10^{-3}	5.562×10^6	1.818	12.58	2.29
1.275×10^{-3}	4.00	1.3527	1.3782	2.292×10^{-3}	6.439×10^6	1.903	14.64	2.28

$$\frac{Gr}{L^3} = \frac{\rho_{sat} (at depth) \times g [e(T_{surface}, P_{depth}) - e_{sat}]}{\mu_{avg}^2}$$

(2)

head (ft)	Gr L=1 cm 10 cm 100 cm	Gr Pr L=1 10 100	(w/cm ² K)			
			Nu (vert plate) L=1 10 100	Nu (hor plate) L=1 10 100	$\bar{h}$ (vert plate) L=1 10 100	$\bar{h}$ (hor plate) L=1 10 100
0.34	1.616 × 10 ⁵ 1.616 × 10 ⁸ 1.616 × 10 ¹¹	3.878 × 10 ⁵ × 10 ⁸ × 10 ¹¹	13.8 77.9 907	13.5 102.1	.0171 .00967 .0113	.0168 .0127
0.66	3.009 × 10 ⁵ × 10 ⁸ × 10 ¹¹	7.222 × 10 ⁵ × 10 ⁸ × 10 ¹¹	16.2 91.0 1163	15.7 125.6	.0201 .0113 .0144	.0195 .0156
0.97	4.407 × 10 ⁵ × 10 ⁸ × 10 ¹¹	10.53 × 10 ⁵ × 10 ⁸ × 10 ¹¹	17.8 100.0 1353	17.3 142.4	.0221 .0124 .0168	.0215 .0177
1.29	5.816 × 10 ⁵ × 10 ⁸ × 10 ¹¹	13.90 × 10 ⁵ × 10 ⁸ × 10 ¹¹	19.1 107.2 1512	18.5 156.2	.0237 .0133 .0188	.0230 .0194
1.63	7.232 × 10 ⁵ × 10 ⁸ × 10 ¹¹	17.21 × 10 ⁵ × 10 ⁸ × 10 ¹¹	20.1 113.0 1646	19.6 167.8	.0250 .0141 .0205	.0244 .0209
1.94	8.883 × 10 ⁵ × 10 ⁸ × 10 ¹¹	21.14 × 10 ⁵ × 10 ⁸ × 10 ¹¹	21.2 119.0 1788	20.6 179.7	.0264 .0148 .0223	.0256 .0224
2.26	1.032 × 10 ⁶ 1.032 × 10 ⁹ 1.032 × 10 ¹²	2.446 × 10 ⁶ 2.446 × 10 ⁹ 2.446 × 10 ¹²	21.9 123.4 1895	21.4 188.6	.0273 .0154 .0236	.0267 .0235
2.60	1.175 × 10 ⁶ × 10 ⁹ × 10 ¹²	2.773 × 10 ⁶ × 10 ⁹ × 10 ¹²	22.6 127.4 1993	22.0 196.7	.0282 .0159 .0249	.0274 .0245
2.91	1.319 × 10 ⁶ × 10 ⁹ × 10 ¹²	3.113 × 10 ⁶ × 10 ⁹ × 10 ¹²	23.3 131.7 2087	22.7 204.4	.0291 .0164 .0260	.0283 .0255
3.25	1.489 × 10 ⁶ × 10 ⁹ × 10 ¹²	3.499 × 10 ⁶ × 10 ⁹ × 10 ¹²	24.0 138.0 2187	23.4 212.5	.0300 .0172 .0273	.0292 .0265
4.98	2.246 × 10 ⁶ × 10 ⁹ × 10 ¹²	5.256 × 10 ⁶ × 10 ⁹ × 10 ¹²	26.6 162.4 2573	25.9 243.4	.0333 .0203 .0322	.0325 .0305
6.80	3.082 × 10 ⁶ × 10 ⁹ × 10 ¹²	7.150 × 10 ⁶ × 10 ⁹ × 10 ¹²	28.7 183.6 2910	27.9 269.7	.0361 .0231 .0366	.0351 .0339
8.67	3.868 × 10 ⁶ × 10 ⁹ × 10 ¹²	8.935 × 10 ⁶ × 10 ⁹ × 10 ¹²	30.3 200.8 3182	29.5 290.5	.0382 .0253 .0401	.0372 .0366

(3)

head (ft)	Gr L=1 cm 10 cm 100 cm	Gr Pr L=1 10 100	Nu (vert) L=1 10 100	Nu (hor) L=1 10 100	$\bar{h}$ (vert) L=1 10 100	$\bar{h}$ (hor) L=1 10 100
10.59	4.697×10^6 $\times 10^9$ $\times 10^{12}$	10.80×10^6 $\times 10^9$ $\times 10^{12}$	31.8 217 3432	31.0 309.5	.0403 .0275 .0434	.0392 .0392
12.58	5.562×10^6 $\times 10^9$ $\times 10^{12}$	12.74×10^6 $\times 10^9$ $\times 10^{12}$	33.2 231 3667	32.3 327.0	.0422 .0293 .0466	.0410 .0415
14.64	6.439×10^6 $\times 10^9$ $\times 10^{12}$	14.68×10^6 $\times 10^9$ $\times 10^{12}$	34.4 245 3881	33.4 342.8	.0439 .0312 .0495	.0426 .0437

Vertical plate correlations: (kreith, Heat Transfer, fig 7-4)

$$Nu = .555 (Gr Pr)^{1/4} \quad (10^5 < Gr < 1.3 \times 10^9) \\ \text{(laminar flow)}$$

$$Nu = .021 (Gr Pr)^{2/5} \quad (1.3 \times 10^9 < Gr < 10^{12}) \\ \text{(turbulent flow)}$$

Horizontal plate correlations: (Kreith, eqs 7-24, 7-25)

$$Nu = .54 (Gr Pr)^{1/4} \quad (10^5 < Gr < 2 \times 10^7) \\ \text{(laminar flow)}$$

$$Nu = .14 (Gr Pr)^{1/3} \quad (2 \times 10^7 < Gr < 3 \times 10^{10}) \\ \text{(turbulent flow)}$$

$$Nu \equiv \frac{hL}{k}$$

Low Velocity Forced Convection in Liquid Argon over a Heated Plate

First check what flat plate length and fluid velocities result in turbulent flow:

$$Re = \frac{\rho v x}{\mu}$$

$$\rho (1.3 \text{ bar liquid argon}) = 1.4 \text{ g/cm}^3$$

$$\mu (1.3 \text{ bar liquid argon}) = 2.4 \times 10^{-3} \text{ g/cm sec}$$

$$Re = \frac{(1.4 \text{ g/cm}^3) v x}{(2.4 \times 10^{-3} \text{ g/cm sec})}$$

$$\text{for } Re \geq 5 \times 10^5 \text{ need } v x \geq 857$$

looking at $v \leq 10 \text{ cm/sec}$, x arbitrary so start with laminar flow equation.

Use $x = 10 \text{ cm}$ as for free convection calc.

The local convective-heat-transfer coefficient is

$$h_{cx} = 0.332 \frac{k}{x} Re_x^{1/2} Pr^{1/3} \quad (\text{Krieth pg 268})$$

$$k (\text{sat liq at } 1.3 \text{ bar}) = 1.2399 \times 10^{-3} \text{ W/cmK}$$

$$\rho (" " " ") = 1.378 \text{ g/cm}^3$$

$$\mu (" " " ") = 2.4186 \times 10^{-3} \text{ g/cm sec}$$

$$Pr (" " " ") = 2.397$$

$$h_{cx} = (0.332) \frac{(1.2399 \times 10^{-3} \text{ W/cmK})}{(10 \text{ cm})} \left[\frac{(1.378 \text{ g/cm}^3) \left(v \frac{\text{cm}}{\text{s}} \right) (10 \text{ cm})}{2.4186 \times 10^{-3} \text{ g/cm s}} \right]^{1/2} \times (2.397)^{1/3}$$

$$= \left(4.158 \times 10^{-3} \frac{\text{W}}{\text{cm}^2 \text{K}} \right) \left(v \frac{\text{cm}}{\text{sec}} \right)^{1/2}$$

(2)

For $v = 10 \text{ cm/sec}$ get $h_{cx} = 13.15 \times 10^{-3} \text{ W/cm}^2 \text{ K}$

$v = 1 \text{ cm/sec}$ get $h_{cx} = 4.16 \times 10^{-3} \text{ W/cm}^2 \text{ K}$

$v = 0.1 \text{ cm/sec}$ get $h_{cx} = 1.315 \times 10^{-3} \text{ W/cm}^2 \text{ K}$

$\therefore$ Free convection heat transfer coefficients, ^{from the horizontal plate} are like 0.2 cm/sec to 2 cm/sec flow velocities with forced convection.

Liquid Argon head is $\sim 185''$ at bottom
or $h = 469.9 \text{ cm}$

$$\rho g h = (1.4 \text{ g/cm}^3) (980.665 \text{ cm/s}^2) (469.9 \text{ cm}) \times 1.45 \times 10^{-5} \frac{\text{psi}}{\text{g/cm}^2} = 9.355 \text{ psid} = 0.64 \text{ bar}$$

so could have boiling temp at bottom of 93.9 K (1.94 bar) and 89.7 K (1.30 bar) at top, for a ΔT of 4.2 K .

ΔT of 2.1 K subcooling at center is possible, but on sides where free convection works better and velocities are higher.

For bottom, 2 cm/sec flow $h_c = 6 \times 10^{-3} \text{ W/cm}^2$
and $\Delta T = 4.2 \text{ K}$ get $2.5 \times 10^{-2} \text{ W/cm}^2$
max heat flux

On our $10 \text{ cm} \times 10 \text{ cm}$ square this is 2.5 W

So convectively can safely remove $.03 \text{ W/cm}^2$
but boil at 0.03 W/cm^2

Second Look at Low Velocity Forced Convection

$$\overline{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3} = .664 \left(\frac{\rho_{avg} v L}{\mu_{avg}} \right)^{1/2} (Pr_{avg})^{1/3}$$

$$Re_L = \frac{\rho v L}{\mu}$$

where avg is wall to ∞ at depth.

Use data on 5 Feb 88 table for avg μ and ρ under 1.3 bar surface at various depth.

ΔT (K)	head (ft)	Nu for $vL =$				
		.1	1	10	100	1000
.10	0.34	6.71	21.22	67.09	212.16	670.9
.20	0.66					
.30	0.97					
.40	1.29					
.50	1.63	6.71	21.21	67.08	212.13	670.8
.60	1.94					
.70	2.26					
.80	2.60					
.90	2.91					
1.00	3.25	6.70	21.19	67.01	211.9	670.1
1.50	4.98					
2.00	6.80	6.72	21.24	67.17	212.42	671.7
2.50	8.67					
3.00	10.51	6.73	21.28	67.29	212.79	672.9
3.50	12.58					
4.00	14.64	6.75	21.33	67.45	213.31	674.54

(2)

want $L = 1, 10, 100$ cm

and $v = 0.1, 1, 10$ cm/sec

$\therefore vL = .1, 1, 10, 100, 1000$

for $L = 1 \quad 10, 10 \quad 1, 10, 100 \quad 10, 100 \quad 100$

$$\bar{h} = \frac{Nu k_{avg}}{L}$$

$$q \text{ (w/cm}^2\text{)} = \bar{h} \Delta T$$

$$.664 \left[\left(\frac{\rho_{avg}}{\mu_{avg}} \right) vL \right]^{1/2} (Pr_{avg})^{1/3} = Nu$$

(3)

 $h (w/cm^2k)$

ΔT (K)	head (ft)									
		$N = 0.1$			$N = 1.0$			$N = 10 \text{ cm/s}$		
		$L = 1 \text{ cm}$	10	100	1	10	100	1	10	100
.1	0.34	.008	.003	.001	.026	.008	.003	.023	.026	.008
.5	1.63									
1.0	3.25									
2.0	6.80									
3.0	10.59									
4.0	14.64	.0086	.0027	.0009	.0272	.0086	.0027	.026	.0272	.0086
		low	low	low		low	low			low