PROBABILISTIC SIMULATION OF SUBSURFACE FLUID FLOW:
A STUDY USING A NUMERICAL SCHEME

Timothy Eric Buscheck
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PROBABILISTIC SIMULATION OF SUBSURFACE FLUID FLOW:

A STUDY USING A NUMERICAL SCHEME

Timothy Eric Buscheck

Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

ABSTRACT

There has been an increasing interest in probabilistic modeling of hydrogeologic systems. The classical approach to groundwater modeling has been deterministic in nature, where individual layers and formations are assumed to be uniformly homogeneous. Even in the case of complex heterogeneous systems, the heterogeneities describe the differences in parameter values between various layers, but not within any individual layer. In a deterministic model a single-number is assigned to each hydrogeologic parameter, given a particular scale of interest.

However, physically there is no such entity as a truly uniform and homogeneous unit. Single-number representations or deterministic predictions are subject to uncertainties. The approach used in this work models such uncertainties with probabilistic parameters. The resulting statistical distributions of output variables are analyzed.

A numerical algorithm, based on axiomatic principles of probability theory, performs arithmetic operations between probability distributions. Two subroutines are developed from the algorithm and incorporated into the computer program TERZAGI, which solves groundwater flow problems in saturated, multi-dimensional systems. The probabilistic computer program is given the name, PROGRES.

The algorithm has been applied to study the following problems: one-dimensional flow through homogeneous media, steady-state and transient flow conditions, one-dimensional flow through heterogeneous media, steady-state and transient flow conditions, and two-dimensional steady-state flow through heterogeneous media. The results are compared with those available in the literature.

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NOMENCLATURE

Arabic Symbols

$A_{\ell m}$	area of the interface of volume elements ℓ and m (L^2)
F	factor used in calculation of stable time step
$D_{\ell m}$	distance from the center of both elements, ℓ and m (L)
$K_{\ell m}$	hydraulic conductivity at the interface of volume elements, ℓ and m (L/T)
K	hydraulic conductivity (L/T)
$M_{c,\ell}$	fluid mass capacity of volume element, ℓ (M)
n	porosity (l)
S_s	coefficient of specific storage (l/L)
$U_{\ell m}$	conductance of the bounding surface of volume elements, ℓ and m (M/LT)
Δt	time step (T)
Δt_{crit}	critical time step (T)
Δt_{stab}	stable time step (T)

Greek Symbols

α	compressibility of the medium (LT^2/M)
β	compressibility of the fluid (LT^2/M)
γ	specific weight of the fluid (M/L^2T^2)
ϕ	hydraulic head (L)
$\bar{\phi}$	mean hydraulic head (L)
ϕ_{ℓ}^0	initial potential head of volume element (L)
$\Delta\phi_{\ell}$	change in potential at volume element (L) during time step t
ϕ_m^0	initial potential of volume element, m (L)
ρ	density (M/L^3)

FORTRAN Symbols Found in Text

CAP	fluid mass capacity of denoted volume element (M)
CON	probabilistic array for permeability of specified volume element (L/T)
CONPRD	probabilistic array representing the product $K_1 K_2$ (L^2/T^2)
CONSUM	probabilistic array representing the expression $K_1 L_2 + K_2 L_1$ (L^2/T)
CONT	probabilistic array input as the permeability of a volume element (L/T)
DELT	stable time step (T)
DELTF	factor used to scale the critical time step in order to assure a stable time step
DFI	probabilistic array for mass flow rate (M/T) and later redefined as simply mass (M)
DPSI	probabilistic array for change in potential of a specified volume element (L)
MAGPOT	denotes whether or not minimum and maximum limits for potential should be considered
MSIGN	denotes the choice of the arithmetic operation
PHI	probabilistic array for potential of specified volume element (L)
PHID	probabilistic array representing the potential difference between two adjacent volume elements (L)
TOLPR	minimum allowable probability value

TRAN probabilistic conductance array for a determined connection
 (M/LT)

RHOZ density (M/L^3)

ZIPMAX maximum conductance (M/LT)

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I. INTRODUCTION

A. Previous Work

Long term decision-making tasks in regard to the hydrogeologic environment must rely heavily on output generated from mathematical models. However accurate the computational scheme may be, the reliability of these results is a function of inherent uncertainties in input data and parameters. It is now being well-recognized that an appreciation of the output uncertainty is of great practical value in arriving at optimal decisions. As a result, there is increased emphasis on incorporating parameter uncertainty into the computational model in such a manner as to obtain confidence limits on the model output.

The methods used to stochastically model groundwater systems have generally included a Monte Carlo simulation.* The approach in this study is to model distributions in probability with discrete values. This technique is thought to be unlike any others employed thus far.

Probabilistic modeling of subsurface flow models on the macroscopic scale can be associated with one of three sources of error in input parameters, as given by Freeze [1975]: (1) those resulting from measurement errors in the input parameters, (2) those caused by spatial averaging of input parameters, and (3) those related to an inherent stochastic description of nonuniform or heterogeneous porous media.

* A glossary is given in Appendix A.

Sagar and Kisiel [1972] made a study of error propagation due to input parameters (cause 1, above). Their work considered the influence of errors in initial hydraulic head, transmissibility, and storage coefficient on the drawdown patterns predicted by the Theis equation for pumpage from a homogeneous, isotropic, confined aquifer. They used a Bayesian "know nothing" approach by modeling input parameters with uniform frequency distributions. Varying the input errors, they studied the error in hydraulic head with time at different radial distances from the pumping well.

Bibby and Sunada [1971] considered the combination of sources (1) and (2) above. They used a numerical simulation model of transient flow to a well in a confined aquifer. A Monte Carlo simulation was employed to analyze the effect of normally distributed measurement errors in initial head, boundary heads, pumping rate, aquifer thickness, hydraulic conductivity, and storage coefficient. They also studied the uncertainties associated with choosing spatially averaged parameter values at each element in the mesh used in the numerical computation. The input parameters were represented by general linear functions which describe the spatial trends in each element. This method applies a normal distribution for hydraulic conductivity values, and other spatially averaged input parameters. The application of a normal distribution makes this approach more compatible with the investigation of measurement errors (case 1) than with stochastically modeled media (case 3) in which hydraulic conductivity is generally considered as being log normally distributed.

Warren and Price [1961] investigated case (3) by implementing a numerical model to study steady-state and transient flow through quasi-linear and quasi-radial systems of "homogeneously heterogeneous" porous media. They modeled hydraulic conductivity using frequency distributions and studied the resulting behavior of the solutions.

Freeze [1975] considered case (3) in the analysis of one-dimensional steady-state saturated flow between two specified boundary heads. The equation governing such flow contains one hydrogeologic parameter, the hydraulic conductivity, assumed to be log normally distributed. Freeze's stochastic model uses a large number of Monte Carlo runs for parameters and boundary conditions, chosen from probabilistic distributions.

Freeze also studied transient flow problems, which included two additional hydrogeologic parameters, porosity and compressibility. Based on the reasoning that these three parameters are not independent, and may be highly correlated, he applied a multivariate normal density function to generate parameter values. Freeze [1977] later examined the concept that geological data display spatially statistical properties, by developing a multivariate normal generator with linear dependence.

In Freeze's work the expression "Monte Carlo simulation" refers to a set of repetitive simulations with a mathematical model and the associated statistical analysis of the results. This term has also been used to relate to a random walk method of solving specific boundary value problems [Shih, 1973; Kelley, 1971].

Probabilistic analysis is common in fields such as numerical weather prediction where it is known as the statistical dynamical approach [Epstein, 1969; Gleeson, 1970; Fleming, 1971]. Chiu [1968] has analyzed open channel flow by studying the integration of statistically defined channel geometries in streamflow routing models. Morse 1972 has developed a stream temperature model with stochastic elements.

In applied mathematics and fluid mechanics, literature exists on the solution of "stochastic differential equations" [Beram, 1968; Bharucha-Reid, 1968]. Buyerich et al. [1969] have studied the effects of random macroscopic porosity variations on transport phenomena associated with steady-state fluid flow. The approach used in this study to model uncertainties in input parameters deviates from those techniques outlined above.

B. Purpose and Scope

The present work investigates the sources of error in output variables resulting from probabilistic structures of input parameters in non-uniform or heterogeneous media (case 3, above). The purpose of this work is to better understand the statistical distributions of output results. The approach used is to model continuous frequency distributions as histograms with discrete values, and perform arithmetic operations directly on the distributions rather than applying a Monte Carlo simulator. Parametric studies are made varying the standard deviations of the log normal distribution for the hydraulic conductivity. Based on these studies, the behavior of the resulting statistical distributions of output variables can be analyzed. The scope of these

problems includes steady-state flow in one- and two-dimensions, as well as one-dimensional transient flow.

A numerical algorithm is developed to perform arithmetic operations between frequency distributions. The calculation is carried out in the explicit mode. Because of the nature of an explicit calculation, within each time step the potentials of adjacent nodes are independent of one another. It is therefore appropriate to assume the axioms of independent probability theory, and these principles are applied to the algorithm.

The parameter, hydraulic conductivity, is modeled in a probabilistic manner. Parametric studies varying the standard deviation on the hydraulic conductivity statistical distribution allow for the analysis of the distributions of output head. The lumped parameter, specific storage (S_g) is treated deterministically. The generality of the algorithm developed herein is such that specific storage could be modeled probabilistically by redimensioning this parameter's array. The aim of this study is principally to consider the most simple problem and for that reason only the hydraulic conductivity is represented by a statistical distribution.

II. THEORY

A. Problem Statement and System Definition

Given a transient or steady-state groundwater system, in which the input parameters are represented statistically, the problem is to generate an output in which the spatial and temporal variations of fluid fluxes and potentials are probability distributions.

A system can be defined as a set of physical processes which convert an input variable (or variables) to an output variable (or variables). A variable is a characteristic of the system which can be measured and which assumes different numerical values at different times. A variable is differentiated from a parameter, which is a quantity characterizing the system which does not change with time. In hydrologic modeling one must be able to mathematically model the relationships between the variables which describe the system behavior. The general form of a hydrologic model is given by Freeze [1975] as

$$y_t = f(x_{t-1}, x_{t-2}, \dots, y_{t-1}, y_{t-2}, \dots, a_1, a_2, \dots) + \epsilon_t \quad (1)$$

The vector variable $\{x_t\}$ ($t = \dots, -1, 0, 1, 2, \dots$) is the input, and the vector variable $\{y_t\}$ ($t = \dots, -1, 0, 1, 2, \dots$) is the output. The system parameters are denoted by "a". The function f defines the nature of the model and the error ϵ_t refers to the lack-of-fit with reality.

A model is defined as probabilistic or deterministic depending on whether or not it contains random variables. In the case where any of the variables (x_t , y_t , or ϵ_t) has a distribution in probability, the model is probabilistic. One possible source of a probabilistic

structure for the output variable, y_t , could be a probabilistic distribution of one or more of the parameters $a_1, a_2, \dots$. Clark [1973] defines the terms conceptual and empirical according to whether or not the form of the function f is suggested by consideration of physical processes acting on the input variables. The terms conceptual and empirical therefore refer to the function f , whereas the terms probabilistic and deterministic refer to the existence of a probabilistic structure for any of the variables.

Groundwater flow modeling has classically been conceptual and deterministic. The function f is usually an initial-boundary value problem based on partial differential equations of flow through porous media. For a one-dimensional transient flow problem one can define the output vector $\{y_t\}$ from (1) above as the values of the hydraulic head at any time t . On the righthand side of the equation y_0 represents the initial conditions of $\phi(x)$ at time zero and the values of $\{x_t\}$ identify the boundary conditions at $x = 0$ and $x = L$ at time t . The parameters a_1, a_2, a_3 are hydraulic conductivity, compressibility, and porosity of the porous medium in saturated flow problems. In the classical approach none of the variables or parameters are expressed in terms of a probability distribution.

Freeze [1975, 1977] takes a stochastic-conceptual approach to one-dimensional groundwater flow. His model is made stochastic by applying a large number of Monte Carlo simulations for parameters and boundary conditions originating from probabilistic distributions. A set of repetitive simulations is made with the mathematical model.

The approach used in this work to probabilistically model a groundwater flow system differs from the Monte Carlo approach followed by Freeze. The technique used in this present work applies a well-defined probabilistic structure. The probability distribution is modeled by a histogram consisting of discrete values and their associated probabilities. Given a "continuous" distribution, one can generate any number of discrete intervals, and their associated probabilities, using a graphical technique. The accuracy of modeling a distribution will presumably improve with the number of discrete intervals chosen. Each probabilistic distribution is therefore modeled by a determined number of pairs of values; a discrete value for the magnitude and one for its associated probability. These probabilistic structures represent the inherent uncertainty of the variable or parameter in questions.

B. The Nature of Distributions

1. The Physical Nature of Distributions for Hydrogeologic Parameters

The governing equation for one-dimensional, transient, saturated flow through a non-uniform porous medium, given by Narasimhan and Witherspoon [1977] can be written: (see nomenclature for description of symbols)

$$\frac{\partial}{\partial x} \left[K(x) \frac{\partial \phi}{\partial x} \right] = \gamma S_s(x) \frac{\partial \phi}{\partial t} \quad (2)$$

where the coefficient of specific storage is given as: $S_s(x) = (\alpha(x) + n(x)\beta)$.

There are two hydrogeologic parameters of interest in the above equation, the hydraulic conductivity, K , and the coefficient of specific

storage, S_s . Freeze [1975] chooses to consider two parameters, compressibility, α , and porosity, n , rather than the lumped parameter, S_s . In Freeze's analysis of transient flow, the three hydrogeologic parameters are correlated, reflecting their interdependence.

If the hydraulic head is not a function of time, $\partial\phi/\partial t = 0$, and we have the steady-state condition given as:

$$\frac{\partial}{\partial x} \left[K(x) \frac{\partial \phi}{\partial x} \right] = 0 \quad (3)$$

In this case, one must only be concerned with the parameter, hydraulic conductivity, K .

The parameter of primary interest in the present work is hydraulic conductivity. K is modeled as a statistical distribution; the parametric studies consider variation of its standard deviation. In order to simplify the problem, S_s is modeled deterministically. In the steady-state problem the potential boundary condition is prescribed. For the transient simulation we have, in addition, an initial condition.

Greenkorn and Kessler [1969] define the terms non-uniformity, heterogeneity, and anisotropy on a statistical basis. Using the example of hydraulic conductivity, they note that in general its probability density is dependent on location and orientation. This function can be given by three rectangular coordinates for location and two angular coordinates for orientation. If the probability density function is independent of orientation, the medium is isotropic. In the case of an anisotropic medium the probability distribution is dependent on orientation. A uniform medium will have a distribution which is

expressible by a finite linear combination of delta functions, while a non-uniform system will not. If the distribution is mono-modal the medium is homogeneous. A heterogeneous medium is characterized by a multi-modal distribution. However, in this work a heterogeneous system is modeled by varying distributions at the appropriate volume elements, rather than applying a multi-modal distribution. One would not expect to find a uniform medium in the real world, so Figures 1C and 1D are more realistic. In this study we have considered both the cases of non-uniform homogeneous media (Figure 1C) and non-uniform heterogeneous media.

In classical terminology Figure 1A is defined as homogeneous and 1C as heterogeneous. However, the classification given by Greenkorn and Kessler [1969] is more consistent with this paper.

Greenkorn and Kessler [1969] also note that the statistical variance of the frequency distributions could be dependent on spatial coordinates or orientation while the means are not. These effects are known as second-order non-idealities. Formations may also show permeability distributions having constant standard deviations through space but mean values which vary with spatial location.

The problems of flow through homogeneous and heterogeneous media are included in this study. In the model of a homogeneous material, the same probability distribution is assigned throughout the entire system. In the case of a heterogeneous system, the permeability will vary from one layer to another (or, for linear flow in one dimension, the variation is simply element to element). This variation in permeability requires different probabilistic structures assigned to

Figure 1. Definition of Uniformity and Homogeneity.

XBL 805-824

appropriate volume elements, depending on the location of the heterogeneities.

The statistical model of hydraulic conductivity is some distribution in probability, a representation of its associated uncertainty. There is direct field evidence that the probability density function is log normal. Law [1944] was the first to suggest hydraulic conductivities are log normally distributed, based on core analyses data from a carbonate oil field reservoir. McMillan [1966] studied the California Department of Water Resources transmissibility maps for the Los Angeles basin and learned that permeabilities there were log normally distributed. Bennion and Griffiths [1966], working with 60,000 cores from 2,000 wells in a sand and conglomerate oil field reservoir and 24,000 cores from 430 wells in a limestone reservoir, discovered permeabilities to be distributed log normally, though somewhat skewed in certain cases.

There is also indirect evidence which would indicate that hydraulic conductivity is log normally distributed. Davis [1969] and other authors point out that the specific capacities of water wells, which are directly related to formation permeabilities, are characterized by log normal distributions. It is generally accepted that grain size distributions are log normal and hydraulic conductivity can be related in a functional way to some representative grain size diameter. Hydraulic conductivity can also be empirically calculated as the exponential function of porosity. Porosity is usually accepted as being normally distributed, so the exponential relationship would indicate a log normal distribution for hydraulic conductivity.

2. The Numerical Representation of Log Normal Distributions

As discussed earlier, hydraulic conductivities are generally regarded as being log normally distributed. Assuming such a frequency distribution to model the uncertainties associated with this parameter, it is necessary to construct the log normal distribution, which we shall now proceed to consider.

The random variable x has a normal distribution if its density function is given by the formula in Fisz [1967]:

$$f(r) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(r-m)^2}{2\sigma^2}} . \quad (4)$$

It is possible to generate a normal probability distribution from the above expression. Given the mean and standard deviation of the distribution, the non-normalized probability that any value x should occur is given by $P(x)$.

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} . \quad (5)$$

Freeze [1975] employs the following expression for a log normal distribution of hydraulic conductivity, K :

$$x = \log K . \quad (6)$$

The random variable, x , is generated with the Monte Carlo technique in Freeze's model. The approach employed in the present work is to use values of x which lie within three standard deviations of the mean, μ_K . Assuming x is known, the log normally distributed values of K are obtained with the equation:

$$K = 10^x \quad (7)$$

This expression is used to generate the hydraulic conductivities for the log normal distribution. The probabilities associated with each discrete value for hydraulic conductivity are determined from the above equation for $P(x)$.

Appendix B illustrates the technique used to develop the log normal distributions for $\sigma_K = 0.5$ and $\sigma_K = 0.25$.

The results for calculations ranging from -3σ to $+3\sigma$ about the mean are tabulated in Table 1 for Cases 1 and 2. A 99% certainty is generally associated with a distribution including 2.58 σ 's. The calculation at 3 σ 's indicates that $K = 316.2$ (Case 1, $\sigma_K = 0.5$) and $K = 56.23$ (Case 2, $\sigma_K = 0.25$).

The log normal distribution for $\mu_K = 1.0$, $\sigma_K = 0.5$ ($K_{STD} = 0.5$) is illustrated in Figure 2. The ordinate represents the non-normalized probability of some K occurring. The hydraulic conductivity, K , is plotted on the abscissa. The mean value ($K = 10$) and values of K at one, two, and three standard deviations greater than the mean are denoted. The distribution indicates that the non-normalized probability of mean K occurring is 0.800.

The log normal distribution for the second case, $\mu_K = 1.0$, $\sigma_K = 0.25$ ($K_{STD} = 0.25$) is shown in Figure 3. The non-normalized probability associated with the mean K is 1.60, twice that in the previous case. As noted earlier, $K = 56.23$ at 3σ for $K_{STD} = 0.25$, while $K = 316.2$ at 3σ for $K_{STD} = 0.5$. The result is a steeper distribution which is not as greatly skewed. The shape of the log normal distribution for

TABLE 1

Summary of Log Normally Generated Hydraulic Conductivity

A. Case 1: $K_{STD} = 0.5$

<u>Discrete Value, x/ Standard Deviation</u>	<u>Hydraulic Conductivity $K = 10^x$</u>	<u>Probability, $P(x)$ (see Appendix B)</u>
-0.5/-3 σ	0.316	0.008
0.0/-2 σ	1.0	0.108
0.5/- σ	3.26	0.485
1.0/ μ	10.0	0.800
1.5/+ σ	31.62	0.485
2.0/+2 σ	100.0	0.108
2.5/+3 σ	316.2	0.008

B. Case 2: $K_{STD} = 0.25$

<u>Discrete Value, x/ Standard Deviation</u>	<u>Hydraulic Conductivity $K = 10^x$</u>	<u>Probability, $P(x)$ (see Appendix B)</u>
0.25/-3 σ	1.78	0.018
0.5/-2 σ	3.26	0.216
0.75/- σ	5.62	0.970
1.0/ μ	10.0	1.700
1.25/ σ	17.78	0.970
1.50/2 σ	31.62	0.216
1.75/3 σ	56.23	0.018

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Figure 2. Log Normal Distribution for $\mu = 1.0$, $\sigma = 0.5$.

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Figure 3. Log Normal Distribution for $\mu = 1.0$, $\sigma = 0.25$.

$K_{STD} = 0.25$ indicates that there is less uncertainty associated with this probability density function than with that for $K_{STD} = 0.5$.

With these log normal distributions plotted, the next task is to convert them to the form of two-dimensional arrays which can be input to the computer program. The method used is to discretize the "continuous" distribution into intervals. The distributions are modeled with nine intervals. Each interval is represented by a pair of discrete values, an arithmetic average, and the probability of occurrence associated with the interval. The probabilities are normalized using the following expression.

$$P(\Delta x_n) = \frac{\int_{K_n}^{K_{n+1}} P(x) dx}{\int_{-3\sigma}^{+3\sigma} P(x) dx} \quad (8)$$

The discrete pairs of numbers (interval average, probability) comprise an array. The log normal distribution corresponding to $K_{STD} = 0.5$ (Figure 2) is tabulated in Table 2A (Case 1). For purposes of the computer simulation, the mean values above are scaled down by a factor of ten (10.0). The justification for this scaling procedure stems from the fact that the mean hydraulic conductivity is unity ($\mu_K = 1.0$) prior to implementing the log normal generator, $K = 10^x$. The simulation is carried out using the original log normal array, above, and the results are identical to those obtained using the scaled version.

The log normal distribution for $K_{STD} = 0.25$ (Figure 3) is represented in Table 2B (Case 2).

3. The Numerical Representation of Other Distributions

The method used to generate the probabilistic arrays for other types of distributions (normal, mono-modal, multi-modal, etc.) is the same as that outlined above for log normal distributions. The general technique entails the discretization of a well-defined "continuous" distribution.

C. Algebra of Distributions

We shall begin our treatment of the "algebra of distributions" by first reviewing the basic concepts of sample space and subset and the algebra of sets.

1. Probability - The Discrete Case

Freund [1962] defines a sample space as "a set whose elements represent all possible outcomes of an experiment." The terms "universal set", "universe of discourse" and "possibilities space" are also used. Sample spaces are usually defined by the number of elements which they contain. If a sample space contains a finite number of elements it is said to be discrete. If the elements of a sample space lie on a line (in a two-dimensional space) or in a plane (in a three-dimensional space), the sample space is said to be continuous. The approach used in this study is to model "sample spaces" or frequency distributions in a discrete manner.

TABLE 2

Probabilistic Arrays

A. Case 1: $K_{STD} = 0.5$

<u>Interval Size Δx</u>	<u>Average Value $\bar{x}$</u>	<u>$\bar{x}/10$</u>	<u>Probability, $P(\Delta x)$</u>
5.0	2.5	0.250	0.040
20.0	15.0	1.500	0.329
17.5	33.75	3.375	0.186
17.5	51.25	5.125	0.120
15.0	67.50	6.750	0.072
20.0	85.0	8.500	0.069
30.0	110.0	11.000	0.064
55.0	152.5	15.250	0.063
136.0	250.0	25.000	0.057

B. Case 2: $K_{STD} = 0.25$

4.0	3.00	0.300	0.039
12.5	11.25	1.125	0.610
7.5	21.25	2.125	0.196
7.5	28.75	2.875	0.085
5.0	35.00	3.500	0.029
5.0	40.00	4.000	0.018
7.5	46.25	4.625	0.014
5.0	52.50	5.250	0.005
7.5	58.75	5.875	0.004

According to Freund [1962], set A is called a "subset" of set B if and only if each element of A is also an element of B. All events which are concerned in probability are represented by subsets of appropriate sample spaces.

We shall assume that the sets on which certain operations are performed belong to a given sample space S. Freund [1962] defines the union or logical sum of two sets A and B to be the set which contains all elements, each of which belongs to A or to B or to both. The union of two sets A and B is written $A \cup B$. Other notations used for the union of A and B include $(A + B)$ and $(A \text{ or } B)$, or referring to inclusive, meaning A or B or both. Freund[1962] explains that the intersection or logical product of two sets A and B is the set which consists of all elements, each of which belongs to both A and B. The intersection of two sets A and B is written $A \cap B$. Other expressions are A and B , $A \cdot B$, AB .

2. The Algebra of Sets

If two sets are equal we can write $A = B$ if and only if each element of A is an element of B and each element of B is an element of A. Two sets A and B are said to be disjoint if and only if $A \cap B = 0$. A' is the complement of A relative to the sample space S if and only if both $A \cap A' = 0$ and $A \cup A' = S$.

In the Algebra of Sets or Boolean Algebra there are some basic postulates found in Freund [1962].

P_1 (laws of closure)	for every pair of sets A and B there exist unique sets $A \cup B$ and $A \cap B$
P_2 (commutative laws)	$A \cup B = B \cup A$ $A \cap B = B \cap A$
P_3 (associative laws)	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
P_4 (distributive laws)	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
P_5 (identity laws)	there exists unique sets ϕ and S such that for each A, $A \cap S = A$ and $A \cup \phi = A$
P_6 (complementation law)	corresponding to each set A there exists a unique set A' such that both $A \cap A' = \phi$ and $A \cup A' = S$

3. A Formal Definition of Probability

An event (or subset) can be represented as a set of elements from a sample space. Each element of that space represents one of the possible outcomes of an experiment. The occurrence of an event A corresponds to the choice of an element from that subset of the sample space which represents the event.

It is helpful to include a third concept in this model, a mathematical representation of probability. Probability is associated with the events rather than the individual outcomes. The classical definition of the probability of an event A is the ratio of the number of elements in A (the number of ways A can occur) to the total number of elements

in the sample space S (the total number of possible outcomes). Therefore, probability must be a function of sets, associating a number with a set of elements rather than with an individual element.

From Pfeiffer [1965] we have the following definition of probability.

Definition 2-3a

A probability system (or probability space) consists of the triple

- 1) A basic space S of elementary outcomes (elements) ξ
- 2) A class $\mathcal{E}$ of events (a completely additive class of subsets of S)
- 3) A probability measure $P(\cdot)$ defined for each event A in the class $\mathcal{E}$ and having the following properties:
(P1) $P(S) = 1$ (probability of the sure event is unity)
(P2) $P(A) \geq 0$ (probability of an event is non-negative)
(P3) If $a = \{A_i : i \in J\}$ [a mutually exclusive (disjoint) class of events] is a finite or countably infinite partition of A (i.e., a mutually exclusive class whose union is A) then

$$P(A) = \sum_{i \in J} P(A_i) \text{ (additivity property)}$$

The additivity property (P3) is the basis of a fundamental concept of a probability measure $P(\cdot)$. Pfeiffer [1965] states this concept: The probability of the union of mutually exclusive events is the sum of the probabilities of the individual events.

In order to understand how probabilistic structures are used in arithmetic operations, it will be helpful to review the subject of conditional probability theory.

4. Conditional Probability

It is meaningful to speak of the probability of an event only if a sample space is specified. Within this sample space the event is represented by a subset, and some appropriate probability is given to that event. There exist many problems where one is interested in probabilities relative to other related events. Therefore, one can define the symbol $P(A/B)$ and refer to it as the "conditional probability of A relative to B."

Freund [1965] defines conditional probability as follows: If A and B are subsets of a discrete sample space and $P(B) \neq 0$, the conditional probability of A relative to B is

$$P(A/B) = \frac{P(A \cap B)}{P(B)} . \quad (9)$$

If we multiply both sides of (9) by $P(B)$, we can derive the first part of the following theorem relating to the definition of conditional probability. This theorem is known as the "General Rule of Multiplication," given by Freund [1965].

$$\text{Theorem 2.14} \quad P(A \cap B) = P(B) \cdot P(A/B) \quad (10)$$

$$P(A \cap B) = P(A) \cdot P(B/A) \quad (11)$$

The above theorem states that the probability of events A and B both occurring is equal to the product of the probability that one of them will occur and the probability that the other will occur, given that the first has occurred. For example, the probability of drawing two hearts in succession from a deck of 52 cards, assuming the first card is not replaced before the second is drawn, is

$$\frac{13}{52} \times \frac{12}{51} = \frac{13}{221} .$$

In the above example, the probability of drawing a heart on the second draw is conditional upon the first draw. This illustrates the concept of dependent probability.

Two events are independent if the probability of the occurrence of either is not affected by the occurrence of the other. For instance, the probability of getting heads in the second flip of a coin is independent of what happened in the first flip. The independence of A and B implies that $P(A) = P(A/B)$. Substituting $P(A)$ for $P(A/B)$ in the first equation of the "General Rule of Multiplication," one can write:

$$P(A \cap B) = P(A) \cdot P(B) \quad (12)$$

For example, if two successive rolls of balanced die are independent, the probability of getting two 1's is

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36} .$$

(In fact, $1/36$ is the probability of rolling two of any number).

The above example illustrates the concept of independent probability.

The definition of independence leads to the next theorem, found in Freund [1962].

Theorem 2.15: If A and B are independent and $P(B) \neq 0$ then $P(A) = P(A/B)$.

This theorem can be derived from the definition of independence and Theorem 2.14 above.

$$P(A \cap B) = P(B) \cdot P(A/B) = P(A) \cdot P(B) \quad (13)$$

Because $P(B) \neq 0$, dividing by this term one obtains: $P(A) = P(A/B)$.

The concept of independent probability is significant in the development of the algorithm for the explicit mode. In the explicit calculation the potentials of adjacent elements are independent of potential changes in one another within any time step. The independence of the potential distributions is the justification for the application of Theorem 2.15 above into the algorithm. In the implicit calculation the potential at a node depends on the potential changes at its neighbors. Therefore, Theorem 2.15 is not applicable and one must incorporate the concept of dependent probability.

5. Arithmetic Operations on Distributions

A numerical algorithm is developed, based on axiomatic foundations of probability theory, to perform arithmetic operations between probability distributions. The algorithm accepts two random variables A and B with distributions specified as histograms with N discrete values and associated probabilities. We are given A_i , $P(A_i)$, B_i , and $P(B_i)$, $i = 1, 2, 3, \dots, N$ where A_i and B_i denote the arithmetic average of the i^{th} interval in the histogram and $P(A_i)$ and $P(B_i)$ refer to the associated probabilities. We can write

$$\sum_{i=1}^N P(A_i) = \sum_{i=1}^N P(B_i) = 1.0 \quad (14)$$

Suppose we wish to add the two distributions to obtain their sum in the form of a third variable, C_i , with associated probabilities $P(C_i)$. C_i is obtained as follows

$$\begin{aligned} C'_{ij} &= A_i + B_j & i &= 1, 2, 3 \dots N \\ & & j &= 1, 2, 3 \dots N \end{aligned} \quad (15)$$

where C'_{ij} is the outcome if events A_i and B_j were to be realized simultaneously. According to the axiomatic foundations of probability theory, the probability of two independent events A_i and B_j occurring simultaneously is

$$\begin{aligned} P(C'_{ij}) &= P(A_i) \cdot P(B_j) & i &= 1, 2, 3 \dots N \\ & & j &= 1, 2, 3 \dots N \end{aligned} \quad (16)$$

and

$$\sum_{i=1}^N \sum_{j=1}^N P(C'_{ij}) = 1.0 \quad (17)$$

Therefore, the outcomes C'_{ij} consist of N^2 discrete values and associated probabilities. In order that the computational efforts are reasonable, it is desirable to express the variable C'_{ij} as an equivalent variable C_i with N discrete values, rather than the N^2 discrete values of the former. This is achieved by dividing the range from the minimum C'_{ij} to the maximum C'_{ij} into N equal intervals. The mean magnitude, C_i , of the i^{th} interval ($i = 1, 2, 3 \dots N$) can be defined in one of two ways. It can be the midpoint of that interval, or, as assumed in this paper, it can be the weighted average of all

the C'_{ij} values falling within that interval. The probability of the i^{th} interval is the sum of the probabilities of all the C'_{ij} 's falling within that interval.

III. THE NUMERICAL MODEL

A. The Explicit Equation

The output variable of primary interest is the time derivative of the potential head. The potential head at the end of the first time step, ϕ_m^1 , is the sum of the accumulation of fluxes across the bounding surfaces divided by the fluid mass capacity, and the initial potential head, ϕ_m^0 .

In keeping consistent with the expression in Figure 4, we shall define the conductance of the interface of volume elements ℓ and m , $U_{\ell m}$, as follows.

$$U_{\ell m} = \frac{K_{\ell m} A_{\ell m} \rho}{D_{\ell m}}$$

The values of $A_{\ell m}$, ρ , and $D_{\ell m}$ are deterministic and the expression $(A_{\ell m} \rho / D_{\ell m})$ is simply a constant. Because the probabilistic form of hydraulic conductivity, the resulting $U_{\ell m}$ array is also a stochastic structure.

The essential feature of the explicit formulation is that ϕ_m^0 is used to compute the the fluxes into node ℓ during Δt . This reasoning is justified under the restriction that Δt be sufficiently small. It can be demonstrated that the critical limit for Δt in the explicit calculation is given as:

$$\Delta t_{\text{crit}} = \frac{M_{c,\ell}}{U_{\ell,m}}$$

The Explicit Expression

$$\frac{\Delta t}{M_{c,l}} \sum_m K_{lm} A_{lm} \rho \left(\frac{\phi_m - \phi_l}{D_{lm}} \right) = \Delta \phi_l$$

Δt = time step (T)

$M_{c,l}$ = fluid mass capacity (M/L)

K_{lm} = hydraulic conductivity (L/T)

A_{lm} = boundary surface of the elements (L²)

D_{lm} = distance from the center of each element (L)

ρ = density (M/L³)

ϕ_m = potential head at node(s) m (L)

ϕ_l = potential head at node l (L)

$\Delta \phi_l$ = potential difference at node l (L)

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Figure 4.

For $\Delta t \geq \Delta t_{\text{crit}}$, the solution will oscillate eventually becoming unbounded. Therefore, $\Delta t < \Delta t_{\text{crit}}$ in order to perform the explicit calculation.

From the above, Δt_{crit} is directly related to the fluid mass capacity, $M_{c,l}$, of the volume element, l . It is also inversely proportional to the conductance of its bounding surface, U_{lm} . In general, a volume element may be communicating with more than one other element. In addition, the fluid mass capacity of one volume element may vary from that of an adjacent element. Therefore, the critical time step may differ from element to element.

In the routine developed for the probabilistic model, a constant stable time step is determined and applied for the duration of the problem throughout the mesh. The stable time step is calculated as a quotient of the minimum fluid mass capacity and the maximum conductance of the system. This represents the smallest critical time step for any volume element in the system. As a further precaution against oscillation, the critical time step is multiplied by some factor less than 1.0 (0.6 ~ 0.9). Thus we obtain the stable time step:

$$\Delta t_{\text{stab}} = F \cdot \frac{M_{c,l}}{\sum_m U_{lm}}$$

The stochastic form of the explicit equation can be written as follows:

$$\frac{\Delta t_{\text{stab}}}{M_{c,l}} \sum_m U_{lm}(\cdot) [\phi_m^o(\cdot) - \phi_l^o(\cdot)] = \Delta \phi_l(\cdot)$$

(•) denotes there is a probability associated with the stochastic variable or parameter

The parameter hydraulic conductivity, $K_{\ell m}$, is input as a statistical distribution, resulting in the stochastic conductance array. The initial heads, ϕ_{ℓ}^0 and ϕ_m^0 , may also be input as probabilistic structure. Whether they are or not, if $K_{\ell m}$ is in the form of a stochastic array, after the calculation of the first time step statistical distributions will begin to appear in ϕ_m^1 and ϕ_{ℓ}^1 . The temporal change of ϕ at ℓ , denoted by $\Delta\phi_{\ell}$, is the output variable.

B. Representation of Probabilistic Structures

The method used to represent the probabilistic structures is fundamental to the understanding of the algorithm. Consider the following "continuous" distribution.

The above distribution is divided into five discrete (not necessarily equal) intervals. This distribution is then "converted" to a histogram, where the size of each interval (ΔX_n) and the probability associated with that interval are illustrated. The likelihood that any one value from the entire distribution should fall within a given interval is the sum of the probabilities associated with all the discrete values which lie in that interval. After calculating the average (or mean)

values of each interval, the histogram can be expanded in terms of an array.

The average value of each interval could be defined as the midpoint of the particular interval, or, a weighted average of all the discrete values falling within that interval. The approach used in this work is the latter.

The following histograms represent the distribution of potentials at nodes 1 and 2 in a simplified manner.

These histograms can be expressed in the form of the following three dimensional arrays. (rows, columns, layers)

$$\begin{array}{ccc}
 \text{Node 1} & & \text{Node 2} \\
 \left[\begin{array}{c} n_{111}, n_{121} \\ n_{211}, n_{221} \\ n_{311}, n_{321} \end{array} \right] & = & \left[\begin{array}{c} 1.0, 0.25 \\ 3.0, 0.50 \\ 5.0, 0.25 \end{array} \right] \qquad \left[\begin{array}{c} n_{112}, n_{122} \\ n_{212}, n_{222} \\ n_{312}, n_{322} \end{array} \right] = \left[\begin{array}{c} 9.0, 0.25 \\ 11.0, 0.5 \\ 13.0, 0.25 \end{array} \right]
 \end{array}$$

In the terminology developed in this algorithm, each three-dimensional array is dimensioned as: A(NDISCR, MORP, N). The indices are defined as follows:

NDISCR = number of discrete values used to represent the probabilistic structure

MORP = magnitude or probability, this index is always equal to one (1) or two (2).

N = used to "identify" the array with its appropriate element (in the case of a potential or permeability) or its appropriate connection (in the case of a parameter value).

It should be noted that the two arrays shown above are actually two-dimensional arrays which make up a larger three-dimensional array. The array associated with node 1 is the first "layer" of this three-dimensional array, as noted by its third index. The node 2 array comprises the second layer of the same three-dimensional array.

Consider the following histograms:

Distribution of Potential Difference
Between Nodes on Either Side of
the Fourth Connection

Distribution of Conductance at
the Interface of Nodes 4 and 5

These histograms can be expressed in terms of the following arrays

$$\begin{array}{c} \text{A} \\ \left[\begin{array}{cc} a_{114}, & a_{124} \\ a_{214}, & a_{224} \\ a_{314}, & a_{324} \end{array} \right] = \left[\begin{array}{cc} 5.0, & 0.25 \\ 7.0, & 0.50 \\ 9.0, & 0.25 \end{array} \right] \end{array}
 \qquad
 \begin{array}{c} \text{B} \\ \left[\begin{array}{cc} b_{114}, & b_{124} \\ b_{214}, & b_{224} \\ b_{314}, & b_{324} \end{array} \right] = \left[\begin{array}{cc} 0.5, & 0.25 \\ 1.0, & 0.50 \\ 1.5, & 0.25 \end{array} \right]
 \end{array}$$

As noted by its third index, array A is the fourth layer of a three-dimensional array. This array is the probabilistic representation of the potential difference between nodes 4 and 5. Array B is the fourth layer of the three-dimensional conductance array, referring to the fourth connection.

With an understanding of the technique used to model a probability distribution, we shall move on to the algorithm which applies these arrays in the desired arithmetic operations.

In the discussion of probability theory we developed the "general rule of multiplication" for independent events or sets. We have also defined probability as the measure of the occurrence of an event. Therefore, the "general rule of multiplication" applies similarly for probabilistic distributions. In applying this principle, it becomes

clear that whether we are concerned with the sum (union) or logical product (intersection) of two probability distributions, the probability of these two events occurring simultaneously is the product of the discrete probabilities. The appropriate operation is performed on the magnitudes of the two distributions.

Let us suppose we are interested in finding the sum of two probability distributions represented by the arrays below.

$$\begin{array}{cc} \text{Array A} & \text{Array B} \\ \begin{bmatrix} a_{111} & a_{121} \\ a_{211} & a_{221} \\ a_{311} & a_{321} \end{bmatrix} & \begin{bmatrix} b_{111} & b_{121} \\ b_{211} & b_{221} \\ b_{311} & b_{321} \end{bmatrix} \end{array}$$

The Magnitude of Array A + Array B = COP(i,j,1)

$$\begin{bmatrix} a_{111} \\ a_{211} \\ a_{311} \end{bmatrix} + \begin{bmatrix} b_{111} \\ b_{211} \\ b_{311} \end{bmatrix} = \begin{bmatrix} (a_{111} + b_{111}) & (a_{211} + b_{111}) & (a_{311} + b_{111}) \\ (a_{111} + b_{211}) & (a_{211} + b_{211}) & (a_{311} + b_{211}) \\ (a_{111} + b_{311}) & (a_{211} + b_{311}) & (a_{311} + b_{311}) \end{bmatrix}$$

$$i = 1, 2, 3$$

$$j = 1, 2, 3$$

The Probability of Array A + Array B = COP(i,j,2)

$$\begin{bmatrix} a_{121} \\ a_{221} \\ a_{321} \end{bmatrix} \cdot \begin{bmatrix} b_{121} \\ b_{221} \\ b_{321} \end{bmatrix} = \begin{bmatrix} (a_{121} \cdot b_{121}) & (a_{221} \cdot b_{121}) & (a_{321} \cdot b_{121}) \\ (a_{121} \cdot b_{221}) & (a_{221} \cdot b_{221}) & (a_{321} \cdot b_{221}) \\ (a_{121} \cdot b_{321}) & (a_{221} \cdot b_{321}) & (a_{321} \cdot b_{321}) \end{bmatrix}$$

$$i = 1, 2, 3$$

$$j = 1, 2, 3$$

The "sum" of arrays A and B involves the repeated sum of each element of A with each element of B.

$$\begin{bmatrix} a_{111} \\ a_{211} \\ a_{311} \end{bmatrix} + \begin{bmatrix} b_{111} \\ b_{211} \\ b_{311} \end{bmatrix}$$

The result is a two dimensional array which can be thought of as the n^{th} layer of the larger three-dimensional COP array, where n is the third index of the subscript denoting the node number or the connection number.

We have to associate a probability with each element of the aforementioned two-dimensional array. This is achieved by generating a "product" of the two one-dimensional arrays:

$$\begin{bmatrix} a_{121} \\ a_{221} \\ a_{321} \end{bmatrix} \cdot \begin{bmatrix} b_{121} \\ b_{221} \\ b_{321} \end{bmatrix}$$

Analogous to the previous case, we perform a repeated multiplication, resulting in a two-dimensional array of associated probabilities.

In this probabilistic model the second index of arrays A and B is always 1 or 2, referring to the magnitude or probability.

1. Subroutine PRODIST

The part of subroutine PRODIST which computes the sum of two probabilistic structures is given below. A more complete description of this subroutine can be found in the next section.

C DETERMINE THE SUM OF THE TWO DISTRIBUTIONS, A AND B

DO 110 I = 1, NDISCR

DO 120 J = 1, NDISCR

C CALCULATE THE SUM OF THE MAGNITUDES

COP(I,J,1) = A(I,1,1) + B(J,1,1)

C CALCULATE THE PRODUCT OF THE PROBABILITIES

COP(I,J,2) = A(I,2,1)*B(J,2,1)

120 CONTINUE

110 CONTINUE

NDISCR is an integer read into the program which defines the number of discrete values used to model the probabilistic structures.

A similar approach is applied to calculate the "difference", the "product" or the "quotient" of two probabilistic distributions. As a matter of convenience, the quotient is treated as reciprocal product, with the condition that the denominator in the reciprocal is non-zero.

2. Subroutine COLAPS

The resulting three-dimensional COP(NDISCR, NDISCR, 2) array is in a form difficult to use and manipulate. In the call to PRODIST from FINK the arrays A(NDISCR, 2, 1) and B(NDISCR, 2, 1) are sent, operating on which, PRODIST generates COP(NDISCR, NDISCR, 2). PRODIST, in turn, calls subroutine COLAPS, in which the array COP is reduced to a smaller array, C(NDISCR,2,1), having the same dimensions as arrays A and B. In fact, array C denotes the result of the prescribed arithmetic operations on arrays A and B. The subroutine COLAPS performs the "collapsing" operation.

```
C  CALL COLAPS SUBROUTINE WHICH CONVERTS NDISCR*2 NO. OF POINTS TO
C  NDISCR NO. OF POINTS

      CALL COLAPS (COP, C, MAGPOT)

      GO TO 500
      .
      .
      .
      .
      .
500  RETURN
```

The integer MAGPOT is sent in the call for COLAPS (as well as the call for PRODIST) in order to determine whether a minimum and maximum tolerance level on the output variable (magnitude of COP array) is required. Such a tolerance (PSIMIN, PSIMAX) is associated with any boundary condition which might be specified. In the analysis of steady-state flow between two specified boundary heads the minimum and maximum values would represent the boundary conditions at $x = 0$ and $x = L$. This routine is required to avoid the propagation of physically meaningless values beyond the boundary conditions. When MAGPOT is given as an integer value greater than one (1) the appropriate tolerances are considered in subroutine COLAPS. $MAGPOT > 1$ precedes the calculation for potential at each node.

The operation on A and B generates the COP array having $(NDISCR)^2$ events with associated magnitudes and probabilities. However, the desired form of the resulting array should have NDISCR magnitudes and probabilities, consistent with arrays A and B.

The COLAPS routine accepts the three-dimensional COP array and splits it into two one-dimensional arrays. The first layer is assigned to CMAG(NDISQU) (magnitude elements) and the second layer becomes CPROB(NDISQU) (probability elements).

```
C  CONVERT 3-DIM ARRAY TO 1-DIM ARRAY

      NC = 0

      DO 10  I=1,NDISCR

      DO 10  J=1,NDISCR

      NC = NC+1

C  ASSIGN COP TO CMAG AND CPROB, 1-DIM ARRAYS

      CMAG(NC) = COP(I,J,1)

      10 CPROB(NC) = COP(I,J,2)
```

The three-dimensional array is converted to the two more simplified one-dimensional arrays which represent the discrete magnitudes [CMAG(NDISQU)] and their corresponding probabilities [CPROB(NDISQU)]. In other words, the probability of CMAG(1) is the element CPROB(1), and so on. The length of these one-dimensional arrays is the square of the dimension of the input arrays A and B, $NDISQU = NDISCR^2$.

The next task is to reduce the dimensions of the arrays CMAG and CPROB from NDISQU to NDISCR, because this is the desired dimension for the final C array. This routine requires the minimum and maximum values of CMAG. Using these values, the probabilistic structure is divided into NDISCR equal intervals. The upper value of each interval is determined (CMAGIN) as well as the midpoint value (CMID).

The code for the procedure follows.

```
C  DETERMINE THE DIFFERENCE BETWEEN XMAX AND XMIN

      DIFF = XMAX - XMIN

      IF (DIFF .EQ. 0.0) GO TO 600

C  DIVIDE THE DIFFERENCE BETWEEN XMAX AND XMIN INTO NDISCR

C  EQUAL INTERVALS

      DINCR = DIFF/NDISCR

C  INITIALIZE INTERVAL MAGNITUDE ENDPOINTS

      CMAGIN(1) = XMIN + DINCR

C  INITIALIZE INTERVAL MIDPOINT VALUES

      CMID(1) = XMIN + (DINCR/2.0)

C  ASSIGN ENDPOINT AND MIDPOINT VALUES

      DO 39 M =1, NDISCR - 1

      CMAGIN(M+1) = CMAGIN(M) + DINCR

      CMID(M+1) = CMID(M) + DINCR

39  CONTINUE
```

In the next part of the routine the probability of each interval [PROB(I)] is initialized to zero. The same is done with the product of the magnitude and probability of each interval [PRODIT(I)]. With these intervals defined, the next task is to run through the magnitude (CMAG) array and place each discrete value in the interval which it falls.

```
      DO 40 I = 1, NDISCR
C   INITIALIZE THE PROBABILITY OF EACH INTERVAL
      PROB(I) = 0.0
C   INITIALIZE THE PRODUCT OF THE PROBABILITY AND MAGNITUDE
C   OF EACH INTERVAL
      PRODIT(I) = 0.0
40  CONTINUE
      DO 45  NC = 1, NDISQU
      DO 50  M = 1, NDISCR
C   DETERMINE WHETHER THE MAGNITUDE FALLS WITHIN THE INCREMENT
      IF (CMAG(NC) .GT. CMAGIN(M)) GO TO 50
C   ADD THE PROBABILITY TO THE MOST RECENT VALUE
      PROB(M) = PROB(M) + CPROB(NC)
C   DETERMINE THE PRODUCT OF THE PROBABILITY AND THE MAGNITUDE
      PROD(M) = CPROB(NC) * CMAG(NC)
C   ADD THE PRODUCT TO THE MOST RECENT VALUE
      PRODIT(M) = PRODIT(M) + PROD(M)
      GO TO 45
50  CONTINUE
45  CONTINUE
```

In the following routine the magnitude of each increment is determined.
From the preceeding code,

$$\text{PRODIT}(M) = \sum \text{CPROB}(\text{NC}) \cdot \text{CMAG}(\text{NC})$$

for each interval, M. With PRODIT(M) defined, it is possible to calculate the weighted average of each interval, by division with its respective probability, PROB(M). If the probability of an interval is zero, the magnitude is assigned to zero. If the interval's probability is less than the specified "meaningful" minimum, TOLPR, the magnitude and probability of that interval are set equal to zero. The elements in the probability array are normalized by a summation routine. Finally, the magnitudes and probabilities are assigned to the three-dimensional C array, which is sent back to subroutine FINK. The FORTRAN follows:

```
C  DETERMINE THE MAGNITUDE OF EACH INTERVAL

      DO 55 M=1, NDISCR

      IF (PROB(M) .EQ. 0.0) GO TO 60

      AVGMAG(M) = PRODIT(M)/PROB(M)

      GO TO 55

C  ASSIGN ZERO MAGNITUDES TO THE INTERVAL AVERAGE VALUES IF

C  THE PROBABILITY FOR THAT INTERVAL IS ZERO

      60 AVGMAG(M) = 0.0

      55 CONTINUE

C  SEARCH FOR PROBABILITIES LESS THAN TOLPR

      DO 80 NHIST = 1, NDISCR

      IF (PROB(NHIST) .LT. TOLPR) GO TO 85

      GO TO 80

      85 PROB(NHIST) = 0.0

      AVGMAG(NHIST) = 0.0

      80 CONTINUE
```

```
C  NORMALIZE THE PROBABILITIES

      PROBSUM(1) = 0.0

      DO 75  M=1,NDISCR

      PROBSUM(M+1) = PROBSUM(M) + PROB(M)

75  CONTINUE

C  ASSIGN AVGMAG AND PROB ARRAYS TO THE C ARRAY

      DO 95  M=1,NDISCR

      C(M,1,1) = AVGMAG(M)

      C(M,2,1) = PROB(M)/PROBSUM(MDISCR)

95  CONTINUE

      GO TO 1000

      .
      .
      .
      .
      .
      .

1000 RETURN

      END
```

The subroutine COLAPS is outlined in the flow chart given in Appendix C.

C. Computer Program PROGRES

The subroutine PRODIST and COLAPS described in the numerical model are incorporated into the computer program TERZAGI, to give rise to a new code called PROGRES. TERZAGI is an integrated finite-difference [Narasimhan and Witherspoon, 1976] computer program for solving steady-state or transient potential distributions in general three dimensions with one-dimensional consolidation under saturated conditions of flow. The geometry may involve complex shapes and

structures. Initial conditions may vary with spatial position, and upper and lower limits may be placed on the size of the primary dependent variable. Upper limits may also be assigned to the time of the problem, on the number of steps, or on the computer time. Solutions are obtained by use of explicit or implicit-difference equations, or by an optimized combination of both.

1. Nature of Modifications

The "original" TERZAGI program solves steady-state and transient flow problems using a deterministic technique. The task of generalizing the program in order that a probabilistic problem can be solved requires the insertion of statistical distributions for input parameters, input variables and output variables. An input or output variable (pressure head) which is deterministically modeled is represented as a one-dimensional array, the length of the array dependent upon the number of nodes in the mesh. However, in the probabilistic model there is a statistical distribution associated with the pressure head, and this requires a three-dimensional array, PHI(NDISCR, MORP, N). The first index indicates the number of discrete values used to model the probabilistic structure (in other words, the number of intervals which comprise the histogram); NDISCR. The second index refers to the magnitude (1) or probability (2), associated with each interval; MORP. The third index is used to identify the node number which the distribution is associated with. Therefore, while the deterministic TERZAGI has one-dimensional arrays for input and output variables PROGRES includes three-dimensional arrays for these variables.

The deterministic TERZAGI program models many of the parameters in two-dimensional arrays. The problem of heterogeneities makes such two-dimensional arrays necessary, the first index referring to the discrete value and the second to the material number. In the probabilistic model the parameters (e.g., permeability CON(NDISCR, MORP, N)) are represented by three-dimensional arrays, in much the same way as the variables. The first and second indices have the same meaning as described above. The third index refers to the node number permeability (input parameters) or in the case of conductance, the connection number (parameter calculated within program).

The insertion of the probabilistic variables and parameters require that they be redimensioned as three-dimensional arrays. The arithmetic operations performed between these three-dimensional arrays take place in the subroutine, PRODIST (Probability Distribution).

2. Subroutine PRODIST

The subroutine PRODIST is called within the subroutine FINK every time two probabilistic arrays are used in an arithmetic operation. Five arguments are used in the call for PRODIST: CALL PRODIST(A,B,C, MAGPOT, MSIGN).

The arrays A and B represent the input distributions which are used in the arithmetic operation. As these distributions refer to a specific node or connection, the third index will never vary within the subroutine PRODIST. Therefore, A and B are in effect two-dimensional arrays. The sum, difference, or product of A and B will be a three dimensional array, COP, referred to in the numerical model. MSIGN denotes the arithmetic mode desired. The COP array is reduced

in the following subroutine COLAPS, to the two-dimensional C array which eventually is sent back to subroutine FINK. The argument MAGPOT is used in the call for COLAPS as well.

3. Subroutine COLAPS

The COLAPS subroutine is called from PRODIST: CALL COLAPS(COP, C, MAGPOT).

MAGPOT specifies whether the maximum and minimum tolerances for output variable are required. In this subroutine the three-dimensional array COP(NDISCR, NDISCR, MORP) is converted to C(NDISCR, MORP, N), which is of the same dimensions as the input arrays A and B. The third index in the C array remains constant within COLAPS, as is the case of A and B; array C is in effect a two-dimensional array when it is sent back to FINK. In FINK, array C is assigned to its respective node or connection number, and the third index takes on its appropriate value.

4. Description of Original Subroutines

The program TERZAGI is subdivided into modular subroutines, each subroutine having an important function. Modifications are required in certain subroutines, because of the insertion of the three-dimensional arrays described earlier. Subroutines not required in the explicit calculation have been detached in generating PROGRES. Associated with some of the original TERZAGI subroutines are "blocks" which are used for the input of data. A description of those subroutines fundamental to the understanding of these input blocks follows.

a. PROGRES (formerly TERZAGI)

This subroutine "controls" the main program. Within PROGRES the other subroutines are called to read input data and to do time step calculations. When the calculations are completed PROGRES stops the problem.

b. THERM

The subroutine THERM accepts input data in BLOCK 2 for material properties. It is in BLOCK 2 that the probabilistic distribution (or, for the heterogeneous problem, distributions) is read in for permeability as the array CONT(NHIST, MORP, 1). A control card assigns these arrays to their appropriate nodes, redefined as the CON(NHIST, MORP, N) array.

The volumetric properties of the elements are input in BLOCK 4. There are no revisions required in this block.

c. FINK

In subroutine FINK the explicit calculation for Darcian gradient is found (see Figure 4). As a result, this subroutine has undergone the maximum change in creating PROGRES.

The properties of the internal connections are read into BLOCK 5. The format for this block is unchanged. Using the internal connection data, and the permeability arrays, CON(NDISCR, MORP, N) of each element, the conductance of each connection is calculated as the TRAN(NDISCR, MORP, N) array.

The explicit calculation is made for each connection, in every time step. The size of the stable time step (DELT) is calculated by dividing the capacitance of a "typical" or interior volume element

[CAP(N)] by the maximum conductance [ZIPMAX], and scaling by some predetermined input factor, DELTF (0.6 ~ 0.9). The maximum conductance is determined by a technique which screens the conductance arrays for the largest value in the distribution. An appropriate choice of DELTF will allow for an even-valued time step.

The connection number is the counter for the first loop which calculates the change of mass in each element. The mass flow rate is computed for every time step based on the preassigned potentials at each volume element. The first call for PRODIST within this loop calculates the difference in potentials at connected nodes N1 and N2 [PHI(NDISCR, MORP, N1), PHI(NDISCR, MORP, N2)]. (The statistical arrays for potential at each node are input in BLOCK 9, subroutine TALLY.) The subtraction operation is achieved in this call for PRODIST with MSIGN = 2. The result is assigned to the array PHID(NDISCR, MORP, 1). The third index in this array will always be one (1). It is insignificant to assign the potential difference to a particular connection because it is used in the next call to PRODIST. Therefore, the values of this array change as one marches through the mesh.

The second call for PRODIST involves the calculation of the mass flow rate [DFI(NDISCR, MORP, N)], the product of the conductance (TRAN) and potential difference (PHID) arrays for the connection in question. MSIGN = 3 for this operation.

Before PRODIST is called a third time the product of the mass flow rate (DFI) and time step (DELT) is computed. Because DELT is a deterministic value, this calculation entails scaling DFI(NDISCR, 1, N), the magnitudes of the mass flow rate array (MORP = 1) by DELT.

This "time-scaled" array represents the change in mass between the two particular elements, and is reassigned to $DFI(NDISCR, 1, N)$.

The probability elements of this array remain unchanged.

PRODIST is next called to determine the change in mass in each individual volume element. The array $DF(NDISCR, MORP, N)$ representing the volume of fluid in each element is probabilistically initialized to zero before the first time step. This entails associating a probability of 1.0 with a magnitude of 0.0 for one pair of elements in the array, all the rest of the magnitude and probability elements in the array are set equal to zero (0.0). The mass change at the first connection ($N1 = 1$), $DFI(NDISCR, MORP, N1)$ is added to the mass in node $N1$, $DF(NDISCR, MORP, N1)$ in PRODIST ($MSIGN = 1$). The same mass change array is subtracted from the mass in node $N2$ ($N2 = 2$), in another call for PRODIST ($MSIGN = 2$). With these two calls for PRODIST the mass of fluid in each node is adjusted for the potential differences of its connections.

This procedure is repeated for every connection in the mesh, for the specified time step. This completes the loop which uses the connection number as its counter. The node number is the counter for the next loop. Within this loop the potential gradient of each element is computed and the new potentials for every node are computed for that time step.

The change in potential is calculated for each node by dividing the mass array [$DFI(NDISCR, 1, N)$] by its respective capacitance [$CAP(N)$]. Because the capacitance is a single-number representation, this calculation involves scaling $DFI(NDISCR, 1, N)$, the magnitudes

of the statistical distribution. The result is assigned to $DPSI(NDISCR, MORP, N)$, the potential across node N . The change in potential ($DPSI$) is added to the initialized potentials $[PHI(NDISCR, MDRP, N)]$ by calling $PRODIST (MSIGN = 1)$. The adjusted potential is reassigned to the PHI array. The distributions from the PHI and $DPSI$ arrays are printed out at the desired time steps.

d. TALLY

Three input blocks are governed in subroutine $TALLY$. The control parameters are read in $BLOCK 1$. The original program includes three data cards in this block. Two new input control parameters have been added to the first data card. The number of discrete values chosen to model the probabilistic distributions, $NDISCR$, is a new parameter. The factor $DELTF$ used in the calculation for time step is also included in this data card. The second and third input cards remain unchanged. A fourth data card is added to $BLOCK 1$, where the value of $TOLPR$ is read. $TOLPR$ sets the tolerance on the minimum probability which will be permitted in the problem. This tolerance is required to prevent the propagation of very small probability values which result from repeated multiplications. The smallest numerical value for probability allowed in these simulations is 1×10^{-5} or 0.001%. Probabilities less than 0.001% are assigned to zero and the resulting distribution is normalized.

The fluid properties are designated in $BLOCK 3$. There are no revisions required in this block. The initial conditions are read in $BLOCK 9$. In this block the initial distributions for the potentials at each node are assigned, requiring revision of the input format.

The final conditions for potential distributions can be punched in the input format and used in BLOCK 9 in restarting the problem.

IV. APPLICATIONS AND RESULTS

Section A of this chapter is concerned with the verification of the algorithm. This validation procedure involves the application of some basic principles of Boolean Algebra. Section B is devoted to a discussion of the results obtained in the study.

A. Verification of Arithmetic Operations

In our discussion of the "Algebra of Distributions" the laws of Boolean Algebra were set forth. These postulates must be considered in conjunction with the algorithm developed for arithmetic operations between probabilistic structures. The subroutines PRODIST and COLAPS were employed during the early part of this study to investigate the relationship of the algorithm to the various laws. A discussion of these studies follows.

1. Commutative Law of Addition

$$A + B = B + A$$

The commutative law of addition applied to the probabilistic model states that the sum of the probabilistic distributions A and B is independent of their order in the operation. In other words, in the call for PRODIST, the result is not dependent upon the order in which arrays A and B are sent, if commutativity holds for addition with the algorithm. In Figure 5 histograms are shown for the results, $A + B$ (upper figure) and $B + A$ (lower figure). These histograms illustrate the graphic form of the resulting array, for magnitude versus probability. The results for the two operations are identically equal, indicating that commutativity applies for addition in the algorithm. The values of mean (μ) and standard deviation (S_0) for the probabilistic structure

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Figure 5. The Commutative Law of Addition.

are also given. The mean is determined using a graphical technique. The mean μ , of the distribution is defined in Appendix A. This technique uses the computed weighted averages of each interval as the average value for the respective interval. The standard deviation defines the magnitude of the increment above and below the mean in which range 68% of the area of the histogram lies.

2. Commutative Law of Multiplication

$$A \cdot B = B \cdot A$$

The commutative law of multiplication asserts that the product of the probabilistic arrays A and B is independent of their order in the call for PRODIST. In Figure 6 the histograms for $A \cdot B$ (upper figure) and $B \cdot A$ (lower figure) are shown. These histograms represent the form of the resulting arrays. Because these distributions are graphically and numerically identical for the two cases, commutativity for multiplication holds for the algorithm. The mean values and standard deviations associated with these histograms are necessarily equal.

3. Associative Law of Addition

$$(A + B) + C = A + (B + C)$$

Consideration of this postulate involves the order in which the three probabilistic distributions are summed. On the left side of this equation the sum of $A + B$ is first determined. The resulting distribution is then added to distribution C. The expression to the right of the equal sign entails the sum of $B + C$ initially. This result is then added to distribution A. Figure 7 indicates that the associative law for addition applies to the algorithm, because the histograms for $[(A + B) + C]$ (upper figure) and $[A + (B + C)]$ are

Figure 6. The Commutative Law of Multiplication.

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Figure 7. The Associative Law of Addition.

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identical. Therefore, the mean and standard deviations are also equal. We can conclude that the sum of a series of probabilistic structures is independent of the order of operations.

4. Associative Law of Multiplication

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

The associative law of multiplication asserts that the repeated product of any number of probabilistic structures is independent of the order of operation. The product of distributions A and B is calculated and multiplied by distribution C, for the arithmetic operation on the left side of the equal sign. The operation to the right of the equal sign considers the product of B and C initially. The distribution A is then multiplied by the resulting distribution of B·C. In Figure 8 the upper histogram represents [(A·B)·C] and the lower figure is the result of [A·(B·C)]. These histograms are not graphically identical. However, it is more significant to investigate the mean and standard deviation associated with each distribution in the analysis of associativity. The interval size and their corresponding probabilities are not consistent from one histogram to the other. Whereas associativity is well-defined for addition, multiplication introduces numerical error propagation with repeated operations of distributions. The representations of the resulting histograms vary somewhat, but the mean values and standard deviations do compare quite favorably. This fact is thought to be more meaningful than the graphical comparison. Therefore, associativity for multiplication is assumed to apply for the algorithm.

Figure 8. The Associative Law of Multiplication.

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5. Distributive Law

$$(A \cdot B) + (A \cdot C) = A \cdot (B + C)$$

The distributive law is also significant in the algorithm. Within the explicit expression a similar operation takes place where the difference of the potential distributions of adjacent nodes is multiplied by the conductance array. The expression to the left side of the equal sign above requires three operations (two products and one sum) while only two operations (one sum and one product) are involved on the right side of the equation. Therefore, the expression to the right is more desirable as it is less costly to perform.

The upper histogram in Figure 9 represents the resulting array of $(A \cdot B) + (A \cdot C)$. The lower figure illustrates that distribution obtained for $A \cdot (B + C)$. There are obvious graphical discrepancies between the two histograms. As noted in the discussion of associativity of multiplication earlier, it is more meaningful to compare the mean values and standard deviations of these histograms, than the explicit widths or heights of the individual bars. The distributions' means and standard deviations for the distributive law compare favorably with each other. This agreement is more significant than the graphical differences, and therefore we shall conclude that the distributive law applies to the algorithm. In the explicit expression found in subroutine FINK an analogous operation to that on the right side of the distributive equation is performed.

Figure 9. The Distributive Law.

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B. Solution of Flow Problems

The computer program PROGRES is applied to solve five general problems. The complexity of the analyses increases somewhat with each problem. The parametric studies of probabilistic simulations involve variation of the standard deviation associated with the probability distributions on hydraulic conductivity (K_{STD}). As explained in the section, The Explicit Equation, the length of the time step is of great importance. Because the critical time step is inversely proportional to the maximum conductance, variation of the hydraulic conductivity distribution will have an effect on the length of the time step. Therefore, the length of the time step will vary in the deterministic and probabilistic simulations. The constant (DELTF) is specified to insure $\Delta t_{stab} < \Delta t_{crit}$. A list of the problems studies, their respective DELTF's and time steps(DELTF) follow.

<u>Nature of Problem</u>	<u>Constant (DELTF)</u>	<u>Time Step (DELTF)</u>
1. One-Dimensional, Steady-State Flow		
Through Homogeneous Media		
A. Deterministic Solution	0.30	0.10
B. Probabilistic Solution, $K_{STD} = 0.5$	0.75	0.01
C. Probabilistic Solution, $K_{STD} = 0.25$	0.88125	0.05
2. One-Dimensional, Transient Flow Through		
Homogeneous Media		
A. Deterministic Solution	0.30	0.10
B. Probabilistic Solution, $K_{STD} = 0.5$	0.75	0.01
C. Probabilistic Solution, $K_{STD} = 0.25$	0.88125	0.05

<u>Nature of Problem</u>	<u>Constant (DELTF)</u>	<u>Time Step (DELT)</u>
3. One-Dimensional, Steady-State Flow Through Heterogeneous Media		
A. Deterministic Solution	0.40	0.10
B. Probabilistic Solution, $K_{STD} = 0.25$	0.88125	0.025
4. One-Dimensional, Transient Flow Through Heterogeneous Media		
A. Deterministic Solution	0.40	0.10
B. Probabilistic Solution, $K_{STD} = 0.25$	0.88125	0.025
5. Two-Dimensional, Steady-State Flow Through Heterogeneous Media		
A. Deterministic	0.40	0.10
B. Probabilistic Solution, $K_{STD} = 0.25$	0.88125	0.025

1. One-Dimensional, Steady-State Flow Through Homogeneous Media

The first analysis is of one-dimensional, steady-state flow between two specified boundary heads in a homogeneous medium. The 100.0 cm. linear system consists of internal nodes 2-11, each with dimensions 10.0 x 10.0 x 10.0 cm (see Figure 10). Boundary nodes 1 and 12 are of "infinite" volume relative to the interior elements, so that their potentials remain constant. The initial conditions are such that the potential at each interior node is $\phi_2 = 10.0$ cm, $\phi_3 = 20.0$ cm, $\phi_4 = 30.0$ cm, $\phi_5 = 40.0$ cm, $\phi_6 = 50.0$ cm, $\phi_7 = 60.0$ cm, $\phi_8 = 70.0$ cm, $\phi_9 = 80.0$ cm, $\phi_{10} = 90.0$ cm, and $\phi_{11} = 100.0$ cm. The initial conditions are assigned in this manner (versus $\phi_2 - \phi_{11} = 100.0$ cm) to facilitate

The Physical Model for One-Dimensional Steady-State Flow Through Homogeneous Media

Initial Conditions:

$\phi_2 = 10.0 \text{ cm.}$ $\phi_7 = 60.0 \text{ cm.}$
 $\phi_3 = 20.0 \text{ cm.}$ $\phi_8 = 70.0 \text{ cm.}$
 $\phi_4 = 30.0 \text{ cm.}$ $\phi_9 = 80.0 \text{ cm.}$
 $\phi_5 = 40.0 \text{ cm.}$ $\phi_{10} = 90.0 \text{ cm.}$
 $\phi_6 = 50.0 \text{ cm.}$ $\phi_{11} = 100.0 \text{ cm.}$
 $K = 1.0 \text{ cm./sec.}$

Figure 10. The Physical Model for One-Dimensional Steady-State Flow Through Homogeneous Media.

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rapid approach to the steady-state solution. The boundary conditions state that $\phi_1 = 0.0$ cm and $\phi_{12} = 100.0$ cm.

The parameter of interest is the statistical variance (standard deviation) of the hydraulic conductivity, K . The variation of this parameter allows for the study of the resulting statistical distributions of the output head. The deterministic problem assumes a value of $K = 1.0$ cm/sec and an associated probability of 1.0. In probabilistic terms these conditions correspond to $\mu_K = 1.0$ cm/sec (mean) and $\sigma_K = 0.0$ cm/sec (standard deviation). The first probabilistic case models the log normal probability density function for hydraulic conductivity with $\mu_K = 1.0$ cm/sec and $\sigma_K = 0.50$ cm ($K_{STD} = 0.50$), the second case considers the values of $\mu_K = 1.0$ cm/sec and $\sigma_K = 0.25$ cm/sec ($K_{STD} = 0.25$). In both cases the coefficient of specific storage was assumed to be a deterministic quantity ($S_g = 1.0$).

The results of the deterministic solution indicate a linear drop in hydraulic head between the two boundary heads. In Figure 11 the hydraulic head is plotted versus distance, x . The 45° line represents the steady-state deterministic solution obtained after 780 cycles. The probabilistic solution (also obtained after 780 cycles) for $K_{STD} = 0.5$ of mean hydraulic head is denoted by the darkened points, which tend to fall on or near the deterministic solution. These points represent the mean value of the resulting distributions of output head. The cross-bars indicate the spread of one standard deviation above and below the mean value (where 68% of the values lie). The plot of standard deviation (S_o) of output head distribution versus distance will be considered later.

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Figure 11. One-Dimensional Steady-State Flow Through Homogeneous Media:
Probabilistic Solution for $K_{STD} = 0.5$ (cycle No. 780).

In Figure 12 the probabilistic solution for $K_{STD} = 0.25$, ϕ versus distance is illustrated for cycle number 780. The 45° line representing the deterministic solution is shown for purposes of reference. It can be observed that the mean values of hydraulic head ($\bar{\phi}$) lie closer to the deterministic solution than in the previous probabilistic case of $K_{STD} = 0.5$. Such behavior occurs because the level of uncertainty associated with the hydraulic conductivity has been decreased. In addition, the cross-bars denoting the standard deviation of the distributions are of lesser length than those generated with $K_{STD} = 0.5$.

Figure 13 is an excerpt from Freeze's [1975] paper. The upper insert shows the frequency distribution of output head at two values of distance, x . Similar behavior is noted in results of this study; normal distributions in the middle of the mesh, skewed distributions near the boundaries reflecting truncation of the range of permissible values that occur there (see Figure 14). The lower left graph of Figure 13 is a plot of mean hydraulic head versus distance, for $K_{STD} = 0.5$. On the right, the standard deviation of output head (S_o) is plotted versus distance. In Figure 15 the influence of input K_{STD} on the standard deviation (S_o) of the output head is illustrated for Freeze's results. Greater input uncertainties result in larger standard variances of output distributions.

S_o versus distance for the results of this study are shown in Figures 16 and 17 for the case of $K_{STD} = 0.5$ and $K_{STD} = 0.25$, respectively. The solutions for cycle number 780 correspond to the plots for ϕ versus distance in Figures 11 and 12. The solution at

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Figure 12. One-Dimensional Steady-State Flow Through Homogeneous Media:
Probabilistic Solution for $K_{STD} = 0.25$ (Cycle No. 780).

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Figure 13. Output plots of the mean value of predicted hydraulic head $\bar{\phi}(x)$ and its standard deviation $S_\phi(x)$ for stochastic-conceptual analysis of one-dimensional steady state flow between two specified boundary heads. Upper inset shows the frequency distribution of ϕ at two values of x , after Freeze [1975].

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Figure 14. Frequency Distributions of Hydraulic Head for Various Nodes Throughout the System ϕ vs. Probability.

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Figure 15. Freeze: The Influence of Standard Deviation of Hydraulic Conductivity on the Standard Deviation of the Output Head.

cycle numbers 1030 and 1240 are also shown to help illustrate the problem that exists in defining "steady-state".

As noted earlier, cycle number 780 is chosen as steady state because it is at this time cycle that the deterministic steady-state solution is obtained (which is equivalent to what is accepted as the "analytical" solution for this problem). Therefore, in the probabilistic simulations, the values of mean and standard deviation for the output head distributions obtained at cycle number 780 are regarded as the "steady-state" solution. However, the behavior observed in Figures 16 and 17 for S_o versus distance do not agree well with Freeze's [1975] results (Figure 15). Our probabilistic results (Figures 16 and 17) at cycle number 780 indicate smaller S_o values in the middle of the mesh than Freeze would predict. Especially in the case of $K_{STD} = 0.25$ (Figure 17) there is a marked deviation from the parabolic behavior observed with Freeze's work.

This apparent discrepancy leads to the consideration of a larger number of time cycles. Perhaps steady-state can be better defined at higher cycle numbers, if it has not yet been reached. In Figure 16 ($K_{STD} = 0.5$) the solutions for cycle numbers 1030 and 1240 indicate that perhaps time cycle 780 is not steady-state. The solution for cycle number 1240 compares quite favorably with the results obtained by Freeze [1975]. It would appear, however, that steady state is not well-defined. In the deterministic simulation steady-state is achieved at cycle number 780, while in the probabilistic problem steady-state is not well defined and in 1240 cycles must be simulated before the predicted steady-state behavior is obtained. This should perhaps be

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Figure 16. One-Dimensional Steady-State Flow Through Homogeneous Media:
 S_0 vs. Distance for $K_{STD} = 0.5$.

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Figure 17. One-Dimensional Steady-State Flow Through Homogeneous Media:
 S_0 vs. Distance for $K_{STD} = 0.25$.

expected since the probabilistic problem includes values of $K = 1.0$. These lower values of K should slow down the attainment of steady-state.

Figure 17 illustrates the probabilistic solution for $K_{STD} = 0.25$ at cycle numbers 780, 1030, and 1240, as well. The plots do not indicate the same kind of behavior observed for $K_{STD} = 0.5$ for greater time cycles. For $K_{STD} = 0.5$, the values of S_o tend to increase with time. However, for $K_{STD} = 0.25$, S_o is smaller in the middle of the mesh than at either end. The values of S_o do tend to increase with time near the boundaries, but in the middle of the system they appear to have reached constant values, for the cycle numbers represented. This behavior may indicate that simulations are required at further cycles before "steady-state" is achieved. There is also the possibility that the solutions at the cycle numbers illustrated very closely represent the "steady-state" solution, and the results obtained for $K_{STD} = 0.25$ by Freeze [1975] will not agree with those generated in this study. It is clear that the concept of "steady-state" is not well-defined in the probabilistic model as applied in this work. It appears that in general, there will be some propagation of the standard deviations associated with the distributions for the output variable. This propagation is a consequence of the repeated operations between the distributions in probability.

In Table 3 the mean and standard deviation are given for various time cycles for both the probabilistic simulations. In the $K_{STD} = 0.25$ case, the values of mean are approaching constant values quite close to the deterministic solution, while the S_o values near the boundaries are increasing. Steady-state is therefore well understood for the mean values of the output distributions, but in regards to standard

TABLE 3

Summary of Time Cycle Parameter in Steady-State Flow Solutions

A. Probabilistic Solution for $K_{STD} = 0.5$

Cycle Number	Mean, μ			Standard Deviation, σ		
	780	1030	1240	780	1030	1240
Node						
2	7.59	7.6	6.67	6.48	7.50	7.92
3	16.76	17.32	18.67	14.82	14.66	16.15
4	26.34	26.97	29.13	16.33	22.03	23.14
5	37.41	37.08	38.53	16.17	25.47	26.89
6	46.92	46.61	47.01	12.44	26.29	27.15
7	54.58	55.27	54.08	9.65	22.72	27.10
8	63.17	63.94	62.39	14.63	20.98	26.84
9	74.64	74.14	71.67	16.00	19.95	22.19
10	84.14	84.06	81.97	12.92	14.34	15.61
11	94.26	93.79	92.79	7.06	7.53	7.46

B. Probabilistic Solution for $K_{STD} = 0.25$

Cycle Number	Mean, μ			Standard Deviation, σ		
	780	1030	1240	780	1030	1240
Node						
2	5.15	5.17	5.46	4.33	4.72	4.91
3	14.70	14.73	14.74	9.90	10.73	10.06
4	26.08	25.65	25.80	6.94	9.44	11.29
5	35.30	34.44	35.80	3.87	4.38	1.70
6	45.27	44.95	45.33	4.53	5.00	4.24
7	54.90	54.83	54.79	4.47	5.30	5.09
8	64.96	64.73	63.98	4.44	5.97	5.63
9	74.99	73.83	74.00	4.56	7.25	10.50
10	84.92	85.66	85.70	7.32	10.60	11.12
11	95.00	94.98	94.76	4.37	4.98	4.31

deviation it is not. The mean values in the $K_{STD} = 0.5$ case do not approach the deterministic solution as consistently as they did for $K_{STD} = 0.25$, because of the greater uncertainty associated with this simulation. The propagation of S_0 is quite apparent. It would appear from these results and the figures discussed earlier, that steady-state is not well-defined with reference to the standard deviation of probabilistic distributions for output variables.

2. One-Dimensional, Transient Flow Through Homogeneous Media

The next series of cases simulate one-dimensional, transient flow through a homogeneous medium. The same 100.0 cm, ten node linear mesh is applied, however boundary node 12 is eliminated (See Figure 18). The initial conditions are such that $\phi_2 - \phi_{11} = 100.0$ cm. The boundary condition sets $\phi_1 = 0.0$ cm. This represents the problem of transient drainage of a system. The time parameter is an important consideration in the analysis of transient flow.

The deterministic solution for the transient problem, using $K = 1.0$, is illustrated in Figure 19. The cycle numbers 50, 300, and 600, are plotted as the parameter. The cycle number is considered a more appropriate parameter than time because the length of the time step changes between the probabilistic simulations. The propagation of output hydraulic head approaching zero with increased cycle number is observed.

The probabilistic case for $K_{STD} = 0.5$ is illustrated in Figure 20. The results of mean output head, $\bar{\phi}$, versus distance are shown for the same cycle numbers as in the deterministic solution (Figure 19). A "lag" can be observed between the probabilistic and deterministic

The Physical Model for One-Dimensional Transient Flow Through Homogeneous Media

Initial Conditions:

$$\phi_2 - \phi_{11} = 100.0 \text{ cm.}$$

$$K = 1.0 \text{ cm./sec.}$$

XBL 805-1081

Figure 18. The Physical Model for One-Dimensional Transient Flow Through Homogeneous Media.

XBL 805-014

Figure 19. One-Dimensional Transient Flow Through Homogeneous Media: Deterministic Solution.

XBL 805 - 815

Figure 20. One-Dimensional Transient Flow Through Homogeneous Media: Probabilistic Solution for $K_{STD} = 0.5$.

solutions at equivalent cycle numbers. The "lag" between the probabilistic case for $K_{STD} = 0.25$ (Figure 21) and the deterministic solution is not as big. Freeze [1975] shows similar plots (left figures) for $K_{STD} = 0.5$ and $K_{STD} = 0.25$ in Figures 22 A and B, respectively. The same behavior is apparent in Freeze's results. Larger uncertainties in hydraulic conductivity result in greater differences between solutions for mean output head when the time parameter is considered. In addition, the propagation of the head towards zero occurs more rapidly with smaller input uncertainties. The mean output hydraulic head at various points in the system is tabulated in Table 4 to illustrate the "lag" behavior.

Freeze's [1975] results in Figures 22 A and B indicate that he has simulated drainage from both ends of the system. In other words, the problem considered in this work assumes symmetry and is one-half of the simulation Freeze has analyzed. The right-hand plots in Figures 22 A and B show S_o versus distance. At early time there is oscillation with large values of S_o near the boundaries, and $S_o = 0.0$ in the middle of the mesh. With time, S_o tends to decrease near the boundaries, and increase in the middle of the system, as these elements "see" the uncertainty associated with the input parameter. At still later time, S_o everywhere in the system approaches zero.

Similar plots of S_o vs. distance generated in the present study for $K_{STD} = 0.5$ and $K_{STD} = 0.25$ are given in Figures 23 and 24, respectively. The cycle numbers 50, 300, 600, and 1000 are shown. The trend illustrated in these figures is consistent with the early time behavior of Freeze's results. However, the later time behavior is not observed, the values of S_o remain close to zero throughout the entire system.

XBL 805 - 816

Figure 21. One-Dimensional Transient Flow Through Homogeneous Media: Probabilistic Solution for $K_{STD} = 0.25$.

Figure 22A. Output plots of $\phi(x,t)$ and $S_o(x,t)$ for $\sigma_y = 0.50$.

Figure 22B. Output plots of $\phi(x,t)$ and $S_o(x,t)$ for $\sigma_y = 0.25$.

XBL 805-828

Figure 22. One-Dimensional Transient Flow through homogeneous Media: Freeze: Results for ϕ vs. x and S_o vs. x .

TABLE 4

Summary of Lag Behavior in Transient Flow Problems

A. Solution Lag for Transient Flow through Homogeneous Media

	300 Cycles		600 Cycles	
	(x = 50.0 cm)cm	(x = 100.0 cm)cm	(x = 50.0 cm)cm	(x = 100.0 cm)cm
Deterministic	39.0	51.0	16.0	20.5
Probabilistic, $K_{STD} = 0.25$	46.5	60.5	21.5	28.0
Probabilistic, $K_{STD} = 0.50$	60.0	75.5	39.0	47.0

B. Solution Lag for Transient Flow through Heterogeneous Media

	300 Cycles		600 Cycles	
	(x = 50.0 cm)cm	(x = 100.0 cm)cm	(x = 50.0 cm)cm	(x = 100.0 cm)cm
Deterministic	39.0	45.5	14.5	16.5
Probabilistic, $K_{STD} = 0.25$	57.0	66.0	32.0	36.0

XBL 805-810

Figure 23. One-Dimensional Transient Flow Through Homogeneous Media:
 S_o vs Distance for $K_{STD} = 0.5$.

XBL 805-811

Figure 24. One-Dimensional Transient Flow Through Homogeneous Media:
 S_0 vs. Distance for $K_{STD} = 0.25$.

It would appear that the end of the mesh never "sees" the distribution on hydraulic conductivity to the extent that the elements near the discharge do. (Note: The end of the mesh in this study is the middle of the doubling draining column in Freeze's simulation.) Cycle number 1000 may not represent a sufficiently long enough time for such behavior to occur. Another explanation is that the nodes furthest from the sink have seen as much disturbance as they will ever see, and S_o values will remain fairly constant as time increases.

3. One-Dimensional, Steady-State Flow Through Heterogeneous Media

The problems considered thus far involve homogeneous systems. The probabilistic structures input for hydraulic conductivity are consistent throughout the mesh. In the analysis of flow in a heterogeneous medium, the hydraulic conductivity distributions are varied, depending on the location of inhomogeneities.

The simulation of one-dimensional, steady-state flow through a heterogeneous medium (see Figure 25) employs the same mesh as that used in the homogeneous problem. The boundary conditions are specified as $\phi_1 = 0.0$ cm and $\phi_{12} = 100.0$ cm. The initial conditions of the internal nodes are $\phi_2 = 10.0$ cm, $\phi_3 = 20.0$ cm, $\phi_4 = 30.0$ cm, $\phi_5 = 40.0$ cm, $\phi_6 = 50.0$ cm, $\phi_7 = 60.0$ cm, $\phi_8 = 70.0$ cm, $\phi_9 = 80.0$ cm, $\phi_{10} = 90.0$ cm, and $\phi_{11} = 100.0$ cm. The permeabilities are assigned such that $\mu_{K1} = 1.0$ cm/sec in nodes 1-6 (0.0-50.0 cm) while $\mu_{K2} = 2.0$ cm/sec in nodes 7-12 (50.0-100.0 cm). This corresponds to the case where K_2 (nodes 7-12) = $2K_1$ (nodes 1-6). In the deterministic problem the standard deviation for both K_1 and K_2 is necessarily zero

The Physical Model for One-Dimensional Steady-State Flow Through Heterogeneous Media

Initial Conditions:

$\phi_2 = 10.0 \text{ cm.}$	$\phi_7 = 60.0 \text{ cm.}$
$\phi_3 = 20.0 \text{ cm.}$	$\phi_8 = 70.0 \text{ cm.}$
$\phi_6 = 30.0 \text{ cm.}$	$\phi_9 = 80.0 \text{ cm.}$
$\phi_4 = 40.0 \text{ cm.}$	$\phi_{10} = 90.0 \text{ cm.}$
$\phi_5 = 50.0 \text{ cm.}$	$\phi_{11} = 100.0 \text{ cm.}$

Hydraulic Conductivity Legend

- ☐ $K_1 = 1.0 \text{ cm./sec.}$
- ☐ $K_2 = 2.0 \text{ cm./sec.}$

XBL 805-1083

Figure 25. The Physical Model for One-Dimensional Steady-State Flow Through Heterogeneous Media.

($K_{STD1} = K_{STD2} = 0.0$) while the probabilistic simulation is such that $K_{STD1} = K_{STD2} = 0.25$.

Figure 26 illustrates output head versus distance for both the deterministic and probabilistic steady-state solutions. The linear drop in output head between 0.0 and 50.0 cm is exactly twice that between 50.0 and 100.0 cm, a result of $K_2 = 2K_1$. The darkened points represent the mean output head resulting from $K_{STD} = 0.25$ in the probabilistic simulation. The cross bars denote one standard deviation above and below the mean output head. The values of $\bar{\phi}$ lie very close to the deterministic solution and the cross-bars are short in length, as the uncertainty associated with the input hydraulic conductivity is small.

4. One-Dimensional, Transient Flow Through Heterogeneous Media

The analysis of one-dimensional transient flow through a heterogeneous medium applies the same 100.0 cm, ten node mesh as that used in the homogeneous problem (Figure 27). The boundary node 1 is specified at $\phi_1 = 0.0$ cm with initial conditions of $\phi_2 - \phi_{11} = 100.0$ cm. The same drainage problem simulated in the homogeneous system earlier is considered in the heterogeneous system. The nodes are assigned the hydraulic conductivities as in the previous steady-state problem, except for the fact that boundary node 12 is disconnected.

Figure 28 illustrates the deterministic solution for transient flow through a heterogeneous system, ϕ versus distance. The solutions are plotted at cycle numbers 100, 200, 300, and 600. With time, the output head moves toward zero. The same "lag" behavior observed between the deterministic and probabilistic solutions in transient flow through

XBL 805 - 808

Figure 26. One-Dimensional Steady-State Flow Through Heterogeneous Media:
Deterministic and Probabilistic ($K_{STD} = 0.25$) Solutions.

The Physical Model for One-Dimensional Transient Flow Through Heterogeneous Media

Initial Conditions:

$$\phi_2 - \phi_{1,1} = 100.0 \text{ cm.}$$

Hydraulic Conductivity Legend:

- ☐ $K_1 = 1.0 \text{ cm./sec.}$
- ☐ $K_2 = 2.0 \text{ cm./sec.}$

XBL 805 - 1084

Figure 27. The Physical Model for One-Dimensional Transient Flow Through Heterogeneous Media.

XBL 805-807

Figure 28. One-Dimensional Transient Flow Through Heterogeneous Media:
Deterministic Solution.

homogeneous media is apparent in the heterogeneous case as well. The probabilistic solution ($K_{STD} = 0.25$) is shown in Figure 29. This lag behavior is tabulated in Table 4B for the heterogeneous simulations.

5. Two-Dimensional Steady-State Flow Through Heterogeneous Media

The problem of two-dimensional steady-state flow in heterogeneous media is the last case studied. The purpose of this simulation is to verify the generality of the algorithm. The square mesh with dimensions 50.0 x 50.0 cm, consisting of the boundary "sink" node 1, the boundary node 99, and 24 interior nodes (10.0 x 10.0 cm) is illustrated in Figure 30. This configuration is numerically modeled assuming symmetry by simulating only half the actual mesh shown in the figure. The boundary conditions are assigned such that the potential of the sink node 1 is $\phi_1 = 0.0$ cm and the boundary node 99 is $\phi_{99} = 100.0$ cm. The initial conditions are for $\phi_2 - \phi_{25} = 100.0$ cm. Two different values of hydraulic conductivity are assigned throughout the mesh, $K_1 = 1.0$ cm/sec and $K_2 = 2.0$ cm/sec in the deterministic problem. The locations of these heterogeneities are denoted in Figure 30.

The steady-state solution for the deterministic simulation is obtained during cycle number 300, and shown in Figure 31. The potential distribution at this time cycle is defined as the steady-state solution because the values of the output head remain constant for increased time cycles (at least as high as cycle number 600). In the probabilistic simulation $\mu_{K1} = 1.0$ cm/sec, $\sigma_{K1} = 0.25$ cm/sec, $\mu_{K2} = 2.0$ cm/sec, and $\sigma_{K2} = 0.25$ cm/sec. The values for the probabilistic solution are denoted in parenthesis in Figure 31. They indicate the mean values

XBL 805-806

Figure 29. One-Dimensional Transient Flow Through Heterogeneous Media:
Probabilistic Solution for $K_{STD} = 0.25$.

The Physical Model for Two-Dimensional Steady-State Flow Through Heterogeneous Media

Hydraulic Conductivity Legend

- ☐ $K_1 = 1.0$ cm./sec.
- ☐ $K_2 = 2.0$ cm./sec.

XBL 805-833

Figure 30. The Physical Model for Two-Dimensional Steady-State Flow Through Heterogeneous Media.

Figure 31

**Two-Dimensional Steady-State
Flow Through Heterogeneous Media:
Deterministic Solution (Cycle No. 300)
Probabilistic Solution for $K_{STD} = 0.25$
($\bar{\phi}$ appears in parenthesis)**

Potential Distribution
(all potentials have units of centimeters)

Boundary Node 1 $\phi_1 = 0.0$ cm.	$\phi_2 = 29.23$ ($\bar{\phi} = 32.45$)	$\phi_3 = 62.14$ ($\bar{\phi} = 62.6$)	$\phi_4 = 82.75$ ($\bar{\phi} = 82.47$)	$\phi_5 = 94.72$ ($\bar{\phi} = 94.83$)
$\phi_6 = 29.23$ ($\bar{\phi}_6 = 32.45$)	$\phi_7 = 48.41$ ($\bar{\phi}_7 = 48.97$)	$\phi_8 = 67.58$ ($\bar{\phi}_8 = 68.13$)	$\phi_9 = 85.39$ ($\bar{\phi}_9 = 85.16$)	$\phi_{10} = 96.14$ ($\bar{\phi}_{10} = 96.06$)
$\phi_{11} = 62.14$ ($\bar{\phi}_{11} = 62.6$)	$\phi_{12} = 67.58$ ($\bar{\phi}_{12} = 68.13$)	$\phi_{13} = 80.76$ ($\bar{\phi}_{13} = 79.51$)	$\phi_{14} = 90.65$ ($\bar{\phi}_{14} = 90.74$)	$\phi_{15} = 96.99$ ($\bar{\phi}_{15} = 96.89$)
$\phi_{16} = 82.75$ ($\bar{\phi}_{16} = 82.47$)	$\phi_{17} = 85.39$ ($\bar{\phi}_{17} = 85.16$)	$\phi_{18} = 90.65$ ($\bar{\phi}_{18} = 90.74$)	$\phi_{19} = 94.40$ ($\bar{\phi}_{19} = 94.10$)	$\phi_{20} = 98.15$ ($\bar{\phi}_{20} = 98.04$)
$\phi_{21} = 94.72$ ($\bar{\phi}_{21} = 94.83$)	$\phi_{22} = 96.14$ ($\bar{\phi}_{22} = 96.06$)	$\phi_{23} = 96.99$ ($\bar{\phi}_{23} = 96.89$)	$\phi_{24} = 98.15$ ($\bar{\phi}_{24} = 98.04$)	$\phi_{25} = 99.38$ ($\bar{\phi}_{25} = 99.31$)

$\phi_{99} = 100.0$ cm.

Boundary Node 99

of the output head distributions for cycle number 300. The distribution of potential mean values does not deviate far from the deterministic solution because of the relatively small uncertainty associated with the standard deviation, $K_{STD} = 0.25$ for $K_1 = 1.0$ cm/sec and $K_2 = 2.0$ cm/sec.

V. CONCLUSIONS AND FURTHER WORK

The purpose of this study is to develop a probabilistic model to simulate the flow of fluids through porous media. The popular approach in stochastic modeling has employed a Monte Carlo simulation. Freeze's [1975] application of this random number technique involves a set of repetitive simulations with a mathematical model. The reliability of the model improves with the number of simulations performed.

The method followed in this work differs from earlier probabilistic models. We have attempted to model the inherent uncertainties associated with input parameters on the basis of the axiomatic foundations of probability theory. Probability density functions are represented by two-dimensional arrays consisting of discrete values and their associated probabilities. The reliability of these probabilistic arrays is expected to improve with the number of discrete values employed to model the distributions. The cost of the computer program increases as the square of the number of values. The optimum number of discrete values has been chosen to be nine in the present work. When fewer than nine values are used, the number of terms in the probabilistic array becomes too few to meaningfully represent the distribution. If many more than nine terms are employed, the program becomes quite costly.

An early problem encountered with the algorithm was the propagation of erroneous values for probability. According to the axiomatic principles of probability theory the sum of the probabilities in a given statistical array must equal one (1.0). Due to numerical

"round-off" errors in individual probability terms, the sum of these values in a given array could easily fall below or above 1.0. In this event, with repeated multiplications (10^5 per time step) the propagation of very large values (where round-off has been up) or very small values (where round-off has been down) takes place quite rapidly. Therefore, a "normalization" routine is inserted in the COLAPS subroutine. Before the resulting C array is sent back to FINK, the sum of the probabilities is normalized to 1.0.

There is little significance associated with probabilities smaller than 0.001% (10^{-5}), especially when only nine values are generated in the representation of the distributions. The greater the number of "meaningful" values in an array, the better. The propagation of very small probabilities is avoided by specifying the minimum probability desired. TOLPR is the input parameter which designates the minimum allowable probability. When a probability term in the C array falls below TOLPR, it is set equal to zero (0.0) and the distribution is normalized. In these simulations $TOLPR = 10^{-5}$.

The postulates for the commutative, associative, and distributive laws are investigated with the two subroutines developed from the algorithm. The equations governing these postulates are examined with the subroutines for symmetric distributions, in order to demonstrate the validity of the algorithm. In this validation the significance of strict graphical comparisons of various histograms is considered secondary to the analysis of the numerical means and standard deviations corresponding to the distributions.

Some fundamental problems are studied with the model. One-dimensional, steady-state flow in homogeneous media is investigated. The results are compared with those obtained by Freeze [1975]. On the basis of this analysis, it would appear that steady-state is not a well-defined condition. Due to the natural propagation of distributions with repeated operations, there is no "steady-state" condition for the value characterizing the uncertainty of a distribution in probability. Freeze's [1975] results indicate that there is a steady-state solution for the standard deviation of the output head distribution versus distance in the mesh. The results obtained in this study would suggest the contrary. The steady-state solution for mean output head versus distance is a better defined condition.

The simulation of one-dimensional, transient flow in homogeneous media is performed. Freeze's [1975] results are available for comparison. The problem of drainage from a system is simulated, thus the solution for output head moves toward zero with time everywhere in the mesh. The interesting phenomenon observed in these solutions is the effect of input uncertainties on the propagation of the mean output head. The propagation of the output head is abated when the uncertainty associated with the input hydraulic conductivity is increased. This same behavior is observed in Freeze's [1975] results. The source of the "lag" in mean output variable propagation is the log normal nature of the input parameter distribution. Such a lag might not occur if the uncertainty were characterized by a normal distribution. This introduces some questions in our understanding of classical deterministic modeling of transient flow systems. At increased times the predictions

obtained from deterministic models could significantly vary from probabilistic forecasting.

The solutions for the standard deviation of output head distributions are also generated for the transient flow problem. The results obtained in this study appear to agree with Freeze's [1975] solutions for early time cycles. At later cycle numbers the agreement is not as good. There is some doubt, however, whether the simulations in this study have been carried out for a sufficient number of time cycles to make such comparisons. There is poor agreement in the output standard deviation solutions for the steady-state problem, as well.

The next series of problems involve the investigation of flow in heterogeneous media. There are no analogous studies in Freeze's [1975, 1977] work, therefore the only analysis which can take place is internal. Varying probabilistic arrays for permeability are assigned throughout a mesh having an identical configuration as that used in the homogeneous model. The steady-state deterministic solution agrees with the analytical solution. The transient-flow problem is also studied. One probabilistic solution is generated in both simulations.

The slightly more complex problem of two-dimensional flow in heterogeneous media is also considered. The primary purpose of this simulation is to demonstrate the generality of the algorithm. Both deterministic and probabilistic solutions are obtained.

The scope of the parametric studies in homogeneous media include standard deviation of 0.5 cm/sec and 0.25 cm/sec associated with a mean value of 1.0 cm/sec for hydraulic conductivity. The effect of

larger uncertainties could be studied in future work. Freeze 1975 employed values of $K_{STD} = 1.0$ and $K_{STD} = 2.0$ in his model.

Future application of this algorithm should entail probabilistically modeling the parameter, specific storage, as well as hydraulic conductivity. Freeze [1975, 1977] considers the dependence of the three hydrogeologic parameters (hydraulic conductivity, porosity, compressibility) in an autocorrelation routine. The concept of spatial dependence of these parameters (heterogeneities), also studied by Freeze [1975, 1977], should be investigated with this model.

As this approach has applied computer program TERZAGI in only the explicit mode, the implicit calculation still remains to be studied. It appears that though the solution may be final, it is not the most economical. If the approach is generalized to the implicit calculation, the length of the time step could be increased, and may be more economical. The implicit calculation introduces the concept of dependent probability, which will require additions and/or changes in the algorithm. An implicit approach would involve the solution of simultaneous equations by a direct solving routine. Whereas in the conventional deterministic method one is concerned with single-valued numbers in a large array, these values will take the form of arrays themselves arising from the statistical structures in the application of the probabilistic model.

The generality of the algorithm is such that this model should be appropriate in other applications of the diffusion equation besides flow through porous media. The subroutines could be used in modeling any system in which a distribution in probability is known to be

associated with the output variable and any of the input parameters can be expressed in the form of a probability density function. The simulation of chemical transport could be investigated by modeling the chemical dispersion as a probabilistic structure. Problems in heat conduction may be probabilistically modeled if there is some uncertainty associated with the thermal conductivity, and this parameter is modeled as a probability density function.

The approach used in this study models a probabilistic distribution in the form of a histogram, containing discrete values for magnitude and associated probability. These statistical representations are used for parameters whose probability density function are known. The classical approach is to use deterministic values for such parameters. These probabilistic structures allow for the analysis of the statistical behavior of output variables.

VI. APPENDICES

APPENDIX A

GLOSSARY OF TERMS AND DEFINITIONS

- Monte Carlo Simulation: An empirical method of learning about the probability distribution of a statistic. It consists of drawing a large number of samples from the population in question, computing the value of the interacting statistic for each sample, and recording the empirically observed sampling distribution of that statistic. [Christ, 1966]
- Stochastic Process: A random phenomenon that arises through a process which is developing in time (or space) in a manner controlled by probabilistic laws. [Parzen, 1962]
- $f(r)$ The density function of the random variable, r , has a normal distribution if it is given by the formula in Fisz [1967].

$$f(r) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-(r-m)^2/2\sigma^2}$$

- r is a random variable
- m, μ is the mean value of the distribution = $[(x_{16} + x_{50} + x_{84})/3]$
- $+\sigma$ is the one standard deviation above the mean, where 34% of the values lie, in a normal distribution = $(x_{84} - \mu)$

$-\sigma$ is one standard deviation below the mean, where 34% of the values lie, in a normal distribution $= (\mu - x_{16})$

σ is the larger of the above two values for standard deviation
(Note: in a normal distribution, $(x_{84} - \mu) = (\mu - x_{16})$, and $+\sigma = -\sigma$. Therefore, the mean is more simply defined $\mu = x_{50}$.)

$P(x)$ the non-normalized probability that any value x should occur in the normal distribution

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

x is the some discrete value

$P(\Delta x_n)$ is the normal probability of the interval Δx_n falling in the distribution

$$P(\Delta x_n) = \frac{\int_{K_n}^{K_{n+1}} P(\bar{x}_n) dx}{\int_{-3\sigma}^{+3\sigma} P(\bar{x}_n) dx}$$

Δx_n is some discrete interval

K_n is the lower endpoint of the interval Δx_n

K_{n+1} is the upper endpoint of the interval Δx_n

$+3\sigma$ is three standard deviations above the mean

-3σ is three standard deviations below the mean

A_i is the arithmetic average associated with the i^{th} interval defined in a distribution of A

$P(A_i)$	is the normalized probability associated with the i^{th} interval in the A distribution
C'_{ij}	is the arithmetic average of the ij interval in the distribution of C
$P(C'_{ij})$	is the normalized probability associated with the ij interval in the C distribution
C_i	the weighted average of the i^{th} interval in the C distribution
$\bar{X}_n$	the arithmetic average of the interval Δx_n
S_o	the standard deviation as defined for μ above
K_{STD}	the standard deviation associated with the distribution of hydraulic conductivity
ϕ_n	the potential at element n.
μ_{K1}	the mean value of the distribution of K_1
μ_{K2}	the mean value of the distribution of K_2
K_{STD1}	the standard deviation associated with the distribution of K_1
K_{STD2}	the standard deviation associated with the distribution of K_2

APPENDIX B

EXAMPLE CALCULATIONS FOR LOG NORMAL DISTRIBUTIONS

Given a mean value, μ , and a standard deviation, σ , the following equation is applied to calculate the non-normalized probability, $P(x)$, for discrete values of x .

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The log normal generator is employed to convert the normal distribution to a log normal distribution by obtaining values for K as follows.

$$K = 10^x$$

Two cases are investigated, $\sigma_K = 0.5$ and $\sigma_K = 0.25$. In both cases $\mu = 1.0$

Example Calculations:

Case 1, $\sigma_K = 0.50$

$x = 0.0$ (-2σ)

$$P(x = 0.0) = \frac{1}{\sqrt{2\pi}(0.5)} e^{-\frac{(0.0-1.0)^2}{2(0.5)^2}}$$

$$P(x = 0.0) = 0.108$$

$$K = 10^x = 10^{0.0}$$

$$K = 1.0$$

Case 2, $\sigma_K = 0.25$

$$x = 1.75 \ (+3\sigma)$$

$$P(x = 1.75) = \frac{1}{\sqrt{2\pi} (0.25)} e^{-\frac{(0.0-1.75)^2}{2(0.25)^2}}$$

$$P(x = 1.75) = 0.018$$

$$K = 10^x = 10^{1.75}$$

$$K = 56.23$$

APPENDIX C

COLAPS FLOWCHART/PROGRAM NOMENCLATURE

Subroutine COLAPS - Program Nomenclature

(expression in parenthesis is FORTRAN equivalent)

I(NDISCR)	number of intervals contained in probabilistic distributions
q(NDISQU)	$NDISCR^2$, number of intervals in resulting COP distribution
y(MDISCR)	(NDISCR+1)
r(MDISQU)	(NDISQU-1)
h(NHLDIS)	(NDISCR+1)/2, midpoint of probabilistic array
k(K,NC)	counter
i(I,NC)	counter
j(J,NN)	counter
M(CMAG)	magnitude array
P(CPROB)	probability array
$F_{i,j,1}$ COP(I,J,1)	first layer of three-dimensional array containing magnitudes elements
$F_{i,j,2}$ COP(I,J,2)	second layer of three-dimensional array containing probability elements
G(MAGPOT)	code to indicate whether tolerance or maximum and minimum potentials should be considered
N(PSIMIN)	minimum value of potential set by boundary conditions
X(PSIMAX)	maximum value of potential set by boundary conditions
e(MIN,MAX)	counter

U(CMIN)	minimum value of magnitude array, CMAG
W(XMIN)	reassigned CMIN
V(CMAX)	maximum value of magnitude array, CMAG
Y(XMAX)	reassigned CMAX
C(DIFF)	difference (XMAX-XMIN)
H(DINCR)	interval size
E[CMAGIN(M)]	upper limit of interval, M
B[CMID(M)]	midpoint of interval, M
R[PROB(M)]	probability associated with interval, M
D[PRODIT(M)]	product of probability and magnitude of interval, M
T[PROD(M)]	product of probability and magnitude of discrete value, M
A[AVGMAG(M)]	average magnitude of interval, M
Z(TOLPR)	minimum value of probability allowed
S(PROBSUM)	summation of probabilities, $\sum_i R_i [\sum_m \text{PROB}(M)]$ over all the intervals
$C_{i,j,1}[C(I,1,1)]$	1st column of two-dimensional array C containing magnitude elements
$C_{i,j,2}[C(I,2,1)]$	2nd column of two-dimensional array C containing probability elements

APPENDIX D

PROGRES PROGRAM LISTING AND SAMPLE PROBLEM

XBL 805-1082

APPENDIX D

PROGRES PROGRAM LISTING AND SAMPLE PROBLEM


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      $      MAX,MID,MODE,NADD,NADG,NADS,NADSB,NAD1,NAD2,NCHECK,NODMS, BLINK 8
      $      NON,NPROB,NSAVE,NSEQ,NSTOP,MUTS,KSPEC BLINK 9
CCC  END BLINK BLINK 10
CCC  TEMPORARY VALUES OF INTEGER AND REAL VARIABLES BLISC 2
      COMMON A1,DX,II,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,WND,M1,M2, BLISC 3
      $      P1,P2,P3,P4,P5,TX,X1,X2,X3 BLISC 4
CCC  END BLISC BLISC 5
CCC  REAL VARIABLES BLARE 2
      COMMON ADATA,ADD,AFLUID,APATH,BET,BETA,BETH,BPRIME,CAPNS, BLARE 3
      $      CAPS,CAPT,CONST,DAY,DELT,DELTS,DELTSS,DEX,DLONG,DNDP, BLARE 4
      $      DPMAXS,DPRES,DPSIYAX,DRAD,DRADS,DWIDE,EAVG,EC,ELT,ESTRESS, BLARE 5
      $      ESTRIN,EX,FEX,FLEX,FLUID,FLUX,FLUXS,FLX,FOF,FORD, BLARE 6
      $      GBYMU,GEE,GEAS,GECH,GHASS,GONE,GX,HONE,MX,PHIBS, BLARE 7
      $      PHIONE,PORS,PSIVARY,PSI1,PSI2,RD,RHOZ,SAVG,SCALE,SET, BLARE 8
      $      SETD,SET1,SMALL,SUMTIM,TAU,THICK,TIME,TZERO,VISC,VOLMS, BLARE 9
      $      VOLS,WMS,PCONE,TIMAX,PCX,DELYF,XX2,XX3 BLARE 10
CCC  END BLARE BLARE 11
CCC  ARRAYED VARIABLES BLARY 2
CCC  BLARY 3
CCC  THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE BLARY 4
CCC  THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC BLARY 5
CCC  DISTRIBUTIONS. THE FIRST INDEX REFERS TO THE DISCREET VALUE, BLARY 6
CCC  THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR BLARY 7
CCC  PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR BLARY 8
CCC  CONNECTION NUMBER. BLARY 9
CCC  BLARY 10
      LEVEL2,CAP,CON,DFI,DPSI,DF,PHI,PPII,ZIP,TRAN BLARY 11
      COMMON/LCH/CAP(100),CON(20,2,100),DFI(20,2,200),DPSI(20,2,200), BLARY 12
      $      DF(20,2,200),PHI(20,2,200),PPII(20,2,200), BLARY 13
      $      ZIP(200),TRAN(20,2,200) BLARY 14
      COMMON F1(12),F2(12),F3(12),MOD1(200),MOD2(200),MODE(100), BLARY 15
      $      FI(11),FX(11),DDFI(11), BLARY 16
      $      DDFSI(11),DPSIS(11),E(11),ITEMS(15),LTABK(10),NAME(20), BLARY 17
      $      NAMES(20),NEWBL(15),NTYPE(100),PC(1),PCH(1),SLIN(1), BLARY 18
      $      TOTSTRS(1),VOL(100),W(1),Z(100),CK(10),NODTD(1),R(1), BLARY 19
      $      NODIM,TLUMP,PLUMP,XF,Q,PSIVARC(12,10),PZ(10),KODANIS, BLARY 20
      $      CON2(1),CON3(1),CONST1(10),CONST2(10),KODCON(1), BLARY 21
      $      KSTDATA,LANISK(10),LTABC(10),PPC(1), BLARY 22
      $      NODMAT(100),NOXMAT(100),MX1(10),PSIVARK(12,10) BLARY 23
CCC  END BLARY BLARY 24
      COMMON /N/ M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, M11, M12, M13 M 2
CCC  M 3
      COMMON/STRAIN/VOLID(100),DELVOL(100) HEART 63
      INTEGER ABLOCK(12) HEART 64
C HEART 65
      DIMENSION KWITMSG(91),KWITLOC(13),NDUMHY(15) HEART 66
      EQUIVALENCE (NDUMHY,NOSPEC) HEART 67
C HEART 68
      DATA M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, M11, M12, M13 HEART 69
      $/1,10,1,300,600,20,20,300,12,0,0,0,0/ HEART 70
CCC HEART 71
      DATA KWITLOC/0,4,9,15,19,26,35,39,49,60,74,81,91/ HEART 72
CCC HEART 73
      DATA KWITMSG / HEART 74
      1 36H*PROBLEM TIME *SUMTIM* REACHED TIMAX., HEART 75
      2 48HA PORE PRESSURE EXCEEDED PSIMAX + 0.001*PSIVARY., HEART 76
      3 53HA PORE PRESSURE WAS LESS THAN PSIMIN - 0.001*PSIVARY., HEART 77
      4 37HSTEADY-STATE CRITERIA WERE SATISFIED., HEART 78
      5 63HAN UNSPECIFIED PATERIAL, NODE OR BOUNDARY NODE WAS REFERRED TO HEART 79
      $., HEART 80
      6 81H-2 PUNCHED IN COL 6-7 OF DATA END CARD OR *CHECK CARD PRECEDED HEART 81
      8 PROBLEM NAME CARD., HEART 82

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7 30H THE NUMBER OF TIME STEPS REACHED NCYC., HEART 87
8 90H THE NUMBER OF SECONDS OF ELAPSED MACHINE TIME USED SINCE THE S HEART 88
START OF THIS HEART JOB REACHED NSEC., HEART 89
9 10H NO MATERIAL LIST (BLOCK 2) OR NODE LIST (BLOCK 4) WAS SPECIF HEART 90
SED OR CARRIED OVER FROM THE PRECEDING PROBLEM., HEART 91
0130H ITERATIVE SCHEME FOR PRESSURE HEAD CHANGES IN CONNECTED SPECIA HEART 92
8L NODES FAILED TO CONVERGE IN 88 ITERATIONS BY USE OF A TIME STEP HEART 93
88 2*SMALL., HEART 94
1 62H THE NUMBER OF ITEMS IN A DATA BLOCK EXCEEDED THE MAXIMUM SIZE. HEART 95
2,97H THE LENGTH OF A TABLE FOR A TIME OR PRESSURE DEPENDANT INPUT ONE HEART 96
QUANTITY EXCEEDED THE MAXIMUM SIZE .# HEART 97
CCC HEART 98
CCC HEART 99
CCC HEART 100
C*****INITIALIZE DATE AND ELAPSED TIME. HEART 101
CCC HEART 102
      CALL SECOND(TZERO) HEART 103
      CALL DATE(DAY) HEART 104
      PRINT 5045, TZERO HEART 105
CCC HEART 106
C*****START HERE FOR NEW SERIES OF PROBLEMS. HEART 107
CCC HEART 108
2110 NPROB = NSAVE = 0 HEART 109
CCC HEART 110
CCC INITIALIZE VOLUME CHANGE HEART 111
CCC HEART 112
      DO 2114 N = 1,100 HEART 113
      2114 DELVOLIN) = 0. HEART 114
      MODIN = 0 HEART 115
CCC HEART 116
CCC START HERE FOR EACH ADDITIONAL PROBLEM IN A STACK OF DATA DECKS. HEART 117
CCC HEART 118
2115 NPROB = NPROB + 1 HEART 119
      KCYC = -1 HEART 120
CCC HEART 121
CCC INITIALIZE CONTROL PARAMETERS. HEART 122
CCC HEART 123
      ISBLOCK = KMIT = NSTOP=NON=NCHECK=0 HEART 124
      DO 2120 N = 1,15 HEART 125
      2120 NEWBLIN) = 0 HEART 126
C HEART 127
C-----READ IN CONTROL CARDS AND PROBLEM NAME CARD. HEART 128
C HEART 129
2125 READ 5075, NAME HEART 130
C HEART 131
CCC HEART 132
CCC IGNORE IT IF THERE IS NO * IN COL. 1. HEART 133
CCC HEART 134
      IF(NAME(1).EQ.1R*) GO TO 2130 HEART 135
      PRINT 5075, NAME HEART 136
      GO TO 2125 HEART 137
CCC HEART 138
CCC CHECK FOR *SPLIT CARD. HEART 139
CCC HEART 140
      2130 IF(NAME(2).EQ.5MSPLIT) GO TO 4000 HEART 141
      IF(NAME(2).NE.5NCHECK) GO TO 2135 HEART 142
      NCHECK = 1 HEART 143
      GO TO 2125 HEART 144
C HEART 145
C-----CONTINUE HERE WHEN A PROBLEM NAME CARD HAS BEEN READ IN. HEART 146
C HEART 147
      2135 CALL HOUR(TIME) HEART 148

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CALL SECOND(ELY)
ELT = ELT - TZERO
PRINT 5830, TIME, DAY, NPROB, NAME
PRINT 5846, ELT
CALL LINE
HEART145
HEART146
HEART147
HEART148
HEART149
HEART150
HEART151
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HEART206

CCC
CCC CONTROL CARRYOVER OF DATA FROM PRECEDING PROBLEM.
CCC
CCC 2200 IF(NAME(16) .NE. 1R2) GO TO 2205
CCC
CCC HERE, USE DATA AND INIT. COND. FROM PRECEDING PROBLEM.
CCC
      NOW = 2
      PRINT 5895
      GO TO 2230
      2205 IF(NAME(16) .NE. 1R3) GO TO 2210
CCC
CCC HERE, USE DATA AND FINAL COND. FROM PRECEDING PROBLEM.
CCC
      NOW = 3
      GO TO 2215
      2210 IF(NAME(16) .NE. 1R4) GO TO 2220
CCC
CCC HERE, USE DATA AND FINAL COND. FROM PRECEDING PROBLEM, W/ OLD TAU.
CCC
      NOW = 4
      TAU = SUNTIM
      2215 PRINT 5900, TAU
      GO TO 2230
CCC
CCC ZERO ALL COUNTERS WHEN NO DATA IS CARRIED OVER FROM LAST PROBLEM.
CCC
      2220 PRINT 5840
      DO 2225 N = 1,15
      2225 NDUMNY(N) = 0
CCC
CCC SAVE INITIAL NUMBER OF ITEMS IN EACH BLOCK.
CCC
      2230 DO 2235 N = 1,14
      2235 ITEMS(N) = NDUMNY(N)
CCC
CCC TEST CARRYOVER.
CCC
      IF(NOW .EQ. 0) GO TO 2245
CCC
CCC CARRYOVER.
CCC
      IF(NAMES(1) .EQ. 1R*) GO TO 2240
      PRINT 5960
      GO TO 2220
      2240 PRINT 5935, NAMES
      CALL TALLY
      NOW = 0
C
C THESE 2 CONTINUES ARE FROM THE COMPUTED GCTO, BELOW.
C
      2290 CONTINUE
      2315 CONTINUE
      2245 CALL LINE
C
C-----READ IN BLOCK NUMBER CARDS.
C
```

2250 READ 5869,ADATA,IBLOCK,MODE,ABLOCK	WEART207
CCC	WEART208
CCC TEST COLUMNS 6-7 BLANK OR ZERO.	WEART209
CCC	WEART210
IF (IBLOCK .NE. 0) GO TO 2255	WEART211
CCC	WEART212
CCC COLUMNS 8-80.	WEART213
CCC	WEART214
PRINT 5870, ABLOCK	WEART215
GO TO 2250	WEART216
2255 PRINT 5865,ADATA,IBLOCK,MODE,ABLOCK	WEART217
CCC	WEART218
CCC CHECK FOR DATA END CARD.	WEART219
CCC	WEART220
IF (IBLOCK .LT. 0) GO TO 2320	WEART221
CCC	WEART222
CCC NO DATA END CARD, CHECK FOR ADD/MODIFY.	WEART223
CCC	WEART224
N = 0	WEART225
IF (MODE.NE.1RA) GO TO 2260	WEART226
CCC	WEART227
CCC DATA ADDS TO DATA ALREADY READ IN.	WEART228
CCC	WEART229
N = 1	WEART230
PRINT 5850	WEART231
GO TO 2265	WEART232
2260 IF (MODE.NE.1RB) GO TO 2265	WEART233
CCC	WEART234
CCC DATA MODIFIES OR ADDS TO DATA ALREADY READ IN.	WEART235
CCC	WEART236
N = -1	WEART237
PRINT 5855	WEART238
2265 MODE = N	WEART239
IF (MODE.EQ.0) ITEMS (IBLOCK) = 0	WEART240
NEWBL (IBLOCK) = NEWBL (IBLOCK) + 1	WEART241
C	WEART242
C-----GO TO SUBROUTINES TO READ IN BLOCK ITEMS.	WEART243
C	WEART244
2275 GO TO (2310,2280,2310,2280,2295,2245,2265,2245,2310,2245), IBLOCK	WEART245
CCC	WEART246
CCC BLOCK 2 OR 4	WEART247
CCC	WEART248
2280 CALL THERM	WEART249
GO TO 2245	WEART250
CCC	WEART251
CCC BLOCK 5.	WEART252
CCC	WEART253
2295 CALL FINK	WEART254
GO TO 2245	WEART255
CCC	WEART256
CCC BLOCK 1,3 OR 9	WEART257
CCC	WEART258
2310 CALL TALLY	WEART259
GO TO 2245	WEART260
C	WEART261
C-----COME HERE AFTER ALL DATA AND DATA END CARD HAVE BEEN READ IN.	WEART262
C-----INITIALIZE BEFORE FIRST TIME STEP.	WEART263
C	WEART264
2320 CALL LINE	WEART265
CALL LINE	WEART266
CCC	WEART267
CCC SAVE NAME FOR IDENTIFICATION OF DATA FOR CONTINUATION PROBLEMS.	WEART268

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CCC      DO 2325  N = 1,20
2325     NAMES(N) = NAME(N)
CCC
CCC      FIND ELAPSED TIME FOR INPUT PHASE OF PROBLEM.
CCC
      CALL SECOND(ELT)
      ELT = ELT - TZERO
      PRINT 5846, ELT
CCC
CCC      SUMMARIZE INPUT DATA.
CCC
      CALL TALLY
CCC
CCC      CHECK FOR MISSING BLOCK 2 OR 4.
CCC
      IF(KWIT .EQ. 9) GO TO 4500
C
C-----INCREMENT CYCLE COUNTER, DO NEXT TIME STEP.
C
4280     KCYC = KCYC + 1
CCC
CCC      GO TO SUBROUTINES TO DO TIME STEP CALCULATIONS AND WRITE OUT DATA.
CCC
C-----RETURN HERE AFTER A TIME STEP IS REJECTED.
C
4285     CALL THERM
      IF(NOCOR .NE. 0) CALL FINK
      IF(NSTOP) 4500,4210,4500
4210     IF(KWIT) 4205,4380,4215
4215     IF((KWIT - 5)*(KWIT - 9) .EQ. 0) GO TO 4500
4220     NSTOP = 1
      GO TO 4280
C
C-----TEST FOR PROBLEM PREPROCESSING ONLY.
C
4380     IF(NCHECK.LE.0) GO TO 4280
      KWIT = 6
      GO TO 4220
C
C-----COME HERE AT END OF PROBLEM. (OTHERS MAY FOLLOW.)
C
4500     KCYC = KCYC - 1
      N = KWITLOC(KWIT)
      KWIT = KWITLOC(KWIT + 1) - N
      PRINT 5805, (KWITNSG(N + K), K = 1,KWIT)
CCC
CCC      GO BACK FOR ANOTHER PROBLEM.
CCC
      GO TO 2115
C
C-----COME HERE WHEN A /SPLIT/ CARD HAS BEEN READ IN FROM DATA DECK.
C
4600     CALL SECOND(ELT)
      ELT = ELT - TZERO
      PRINT 5846, ELT
      PRINT 5955
      STOP
CCC
CCC      END OF HEART, STOP PROCESSING.
CCC
5825     FORMAT(/10X,10HENDED PROB,1I5,5X,6HKCYC =,1I6,5X,8HNSUMYIN =,E13.5)

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HEART269
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 HEART329
 HEART330


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5830 FORMAT(1H1,9X,17HTRUST OUTPUT DATA,13X,12HPROGRAM DATA,2(2X,A10), HEART331
      1 20X,9HDATA DECK,115//10X,1R1,14A5,1R1,4A2) HEART332
5840 FORMAT(//10X,42HNO DATA CARRY-OVER FROM PRECEDING PROBLEM.) HEART333
5845 FORMAT(50H1 ELAPSED TIME BEFORE START OF TRUST JOB =, HEART334
      1 1F7.3,9H SECONDS.) HEART335
5846 FORMAT(//10X,40H ELAPSED TIME SINCE START OF TRUST JOB = ,1F7.3, HEART336
      8 9H SECONDS.) HEART337
5850 FORMAT(30X,34HDATA ADDS TO DATA ALREADY READ IN.) HEART338
5855 FORMAT(30X,46HDATA MODIFIES OR ADDS TO DATA ALREADY READ IN.) HEART339
5860 FORMAT(1A5,1I2,1R1,12A6) HEART340
5865 FORMAT(10X,5HDATA ,1A5,1I3,1R1,14X,12A6) HEART341
5870 FORMAT(30X,12A6) HEART342
5875 FORMAT(1R1,14A5,1R1,4A2) HEART343
5880 FORMAT(10X,12A6) HEART344
5885 FORMAT(//10X,27HREASON FOR ENDING PROBLEM -//10X,11A10,//10X,3A10) HEART345
5895 FORMAT(//,10X,36H WILL USE DATA AND INITIAL CONDITIONS, HEART346
      1 43H FROM PRECEDING PROBLEM WITH NAME, DATE --- ) HEART347
5900 FORMAT(//,10X,46H WILL USE DATA AND FINAL CONDITIONS, WITH YAU =, HEART348
      1 E13.5,43H FROM PRECEDING PROBLEM WITH NAME, DATE --- ) HEART349
5905 FORMAT(12A5) HEART350
5910 FORMAT(1I2,1X,1A5) HEART351
5915 FORMAT(1I6,2(2I3,1X,1A10,1I7)) HEART352
5920 FORMAT(1I4) HEART353
5935 FORMAT(10X,1R1,14A5,1R1,4A2) HEART354
5955 FORMAT(//10X,17HEND OF TRUST JOB.) HEART355
5960 FORMAT(32H *****NO DATA IS IN MEMORY.) HEART356
      END HEART357
C HEART358
C HEART359
C HEART360
C HEART361
      SUBROUTINE THERM THERM 2
C THERM 3
C*****VERSION 5/20/69. THERM 4
C*****REVISED 3/73. THERM 5
C THERM 6
C*****THERM IS A REQUIRED SUBROUTINE. THERM 7
CCC VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES BLINK 2
      COMMON DINC,NOISCR,PSIPIN,PSINAX,TOLPR,XMIN,XMAX,IRESTRY,INT BLINK 3
CCC INTEGER VARIABLES BLINK 4
      COMMON NOSPEC,NHAT,NFLUID,MODES,NOCON,NOSCON,NODBS,NOGEN,NIT, BLINK 5
      8 NVARC,NVARG,NVARH,NVARK,NVART,NZ,IPRINT,TIMEP BLINK 6
      COMMON IBLOCK,KCYC,KDATA,KS,KSECS,KSVM,KWIT,LABEL,LTAD,LXX, BLINK 7
      8 MAX,NID,NODE,NADD,NADG,NADS,NADSB,NAD1,NAD2,NCHECK,NODMS, BLINK 8
      8 NOW,NPROB,NSAVE,NSEQ,NSTOP,NUTS,KSPEC BLINK 9
CCC END BLINK BLINK 10
CCC TEMPORARY VALUES OF INTEGER AND REAL VARIABLES BLISC 2
      COMMON A1,DX,II,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,WND,W1,NZ, BLISC 3
      8 P1,P2,P3,P4,P5,TX,X1,X2,X3 BLISC 4
CCC END BLISC BLISC 5
CCC REAL VARIABLES BLARE 2
      COMMON ADATA,ADD,AFLUID,APATH,BEY,BETA,BETH,BPRIME,CAPNS, BLARE 3
      8 CAPS,CAPT,CONST,DAY,DELT,DELTS,DELYSS,DEX,DLONG,DNDP, BLARE 4
      8 DPHAXS,DPRES,DPSIPAX,DRAD,ORADS,OWIDE,EAVG,EC,ELY,ESTRESS, BLARE 5
      8 ESTRIN,EX,FEX,FLEX,FLUID,FLUX,FLXS,FLX,FOR,FORD, BLARE 6
      8 GBYNU,GEE,GENS,GECH,GHASS,GONE,GX,HONE,HX,PHIBS, BLARE 7
      8 PHIONE,PORS,PSIVARY,PSI1,PSI2,RD,RNOZ,SAVG,SCALE,SET, BLARE 8
      8 SEYD,SET1,SMALL,SUNYTH,YAU,THICK,TIME,YZERO,VISC,VOLHS, BLARE 9
      8 VOLS,WNS,PCONE,TIMAX,PCX,DELYF,XX2,XX3 BLARE 10
CCC END BLARE BLARE 11
CCC ARRAYED VARIABLES BLARY 2
CCC BLARY 3

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CCC THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE
CCC THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC
CCC DISTRIBUTIONS. THE FIRST INDEX REFERS TO THE DISCREET VALUE,
CCC THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR
CCC PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR
CCC CONNECTION NUMBER.
CCC
      LEVEL2,CAP,CON,DFI,DPSI,DF,PHI,PPHI,ZIP,TRAN
      COMMON/LCM/CAP(100),CON(20,2,100),DFI(20,2,200),DPSI(20,2,200),
$      DF(20,2,200),PHI(20,2,200),PPHI(20,2,200),
$      ZIP(200),TRAN(20,2,200)
      COMMON F1(12),F2(12),F3(12),NOD1(200),NOD2(200),NODE(100),
$      FX(1),FY(1),DDFI(1),
$      DPSI(1),DPSIS(1),E(1),ITEMS(15),LTABK(10),NAME(20),
$      NAMES(20),NEWBL(15),NTYPE(100),PC(1),PCH(1),SLIN(1),
$      TOTSTRS(1),VOL(100),W(1),Z(100),CK(10),NODTO(1),R(1),
$      MODIN,TLUMP,PLUMP,XF,Q,PSIVARC(12,10),PZ(10),KODANIS,
$      COA2(1),CON3(1),CONST1(10),CONST2(10),KODCON(1),
$      KSTDATA,LANISK(10),LTABG(10),PPC(1),
$      NODHAT(100),NOXHAT(100),NX1(10),PSIVARK(12,10)
CCC END BLARY
C
C      COMMON/AREA/APEA(200),DEL1(200),DEL2(200),NCX1(200),NOX2(200)
      DIMENSION AMAT(10),AV(10),CC(10),CS(10),CONT(20,2,10),EK(10),
$      COA2(10),EZ(10),SLOG(12,10),SS(10),MAT(10)
      COMMON/AGEN/GT(1),LTABG(1),NODG(1),NOXG(1),SLOG(1),
$      TVARG(1),G(1),FG(1),DFG(1),GG(1)
C
      COMMON /M/ M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, M11, M12, M13
CCC
CCC
      COMMON/STRAIN/VOLID(100),DELVOL(100)
CCC
      DIMENSION CON1(1,20,2)
      EQUIVALENCE (CON1,CON)
C      DIMENSION CONT1(10),CONT2(10),CONT3(10)
      DIMENSION AVS(10)
      EQUIVALENCE (SS,AVS)
      NMIN1(I) = MIN0(I, N - 1)
C
      IF(KCYC) 2000,3100,4200
      2000 IF(10LOCK.EQ. 2) GO TO 2135
      IF(10LOCK.EQ. 4) GO TO 2275
C
CCC INPUT DATA BLOCK 2. MATERIAL PROPERTY LIST.
C
      2135 LABEL = 1
      IF(MODE.EQ.0)NVARC = NVARK=NNAT=0
      N = NNAT
C
      2140 READ 5800,A1,M1,L1,L2,L3,L4,P1,P2,P3,P5
      IF(A1.NE.5HSYSTEM) GO TO 2150
      READ 5790,THICK,RO,XF,Q
      THICK = THICK * SCALE
      PRINT 5795,THICK,RO,XF,Q
      CALL LINE
      GO TO 2140
C
      2150 IF(M1.EQ.0) GO TO 2270
      DO 2141 NHI=1,NDISCR
CCC READ IN THE PROBABILISTIC ARRAY, CONT, FOR PERMEABILITY.

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BLARY 4
 BLARY 5
 BLARY 6
 BLARY 7
 BLARY 8
 BLARY 9
 BLARY 10
 BLARY 11
 BLARY 12
 BLARY 13
 BLARY 14
 BLARY 15
 BLARY 16
 BLARY 17
 BLARY 18
 BLARY 19
 BLARY 20
 BLARY 21
 BLARY 22
 BLARY 23
 BLARY 24
 THERM 12
 THERM 13
 THERM 14
 THERM 15
 THERM 16
 AGEN 2
 AGEN 3
 AGEN 4
 N 2
 N 3
 THERM 19
 THERM 20
 THERM 21
 THERM 22
 THERM 23
 THERM 24
 THERM 25
 THERM 26
 NMIN1 2
 NMIN1 3
 THERM 28
 THERM 29
 THERM 30
 THERM 31
 THERM 32
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 THERM 34
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 THERM 36
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 THERM 38
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 THERM 41
 THERM 42
 THERM 43
 THERM 44
 THERM 45
 THERM 46
 THERM 47
 THERM 48

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2141 READ 5801,CONT(NHIST,1,N1),CONT(NHIST,2,N1)          THERN 49
2142 READ 5802,N3,NSEQ,NADD,N4                             THERN 50
DO 2145 J1=N3,N3+NSEQ,NADD                                THERN 51
DO 2146 NHIST=1,NDISCR                                     THERN 52
DO 2147 MORP=1,2                                           THERN 53
CCC ASSIGN CONT ARRAY TO CON ARRAY                        THERN 54
CON(NHIST,MORP,J1)=CONT(NHIST,MORP,N1)                   THERN 55
2147 CONTINUE                                              THERN 56
2146 CONTINUE                                              THERN 57
2145 CONTINUE                                              THERN 58
IF(N4.EQ.1) GO TO 2142                                     THERN 59
IF(MODE.GE.0) GO TO 2170                                    THERN 60
CALL SEEK1(N,N1,MAT,NHAT,K)                                THERN 61
IF(K.EQ.0) GO TO 2165                                       THERN 62
IF(LTABC(N).NE.0) NVARC = NVARC - 1                       THERN 63
IF(LTABK(N).NE.0) NVARK = NVARK - 1                       THERN 64
2165 ITEMS(2) = NMIN1(ITEMS(2))                            THERN 65
GO TO 2175                                                  THERN 66
2170 N = NHAT = N + 1                                       THERN 67
2175 IF(N.GT.N2) CALL MORTHAN(N,NPAT,N2)                  THERN 68
ANAT(N) = A1                                                THERN 69
MAT(N) = N1                                                 THERN 70
LTABC(N) = L1                                               THERN 71
LTABK(N) = L2                                               THERN 72
LANISK(N) = L3                                              THERN 73
AV(N) = ABS(P1)                                             THERN 74
EZ(N) = P2                                                  THERN 75
PZ(N) = P3                                                  THERN 76
SS(N) = P5                                                  THERN 77
LABEL = 0                                                  THERN 78
CK(N) = CS(N) = CC(N) = 0.                                  THERN 79
CONST1(N) = EZ(N) + AV(N) * PZ(N)                         THERN 80
IF(SS(N).LE.0..OR.AV(N).GT.0.)GO TO 2180                 THERN 81
PRINT 5805                                                  THERN 82
PRINT 5806,ANAT(N),MAT(N),N,LTABC(N),LTABK(N),LANISK(N),SS(N) THERN 83
DO 2176 NHIST=1,NDISCR                                     THERN 84
CCC PRINT OUT PERMEABILITY DISTRIBUTIONS                 THERN 85
PRINT 5807,CONT(NHIST,1,N),CONT(NHIST,2,N)               THERN 86
2176 CONTINUE                                              THERN 87
GO TO 2183                                                  THERN 88
2180 PRINT 5810                                             THERN 89
PRINT 5813,ANAT(N),MAT(N),N,LTABC(N),LTABK(N),LANISK(N),AV(N), THERN 90
SEZ(N),PZ(N),AVS(N)                                       THERN 91
DO 2181 NHIST=1,NDISCR                                     THERN 92
CCC PRINT OUT PERMEABILITY DISTRIBUTIONS                 THERN 93
PRINT 5814,CONT(NHIST,1,1),CONT(NHIST,2,1)               THERN 94
CON(NHIST,1,1)=CONT(NHIST,1,1)                            THERN 95
CON(NHIST,2,1)=CONT(NHIST,2,1)                            THERN 96
2181 CONTINUE                                              THERN 97
IF(AV(N).GT.0.0) GO TO 2220                                THERN 98
CCC THE ROUTINE WHICH NORMALLY FOLLOWS READS IN THE PERMEABILITIES THERN 99
CCC FOR THE ANISOTROPIC SYSTEM. THE CURRENT PROGRAM      THERN100
CCC DOES NOT PERFORM THIS PROBLEM. THE CONT1, CONT2, AND CONT3 THERN101
CCC ARRAYS MUST BE REDIMENSIONED.                        THERN102
CCC                                                       THERN103
2183 IF(LANISK(N).LE.0) GO TO 2185                         THERN104
2184 KODANIS=1                                             THERN105
2185 IF(SS(N).NE.0..AND.AV(N).LE.0.)GO TO 2140           THERN107
2220 CONTINUE                                              THERN108
CCC                                                       THERN109
CCC CALCULATE SLOPES FOR CONDUCTIVITY TABLE WHEN FLOW REGION IS ALWAYS THERN110

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CCC      TO REMAIN SATURATED
CCC      THE FORM OF THIS PROGRAM DOES NOT SOLVE THIS PROBLEM.
CCC
  2250  LTAB = IABS(LTABK(N))
        IF(LTAB.GT. 1) GO TO 2255
        LTABK(N) = 0
        GO TO 2140
  2255  NVAR=NVAR+1
        LABEL = 1
  2270  NEWBL(4) = NEWBL(4) + 1000
        RETURN

C
C COMPLETED BLOCK 2.
C
CCC
CCC INPUT DATA BLOCK 4.  NODE LIST.
CCC
  2275  L = 0
        IF(MODE.EQ.0) NODS=NOSPEC=0
        N = NODS
C
        KODEZ = 0
  2280  READ 5020,N1,NSEQ,NADD,N2,KS,DLONG,DWIDE,DRAD,P2,P1
C
        IF(N1.EQ. 0) GO TO 2330
        DLONG = DLONG*SCALE
        DWIDE = DWIDE*SCALE
        DRAD = DRAD*SCALE
        P1 = P1*SCALE
        P2 = P2*SCALE
        ADD = 0.0
        IF(DRAD.GE. 0.) GO TO 2285
        ADD = -(DRAD + DRADS)
        DRAD = DRADS
  2295  IF(MODE.GE.0) GO TO 2295
        CALL SEEK1(N, N1, NODE, NODS, K)
        IF(K*NTYPE(N).NE. 0) NOSPEC = NOSPEC - 1
        ITEMS(4) = MIN1(ITEMS(4))
        GO TO 2300
  2295  N = NODS = N + 1
  2300  IF(N.GT. N4) CALL MORTHAN(N,NODS,N4)
        MODMAT(N) = NOXMAT(N) = N2
        NODE(N) = N1
        NTYPE(N) = 0
        IF(KODEZ.LE.0) GO TO 2305
        Z(N) = Z(N-1) + P2
        GO TO 2310
  2305  Z(N) = P1
  2310  CONTINUE
        G(1)=GONE
        PC(1)=PCONE
        DRADS = DRAD + ADD
        VOL(N) = GEOM*DLONG*DWIDE*DRAD**KSYM
        IF(VOL(N).NE. 0.) GO TO 2315
        VOL(N) = 1.E-24
        NTYPE(N) = 2
        GO TO 2319
  2315  IF(KS.EQ. 0) GO TO 2320
        NTYPE(N) = 3
  2319  NOSPEC = NOSPEC + 1
  2320  IF(MOD(L,57).EQ. 0) PRINT 5025
        L = L + 1

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THERM111
 THERM112
 THERM113
 THERM114
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 THERM122
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 THERM170
 THERM171
 THERM172

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      PRINT 5830, NODE(N), N, NODNAT(N), MTYPE(N), DLONG, DOWIDE, DRAD, VOL(N),
$Z(N)
      IF (INSEQ.LE.0) GO TO 2325
      NSEQ = NSEQ + 1
      N1 = N1 + NADD
      KODEZ = 1
      GO TO 2285
2325 KODEZ = 0
      GO TO 2280
2330 DO 2335 N = 5,12
2335 NEWBL(N) = NEWBL(N) + 1000
      NEWBL(1) = NEWBL(1) + 1000
      NEWBL(7) = NEWBL(7) - 1000
      RETURN
C
C COMPLETED BLOCK 4. RETURN TO HEART.
C
3100 IBLOCK = 4
      CALL REFER(NODNAT, NOXNAT, NODES, NAT, MNAT)
CCC
CCC CALC. NODE MASSES, CAPACITIES, CONDUCTIVITIES.
CCC
3110 IF (KNIT.NE.0) RETURN
CCC
CCC DETERMINE THE CAPACITANCE OF EACH ELEMENT
CCC
3115 DO 3125 M=1,NODES
      J = NODNAT(N)
      IF (NAMES(16).NE.1R 180 TO 6600
      FACT = 1.
      CAP(N) = VOL(N) * SS(J) * FACT * RHOZ
      E(N) = 0.4
6600 IF (CAP(N).LE.0.) CAP(N) = 1.E-36
3125 CONTINUE
CCC
CCC FIND OUT AND WRITE TOTAL NODES, VOLUME, CAP OF EACH MATERIAL
CCC
      PRINT 5870
      CAPS = FLUID = VOLS = 0.
      DO 3135 K = 1,MNAT
      NODNS = 0
      VOLNS = CAPNS = WMS = 0.
      DO 3130 N = 1,NODES
      IF (NODNAT(N).NE.K) GO TO 3130
      NODNS = NODNS + 1
      VOLNS = VOLNS + VOL(N)
      CAPNS = CAPNS + CAP(N)
      WMS = WMS + W(N)
3130 CONTINUE
      CAPS = CAPS + CAPNS
      VOLS = VOLS + VOLNS
C
      FLUID = FLUID + WMS
      IF (CAPNS.GT.0) PRINT 5875, ANAT(K), NAT(K), NODNS, VOLNS, CAPNS
3135 CONTINUE
      PRINT 5890, NODES, VOLS, CAPS
4200 IF (KSPEC.LY.0) RETURN
5790 FORMAT(16E10.3)
5795 FORMAT(44X, 22NFLOW REGION PROPERTIES, /,
      825X, 56H THICKNESS REL DEN 6/0, THERN231
      825X, 68H OF FLOW REGION OF SATURATED XF THERN232
      8 Q. /, THERN233
      825X, 56H SCIL 6/0, THERN234

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      540X,4E15.4)
5800 FORMAT(A5,5I5,4E10.3)
5801 FORMAT(2E10.4)
5802 FORMAT(4I5)
5805 FORMAT(/,10X,77H AHAT NATL INDEX LTABC LTABK LANISK PERHEABTHERN239
      $ILITY SPECIFIC STORAGE,/,49X,21NMAGNITUDE PROBABILITY,/) THERN240
5806 FORMAT(10X,A6,4I6,I7,24X,E15.4) THERN241
5807 FORMAT(47X,2E12.4) THERN242
5810 FORMAT(10X,119H AHAT NATL INDEX LTABC LTABK LANISK AVTHERN243
      $ EZ PZ PERHEABILITY AVTHERN244
      $S,/,94X,22NMAGNITUDE PROBABILITY) THERN245
5813 FORMAT(8X,A6,3I6,2I7,6X,3E14.4,24X,E14.4) THERN246
5814 FORMAT(94X,2E14.4) THERN247
5815 FORMAT (8E10.3) THERN248
5820 FORMAT(5I5,5X,5E10.3) THERN249
5825 FORMAT(/,12X,22HNODE INDEX NATL NTYPE,4X,5HDLONG,10X,5HWDIDE,
      $10X,4HDRAD,11X,6HVOLUME,11X,1HZ) THERN250
5830 FORMAT(10X,4I6,5E14.5) THERN251
5865 FORMAT(10X,4I6,2X,A6) THERN252
5870 FORMAT(/,10X,16HMATERIAL SUPMARY,/,
      $12X,52HNAME NATL NODES TOT VOL TOT CAP TOT FLUID) THERN253
5875 FORMAT(10X,A6,2I6,2E13.5) THERN254
5890 FORMAT(/,10X,12HSYSTEM TOTAL,16,2E12.5) THERN255
      END THERN256
C THERN257
C THERN258
C THERN259
C THERN260
C THERN261
C THERN262
      SUBROUTINE FINK FINK 2
C FINK 3
C*****VERSION 5/20/69. FINK 4
C*****REVISED 6/73. FINK 5
C FINK 6
C*****FINK IS A REQUIRED SUBROUTINE FOR HEAT CONDUCTION PROBLEMS. FINK 7
C*****A NUMBER OF PROBLEMS INVOLVING FLOW, CHEMICAL REACTION, HEAT FINK 8
C*****GENERATION, CONVECTIVE AND RADIATIVE TRANSPORT, ETC., MAY BE FINK 9
C*****SOLVED WHICH DO NOT REQUIRE FINK. FINK 10
CCC VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES BLINK 2
      COMMON DIMCR,NDISCR,PSI,FIN,PSIMAX,TOLPR,XFIN,XMAX,IRESTRT,INT BLINK 3
CCC INTEGER VARIABLES BLINK 4
      COMMON NCSPEC,NMAT,NFLUID,NODES,NOCON,NOSCON,NODBS,NOGEN,NIT, BLINK 5
      $ NVARC,NVARG,NVARH,NVARK,NVART,NZ,IPRINT,TIMEP BLINK 6
      COMMON IBLOCK,KCYC,KDATA,KS,KSECS,KSVM,KHIT,LABEL,LTAB,LXX, BLINK 7
      $ MAX,MID,MODE,NADD,NADG,NADS,NADSB,AD1,NAD2,NCHECK,MODMS, BLINK 8
      $ NOW,NPROB,NSAVE,NSEQ,NSTOP,NUTS,KSPEC BLINK 9
CCC END BLINK BLINK 10
CCC TEMPORARY VALUES OF INTEGER AND REAL VARIABLES BLISC 2
      COMMON A1,DX,II,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,NND,N1,N2, BLISC 3
      $ P1,P2,P3,P4,P5,TX,X1,X2,X3 BLISC 4
CCC END BLISC BLISC 5
CCC REAL VARIABLES BLARE 2
      COMMON ADATA,ADD,AFLUID,APATH,BET,BETA,BETH,BPRIME,CAPHS, BLARE 3
      $ CAPS,CAPT,CONST,DAY,DELT,DELTS,DELTSS,DEX,DLONG,DNDP, BLARE 4
      $ DPHXS,DPRES,DPSIPAX,DRAD,DRADS,DWIDE,EAVG,EC,ELT,ESTRESS, BLARE 5
      $ ESTRIN,EX,FEX,FLEX,FLUID,FLUX,FLUXS,FLX,FOR,FORD, BLARE 6
      $ GBYMU,GEE,GENS,GECH,GHASS,GONE,GX,HONE,HX,PHIBS, BLARE 7
      $ PHIONE,PORS,PSIVARY,PSI1,PSI2,RO,RHOZ,SAVG,SCALE,SET, BLARE 8
      $ SEYO,SET1,SHALL,SUMTIN,TAU,THICK,TIME,TZERO,VISC,VOLMS, BLARE 9
      $ VOLS,WMS,PCONE,TIMAX,PCX,DELTF,XX2,XX3 BLARE 10
CCC END BLARE BLARE 11
CCC ARRAYED VARIABLES BLARY 2
CCC BLARY 3

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CCC THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE BLARY 4
CCC THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC BLARY 5
CCC DISTRIBUTIONS. THE FIRST INDEX REFERS TO THE DISCREET VALUE. BLARY 6
CCC THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR BLARY 7
CCC PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR BLARY 8
CCC CONNECTION NUMBER. BLARY 9
CCC BLARY 10
      LEVEL2,CAP,CON,DFI,DPSI,DF,PHI,PPHI,ZIP,TRAN
      COMMON/LCH/CAP(100),CON(20,2,100),DFI(20,2,200),DPSI(20,2,200),
$      DF(20,2,200),PHI(20,2,200),PPHI(20,2,200),
$      ZIP(200),TRAN(20,2,200)
      COMMON F1(12),F2(12),F3(12),MOD1(200),MOD2(200),MODE(100),
$      FI(1),FX(1),DDFI(1),
$      DDPSI(1),DPSIS(1),E(1),ITEMS(15),LTABK(10),NAME(20),
$      NAMES(20),NEWEL(15),NTYPE(100),PC(1),PCH(1),SLIN(1),
$      TOTSYS(1),VOL(100),W(1),Z(100),CK(10),MODTO(1),R(1),
$      MODIN,TLUMP,PLUMP,XF,Q,PSIVARC(12,10),PZ(10),KODANIS,
$      CON2(1),CON3(1),CONST1(10),CONST2(10),KODCON(1),
$      KSTDATA,LANISK(10),LTABC(10),PPC(1),
$      NODMAY(100),NOXHAT(100),NOX1(10),PSIVARK(12,10)
CCC END BLARY BLARY 23
      COMMON/AREA/AREA(200),DEL1(200),DEL2(200),NOX1(200),NOX2(200)
      COMMON /P/ M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, M11, M12, M13
CCC THE FOLLOWING VARIABLES ARE THREE-DIMENSIONAL AREA 2
CCC PROBABILISTIC ARRAYS. W 2
CCC W 3
      LEVEL2,CEN1,PHID,C,COMPRD,CONSUN,PHINT,XNINPHI,XNAXPHI
      COMMON/TOA/PHID(20,2,1),C(20,2,1),CEN1(20,2,100),COMPRD(20,2,1),
$      CONSUN(20,2,1),PHINT(100),OPSINT(100),XNINPHI(100),XNINDPS(100),
$      SXNAXPHI(100),YNAXDPS(100)
      NPIN1(1) = MIN0(1, N - 1)
C
      IF(KCYC) 2000,3100,4200
CCC INPUT DATA BLOCK 5. INTERNAL CONNECTION LIST FINK 17
CCC FINK 18
CCC FINK 19
      LEVEL2,CEN1,PHID,C,COMPRD,CONSUN,PHINT,XNINPHI,XNAXPHI
      COMMON/TOA/PHID(20,2,1),C(20,2,1),CEN1(20,2,100),COMPRD(20,2,1),
$      CONSUN(20,2,1),PHINT(100),OPSINT(100),XNINPHI(100),XNINDPS(100),
$      SXNAXPHI(100),YNAXDPS(100)
      NPIN1(1) = MIN0(1, N - 1)
C
      IF(KCYC) 2000,3100,4200
CCC INPUT DATA BLOCK 5. INTERNAL CONNECTION LIST FINK 20
CCC FINK 21
CCC FINK 22
      2000 L = 0
      IF(MODE.EQ.0) N=NOCON=0
      N = NOCON
C
      2100 READ 5005,N1,N2,NSEQ,NAD1,NAD2,NZ,P1,P2,DLONG,DRAD
C
      IF(N1.EQ.0) RETURN
      P1 = P1*SCALE
      P2 = P2*SCALE
      DLONG = DLONG*SCALE
      DRAD = DRAD*SCALE
      IF(NZ.LE.0) NZ = 1
      2105 CONTINUE
      2120 ADD = 0.0
      IF(DRAD.GE.0.) GO TO 2125
      ADD = -(DRAD + DRADS)
      DRAD = DRADS
      2125 IF(MODE.GE.0) GO TO 2135
      CALL SEEK2(N, N1, N2, NOX1, NOX2, NOCON, K)
      ITEMS(5) = NMN1(ITEMS(5))
      GO TO 2140
      2135 N = NOCON = N + 1
      KODCON(N) = NZ
      2140 IF(N.GT. M5) CALL MORTNAN(N,NOCON,M5)
      MOD1(N) = NOX1(N) = M1

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MOD2(N) = NOX2(N) * N2
DEL1(N) = P1
DEL2(N) = P2
DRAD = DRADS = DRAD + ADD
AREA(N) = GEOM*DLONG*DRAD**KSYM
IF(MOD(L,57) .EQ. 0) PRINT 5010
L = L + 1
PRINT 5000,MOD1(N),MOD2(N),N,DEL1(N),DEL2(N),DLONG,DRAD,AREA(N),
SKODCON(N)
IF(NSEQ .LE. 0) GO TO 2100
NSEQ = NSEQ - 1
N1 = N1 + NAD1
N2 = N2 + NAD2
GO TO 2125
C
C COMPLETED BLOCK 5.
C
3100  IBLOCK = 5
      NPRINT=0
      CALL REFER(MOD1, NOX1, NOCON, NODE, NODES)
      CALL REFER(MOD2, NOX2, NOCON, NODE, NODES)
      IF(KWIT .NE. 0) RETURN
CCC
CCC  INITIALIZE THE MASS (DF) AND THE CHANGE IN POTENTIAL (DPSI).
CCC
      NHLDIS=(NDISCR+1)/2
      DO 3102 N=1,NODES
      DO 3103 NHIST=1,NDISCR
C INITIALIZE MAGNITUDE ELEMENTS
      DF(NHIST,1,N)=DPSI(NHIST,1,N)=DFI(NHIST,1,N)=0.0
      3103 CONTINUE
      DO 3104 NHIST=1,NHLDIS-1
C INITIALIZE PROBABILITY ELEMENTS
      DF(NHIST,2,N)=DPSI(NHIST,2,N)=DFI(NHIST,2,N)=0.0
      3104 CONTINUE
      DF(NHLDIS,2,N)=DPSI(NHLDIS,2,N)=DFI(NHLDIS,2,N)=1.0
      DO 3105 NHIST=NHLDIS+1,NDISCR
      DF(NHIST,2,N)=DPSI(NHIST,2,N)=DFI(NHIST,2,N)=0.0
      3105 CONTINUE
      3102 CONTINUE
CCC
CCC  CALCULATE CONDUCTANCE OF INTERNAL CONNECTIONS.
CCC
      NS=0
      DO 3150 N = 1,NOCON
      N1 = MOD1(N)
      N2 = MOD2(N)
      X1 = GBYHU * AREA(N)
      IF(KODANIS.LE.0) GO TO 3140
      IF(KODCON(N)-2) 3140,3125,3130
CCC
CCC  THE ROUTINE FOR AN ANISOTROPIC MATERIAL FOLLOWS. THE EXISTING
CCC  FORM OF THE PROGRAM DOES NOT SOLVE THE ANISOTROPIC PROBLEM.
CCC  IF THIS IS DESIRED, THE APPROPRIATE DIMENSIONS ARE REQUIRED
CCC  FOR ARRAYS CON2 AND CON3.
CCC
      3125 C1=CON2(N1)
          C2=CON2(N2)
          GO TO 3145
      3130 C1=CON3(N1)
          C2=CON3(N2)
          GO TO 3145

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FINK 54
 FINK 55
 FINK 56
 FINK 57
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 FINK 111
 FINK 112
 FINK 113
 FINK 114
 FINK 115


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CCC      DETERMINE WHETHER A HETEROGENEITY EXISTS.                                FINK 116
3140 IF (NODHAT(N1) .EQ. NODHAT(N2)) GO TO 3146                                FINK 117
CCC                                             FINK 118
CCC      THE HARMONIC MEAN PERMEABILITY IS CALCULATED USING                      FINK 119
CCC      PROBABILISTIC PERMEABILITIES AT THE INTERFACE OF A                     FINK 120
CCC      HETEROGENEITY.                                                            FINK 121
CCC                                             FINK 122
          MAGPOT=1                                                                FINK 123
          NSIGN=3                                                                  FINK 124
CCC      CALL PRODIST TO CALCULATE CONPRD = K1 * K2.                            FINK 125
          CALL PRODIST(CON(1,1,N1),CON(1,1,N2),CONPRD(1,1,1),MAGPOT,NSIGN)      FINK 126
          DO 3141 NHIST=1,NDISCR                                                  FINK 127
CCC      CALCULATE THE PRODUCT OF CONPRD AND RHOZ (DENSITY) BY                    FINK 128
CCC      SCALING THE MAGNITUDE ELEMENTS.                                          FINK 129
          CONPRD(NHIST,1,1)=CONPRD(NHIST,1,1)*RHOZ                             FINK 130
3141 CONTINUE                                                                    FINK 131
          DO 3142 NHIST=1,NDISCR                                                  FINK 132
CCC      CALCULATE CON1(N1) = K1 * L2. SCALE THE MAGNITUDE                      FINK 133
CCC      ELEMENTS BY L2.                                                          FINK 134
          CON1(NHIST,1,N1)=CON(NHIST,1,N1)*DEL2(N)                             FINK 135
CCC      ASSIGN THE PROBABILITY ELEMENTS.                                         FINK 136
          CON1(NHIST,2,N1)=CON(NHIST,2,N1)                                       FINK 137
CCC      CALCULATE CON1(N2) = K2 * L1. SCALE THE MAGNITUDE                      FINK 138
CCC      ELEMENTS BY L1.                                                          FINK 139
          CON1(NHIST,1,N2)=CON(NHIST,1,N2)*DEL1(N)                             FINK 140
CCC      ASSIGN THE PROBABILITY ELEMENTS.                                         FINK 141
          CON1(NHIST,2,N2)=CON(NHIST,2,N2)                                       FINK 142
3142 CONTINUE                                                                    FINK 143
          NSIGN=1                                                                  FINK 144
CCC      SUBROUTINE PRODIST IS CALLED TO CALCULATE (K1*L2) + (K2*L1).            FINK 145
CCC      WHICH IS DESIGNATED TO THE CONSUM ARRAY.                                FINK 146
          CALL PRODIST(CON1(1,1,N1),CON1(1,1,N2),CONSUM(1,1,1),MAGPOT,NSIGN)    FINK 147
          DO 3143 NHIST=1,NDISCR                                                  FINK 148
CCC      IF THE MAGNITUDE ELEMENT OF THE CONSUM ARRAY IS ZERO.                   FINK 149
CCC      BYPASS DIVISION WITH RHOZ AND RETAIN ZERO VALUE.                        FINK 150
          IF (CONSUM(NHIST,1,1) .EQ. 0.0) GO TO 3143                             FINK 151
CCC      PLACE (K1*L2)+(K2*L1) IN THE DENOMINATOR OF THE EXPRESSION              FINK 152
CCC      BY DIVISION WITH RHOZ.                                                    FINK 153
          CONSUM(NHIST,1,1)=RHOZ/CONSUM(NHIST,1,1)                             FINK 154
3143 CONTINUE                                                                    FINK 155
          DO 3144 NHIST=1,NDISCR                                                  FINK 156
CCC      SCALE MAGNITUDE ELEMENTS OF CONSUM ARRAY BY THE AREA (X1).              FINK 157
          CONSUM(NHIST,1,1)=CONSUM(NHIST,1,1)*X1                                FINK 158
3144 NSIGN=3                                                                      FINK 159
CCC      CALL PRODIST TO DETERMINE THE HARMONIC MEAN PERMEABILITY.                FINK 160
CCC      TRAN = ((K1 * K2)/((K1 * L2) + (K2 * L1))).                            FINK 161
          CALL PRODIST(CONPRD(1,1,1),CONSUM(1,1,1),TRAN(1,1,N),MAGPOT,NSIGN)    FINK 162
          GO TO 3146                                                              FINK 163
CCC                                             FINK 164
CCC      THE HARMONIC MEAN PERMEABILITY IS CALCULATED AT THE INTERFACE            FINK 165
CCC      OF ADJACENT HOMOGENEOUS ELEMENTS.                                       FINK 166
CCC                                             FINK 167
CCC      DETERMINE RHOZ/(L1 + L2)                                                 FINK 168
3146 X2=RHOZ/(DEL1(N)+DEL2(N))                                                  FINK 169
          X3 = RHOZ                                                                FINK 170
CCC      CALCULATE THE EXPRESSION, AREA*(RHOZ**2)/(L1 + L2)                     FINK 171
          X1X2X3=X1*X2*X3                                                        FINK 172
          DO 3147 NHIST=1,NDISCR                                                  FINK 173
CCC      SCALE THE MAGNITUDE ELEMENTS OF PERMEABILITY BY                       FINK 174
CCC      (AREA*(RHOZ**2)/(L1 + L2)) TO DETERMINE THE CONDUCTANCE AT              FINK 175
CCC      THE INTERFACE, N1.                                                       FINK 176
          TRAN(NHIST,1,N)=X1X2X3*CON(NHIST,1,N1)                               FINK 177

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CCC	THE PROBABILITY ELEMENTS OF THE PERMEABILITY ARRAY ARE	FINK	178
CCC	ASSIGNED TO THE CONDUCTANCE ARRAY, TRAN.	FINK	179
	TRAN(NHIST,2,N)=CON(NHIST,2,N1)	FINK	180
3147	CONTINUE	FINK	181
3148	CONTINUE	FINK	182
CCC	BYPASS CALCULATION FOR THE SUM OF THE CONDUCTANCES IF	FINK	183
CCC	ONLY THE FIRST ELEMENT CONDUCTANCE HAS BEEN DETERMINED.	FINK	184
	IF(N .EQ. 1) GO TO 3158	FINK	185
	N5=N5+1	FINK	186
CCC	CALCULATE THE SUM OF THE CONDUCTANCES (ZIP).	FINK	187
	ZIP(N5)=TRAN(NDISCR,1,N) + TRAN(NDISCR,1,N-1)	FINK	188
3150	CONTINUE	FINK	189
CCC	FIND THE MAXIMUM VALUE OF THE SUM OF THE CONDUCTANCES.	FINK	190
	ZIPMAX=ZIP(1)	FINK	191
	DO 3152 N6=2,N5	FINK	192
	IF(ZIP(N6) .LE. ZIPMAX) GO TO 3152	FINK	193
	ZIPMAX=ZIP(N6)	FINK	194
3152	CONTINUE	FINK	195
CCC	DETERMINE THE TIME STEP = SCALING FACTOR * CAPACITANCE/ MAXIMUM	FINK	196
CCC	SUM OF THE CONDUCTANCES.	FINK	197
	DELT=DELYF*CAP(INT)/ZIPMAX	FINK	198
CCC	INITIALIZE THE TOTAL TIME.	FINK	199
	SUMTIM=TAU	FINK	200
	GO TO 4210	FINK	201
CCC		FINK	202
CCC	FIND TOTAL FLUID FLUX ACROSS EACH INTERNAL CONNECTION	FINK	203
CCC		FINK	204
CCC	DETERMINE THE TIME STEP = SCALING FACTOR * CAPACITANCE/ MAXIMUM	FINK	205
CCC	SUM OF THE CONDUCTANCES.	FINK	206
CCC		FINK	207
4200	DELT=DELYF*CAP(INT)/ZIPMAX	FINK	208
	SUMTIM=SUMTIM+DELT	FINK	209
	IF(SUMTIM .GT. TIMAX) GO TO 4450	FINK	210
CCC		FINK	211
CCC	INITIALIZE THE MASS OR HEAT (DF).	FINK	212
	DO 4205 N=1,NCON	FINK	213
CCC	INITIALIZE THE MAGNITUDE ELEMENTS.	FINK	214
	DO 4201 NHIST=1,NDISCR	FINK	215
4201	DF(NHIST,1,N)=0.0	FINK	216
CCC	INITIALIZE PROBABILITY ELEMENTS.	FINK	217
	DO 4202 NHIST=1,NMLDIS-1	FINK	218
4202	DF(NHIST,2,1)=0.0	FINK	219
	DF(NMLDIS,2,N)=1.0	FINK	220
	DO 4203 NHIST=NMLDIS+1,NDISCR	FINK	221
4203	DF(NHIST,2,N)=0.0	FINK	222
4205	CONTINUE	FINK	223
4210	IF(IPRINT .EQ. 0) GO TO 4300	FINK	224
	IF(MOD(KCYC,IPRINT) .GT. 0) GO TO 4300	FINK	225
CCC		FINK	226
CCC	WRITE OUT PROPERTIES OF EACH INTERNAL CONNECTION.	FINK	227
CCC		FINK	228
	NPRINT=NPRINT+1	FINK	229
	PRINT 5020	FINK	230
	PRINT 5021,NPRINT,KCYC,SUMTIM,DELT	FINK	231
	TX = ANAX1(SUMTIM - TAU, 1.E-12)	FINK	232
	DO 4222 N=1,NODES	FINK	233
	IF(MOD(N,57) .EQ. 1) PRINT 5027	FINK	234
	PRINT 5028,N	FINK	235
	DO 4223 NHIST=1,NDISCR	FINK	236
CCC	PRINT OUT THE DISTRIBUTIONS OF POTENTIAL (PHI) AND THE	FINK	237
CCC	CHANGE IN POTENTIAL AT EACH NODE.	FINK	238
	PRINT 5029,PHI(NHIST,1,N),PHI(NHIST,2,N),DPSI(NHIST,1,N),	FINK	239

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      SDPSI(NHIST,2,N)
4223 CONTINUE
CCC PRINT OUT THE INTERVAL SIZE, AND MINIMUM AND MAXIMUM VALUES
CCC OF EACH POTENTIAL DISTRIBUTION.
      PRINT 5033,PHINT(N)
      PRINT 5034,XMINPHI(N)
      PRINT 5035,XMAXPHI(N)
4222 CONTINUE
      CALL LINE
CCC
CCC FIND NEW CONDUCTANCES OF INTERNAL CONNECTIONS.
CCC
4300 IF(NVARK.LE.0) GO TO 4400
CCC
CCC THE FOLLOWING ROUTINE CALCULATES THE TIME-DEPENDENT CONDUCTANCE.
CCC IF THIS ROUTINE IS DESIRED, THE ARRAYS CON1, CON2, CON3 MUST
CCC BE REDIMENSIONED APPROPRIATELY.
CCC
C DO 4350 N=1,NCON
C N1 = NOD1(N)
C N2 = NOD2(N)
C4305 X1 = GBHYU * AREA(N)
C IF(KODANIS.LE.0) GO TO 4340
C IF(KODCON(N) - 2) 4320,4325,4330
C4320 C1 = CON1(N1)
C C2 = CON1(N2)
C GO TO 4335
C4325 C1 = CON2(N1)
C C2 = CON2(N2)
C GO TO 4335
C4330 C1 = CON3(N1)
C C2 = CON3(N2)
C4335 X2 = C1*C2*RHOZ/(C1*DEL2(N) + C2*DEL1(N))
C X3 = RHOZ
C TRAN(N) = X1 * X2 * X3
C4340 X2 = CON(N1)*CON(N2)*RHOZ/(CON(N1)*DEL2(N) + CON(N2)*DEL1(N))
C X3 = RHOZ
C TRAN(N) = X1 * X2 * X3
C4350 CONTINUE
CCC
CCC FIND PRESSURE HEAD CHANGES IN NODES DUE TO FLUID FLOW
CCC
4400 DO 4405 N = 1,NCON
      N1 = NOD1(N)
      N2 = NOD2(N)
      NSIGN=2
      MAGPOT=1
CCC CALL PRODIST TO DETERMINE THE DIFFERENCE IN POTENTIALS
CCC PHI(N1) AND PHI(N2).
      CALL PRODIST(PHI(1,1,N2),PHI(1,1,N1),PHID(1,1,1),MAGPOT,NSIGN)
      NSIGN=3
CCC CALL PRODIST TO DETERMINE THE FLUX (DFI) AS THE PRODUCT OF
CCC THE CONDUCTANCE (TRAN) AND THE POTENTIAL DIFFERENCE (PHID).
      CALL PRODIST(TRAN(1,1,N),PHID(1,1,1),DFI(1,1,N),MAGPOT,NSIGN)
      DO 30 I=1,NDISCR
CCC SCALE THE MAGNITUDE ELEMENTS OF THE FLUX ARRAY BY THE TIME
CCC STEP (DELT) TO DETERMINE THE ABSOLUTE MASS (OR HEAT) AND
CCC ASSIGN THIS TO THE DFI ARRAY.
      30 DFI(I,1,N)=DFI(I,1,N)*DELT
      NSIGN=1
CCC CALL PRODIST TO SUM THE INITIAL MASS (OR HEAT), DF, AND THE
CCC CALCULATED MASS (OR HEAT), DFI, RESULTING FROM THE CHANGE

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CCC      IN POTENTIALS, FOR  NODE=N1.                                FINK 302
      CALL PRODIST(DF(1,1,N1),DF(1,1,N),DF(1,1,N1),MAGPOT,MSIGN)    FINK 303
      NSIGN=2                                                         FINK 304
CCC      CALL PRODIST TO SUM THE INITIAL MASS (OR HEAT), DF, AND THE  FINK 305
CCC      CALCULATED MASS (OR HEAT), DF1, RESULTING FROM THE CHANGE    FINK 306
CCC      IN POTENTIALS, FOR  NODE=N2.                                FINK 307
      CALL PRODIST(DF(1,1,N2),DF(1,1,N),DF(1,1,N2),MAGPOT,MSIGN)    FINK 308
4405  CONTINUE                                                         FINK 309
      DO 4406 N=1,NODES                                              FINK 310
      DO 80 NHIST=1,NDISCR                                           FINK 311
CCC      CALCULATE THE CHANGE IN POTENTIAL, MASS/CAPACITANCE.        FINK 312
      DPSI(NHIST,1,N)=DF(NHIST,1,N)/CAP(N)                          FINK 313
      DPSI(NHIST,2,N)=DF(NHIST,2,N)                                  FINK 314
      80 CONTINUE                                                    FINK 315
CCC      SET DPSI EQUAL TO ZERO IF LESS THAN 1.E-12 AND GREATER      FINK 316
CCC      THAN -1.E-12.                                               FINK 317
      DO 90 NHIST=1,NDISCR                                           FINK 318
      IF(DPSI(NHIST,1,N) .GT. -1.E-12) GO TO 94                      FINK 319
      GO TO 90                                                        FINK 320
      94 IF(DPSI(NHIST,1,N) .LT. 1.E-12) GO TO 95                    FINK 321
      GO TO 90                                                        FINK 322
      95 DPSI(NHIST,1,N)=0.0                                          FINK 323
      90 CONTINUE                                                    FINK 324
      NSIGN=1                                                         FINK 325
      MAGPOT=2                                                         FINK 326
CCC      CALL PRODIST TO DETERMINE THE NEW VALUES OF POTENTIAL.    FINK 327
      CALL PRODIST(PHI(1,1,N),DPSI(1,1,N),PHI(1,1,N),MAGPOT,MSIGN)  FINK 328
CCC      DETERMINE THE INTERVAL SIZE AND MINIMUM AND MAXIMUM VALUES  FINK 329
CCC      THE POTENTIAL DISTRIBUTION.                                  FINK 330
      PHINT(N)=DINCR                                                  FINK 331
      XMINPHI(N)=XWIN                                                 FINK 332
      XMAXPHI(N)=XMAX                                                 FINK 333
4406  CONTINUE                                                         FINK 334
      GO TO 1000                                                      FINK 335
CCC      DETERMINE WHETHER FINAL CONDITIONS SHOULD BE PUNCHED.    FINK 336
4450  IF(IRESTRY .EQ. 0) GO TO 4454                                  FINK 337
CCC      PUNCH THE FINAL CONDITIONS FOR NODE, VOLUMETRIC GENERATION,  FINK 338
CCC      PRECONSOLIDATION (THE LATTER TWO VALUES ARE NOT          FINK 339
CCC      USED IN THIS FORM OF THE PROGRAM AND THEREFORE APPEAR AS    FINK 340
CCC      THIS PUNCHED CARD CAN BE USED AS INPUT FOR BLOCK 9 IN     FINK 341
CCC      RESTARTING THE PROBLEM.                                     FINK 342
      DO 4451 N=1,NODES                                              FINK 343
      PUNCH 5031,N,XX2,XX3                                           FINK 344
      DO 4452 NHIST=1,NDISCR                                           FINK 345
CCC      PUNCH FINAL CONDITIONS FOR POTENTIAL DISTRIBUTIONS WHICH   FINK 346
CCC      CAN BE USED AS INPUT FOR BLOCK 9 IN RESTARTING THE PROBLEM. FINK 347
      PUNCH 5032,PHI(NHIST,1,N),PHI(NHIST,2,N)                     FINK 348
4452  CONTINUE                                                         FINK 349
4451  CONTINUE                                                         FINK 350
4454  CONTINUE                                                         FINK 351
      NSTOP=1                                                         FINK 352
      KNTT=1                                                         FINK 353
      1000 RETURN                                                    FINK 354
CCC                                                         FINK 355
CCC      COMPLETED FINK, RETURN TO HEART.                            FINK 356
CCC                                                         FINK 357
5000  FORMAT(10X,I5,I6,I5,5E12.4,I5)                                FINK 358
5005  FORMAT(2I5,3I3,I1.4E10.3)                                     FINK 359
5010  FORMAT(//,11X,59HNO01 MOD2 INDEX DEL1          DEL2          DLONG FINK 360
      S      DRAD          AREA,5X,6HKODCON)                      FINK 361
5020  FORMAT(10X,53HINTERNAL CONNECTION DATA. CHECK CONDUCTANCES (TRAN)FINK 362
      1.,/,15X,94HLARGE DIFFERENCES BETWEEN CONNECTIONS MAY BE DUE TO POOFINK 363

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2R ZONING, AND MAY PRODUCE POOR RESULTS.)
5021 FORMAT(//,10X,44HPRINTOUT CYCLE TOTAL TIME TIME STEP/,11X
      5,15,3X,15,1X,2E15.5,/)
5027 FORMAT(//,57M
      $PSI/,15X,52M MAGNITUDE PROBABILITY PHI MAGNITUDE PROBABILITY)
5028 FORMAT(2X,4HNODE,15)
5029 FORMAT(13X,2E12.4,5X,2E12.4)
5031 FORMAT(15,15X,2E10.3)
5032 FORMAT(2E12.4)
5033 FORMAT(//,14X,10HINTERVAL =,E12.4)
5034 FORMAT(14X,10HMINIMUM = ,E12.4)
5035 FORMAT(14X,10HMAXIMUM = ,E12.4)
      END
C
C
C
C
      SUBROUTINE PRODIST(A,B,C,MAGPOT,MSIGN)
CCC
CCC WRITTEN 11/79.
CCC
CCC PRODIST IS A SUBROUTINE WHICH PERFORMS ARITHMETIC OPERATIONS
CCC ON PROBABILISTIC ARRAYS. THE ARRAYS A AND B ARE SENT TO
CCC PRODIST, ARRAY COP IS GENERATED AND SENT TO SLBRoutine COLAPS.
CCC
CCC VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES
      COMMON DINC,NDISC,PSINH,PSIMAX,TOLPR,XMIN,XMAX,IRESTRY,INT
      LEVEL2,A,B,C,COP
      COMMON/TOV/COP(20,20,2)
      DIMENSION A(20,2,1),B(20,2,1),C(20,2,1)
C IF MULTIPLICATION IS DESIRED GO TO 300.
      IF(MSIGN .EQ. 3) GO TO 300
      DO 53 NHIST=1,NDISC
C SEARCH FOR IDENTITY MATRICES.
      IF(A(NHIST,1,1) .NE. 0.0) GO TO 62
      53 CONTINUE
      GO TO 80
      62 DO 63 NHIST=1,NDISC
      IF(B(NHIST,1,1) .NE. 0.0) GO TO 100
      63 CONTINUE
      GO TO 90
C A IS AN IDENTITY MATRIX
C
C IF ADDITION IS DESIRED GO TO 81. IF SUBTRACTION IS DESIRED GO TO 83.
      80 IF(MSIGN .EQ. 1) GO TO 81
      GO TO 83
      81 DO 82 NHIST=1,NDISC
C SET THE MAGNITUDE ELEMENTS OF ARRAY C EQUAL TO THOSE OF ARRAY B.
      82 C(NHIST,1,1)=B(NHIST,1,1)
      GO TO 85
      83 DO 84 NHIST=1,NDISC
C SET THE MAGNITUDE ELEMENTS OF ARRAY C EQUAL TO THE NEGATIVE
C VALUES OF ARRAY B.
      84 C(NHIST,1,1)=-B(NHIST,1,1)
      85 DO 86 NHIST=1,NDISC
C SET THE PROBABILITY ELEMENTS OF ARRAY C EQUAL TO THOSE OF ARRAY B.
      86 C(NHIST,2,1)=B(NHIST,2,1)
C SET THE VALUES OF XMIN, XMAX, AND DINC
      XMIN=C(1,1,1)
      XMAX=C(NDISC,1,1)
      DINC=(XMAX-XMIN)/NDISC
      GO TO 500

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C B IS AN IDENTITY MATRIX
90 DO 91 NHIST=1,NDISCR
C SET THE MAGNITUDES AND PROBABILITIES OF ARRAY C EQUAL TO THOSE
C OF ARRAY A.
    C(NHIST,1,1)=A(NHIST,1,1)
    C(NHIST,2,1)=A(NHIST,2,1)
91 CONTINUE
C SET THE VALUES OF XMIN, XMAX, AND DINCR
    XMIN=C(1,1,1)
    XMAX=C(NDISCR,1,1)
    DINCR=(XMAX-XMIN)/NDISCR
    GO TO 500
C IF SUBTRACTION IS DESIRED GO TO 200.
100 IF(MSIGN .EQ. 2) GO TO 200
C DETERMINE THE SUM OF THE TWO DISTRIBUTIONS, A AND B
    DO 110 I=1,NDISCR
    DO 120 J=1,NDISCR
C CALCULATE THE SUM OF THE MAGNITUDES
    COP(I,J,1)=A(I,1,1)+B(J,1,1)
C CALCULATE PRODUCT OF PROBABILITIES
    COP(I,J,2)=A(I,2,1)*B(J,2,1)
120 CONTINUE
110 CONTINUE
C CALL COLAPS SUBROUTINE WHICH CONVERTS NDISCR**2 NO. OF POINTS TO
C NDISCR NO. OF POINTS
    CALL COLAPS(COP,C,MAGPOT)
    GO TO 500
200 CONTINUE
C DETERMINE THE DIFFERENCE OF THE TWO DISTRIBUTIONS, A AND B
    DO 210 I=1,NDISCR
    DO 220 J=1,NDISCR
C CALCULATE DIFFERENCE OF MAGNITUDES
    COP(I,J,1)=A(I,1,1)-B(J,1,1)
C CALCULATE PRODUCT OF PROBABILITIES
    COP(I,J,2)=A(I,2,1)*B(J,2,1)
220 CONTINUE
210 CONTINUE
C CALL COLAPS SUBROUTINE
    CALL COLAPS(COP,C,MAGPOT)
    GO TO 500
300 CONTINUE
    DO 10 NM=1,NDISCR
    IF(A(NM,2,1) .EQ. 1.0) GO TO 15
10 CONTINUE
    GO TO 301
15 DO 20 NHIST=1,NDISCR
C SCALE THE B MAGNITUDE ELEMENTS BY THE DETERMINISTIC A ARRAY
    C(NHIST,1,1)=A(NM,1,1)*B(NHIST,1,1)
C SET THE PROBABILITY ELEMENTS OF ARRAY C EQUAL TO THOSE OF ARRAY B.
    C(NHIST,2,1)=B(NHIST,2,1)
20 CONTINUE
    GO TO 500
301 CONTINUE
    DO 30 NT=1,NDISCR
    IF(B(NT,2,1) .EQ. 1.0) GO TO 35
30 CONTINUE
    GO TO 302
35 DO 40 NHIST=1,NDISCR
C SCALE THE A MAGNITUDE ELEMENTS BY THE DETERMINISTIC B ARRAY
    C(NHIST,1,1)=B(NT,1,1)*A(NHIST,1,1)
C SET THE PROBABILITY ELEMENTS OF ARRAY C EQUAL TO THOSE OF ARRAY A.
    C(NHIST,2,1)=A(NHIST,2,1)

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      40 CONTINUE
      GO TO 500
    302 CONTINUE
C DETERMINE THE PRODUCT OF THE TWO DISTRIBUTIONS, A AND B
      DO 310 N=1,2
      DO 320 J=1,NDISCR
      DO 330 I=1,NDISCR
        COP(I,J,N)=A(I,N,1)*B(J,N,1)
    330 CONTINUE
    320 CONTINUE
    310 CONTINUE
C CALL COLAPS SUBROUTINE
      CALL COLAPS(COP,C,MAGPOT)
    500 RETURN
      END

C
C
C
C
      SUBROUTINE COLAPS(COP,C,MAGPOT)
CCC
CCC   WRITTEN 11/79.
CCC
CCC   COLAPS IS A SUBROUTINE WHICH ACCEPTS THE THREE-DIMENSIONAL
CCC   ARRAY COP, CONTAINING N**2 VALUES AND COLAPSES IT TO
CCC   ARRAY C, CONSISTING OF N VALUES.
CCC
CCC   VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES
      COMMON DINC,NDISCR,PSIMIN,PSIMAX,TOLPR,XMIN,XMAX,TRESTRT,INT
CCC   THE FOLLOWING VARIABLELES ARE THREE-DIMENSIONAL ARRAYS.
      LEVEL2,C,COP
      DIMENSION COP(20,20,2),CHAG(400),CPROB(400),PROB(21),PRODIT(21),
      $PROD(21),AVGHAG(21),CHAGIN(21),C(20,2,1),CHID(21),PROBSUM(21)
C THE NO. OF GENERATED POINTS IS THE ORIGINAL NO. OF DATA POINTS SQUARED
C SET UPPER LIMITS FOR LOOP COUNTERS
      NDISQU=NDISCR**2
      NDISCR=NDISCR+1
      NDISQU=NDISQU-1
      NMLDIS=(NDISCR+1)/2
C CONVERT 3-DIM ARRAY TO 1-DIM ARRAY
      NC=0
      DO 10 I=1,NDISCR
      DO 10 J=1,NDISCR
        NC=NC+1
C ASSIGN COP TO CHAG AND CPROB, 1-DIM ARRAYS
        CHAG(NC)=COP(I,J,1)
        10 CPROB(NC)=COP(I,J,2)
C DETERMINE WHETHER THE MIN-MAX ROUTINE FOR POTENTIAL IS REQUIRED.
      IF(MAGPOT .GT. 1) GO TO 19
      GO TO 30
    19 DO 20 NC=1,NDISQU
C DETERMINE WHETHER THE POTENTIAL FALLS WITHIN THE SPECIFIED
C MINIMUM AND MAXIMUM VALUES.
      IF(CHAG(NC) .LT. PSIMIN .OR. CHAG(NC) .GT. PSIMAX) GO TO 22
      GO TO 20
C IF THE POTENTIAL IS LESS THAN PSIMIN OR GREATER THAN PSIMAX, SET
C THE ASSOCIATED PROBABILITY EQUAL TO ZERO.
      22 CPROB(NC)=0.0
      20 CONTINUE
C DETERMINE MINIMUM VALUE OF MAGNITUDE
      30 DO 11 MIN=1,NDISQU
      IF(CPROB(MIN) .GT. 0.0) GO TO 12

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 COLAPS44

11 CONTINUE	COLAPS45
12 CHIN=CHAG(MIN)	COLAPS46
IF(MIN .EQ. NDISQU) GO TO 610	COLAPS47
IF(CPROB(MIN) .EQ. 1.0) GO TO 610	COLAPS48
DO 100 NC=MIN+1,NDISQU	COLAPS49
IF(CHIN .LE. CHAG(NC)) GO TO 100	COLAPS50
IF(CPROB(NC) .GT. 0.0) GO TO 210	COLAPS51
GO TO 100	COLAPS52
210 CHIN=CHAG(NC)	COLAPS53
100 CONTINUE	COLAPS54
C REASSIGN MINIMUM MAGNITUDE TO XMN	COLAPS55
XMN=CHIN	COLAPS56
C DETERMINE MAXIMUM VALUE OF MAGNITUDE	COLAPS57
DO 13 MN=1,NDISQU	COLAPS58
MAX=MN+1+NDISQU-(2*MN)	COLAPS59
IF(CPROB(MAX) .GT. 0.0) GO TO 14	COLAPS60
13 CONTINUE	COLAPS61
14 CHAX=CHAG(MAX)	COLAPS62
DO 400 NC=MN,NDISQU	COLAPS63
K=NC+NDISQU-(2*NC)	COLAPS64
IF(CHAX .GE. CHAG(K)) GO TO 400	COLAPS65
IF(CPROB(K) .GT. 0.0) GO TO 310	COLAPS66
GO TO 400	COLAPS67
310 CHAX=CHAG(K)	COLAPS68
400 CONTINUE	COLAPS69
C REASSIGN MAXIMUM MAGNITUDE TO XMAX	COLAPS70
XMAX=CHAX	COLAPS71
C DETERMINE THE DIFFERENCE OF XMAX AND XMN	COLAPS72
DIFF=XMAX-XMN	COLAPS73
C CONFIRM NON-ZERO VALUE FOR DIFFERENCE	COLAPS74
IF(DIFF .EQ. 0.0) GO TO 600	COLAPS75
C DIVIDE DIFFERENCE INTO NDISCR EQUAL INTERVALS	COLAPS76
DINCR=DIFF/NDISCR	COLAPS77
C INITIALIZE THE FIRST INTERVAL ENDOPOINT.	COLAPS78
CHAGIN(1)=XMN+DINCR	COLAPS79
C INITIALIZE THE FIRST INTERVAL MIDPOINT.	COLAPS80
CHID(1)=XMN+(DINCR/2.0)	COLAPS81
DO 39 M=1,NDISCR-1	COLAPS82
C INITIALIZE THE INTERVAL ENDOPOINTS.	COLAPS83
CHAGIN(M+1)=CHAGIN(M)+DINCR	COLAPS84
C INITIALIZE THE INTERVAL MIDPOINTS.	COLAPS85
CHID(M+1)=CHID(M)+DINCR	COLAPS86
39 CONTINUE	COLAPS87
DO 40 I=1,NDISCR	COLAPS88
C INITIALIZE PROBABILITY OF EACH INTERVAL	COLAPS89
PROB(I)=0.0	COLAPS90
C INITIALIZE THE PRODUCT OF THE PROBABILITY AND MAGNITUDE OF EACH	COLAPS91
C INTERVAL	COLAPS92
PRODIT(I)=0.0	COLAPS93
40 CONTINUE	COLAPS94
DO 45 NC=1,NDISQU	COLAPS95
DO 50 M=1,NDISCR	COLAPS96
C DETERMINE WHETHER THE MAGNITUDE FALLS WITHIN THE MTH INTERVAL	COLAPS97
IF(CHAG(NC) .GT. CHAGIN(M)) GO TO 50	COLAPS98
C ADD THE PROBABILITY TO THE MOST RECENT VALUE	COLAPS99
PROB(M)=PROB(M)+CPROB(NC)	COLAP100
C DETERMINE THE PRODUCT OF THE PROBABILITY AND MAGNITUDE	COLAP101
PROD(M)=CPROB(NC)*CHAG(NC)	COLAP102
C ADD THE PRODUCT TO THE MOST RECENT VALUE	COLAP103
PRODIT(M)=PRODIT(M)+PROD(M)	COLAP104
GO TO 45	COLAP105
50 CONTINUE	COLAP106


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45 CONTINUE
C DETERMINE THE AVERAGE MAGNITUDE OF EACH INTERVAL
DO 55 M=1,NDISCR
  IF (PROB(M) .EQ. 0.0) GO TO 60
  AVGMAG(M)=PRODIT(M)/PROB(M)
  GO TO 55
C ASSIGN ZERO MAGNITUDES TO THE INTERVAL AVERAGE IF
C THE PROBABILITY ASSOCIATED WITH THAT INTERVAL IS ZERO
60 AVGMAG(M)=0.0
55 CONTINUE
C SEARCH FOR PROBABILITIES LESS THAN TOLPR
DO 80 NHIST=1,NOISCR
  IF (PROB(NHIST) .LT. TOLPR) GO TO 85
  GO TO 80
C ASSIGN PROBABILITY AND MAGNITUDE TO ZERO
85 PROB(NHIST)=0.0
  AVGMAG(NHIST)=0.0
80 CONTINUE
C NORMALIZE THE PROBABILITIES
PROBSUM(1)=0.0
DO 75 M=1,NOISCR
  PROBSUM(M+1)=PROBSUM(M)+PROB(M)
75 CONTINUE
C ASSIGN AVGMAG AND PROB ARRAYS TO THE C ARRAY
DO 95 M=1,NOISCR
  C(M,1,1)=AVGMAG(M)
  C(M,2,1)=PROB(M)/PROBSUM(NOISCR)
95 CONTINUE
GO TO 1000
C SET MAGNITUDE MIDPOINT EQUAL TO THE CURRENT CHAG VALUE
600 C(NHLDIS,1,1)=CHAG(K)
C SET PROBABILITY MIDPOINT EQUAL TO THE CURRENT CPROB VALUE (1.0)
C(NHLDIS,2,1)=CPROB(K)
GO TO 607
C SET MAGNITUDE MIDPOINT EQUAL TO CHAG(MIN)
610 C(NHLDIS,1,1)=CHAG(MIN)
C SET PROBABILITY MIDPOINT EQUAL TO CPROB(MIN) (1.0)
C(NHLDIS,2,1)=CPROB(MIN)
607 CONTINUE
DO 608 NHIST=1,NHLDIS-1
C SET ALL OTHER MAGNITUDE AND PROBABILITY ELEMENTS EQUAL
C TO ZERO
C(NHIST,1,1)=0.0
C(NHIST,2,1)=0.0
608 CONTINUE
DO 609 NHIST=NHLDIS+1,NOISCR
  C(NHIST,1,1)=0.0
  C(NHIST,2,1)=0.0
609 CONTINUE
C SET INTERVAL SIZE EQUAL TO ZERO
DINCR=0.0
C SET XMAX AND XMIN EQUAL TO THE MIDPOINT VALUE
XMAX=XMIN=C(NHLDIS,1,1)
1000 RETURN
END
C
C
C
C
SUBROUTINE TALLY
C
C*****VERSION 5/20/69.

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COLAP107
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COLAP165
TALLY 2
TALLY 3
TALLY 4

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C*****REVISED 3/73.
C
C*****TALLY IS A REQUIRED SUBROUTINE.
C
CCC  VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES
COMMON DINC,NOISCR,PSIPIN,PSIMAX,TOLPR,XPIN,XMAX,IRESTRT,INT
CCC  INTEGER VARIABLES
COMMON NOSPEC,NMAT,NFLUID,MODES,NOCON,NOCONA,NOCBS,NOGEN,NIT,
$  NVARC,NVARG,NVARH,NVARK,NVART,NZ,TPRINT,TIMEP
COMMON IBLOCK,KCYC,KDATA,KS,KSECS,KSYN,KHIT,LABEL,LTAB,LXX,
$  MAX,MID,MODE,NADD,NADG,NADS,NADSB,NAD1,NAD2,NCHECK,NOOMS,
$  NOW,NPROB,NSAVE,NSEQ,NSTOP,NUTS,KSPEC
CCC  END BLINK
CCC  TEMPORARY VALUES OF INTEGER AND REAL VARIABLES
COMMON A1,DX,I1,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,NND,N1,N2,
$  P1,P2,P3,P4,P5,TX,X1,X2,X3
CCC  END BLISC
CCC  REAL VARIABLES
COMMON ADATA,ADD,AFLUID,APATH,BEY,BETA,BEYH,BPRIME,CAPHS,
$  CAPS,CAPT,CONST,DAY,DELT,DELYS,DELYSS,DEX,DLONG,DNDP,
$  DPHAXS,DPRES,DPSIPAX,DRAD,DRADS,DWIDE,EAVG,EC,ELT,ESTRESS,
$  ESTRIN,EX,FEX,FLEX,FLUID,FLUX,FLXS,FLX,FOR,FORD,
$  GBYMU,GEE,GEKS,GECH,GMASS,GONE,GX,MONE,MX,PHIBS,
$  PHIONE,PORS,PSIVARY,PSI1,PSI2,RD,RHOZ,SAVG,SCALE,SET,
$  SETD,SET1,SMALL,SUMTIM,TAU,THICK,TIME,TZERO,VISC,VOLMS,
$  VOLS,WMS,PCONE,TIMAX,PCX,DELYF,XX2,XX3
CCC  END BLARE
CCC  ARRAYED VARIABLES
CCC
CCC  THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE
CCC  THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC
CCC  DISTRIBUTIONS.  THE FIRST INDEX REFERS TO THE DISCREET VALUE,
CCC  THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR
CCC  PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR
CCC  CONNECTION NUMBER.
CCC
LEVEL2,CAP,CON,DFI,DPSI,DF,PHI,PPHI,ZIP,TRAN
COMMON/LCH/CAP(100),CON(20,2,100),DFI(20,2,200),DPSI(20,2,200),
$  DF(20,2,200),PHI(20,2,200),PPHI(20,2,200),
$  ZIP(200),TRAN(20,2,200)
COMMON F1(12),F2(12),F3(12),NOD1(200),NOD2(200),NODE(100),
$  FI(1),FX(1),DDFI(1),
$  DDPSI(1),DPSIS(1),E(1),ITEMS(15),LTAB(10),NAME(20),
$  NAMES(20),NEHBL(15),NTYPE(100),PC(1),PCM(1),SLIN(1),
$  TOTSTRS(1),VOL(100),W(1),Z(100),CK(10),NCDYD(1),R(1),
$  NODIN,TLUMP,PLUMP,XF,G,PSIVARC(12,10),PZ(10),KODANIS,
$  CON2(1),CON3(1),CONST1(10),CONST2(10),KODCON(1),
$  KSTDATA,LANISK(10),LTAB(10),PPC(1),
$  NODMAT(100),NOXMAT(100),WX1(10),PSIVARK(12,10)
CCC  END BLARY
DIMENSION F(1),M(1),NOTE(100),NOXE(100),NTYPES(100)
COMMON /M/ M1, M2, M3, M4, M5, M6, M7, M8, M9, M10, M11, M12, M13
CCC
COMMON/AGEN/GT(1),LTAB(1),NODG(1),NOXG(1),SLOG(1),
$  TVARG(1),G(1),FG(1),DFG(1),GG(1)
C
DIMENSION NDUMMY(15)
EQUIVALENCE (NDUMMY,NOSPEC)
NMIN1(I) = MIN(I, N - 1)
C
IF (KCVC) 2000,3100,4200
2000 IF (IBLOCK) 2400,2005,2200

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TALLY 5
TALLY 6
TALLY 7
TALLY 8
BLINK 2
BLINK 3
BLINK 4
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BLISC 2
BLISC 3
BLISC 4
BLISC 5
BLARE 2
BLARE 3
BLARE 4
BLARE 5
BLARE 6
BLARE 7
BLARE 8
BLARE 9
BLARE 10
BLARE 11
BLARY 2
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BLARY 20
BLARY 21
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BLARY 23
BLARY 24
TALLY 13
M 2
M 3
AGEN 2
AGEN 3
AGEN 4
TALLY 16
TALLY 17
NMIN1 2
NMIN1 3
TALLY 19
TALLY 20

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CCC		TALLY 21
CCC	START HERE AFTER PROBLEM NAME CARD HAS BEEN READ IN WITH 2, 3, OR	TALLY 22
CCC	4 IN COLUMN 72, FOR SAVING DATA FROM PRECEDING PROBLEM.	TALLY 23
CCC		TALLY 24
	2005 PRINT 5950, (1, I=2, 9)	TALLY 25
	NIT = NODES	TALLY 26
CCC		TALLY 27
CCC	FIND OUT IF NODE TYPES AND PHI WILL BE INITIAL OR FINAL	TALLY 28
CCC	VALUES FROM PRECEDING PROBLEM (NAME(16) = 2, OR 3 OR 4, RESP.).	TALLY 29
CCC		TALLY 30
	IF(NOW .NE. 2) GO TO 2105	TALLY 31
	DO 2100 N = 1, NODES	TALLY 32
	IF(NTYPE(N) .EQ. NTYPES(N)) GO TO 2100	TALLY 33
	NTYPE(N) = NTYPES(N)	TALLY 34
	NOSPEC = NOSPEC - 1	TALLY 35
2100	CONTINUE	TALLY 36
	RETURN	TALLY 37
CCC		TALLY 38
CCC	CHANGE INTERNAL BLOCK 9 TO FINAL PHI OF PREVIOUS PROBLEM	TALLY 39
CCC		TALLY 40
2105	DO 2110 N = 1, NODES	TALLY 41
	DO 2111 NHIST=1, NDISCR	TALLY 42
	DO 2112 MORP=1, 2	TALLY 43
	PPHI(NHIST, MORP, N) = PHI(NHIST, MORP, N)	TALLY 44
	2112 CONTINUE	TALLY 45
	2111 CONTINUE	TALLY 46
2110	CONTINUE	TALLY 47
	RETURN	TALLY 48
	2200 IF(IBLOCK .EQ. 9) GO TO 2300	TALLY 49
	IF(IBLOCK .EQ. 10) GO TO 2260	TALLY 50
	IF(IBLOCK .EQ. 3) GO TO 2350	TALLY 51
	IF(IBLOCK .NE. 1) RETURN	TALLY 52
CCC		TALLY 53
CCC	START HERE AFTER THE BLOCK NUMBER CARD FOR BLOCK 1 IS READ IN.	TALLY 54
CCC		TALLY 55
CCC	INPUT DATA BLOCK 1. CALCULATION CONTROLS, LIMITS, AND CONSTANTS.	TALLY 56
CCC	NDISCR SPECIFIES THE NUMBER OF DISCREET VALUES MODELING	TALLY 57
CCC	THE PROBABILISTIC ARRAY. DELTF IS A FACTOR USED TO SCALE	TALLY 58
CCC	THE TIME STEP.	TALLY 59
CCC		TALLY 60
	READ 5865, IPRINT, NUM, KDATA, KSPEC, NCYC, NSEC, NPUNCH, KSTDATA, KONSTIN,	TALLY 61
	8NDISCR, TIMEP, SCALE, DELTF	TALLY 62
CCC		TALLY 63
	NUMX = NUM	TALLY 64
	KDATAX = KDATA	TALLY 65
	IF(SCALE .LE. 0.0) SCALE = 1.0	TALLY 66
	PRINT 5890, IPRINT, NUM, KDATA, KSPEC, NCYC, NSEC, NPUNCH, KSTDATA,	TALLY 67
	8KONSTIN, NDISCR, TIMEP, SCALE, DELTF	TALLY 68
CCC		TALLY 69
CCC	PSIMIN AND PSIMAX REPRESENT THE MINIMUM AND MAXIMUM POTENTIALS	TALLY 70
CCC	AS GIVEN BY THE BOUNDARY CONDITIONS.	TALLY 71
	READ 5870, KD, DELTO, SHALL, PSIVARY, TAU, TIMAX, PSIMIN, PSIMAX	TALLY 72
CCC		TALLY 73
CCC	THE MINIMUM ALLOWABLE PROBABILITY VALUE IS INPUT.	TALLY 74
CCC	SPECIFY WHETHER THE FINAL CONDITIONS SHOULD BE PUNCHED.	TALLY 75
CCC	GIVE THE NODE NUMBER OF AN INTERIOR ELEMENT FOR THE	TALLY 76
CCC	TIME STEP CALCULATION.	TALLY 77
CCC		TALLY 78
	READ 5871, TOLPR, IRESTRT, INT	TALLY 79
CCC		TALLY 80
	KD = MAX0(1, KD)	TALLY 81
	KSYN = 1 + KD/3	TALLY 82

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      GEON = FLOAT(LEFT(1, KD - 1))*3.14159265358979*(KD/2)
      IF((DELTO - 1.E-10)*(DELTO - 1.E12) .GT. 0.0) DELTO = 1.E12
      SHALL = SMALL = AMAX1(SPALL, 1.E-12)
      IF(PSIMAX.GT.PSIMIN) GO TO 2225
      PSIMAX = 1.E12
      PSIMIN = -PSIMAX
2225  IF(PSIVARY.LE.0.) PSIVARY = AMIN1(5.,.005*(PSIMAX-PSIMIN))
      X1 = PSIVARY*.001
      PRINT 5875,KD,DELTO,SHALL,PSIVARY,TAU,TIMAX,PSIMIN,PSIMAX
      PRINT 5880,KD,KSYM,GEON,TOLPR,IRESYRT,INT
C
C
      READ 5810,PHIONE,GONE,HONE,PCONE
      PHIONE = PHIONE * SCALE
      PRINT 5815,PHIONE,GONE,HONE,PCONE
      RETURN
C
C COMPLETED BLOCK 1. RETURN TO HEART.
C
CCC
CCC  START HERE AFTER THE BLOCK NUMBER CARD FOR BLOCK 9 IS READ IN.
CCC
CCC INPUT DATA BLOCK 9. INITIAL CONDITIONS LIST.
CCC
2300 L = 0
      IF(MODE.EQ.0) NIT = 0
      N = NIT
CCC
2305 READ 5840,N1,NSEQ,NADD,XX2,XX3
      IF(N1 .EQ. 0) RETURN
      DO 2306 NHIST=1,NDISCR
2306  READ 5842,PPHI(NHIST,1,N1),PPHI(NHIST,2,N1)
      IF(NADD .EQ. 0) GO TO 2307
      DO 2311 J1=N1,N1+NSEQ,NADD
      DO 2312 NHIST=1,NDISCR
      DO 2313 MORP=1,2
      PPHI(NHIST,MORP,J1)=PPHI(NHIST,MORP,N1)
2313  CONTINUE
2312  CONTINUE
2311  CONTINUE
      GO TO 2310
2307  DO 2308 NHIST=1,NDISCR
      DO 2309 MORP=1,2
      PPHI(NHIST,MORP,N1)=PPHI(NHIST,MORP,N1)
2309  CONTINUE
2308  CONTINUE
CCC
2310 IF(MODE.GE.8) GO TO 2315
      CALL SEEK1(N, N1, NOXE, NIT, K)
      ITEMS(9) = NHIN1(ITEMS(9))
      GO TO 2320
2315 N = NIT = N + 1
2320 IF(N .GT. N4) CALL MORTHAN(N,NIT,N4)
      NOTE(N) = NOXE(N) = N1
      IF(MOD(L, 57) .EQ. 0) PRINT 5845
      L = L + 1
      PRINT 5850,N1,N,XX2,XX3
      DO 2321 NHIST=1,NDISCR
2321  PRINT 5851,PHI(NHIST,1,N1),PHI(NHIST,2,N1)
CCC
CCC  TEST THROUGH WITH CARD.
CCC

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IF(NSEQ.LE. 0) GO TO 2305
NSEQ = NSEQ - 1
N1 = N1 + NADD
GO TO 2310
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CCC
CCC START HERE AFTER BLOCK NUMBER CARD FOR BLOCK 3 HAS BEEN READ IN.
CCC FLUID PROPERTIES
CCC
2350 READ 5790,AFLUID,VISC,BETA,RHOZ,GEE
5790 FORMAT(A10,4E10.3)
GRYNU = GEE/VISC
BPRIME = BETA * RHOZ * GEE
PRINT 5795,AFLUID,VISC,BETA,RHOZ,GEE,GRYNU,BPRIME
5795 FORMAT(/,10X,99H FLUID VISCOSITY BETA RHOZ,TALLY158
S GEE G/VISCOSITY BETA PRIME ,/10X,A10,6E15.5)TALLY159
NEMBL(2) = NEMBL(2) + 1000
RETURN
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C
C COMPLETED BLOCK 3. RETURN TO TRUST
C

CCC
CCC READ IN BLOCK 10. DATA FOR DIMENSIONLESS VARIABLES
CCC
2260 PRINT 5775
2265 READ 5780,N3,NSEQ,NADD,X1,X2,X3,DELY,DELY,DELY
5780 FORMAT(3I5,5X,6E10.3)
IF(N3.EQ.0) RETURN
2270 NODIM = NODIM + 1
N2 = N3
CALL REFER(N2,N3,1,NODE,NODES)
5775 FORMAT(/,10X,57HMODTO INDEX X Y Z
S R,/)
MODTO(NODIM) = N2
R(N2) = SQRT(X1**2 + X2**2 + X3**2)
IF(MOD(NODIM+13,50).EQ.0)PRINT 5775
PRINT 5785,MODTO(NODIM),NODIM,X1,X2,X3,R(N2)
5785 FORMAT(10X,2I6,4E13.4)
IF(NSEQ.LE.0)GO TO 2265
NSEQ = NSEQ - 1
N3 = N3 + NADD
X1 = X1 + DELX
X2 = X2 + DELY
X3 = X3 + DELZ
GO TO 2270
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TALLY200
TALLY201
TALLY202
TALLY203
TALLY204
TALLY205
TALLY206

CCC
CCC COMPLETED BLOCK 10
CCC
CCC START HERE AFTER THE DATA END CARD IS READ IN.
CCC SUMMARIZE INPUT DATA, AND INITIALIZE PRIOR TO TIME STEP ZERO.
CC
2400 PRINT 5950,(I,I=2,9)
PRINT 5985,(NEMBL(I),I=2,9)
PRINT 5975
PRINT 5955,N2,N3,N4,N5,N6,N7,N8,N9
PRINT 5965,(ITEMS(I),I=2,9)
PRINT 5970,(NDUMMY(I),I=2,9)
NTABLE = NVARC+NVARL+NVARH+NVARV+NVARC
PRINT 5945,NVARC,NVARL,NVARH,NVARV,NVARC,NTABLE
PRINT 5930, N9
IF(NHAT*NODES .GE. 1) GO TO 2410
C

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C MISSING BLOCK 2 OR 4. RETURN TO HEART.
C
    PRINT 5805, WHAT, NODES
    KMIT = 9
    RETURN
2410 IF(NUM.EQ. 0) GO TO 2415
    IBLOCK = 1
    CALL REFER(NUM, NUMX, 1, NODE, NODES)
2415 IF(NIT.LE. 0) GO TO 2425
    IBLOCK = 9
    IF(KMIT.NE. 0) GO TO 2500
CCC
CCC    SET UP INTERNAL BLOCK 9 WITH INITIAL CONDITIONS OF THIS PROBLEM.
CCC
    2425 DO 2430 N=1,NODES
        NOTE(N) = N
        NOXE(N) = NODE(N)
    2430 NTYPE(N) = NTYPE(N)
C
C MAKE ALL NODES SPECIAL IF KSPEC IS POSITIVE.
C
    IF(KSPEC.LE. 0) GO TO 2500
    DO 2435 N = 1,NODES
    IF(NTYPE(N).NE. 0) GO TO 2435
        NTYPE(N) = 5
        NOSPEC = NOSPEC + 1
2435 CONTINUE
CCC
CCC    INITIALIZE BEFORE FIRST TIME INCREMENT
CCC
    2500 NOGEN = 0
        F(1)=H(1)=0.0
        NOGEN = NOGEN + 1
2515 CONTINUE
CCC
CCC    CALCULATE STORAGE REQUIREMENTS FOR PARAMETER-ADJUSTED ARRAYS.
CCC
CCC    LEFT OUT, 6/73. USEFUL ONLY AS COMPILE-TIME DIRECTIVES.
CCC
    PRINT 5900,NOSPEC,NOGEN
CCC
CCC    INITIALIZE BEFORE FIRST TIME STEP (KCYC = 0, DELT = 1.0E-12).
CCC
    SUNTIM = TAU
    FOR = 1.
    DELT = 1.0E-12
    FORD = FOR*DELT
    SUNTAU = DELTS = DPSIMAX = 0.
    GO TO 4020
CCC
CCC    START HERE AT THE END OF TIME STEP ZERO (DELT = 1.0E-12).
CCC
    3100 IF(KSPEC.LT. 0) RETURN
    4200 IF(KSPEC.LT. 0) RETURN
4020 DO 4025 N = 1,NODES
    DOPSI(N) = 0.
4025 CONTINUE
    RETURN
CCC
CCC    COMPLETED TALLY. RETURN TO HEART.
CCC
    5805 FORMAT(//10X,25HINPUT ERROR, MATERIALS = ,I5,10H, NODES = ,I5,10H)TALLY260
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5810 FORMAT (SE10.3) TALLY269
5815 FORMAT(//,12X,41H PHONE GONE HONE PCONE,/,10X,4TALLY270
SE12.4) TALLY271
5820 FORMAT(11H1,9X,17HTRUST OUTPUT DATA,69X,9HDATA DECK,115// TALLY272
S 10X,1R1,14A5,2R1,4A2,4X,2(2X,1A6)) TALLY273
5840 FORMAT(3I5,5X,2E10.3) TALLY274
5841 FORMAT(1X) TALLY275
5842 FORMAT(2E12.4) TALLY276
5845 FORMAT(//,12X,10HNOTE INDEX,13X,4HPPHI,15X,2HGG,13X,2HPC,/,27X,22HTALLY277
SMAGNITUDE PROBABILITY) TALLY278
5850 FORMAT(10X,2I5,29X,2E15.6) TALLY279
5851 FORMAT(25X,2E12.4) TALLY280
5865 FORMAT(10I5,2E10.3,E10.7) TALLY281
5870 FORMAT(15,5X,7E10.3) TALLY282
5871 FORMAT(5X,E15.5,5X,2I5) TALLY283
5875 FORMAT(//,11X,92N KD DELTO SMALL PSIVARY TALLY284
SAU TIMAX PSIMIN PSIMAX,/,10X,15,7E13.5) TALLY285
5880 FORMAT(//,10X,57H KD KSYM GEOM TOLPR IRESTRYTALLY286
S INT,/,10X,2I5,E13.5,E15.5,5X,15,5X,15) TALLY287
5890 FORMAT(//10X,51HPRINT NUM KOATA KSPEC MCYC WSEC NPUNCH KSTDAYTALLY288
8A,8H KONSTIN,7H NDISCR,4X,5HTIMEP,8X,5HSCALE,8X,5HDELTF,/,10X,6I6, TALLY289
81X,16,2X,16,2X,16,1X,16,2E13.5,E13.7) TALLY290
5930 FORMAT(//,26X,31HMAXIMUM ALLOWED TABLE LENGTH IS,14,1H.) TALLY291
5945 FORMAT(//,10X,5HTABLES,10X,36H CAPN CCAT NSURT PHIB BY TOTALTALLY292
S,/,26X,6I6) TALLY293
5950 FORMAT(//10X,21HSUMMARY OF INPUT DATA,/, TALLY294
810X,12HBLOCK NUMBER,4X,8I6) TALLY295
5955 FORMAT(10X,12HMAXIMUM SIZE,4X,8I6) TALLY296
5960 FORMAT(10X,12HINITIAL SIZE,4X,8I6) TALLY297
5965 FORMAT(10X,15HUNMODIFIED SIZE,1X,8I6) TALLY298
5970 FORMAT(10X,10HFINAL SIZE,6X,8I6) TALLY299
5975 FORMAT(10X,12HITEM NAME ,4X, TALLY300
848H MAT FLUID NODE NOD1 NODS NODS8 NODG NODT) TALLY301
5980 FORMAT(//10X,12HOTHER TOTALS,4X,12HNOSPEC NOGEN,/, TALLY302
826X,2I6) TALLY303
5985 FORMAT(10X,13HTIMES READ IN,3X,8(3X,13)) TALLY304
END TALLY305
C TALLY306
C TALLY307
C TALLY308
C TALLY309
SUBROUTINE REFER(LIST, LISTX, MAXE, LISTR, VAR) REFER 2
C REFER 3
C*****VERSION 5/20/69. REFER 4
C*****REVISED 2/73. REFER 5
C REFER 6
C*****REFER IS A REQUIRED SUBROUTINE. REFER 7
C REFER 8
C REFER IS USED FOR CROSS-REFERENCING NODE, MATERIAL, REACTANT WDS. REFER 9
C FINDS J FOR WHICH LISTX(N) = LISTR(J), MAKES LIST(N) = J. REFER 10
C REFER 11
C DIMENSION LIST(1), LISTR(1), LISTX(1) REFER 12
C REFER 13
CCC VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES BLINK 2
COMMON DINC, NDISCR, PSIMIN, PSIMAX, TOLPR, XMIN, XMAX, IRESTRY, INT BLINK 3
CCC INTEGER VARIABLES BLINK 4
COMMON NOSPEC, NMAT, NFLUX, NODES, NOCON, NCSCON, NODS, NOGEN, NIT, BLINK 5
S NVARC, NVARG, NVARH, NVARK, NVART, NZ, IPRINT, TIMEP BLINK 6
COMMON IBLOCK, KCYC, KOATA, KS, KSECS, KSYM, KWIT, LABEL, LTAB, LXX, BLINK 7
S MAX, MID, MODE, NADD, NADG, NADS, NADSB, NAD1, NAD2, NCHECK, NODMS, BLINK 8
S NCW, NPROB, NSAVE, NSEQ, NSTOP, NUTS, KSPEC BLINK 9
CCC END BLINK BLINK 10

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CCC	TEMPORARY VALUES OF INTEGER AND REAL VARIABLES	BLZSC	2
	COMMON A1,DX,YI,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,MND,N1,N2,	BLZSC	3
8	P1,P2,P3,P4,P5,TX,X1,X2,X3	BLZSC	4
CCC	END BLZSC	BLZSC	5
CCC	REAL VARIABLES	BLARE	2
	COMMON ADATA,ADD,AFLUID,APATH,BET,BETA,BETW,DPRIME,CAPMS,	BLARE	3
8	CAPS,CAPT,CONST,DAY,DELT,DELTS,DELTSS,DEX,DLONG,DNDP,	BLARE	4
8	OPMAXS,DPRES,DPSIMAX,DRAD,DRADS,DWIDF,EAVG,EC,ELY,ESTRESS,	BLARE	5
8	ESTRIN,EX,FEX,FLEX,FLUID,FLUX,FLUXS,FLX,FOF,FORD,	BLARE	6
8	GBYMU,GEE,GENS,GEOM,GMASS,GONE,GX,HONE,HX,PHIBS,	BLARE	7
8	PHIONE,PORS,PSIVARY,PSI1,PSI2,RD,RHOZ,SAVG,SCALE,SET,	BLARE	8
8	SETD,SET1,SHALL,SUMIN,TAU,THICK,TIME,TZERO,VISC,VOLMS,	BLARE	9
8	VOLS,WMS,PCONE,TIMAX,PCX,DELYF,XX2,XX3	BLARE	10
CCC	END BLARE	BLARE	11
CCC	ARRAYED VARIABLES	BLARY	2
CCC		BLARY	3
CCC	THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE	BLARY	4
CCC	THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC	BLARY	5
CCC	DISTRIBUTIONS. THE FIRST INDEX REFERS TO THE DISCREET VALUE,	BLARY	6
CCC	THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR	BLARY	7
CCC	PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR	BLARY	8
CCC	CONNECTION NUMBER.	BLARY	9
CCC		BLARY	10
	LEVEL2,CAP,CON,DFI,OPSI,OF,PHI,PPII,ZIP,TRAN	BLARY	11
	COMMON/LCM/CAP(100),CON(20,2,100),DFI(20,2,200),OPSI(20,2,200),	BLARY	12
8	DF(20,2,200),PHI(20,2,200),PPII(20,2,200),	BLARY	13
8	ZIP(200),TRAN(20,2,200)	BLARY	14
	COMMON F1(12),F2(12),F3(12),NOD1(200),NOD2(200),NODE(100),	BLARY	15
8	FI(1),FX(1),DOFI(1),	BLARY	16
8	DOPSI(1),DPSIS(1),E(1),ITEMS(15),LTABK(10),NAME(20),	BLARY	17
8	NAMES(20),NEWBL(15),NTYPE(100),PC(1),PCM(1),SLIN(1),	BLARY	18
8	TOTSTRS(1),VOL(100),W(1),Z(100),CK(10),NCDTD(1),R(1),	BLARY	19
8	NODIN,TLUMP,PLUMP,XF,Q,PSIVARC(12,10),PZ(10),KODANIS,	BLARY	20
8	CON2(1),CON3(1),CONST1(10),CONST2(10),KODCON(1),	BLARY	21
8	KSTDATA,LANISK(10),LTABC(10),PPC(1),	BLARY	22
8	NCDMAT(100),NOXMAT(100),WX1(10),PSIVARK(12,10)	BLARY	23
CCC	END BLARY	BLARY	24
	IF((MAR * MAXE) .LE. 0 .OR. NEWBL(1BLOCK) .LE. 0 .OR. 1BLOCK .LE.	REFER	18
8	0) RETURN	REFER	19
	N1 = 1	REFER	20
	IF(1BLOCK .NE. 1 .AND. NEWBL(1BLOCK) .LT. 1000)	REFER	21
	8 N1 = MIN0(1 + ITEMS(1BLOCK), MAXE)	REFER	22
	DO 3120 N = N1,MAXE	REFER	23
	J = MAR	REFER	24
	DO 3105 L = 1,MAR	REFER	25
	IF(LISTX(N) .EQ. LISTR(J)) GO TO 3115	REFER	26
3105	J = J - 1	REFER	27
	IF(1BLOCK .EQ. 2 .AND. LISTX(N) .EQ. 0) GO TO 3120	REFER	28
	PRINT 5000, N, 1BLOCK, LISTX(N)	REFER	29
	KWIT = 5	REFER	30
	GO TO 3120	REFER	31
3115	LIST(N) = J	REFER	32
3120	CONTINUE	REFER	33
	RETURN	REFER	34
C		REFER	35
C	END OF REFER, RETURN TO CALLER.	REFER	36
C		REFER	37
	5000 FORMAT(/1X,3(5H****),0HERROR IN,1I5,17H2TH ITEM IN BLOCK,1I3,	REFER	38
	840H REFERS TO UNSPECIFIED MATL OR NODE NO,16,1H.)	REFER	39
	END	REFER	40
C		REFER	41
C		REFER	42

C		REFER 43
C		REFER 44
	SUBROUTINE SEEK1(N, ITEM, NB1, MAXE, K)	SEEK1 2
C		SEEK1 3
C*****VERSION 5/20/69.		SEEK1 4
C*****REVISED 2/73.		SEEK1 5
C		SEEK1 6
C*****SEEK1 IS A REQUIRED SUBROUTINE.		SEEK1 7
C		SEEK1 8
C	SEEK1 IS USED FOR FINDING DATA BLOCK ITEMS TO BE ALTERED.	SEEK1 9
C	SEEK1 LOOKS FOR N1 IN ARRAY NB1. SETS N TO SUBSCRIPT OF NB1 IF	SEEK1 10
C	N1 IS FOUND, OTHERWISE SETS N TO ARRAY SIZE PLUS 1.	SEEK1 11
C		SEEK1 12
	DIMENSION NB1(1)	SEEK1 13
C		SEEK1 14
CCC	VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES	BLINK 2
	COMMON DINC,NDISC,PSIPIN,PSINAX,TOLPR,XPIN,XMAX,INESTRY,INT	BLINK 3
CCC	INTEGER VARIABLES	BLINK 4
	COMMON NOSPEC,NHAT,NFLUID,NODES,NOCON,NCSCON,NODBS,MOGEN,NIT,	BLINK 5
\$	NVARC,NVARG,NVARH,NVARK,NVART,NZ,IPRINT,TIMEP	BLINK 6
	COMMON IBLOCK,KCYC,KDATA,KS,KSECS,KSVM,KNIT,LABEL,LTAB,LXX,	BLINK 7
\$	MAX,MID,MODE,NADD,NADG,NADS,NADSB,NAD1,NAD2,NCHECK,NODMS,	BLINK 8
\$	NCH,NPROB,NSAVE,NSEQ,NSTOP,NUTS,KSPEC	BLINK 9
CCC	END BLINK	BLINK 10
CCC	TEMPORARY VALUES OF INTEGER AND REAL VARIABLES	BLISC 2
	COMMON A1,DX,II,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,NND,N1,N2,	BLISC 3
\$	P1,P2,P3,P4,P5,YX,X1,X2,X3	BLISC 4
CCC	END BLISC	BLISC 5
CCC	REAL VARIABLES	BLARE 2
	COMMON ADATA,ADD,AFLUID,APATH,BET,BETA,BETW,BPRIME,CAPMS,	BLARE 3
\$	CAPS,CAPT,CONST,DAY,DELT,DELT5,DELTSS,DEX,DLONG,DNOP,	BLARE 4
\$	OPMAXS,OPRES,DPSIPAX,ORAD,ORADS,OWIDE,EAVG,EC,ELT,ESTRESS,	BLARE 5
\$	ESTRIN,EX,FEX,FLEX,FLUID,FLUX,FLUXS,FLX,FOR,FORD,	BLARE 6
\$	GBYNU,GEE,GENS,GECH,GHASS,GONE,GX,NONE,HX,PHIS,	BLARE 7
\$	PHONE,PORS,PSIVARY,PSI1,PSI2,RO,RHOZ,SAVG,SCALE,SET,	BLARE 8
\$	SETD,SET1,SMALL,SUPTIN,TAU,THICK,TIME,TZERO,VISC,VOLWS,	BLARE 9
\$	VOLS,WMS,PCONE,TIPAX,PCX,DELTF,XX2,XX3	BLARE 10
CCC	END BLARE	BLARE 11
CCC	ARRAYED VARIABLES	BLARY 2
CCC		BLARY 3
CCC	THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE	BLARY 4
CCC	THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC	BLARY 5
CCC	DISTRIBUTIONS. THE FIRST INDEX REFERS TO THE DISCREET VALUE,	BLARY 6
CCC	THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR	BLARY 7
CCC	PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR	BLARY 8
CCC	CONNECTION NUMBER.	BLARY 9
CCC		BLARY 10
	LEVEL2,CAP,CON,DFI,DPSI,DF,PHI,PPHI,ZIP,TRAN	BLARY 11
	COMMON/LCH/CAP(100),CON(20,2,100),DFI(20,2,200),DPSI(20,2,200),	BLARY 12
\$	DF(20,2,200),PHI(20,2,200),PPHI(20,2,200),	BLARY 13
\$	ZIP(200),TRAN(20,2,200)	BLARY 14
	COMMON F1(12),F2(12),F3(12),NOD1(200),NOD2(200),MODE(100),	BLARY 15
\$	FI(1),FX(1),DDFI(1),	BLARY 16
\$	DDPSI(1),DPSIS(1),E(1),ITEMS(15),LYARK(10),NAME(20),	BLARY 17
\$	NAME5(20),NEWBL(15),NTYPE(100),PC(1),PCH(1),SLIN(1),	BLARY 18
\$	TOTSTRS(1),VOL(100),W(1),Z(100),CK(10),NODTO(1),R(1),	BLARY 19
\$	NODIN,TLUMP,PLUMP,XF,Q,PSIVARC(12,10),PZ(10),KODANIS,	BLARY 20
\$	CON2(1),CON3(1),CCAST1(10),CONST2(10),KODCON(1),	BLARY 21
\$	KSTDATA,LANISK(10),LTABC(10),PFC(1),	BLARY 22
\$	NODHAT(100),NOXHAT(100),MX1(10),PSIVARK(12,10)	BLARY 23
CCC	END BLARY	BLARY 24
	IF(MAXE .LE. 0) GO TO 2105	SEEK1 19

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J = MAXE
DO 2100 L = 1,MAXE
IF(NB1(J) .EQ. ITEM) GO TO 2110
2100 J = J - 1
2105 N = MAXE - MAXE + 1
K = 0
PRINT 5000, ITEM, IBLOCK
RETURN
2110 N = J
K = 1
RETURN
5000 FORMAT(1X,3(5H****),5HITEM ,1X6.33H NOT FOUND, ADDED TO END OF BL
80CK, 1I4,1N.)
END

C
C
C
C
SUBROUTINE SEEK2(N, ITEM1, ITEM2, NB1, NB2, MAXE, K)
C
C*****VERSION 5/20/69.
C*****REVISED 3/73.
C
C*****SEEK2 IS A REQUIRED SUBROUTINE.
C
C SEEK2 IS USED FOR FINDING DATA BLOCK ITEMS TO BE ALTERED.
C SEEK2 LOOKS FOR PAIR OF NUMBERS N1, N2 IN PAIR OF ARRAYS NB1, NB2.
C SETS N TO SUBSCRIPT OF NB1, NB2 IF FOUND, OTHERWISE SETS N TO
C ARRAY SIZE PLUS 1.
C
C DIMENSION NB1(1), NB2(1)
CCC VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES
COMMON DINC,NDISCR,PSIPIN,PSIMAX,TOLPR,XMIN,XMAX,IRESTRY,INT
CCC INTEGER VARIABLES
COMMON NOSPEC,NMAT,NFLUID,MODES,NOCON,NCSCON,NODBS,NOGEN,NIT,
S NVARC,NVARG,NVARH,NVARK,NVARY,NZ,IPRINT,TIMEP
COMMON IBLOCK,KCYC,KDATA,KS,KSECS,KSVM,KWIT,LABEL,LTAB,LXX,
S MAX,HID,MODE,NADD,NADG,NADS,NADSB,AA01,NAD2,NCHECK,NODMS,
S NOW,NPROB,NSAVE,NSEQ,NSTOP,NUTS,KSPEC
CCC END BLINK
CCC TEMPORARY VALUES OF INTEGER AND REAL VARIABLES
COMMON A1,DX,II,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,NND,N1,N2,
S P1,P2,P3,P4,P5,TX,X1,X2,X3
CCC END BLISC
CCC REAL VARIABLES
COMMON ADATA,ADD,AFLUID,APATH,BET,BETA,BETH,BPRIME,CAPMS,
S CAPS,CAPT,CONST,DAY,DELT,DELTS,DELTS,DEX,DLONG,DNDP,
S DPHAXS,DPRES,OPSMAX,OFAD,ORADS,OWIDE,EAVG,EC,ELT,ESTRESS,
S ESTRIN,EX,FEX,FLEX,FLUID,FLUX,FLUXS,FLX,FOR,FORD,
S GBYHU,GEE,GENS,GECM,GHASS,GONE,GX,HONE,HX,PHIBS,
S PHIONE,PORS,PSIVARY,PSI1,PSI2,RO,RHOZ,SAVG,SCALE,SET,
S SETD,SET1,SMALL,SUNTIN,TAU,THICK,TIME,TZERO,VISC,VOLNS,
S VOLS,WMS,PCONE,TIMAX,PCX,DELTF,XX2,XX3
CCC END BLARE
CCC ARRAYED VARIABLES
CCC
CCC THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE
CCC THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC
CCC DISTRIBUTIONS. THE FIRST INDEX REFERS TO THE DISCREET VALUE,
CCC THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR
CCC PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR
CCC CONNECTION NUMBER.

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SEEK1 20
 SEEK1 21
 SEEK1 22
 SEEK1 23
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 SEEK1 29
 SEEK1 30
 SEEK1 31
 SEEK1 32
 SEEK1 33
 SEEK1 34
 SEEK1 35
 SEEK1 36
 SEEK1 37
 SEEK2 2
 SEEK2 3
 SEEK2 4
 SEEK2 5
 SEEK2 6
 SEEK2 7
 SEEK2 8
 SEEK2 9
 SEEK2 10
 SEEK2 11
 SEEK2 12
 SEEK2 13
 SEEK2 14
 BLINK 2
 BLINK 3
 BLINK 4
 BLINK 5
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 BLINK 7
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 BLISC 2
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 BLARE 2
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 BLARE 5
 BLARE 6
 BLARE 7
 BLARE 8
 BLARE 9
 BLARE 10
 BLARE 11
 BLARY 2
 BLARY 3
 BLARY 4
 BLARY 5
 BLARY 6
 BLARY 7
 BLARY 8
 BLARY 9

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CCC      LEVEL2,CAP,CON,DFI,DPSI,DF,PHI,PPHI,ZIP,TRAN          BLARY 10
COMMON/LCM/CAP(100),CON(20,2,100),DFI(20,2,200),DPSI(20,2,200), BLARY 11
$      DF(20,2,200),PHI(20,2,200),PPHI(20,2,200),          BLARY 12
$      ZIP(200),TRAN(20,2,200)                                BLARY 13
COMMON F1(12),F2(12),F3(12),MOD1(200),MOD2(200),MODE(100),    BLARY 14
$      F1(1),FX(1),DOFI(1),                                  BLARY 15
$      DDPSI(1),DPSIS(1),E(1),ITEMS(15),LTABK(10),NAME(20),    BLARY 16
$      NAMES(20),NEWBL(15),NTYPE(100),PC(1),PCM(1),SLIN(1),    BLARY 17
$      TOTSTRS(1),VOL(100),W(1),Z(100),CK(10),NCOTD(1),R(1),    BLARY 18
$      MODIN,TLUMP,PLUMP,XF,Q,PSIVARC(12,10),PZ(10),KODANIS,    BLARY 19
$      CCN2(1),COA3(1),COAST1(10),CONST2(10),KODCON(1),        BLARY 20
$      KSTDATA,LANISK(10),LTABC(10),PPC(1),                    BLARY 21
$      MODMAT(100),NOXMAT(100),MX1(10),PSIVARK(12,10)          BLARY 22
CCC      END BLARY                                              BLARY 23
IF(MAXE .LE. 0) GO TO 2105                                     BLARY 24
J = MAXE                                                       SEEK2 19
DO 2100 L = 1,MAXE                                             SEEK2 20
IF(NB1(J) .EQ. ITEM1 .AND. NB2(J) .EQ. ITEM2) GO TO 2110     SEEK2 21
2100 J = J - 1                                                 SEEK2 22
2105 N = MAXE = MAXE + 1                                       SEEK2 23
K = 0                                                         SEEK2 24
PRINT 5000, ITEM1, ITEM2, IBLOCK                             SEEK2 25
RETURN                                                         SEEK2 26
2110 N = J                                                     SEEK2 27
K = 1                                                         SEEK2 28
RETURN                                                         SEEK2 29
5000 FORMAT(1X,3(5H****),5HITEMS,2I6,3H NOT FOUND, ADDED TO END OF BL,SEEK2 31
$OCK, 1I4,1H.)                                               SEEK2 32
END                                                            SEEK2 33
C                                                            SEEK2 34
C                                                            SEEK2 35
C                                                            SEEK2 36
C                                                            SEEK2 37
SUBROUTINE LINE                                              LINE 2
PRINT 1                                                       LINE 3
RETURN                                                         LINE 4
1 FORMAT(10X,10(10H=====))                                LINE 5
END                                                            LINE 6
C                                                            LINE 7
C                                                            LINE 8
C                                                            LINE 9
C                                                            LINE 10
SUBROUTINE MORTHAN(N, NAT, NSUBI)                            MORTHAN2
CCC      VARIABLES IN COMMON WITH PROBABILISTIC SUBROUTINES    BLINK 2
COMMON DINC,NOISCR,PSIPI,PSIMAX,TOLPR,XMIN,XMAX,IRESYR,INT     BLINK 3
CCC      INTEGER VARIABLES                                       BLINK 4
COMMON NOSPEC,NMAT,NFLUID,NODES,NOCON,NCSCON,NODBS,NOGEN,NIT,    BLINK 5
$      NVARC,NVARG,NVARH,NVARK,NVART,NZ,IPRINT,TIMEP          BLINK 6
COMMON IBLOCK,KCYC,KDATA,KS,KSECS,KSVM,KWIT,LABEL,LTAB,LXX,    BLINK 7
$      MAX,MID,MODE,NADD,NAOC,NADS,NAOSB,NAD1,NAD2,NCHECK,NODNS, BLINK 8
$      NOW,NPROB,NSAVE,NSEQ,NSTOP,NUTS,KSPEC                  BLINK 9
CCC      END BLINK                                              BLINK 10
CCC      TEMPORARY VALUES OF INTEGER AND REAL VARIABLES        BLISC 2
COMMON A1,DX,II,J,JJ,KK,K1,K2,L,L1,L2,L3,L4,NND,N1,N2,        BLISC 3
$      P1,P2,P3,P4,P5,FX,X1,X2,X3                             BLISC 4
CCC      END BLISC                                              BLISC 5
CCC      REAL VARIABLES                                          BLARE 2
COMMON ADATA,ADD,AFLUID,APATH,BET,BETA,BETH,BPRIME,CAPHS,      BLARE 3
$      CAPS,CAPT,CONST,OAY,DELT,DELTSS,DEX,DLONG,DNDP,        BLARE 4
$      DPMAXS,DPRES,DPSIMAX,DRAD,DRADS,DWIDE,EAVG,EC,ELT,ESTRESS, BLARE 5
$      ESYRIN,EX,FEX,FLEX,FLUID,FLUX,FLUXS,FLX,FOF,FORD,      BLARE 6

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$      GBYNU,GEE,SENS,GECM,GHASS,GONE,GX,HONE,NX,PHIBS,      BLARE 7
$      PHIONE,PORS,PSIVARY,PSI1,PSI2,RD,RHOZ,SAVG,SCALE,SET,   BLARE 8
$      SETD,SET1,SMALL,SUMTIN,TAU,THICK,TINE,TZERO,VISC,VOLMS, BLARE 9
$      VCLS,WMS,PCONE,TIPAX,PCX,DELTF,XX2,XX3                 BLARE 10
CCC      END BLARE                                             BLARE 11
CCC      ARRAYED VARIABLES                                     BLARY 2
CCC                                                    BLARY 3
CCC      THE FOLLOWING VARIABLES (WITH THE EXCEPTION OF ZIP AND CAP) ARE BLARY 4
CCC      THREE-DIMENSIONAL ARRAYS REPRESENTING PROBABILISTIC   BLARY 5
CCC      DISTRIBUTIONS. THE FIRST INDEX REFERS TO THE DISCREET VALUE, BLARY 6
CCC      THE SECOND INDICATES WHETHER IT IS THE MAGNITUDE (1) OR BLARY 7
CCC      PROBABILITY (2), AND THE THIRD IDENTIFIES THE NODE OR   BLARY 8
CCC      CONNECTION NUMBER.                                       BLARY 9
CCC                                                    BLARY 10
      LEVEL2,CAP,CON,DFI,DPSI,DF,PHI,PPHI,ZIP,TRAN            BLARY 11
      COMMON/LCN/CAP(100),CON(20,2,100),DFI(20,2,200),DPSI(20,2,200), BLARY 12
$      DFI(20,2,200),PHI(20,2,200),PPHI(20,2,200),           BLARY 13
$      ZIP(200),TRAN(20,2,200)                                  BLARY 14
      COMMON F1(12),F2(12),F3(12),MOD1(200),MOD2(200),NODE(100), BLARY 15
$      F1(1),FX(1),ODFI(1),                                     BLARY 16
$      DDPSI(1),DPSIS(1),E(1),ITEMS(15),LTABK(10),NAME(20),   BLARY 17
$      NAMES(20),MENBL(15),NTYPE(100),PC(1),PCM(1),SLTH(1),    BLARY 18
$      TOTSTRS(1),VOL(100),W(1),Z(100),CK(10),MODTD(1),R(1),  BLARY 19
$      MODIN,TLUMP,PLUMP,XF,Q,PSIVARC(12,10),PZ(10),KODANIS,   BLARY 20
$      CON2(1),CON3(1),CONST1(10),CONST2(10),KODCON(1),       BLARY 21
$      KSTDATA,LANISK(10),LTABC(10),PPC(1),                   BLARY 22
$      MODMAT(100),NOXNAT(100),NX1(10),PSIVARK(12,10)         BLARY 23
CCC      END BLARY                                             BLARY 24
C      NORTHAN7
      KWIT = 11                                                NORTHAN8
      N = MAT = NSUBI                                         NORTHAN9
      PRINT 1, W, IBLOCK                                     NORTHAN10
      RETURN                                                  NORTHAN11
1  FORNAT(32H *****MORE THAN ALLOWED,115,15HITEMS IN BLOCK, NORTHAN12
$  114)                                                       NORTHAN13
      END                                                    NORTHAN14
C      NORTHAN15
C      NORTHAN16
C      NORTHAN17
C      NORTHAN18

```

ELAPSED TIME BEFORE START OF TRUST JOB = .111 SECONDS.
TRUST OUTPUT DATA PROGPAM DATA 12.33.24 18 MAY 80

DATA DECK 1

*ONE-DIM STEADY-STATE FLOW THROUGH A HETEROGENEOUS MEDIUM

ELAPSED TIME SINCE START OF TRUST JOB = .001 SECONDS.

NO DATA CARRY-OVER FROM PRECEDING PROBLEM.

DATA BLOCK 1

IPRINT	NUM	KDATA	KSPEC	MCYC	MSEC	NPUNCH	KSTDATA	KONSTIM	NDISCR	TIMEP	SCALE	DELTF
10	-0	-1	-1	20	-0	-0	-0	1	9	-0.	.10000E+01	.8812500E+01

KD	DELTO	SMALL	PSIVARY	TAU	TIMAX	PSIMIN	PSIMAX
1	.50000E-01	.10000E-11	.50000E+00	0.	.80000E+00	0.	.10000E+03

KD	KSYM	GEOM	TOLPR	IRESTRY	INT
1	1	.10000E+01	.10000E-04	1	3

PHIONE	GONE	NONE	PCONE
0.	-0.	-0.	-0.

DATA BLOCK 2

FLOW REGION PROPERTIES			
THICKNESS	REL DEN	OF FLOW REGION	OF SATUPATED SOIL
.1000E+01	-0.	-0.	-0.

AMAT	MATL	INDEX	LTABC	LTABK	LANISK	PERMEABILITY	SPECIFIC STORAGE
						MAGNITUDE	PROBABILITY
STUFF	1	1	-0	-0	-0	.6000E+00	.1000E+01
						.2250E+01	.3900E-01
						.4250E+01	.6100E+00
						.5750E+01	.1960E+00
						.7000E+01	.8500E-01
						.8000E+01	.2900E-01
						.9250E+01	.1800E-01
						.1050E+02	.1400E-01
						.1175E+02	.5000E-02
						.4000E-02	

AMAT	MATL	INDEX	LTABC	LTABK	LANISK	PERMEABILITY	SPECIFIC STORAGE
						MAGNITUDE	PROBABILITY
CLAY	2	2	-0	-0	-0	.3000E+00	.1000E+01
						.1125E+01	.3900E-01
						.2125E+01	.6100E+00
						.2875E+01	.1960E+00
						.3500E+01	.8500E-01
						.4000E+01	.2900E-01
						.4625E+01	.1800E-01
						.5250E+01	.1400E-01
						.5000E-02	

.5975E+01 .4000E-02

DATA BLOCK 3

FLUID	VISCOSITY	BETA	RHOZ	GEE	G/VISCOSITY	BETA PRIME
GROUNDWATER	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01	.1000E+01

DATA BLOCK 4

NODE	INDEX	MATL	NTYPE	DLONG	DWIDE	DRAD	VOLUME	Z
1	1	1	0	.1000E+17	.1000E+01	.1000E+01	.1000E+17	0.
2	2	1	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
3	3	1	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
4	4	1	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
5	5	1	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
6	6	1	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
7	7	2	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
8	8	2	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
9	9	2	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
10	10	2	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
11	11	2	0	.1000E+01	.1000E+01	.1000E+01	.1000E+01	0.
12	12	2	0	.1000E+17	.1000E+01	.1000E+01	.1000E+17	0.

DATA BLOCK 5

NOD1	NOD2	INDEX	DEL1	DEL2	DLONG	DRAD	AREA	KODCON
1	2	1	0.	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
2	3	2	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
3	4	3	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
4	5	4	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
5	6	5	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
6	7	6	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
7	8	7	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
8	9	8	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
9	10	9	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
10	11	10	.5000E+00	.5000E+00	.1000E+01	.1000E+01	.1000E+01	1
11	12	11	.5000E+00	0.	.1000E+01	.1000E+01	.1000E+01	1

DATA BLOCK 9

NOTE	INDEX	PPHI	GG	PC
		MAGNITUDE	PROBABILITY	
1	1	0.	0.	0.
		0.	0.	
		0.	0.	
		0.	0.	
		0.	.1000E+01	
		0.	0.	
		0.	0.	
		0.	0.	
		0.	0.	
2	2	0.	0.	0.
		0.	0.	
		0.	0.	
		0.	0.	
		.1000E+03	.1000E+01	

		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
3	3			0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
4	4			0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
5	5			0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
6	6			0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
7	7			0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
8	8			0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		

9	9	0.	0.	0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
10	10	0.	0.	0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
11	11	0.	0.	0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
12	12	0.	0.	0.	0.
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		
		.1000E+03	.1000E+01		
		0.	0.		
		0.	0.		
		0.	0.		
		0.	0.		

DATA ENDED -1

ELAPSED TIME SINCE START OF TRUSTY JOB = .057 SECONDS.

SUMMARY OF INPUT DATA

BLOCK NUMBER	2	3	4	5	6	7	8	9
TIMES READ IN	***	1	***	***	***	0	***	***
ITEM NAME	9AY FLUID	NODE	NOD1	NODS	NODSB	NODG	NOYE	
MAXIMUM SIZE	10	1	300	600	20	20	300	12
UNMODIFIED SIZE	0	0	0	0	0	0	0	0
FINAL SIZE	2	0	12	11	0	0	0	12

TABLES	CAPM	CONT	MSURT	PHIB	GY TOTAL
	0	0	0	0	0

MAXIMUM ALLOWED TABLE LENGTH IS 12.

OTHER TOTALS NOSPEC NOGEN
0 1

MATERIAL SUMMARY

NAME	MATL	NODES	TOT VOL	TOT CAP	TOT FLUID
STUFF	1	6	.10000E+17	.10000E+17	
CLAY	2	6	.10000E+17	.10000E+17	

SYSTEM TOTAL 12 .20000E+17 .20000E+17

INTERNAL CONNECTION DATA. CHECK CONDUCTANCES (TRAN).

LARGE DIFFERENCES BETWEEN CONNECTIONS MAY BE DUE TO POOR ZONING, AND MAY PRODUCE POOR RESULTS.

PRINTOUT	CYCLE	TOTAL TIME	TIME STEP
1	0	0.	.25000E-01

NODE		PHI		DPSI	
		MAGNITUDE	PROBABILITY	MAGNITUDE	PROBABILITY
1	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	.1000E+01	0.	.1000E+01	
	0.	0.	0.	0.	
	0.	0.	0.	0.	
	0.	0.	0.	0.	
	0.	0.	0.	0.	

INTERVAL =-INDEF
MINIMUM = -INDEF
MAXIMUM = -INDEF

NODE		PHI		DPSI	
		MAGNITUDE	PROBABILITY	MAGNITUDE	PROBABILITY
2	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	.1000E+03	.1000E+01	0.	.1000E+01
	0.	0.	0.	0.	
	0.	0.	0.	0.	
	0.	0.	0.	0.	
	0.	0.	0.	0.	

INTERVAL =-INDEF
MINIMUM = -INDEF
MAXIMUM = -INDEF

NODE		PHI		DPSI	
		MAGNITUDE	PROBABILITY	MAGNITUDE	PROBABILITY
3	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.
	0.	.1000E+03	.1000E+01	0.	.1000E+01
	0.	0.	0.	0.	
	0.	0.	0.	0.	
	0.	0.	0.	0.	
	0.	0.	0.	0.	

INTERVAL =-INDEF

```

MINIMUM = -INDEF
MAXIMUM = -INDEF
NODE 4
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
.1000E+03 .1000E+01 0. .1000E+01
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.

INTERVAL = -INDEF
MINIMUM = -INDEF
MAXIMUM = -INDEF
NODE 5
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
.1000E+03 .1000E+01 0. .1000E+01
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.

INTERVAL = -INDEF
MINIMUM = -INDEF
MAXIMUM = -INDEF
NODE 6
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
.1000E+03 .1000E+01 0. .1000E+01
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.

INTERVAL = -INDEF
MINIMUM = -INDEF
MAXIMUM = -INDEF
NODE 7
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
.1000E+03 .1000E+01 0. .1000E+01
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.

INTERVAL = -INDEF
MINIMUM = -INDEF
MAXIMUM = -INDEF
NODE 8
0. 0. 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.

```

		0.	0.	0.	0.
		.1000E+03	.1000E+01	0.	.1000E+01
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		INTERVAL = -INDEF			
		MINIMUM = -INDEF			
		MAXIMUM = -INDEF			
NODE	9				
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		.1000E+03	.1000E+01	0.	.1000E+01
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		INTERVAL = -INDEF			
		MINIMUM = -INDEF			
		MAXIMUM = -INDEF			
NODE	10				
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		.1000E+03	.1000E+01	0.	.1000E+01
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		INTERVAL = -INDEF			
		MINIMUM = -INDEF			
		MAXIMUM = -INDEF			
NODE	11				
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		.1000E+03	.1000E+01	0.	.1000E+01
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		INTERVAL = -INDEF			
		MINIMUM = -INDEF			
		MAXIMUM = -INDEF			
NODE	12				
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		.1000E+03	.1000E+01	0.	.1000E+01
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.

INTERVAL = INDEF
MINIMUM = INDEF
MAXIMUM = INDEF

INTERNAL CONNECTION DATA. CHECK CONDUCTANCES (TRAN).
LARGE DIFFERENCES BETWEEN CONNECTIONS MAY BE DUE TO POOR ZONING, AND MAY PRODUCE POOR RESULTS.

PRINTOUT CYCLE TOTAL TIME TIME STEP
2 10 .25000E+00 .25000E-01

NODE		PHI		DPSI	
		MAGNITUDE	PROBABILITY	MAGNITUDE	PROBABILITY
1	0.	.6627E+00	0.	.6627E+00	
	0.	.2326E+00	0.	.2326E+00	
	0.	.8135E-01	0.	.8135E-01	
	0.	.1688E-01	0.	.1688E-01	
	0.	.4798E-02	0.	.4798E-02	
	0.	.1395E-02	0.	.1395E-02	
	0.	.2749E-03	0.	.2749E-03	
	0.	0.	0.	0.	
	0.	0.	0.	0.	
		INTERVAL = 0.			
	MINIMUM = 0.				
	MAXIMUM = 0.				
2	.5503E+01	.8675E-01	-.3415E+02	.2469E-03	
	.1620E+02	.1244E+00	-.2856E+02	.1265E-02	
	.2662E+02	.1922E+00	-.2509E+02	.4410E-02	
	.3935E+02	.3843E+00	-.1761E+02	.1563E-01	
	.5125E+02	.1441E+00	-.1125E+02	.7371E-01	
	.6016E+02	.5774E-01	-.2903E+01	.5927E+00	
	.6967E+02	.9756E-02	.1440E+01	.2873E+00	
	.7996E+02	.6750E-03	.7980E+01	.2311E-01	
	.9127E+02	.1819E-04	.1426E+02	.1605E-02	
		INTERVAL = .1038E+02			
	MINIMUM = .3791E+01				
	MAXIMUM = .9719E+02				
3	0.	0.	-.1702E+02	.2012E-02	
	.4025E+02	.2326E-04	-.1298E+02	.8564E-02	
	.4745E+02	.3230E-03	-.9947E+01	.1932E-01	
	.5576E+02	.4543E-02	-.6762E+01	.8452E-01	
	.6370E+02	.3799E-01	-.3470E+01	.2659E+00	
	.7086E+02	.1688E+00	-.9496E+00	.6047E+00	
	.7818E+02	.4842E+00	.2360E+01	.1454E-01	
	.8351E+02	.3040E+00	.5570E+01	.4400E-03	
	.9183E+02	.1223E-03	.8883E+01	.1140E-04	
		INTERVAL = .8081E+01			
	MINIMUM = .2547E+02				
	MAXIMUM = .9819E+02				
4	0.	0.	-.1065E+02	.1878E-04	
	0.	0.	-.8774E+01	.1102E-03	
	0.	0.	-.7146E+01	.5330E-03	
	.8414E+02	.4169E-04	-.5477E+01	.3815E-02	

		.8623E+02	.3556E-03	-.4052E+01	.1455E-01
		.8899E+02	.6623E-02	-.3588E+01	.3097E-02
		.9129E+02	.4197E-01	-.2276E+01	.1245E+00
		.9378E+02	.4393E+00	-.6672E+00	.8505E+00
		.9548E+02	.5117E+00	.4986E+00	.2930E-02
		INTERVAL =	.2352E+01		
		MINIMUM =	.7548E+02		
		MAXIMUM =	.9665E+02		
NODE	5	0.	0.	-.2639E+01	.1473E-04
		0.	0.	-.2305E+01	.3841E-04
		0.	0.	-.1843E+01	.4402E-03
		.9646E+02	.1544E-04	0.	0.
		.9700E+02	.2540E-03	-.1325E+01	.2939E-02
		.9750E+02	.1632E-02	-.9784E+00	.2049E-01
		.9793E+02	.1020E-01	-.9151E+00	.1586E+00
		.9834E+02	.5971E-01	-.1741E+00	.8059E+00
		.9896E+02	.9282E+00	-.1067E-01	.1154E-01
		INTERVAL =	.5259E+00		
		MINIMUM =	.9450E+02		
		MAXIMUM =	.9923E+02		
NODE	6	0.	0.	-.4814E+00	.1190E-04
		0.	0.	-.3905E+00	.1903E-03
		0.	0.	0.	0.
		.9940E+02	.2785E-04	-.2900E+00	.1322E-02
		.9949E+02	.3111E-03	-.2034E+00	.1263E-01
		.9958E+02	.2078E-02	-.1184E+00	.1171E+00
		.9967E+02	.1805E-01	-.5474E-01	.1022E-03
		.9979E+02	.4528E+00	-.4196E-01	.8678E+00
		.9985E+02	.5268E+00	.2579E-01	.8170E-03
		INTERVAL =	.9513E-01		
		MINIMUM =	.9906E+02		
		MAXIMUM =	.9992E+02		
NODE	7	0.	0.	-.1141E+00	.7655E-04
		0.	0.	0.	0.
		.9983E+02	.1456E-04	0.	0.
		.9985E+02	.6846E-04	-.6816E-01	.8638E-03
		.9987E+02	.1189E-03	0.	0.
		.9989E+02	.5400E-02	0.	0.
		.9992E+02	.3542E-02	0.	0.
		.9993E+02	.1375E+00	0.	0.
		.9996E+02	.8534E+00	-.4143E-02	.9991E+00
		INTERVAL =	.2090E-01		
		MINIMUM =	.9978E+02		
		MAXIMUM =	.9997E+02		
NODE	8	0.	0.	-.1432E-01	.4201E-04
		0.	0.	-.1197E-01	.6535E-04
		.9998E+02	.4327E-04	-.1047E-01	.9820E-04
		.9999E+02	.6758E-04	-.8845E-02	.7528E-03
		.9999E+02	.6543E-03	-.7181E-02	.3074E-02
		.9999E+02	.3085E-02	-.5499E-02	.7784E-02
		.9999E+02	.9061E-02	-.4243E-02	.2259E-01
		.9999E+02	.3092E-01	-.7409E-02	.1816E+00
		.1000E+03	.9560E+00	-.9703E-03	.7848E+00

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INTERVAL = .1933E-02
MINIMUM = .9998E+02
MAXIMUM = .1000E+03

NODE 9
0. 0. -.3526E-03 .2153E-03
0. 0. -.2829E-03 .9661E-03
0. 0. -.2706E-03 .6157E-04
.1000E+03 .8604E-04 -.2136E-03 .2588E-02
.1000E+03 .1311E-02 0. 0.
.1000E+03 .6534E-02 -.1198E-03 .7571E-01
.1000E+03 .1780E-01 -.1074E-03 .4928E-02
.1000E+03 .1299E+00 -.3844E-04 .8596E+00
.1000E+03 .8443E+00 -.2608E-04 .5593E-01

INTERVAL = .7574E-04
MINIMUM = .1000E+03
MAXIMUM = .1000E+03

NODE 10
0. 0. -.5160E-04 .2163E-04
0. 0. -.4415E-04 .1272E-03
0. 0. -.3671E-04 .5680E-03
.1000E+03 .2748E-04 -.3005E-04 .1517E-02
.1000E+03 .2061E-03 -.2933E-04 .1766E-04
.1000E+03 .1024E-02 -.2319E-04 .9434E-02
.1000E+03 .1226E-01 -.1643E-04 .4942E-01
.1000E+03 .8007E-01 -.9982E-05 .2218E+00
.1000E+03 .9064E+00 -.4379E-05 .7171E+00

INTERVAL = .9063E-05
MINIMUM = .1000E+03
MAXIMUM = .1000E+03

NODE 11
.1000E+03 .2641E-02 -.1153E-06 .9885E+00
0. 0. -.7641E-06 .1047E-01
.1000E+03 .1407E-01 -.1395E-05 .8515E-03
0. 0. -.1975E-05 .1619E-03
.1000E+03 .7399E-01 -.2606E-05 .3040E-04
0. 0. 0. 0.
.1000E+03 .9093E+00 0. 0.
0. 0. 0. 0.
0. 0. 0. 0.

INTERVAL = .2768E-06
MINIMUM = .1000E+03
MAXIMUM = .1000E+03

NODE 12
0. 0. 0. .3900E-01
0. 0. 0. .6100E+00
0. 0. 0. .1960E+00
0. 0. 0. .8500E-01
.1000E+03 .1000E+01 0. .2900E-01
0. 0. 0. .1800E-01
0. 0. 0. .1400E-01
0. 0. 0. .5000E-02
0. 0. 0. .4000E-02

INTERVAL = 0.
MINIMUM = 0.
MAXIMUM = 0.

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INTERNAL CONNECTION DATA. CHECK CONDUCTANCES (TRAN).
 LARGE DIFFERENCES BETWEEN CONNECTIONS MAY BE DUE TO POOR ZONING, AND MAY PRODUCE POOR RESULTS

PRINTOUT		CYCLE	TOTAL TIME	TIME STEP	
3		20	.50000E+00	.25000E-01	
NODE 1		PHI		DPSI	
		MAGNITUDE	PROBABILITY	MAGNITUDE	PROBABILITY
	0.	.7484E+00	0.	.7484E+00	
	0.	.2049E+00	0.	.2049E+00	
	0.	.3053E-01	0.	.3053E-01	
	0.	.1181E-01	0.	.1181E-01	
	0.	.3165E-02	0.	.3165E-02	
	0.	.1080E-02	0.	.1080E-02	
	0.	.1465E-03	0.	.1465E-03	
	0.	.2550E-04	0.	.2550E-04	
	0.	0.	0.	0.	
	INTERVAL = 0.				
	MINIMUM = 0.				
	MAXIMUM = 0.				
	NODE 2				
.6714E+01		.1962E+00	-.3839E+02	.1168E-04	
.1690E+02		.2100E+00	-.3326E+02	.8000E-04	
.2628E+02		.2833E+00	-.2701E+02	.5697E-03	
.3649E+02		.1808E+00	-.2077E+02	.3409E-02	
.4601E+02		.9062E-01	-.1521E+02	.1096E-01	
.5547E+02		.3115E-01	-.7193E+01	.1244E+00	
.6561E+02		.7235E-02	-.6774E+00	.7759E+00	
.7431E+02		.6360E-03	.4740E+01	.8013E-01	
.8537E+02		.1043E-04	.1095E+02	.4968E-02	
INTERVAL = .1072E+02					
MINIMUM = .1404E+01					
MAXIMUM = .9158E+02					
NODE 3					
		0.	0.	-.1289E+02	.3431E-02
	.2193E+02	.7995E-04	-.8832E+01	.1706E-01	
	.3199E+02	.2548E-02	-.5617E+01	.7640E-01	
	.4137E+02	.1872E-01	-.1946E+01	.4029E+00	
	.4984E+02	.9482E-01	.3601E+00	.4878E+00	
	.5933E+02	.4036E+00	.4259E+01	.1006E-01	
	.6628E+02	.4768E+00	.7515E+01	.1481E-02	
	.7623E+02	.3413E-02	.1172E+02	.3075E-04	
	.8251E+02	.3338E-04	0.	0.	
	INTERVAL = .9103E+01				
	MINIMUM = .7636E+01				
	MAXIMUM = .8956E+02				
	NODE 4				
		0.	0.	-.1237E+02	.6462E-04
0.		0.	-.1008E+02	.4246E-03	
.6754E+02		.2000E-03	-.7826E+01	.2153E-02	
.7142E+02		.2244E-02	-.5896E+01	.9820E-02	
.7527E+02		.1838E-01	-.3586E+01	.7743E-01	
.7910E+02		.1094E+00	-.1571E+01	.3087E+00	
.8336E+02		.5161E+00	-.3618E+00	.9762E+00	
.8654E+02		.3086E+00	.1422E+01	.2501E-01	
.8956E+02		.4512E-01	.3792E+01	.2665E-03	
INTERVAL = .4225E+01					

	MINIMUM =	.5561E+02		
	MAXIMUM =	.9364E+02		
NODE 5	0.	0.	-.5422E+01	.6386E-04
	0.	0.	-.4439E+01	.4307E-03
	.8713E+02	.1222E-04	-.3507E+01	.2320E-02
	.8851E+02	.1709E-03	-.2683E+01	.1058E-01
	.9021E+02	.2854E-02	-.1912E+01	.3672E-01
	.9185E+02	.3287E-01	-.1060E+01	.2065E+00
	.9357E+02	.3722E+00	-.3178E+00	.7141E+00
	.9501E+02	.5860E+00	.3141E+00	.2878E-01
	.9626E+02	.7593E-01	.1030E+01	.5597E-03
	INTERVAL =	.1651E+01		
	MINIMUM =	.8272E+02		
	MAXIMUM =	.9758E+02		
NODE 6	0.	0.	-.1901E+01	.6026E-04
	0.	0.	-.1496E+01	.7363E-03
	.9635E+02	.6364E-04	-.1143E+01	.4493E-02
	.9688E+02	.1172E-02	-.7678E+00	.3663E-01
	.9739E+02	.1549E-01	-.4497E+00	.1665E+00
	.9802E+02	.2624E+00	-.1560E+00	.7835E+00
	.9843E+02	.5806E+00	.1705E+00	.7631E-02
	.9877E+02	.1401E+00	.5014E+00	.3536E-03
	.9945E+02	.1162E-03	.8304E+00	.5871E-04
	INTERVAL =	.5588E+00		
	MINIMUM =	.9481E+02		
	MAXIMUM =	.9984E+02		
NODE 7	0.	0.	-.1283E+01	.1190E-04
	0.	0.	0.	0.
	0.	0.	-.9713E+00	.5991E-04
	.9820E+02	.1328E-04	-.6970E+00	.3653E-03
	0.	0.	-.6740E+00	.6904E-04
	.9855E+02	.1133E-03	0.	0.
	.9880E+02	.8769E-03	-.3616E+00	.9566E-02
	.9913E+02	.2804E-01	0.	0.
	.9945E+02	.9710E+00	-.3541E-01	.9899E+00
	INTERVAL =	.2372E+00		
	MINIMUM =	.9731E+02		
	MAXIMUM =	.9945E+02		
NODE 8	0.	0.	-.1163E+00	.2112E-04
	0.	0.	-.9686E-01	.1945E-03
	0.	0.	0.	0.
	.9979E+02	.6018E-04	-.7450E-01	.6612E-03
	.9981E+02	.5110E-03	-.5112E-01	.1614E-01
	.9983E+02	.3264E-02	-.4789E-01	.8926E-02
	.9985E+02	.1901E-01	-.3067E-01	.1325E+00
	.9987E+02	.8827E-01	-.1204E-01	.5420E+00
	.9990E+02	.8990E+00	-.9803E-02	.2996E+00
	INTERVAL =	.2229E-01		
	MINIMUM =	.9971E+02		
	MAXIMUM =	.9991E+02		
NODE 9	0.	0.	-.1724E-01	.8257E-04
	0.	0.	-.1472E-01	.9272E-03
	.9997E+02	.3179E-04	-.1215E-01	.3196E-02

		.9997E+02	.5209E-03	-.1136E-01	.6449E-04
		.9998E+02	.2973E-02	-.9030E-02	.2684E-01
		.9998E+02	.2018E-01	-.6459E-02	.7184E-01
		.9998E+02	.1154E+00	-.4197E-02	.1634E+00
		.9999E+02	.2599E+00	-.3847E-02	.9029E-01
		.9999E+02	.6010E+00	-.1789E-02	.6438E+00
		INTERVAL =	.2944E-02		
		MINIMUM =	.9996E+02		
		MAXIMUM =	.9999E+02		
NODE	10	0.	0.	-.1978E-02	.5581E-04
		0.	0.	-.1645E-02	.4213E-03
		0.	0.	-.1339E-02	.1857E-02
		.1000E+03	.2082E-03	-.1307E-02	.1024E-02
		.1000E+03	.2147E-02	-.1020E-02	.1642E-01
		.1000E+03	.1815E-01	-.7205E-03	.6967E-01
		.1000E+03	.8143E-01	-.6611E-03	.3933E-02
		.1000E+03	.2521E+00	-.4319E-03	.2646E+00
		.1000E+03	.6460E+00	-.1894E-03	.6420E+00
		INTERVAL =	.3422E-03		
		MINIMUM =	.1000E+03		
		MAXIMUM =	.1000E+03		
NODE	11	0.	0.	-.2054E-03	.3369E-04
		0.	0.	-.1792E-03	.1664E-03
		.1000E+03	.1263E-04	-.1395E-03	.1558E-02
		.1000E+03	.1219E-03	-.1299E-03	.5611E-04
		.1000E+03	.7700E-03	-.1056E-03	.9702E-02
		.1000E+03	.4369E-02	-.7657E-04	.4186E-01
		.1000E+03	.6503E-01	-.4247E-04	.2917E+00
		.1000E+03	.3667E+00	-.1716E-04	.6326E+00
		.1000E+03	.5630E+00	-.7339E-05	.2241E-01
		INTERVAL =	.3461E-04		
		MINIMUM =	.1000E+03		
		MAXIMUM =	.1000E+03		
NODE	12	0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		0.	0.	0.	0.
		.1000E+03	.1000E+01	0.	.2036E-03
		0.	0.	0.	.1415E-02
		0.	0.	0.	.7422E-02
		0.	0.	0.	.3217E-01
		0.	0.	0.	.1933E+00
		0.	0.	0.	.7655E+00
		INTERVAL =	0.		
		MINIMUM =	0.		
		MAXIMUM =	0.		

INTERNAL CONNECTION DATA. CHECK CONDUCTANCES (TRAN).
LARGE DIFFERENCES BETWEEN CONNECTIONS MAY BE DUE TO POOR ZONING, AND MAY PRODUCE POOR RESULTS.

PRINTOUT	CYCLE	TOTAL TIME	TIME STEP
4	30	.75000E+00	.25000E-01

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NODE		MAGNITUDE	PROBABILITY	MAGNITUDE	PROBABILITY
NODE 1		0.	.7830E+00	0.	.7830E+00
		0.	.1734E+00	0.	.1734E+00
		0.	.3250E-01	0.	.3250E-01
		0.	.8223E-02	0.	.8223E-02
		0.	.1967E-02	0.	.1967E-02
		0.	.6618E-03	0.	.6618E-03
		0.	.1993E-03	0.	.1993E-03
		0.	.5422E-04	0.	.5422E-04
		0.	0.	0.	0.
		INTERVAL =	0.		
		MINIMUM =	0.		
		MAXIMUM =	0.		
NODE 2		.5537E+01	.2539E+00	0.	0.
		.1510E+02	.2627E+00	-.3127E+02	.8792E-04
		.2373E+02	.2077E+00	-.2453E+02	.6954E-03
		.3214E+02	.1553E+00	-.1979E+02	.1727E-02
		.4112E+02	.7841E-01	-.1336E+02	.1411E-01
		.4994E+02	.2972E-01	-.7611E+01	.5737E-01
		.5914E+02	.1093E-01	-.9689E+00	.7706E+00
		.6673E+02	.1332E-02	.3496E+01	.1513E+00
		.7683E+02	.3384E-04	.9998E+01	.4078E-02
		INTERVAL =	.9138E+01		
		MINIMUM =	.1034E+01		
		MAXIMUM =	.8729E+02		
NODE 3		.7638E+01	.2617E-04	-.1102E+02	.3557E-02
		.1532E+02	.3328E-03	-.7355E+01	.2297E-01
		.2542E+02	.5807E-02	-.4080E+01	.1136E+00
		.3473E+02	.3886E-01	-.4657E+00	.7611E+00
		.4243E+02	.1189E+00	.2784E+01	.9462E-01
		.5316E+02	.6587E+00	.6891E+01	.3963E-02
		.5965E+02	.1632E+00	.1049E+02	.2452E-03
		.6635E+02	.1372E-01	0.	0.
		.7290E+02	.4122E-03	0.	0.
		INTERVAL =	.8830E+01		
		MINIMUM =	.9453E+00		
		MAXIMUM =	.8042E+02		
NODE 4		0.	0.	-.1363E+02	.2599E-04
		.5105E+02	.1130E-04	-.1052E+02	.4412E-03
		.5649E+02	.4050E-03	-.8018E+01	.2414E-02
		.6185E+02	.5020E-02	-.5979E+01	.7637E-02
		.6883E+02	.1306E+00	-.3974E+01	.4728E-01
		.7375E+02	.2176E+00	-.1826E+01	.2501E+00
		.7693E+02	.6393E+00	-.1443E+00	.6828E+00
		.8385E+02	.7026E-02	.2731E+01	.8382E-02
		.8912E+02	.1701E-04	.4380E+01	.8733E-03
		INTERVAL =	.5983E+01		
		MINIMUM =	.3991E+02		
		MAXIMUM =	.9376E+02		
NODE 5		0.	0.	-.6996E+01	.5822E-04
		.7748E+02	.1497E-04	-.5361E+01	.7558E-03
		.7976E+02	.3667E-03	-.4707E+01	.4962E-03
		.8248E+02	.6698E-02	-.3519E+01	.1071E-01

		.8438E+02	.2490E-01	-.2359E+01	.4668E-01
		.8706E+02	.3449E+00	-.1282E+01	.2030E+00
		.8927E+02	.4004E+00	-.2628E+00	.7222E+00
		.9099E+02	.2164E+00	.1006E+01	.1573E-01
		.9329E+02	.6307E-02	.2140E+01	.3076E-03
		INTERVAL =	.2505E+01		
		MINIMUM =	.7312E+02		
		MAXIMUM =	.9566E+02		
NODE	6	0.	0.	-.2767E+01	.3990E-03
		.9055E+02	.1317E-04	-.2070E+01	.3948E-02
		.9188E+02	.6864E-03	-.1537E+01	.1610E-01
		.9290E+02	.7782E-02	-.1010E+01	.7965E-01
		.9413E+02	.1159E+00	-.4342E+00	.4494E+00
		.9514E+02	.6616E+00	-.1264E+00	.4345E+00
		.9612E+02	.2117E+00	.4248E+00	.1462E-01
		.9725E+02	.2321E-02	.1151E+01	.1259E-02
		.9844E+02	.2973E-04	.2022E+01	.1059E-03
		INTERVAL =	.1243E+01		
		MINIMUM =	.8834E+02		
		MAXIMUM =	.9953E+02		
NODE	7	0.	0.	-.2386E+01	.1180E-03
		0.	0.	0.	0.
		0.	0.	-.1536E+01	.9002E-03
		.9530E+02	.3246E-04	-.1474E+01	.4927E-03
		.9588E+02	.4929E-03	0.	0.
		.9647E+02	.2718E-02	-.6568E+00	.2158E-01
		.9690E+02	.1183E-01	-.4953E+00	.1191E-01
		.9752E+02	.1482E+00	-.8285E-01	.8856E+00
		.9808E+02	.8367E+00	.1744E-01	.7954E-01
		INTERVAL =	.5158E+00		
		MINIMUM =	.9353E+02		
		MAXIMUM =	.9817E+02		
NODE	8	0.	0.	-.3670E+00	.5279E-04
		0.	0.	-.3017E+00	.1819E-03
		0.	0.	-.2594E+00	.1285E-02
		.9913E+02	.1352E-03	-.1937E+00	.8095E-02
		.9922E+02	.1236E-02	-.1797E+00	.4560E-02
		.9932E+02	.1391E-01	-.1282E+00	.6797E-01
		.9940E+02	.6532E-01	-.7249E-01	.2663E+00
		.9950E+02	.4040E+00	-.2691E-01	.6389E+00
		.9958E+02	.5154E+00	.6060E-02	.1267E-01
		INTERVAL =	.9400E-01		
		MINIMUM =	.9078E+02		
		MAXIMUM =	.9963E+02		
NODE	9	0.	0.	-.7701E-01	.7231E-04
		0.	0.	-.6560E-01	.4448E-03
		.9982E+02	.1332E-04	-.5306E-01	.3443E-02
		.9984E+02	.3099E-03	-.4152E-01	.1451E-01
		.9986E+02	.2887E-02	-.3727E-01	.1133E-02
		.9988E+02	.2453E-01	-.2790E-01	.8762E-01
		.9989E+02	.9157E-01	-.1648E-01	.2498E+00
		.9991E+02	.4452E+00	-.6852E-02	.6319E+00
		.9993E+02	.4355E+00	-.7683E-03	.1105E-01

```

      INTERVAL = .1845E-01
      MINIMUM = .9977E+02
      MAXIMUM = .9994E+02

NODE 10
      0.          0.          -.1566E-01  .1869E-04
      0.          0.          -.1339E-01  .1364E-03
      0.          0.          -.1144E-01  .6229E-03
      .9997E+02  .6403E-04  -.9359E-02  .3081E-02
      .9997E+02  .7397E-03  -.7465E-02  .1341E-01
      .9998E+02  .4394E-02  -.5374E-02  .5466E-01
      .9998E+02  .6271E-01  -.3054E-02  .2677E+00
      .9998E+02  .1523E+00  -.1260E-02  .6101E+00
      .9999E+02  .7798E+00  -.5121E-03  .5030E-01

      INTERVAL = .3364E-02
      MINIMUM = .9996E+02
      MAXIMUM = .9999E+02

NODE 11
      0.          0.          -.2403E-02  .5428E-04
      0.          0.          -.2037E-02  .3546E-03
      .1000E+03  .2007E-04  -.1645E-02  .2638E-02
      .1000E+03  .3700E-03  -.1273E-02  .1256E-01
      .1000E+03  .2354E-02  -.8953E-03  .5783E-01
      .1000E+03  .3448E-01  -.5323E-03  .1922E+00
      .1000E+03  .1973E+00  -.2453E-03  .5201E+00
      .1000E+03  .6002E+00  -.9765E-04  .2120E+00
      .1000E+03  .1653E+00  .1998E-03  .2289E-02

      INTERVAL = .5762E-03
      MINIMUM = .9999E+02
      MAXIMUM = .1000E+03

NODE 12
      0.          0.          0.          0.
      0.          0.          0.          0.
      0.          0.          0.          0.
      0.          0.          0.          .1981E-03
      .1000E+03  .1000E+01  0.          .9489E-03
      0.          0.          0.          .4685E-02
      0.          0.          0.          .8942E-01
      0.          0.          0.          .2631E+00
      0.          0.          0.          .6416E+00

      INTERVAL = 0.
      MINIMUM = 0.
      MAXIMUM = 0.

```

REASON FOR ENDING PROBLEM -

PROBLEM TIME *SUMTIME* REACHED YIMAX.

ELAPSED TIME SINCE START OF TRUST JOB = 4.355 SECONDS.

END OF TRUST JOB.

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