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Final Report - Summer Visit 2011

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Subject: Summer Visit
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During my visit to LLNL during the summer of 2011, I worked on algebraic multilevel solvers for large sparse systems of linear equations arising from discretizations of partial differential equations. The particular emphasis this year was on finite elements matrices arising from hp-adaptive finite element computations. Such matrices are interesting in several respects. First, higher order polynomials give rise to dense blocks within the sparse matrix. For this reason I implemented a general sparse block data structure, in which the elements are blocks of arbitrary size and shape. The blocks are referenced via a ja array (pointers and block column indices) similar to a standard sparse matrix. However, *dense* matrix operations (matrix multiplies, ILU factorizations) can now be done more efficiently using dense matrix techniques with no indirect addressing.

Within this new data structure, I implemented several solvers. The first was a block ILU factorization using (block) minimum degree orderings, and a block version of drop tolerance. This was a generalization of a similar method I implemented for standard sparse matrices. The second basic method was a two-level iteration. The smoother in this scheme is a simple block symmetric Gauss-Seidel iteration. The coarse space is hierarchical in nature. For each vertex in the mesh, we associate the linear nodal basis function. For each edge in the mesh with degree $p \geq 2$, we associate a quadratic bump function. For each element with interior degree $p \geq 3$, we associate a cubic bubble function. If all basis functions are present, the maximum degree of this coarse space is approximately $6N$, where N is the number of vertices in the triangulation (2D). While this hierarchical coarse space cannot be sufficient to achieve convergence rates independent of both N and p as $p \to \infty$, $p \leq 9$ in my finite element program (due to availability of quadrature rules), so as a practical matter this is not an issue.

In addition to the work on sparse block solvers, we had several discussions related to the algebraic domain decomposition solver project. The idea here is to give each processor some portion of the fine matrix, and a coarse description of the remaining matrix. In this sense it is an algebraic version of the (geometric grid based) DD solver in $PLTMG$.