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**PASSIVE COOLING OF
RTG AND STORAGE CONTAINER**

DESIGN AND ANALYSIS

**A. Schock
Fairchild Space Company
Germantown, Md.**

August 27, 1984

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1. INTRODUCTION AND CONCLUSIONS

1.1 Background

RTGs contain components which could suffer significant degradation if exposed to air at elevated temperatures. This is particularly true of the molybdenum multifoil insulation, whose reflectivity diminishes upon oxidation. For this reason, an RTG must have a hermetic housing while exposed to an air environment.

But hermeticity of such a housing is difficult to maintain, because of unavoidable air inleakage through the large number of metallic C-seals present in a typical RTG. Recent experience has shown that such inleakage cannot be prevented simply by maintaining a hyperatmospheric inert gas pressure inside the RTG housing. Even when the dominant leakage is outward, significant inleakage of air can occur in a matter of weeks. This has been confirmed by analyses performed by the author.

Based on this experience, it was concluded that air exposure is permissible for relatively short periods, e.g. for a series of vibration tests or for shipment to the launch site; but that for long-term pre-launch storage, which may extend for two years or more, not only the inside but also the outside of the RTG must be protected by an inert-gas atmosphere.

In designing an inert-gas-filled storage container, it was specified that the equilibrium RTG housing temperature not exceed 180 C. Above that temperature, the mechanical properties of the aluminum alloy reportedly degrade below the level required for launch. Once in space, the RTG housing operates much above 180 C, but at that point -- after launch -- the resultant degradation of mechanical properties is acceptable.

1.2 Design Concepts

The simplest storage arrangement would be to place the RTG inside a helium-filled bell jar, but this would lead to equilibrium temperatures much above 180 C. Analysis indicates that even if the bell jar were water cooled, the RTG temperature would still be excessive.

The next simplest arrangement would be to maintain the desired RTG temperature by utilizing the auxiliary cooling ducts at the root of the eight RTG fins. Those ducts are provided to permit RTG cooling during the launch phase, while the unit is enclosed within the shuttle bay.

But using those ducts for cooling during storage would raise two major concerns. One is the possible corrosive effect of the recirculating coolant on the duct walls after prolonged operation. Unless it can be conclusively demonstrated that such damage will not occur, the resultant risk to a very expensive piece of flight hardware and to the planned Galileo and Solar Polar launch schedules may be unacceptable to the NASA/JPL user agency.

FIG. 1.
PASSIVELY COOLED RTG, WATER-COOLED OUTER SHELL

Even if it can be demonstrated that corrosion will definitely not occur, there is another cause for concern. A recirculating cooling liquid loop is an active device. Any interruption, e.g. due to a power failure, would cause the RTG to overtemperature until the condition is corrected. Since the allowable overtemperature time is rather limited, this would require a closer degree of monitoring and operator attendance than would be the case with a passive cooling system.

To keep the RTG housing below 180 C with only passive cooling, the author had proposed the use of a chimney extending above the RTG for enhanced natural-convection cooling gas circulation and improved heat transfer. An initial version of this concept is illustrated in Figure 1. As shown there, the chimney acts as a baffle between the upcoming and downgoing gas streams. Since the gas is heated at the bottom of the upcomer and cooled at the top of the downcomer, the average temperatures and average gas densities in the two ducts are quite different, and it is that difference in densities which provides the driving force for the circulatory flow pattern depicted in the figure.

To determine whether the desired RTG temperature could be achieved with a chimney of reasonable dimensions, a rather crude preliminary analysis was carried out. The results of that analysis, shown in Figure 2, indicated that for the assumed gas temperatures, a chimney extension of three feet would be sufficient to produce the required driving force for the helium flow rate and velocity shown.

FIG. 2.

RESULTS OF PRELIMINARY ANALYSIS

The design depicted in Figure 2 still depends on an active water loop for cooling the outer shell. Water-cooling the outer shell is less of a concern than water-cooling the RTG itself, because there is much less danger of serious corrosive damage, and because -- if there were an interruption of the recirculating water -- the system could automatically switch to non-recirculating city water until the malfunction is corrected. Nevertheless, it would still be much preferable to have a completely passive cooling system, both inside and outside the outer shell. Such a system, suggested by the author, is shown in Figure 3.

As can be seen, this design is identical to that shown earlier in Figure 2, except for the external fins near the top of the outer shell. These replace the previous water cooling loop. It should be noted that the gas temperatures shown were arbitrary assumptions. No analysis had yet been done to determine whether those temperatures could in fact be achieved with reasonable numbers and sizes of inner and outer fins.

When that analysis was carried out, it was found that the assumed heat transfer was easily achievable at the inner fins (helium), but not at the outer fins (air). It was found that with 36 outer fins, each fin would have to be 6 square feet in size (i.e., much larger than shown in the figure). Such large fins would have to be impractically thick to avoid unattractively low fin efficiencies. It was therefore concluded that the basic concept of concentrating the heat removal near the top of the outer case was not practical. To

FIG. 3.

PASSIVELY COOLED RTG AND OUTER SHELL

achieve reasonable outer-fin dimension, it would be necessary to have the inner and outer fins extend over the full length of the outer shell, as illustrated in the revised design shown in Figure 4.

FIG. 4.

REVISED PASSIVE DESIGN CONCEPT

In the design shown earlier in Figure 3, most of the helium heating takes place near the bottom of the inner duct, and most of the helium cooling occurs near the top of the outer duct. This is the ideal situation for maximizing the density difference which drives the helium circulation. But this is not the situation in the design shown in Figure 4. Nevertheless, the natural circulation in that design -- while less favorable -- can still be adequate, for reasons illustrated in Figure 5.

FIG. 5.

If the temperature profiles in the inner and outer ducts were linear, as illustrated by the dashed line in Figure 5, then the inner and outer gas would have the same average density, and therefore there would be no driving force for the gas circulation. But, in fact, the

respective temperature profiles will not be linear. As shown by the solid curves in Figure 5, the inner-duct gas asymptotically approaches the RTG temperature, and the helium in the outer duct asymptotically approaches the outer-shell temperature. Because of these non-linearities, average gas temperatures and densities in the two ducts are seen to differ significantly, and this difference can provide sufficient driving force to produce the desired gas circulation. However, since we no longer have the ideal situation of heating at the bottom and cooling at the top, a much more accurate temperature profile determination is required to assess the Practicality of the concept.

The required analysis is described in Section 2, and the analytical results are presented in Section 3.

1.3 Conclusions

Based on the results presented in Section 3, it is concluded that:

- o The basic concept for a high-reliability design, in which both the RTG and its helium-filled storage container are passively cooled by natural-convection gas circulation, is feasible.
- o The recommended reference design uses a 5 ft-long chimney extension and 8.6"-long outer fins, resulting in a storage container with an overall height of about 12 feet and a tip-to-tip span of about 4 feet. If desired, the container's outer shell can be built as two 4 ft-long sections, to facilitate fabrication and assembly.
- o If the friction drops and heat transfer coefficients conform to theoretical predictions, the RTG temperature limit can be satisfied with 25 psia in the RTG and 22 psia in the storage container.
- o To accommodate significantly higher-than-theoretical friction drops and lower-than-theoretical heat transfer coefficients, the helium pressure in the storage container must be substantially higher.
- o A design pressure of 44 psia is recommended. With that pressure, the selected reference design will -- for example -- accommodate friction drops up to 60% above theoretical together with heat transfer coefficients down to 28% below theoretical.
- o The higher storage container pressure will require a correspondingly higher RTG pressure (e.g., 47 psia). These higher pressures can be initiated in a two-step process, without ever producing pressure reversals or higher-than-usual pressure differentials between the RTG and its environment.
- o Even after the reference design is built and tested, if the friction losses and heat transfer should be found to be even poorer than the values cited above they could still be accommodated by going to a higher helium pressure (e.g., to 66 psia).
- o The analysis was based on laminar flow, but the helium flow past the RTG may actually be turbulent. If so, this would lower the RTG temperature and/or lower the required helium pressure.

2. ANALYSIS

2.1 Approach

The principal goal of the analysis is to select the four design parameters depicted in Figure 6. These must be chosen to satisfy all thermal and fluid flow constraints, and to yield the desired RTG temperature with reasonable design margin to compensate for analytical approximations.

An exact analysis from first principles of the design depicted in Figure 4 is possible, but would exceed the available time and budget. To make the problem tractable, a number of simplifying approximations will be employed. The more significant ones will be identified where they occur. To compensate for the uncertainties introduced by those approximations, considerable margin will be built into the design. As will be seen, with the selected design the heat transfer coefficients and friction losses can be significantly poorer than the predicted values without exceeding the RTG temperature limit of 180 C.

Even with these simplifying approximations, the analysis of the design shown in Figure 4 is somewhat complex, because all of the variables (including temperatures, heat transfer coefficients, fluid flow, rates and velocities, friction drops, chimney dimensions, buoyant forces, fin numbers and heights) are mutually interrelated. Therefore, there is no deductive sequence for solving for one variable at a time. Instead, it is necessary to derive and then solve a number of simultaneous algebraic and integral equations. Since those

FIG. 6.

CRITICAL DESIGN PARAMETERS

equations are generally non-linear and non-integrable, numerical integration and multi-level trial-and-error procedures must be employed to find consistent solutions satisfying the various thermal and fluid flow constraints.

Natural-convection heat transfer correlations in the literature are generally for a hot object in what is in effect an infinitely large cool environment. This is not representative of the proposed design, and those equations are therefore inapplicable to the present problem.

Figure 7, which is excerpted from Reference [1], illustrates four types of convection flow: Figures 7(a) and (b) depict unbounded flow, while (c) and (d) depict bounded flow. Figures (a) and (c) depict natural convection, while (b) and (d) depict forced convection. Clearly, Figure 7(c) is representative of our present problem. The fluid in the heated and cooled sections is driven by a pressure gradient dp/dz . The fluid does not know or care whether that pressure gradient is produced by a difference in gravity heads, as in 7(c), or by an actual pump, as in 7(d). Therefore, the fluid flow and heat transfer problems in the inner and outer ducts can be treated as forced convection problems. This is valid as long as the difference in gravity heads of the two ducts balances the friction losses around the loop.

FIG. 7. Types of convection flow. (a) Unbounded natural convection. (b) Unbounded forced convection. (c) Bounded natural convection. (d) Bounded forced convection.

The symbols used in the subsequent analysis are defined where they first appear. The principal symbols are depicted in Figure 8.

FIG. 8
DESIGN PARAMETER SYMBOLS USED IN ANALYSIS

2.2 Duct Parameters

The inner duct has a length L_i equal to that of the RTG fins. Its cross-sectional area is bounded by the RTG radius R_R and the chimney radius R_c . It contains eight equispaced radial RTG fins, which extend outward almost to the chimney. Therefore, the duct's area A_i is given by

$$A_i = \pi (R_c^2 - R_R^2). \quad (1)$$

The inner duct's wetted perimeter W_i is given by

$$W_i = 2\pi(R_R + R_c) + 16(R_c - R_R)L, \quad (2)$$

its hydraulic diameter D_i , defined as four times the area divided by the wetted perimeter, is

$$D_i = 4A_i / W_i, \quad (3)$$

and its effective heated perimeter H_i is given by

$$H_i = 2\pi R_R + 16\eta_R(R_c - R_R) \quad (4)$$

where η_R is the efficiency of the RTG fins.

The outer duct has a length L_o equal to the sum of the inner duct length L_i and extension length L_x ,

$$L_o = L_i + L_x. \quad (5)$$

Its cross-sectional area is bounded by the chimney radius R_c and the outer shell radius R_s . It contains n_i inner fins, extending radially inward from the outer wall almost to the chimney. Its

cross-sectional area is

$$A_o = \pi (R_s^2 - R_c^2), \quad (6)$$

its wetted perimeter W_o is

$$W_o = 2\pi (R_c + R_s) + 2n_i (R_s - R_c), \quad (7)$$

its hydraulic diameter D_o is

$$D_o = 4A_o / W_o, \quad (8)$$

and its effective heat transfer perimeter H_o is

$$H_o = 2\pi R_s + 2n_i \eta_i (R_s - R_c), \quad (9)$$

where η_i is the efficiency of the inner fins, which corrects for the temperature drop-off from fin root to fin tip.

The efficiency η of a convectively cooled rectangular fin of thickness δ and thermal conductivity k_F is given by

$$\eta = \tanh \left[\left(\frac{2\bar{h}}{\delta k_F} \right)^{\frac{1}{2}} L \right] / \left[\left(\frac{2\bar{h}}{\delta k_F} \right)^{\frac{1}{2}} L \right], \quad (10)$$

where $\bar{h}$ is the average value of the heat transfer coefficient.

Therefore, the respective efficiencies of the RTG fins and inner fins are

$$\eta_R = \tanh \left[\left(\frac{2\bar{h}_i}{\delta_R k_R} \right)^{\frac{1}{2}} (R_c - R_R) \right] / \left[\left(\frac{2\bar{h}_i}{\delta_R k_R} \right)^{\frac{1}{2}} (R_c - R_R) \right], \quad (11)$$

$$\eta_i = \tanh \left[\left(\frac{2\bar{h}_o}{\delta_i k_i} \right)^{\frac{1}{2}} (R_s - R_c) \right] / \left[\left(\frac{2\bar{h}_o}{\delta_i k_i} \right)^{\frac{1}{2}} (R_s - R_c) \right], \quad (12)$$

where $\bar{h}_i$ and $\bar{h}_o$ are the average heat transfer coefficients in the inner and outer ducts.

2.3 Helium Heating and Cooling

Any inert gas could be used in the storage container. A high molecular weight is advantageous for natural convection, but it was found that this advantage is outweighed by helium's higher thermal conductivity, which leads to higher heat transfer coefficients. Therefore, helium is employed in the present design.

The axial position z in each duct is measured from its inlet end. Helium, with a mass flow rate $\dot{m}$, specific heat C_p , viscosity μ , and thermal conductivity k , enters the inner duct at temperature T_1 , flows upward, and exits at temperature T_2 . Conversely, helium enters the outer duct at temperature T_2 , flows downward, and exits at temperature T_1 . The axial temperature variation of helium in the inner duct is obtained from the differential equation

$$\dot{m} C_p (dT/dz) = H_i h_i (T_R - T), \quad (13)$$

where T is the mixed mean gas temperature at axial position z , T_R is the RTG wall temperature (assumed to be uniform), and h_i is the local heat transfer coefficient in the inner duct. Similarly, the axial temperature variation of helium in the outer duct is obtained from the differential equation

$$\dot{m} C_p (dT/dz) = - H_o h_o (T - T_s), \quad (14)$$

where T_s is the outer shell temperature (assumed to be uniform), and h_o is the local heat transfer coefficient in the outer duct.

Applying the inlet boundary condition for each duct to Eqs. (13) and (14), we obtain

$$\int_{T_1}^T \frac{dT}{T_R - T} = \frac{H_i}{\dot{m} C_p} \int_0^z h_i dz, \quad (15)$$

and

$$\int_{T_2}^T \frac{dT}{T - T_s} = - \frac{H_o}{\dot{m} C_p} \int_0^z h_o dz. \quad (16)$$

The local heat transfer coefficient h can be expressed in terms of the local Nusselt number Nu by

$$h = (k/D) Nu, \quad (17)$$

where D is the duct's hydraulic diameter and k is the fluid's thermal conductivity. From Eq. (17), it follows that

$$\int_0^z h dz = (k/D) \int_0^z Nu dz = (kz/D) \bar{Nu}, \quad (18)$$

where $\bar{Nu}$ is the average value of the Nusselt number from inlet to position z .

The Nusselt number Nu is a function of the local flow conditions. Fluid entering a duct has a uniform velocity profile and temperature profile. After a transition region, the flow changes to "fully developed" velocity and temperature profiles, as illustrated in Figure 9, excerpted from Reference [2]. Figure 9(a) shows the transition from slug flow, to developing flow, to fully developed flow, with accompanying development of the velocity boundary layer δ_v . Similarly, Figure 9(b) shows the transition from uniform

temperature, to developing temperature profile, to fully developed temperature profile, with accompanying development of the thermal boundary layer δ_T .

FIG. 9.

The development of velocity and temperature profiles in the entrance region between parallel plates.

transition from slug flow, to developing flow, to fully developed flow, with accompanying development of the velocity boundary layer δ_v . Similarly, Figure 9(b) shows the transition from uniform temperature, to developing temperature profile, to fully developed temperature profile, with accompanying development of the thermal boundary layer δ_T .

In many practical problems, with ducts having large length-to-diameter ratios, this transition zone is negligible in comparison to the duct length, and entrance effects can be ignored. In the present problem the ducts -- particularly the inner duct -- have very small L/D ratios, and entrance effects cannot be neglected.

Preliminary analysis indicated that the current problem is in the laminar flow regime. The problem of analyzing simultaneously developing velocity and temperature profiles has been solved for laminar flow in circular ducts and in ducts with parallel walls. But we were unable to find equivalent solutions for annular ducts partitioned by radial fins, as in the present case. Therefore, the solution for circular ducts will be used. These are expected to be good approximations for the present case, as long as the equivalent hydraulic diameter is used.

Table 1, reproduced from Reference [3] (except for the last two columns), shows the theoretical variation of Nusselt number as a function of x^+ , which is a dimensionless distance from the inlet, defined by

$$x^+ = \frac{z}{\frac{1}{2} D Re Pr} \quad (19)$$

Here Re is the Reynolds number,

$$Re = \frac{D V \rho}{\mu} = \frac{D \dot{m}}{\mu A} \quad (20)$$

and Pr is the fluid's Prandtl number,

$$Pr = C_p \mu / k. \quad (21)$$

Therefore, the dimensionless distance x^+ can be expressed as

$$x^+ = z / Z, \quad (22)$$

where Z for the inner duct is given by

$$Z_i = \frac{\dot{m} C_p D_i^2}{2 k A_i}, \quad (23)$$

and Z for the outer duct is given by

$$Z_o = \frac{\dot{m} C_p D_o^2}{2 k A_o}. \quad (24)$$

TABLE 1

*Nusselt numbers for the circular tube; constant surface temperature;
combined thermal and hydrodynamic entry length*

(Pr = 0.7)

x^+	Nu_x	Nu_m	$\overline{Nu}^*$	% Deviation
0.005	11.3	17.4	17.4	0.0
0.010	7.90	13.4	13.2	1.5
0.020	5.82	10.0	10.0	0.0
0.040	4.57	7.53	7.63	1.3
0.10	3.77	5.43	5.46	0.5
0.20	3.67	4.56	4.47	2.0
	3.66	3.66	3.66	0.0

*Equation (25) curve fit

Table 1 presents values of both the local Nusselt number Nu_x and the mean Nusselt number Nu_m (averaged from the inlet to position x).

E. Vogel curve-fitted the latter values with the equation

$$\overline{Nu} = 3.66 \left[1 + 0.080 (x/Z)^{-1.2} \right]^{2/9}, \quad (25)$$

where 3.66 is the Nusselt number for fully developed flow in a circular duct with constant-temperature walls, and the square bracket is the entrance effect correction factor. As shown by the last two columns of Table 1, this equation provides an excellent fit to the data from Reference [1].

Inserting Eq. (25) into (18), we obtain

$$\int_0^z h_i dz = 3.66 M_H \frac{kz}{D_i} \left[1 + 0.080 \left(\frac{z}{Z_i} \right)^{-1.8} \right]^{2/9} \quad (26)$$

and

$$\int_0^z h_o dz = 3.66 M_H \frac{kz}{D_o} \left[1 + 0.080 \left(\frac{z}{Z_o} \right)^{-1.8} \right]^{2/9}, \quad (27)$$

where M_H is a heat transfer multiplier, added for conservatism. It is defined as the ratio of the assumed value of the helium heat transfer coefficient to its theoretical value, and is inserted here for use in subsequent sensitivity studies, to assess the effect of less-than-theoretical heat transfer coefficients, and to allow us to provide design margin to compensate for analytical approximations.

Integrating Eqs. (26) and (27) over the length of the inner and outer ducts, we obtain expressions for the average heat transfer coefficients in those ducts:

$$\bar{h}_i = 3.66 M_H \frac{k}{D_i} \left[1 + 0.080 \left(\frac{L_i}{Z_i} \right)^{-1.8} \right]^{2/9}, \quad (28)$$

$$\bar{h}_o = 3.66 M_H \frac{k}{D_o} \left[1 + 0.080 \left(\frac{L_o}{Z_o} \right)^{-1.8} \right]^{2/9}. \quad (29)$$

These expressions are needed to compute the RTG fin efficiency η_R and the inner-fin efficiency η_i from Eqs. (11) and (12).

Integrating Eqs. (15) and (16), and inserting Eqs. (26) and (27), we obtain expressions for the helium temperature profile in the inner

duct,

$$T = T_R - (T_R - T_1) \exp \left\{ -7.32 M_H \frac{H_i}{W_i} \left[\left(\frac{z}{Z_i} \right)^{4.5} + 0.080 \left(\frac{z}{Z_i} \right)^{2.7} \right]^{2/9} \right\}, \quad (30)$$

and in the outer duct,

$$T = T_S + (T_2 - T_S) \exp \left\{ -7.32 M_H \frac{H_o}{W_o} \left[\left(\frac{z}{Z_o} \right)^{4.5} + 0.080 \left(\frac{z}{Z_o} \right)^{2.7} \right]^{2/9} \right\}. \quad (31)$$

These two expressions will be used in finding solutions satisfying the energy balance and the pressure balance.

2.4 Energy Balance

The RTG has a thermal power Q of 4500 watts. Assuming that it will be stored in a shorted condition, all of this energy is delivered to the outer shell, which in turn rejects it to the ambient air. Heat transport from the RTG to the outer shell is primarily by convection and secondarily by radiation:

$$Q = Q_{\text{conv}} + Q_{\text{rad}}. \quad (32)$$

The heat transported convectively raises the helium temperature in the inner duct from T_1 to T_2 , so that

$$Q_{\text{conv}} = \dot{m} C_p (T_2 - T_1). \quad (33)$$

The radiative heat transport consists of two sequential steps: radiation from the RTG to the chimney, given by

$$Q_{\text{rad}} = \frac{2\pi R_c L_i \sigma \epsilon (T_R^4 - T_c^4)}{2 - \epsilon}, \quad (34)$$

and radiation from the chimney to the outer shell, given by

$$Q_{rad} = \frac{2\pi R_c R_s L_i \sigma \epsilon (T_c^4 - T_s^4)}{R_s + (1-\epsilon) R_c} \quad (35)$$

where ϵ is the surface emissivity. Note that the chimney extension was assumed to be adiabatic, and that the "string method" was used to determine the RTG's effective radiating area. Combining the two equations to eliminate T_c , we obtain

$$Q_{rad} = \frac{2\pi R_c R_s L_i \sigma \epsilon (T_R^4 - T_s^4)}{(3-\epsilon) R_s + (1-\epsilon) R_c} \quad (36)$$

Inserting Eqs. (33) and (36) into (32), we obtain

$$Q = \dot{m} C_p (T_2 - T_1) + \frac{2\pi R_c R_s L_i \sigma \epsilon (T_R^4 - T_s^4)}{(3-\epsilon) R_s + (1-\epsilon) R_c} \quad (37)$$

The outlet temperature T_2 of the inner duct must satisfy Eq. (30) for $z = L_i$, so that

$$T_2 = T_R - (T_R - T_1) E_i, \quad (38)$$

where

$$E_i = \exp \left\{ -7.32 M_H \frac{H_i}{W_i} \left[\left(\frac{L_i}{Z_i} \right)^{4.5} + 0.080 \left(\frac{L_i}{Z_i} \right)^{2.7} \right]^{2/9} \right\}. \quad (39)$$

Similarly, the outlet temperature T_1 of the inner duct must satisfy Eq. (31) for $z = L_o$, so that

$$T_1 = T_s + (T_2 - T_s) E_o, \quad (40)$$

where

$$E_o = \exp \left\{ -7.32 M_H \frac{H_o}{W_o} \left[\left(\frac{L_o}{Z_o} \right)^{4.5} + 0.080 \left(\frac{L_o}{Z_o} \right)^{2.7} \right]^{2/9} \right\}. \quad (41)$$

For any set of design parameters and an assumed value of $\dot{m}$, Equations (37), (38), and (40) form a set of three simultaneous equations for the unknown temperatures T_1 , T_2 , and T_s . By combining these, T_1 and T_2 can be eliminated to yield a single equation for the unknown shell temperature T_s :

$$T_s = T_R - \left(\frac{(1-E_i)^{-1} + (E_o^{-1} - 1)^{-1}}{\dot{m} C_p} \right) \left[Q - \frac{2\pi R_c R_s L_i \sigma \epsilon (T_R^4 - T_s^4)}{(3-\epsilon) R_s + (1-\epsilon) R_c} \right] \quad (42)$$

Although this is a fourth-degree equation in T_s , it can be solved quite rapidly by inserting the last value of T_s into the square bracket. Convergence is rapid because the T_s^4 term is only a very small fraction of the square bracket. After Eq. (42) is solved for T_s , Equations (38) and (40) can be solved for T_1 and T_2 :

$$T_1 = [E_o(1-E_i)T_R + (1-E_o)T_s] / (1-E_i E_o), \quad (43)$$

$$T_2 = [(1-E_i)T_R + E_i(1-E_o)T_s] / (1-E_i E_o). \quad (44)$$

Once T_s , T_1 , and T_2 have been computed, they can then be used in the pressure balance, to determine whether the outer duct length L_o is large enough to drive the assumed mass flow rate $\dot{m}$. If not, the whole procedure is repeated with different values of $\dot{m}$ until a consistent solution is found.

2.5 Pressure Balance

The pressure balance is satisfied when the sum of the friction drops in the helium loop matches the difference between the gravity heads in the inner and outer ducts.

The friction drop Δp for a fluid of density ρ flowing at velocity V through a duct of hydraulic diameter D is obtained from

$$\frac{dp}{dz} = -4f \frac{\rho V^2}{2D}, \quad (45)$$

where f is the friction factor. The velocity is related to the mass flow rate by

$$\dot{m} = \rho A V, \quad (46)$$

and the density of a perfect gas is

$$\rho = p / RT \quad (47)$$

where p is the pressure and R is the gas constant. Therefore, Eq. (42) can be rewritten as

$$\frac{dp}{dz} = -2f \frac{\dot{m}^2 RT}{D p A^2}. \quad (48)$$

For fully developed laminar flow in a circular duct, the friction factor is given by

$$f = 16 / Re. \quad (49)$$

For the present problem, we must apply shape-factor corrections for the non-circular duct cross-sections, and a correction for developing flow (i.e., the entrance effect).

Laminar flow through internally-finned annular ducts has been analyzed, with the results shown in Figure 10 and Table 2 (excerpted from Reference [4]). For both the inner and outer ducts, the ratio of inner to outer radius is about 0.5. In the inner duct there are 8

FIG. 10.

Friction factor results for fully developed laminar flow in internally finned annular ducts.

fins, so that $\theta_0 = 45^\circ$, and fRe (from Table 2) is 14.4. Therefore, a shape correction factor of $14.4/16 = 0.90$ will be used in the inner duct friction loss. In the outer duct, the number n_i of inner fins is typically 24, so that $\theta_0 \approx 15^\circ$, and $fRe \approx 16$. Therefore, no shape factor correction was used for the outer duct.

TABLE 2

Friction Factor Reynolds Number Product for Laminar Flow through Internally Finned Annular Ducts

$f \cdot Re$	$\theta_0 \rightarrow$	5°	10°	15°	20°	30°	40°	50°	60°
$r_i/r_o =$	0.05	13.42	13.55	13.68	13.81	14.03	14.22	14.38	14.52
	0.1	14.47	14.49	14.51	14.54	14.59	14.62	14.66	14.69
	0.2	16.34	16.04	15.79	15.57	15.21	14.94	14.75	14.65
	0.3	17.80	17.07	16.47	15.98	15.24	14.78	14.54	14.47
	0.4	18.75	17.51	16.56	15.84	14.90	14.48	14.38	14.49
	0.5	19.14	17.37	16.13	15.30	14.47	14.33	14.52	14.88
	0.6	18.94	16.68	15.35	14.64	14.29	14.60	15.14	15.72
	0.65	18.60	16.16	14.91	14.38	14.40	14.97	15.65	16.32
	0.7	18.08	15.56	14.52	14.26	14.71	15.52	16.33	17.04
	0.8	16.42	14.42	14.33	14.84	16.15	17.28	18.16	18.86
	0.85	15.32	14.26	14.91	15.82	17.39	18.52	19.34	19.95
	0.9	14.35	14.98	16.36	17.51	19.07	20.05	20.71	21.18
	0.95	15.06	17.62	19.17	20.14	21.26	21.88	22.28	22.56

The entrance-effect correction for developing flow has been analyzed, with the results for laminar flow shown in Figure 11 (excerpted from Reference [5]). The figure presents three curves. The lower curve shows the local friction factor as a function of position; the middle curve shows the mean friction factor averaged over the duct length; and the upper curve shows the mean friction factor plus a correction for the increase in the total fluid momentum flux associated with the development of the velocity profile. The present analysis uses the upper curve, for which Vogel obtained an excellent fit with the equation

$$\bar{f} Re = 16 \left[1 + 0.0343 (Re D/L)^{1.28} \right]^{1/3}, \quad (50)$$

as demonstrated by the circled points in Figure 11.

FIG. 11.

Friction coefficients in the entrance of a circular tube with laminar flow.

Combining Eqs. (49) and (50) with the shape-factor correction, and invoking Eqs. (19), (20), (22), and (23), we obtain the following expressions for the mean friction factors in the inner and outer ducts:

$$\bar{f}_i = 14.4 \left(\frac{\mu A_i M_F}{\dot{m} D_i} \right) \left[1 + 0.0834 \left(\frac{z_i}{Pr L_i} \right)^{1.28} \right]^{1/3}, \quad (51)$$

$$\bar{f}_o = 16 \left(\frac{\mu A_o M_F}{\dot{m} D_o} \right) \left[1 + 0.0834 \left(\frac{z_o}{Pr L_o} \right)^{1.28} \right]^{1/3}, \quad (52)$$

where M_F is a friction factor multiplier, analogous to the heat transfer multiplier M_H introduced earlier in Eqs. (26) and (27). It is defined as the ratio of the assumed value of the friction factor to its theoretical value, and is inserted for use in subsequent sensitivity studies. It will enable us to assess the effect of greater-than-theoretical friction losses, and to provide design margin to compensate for analytical approximations..

Inserting Eqs. (51) and (52) into (48), we obtain expressions for the friction loss in the inner and outer ducts:

$$\Delta p_i = 28.8 \left(\frac{\mu \dot{m} R \bar{T}_i L_i}{A_i D_i^2 p} \right) \left[1 + 0.0834 \left(\frac{z_i}{Pr L_i} \right)^{1.28} \right]^{1/3}, \quad (53)$$

$$\Delta p_o = 32 \left(\frac{\mu \dot{m} R \bar{T}_o L_o}{A_o D_o^2 p} \right) \left[1 + 0.0834 \left(\frac{z_o}{Pr L_o} \right)^{1.28} \right]^{1/3}, \quad (54)$$

where $\bar{T}_i$ and $\bar{T}_o$ are the average temperatures in the inner and outer ducts, obtained from Eqs. (27) and (28) by numerical integration:

$$\bar{T}_i = \int_0^{L_i} \left[T_F - (T_F - T_1) \exp \left\{ -7.32 M_H \frac{H_i}{W_i} \left[\left(\frac{z}{z_i} \right)^{4.5} + 0.080 \left(\frac{z}{z_i} \right)^{2.7} \right]^{2/9} \right\} \right] \frac{dz}{L_i}, \quad (55)$$

$$\bar{T}_o = \int_0^{L_o} \left[T_3 + (T_2 - T_3) \exp \left\{ -7.32 M_H \frac{H_o}{W_o} \left[\left(\frac{z}{z_o} \right)^{4.5} + 0.080 \left(\frac{z}{z_o} \right)^{2.7} \right]^{2/9} \right\} \right] \frac{dz}{L_o}. \quad (56)$$

In addition to the two friction losses derived above, there is also a small loss in the chimney extension, given by

$$\Delta p_x = \frac{8 \mu \dot{m} R T_2 L_x}{\pi R_c^4 p} \quad (57)$$

Since the hydraulic diameter of the chimney extension is much larger than those of the inner and outer loops, it makes only a small contribution to the total friction loss around the loop, which is given by

$$\Delta p_{TOT} = \Delta p_i + \Delta p_x + \Delta p_o \quad (58)$$

The total friction drop Δp_{TOT} must be balanced by the difference in gravity heads between the inner duct plus chimney extension and the outer duct. The inner duct gravity head G_i is given by

$$G_i = g \int_0^{L_i} p dz = g p \overline{T_i^{-1}} L_i / R, \quad (59)$$

where g is the gravitational acceleration at the location of the storage container, and $\overline{T_i^{-1}}$ is the inner duct's mean reciprocal temperature, obtained by numerical integration of Eq. (30):

$$\overline{T_i^{-1}} = \int_0^{L_i} \left[T_R - (T_R - T_i) \exp \left\{ -7.32 M_H \frac{H_i}{W_i} \left[\left(\frac{z}{Z_i} \right)^{4.5} + 0.080 \left(\frac{z}{Z_i} \right)^{2.7} \right]^{2/9} \right\} \right]^{-1} \frac{dz}{L_i} \quad (60)$$

Similarly, the outer-duct gravity head is given by

$$G_o = g \int_0^{L_o} p dz = g p \overline{T_o^{-1}} L_o / R,$$

with the mean reciprocal temperature $\overline{T_o^{-1}}$ obtained by numerical integration of Eq. (31):

$$\overline{T_o^{-1}} = \int_0^{L_o} \left[T_S + (T_2 - T_S) \exp \left\{ -7.32 M_H \frac{H_o}{W_o} \left[\left(\frac{z}{Z_o} \right)^{4.5} + 0.080 \left(\frac{z}{Z_o} \right)^{2.7} \right]^{2/9} \right\} \right]^{-1} \frac{dz}{L_o} \quad (62)$$

Finally, the gravity head of the chimney extension is given by

$$G_x = g \rho L_x / R T_2 , \quad (63)$$

and the net gravity head G_{net} in the loop is given by

$$G_{net} = G_o - G_i - G_x . \quad (64)$$

For a consistent solution, this must balance to total friction drop in the loop:

$$G_{net} = \Delta P_{TOT} . \quad (65)$$

If Eq. (65) is not satisfied, the whole procedure is repeated for different trial values of the mass flow rate $\dot{m}$, until a satisfactory solution is found for the desired RTG temperature T_R .

2.6 Outer Fin Dimensions

Once the value of $\dot{m}$ which satisfies Eq. (65) is found, the shell temperature T_s which satisfies the energy balance and pressure balance is automatically determined. The only remaining problem is to determine what outer fins are required to produce that shell temperature; i.e., to define the relationship between the number of outer fins (n_o) and the required fin height L_f (i.e., their tip-to-root dimension).

For a shorted RTG, all of its thermal power must be rejected to the surrounding air by the finned outer shell. This heat rejection consists of two parts: radiation and natural convection.

$$Q = Q_{\text{rad}}^A + Q_{\text{conv}}^A \quad (66)$$

The radiation term is obtained by using the "string method" for the effective radiating area, so that

$$Q_{\text{rad}}^A = 2\pi(R_s + L_F \eta_o) L_o \sigma \epsilon (T_s^4 - T_A^4), \quad (67)$$

where T_A is the ambient air temperature and η_o is the fin efficiency of the outer fins, which -- for rectangular fins of thickness δ_o and thermal conductivity k_F -- is given by:

$$\eta_o = \tanh \left[\left(\frac{2\bar{h}_A}{\delta_o k_F} \right)^{\frac{1}{2}} L_F \right] / \left[\left(\frac{2\bar{h}_A}{\delta_o k_F} \right)^{\frac{1}{2}} L_F \right] \quad (68)$$

The air convection term in Eq. (60) is given by

$$Q_{\text{conv}}^A = (2\pi R_s + 2n_o \eta_o L_F) L_o \bar{h}_A (T_s - T_A). \quad (69)$$

In Eqs. (65) and (66), $\bar{h}_A$ is the average value of the natural-convection heat transfer coefficient to the air. This is a function of the Prandtl number Pr_A and the Grasshof number Gr , which is defined by

$$Gr = g \rho_A^2 L^3 \beta \Delta T / \mu_A^2 \quad (70)$$

where μ_A is the viscosity of the air, ρ_A is its density, and β is its expansion coefficient, which -- for a perfect gas -- is the reciprocal of the absolute temperature. Therefore, for our problem,

$$Gr = g \rho_A^2 L_o^3 (T_s - T_A) / T_A \mu_A^2. \quad (71)$$

For the range of $10^9 < (Pr Gr) < 10^{12}$, which is our range of interest, Reference [6] recommends the equation

$$\bar{h}_A L_o / k_F = 0.13 (Pr Gr)^{1/3}, \quad (72)$$

so that for our problem, the outer-fin heat transfer coefficient is given by

$$\bar{h}_A = 0.13 \left[g \left(\frac{C_p k_A^2 P_A^2}{\mu_A} \right) \left(\frac{T_s - T_A}{T_A} \right) \right]^{1/3}. \quad (73)$$

Inserting Eqs. (67) and (69) into (66), we obtain an expression for the outer-fin height L_F :

$$L_F = \frac{\dot{Q} - 2\pi L_o \sigma \epsilon (T_s^4 - T_A^4) R_s - 2\pi R_s L_o \bar{h}_A (T_s - T_A)}{2\pi L_o \sigma \epsilon (T_s^4 - T_A^4) \eta_o + 2L_o \bar{h}_A (T_s - T_A) n_o \eta_o}. \quad (74)$$

Thus, given the desired RTG design temperature T_R , the chimney extension length L_x , and the number of inner fins n_i , the analysis described in previous sections yields the outer shell temperature T_s required to satisfy the various heat transfer, energy balance, and pressure balance constraints inside the storage container. The temperature T_s is then inserted into Eq. (74) to compute the required outer-fin length L_F for various values of n_o , the number of outer fins.

2.7 Computational Sequence

The steps of the computational sequence, in terms of the previously derived equations, are recapitulated below:

1. Input Constants: $C_p, \mu, k, \sigma, \epsilon, R, Pr, k_R, k_F, Q, R_R, R_e, R_s, L_i, \delta_R, \delta_i, \delta_o$
2. Input Variables: $p, M_F, M_H, n_i, T_R, L_x$
3. Compute L_o from (5)
4. Compute A_i and A_o from (1) and (6)
5. Compute W_i and W_o from (2) and (7)
6. Compute D_i and D_o from (3) and (8)
7. Compute H_i and H_o from (4) and (9)
8. Select trial value of $\dot{m}$
9. Compute Z_i and Z_o from (23) and (24)
10. Compute $\bar{h}_i$ and $\bar{h}_o$ from (28) and (29)
11. Compute η_R and η_i from (11) and (12)
12. Compute E_i and E_o from (39) and (41)
13. Solve (42) for T_s by trial and error
14. Compute T_1 from (43)
15. Compute T_2 from (44)
16. Compute $\bar{T}_i$ and $\bar{T}_o$ by numerical integration of (55) and (56)
17. Compute Δp_i and Δp_o from (53) and (54)
18. Compute Δp_x from (57)
19. Compute $\Delta p_{T_o T}$ from (58)
20. Compute $\bar{T}_i^{-1}$ and $\bar{T}_o^{-1}$ by numerical integration of (57) and (59)

21. Compute G_i and G_o from (59) and (61)
22. Compute G_x from (63)
23. Compute G_{net} from (64)
24. If $\Delta p_{TOT} \neq G_{net}$, select new trial value of $\dot{m}$ and return to step 8.
25. For specified values of n_o , compute L_F from (74)

The above sequence of steps was programmed by E. Vogel, to produce the results presented in the next section.

3. ANALYTICAL RESULTS

This section consists of three subsections: (1) Parametric studies to guide selection of a Reference Design; (2) Selection and detailed characterization of that design; and (3) Sensitivity studies on that design, to determine effect on the RTG temperature of greater-than-theoretical friction drops and less-than-theoretical heat transfer coefficients.

3.1 Parametric Studies

Figure 12 illustrates the effect of the assumed chimney extension length L_x on the outer-shell temperature T_s , and on the helium temperature T_1 at the bottom and T_2 at the top of the ducts. The curves shown are for an RTG design temperature of 180 C, 36 inner fins, an emissivity of 0.9, and a helium pressure of 22 psia. That pressure was chosen for the initial part of the parametric studies because the RTG traditionally has been stored with a maximum internal gas pressure of 25 psia and a minimum of 15.5 psia (to minimize inleakage).

As can be seen, increasing the chimney length L_x raises the shell temperature T_s which satisfies the heat and pressure balances. This is because, with a longer chimney, the gravitational driving force can be produced with a smaller density difference between the inner and outer ducts. Figure 12 also shows that the helium temperature T_1 at the outlet of the outer duct is only 2 to 5 degrees C above the shell temperature. This shows how effective the inner fins are in

cooling the helium. By contrast, the helium temperature T_2 at the outlet of the inner duct is 91 to 97 degrees C below the assumed RTG design temperature of 180 C. The inner duct is a much poorer heat exchanger, because of its shorter length and much smaller number of

FIG. 12.

**EFFECT OF CHIMNEY EXTENSION LENGTH
ON PRINCIPAL TEMPERATURES**

$T_R = 180^\circ\text{C}$, $n_i/n_o = 36/36$, $M_H = M_F = 1$, $p = 21$ psia

fins.

For a given extension length L_x , the shell temperature T_s shown in Figure 12 determines the required outer-fin length L_f , for any given number n_o of outer fins. This is illustrated in Figure 13, which shows the effect of the chimney extension length L_x on the shell temperature T_s and on the required outer-fin height L_f , assuming 36 outer fins. This and subsequent results assume an ambient air temperature T_A of 21 C and fin thicknesses δ_i and δ_o of 0.125 inch.

FIG. 13.

As can be seen, for an assumed RTG design temperature of 180 C and for 36 inner and outer fins, there is a continuum of solutions representing a trade-off between minimizing the extension length L_x

and the outer-fin length L_F . The same trade-off format will be used in subsequent figures, but the implicit shell temperatures T_s will be omitted.

Figure 14 shows similar trade-off curves for RTG design temperatures of 160, 170, and 180 C, again for 36 inner and outer fins. As can be seen, the prescribed RTG temperature limit of 180 C is rather easily satisfied, with storage container designs of quite reasonable dimensions. But if the RTG design temperature is lowered to provide design margin for accommodating analytical uncertainties, the required design dimensions grow quite rapidly.

FIG. 14.

One possible way of alleviating the situation is to change the number of fins on the outer shell. Figures 15 and 16 illustrate the effect of the number of inner and outer fins on the required design dimensions. For an RTG design temperature of 170 C, Figure 15 shows the effect of the number of inner fins (with 48 outer fins), and Figure 16 shows the effect of the number of outer fins (with 24 inner fins).

FIG. 15.

**EFFECT OF NUMBER OF INNER FINS n_i
ON REQUIRED STORAGE CONTAINER DIMENSIONS**

$T_R = 170^\circ\text{C}$, $n_o = 48$, $M_H = M_F = 1$, $p = 22$ psia

As shown by Figure 15, increasing the number of inner fins from 24 to 36 reduces the required design dimensions significantly, but further increases yield very little additional benefit. By contrast, Figure 16 shows that all increases in the number of outer fins are beneficial.

FIG. 16.

**EFFECT OF NUMBER OF OUTER FINS n_o
ON REQUIRED STORAGE CONTAINER DIMENSIONS**

$T_R = 170^\circ\text{C}$, $n_i = 24$, $M_H = M_F = 1$, $p = 22$ psia

All the results presented thus far have assumed values of unity for the friction factor multiplier M_F and the heat transfer multiplier M_H . Other than lowering the RTG design temperature T_R from the design limit of 180 C to 170 C, no margin has been built into the results shown thus far.

The effect of providing additional margin by assuming more pessimistic values of the multipliers M_F and M_H is shown in Figures 17 and 18. For an RTG design temperature of 170 C and 36 inner and outer fins, Figure 17 shows the effect of varying the assumed friction factor multiplier M_F , and Figure 18 shows the effect of varying the assumed heat transfer multiplier M_H .

FIG. 17.

**EFFECT OF FRICTION FACTOR MULTIPLIER M_F
ON REQUIRED STORAGE CONTAINER DIMENSIONS**

$T_R = 170^\circ\text{C}$, $n_i/n_o = 36/36$, $M_H = 1$, $p = 22$ psia

FIG. 18.

**EFFECT OF HEAT TRANSFER MULTIPLIER M_H
ON REQUIRED STORAGE CONTAINER DIMENSIONS**

$T_R = 170^\circ\text{C}$, $n_i/n_o = 36/36$, $M_F = 1$, $p = 22$ psia

As can be seen, the required design dimensions are relatively insensitive to the assumed value of M_F , but quite sensitive to the assumed value of M_H . Figure 17 shows that the friction factors can be increased by as much as 60% with only moderate increase in the design dimensions; but Figure 18 shows that decreasing the helium heat transfer coefficients by only 10% raises the required extension length L_x and/or fin length L_f quite substantially, making the concept much less attractive unless other compensation can be provided.

The most effective way of compensating for less-than-theoretical heat transfer coefficients is to raise the helium pressure p . This is much more effective than raising the number of inner or outer fins. The effect of raising the helium pressure is illustrated in Figure 19, which is identical to Figure 18 except that the pressure has been raised from 22 psia to 44 psia. As can be seen, this has substantially lowered the required design dimensions, and makes it possible to accommodate 80% or less of theoretical heat transfer within acceptable design dimensions.

FIG. 19.

**EFFECT OF HEAT TRANSFER MULTIPLIER M_H
ON REQUIRED STORAGE CONTAINER DIMENSIONS
WITH 44 psia HELIUM**

$T_R = 170^\circ\text{C}$, $n_i/n_o = 36/36$, $M_F = 1$,

The RTG's internal pressure must always be higher than the external pressure in the storage container, to minimize the chance of inleakage and to avoid displacements of the C-seals due to pressure reversals. Therefore, raising the helium pressure in the storage container requires that the internal RTG pressure also be raised. This should not be done in a single step, since the RTGs are designed for a maximum pressure differential to its surroundings of 25 psi. But there is no obvious reason why the RTGs cannot be charged to higher pressures in a stepwise procedure as outlined below:

1. Charge RTG with 25 psia inert gas
2. Place in storage container
3. Connect auxiliary cooling system for temporary
(short-term) use during installation
4. Close storage container and evacuate
5. Charge storage container with 22 psia helium
6. Raise RTG pressure by 22 psi (to 47 psia)
7. Raise storage container pressure by 22 psi (to 44 psia)

In principle, there is no reason why steps 5 and 6 cannot be repeated to raise the pressure in the storage container to 66 or 88 psia, if its walls have been designed with sufficient strength.

Figure 20 combines a number of the conservative factors discussed earlier in a single case, and examines the effect of helium pressure on the required extension length L_x and fin length L_F for that

case. The results shown are for an RTG design temperature T_R of 170 C, 36 inner and outer fins, a friction drop multiplier of 1.40, and a heat transfer multiplier of 0.80.

FIG. 20. **EFFECT OF HELIUM PRESSURE
ON REQUIRED STORAGE CONTAINER DIMENSIONS**

$T_R = 170^\circ\text{C}$, $n_i/n_o = 36/36$, $M_F = 1.40$, $M_H = 0.80$

The 22-psia curve is missing, because it falls outside the field of the figure. Thus, while the $M_F = M_H = 1$ curve in Figure 18 showed that with theoretical values of the heat transfer coefficients and friction factors a helium pressure of 22 psia yields reasonable design dimensions, Figure 20 shows that with the more conservative assumptions of 80% heat transfer and 140% friction, helium pressures much lower than 44 psia would lead to a prohibitively large extension

length L_x or outer-fin length L_F . But, as shown in Figure 20, those dimensions are acceptable at 44 psia, and even more so at 66 psi. The sensitivity of the required design dimensions to helium pressure is clearly shown.

FIG. 21. **EFFECT OF HELIUM PRESSURE
ON REQUIRED STORAGE CONTAINER DIMENSIONS**

$T_R = 170^\circ\text{C}$, $n_i/n_o = 48/48$, $M_F = 1.40$, $M_H = 0.80$

Figure 21 is identical to Figure 20, except that it is for 48 instead of 36 inner and outer fins. Comparison of the two figures shows that for the same helium pressure, the 48-fin design yields appreciably smaller design dimensions. Nevertheless, to simplify the fabrication problem, it was decided to recommend a reference design based on 36 inner and outer fins, specifically on the design point P shown in Figure 20. This calls for an extension length L_x of

60 inches and an outer-fin length L_F of 8.6 inches. As shown by Figure 21, the alternative use of 48 fins would lower the required extension length from 60" to 49", for the same fin length, or would reduce the required fin length from 8.6" to 5.9", for the same extension length.

3.2 Reference Design Characterization

The results presented in this subsection are based on the design assumptions of 140% theoretical friction drops, 80% of theoretical heat transfer coefficients, and 44 psia helium pressure.

Figure 22 depicts the dimensions and number of fins of the Reference Design. As can be seen, its overall size is rather large compared to that of the RTG. This is the price for achieving passive cooling of both the RTG and the storage container. The finned outer shell is shown as consisting of two half-length sections. This would facilitate fabrication and assembly.

Figure 22 shows the average helium velocities in the inner and outer ducts, the respective average heat transfer coefficients, and the corresponding fin efficiencies. These are for the indicated mass flow rate $\dot{m}$ of 38 gm/sec, at which the total friction drop is balanced by the difference in gravitational heads between the inner and outer ducts. This will be demonstrated at the end of this subsection.

Figure 22 shows the values of $\bar{h}_i$, $\bar{h}_o$, and $\bar{h}_A$, which are the average values of the respective heat transfer coefficients. Note the small magnitude of these coefficients, particularly in the case of the natural-convection air-cooled outer fins.

FIG. 22.

REFERENCE DESIGN PARAMETERS

(Helium Pressure = 44 psi)

The figure also shows the fin efficiencies η_r , η_i , and η_o of the RTG fins and the inner and outer fins. The RTG fins have the lowest efficiency. This is because they are part of the existing RTG, which was designed to minimize the weight of the flight hardware. The inner and outer fins of the storage container are not similarly weight-constrained. Their efficiencies are based on an assumed thickness δ_i and δ_o of 0.125 inch.

In Figure 23, the local heat transfer coefficients h_i and h_o in the inner and outer ducts are plotted as a function of axial position. Note that the inner-duct's coefficient has not leveled off at its outlet end, which shows that fully-developed velocity profiles and temperature profiles are far from established. By contrast, in the much longer outer duct the heat transfer coefficient has almost reached its "fully-developed" asymptotic limit. The figure also shows the average heat transfer coefficients $\bar{h}_i$ and $\bar{h}_o$ for the two ducts.

FIG. 23.

Figure 24 shows the principal operating temperatures for the Reference Design under its assumed operating conditions (140% friction drop, 80% heat transfer, 44 psia). For the specified ambient air temperature T_A of 21 C, the shell temperature T_s is 43 C, and the inner duct's helium inlet temperature T_1 is 50 C and its outlet temperature T_2 is 67 C. The inlet and outlet temperatures are reversed for the outer duct. Finally, the RTG temperature T_R is 169 C.

FIG. 24.
REFERENCE DESIGN TEMPERATURES
 $M_F = 1.40, M_H = 0.80, p = 44 \text{ psia}$

Figure 25 shows the same temperatures, together with the axial temperature profiles in the inner and outer ducts. As can be seen, the shell temperature of 43 C is 22 degrees above the ambient air temperature. The outer duct's outlet temperature of 50 C is only 7 degrees above the shell temperature, which is indicative of the high efficiency of the outer-duct heat exchanger. By contrast, the inner duct's outlet temperature of 67 C is 102 degrees below the RTG temperature, showing its relatively poor heat exchanger efficiency. It should also be noted that the temperature rise of the helium in the inner duct is only 17 degrees, which is quite small compared to the mean temperature difference between the helium and the RTG. Therefore, departure of the RTG from its assumed isothermal condition would only have a small effect on the results.

FIG. 25.

Figure 26 summarizes the heat balance in the Reference Design under the stated operating conditions. It shows how the thermal power of 4.5 kw is transported from the RTG to the outer shell, and from the outer shell to the ambient air. In both cases, convective heat transport is seen to predominate over radiative heat transport, as was expected.

FIG. 26
THERMAL BALANCE (in kw)

Figure 27 shows an expanded version of the axial temperature profile in the helium loop. An exact determination of these temperatures is needed for accurately computing the net gravitational head which drives the helium circulation against the frictional resistance around the loop. As shown, the average temperature $\bar{T}_i$ of the inner duct is 60 C and the average temperature $\bar{T}_o$ of the outer duct is 54 C. The helium temperature T_2 in the chimney extension is 67 C. As will be seen, these small temperature differences, and the correspondingly small density differences, are sufficient to overcome the frictional resistance.

FIG. 27. HELIUM TEMPERATURE PROFILES AROUND LOOP

This is illustrated in Figure 29, which shows the friction drops Δp_i , Δp_x , Δp_o and the gravity heads G_i , G_x , G_o in the inner duct, chimney extension, and outer duct. These are all very

inches of helium at 44 psia. As would be expected, the three gravity heads in these units are close to the actual lengths of the three sections. They are somewhat lower, because the average helium temperatures are above 0 degrees C. As shown at the bottom of the figure, the net gravity head G_{net} balances the total friction drop Δp_{TOT} . This demonstrates the consistency of the solution.

FIG. 28. PRESSURE BALANCE

3.3 Sensitivity Studies

This subsection examines the effect of various parameters on the performance of the selected Reference Design. For this purpose, E. Vogel constructed a second computer program, based on the same analytical equations as the first. In the earlier program, the desired RTG temperature T_R was input, and the critical design dimensions L_x and L_F were the output. This was used to guide the selection of the Reference Design. After that design is selected, the new program is used to determine its performance sensitivity to various parameters. Thus, the dimensions L_x and L_F are specified inputs, and the program solves for the RTG temperature as various assumptions are varied.

All results presented thus far have assumed the use of 0.125 inch-thick copper plates for the inner and outer fins. The effect of changing fin thickness and material is displayed in Figure 30. For the previously used Reference Design assumptions (i.e., friction drops 40% above theoretical, heat transfer 20% below theoretical), the figure compares copper and aluminum fins ranging from 0.060 inch to 0.200 inch in thickness. Note the greatly expanded temperature scale of this figure. As can be seen, both of those variables have only a small effect on the RTG temperature.

FIG. 29.

**EFFECT OF INNER AND OUTER FIN THICKNESS AND MATERIAL
ON RTG TEMPERATURE**

$L_X = 60''$, $L_F = 8.6''$, $n_i/n_o = 36/36$, $M_F = 1.40$, $M_H = 0.80$

For the previously assumed fin thickness of 0.125 inch, Figure 29 shows that changing from copper to aluminum would only raise the RTG temperature by 1.5 degrees. Clearly, this is not a decisive difference. Other factors will influence the choice. The reason the author favors copper over aluminum is that it permits fabrication of the internally and externally finned outer shell in a single furnace-braze operation. In the case of aluminum, the absence of suitable braze alloys would probably require welded joints, with each fin welded individually to the outer shell. While this is not impossible, it would be much more time-consuming than the single furnace-braze operation made possible by copper.

Figure 30 shows the effect on the Reference Design RTG temperature of poorer-than-theoretical friction losses and heat transfer coefficients. As can be seen, varying the friction factors has much less effect than varying the heat transfer coefficients. As shown, the Reference assumptions of 140% friction drop and 80% heat transfer leads to an RTG temperature of 170C. If both of these parameters were at their theoretical values, the RTG temperature would only be 148 C.

Figure 30 also shows a line representing the prescribed design limit of 180 C. As can be seen, for 40% excess friction the design limit is satisfied as long as the heat transfer coefficients exceed 71% of their theoretical values.

FIG. 30.
**EFFECT OF RELATIVE FRICTION FACTORS
 AND HEAT TRANSFER COEFFICIENTS
 ON REFERENCE-DESIGN RTG TEMPERATURE**

$L_X = 60''$, $L_F = 8.6''$, $n_i/n_o = 36/36$, $p = 44$ psia

*Relative to Theoretical Predictions

Figure 30 shows that if the actual heat transfer coefficients turn out to be even lower than about 70% of theoretical, the RTG temperature will exceed 180 C. But even if that should happen, the RTG temperature could still be lowered to the design limit, by increasing the helium pressure further.

This is illustrated in Figure 31, for friction factors 40% above theoretical. The figure confirms that the initially assumed pressure of 22 psia would be adequate for theoretical heat transfer, but would not be adequate below 90% theoretical. It also shows that raising the helium pressure accommodates heat transfer down to 71% theoretical for 44 psia, and down to 62% theoretical for 66 psia.

FIG. 31. **EFFECT OF HEAT TRANSFER COEFFICIENT AND HELIUM PRESSURE ON RTG TEMPERATURE IN REFERENCE DESIGN**

Figure 31 also shows a line representing the prescribed RTG temperature limit of 180 C. The intersections of that line with the respective curves represent the minimum helium pressures required to accommodate various values of the heat transfer coefficients. These intersections are cross-plotted in Figure 32, which shows the minimum helium pressure required to keep the RTG temperature below 180 C, as a function of heat transfer coefficients and friction factors.

FIG. 32. **EFFECT OF HEAT TRANSFER COEFFICIENTS
AND FRICTION FACTORS ON REQUIRED HELIUM PRESSURE
FOR $T_R = 180^\circ\text{C}$**

As can be seen, the option of adjusting the helium pressure after the unit is built and tested provides convenient safety margins for compensating for higher-than-anticipated friction factors and for lower-than-anticipated heat transfer coefficients.

Actually, it seems highly unlikely that some of the very low heat transfer coefficients examined in Figures 30, 31, and 32 will actually occur. There is no obvious reason why those coefficients would be that far below their theoretical values. What seems much more likely is that the heat transfer coefficients will actually turn out to be substantially higher than those predicted for laminar flow, because of the presence of turbulence.

The Reynolds number in the outer duct is about 500, but that in the inner duct is around 2100, which is just at the threshold of the transition zone from laminar to turbulent flow. Laminar flow is most likely in a smooth-walled duct. But the RTG protrusions formed by the uncouple mounting screws and the flow irregularities caused by the periodic RTG fin slots are likely to promote turbulence in the inner duct.

While the presence of turbulence in the inner duct would lead to a substantial favorable increase of its heat transfer coefficient, it would also -- by analogy -- lead to a corresponding unfavorable increase of its friction factor. These two changes have opposite effects on the design's performance.

However, as shown in Figures 30 and 32, the favorable effect of increased heat transfer is much greater than the unfavorable effect of increased friction. Therefore, the presence of turbulence is likely to be a net benefit to the design's performance. As shown by Figure 30, corresponding increases of heat transfer and friction would lead to substantial reductions in RTG temperature. Therefore, as shown by Figure 32, turbulence-induced increases in heat transfer coefficients and friction factors are likely to permit the use of helium pressures substantially below the recommended 44 psia.

REFERENCES

- (1) W. M. Rohsenow, H. Y. Choi, "Heat Mass and Momentum Transfer", Prentice Hall, page 89, Figure 5.1.
- (2) Ibid, page 133, Figure 7.1.
- (3) W. M. Kays, "Convective Heat and Mass Transfer", McGraw Hill, 1966, page 143, Table 8-10.
- (4) W. P. Rohsenow, J. P. Hartnett, Editors, "Handbook of Heat Transfer", McGraw Hill, page 7-139, Figure 121 and Table 39.
- (5) Ibid, page 7-4, Figure 1.
- (6) W. H. McAdams, "Heat Transmissions", 3rd Edition, 1954, McGraw Hill, page 172.