**D-0 END CALORIMETER
WARM TUBE/TEV DRY AIR PURGE**

D-Zero Engineering Note #3740.225-EN-317

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D-0 End Calorimeter Warm Tube/Tev Dry Air Purge

This Engineering Note studies the design of the Dry Air Purge that is going to flow through the Warm Tube of the End Calorimeter of the D-0 Calorimeter. The Tev tubes through the E.C. can be thought of as a cluster of concentric tubes: The Tev tube, the warm (vacuum vessel) tube, 15 layers of superinsulation, the cold (argon vessel) tube, and the Inner Hadronic center support tube.

The Dry Air Purge will involve flowing Dry Air through the annular region between the Warm Tube and the Tev Beam Pipe. This air flow is intended to prevent condensation from forming in this region which could turn to ice under cryogenic temperatures. Any ice formed in this gap, could cause serious problems when these tubes are moved.

The Air will flow through a Nylon Tube Fitting - 1/4" I.D. to 1/8" male pipe thread (Cole Palmer #YB-06465-15) see Drawing MC-295221 (Appendix A). This fitting will be attached to the Nylon 2" Tube - Wiper and Seal Assembly which is clamped to the ends of the Warm Tube (Appendix A).

This note includes drawings and calculations that explain the setup of the Dry Air Purge and give the required information on the pressure drops through the setup. The Equations and properties used in the calculations were obtained from the Applied Fluid Dynamics Handbook by Robert D. Blevins and Fluid Dynamics Second Edition by Frank M. White.

The Dry Air purge can be assumed incompressible
if $Ma < .3$

for Dry Air $a = 1117 \frac{ft}{sec}$

$$Ma = \frac{V}{a} = \frac{V}{1117 \frac{ft}{sec}} < .3$$

$$V < .3 \times 1117 \frac{ft}{sec}$$

$$V < 335 \frac{ft}{sec}$$

For our situation, the velocity will be well below $335 \frac{ft}{sec}$.

So, the assumption that the purge is incompressible is acceptable.

Using Bernoulli Equation

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + w_f$$

$$w_f = K \frac{V^2}{2}$$

$$K = f \left(\frac{L}{D} \right)$$

$$V = \frac{Q}{A}$$

① → ②

$$\frac{P_1}{\rho} + \frac{Q^2}{A_1^3(2)} = \frac{P_2}{\rho} + \frac{Q^2}{A_2^3(2)} + \left[1 - \left(\frac{A_1}{A_2} \right)^2 \right] \frac{Q^2}{A_1^3(2)}$$

$$P_1 - P_2 = \left(\frac{\rho Q^2}{2} \right) \left[\frac{1}{A_2^3} + \frac{\left[1 - \left(\frac{A_1}{A_2} \right)^2 \right]^2 - 1}{A_1^3} \right] \quad (1)$$

② → ③

$$\frac{P_2}{\rho} + \frac{Q^2}{A_2^3(2)} = \frac{P_3}{\rho} + \frac{Q^2}{A_3^3(2)} + \left[1 - \left(\frac{A_2}{A_3} \right)^2 \right] \frac{Q^2}{A_2^3(2)}$$

$$P_2 - P_3 = \left(\frac{\rho Q^2}{2} \right) \left[\frac{1}{A_3^3} + \frac{\left[1 - \left(\frac{A_2}{A_3} \right)^2 \right]^2 - 1}{A_2^3} \right] \quad (2)$$

③ → ④

$$\frac{P_3}{\rho} + \frac{Q^2}{A_3^3(2)} = \frac{P_4}{\rho} + \frac{Q^2}{A_4^3(2)} + K_3 \left(\frac{Q^2}{A_3^3(2)} \right) \quad (A_3 = A_4)$$

$$P_3 - P_4 = \left(\frac{\rho Q^2}{2} \right) \left[\frac{K_3}{A_3^3} \right] \quad (3)$$

④ → ⑤

$$\frac{P_4}{\rho} + \frac{Q^2}{A_4^3(2)} = \frac{P_5}{\rho} + \frac{Q^2}{A_5^3(2)} + (.5) \left(\frac{Q^2}{A_4^3(2)} \right)$$

$$P_4 - P_5 = \left(\frac{\rho Q^2}{2} \right) \left[\frac{1.5}{A_5^3} - \frac{1}{A_4^3} \right] \quad (4)$$

⑤ → ⑥

$$\frac{P_5}{\rho} + \frac{Q^2}{A_5^3(2)} = \frac{P_6}{\rho} + \frac{Q^2}{A_6^3(2)} + (.5) \left(\frac{Q^2}{A_5^3(2)} \right)$$

$$P_5 - P_6 = \left(\frac{\rho Q^2}{2} \right) \left[\frac{1.5}{A_6^3} - \frac{1}{A_5^3} \right] \quad (5)$$

A_1, A_6 = area of inlet of nozzle

$$A_1, A_6 = \pi \left(\frac{.157}{2} \right)^2 = 0.019 \text{ in}^2 \left(\frac{6.45 \times 10^{-4} \text{ m}^2}{\text{in}^2} \right) = \boxed{1.226 \times 10^{-5} \text{ m}^2}$$

A_2, A_5 = area of threaded part of nozzle

$$A_2, A_5 = \pi \left(\frac{.25}{2} \right)^2 = 0.049 \text{ in}^2 \left(\frac{6.45 \times 10^{-4} \text{ m}^2}{\text{in}^2} \right) = \boxed{3.16 \times 10^{-5} \text{ m}^2}$$

A_3, A_4 = area of annulus of tubes

$$A_3, A_4 = \pi (1.1385^2 - 1.0^2) = 0.9305 \text{ in}^2 \left(\frac{6.45 \times 10^{-4} \text{ m}^2}{\text{in}^2} \right) = \boxed{6.00 \times 10^{-4} \text{ m}^2}$$

Density of Dry Air

$$\rho_g = 11.8 \frac{\text{N}}{\text{m}^3}$$

$$\rho = \frac{11.8 \frac{\text{N}}{\text{m}^3}}{9.8 \frac{\text{m}}{\text{sec}^2}} = \boxed{1.204 \frac{\text{kg}}{\text{m}^3}}$$

Loss Coefficient (K_3) through Annulus of pipe

$$K = f \left(\frac{L}{D} \right)$$

$$f Re = 96.00 \quad \left(\text{Applied fluid Dynamics} \right. \\ \left. \text{Hand book, p. 44} \right)$$

$$Re = \frac{V_3 D}{\nu} \quad D = 2(1.1385 - 1.0) = .277 \text{ in} \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) = .023 \text{ ft}$$

$$\nu = 1.58 \times 10^{-4} \frac{\text{ft}^2}{\text{sec}}$$

$$f = \frac{96.00 (1.58 \times 10^{-4} \frac{\text{ft}^2}{\text{sec}})}{V_3 (.023 \text{ ft})}$$

$$f = \frac{0.659 \frac{\text{ft}^2}{\text{sec}} \left(\frac{1.204 \text{ kg}}{\text{m}^3} \right)}{V_3}$$

$$f = \frac{0.200 \frac{\text{ft}^2}{\text{sec}}}{V_3}$$

$$L = 101 \text{ in}$$

$$D = .277 \text{ in}$$

$$K_3 = \frac{.200 \frac{\text{ft}^2}{\text{sec}} \left(\frac{101}{.277} \right)}{V_3}$$

$$\boxed{K_3 = \frac{73.24 \frac{\text{ft}^2}{\text{sec}} (A_2)}{Q}}$$

Plugging in values of $A_1, A_2, A_3, A_4, A_5, A_6, f, K_2$ into equations (1-5) gives the following:

$$(1) \quad P_1 - P_2 = \left(\frac{1.204}{2}\right) Q^2 \left[\frac{1}{(3.16 \times 10^{-5} \text{ m}^2)^2} + \frac{\left[1 - \left(\frac{1.57}{1.226 \times 10^{-5} \text{ m}^2}\right)^2 - 1\right]}{(1.226 \times 10^{-5} \text{ m}^2)^2} \right]$$

$$P_1 - P_2 = Q^2 (-1.933 \times 10^9 \frac{\text{kg}}{\text{m}^2}) \quad (1a)$$

$$(2) \quad P_2 - P_3 = \left(\frac{1.204}{2}\right) Q^2 \left[\frac{1}{(6.0 \times 10^{-4} \text{ m}^2)^2} + \frac{\left[1 - \left(\frac{3.16 \times 10^{-5}}{6 \times 10^{-4}}\right)^2 - 1\right]}{(3.16 \times 10^{-5} \text{ m}^2)^2} \right]$$

$$P_2 - P_3 = Q^2 (-6.916 \times 10^7 \frac{\text{kg}}{\text{m}^2}) \quad (2a)$$

$$(3) \quad P_3 - P_4 = \left(\frac{1.204}{2}\right) Q^2 \left[\frac{73.24 \times (6.0 \times 10^{-4} \text{ m}^2)}{Q (6.0 \times 10^{-4})^2} \right]$$

$$P_3 - P_4 = Q (7.348 \times 10^4 \frac{\text{kg}}{\text{m}^2 \text{ sec}}) \quad (3a)$$

$$(4) \quad P_4 - P_5 = \left(\frac{1.204}{2}\right) Q^2 \left[\frac{1.5}{(3.16 \times 10^{-5})^2} - \frac{1}{(6 \times 10^{-4})^2} \right]$$

$$P_4 - P_5 = Q^2 (9.026 \times 10^8 \frac{\text{kg}}{\text{m}^2}) \quad (4a)$$

$$(5) \quad P_5 - P_6 = \left(\frac{1.204}{2}\right) Q^2 \left[\frac{1.5}{(1.226 \times 10^{-5})^2} - \frac{1}{(3.16 \times 10^{-5})^2} \right]$$

$$P_5 - P_6 = (Q^2) (5.405 \times 10^9 \frac{\text{kg}}{\text{m}^2}) \quad (5a)$$

Adding Equations (1a - 5a) gives:

$$P_1 - P_6 = (4.314 \times 10^9) Q^2 + (7.348 \times 10^4) Q$$

$$\Delta P = (4.314 \times 10^9 \frac{\text{kg}}{\text{m}^2}) Q^2 + (7.348 \times 10^4 \frac{\text{kg}}{\text{m}^2 \text{ sec}}) Q$$

assuming: $Q = 10 \frac{\text{ft}^3}{\text{hr}} \left(\frac{.3048 \text{ m}}{\text{ft}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ sec}} \right) = 7.87 \times 10^{-5} \frac{\text{m}^3}{\text{sec}}$

$$\Delta P = (4.314 \times 10^9) (7.87 \times 10^{-5})^2 + (7.348 \times 10^4) (7.87 \times 10^{-5}) = 32.5 \text{ Pa}$$

$$\Delta P = 32.5 \text{ Pa} \left(\frac{1 \text{ psi}}{6895 \text{ Pa}} \right) = 4.71 \times 10^{-3} \text{ psi}$$

$$\Delta P = .0047 \text{ psi} \left(\frac{0.433'' \text{ of H}_2\text{O}}{0.016 \text{ psi}} \right)$$

$$\Delta P = 0.127'' \text{ of H}_2\text{O}$$

Appendix - A
(Drawings)

tionally than those of Table 6-1. Profiles for laminar flow in noncircular sections can be found in Refs. 6-13 and 6-19, pp. 123-127.

6.3. STRAIGHT UNIFORM PIPES AND DUCTS

6.3.1. General Form for Incompressible Flow

The loss in fluid pressure between two points along a straight, level, uniform pipe or duct, in which the flow is incompressible and fully developed, can be calculated by the Darcy-Weisbach formula:

$$\Delta p = \frac{\rho U^2}{2} \frac{fL}{D}, \quad (6-6)$$

where Δp = loss in both fluid static pressure and total pressure between an upstream and a downstream point (the two losses are identical since velocity is constant through the uniform duct, see Section 6.1.2),

f = dimensionless friction factor (Fig. 6-6),

D = hydraulic diameter (Table 6-2),

L = distance along pipe span separating the two points,

U = average flow velocity over the cross section,

ρ = fluid density.

Consistent sets of units are given in Table 3-1. The above formula permits calculation of the pressure drop if the pipe geometry, flow velocity, and friction factor are known. This formula can easily be inverted to permit calculation of either flow rate or pipe diameter. These inverted forms are given in Table 6-3.

The pressure loss through a pipe increases with increasing flow velocity and decreases with increasing pipe diameter. The details of these relationships depend on the friction factor, which is also a function of flow velocity and pipe diameter.

The friction factor of Eq. (6-6) is known as the D'Arcy, Darcy-Weisbach, or Moody friction factor. A simple force balance shows that the fluid shear stress τ (shear force/unit surface area) imposed on the wall of a uniform duct by the friction of the flowing fluid is proportional to the friction factor (see Section 5.5.2):

$$\tau = \frac{\rho U^2}{2} \frac{f}{4}. \quad (6-7)$$

An alternative definition of friction factor, known as the Fanning friction factor or "small" friction factor, is exactly one-fourth the present friction factor.

The friction factor for fully developed flow in uniform pipes and ducts is a function of the geometry of the cross section, the Reynolds number (based on hydraulic diameter), and, for turbulent flow, the surface roughness of the pipe or duct:

$$f = F(\text{cross section, } UD/\nu, \epsilon/D), \quad (6-8)$$

where ν is the kinematic viscosity of the fluid, ϵ is the equivalent surface roughness, and D is the hydraulic diameter (Table 6-2). The dimensionless ratio UD/ν is called the Reynolds number, and the ratio of the equivalent surface roughness to the hydraulic diameter (ϵ/D) is the relative roughness. The friction factor is substantially independent of Mach number for Mach numbers less than 1 (Ref. 6-3, Vol. II, p. 1131). The friction factor is presented for laminar, transitional, and turbulent flow in the following subsections.

6.3.2. Friction Factor for Laminar Flow ($Re < 2000$)

Below a critical number (based on hydraulic diameter) of approximately 2000, the flow in pipes and ducts is laminar. The laminar flow is characterized by layers of fluid gliding by each other in a well-defined profile (frame 1 of Table 6-1 for circular pipes). Any disturbances introduced into the flow will be damped out. The friction factor for laminar flow is inversely proportional to Reynolds number:

$$f = \frac{k}{Re}. \quad (6-9)$$

$Re = UD/\nu$ is the Reynolds number based on the hydraulic diameter (D), mean flow velocity over the cross section (U), and kinematic viscosity (ν). Since Reynolds number is proportional to flow velocity, the pressure losses in laminar flow increase with flow velocity to the first power. The dimensionless friction coefficient k is found exactly by the solution of Poisson's equation over the cross section, and it is dependent only on the shapes of the cross section. It is largely independent of surface roughness. Values of k for various cross sections are given in Table 6-2 and are plotted in Fig. 6-4.

The hydraulic diameter is proportional to flow area divided by the perimeter of the surface containing the flow:

$$D = \frac{4(\text{flow area of cross section})}{\text{wetted perimeter of cross section}}. \quad (6-10)$$

Table 6-2. Properties of Cross Sections. (Continued)

Cross Section	Area (A) and Hydraulic Diameter (D)	Laminar Flow Friction Coefficient ($k = f Re$) Inlet Resistance Coefficient (K)																																															
5. Annulus	$A = \pi(a^2 - b^2)$ $D = 2(a - b)$ $f Re = \frac{64(a - b)^2}{a^2 + b^2 - \frac{(a^2 - b^2)}{\log_e \frac{a}{b}}}$	<table><tr><th>b/a</th><th>f Re</th><th>K</th></tr><tr><td>0.0</td><td>64.0</td><td>1.25</td></tr><tr><td>0.05</td><td>86.27</td><td>0.830</td></tr><tr><td>0.10</td><td>89.37</td><td>0.784</td></tr><tr><td>0.50</td><td>95.25</td><td>0.688</td></tr><tr><td>0.75</td><td>95.87</td><td>0.678</td></tr><tr><td>1.00</td><td>96.00</td><td>0.674</td></tr></table> <p>Refs. 6-16; 6-19, p. 124.</p>	b/a	f Re	K	0.0	64.0	1.25	0.05	86.27	0.830	0.10	89.37	0.784	0.50	95.25	0.688	0.75	95.87	0.678	1.00	96.00	0.674																										
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0.75	95.87	0.678																																															
1.00	96.00	0.674																																															
6. Eccentric Annulus	$A = \pi(a^2 - b^2)$ $D = 2(a - b)$	<p>$f(c \neq 0)/f(c = 0)$</p> <table><tr><th rowspan="2">$\frac{c}{a-b}$</th><th colspan="5">b/a</th></tr><tr><th>0.1</th><th>0.3</th><th>0.5</th><th>0.7</th><th>0.9</th></tr><tr><td>0.01</td><td>0.9952</td><td>0.9597</td><td>0.9018</td><td>0.8405</td><td>0.7942</td></tr><tr><td>0.1</td><td>0.9909</td><td>0.9256</td><td>0.8246</td><td>0.7219</td><td>0.6391</td></tr><tr><td>0.3</td><td>0.9875</td><td>0.9004</td><td>0.7587</td><td>0.6345</td><td>0.5253</td></tr><tr><td>0.5</td><td>0.9861</td><td>0.8877</td><td>0.7421</td><td>0.5987</td><td>0.4797</td></tr><tr><td>0.7</td><td>0.9855</td><td>0.8829</td><td>0.7315</td><td>0.5827</td><td>0.4594</td></tr><tr><td>0.9</td><td>0.9852</td><td>0.8819</td><td>0.7276</td><td>0.5769</td><td>0.4522</td></tr></table> <p>$f(c = 0)$ from frame 5, Ref. 6-19, p. 127.</p>	$\frac{c}{a-b}$	b/a					0.1	0.3	0.5	0.7	0.9	0.01	0.9952	0.9597	0.9018	0.8405	0.7942	0.1	0.9909	0.9256	0.8246	0.7219	0.6391	0.3	0.9875	0.9004	0.7587	0.6345	0.5253	0.5	0.9861	0.8877	0.7421	0.5987	0.4797	0.7	0.9855	0.8829	0.7315	0.5827	0.4594	0.9	0.9852	0.8819	0.7276	0.5769	0.4522
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7. Sector of Annulus	$A = \frac{\theta}{2}(a^2 - b^2)$ $D = \frac{2\theta(a^2 - b^2)}{\theta(a + b) + 2(a - b)}$	<p>Values of f Re</p> <p>$\theta(\text{deg})$</p> <table><tr><th>$\frac{b}{a}$</th><th>10</th><th>30</th><th>60</th><th>90</th><th>180</th></tr><tr><td>0.1</td><td>57.96</td><td>58.36</td><td>58.76</td><td>59.22</td><td>62.46</td></tr><tr><td>0.3</td><td>68.28</td><td>60.96</td><td>57.88</td><td>59.28</td><td>67.45</td></tr><tr><td>0.5</td><td>69.48</td><td>57.88</td><td>59.52</td><td>64.12</td><td>75.06</td></tr><tr><td>0.7</td><td>62.24</td><td>58.84</td><td>68.14</td><td>74.48</td><td>83.36</td></tr><tr><td>0.9</td><td>59.92</td><td>76.28</td><td>84.72</td><td>88.12</td><td>92.00</td></tr></table> <p>Ref. 6-26.</p>	$\frac{b}{a}$	10	30	60	90	180	0.1	57.96	58.36	58.76	59.22	62.46	0.3	68.28	60.96	57.88	59.28	67.45	0.5	69.48	57.88	59.52	64.12	75.06	0.7	62.24	58.84	68.14	74.48	83.36	0.9	59.92	76.28	84.72	88.12	92.00											
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0.9	59.92	76.28	84.72	88.12	92.00																																												
8. Ellipse	$A = \pi ab$ $D \approx \frac{4ab}{a + b} \left(\frac{64 - 16c^2}{64 - 3c^4} \right)$ $c = (a - b)/(a + b)$ $0.1 < a/b < 10$ Ref. 6-27	$f Re = \frac{8D^2(a^2 + b^2)}{a^2 b^2}$ Ref. 6-19, p. 123.																																															
9. Right Triangle	$A = \frac{1}{2} ab$ $D = \frac{2ab}{a + b + (a^2 + b^2)^{1/2}}$ [$\theta = \tan^{-1}(b/a)$].	<table><tr><th>θ (deg)</th><th>f Re</th><th>K</th></tr><tr><td>10</td><td>49.96</td><td>2.40</td></tr><tr><td>30</td><td>52.14</td><td>1.95</td></tr><tr><td>45</td><td>52.62</td><td>1.88</td></tr><tr><td>60</td><td>52.14</td><td>1.95</td></tr><tr><td>70</td><td>51.32</td><td>2.10</td></tr><tr><td>90</td><td>48.00</td><td>2.971</td></tr></table> <p>Refs. 6-13, p. 237; 6-28.</p>	θ (deg)	f Re	K	10	49.96	2.40	30	52.14	1.95	45	52.62	1.88	60	52.14	1.95	70	51.32	2.10	90	48.00	2.971																										
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6.2.2) and the extent of flow separation within an abrupt contraction is considerably reduced. The viscous component of the losses in laminar inlet flow arise more from the energy required to transform a uniform inlet profile to the laminar parabolic flow profile than from eddy formation. Thus, for laminar flow it is reasonable to apply the formulas of the first column of frames 1 through 13 of Table 6-7, but with the loss coefficient K as given for the laminar flow inlet loss. These laminar flow inlet loss values of K are given in Table 6-2. This procedure yields

reasonably good agreement with experimental data of Refs. 6-76 and 6-77.

6.5.2. Exits and Abrupt Expansions

Incompressible Flow in Exits and Abrupt Expansions. The change in static pressure induced in a turbulent incompressible flow is given in frames 1 through 19 of Table 6-7. The data of this table were primarily taken from experiments on circular pipes. It is reasonable

Table 6-7. Pressure Loss in Abrupt Contractions and Expansions.

Notation: A = cross-sectional flow area; D = diameter for circular pipes, hydraulic diameter (Table 6-2) for noncircular ducts; f = friction factor for fully developed flow in uniform pipe or duct (Section 6.3); K = dimensionless coefficient; L = spanwise length; p = fluid static pressure; R = radius; U = flow velocity averaged over cross section; θ = angle; ρ = fluid density (fluid is incompressible); ν = kinematic viscosity. Results are for turbulent flow $UD/\nu > 10^4$; see text for laminar flow. See Table 3-1 for consistent sets of units.

Description and Static Pressure Change	Non-Recoverable Loss Coefficient, K
<p>1. Entrance Flush with Wall At Right Angle</p> $\frac{p_1 - p_2}{\frac{1}{2} \rho U^2} = 1 + K + \frac{fL}{D}$	<p>$K = 0.5$</p>
<p>2. Entrance Flush with Wall at Arbitrary Angle</p> $\frac{p_1 - p_2}{\frac{1}{2} \rho U^2} = 1 + K + \frac{fL}{D}$	<p>$K = 0.5 + 0.3 \cos \theta + 0.2 \cos^2 \theta$</p> <p>Ref. 6-73</p>

Table 6-7. Pressure Loss in Abrupt Contractions and Expansions. (Continued)

Description and Static Pressure Change	Non-Recoverable Loss Coefficient, K																																																																																									
<p>17. Abrupt Expansion</p> $L \gg D$ $\frac{p_1 - p_2}{\frac{1}{2} \rho U_1^2} = \left(\frac{A_1}{A_2} \right)^2 - 1 + K + \frac{fL}{D}$	$K = \left(1 - \frac{A_1}{A_2} \right)^2$ <p>A_1, A_2 are cross-sectional areas.</p> <p>$U_2 = (A_1/A_2)U_1$.</p> <p>The expansion results in a pressure rise. See text for formulas which take flow distribution into account.</p>																																																																																									
<p>18. Gradual Expansion</p> $L_1, L_2 \gg D_2$ $\frac{p_1 - p_2}{\frac{1}{2} \rho U_1^2} = \left(\frac{A_1}{A_2} \right)^2 - 1 + K + \frac{fL_2}{D_2}$	<table><tr><th rowspan="2">$\frac{A_2}{A_1}$</th><th colspan="8">K</th></tr><tr><th colspan="8">$2l/D_1$</th></tr><tr><th></th><th>0.1</th><th>0.2</th><th>0.3</th><th>0.5</th><th>1.0</th><th>2.0</th><th>3.0</th><th>5.0</th></tr><tr><td>1.2</td><td>0.06</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr><tr><td>1.4</td><td>0.10</td><td>0.09</td><td>0.08</td><td>0.07</td><td>0.06</td><td>-</td><td>-</td><td>-</td></tr><tr><td>1.6</td><td>0.17</td><td>0.13</td><td>0.12</td><td>0.10</td><td>0.08</td><td>0.06</td><td>-</td><td>-</td></tr><tr><td>2.0</td><td>0.25</td><td>0.25</td><td>0.23</td><td>0.20</td><td>0.15</td><td>0.08</td><td>0.06</td><td>-</td></tr><tr><td>2.5</td><td>0.35</td><td>0.35</td><td>0.32</td><td>0.35</td><td>0.25</td><td>0.10</td><td>0.08</td><td>0.06</td></tr><tr><td>3.0</td><td>0.45</td><td>0.45</td><td>0.45</td><td>0.45</td><td>0.37</td><td>0.22</td><td>0.15</td><td>0.10</td></tr><tr><td>4.0</td><td>0.60</td><td>0.60</td><td>0.60</td><td>0.60</td><td>0.55</td><td>0.42</td><td>0.40</td><td>0.30</td></tr></table> <p>$U_2 = (A_1/A_2)U_1$. Ref. 6-4. See Chapter 7 for a diffuser with free discharge.</p> <p>A_1, A_2 are cross-sectional areas.</p>	$\frac{A_2}{A_1}$	K								$2l/D_1$									0.1	0.2	0.3	0.5	1.0	2.0	3.0	5.0	1.2	0.06	-	-	-	-	-	-	-	1.4	0.10	0.09	0.08	0.07	0.06	-	-	-	1.6	0.17	0.13	0.12	0.10	0.08	0.06	-	-	2.0	0.25	0.25	0.23	0.20	0.15	0.08	0.06	-	2.5	0.35	0.35	0.32	0.35	0.25	0.10	0.08	0.06	3.0	0.45	0.45	0.45	0.45	0.37	0.22	0.15	0.10	4.0	0.60	0.60	0.60	0.60	0.55	0.42	0.40	0.30
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4.0	0.60	0.60	0.60	0.60	0.55	0.42	0.40	0.30																																																																																		
<p>19. Expansion from Multi-Channel Core</p> $L \gg D$ $\frac{p_1 - p_2}{\frac{1}{2} \rho U_1^2} = \left(\frac{A_1}{A_2} \right)^2 - 1 + K + \frac{fL}{D}$	$K = \left(1 - \frac{A_1}{A_2} \right)^2$ <p>$A_1 = \sum_i A_{1i}$ = sum of flow areas on left-hand side.</p> <p>A_{1i}, A_2 are cross-sectional areas. All channels on left are identical. $U_2 = (A_1/A_2)U_1$. The expansion results in pressure rise. See text for formulas that take into account flow distribution.</p>																																																																																									

energy correction factor $\alpha_1 = \alpha_2$ and, since $V_1 = V_2$ from (6.23), Eq. (6.24) now reduces to a simple expression for the friction-head loss h_f

$$h_f = \left(z_1 + \frac{p_1}{\rho g} \right) - \left(z_2 + \frac{p_2}{\rho g} \right) = \Delta \left(z + \frac{p}{\rho g} \right) = \Delta z + \frac{\Delta p}{\rho g} \quad (6.25)$$

The pipe-head loss equals the change in the sum of pressure and gravity head, i.e., the change in height of the HGL. Since the velocity head is constant through the pipe, h_f also equals the height change of the EGL. Recall from Fig. 3.18 that the EGL decreases downstream in a flow with losses unless it passes through an energy source, e.g., as a pump or heat exchanger.

Finally apply the momentum relation (3.40) to the control volume in Fig. 6.10, accounting for applied forces due to pressure, gravity, and shear

$$\Delta p \pi R^2 + \rho g (\pi R^2) \Delta L \sin \phi - \tau_w (2\pi R) \Delta L = \dot{m}(V_1 - V_2) = 0 \quad (6.26)$$

This equation relates h_f to the wall shear stress

$$\Delta z + \frac{\Delta p}{\rho g} = h_f = \frac{2\tau_w \Delta L}{\rho g R} \quad (6.27)$$

where we have substituted $\Delta z = \Delta L \sin \phi$ from Fig. 6.10.

So far we have not assumed either laminar or turbulent flow. If we can correlate τ_w with flow conditions, we have resolved the problem of head loss in pipe flow. Functionally, we can assume that

$$\tau_w = F(\rho, V, \mu, d, \epsilon) \quad (6.28)$$

where ϵ is the wall-roughness height. Then dimensional analysis tells us that

$$\frac{8\tau_w}{\rho V^2} = f = F\left(\text{Re}_d, \frac{\epsilon}{d}\right) \quad (6.29)$$

The dimensionless parameter f is called the *Darcy friction factor*, after Henry Darcy (1803-1858), a French engineer whose pipe-flow experiments in 1857 first established the effect of roughness on pipe resistance.

Combining Eqs. (6.27) and (6.29), we obtain the desired expression for finding pipe-head loss

$$h_f = f \frac{L V^2}{d 2g} \quad (6.30)$$

This is the Darcy-Weisbach equation, valid for duct flows of any cross section and for laminar and turbulent flow. It was proposed by Julius Weisbach, a German professor who in 1850 published the first modern textbook on hydrodynamics.

Our only remaining problem is to find the form of the function F in Eq. (6.29) and plot it in the Moody chart of Fig. 6.13.