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RELATIVISTIC QUANTUM FIELD THEORY OF Λ HYPERNUCLEI

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Abstract

The relativistic mean field model of infinite nuclear matter is applied to the study of Λ hypernuclei. A good fit to the single particle levels is obtained, including a small spin-orbit splitting.

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Despite considerable effort, some aspects of the lambda hypernucleus, such as the small spin-orbit splitting, remain unexplained. A similar problem exists for ordinary nuclei, where the large spin-orbit splitting, so essential for the shell model, is unexplained. No doubt these two problems are related. To understand spin-orbit splitting from a nonrelativistic framework is hard indeed. Furthermore, to relate the properties of finite nuclei with those of nucleon-nucleon interaction obtained from scattering data might be just too ambitious and not necessary. We know that inside a nucleus, many aspects of the NN force that appear in the scattering problem average out. The same is expected for the ΛN interaction. Thus it could be easier to study the properties of ordinary nuclei as well as Λ hypernuclei by using infinite matter as the starting point, rather than the free two-body interaction[1)]. A relativistic mean field model incorporating such a point of view has achieved a number of important phenomenological successes in the study of finite nuclei[2)]. By taking the bulk properties of infinite nuclear matter as a starting point, the model predicts reasonable matter distributions in finite nuclei, their single particle energy levels, including spin-orbit splitting, the level density parameter, and energy dependence of the real part of nucleon-nucleus optical potential. All these quantities are interrelated in the relativistic mean field model. This model makes two major assumptions. First, we assume that the nucleons move in an effective mean field (shell model potential) generated by themselves. This effective mean field is composed of two distinct parts $U = U_s + U_\omega$. The first part U_s is just the mean value of a scalar meson field σ and transforms as a Lorentz scalar, while the second part U_ω is the mean value of the time component of the Lorentz vector meson ω_μ. The potentials U_s and U_ω are determined self-consistently by relativistic field equations.

That a relativistic model can account for the small spin-orbit splitting in Λ hypernuclei was first shown by Brockmann and Weise[3]. Our approach, though relativistic, is essentially different from the above work. The model that we deal with is a field theory model of nuclear interactions. This means that the motion of the Λ inside a nucleus will be completely determined by the existing fields $\sigma(r)$ and $\omega_0(r)$ inside the nucleus and the coupling, $g_{\Lambda\Lambda\sigma}$, $g_{\Lambda\Lambda\omega}$, to these fields. The fields $\sigma(r)$ and $\omega_0(r)$ are determined in the Hartree approximation. While Brockmann and Weise had to consider a number of complicated intermediate states in the $N\Lambda$ interaction, we describe the Λ interaction through the fields.

The relativistic quantum field theory of nuclear matter as proposed by Walecka[4] and extended by Boguta and Bodmer[5] assumes that the nuclear interactions are mediated by a scalar σ and vector meson fields ω_μ and that these mesonic degrees of freedom are essential ingredients in the study of nuclear matter. In the Boguta and Bodmer model, the mesonic degrees of freedom cannot be eliminated in favor of nucleon degrees of freedom, and in general they are necessary to define a systematic program to compute higher order corrections[6]. In the mean field approximation, $\sigma \rightarrow \langle\sigma\rangle$, $\omega_\mu \rightarrow \delta_{\mu 0}\langle\omega_0\rangle$, these fields can saturate nuclear matter because of relativistic kinematics[7]. Infinite nuclear matter problem reduces to a simple set of algebraic equations in terms of g_s/m_s, g_v/m_v (where g_v, g_s are the coupling constants of the σ and ω_μ to the nucleon, and m_s, m_v are the masses). Since m_v is known and m_σ can be determined by fitting to the surface thickness of a finite nucleus (m_σ = 500 MeV)[8], all parameters of the model are determined by the gross properties of nuclear matter. We apply this method to the study of Λ hypernuclei. On the basis of the quark model we will argue that $g_s/g_{\Lambda\Lambda\sigma} = g_v/g_{\Lambda\Lambda\omega} = x$, which has to be determined. We need one piece of information about Λ hypernuclei or Λ in nuclear matter to determine the model completely. We fit to one known neutron-lambda single particle energy difference[9].

The hypernucleus is assumed to consist of a Λ moving in the mean fields that are created by all the baryons inside the nucleus. The motion of the Λ is given by the Dirac equation

$$[-i\vec{\alpha}\cdot\vec{\nabla} + \beta(m_\Lambda + g_{\Lambda\Lambda\sigma}\sigma)]\psi_\Lambda = (E - g_{\Lambda\Lambda\omega}\omega_o)\psi_\Lambda \quad . \tag{1}$$

A normal nucleus, consisting of Z protons and N neutrons is described by nucleon wave functions $\psi^{(i)}$ in the presence of the sigma field σ, the omega field ω_μ, the rho field R_μ, and the coulomb field A_μ. They satisfy the following equations:

$$(\nabla^2 - m_s^2)\sigma = +g_s \sum_{i=1}^{A} \bar{\psi}^{(i)}\psi^{(i)} \tag{2a}$$

$$(\nabla^2 - m_v^2)\omega_o = -g_v \sum_{i=1}^{A} \psi^{(i)\dagger}\psi^{(i)} \tag{2b}$$

$$(\nabla^2 - m_v^2)R_o^{(0)} = g_r\left[\sum_{\text{neutrons}} \psi^{(i)\dagger}\psi^{(i)} - \sum_{\text{protons}} \psi^{(i)\dagger}\psi^{(i)}\right] \tag{2c}$$

$$\nabla^2 A_o = e \sum_{\text{protons}} \psi^{(i)\dagger}\psi^{(i)} \tag{2d}$$

$$[-i\vec{\alpha}\vec{\nabla} + \beta(m_N + g_s\sigma)]\psi = \left[E - g_v\omega_o + g_r\tau_3 R_0^{(0)} + e\,\frac{(1+\tau_3)}{2}\,A_o\right]\psi \tag{2e}$$

where the isospin structure of ψ is

$$\psi = \begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}. \tag{2f}$$

For spherically symmetric geometry, the Dirac equation can be reduced to spherical coordinates in the usual way.

$$\psi(\vec{x}) = \frac{1}{r}\begin{bmatrix} iG(r) \\ F(r)\hat{\sigma}\cdot\hat{n} \end{bmatrix} \Omega_{\mathcal{J}\ell n}(\hat{n}) \tag{3a}$$

$$\frac{dF}{dr} = [M - E + g_s\sigma + g_v\omega_o]G + \frac{K}{r}F(r) \tag{3b}$$

$$\frac{dG}{dr} = [M + E + g_s\sigma - g_v\omega_o]F - \frac{K}{r}G(r) \tag{3c}$$

$$K = \pm(j + \frac{1}{2}) \text{ for } j = \ell \mp \frac{1}{2} \ . \tag{4}$$

The spherical equations (eq. 3a–3d) apply to the nucleons as well as the Λ particle.

We have solved the coupled system of equations in the Hartree approximation for O^{16} and Ca^{40}. The details of this work will be reported elsewhere[10]. In this work we shall use the field solutions $\sigma(r)$ and $\omega_o(r)$ to compute Λ single particle levels in O^{16} and Ca^{40}.

Saturation of symmetric ($N = Z$) infinite nuclear matter at a Fermi momentum of $k_F = 1.34\ fm^{-1}$, with a binding energy of -15.75 MeV/particle is achieved with the choice of $C_s = (g_s/m_s)m_N = 17.96$ and $C_v = (g_v/m_v)m_N = 15.6$. This corresponds to $g_s = 9.57$ and $g_v = 12.97$ with the masses taken to be $m_s = 500$ MeV and $m_v = 780$ MeV. The corresponding nucleon-nucleus optical potential well depth is -46.7 MeV.

The Hartree single particle levels for O^{16} and Ca^{40} are shown in Table 1. The agreement with the data is good[11]. Our results are comparable with those of Brockmann and Weise but are more reliable because our density distribution of matter in these nuclei is superior[12]. This is because their choice of g_s and g_v, based on scattering data, overestimates the saturation density and binding energy of infinite nuclear matter ($k_F = 1.5\ fm^{-1}$, $e/\rho = -20$ MeV). We insisted on correct saturation right from the start.

To study Λ hypernuclei, we must determine $g_{\Lambda\Lambda\sigma}$ and $g_{\Lambda\Lambda\omega}$. The quark model content of Λ is (u,d,s), while that of a neutron is (u,d,d). A recent quark model assignment for the lowest 0^{++} nonet is $Q^2\bar{Q}^2$ with the ϵ having a mass of about 650 MeV and quark assignment of $u\bar{u}d\bar{d}$. It has been pointed out by J.J. deSwart that this is the meson that corresponds to the phenomenological σ of mass 500 MeV needed in the NN scattering analysis[8]. The physical omega has a $(u\bar{u} + d\bar{d})$ quark assignment. The presence of a strange quark in the Λ implies that Λ will couple more weakly to the σ and ω_μ meson fields than a nucleon would and the reduction in coupling strength will be proportional for both couplings. That is, on the basis of the quark model, we take $g_s/g_{\Lambda\Lambda\sigma} = g_v/g_{\Lambda\Lambda\omega} = x$. The ratio x is to be determined by fitting to one known property of a hypernucleus.

The difference of the neutron and Λ single particle energy levels in O^{16} and Ca^{40} are known reasonably well experimentally[9]. The Hartree neutron single particle levels are tabulated in Table 1. For a given value of x the Λ single particle levels can be computed in the corresponding nucleus. We fit the $(1d^{-1}_{5/2}, 1d_{5/2})$ energy difference in ${}_{\Lambda}Ca^{40}$. This corresponds to the value of $x = 0.33$ and the removal energy of Λ from infinite nuclear matter is then -28 MeV. In Table 2 we show the Λ single particle energy levels in ${}_{\Lambda}O^{16}$ and ${}_{\Lambda}Ca^{40}$.

In Table 3 we show the single particle energy differences in O^{16} and Ca^{40} as predicted by the model, with $x = 0.33$ and that of experiment[9]. The agreement is not only qualitative but also quantitative. A small variation in the single particle neutron energy levels in O^{16} and Ca^{40} can easily explain the disagreement with the data.

The spin-orbit splittings in Λ hypernucleus is of particular interest. They can be read off from Table 2 and compared to the corresponding neutron spin-orbit splittings given in Table 1. We see that for the Λ hypernucleus, the model predicts an order of magnitude reduction in the spin-orbit splitting when compared with the neutron splitting. The reason for this is twofold. In the Thomas limit, the spin-orbit splitting can be written as

$$s.o \sim \frac{1}{M_B^2 r} \frac{d}{dr} (g_s \sigma + g_v \omega_o) \quad . \tag{5}$$

In comparing the spin-orbit splitting of the Λ to the neutron in the same nucleus, only the coupling constant and mass change; the fields are assumed to be the same. Hence from the above equation we expect that the ratio of the spin-orbit splitting of the Λ to the neutron will be

$$\left(\frac{\Lambda}{N}\right)_{s.o} \sim \left(\frac{m_N}{m_\Lambda}\right)^2 \cdot x \tag{6a}$$

$$= 0.23 \tag{6b}$$

for $x = 0.33$. Thus the smallness of the spin-orbit splitting of the Λ can be considered to be the result of the reduced coupling of the Λ to the scalar and vector fields σ, ω_μ.

The unconventional approach to nuclear physics, that is, assuming the importance of relativity and field type of interactions, does a reasonable, phenomenological job in describing a wide variety of data. In this note we showed that it is also applicable in Λ hypernuclei. The phenomenological success of the relativistic mean field model should stimulate a better theoretical understanding of the theory.

Table Captions

Table 1 Single particle neutron levels in O^{16} and Ca^{40} in the Hartree approximation. $G_s = 9.57$ and $G_v = 12.97$.

Table 2 Single particle Λ levels in $_\Lambda O^{16}$ and $_\Lambda Ca^{40}$ for $x = 0.333$.

Table 3 Energy differences of neutron and Λ single particle levels predicted by the model and experimental data[9]. The experimental numbers are known to an accuracy of about 2 MeV, corresponding to the resolution of the spectrometer.

Table 1

	O^{16}(MeV)	Ca^{40}(MeV)
$1S_{1/2}$	39.4	54.6
$1P_{3/2}$	19.7	37.5
$1p_{1/2}$	11.4	32.0
$1d_{5/2}$		20.9
$2s_{1/2}$		14.3
$1d_{3/2}$		12.8

Table 2

	$_{\Lambda}O^{16}$(MeV)	$_{\Lambda}Ca^{40}$(MeV)
$1S_{1/2}$	10.8	17.1
$1P_{3/2}$	2.1	8.7
$1p_{1/2}$	1.5	8.1
$1d_{5/2}$		1.3
$2s_{1/2}$		1.1
$1d_{3/2}$		0.8

Table 3

$_{\Lambda}Ca^{40}$

	Theory (MeV)	Exp(MeV)
$(1d_{5/2}^{-1}, 1d_{5/2})$	19.6	20
$(1d_{3/2}^{-1}, 1d_{3/2})$	12.0	15
$(1d_{5/2}^{-1}, 1p_{3/2})$	12.2	10
$(1d_{3/2}^{-1}, 1p_{1/2})$	4.7	5

$_{\Lambda}O^{16}$

$(p_{3/2}^{-1}, p_{3/2})$	17.6	18
$(p_{1/2}^{-1}, p_{1/2})$	9.9	12
$(p_{3/2}^{-1}, s_{1/2})$	8.9	8
$(p_{1/2}^{-1}, s_{1/2})$	0.6	2

References

1. R. Serber, Phys. Rev. 14C, 718 (1976)
2. J.D. Walecka, Collective Excitations of Nuclei in a Relativistic Mean-Field Theory, Stanford preprint
   F.E. Serr and J.D. Walecka, Phys. Lett. 79B, 10 (1978)
   B.D. Serot and J.D. Walecka, Phys. Lett. 87B, 172 (1980)
   J. Boguta and J. Rafelski, Phys. Lett. 71B, 22 (1977)
   J. Boguta, Properties of T $\neq$ 0 Nuclei in Relativistic Field Theory of Nuclear Matter, LBL preprint
   J. Boguta, Relativistic Quantum Field Theory of Finite Nuclei, LBL preprint
   J. Boguta, Nucleon-Nucleus Optical Potential in a Relativistic Theory of Nuclear Matter, LBL preprint
   L.N. Savushkin, Sov. J. Nucl. Phys. 30, 349 (1979)
3. R. Brockmann and W. Weise, Phys. Lett. 69B, 167 (1977)
4. J.D. Walecka, Ann. Phys. 83, 491 (1979)
5. J. Boguta and A.R. Bodmer, Nucl. Phys. A292, 413 (1977)
6. S.A. Chin, Ann. Phys. 108, 491 (1977)
7. T.D. Lee and G.C. Wick, Phys. Rev. 9D, 2291 (1974)
8. J.J. deSwart, The ΛN-Interaction and Hypernuclei. THEF-NYM-79.16 preprint.
9. W. Brückner et al., Phys. Lett. 79B, 157 (1978)
   K. Kilian, AIP Conf. Proc. 54, 666 (1979)
   R. Bertini et al., AIP Conf. Proc. 54, 719 (1979)
   B. Povh, Nucl. Phys. A335, 233 (1980)
10. J. Boguta, Ground State Properties of O^{16} and Ca^{40} in a Relativistic Hartree Theory, in preparation
11. K. Nakamura, S. Hiramatsu, T. Kamal, H. Muramatsu, N. Izutsu and Y. Wutase, Phys. Rev. Lett. 33, 853 (1974)
12. R. Brockmann and W. Weise, Phys. Rev. 16C, 1282 (1977)