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REACTOR PHYSICS DEPARTMENT

TECHNICAL ACTIVITIES

QUARTERLY REPORT

APRIL, MAY, JUNE, 1967

JANUARY, 1968

**AEC RESEARCH &
DEVELOPMENT REPORT**

TO		FROM	
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REACTOR PHYSICS DEPARTMENT
TECHNICAL ACTIVITIES QUARTERLY REPORT
APRIL, MAY, JUNE, 1967

by

The Reactor Physics Department Staff

F. G. Dawson, Manager

January 1968

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REACTOR PHYSICS DEPARTMENT
TECHNICAL ACTIVITIES QUARTERLY REPORT
JANUARY, FEBRUARY, MARCH, 1967

FOREWORD

The Technical Activities Quarterly Report replaces the Physics Research Quarterly Report previously published by Battelle-Northwest's Reactor Physics Department. The objective of the report is to inform the scientific community in a timely manner of the technical progress made on the many phases of work within the department. The report contains brief technical discussions of accomplishments in all areas where significant progress has been made during the quarter. The results presented should be considered preliminary, and do not constitute final publication of the work. A list of publications and papers by the department staff is given in the report. Anyone wishing to obtain additional information on the work presented is encouraged to contact the author directly.

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REACTOR PHYSICS DEPARTMENT
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1.0. SUMMARY

REACTOR THEORY AND CODE DEVELOPMENT

Work on optical model potentials is continuing. Several forms of short range correlations in the target nucleus wave function have been investigated.

A fairly extensive code development effort has been underway during the past reporting period. FCC has been modified, and a new version, FCC-IV is now available. The ENDF/B cross section library was successfully converted from card image to the binary form. Program ETOE has been compiled and a test case was successfully executed. Program MC² has been successfully compiled on the 1107. A number of shielding codes have been successfully converted to the newly acquired UNIVAC 1108.

THERMAL REACTORS

Further analysis of the Batch Core critical tests in the PRTR has resulted in curves of the moderator level coefficients of reactivity, boron worth and sensitivity, and fuel element reactivity worths.

Measurements have been made of the effect in an inserted reactivity worth,

coolant void effect, and relative power distribution in the 8-rod FERTF fuel element, when installed in Channel 1550 in PRTR.

A theoretical analysis of a 19-rod cluster UO₂ cell was made using the RBU Monte Carlo Code. This analysis is the last of a series of calculations designed to test the RBU code.

Experiments have continued with the MTR-Phoenix Fuel mockup in the PRCF. Measurements have included shim-free critical loading, banked shim critical height for the fully loaded core, and fission product activity traverse of fuel plates to determine the locations and magnitudes of power peaks.

Three Phoenix-fueled critical approach experiments were analyzed using multigroup diffusion theory. Agreement of the order of 1% was achieved using 17 energy groups. Resonance treatment for ²³⁹Pu, ²⁴⁰Pu, and ²⁴¹Pu was used in the slowing down spectrum calculation. Integral transport theory was used to compute the thermal spectra. The plutonium fuel contained approximately 8% ²⁴⁰Pu.

A simple and direct method for measuring the Doppler coefficients,

$\frac{1}{T} \frac{dT}{dt}$, for fertile absorbers in the HTLTR has been devised.

Program GAMTEC II has been revised to incorporate the Nordheim integration scheme for treating resonance absorption.

FAST REACTORS

Several possible configurations for the FTR split-cone concept have been investigated. Flux levels and fissile fuel inventories have been determined.

The effect of the conical tilt angle on the FTR neutronics has been evaluated. A change of $+0.001 \Delta k$ was observed in going from a vertical (r-z) to the tilted core.

A fairly extensive study of the FTR kinetics characteristics has been complete. The study covers the interrelationships of reactivity insertion rates, control rod acceleration rates, and rod release times. The importance of short rod release times is demonstrated.

As part of a continuing effort to check calculational routines employed in the FTR nuclear design, the analysis of ZPR-III critical experiments has been extended. Comparison of experimental and calculated ^{238}U fission rates for Assembly 47 exhibit good agreement, while the calculated ^{239}Pu fission rate does not agree too well with experiment. Very good agreement has been obtained for the calculated Doppler coefficients for Assemblies 47 and 48.

Comparisons of measured and calculated neutron spectra also show good agreement between measured and calculated values except in the low keV region.

Three-dimensional calculations have been employed in the analysis of some

Phase-A control rod experiments. Rod worths check to within about 5%. Fairly large discrepancies in the radial B-10 reaction rate calculations have still not been resolved.

A major effort is under way in the planning of the FTR Phase-B experiments. Technical bases are being provided and material requirements for the experiments are being established.

A preliminary estimate of heat generation rates in the tube reflector, plate reflector, shroud, and vessel wall for the split-cone reactor concept has been made. A preliminary estimate has been made of the neutron damage flux intensity incident on reactor components in the vicinity of the FTR. Estimates have been made of the radiation environment that might exist in the FTR radial shield at a typical position for a neutron sensing element in the reactor instrument system. Finally, a study has been made to estimate shielding requirements for a cask to handle an IHX.

Criticality safety reviews of various systems are in progress.

CRITICAL MASS PHYSICS

Reduction and analysis of data from a series of recent PCTR experiments has established the minimum critical ^{235}U enrichment of homogeneous hydrogenous uranyl nitrate systems at 2.104 ± 0.01 wt%. Optimum neutron moderation occurs at H/U atomic ratios of 8.00 ± 1.0 and 9.3 ± 0.5 for 2.14 and 2.26 wt% enrichments, respectively, as compared to 10.5 previously measured for 3.04 wt%. These three enrichments have maximum k_{∞} values of 1.013, 1.04, and 1.145, respectively. For the 2.14 wt%, k_{∞}

values greater than unity occur for H/U atomic ratios between 6 and 10.5. For the 2.26 wt%, this increases to between 4 and 15, and for the 3.04 wt% to between 2 and 31.

A system for discriminating between neutron and gamma detections in an organic scintillator has been assembled and proven in low count rate fields with the gamma to neutron detection ratio as high as 80. The method employs the differences in the rise and fall times of the pulses resulting from the detection of the different events.

NEUTRON PHYSICS

A number of exploratory experimental measurements have been made using the

technique of multiple bragg reflections to produce semi-parallel beams of highly monochromatic slow neutrons at fixed wave lengths.

THEORETICAL PHYSICS

Battelle-Northwest program Orbit constructs and plots eight-dimensional phase-space orbits for a relativistic charged particle driven by an electromagnetic plane wave.

2.0. REACTOR THEORY AND CODE DEVELOPMENTCROSS SECTION ANALYSES WITH OPTICAL POTENTIALS

R. E. Schenter

Optical model potentials which describe the elastic scattering of neutrons by nuclei can be used to obtain the angular and energy dependence of elastic scattering cross sections in previously unmeasured regions of momentum space. They also appear as input to theoretical calculations of nonelastic nuclear reaction processes, which include inelastic scattering, capture, and fission. Consequently, a program has been initiated to calculate the optical potentials for the scattering of neutrons off nuclei that are important in fast reactor analysis. The plan is to calculate the optical potential, which is both complex and nonlocal, in terms of the fundamental nucleon-nucleon interaction and the wavefunction of the target nucleus.

Target Correlation Studies in the Optical Potential

Preliminary calculations have been made for the nuclei of oxygen and helium. These nuclei are tightly bound systems which are highly resistant to distortion by the incident neutron, with the consequence that simplifying approximations can be made in the optical model calculations. The essential output from these calculations is the spacial dependence of single-particle wavefunctions that describe the

elastic scattering of the incident neutron. Several forms of short range correlation in the target nucleus wavefunction have been investigated. This correlation appears in optical model calculations via the two-body correlation function of the target and is the part that takes into account the hard core properties of the nucleon-nucleon interaction. Figures 2.1 and 2.2 show typical results for three types of short range correlation and the resulting single-particle wavefunctions generated for an oxygen target. The differences between the curves in Figure 2.2 could have important significance in nuclear reaction calculations since, for example, the nonelastic cross section is proportional to the following integral:

$$\int_0^{\infty} \text{Im } V_{\text{op}}(r_0) r_0^2 |\psi(r_0)|^2 dr_0,$$

where $\text{Im } V_{\text{op}}$ is the imaginary part of the optical potential.

Cross Sections for Shielding Studies

In order to evaluate the Al neutron attenuation experiments in sodium, cross sections for the following activation reactions have been studied and tabulated:

FIGURE 2.1. Short-Range Correlation Functions $g(r)$ ($r = |\vec{r}_1 - \vec{r}_0|$)

FIGURE 2.2. Single-Particle Wavefunctions for Neutron-Nucleus S-Wave Scattering, Oxygen-16 Targets 5, 6, 7 ($E = 3.7$ MeV)

Cross section curves were constructed from data given in recently published experimental papers. Theoretical calculations used to obtain cross sections in unmeasured energy regions were investigated. These involved statistical model analysis that requires as input neutron, proton, and alpha particle optical potentials.

Fission Product Capture Cross Sections

Studies of the capture cross section for fission products with neutron energies typical to a fast reactor system have been initiated. This work involves combining previously measured cross section values with data obtained from optical model calculations.

CODE DEVELOPMENT AND CODE ADAPTATION

FCC-IV - R. W. Hardie and W. W. Little

The coding of FCC-IV, a revision of the FCC fundamental mode fast reactor code, is essentially complete; a formal BNWL report is being prepared. The major features of the revision are the following:

- The maximum number of energy groups has been increased from 26 to 40.
- A more complete scattering matrix has been added.
- An option has been introduced to read the Russian cross section data from tape.
- A flux feedback in the calculation of the elastic scattering cross section has been added.
- An option to calculate and punch perturbation cross sections has been added.
- The $1/T$ interpolation equation on temperature to compute shielded cross sections has been replaced by a logarithmic interpolation scheme.

One of the major changes in a revision of the FCC code is the inclusion

of a feedback loop on flux to recompute the elastic moderation cross section, $\sigma_{d,e}$. Previously, the elastic moderation term was calculated using a fission spectrum weighting in the top three groups and a constant flux [$\phi(u) = \text{constant}$] weighting in the remaining groups. The revised code, FCC-IV, computes $\sigma_{d,e}$ for each group at lethargy u'_i , where

$$u'_i = u_i - c\xi$$

and c is an input parameter, $0 \leq c \leq 1$, (Figure 2.3). When linear interpolation is used on $\xi \bar{\sigma}_e \phi(u)$, $\sigma_{d,e}$ is then recomputed using the expression

$$\sigma_{d,e}^i = \frac{1}{\delta u_i} \left\{ (\xi \bar{\sigma}_e)^i + \frac{u'_i - \bar{u}_i}{\bar{u}_{i+1} - \bar{u}_i} \left[\frac{\phi(\bar{u}_{i+1})}{\phi(\bar{u}_i)} (\xi \bar{\sigma}_e)^{i+1} - (\xi \bar{\sigma}_e)^i \right] \right\}.$$

If the new elastic moderation cross sections are used, the flux can be recalculated and the process repeated. After three or four iterations, it has

FIGURE 2.3. Schematic Diagram of Elastic Removal Model

been found that $\sigma_{d,e}$ converges well. Note that zero-iteration is equivalent to the old FCC method.

Table 2.I demonstrates how the ^{238}U to ^{235}U central fission ratios check with experimental values for 16 critical assemblies using the two methods described above. The data, excluding FCC-IV data, was taken from BNWL-347, Multigroup Analysis of Selected Fast Critical Assemblies.

It is clear that the use of the feedback loop on flux significantly

improves the agreement between the calculated and experimental values.

Table 2.II shows the central flux spectrum and critical buckling in each of the 16 assemblies as calculated by both FCC and FCC-IV. Note that the effect of the feedback loop on flux is to increase the flux in the keV region with a subsequent decrease in critical buckling. The decreased buckling is due, of course, to the U-shaped adjoint flux curve.

TABLE 2.I. Central Reaction Rate Ratios of σ_f^{U238} to σ_f^{U235}

Assembly	Experimental	Calculated		C/E ^(a)	
		FCC-IV	FCC	FCC-IV	FCC
ZPR-III 14	0.060	0.068	0.071	1.133	1.182
29	0.038	0.038	0.040	1.000	1.053
34	0.037	0.037	0.039	1.000	1.054
35 (FERMI B)	0.031	0.032	0.034	1.032	1.100
38	0.033	0.035	0.035	1.061	1.061
39	0.048	0.048	0.050	1.000	1.042
44 (RAPSODIE)	0.082	0.083	0.086	1.012	1.049
45A	0.028	0.028	0.029	1.000	1.036
46B (FARET)	-	0.076	0.078	-	-
47 (SEFOR)	0.026	0.024	0.026	0.923	1.000
48	0.031	0.033	0.034	1.065	1.097
ZPR-VI 2	0.036	0.037	0.038	1.028	1.056
ZEBRA 1	0.041	0.043	0.043	1.049	1.049
2	0.032	0.033	0.035	1.031	1.094
3	0.046	0.046	0.048	1.000	1.043
6A	0.036	0.038	0.040	1.056	1.111

(a) Ratio of calculated value to experimental value.

TABLE 2.II. Calculated Critical Assembly Characteristics

Assembly	Percent of Flux Above Indicated Energy						Critical Buckling	
	0.01 MeV		0.1 MeV		1.4 MeV		B ² x 10 ³	
	FCC-IV	FCC	FCC-IV	FCC	FCC-IV	FCC	FCC-IV	FCC
ZPR-III 14	94.9	94.4	74.1	74.7	21.1	22.2	5.60	5.58
29	98.0	97.2	73.2	73.2	10.8	11.8	2.39	2.42
34	98.0	97.2	74.3	74.5	10.7	11.3	1.99	2.01
35 (FERMI)	97.7	97.0	75.8	75.9	9.4	10.1	1.78	1.80
38	99.9	99.9	85.0	85.0	8.8	8.9	3.19	3.22
39	99.7	99.3	84.0	83.4	12.5	13.1	2.12	2.14
44 (RAPSODIE)	99.7	99.5	88.1	87.1	20.9	22.1	8.51	8.55
45A	94.0	93.2	66.3	67.2	9.36	10.0	1.02	1.07
46B (FARET)	99.5	99.1	86.3	85.7	19.6	20.2	5.85	5.88
47 (SEFOR)	89.8	89.3	59.7	61.0	9.3	10.3	2.26	2.40
48	92.6	92.0	66.4	67.2	11.4	12.1	2.48	2.54
ZPR-VI 2	97.4	96.7	72.9	73.5	10.8	11.4	2.03	2.08
ZEBRA 1	100.0	99.9	87.3	87.1	10.7	10.8	5.60	5.62
2	93.4	92.8	65.2	66.5	11.0	11.8	2.42	2.59
3	99.9	99.8	87.3	87.3	11.5	12.0	8.29	8.29
6A	91.5	90.9	65.9	67.1	13.5	14.6	3.19	3.27

ENDF/B Neutron Cross Section Library

K. B. Stewart

The ENDF/B cross section library was successfully converted from card image form to the two binary forms required as input to the system processing programs. The library contains approximately 42,000 card images.

Program ETOE - K. B. Stewart

The computer program ETOE has been compiled and the test case was successfully executed. Time requirements are excessive since the sample isotope required 6 min of UNIVAC 1107 computer time to modify the data for use in the slowing down computer code MC². The vast majority of the time (4 min) was spent copying and sorting data from temporary drum storage to the new MC² library tape.

Conversion of ETOE to the new UNIVAC 1108 computer should not present any unusual problems. A test case has been successfully executed on the 1108.

Program MC² - K. B. Stewart

The fast reactor slowing down program MC² has been successfully compiled on the UNIVAC 1107 computer. MC² cannot be executed until after ETOE has produced a library tape for MC².

Shielding Code Conversions - D. R. Marr

Shielding codes that have been successfully converted for use on the UNIVAC 1108 include ANISN (one dimensional neutron transport code), MAC (one dimensional removal-diffusion

code with coupled gamma calculation), RIBD (for generation of fission product inventory), and ISOSHLI and -II (point kernel gamma integration programs). At the same time, a correction was made in the ISOSHLI integration routine to calculate intensity at a point off the end of a cylindrical source.

Revisions in BARNS-II - J. L. Carter

BARNS-II is a computer code designed to process neutron cross section data from the BNW Master Library into point or group cross sections. In general, any one of four possible energy variations of the flux can be selected for each group in calculating the group values: constant, 1/E, fission, or Maxwellian. An exception to this general capability has been in the resonance region of certain isotopes, for which the cross sections have been calculated as superpositions of contributions of Breit-Wigner single level resonances. In this combination, only a 1/E flux has been used, and group cross sections have been calculated analytically with no doppler broadening. This limitation has been removed by recent revisions to the code that permit cross sections in this resonance region to be calculated optionally by Simpson's rule for integration. In other revisions, flexibility has been introduced by providing for a convenience routine, DOCTOR, to manipulate the results into nonstandard form. For example, versions of DOCTOR have been produced to permit multiple lower limits of an epithermal group, or multiple temperatures and upper limits

of Maxwellian groups, to be calculated as a single case.

HRG Data Tape - J. L. Carter

Further updating of the HRG data tape has been done to reflect the continuing revisions of the BNW Master Library. Since the last report,⁽¹⁾ data for 57 nuclides have been recalculated. Most of the changes were a consequence of the extended use of resonance parameter tables in the Master Library. Data for H₂O and D₂O were corrected to represent value per molecule, rather than per atom as had

inadvertently been done originally. Data for 304 SS were corrected to represent atomic rather than weight percent of the constituents. Tentative data for ¹⁶⁹Tm and data for two pseudo fission products were added, bringing the total to 240 nuclides.

Reference

1. J. L. Carter. "HRG Data Tape," Reactor Physics Department Technical Activities Quarterly Report, October-December, 1966, BNWL-400. Pacific Northwest Laboratory, Richland, Washington. April 15, 1967.

3.0 THERMAL REACTORSANALYSIS OF BATCH CORE CRITICAL TESTS
IN THE PRTR

J. W. Kutcher and J. H. Lauby

Additional analysis has been made of the critical experiments that have been performed in the Plutonium Recycle Test Reactor (PRTR) with the Batch Core Loading⁽¹⁾. The fuel element worths and boron worths have been determined using the best fit to the data on moderator level coefficients of reactivity. An additional axial flux traverse, using gold foils, has been made.

Fuel element worth was determined for each addition of an element during the Batch Core loading sequence. The curve in Figure 3.1 is a fit to the experimental data, using the critical moderator height for the determination of reactivity.

The integrated reactivity worth of boron in the D₂O moderator has been determined experimentally for five different concentrations of boron. The computer code LEARN⁽²⁾ was used to fit a smooth curve to these experi-

mental points. This curve was then differentiated to obtain a curve for the reactivity worth of boron in milli-k/wppm ¹⁰B. Both of these curves appear in Figure 3.2. The effect of self-shielding in the ¹⁰B can be clearly seen.

Axial flux traverses, using gold foils, have been made at various core sizes and boron concentrations. By use of these data, ratios of axial peak-to-average flux over the length of the fuel have been calculated (Table 3.1).

References

1. J. W. Kutcher and J. H. Lauby. "Batch Core Critical Tests in the PRTR," Reactor Physics Department Technical Activities Quarterly Report, October, November, December, 1966, BNWL-400. Pacific Northwest Laboratory, Richland, Washington, pp. 18-22. April 15, 1967.
2. B. H. Duane. Maximum Likelihood Nonlinear Corrected Fields (Battelle-Northwest Program LIKELY), BNWL-390. Pacific Northwest Laboratory, Richland, Washington, 1965.

TABLE 3.1. Ratios of Axial Peak-to-Average Flux

<u>Number of Fuel Elements</u>	<u>¹⁰B Concentration, wppm</u>	<u>Moderator Level, in.</u>	<u>Peak-to-Average Flux Ratio</u>
9	0	98.5	1.28
12	2.6	99.75	1.29
16	5.6	94.55	1.34
23	10.2	95.15	1.34
31	14.1	96.7	1.32
55	14.1	72.2	1.86
55	20.9	98.0	1.34

FIGURE 3.1. Fuel Element Worth in the PRTR Batch Core

FIGURE 3.2. Reactivity Worth of Boron -
PRTR Batch Core

FERTF ZERO-POWER TESTS IN THE PRTR

J. W. Kutcher and G. J. Konzek

Additional⁽¹⁾ zero-power measurements have been performed in the Plutonium Recycle Test Reactor (PRTR) using the Fuel Element Rupture Test Facility (FERTF). These measurements were performed with the new FERTF element located in the central channel (#1550) with H₂O coolant. This new element (Figure 3.3) consists of eight UO₂-2 wt% PuO₂ rods in a 0.125-in. stainless steel basket tube. A 0.114-in. stainless steel basket liner was used in the element for some of the tests. Plugs of stainless steel and Hastelloy-X, 1 in. in diameter, were used in the center of the element to displace H₂O and reduce the neutron flux. In addition to the FERTF element, the core loading consisted of 54 standard Mark I-R 19-rod clusters of UO₂-2 wt% PuO₂ in D₂O coolant, with 21 wppm ¹⁰B in the D₂O moderator.

The set of measurements reported here involved four configurations of FERTF element: stainless steel and Hastelloy center plugs, both with and without the 0.114-in. stainless steel basket liner. The worth of the FERTF element relative to the channel

filled with H₂O has been measured in all four cases. The results of these measurements are listed in Table 3.II. In all four cases the worth of the FERTF element is positive; the loss of the element would produce a negative reactivity change.

The coolant void worth has been measured for the FERTF element without the stainless steel liner and with the Hastelloy-X plug. This is the worst case, as there is more coolant in the tube when the liner is absent. The loss of H₂O coolant produced a positive reactivity change of 1.6 ± 0.2 milli-k.

Power generation rates in fuel rods in the FERTF element and in the first ring of elements in the Batch Core have been measured by gamma scanning of these rods after a low power PRTR irradiation. These measurements determined the power ratio between rods in the FERTF and the hottest rods in a 19-rod cluster in the first ring of elements in the Batch Core.

Relative power generation rates for these configurations of the FERTF have been calculated with the one-dimensional diffusion theory code HFN,⁽²⁾ using cross sections calculated by THERMOS⁽³⁾ and HRG,⁽⁴⁾ (Table 3.II).

TABLE 3.II. Gamma Scanning Measurement Results

Basket Thickness, in.	Center Rod	FERTF Element Worth, milli-k	Power Ratio (FERTF/1st Ring)	
			Calculated	Measured
0.125	SS	6.8 ± 0.2	1.02	0.95 ± 0.03
0.125	Hast-X	6.5 ± 0.2	1.01	0.95 ± 0.03
0.239	SS	4.6 ± 0.2	0.91	0.89 ± 0.04
0.239	Hast-X	3.7 ± 0.2	0.90	0.90 ± 0.06

FIGURE 3.3. Cross Section of FERTF Basket Element

References

1. J. W. Kutcher and J. H. Lauby. "FERTF Zero Power Tests in the PRTR," Reactor Physics Department Technical Activities Quarterly Report, January, February, March, 1967, BNWL-472. Pacific Northwest Laboratory, Richland, Washington.
2. J. R. Lilley. Computer Code HFN - Multigroup Multiregion Neutron Diffusion Theory in One Space Dimension, HW-71545, Hanford Laboratories, 1961.
3. H. C. Honeck. THERMOS, A Thermalization Transport Theory Code for Reactor Lattice Calculations, BNL-5826, Brookhaven National Laboratory, 1961.
4. J. L. Carter. "Computer Code - HRG," Reactor Physics Department Technical Activities Quarterly Report, July, August, September, 1966, BNWL-340, pp. 66-68, Pacific Northwest Laboratory, Richland, Washington, October 15, 1966.

ANALYSIS OF A UO_2 19-ROD CLUSTER EXPERIMENT WITH THE RBU MONTE CARLO CODE

D. H. Thomsen

The RBU Monte Carlo code (RBU) was used to perform a theoretical analysis of a 19-rod UO_2 cluster cell. A paper⁽¹⁾ on this analysis was presented at the June meeting of the American Nuclear Society in San Diego. This analysis is the last in a series of calculations⁽²⁻⁴⁾ designed to test the RBU code. Part of the analysis included testing the RBU resonance routine. Several problems were uncovered and corrected. Final tests indicate that the RBU resonance routine is working correctly.

The cell for which this analysis was performed consisted of a 19-rod cluster of natural uranium dioxide fuel clad in aluminum. The element was moderated and cooled with D_2O . The cell also contained an aluminum process tube and shroud tube. For this analysis the cell geometry was described completely in two space dimensions, assuming the cell to be infinite in length. Resonance parameter tables were used for ^{238}U . Values were obtained from the latest Sigma Center compilation.⁽⁵⁾ Thermalization was approximated using the ideal gas model with a fictitious heavy scatterer as described in Reference 4.

The lattice constants for this cell were experimentally determined^(6,7) in the Physical Constants Test Reactor (PCTR). Table 3.III compares experimental and calculated values of various lattice parameters for the final calculation of this analysis.

The results are given after 300,000 neutron collisions and after 500,000 neutron collisions. The error bars given with the calculated values are for one statistical standard deviation. All of the earlier RBU calculations gave results consistent with the values at 300,000 collisions. The change between the results for p , k_∞ , and k (tallies) at 300,000 collisions and 500,000 collisions uncovered a mathematical problem in the RBU Monte Carlo code mathematics. Because of the way absorption is treated in RBU, one extra degree of statistical freedom is introduced. Neutron absorption is tied to the random number selections. This is unnecessary and can introduce a small error into the calculation. Even with the possibility of this small error the calculated values are probably valid within their quoted uncertainties. It is seen from Table 3.III that the calculated value of k_∞ is too low by about 2%, while the values of f , p , and ϵ each seem to agree with the experimental value. It is felt that the resonance absorption is too large but this could not be proven.

The results of this analysis and the previous test calculations indicate that the RBU Monte Carlo Code is probably free of gross programming errors and gives reliable results. The results could be improved by correcting the mathematical problem uncovered in this analysis. It is planned to convert the RBU Monte Carlo Code to Fortran for use on the UNIVAC-1108 computer. The above problem will be corrected in the Fortran version.

TABLE 3.III. Calculated and Experimental Lattice Parameter Values

Parameter	Experiment	Calculated	
		300 k Collisions	500 k Collisions
k_{∞}	1.052 ± 0.003	1.026 ± 0.008	1.039 ± 0.006
$f(1/v)$	0.896 ± 0.003	0.896 ± 0.001	0.896 ± 0.001
p	0.868 ± 0.009	0.859 ± 0.006	0.869 ± 0.005
ϵ	1.018 ± 0.002	1.025 ± 0.003	1.024 ± 0.002
$k(\text{tallies})$		1.07 ± 0.23	1.11 ± 0.18

References

1. D. H. Thomsen. "Analysis of a UO₂-19-Rod Cluster Experiment with the RBU Monte Carlo Code," *Trans. Am. Nucl. Soc.*, vol. 10, p. 238, June, 1967.
2. D. H. Thomsen and A. G. Gibbs. *New Analytical Results From Slowing-Down Theory and Their Application to a Check Out of the RBU Slowing-Down Routine*, BNWL-307. Pacific Northwest Laboratory, Richland, Washington.
3. D. H. Thomsen. "Thermalization Calculational Study-Comparison of Results Obtained Using the RBU Monte Carlo, THERMOS, and Program S-XIII Codes," *Quarterly Technical Activities Report*, July, August, September, 1966, BNWL-340. Pacific Northwest Laboratory, Richland, Washington.
4. D. H. Thomsen. "Cell Calculations Using the RBU Monte Carlo Code," *Reactor Physics Department Technical Activities Quarterly Report*, October, November, December, 1966, BNWL-400. Pacific Northwest Laboratory, Richland, Washington.
5. J. R. Stein, et al. *Neutron Cross Section*, BNL-325, 2nd ed., supp. No. 2, vol. 3, Brookhaven National Laboratory. February, 1965.
6. J. R. Lilley. *Correlation of Data on Heavy-Water Moderated Cluster Lattices*, HW-60275, General Electric Company, Richland, Washington.
7. J. J. Oakes. "Determination of Lattice Parameters for a 19-Rod Cluster," *Nuclear Research Quarterly Report*, January, February, March, 1958, HW-55879, General Electric Company, Richland, Washington.

PRCF-PHOENIX FUEL EXPERIMENT

R. I. Smith, E. C. Davis, L. D. Williams
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The beryllium reflector region adjacent to the core tank wall has been restacked to improve the similarity between the PRCF core and the planned MTR-Phoenix core. This modification has increased the beryllium volume fraction in that region from approximately 70% to nearly 94%, with a corresponding reduction in the volume of H_2O . A plane view of the reactor configuration is shown in Figure 3.4.

All fuel assemblies have been boiled in H_2O for 6 to 8 hr to improve the aluminum oxide film on the surfaces of the assemblies. This was done in an attempt to eliminate a possible source of reactivity loss, thought to be due to formation of H_2 bubbles on the assembly surfaces. Subsequent measurements have shown that the problem has not been completely eliminated but has been reduced in magnitude.

After restacking the beryllium and boiling the fuel elements, a new shim-free critical loading was achieved with 14 1/4 fuel boxes arranged in a 3 x 4 3/4 box array. The new critical mass was about 5.6 kg Pu as compared with the critical mass of 6.5 kg Pu before restacking. The worth of a single fuel plate in this core was found to be about 25 cents.

An irradiation was performed with the shim-free loading and the power distribution was determined in the fuel plates by scanning the fission product activities in the plates. These measurements were obtained by utilizing the gamma scanning facility

at the PRCF with an aperture width of $\sim 1/4$ in. resolution. Figure 3.5 illustrates the measured power distribution for a midplane-lateral traverse from east to west across Row 3. Figure 3.6 is a midplane-lateral traverse from north to south across the elements in Position Y-3. Figure 3.7 shows the measured power distribution for an axial traverse at the centerline of the elements in Position Y-3 of this core. These data have been normalized to the previously reported⁽¹⁾ Phoenix power distributions for comparison.

Following the shim-free critical measurements, the fuel elements in the eight shim locations were replaced with the fuel-follower shim assemblies. A critical loading was achieved with a 3 x 9 array (19 elements, 8 shims) with the eight shims 59% withdrawn. The reactivity worth of a 0.1% change of the entire bank from 59% was determined to be 21.9 cents and a 1% change in shim "E" yields a worth of 12.9 cents.

The reactivity worths of the safety sheets were again measured in the shim-free core and in the 3 x 9 shimmed loading, after restacking the beryllium reflector. The worths were determined by comparing the rate of flux decay following a sheet drop to the decay rate calculated for various step decreases in reactivity. The results showed an increased worth of the safety sheets, as compared with the previous configuration, of $\sim 50\%$ in the shim-free core and $\sim 40\%$ and $\sim 30\%$ in the 3 x 9 shimmed core. The results of the sheet measurements for the various Phoenix core configurations are tabulated as follows:

PRCF-PHOENIX CORE

	Original		Restacked Reflector	
	Shim-free 16 1/2 boxes, milli-k	Shimmed 27 boxes, milli-k	Shim-free 14 1/4 boxes, milli-k	Shimmed 27 boxes, milli-k
Total Package	~15	~14	~21	~19 1/2
Strongest Sheet	~5	~3 1/2	~7 1/4	~4 1/2

FIGURE 3.4. PRCF - Phoenix Core Configuration

Lateral Core Traverse in East-West Direction 14 1/4 Element Clean Core.

FIGURE 3.5. East - West Traverse

Lateral Traverse in North-South Direction at Midplane of Element in Position y-3 of 14 1/4 Element Clean Core.

FIGURE 3.6. North - South Traverse

Axial Traverse at Midplane of Center Plate in Position y-3 of 14 1/4 Element
Clean Core.

FIGURE 3.7. Axial Traverse in Fixed Fuel Element

Power distributions in the 3 x 9 shimmed core were measured in the plates of fixed fuel elements and in the plates of a fuel follower of a shim that was withdrawn to 59%. The fission product activities in the fuel plates were measured by utilizing the gamma scanning facility at the PRCF. The aperture width was decreased from $\sim 1/4$ in. to $\sim 1/8$ in. to increase the spatial resolution of the measured power distributions. The relative power distri-

butions inferred from the plate scans are shown in Figure 3.8. The axial traverses of the plates from the fixed elements have shown that the axial peak power on the outside plate is located at ~ 5 in. above the bottom of the fuel meat whereas the power spike appears at the bottom edge of the interior plate. Analysis of the horizontal traverse across the fixed element and the axial traverse along the shim plates is in progress.

Axial Power Profile in Fixed Fuel Element Full Core - Banked Shims at 59% Withdrawn Scan of Fission Product Activities.

FIGURE 3.8. Power Profile in Fixed Fuel Element

Six plates containing tapered fuel cores have been loaded into the reactor. The fuel cores have tapers of 1 1/2 in. and 1/2 in. respectively, at opposite ends of the plates. An irradiation was performed with these plates and gamma scanning measurements were made on the plate having the 1 1/2 in. taper at the bottom of the core. The plates were then rotated to place the 1/2 in. taper at the bottom of the core. An additional irradiation was then performed and gamma scanning measurements were made on the plate having the 1/2 in. taper at the bottom of the core. These measurements will determine the effectiveness of the tapered fuel in reducing the power spikes at the bottom of the core in a fixed fuel element.

"Stuck Shim Rod" experiments were conducted in the 3 x 9 shimmed loading. The reactor was still subcritical with

one outer shim and two inner shims fully withdrawn.

A series of exploratory measurements were initiated to isolate the cause of the slow loss of reactivity observed in the operation of this core. It has been noted that by driving the sheets in (instead of dropping them), the reproducibility of period measurements was improved. There are also indications that part of the reactivity losses may be the result of small changes in the temperature of the H₂O in the core during the course of a day's work.

Reference

1. R. I. Smith, L. D. Williams, and E. C. Davis. "PRCF-Phoenix Fuel Experiment," Reactor Physics Department Technical Activities Quarterly Report, January, February, March, 1967, BNWL-472. Pacific Northwest Laboratory, Richland, Washington.

THE ANALYSIS OF PHOENIX FUEL CRITICAL APPROACH EXPERIMENTS

C. M. Heeb

Small, strongly reflected, high power density water cooled reactors are currently the most interesting Phoenix fuel environment. To gain an understanding of this type of neutronic system, critical approach experiments were conducted by D. G. Albertson and W. P. Stinson in the PNL Critical Approach Facility (CAF). These experiments have been analyzed in detail using multigroup diffusion theory and multigroup neutron transport theory. The results of the transport theory

application for the simplest experimental assembly are contained in the preceeding Quarterly Report.⁽¹⁾ Diffusion theory was used in the analysis of the three basic experimental geometries because better agreement with experiment was obtained than with transport theory for the nuclear data used.

The experiments were conducted with arrays of 2.25 in. OD aluminum cans 30 in. long with 1/8 in. thick wall. The cans were loaded with a fuel-moderator mixture consisting of 20 wt% plutonium-aluminum alloy fuel discs interspersed with pure aluminum discs and moderator discs of polyethylene.

The can arrays were tightly packed with edges touching. The entire assembly was submerged in water. The overall H/Pu ratio was approximately 200, including interstitial water. The plutonium used was 8% ^{240}Pu . An entire array thus represented a plate type core whose metal-to-water ratio is similar to that of the MTR.

Three basic variations on the 19 can array were measured:

1. The extrapolated critical buckling was determined with a water reflected core.
2. The core was reflected with one ring of 1.96 in. diam logs of beryllium metal on the 2.25 in. core pitch. The extrapolated critical height was determined for the 19 can array.
3. The same reflector assembly was used but the central fuel can in the array was replaced by a control rod consisting of a 2 in. outer diameter, 40 mil wall thickness cadmium metal tube. The cadmium tube was filled with polyethylene. The extrapolated critical height was again determined.

For the analysis of these three experiments, it was convenient to interpret the extrapolated critical buckling of the first assembly in terms of an experimental k_{eff} . To do this, an axial buckling corresponding to the 30 in. height and an oversize radius sufficient to make the total buckling equal to the experimental critical buckling was used in a 17 group diffusion theory calculation of k_{eff} . It was found that $k_{\text{eff}} = 0.997$. Thus, the calculated k_{eff} for this assembly was 3 milli-k below the experimental $k_{\text{eff}} = 1.0$. With the same axial buckling, the radius was reduced to that corre-

sponding to 19 cans. A calculated k_{eff} of 0.945 resulted. Assuming that the bias between diffusion theory and experiment remained constant at 3 milli-k low, an experimental k_{eff} of 0.948 ($0.945 + 0.003$) is reasonable.

The nuclear data used will be presented in detail in a formal document. The ^{239}Pu cross sections used were Sherr 1965 values for BNL-325, Supplement 2.

Spectrum average cross sections were computed using HRG,⁽²⁾ a site revision of GAM I and THERMOS.⁽³⁾ The upper boundary of the thermal group was 0.683 eV. Thermal spectra for the core were calculated from two successive passes through THERMOS. First, a calculation was done in slab geometry with the internal fuel and moderator comprising the unit cell. These region average fluxes were used to adjust the nuclear densities for the next calculation. This second calculation in cylindrical geometry was a representation of the unit cell for the core in the axial direction. The can, the can wall, and the surrounding water were the regions used. The can region nuclear densities were adjusted according to the region average to cell average flux ratios obtained from the first case.

The sensitivity of k_{eff} to the number of energy groups is shown in Figure 3.9. The one dimensional diffusion theory code, HFN,⁽⁴⁾ was used on the 4, 7, 10, 13, and 17 group calculations which define Figure 3.9. The seventeen group structure was selected as standard for the analysis.

The calculated and experimental values of k_{eff} are shown in Table 3.IV.

FIGURE 3.9. Calculated k_{eff} Versus Groups Diffusion Theory - Water Reflected Assembly

TABLE 3.IV. Seventeen Group Diffusion Theory

Model	Experimental k_{eff}	Calculated k_{eff}
Water Reflected	0.948	0.945
Beryllium Reflector, Homogeneous Reflector	1.0	1.025
Beryllium Reflected Assembly, Two Region Reflector	1.0	1.012
Beryllium Reflected Assembly, Central Control Rod	1.0	0.991

The poorest agreement was obtained for the partially beryllium reflected assembly with the ring of beryllium metal logs represented as a single homogenized beryllium water region. An improvement was made by using a two-region model for the reflector in which the water and beryllium were separated, half of the interstitial water in the reflector being lumped into a pure water region immediately surrounding the core. The improvement obtained by this more accurate reflector model illustrates the great importance of the region adjacent to the core. The computed value of k_{eff} is heavily dependent upon the space and energy detail in this region because of the high fast and epithermal leak rate and because of in-leaking thermal neutrons. Many of these have first leaked out, been thermalized in the reflector, and then returned to the core.

In the poisoned core model, the cadmium region was smeared into the aluminum can wall region with appropriate self-shielding in the thermal region obtained from cell region fluxes from a THERMOS calculation. The THERMOS cell model included a large

portion of the surrounding core in order to compute the correct spectrum.

In general, the three experiments and the calculational models developed provide a good test of nuclear data for use in small Phoenix reactor studies. The data used in this particular study allows fairly accurate predictions of k_{eff} with a diffusion theory calculation. The diffusion theory models should be reasonably detailed both in energy and in space. This is particularly true of the core-reflector interface regions.

References

1. Reactor Physics Department Technical Activities Quarterly Report, January, February, March 1967, BNWL-472. Pacific Northwest Laboratory, Richland, Washington.
2. HRG (Hanford Revised GAM) is a site revision of the PI Multigroup Slowing Down Code GAM.
3. H. C. Honeck. THERMOS, A Thermalization Transport Theory Code for Reactor Lattice Calculations, BNL-3826. Brookhaven National Laboratory, Upton, Long Island, New York.
4. J. R. Lilley. HFN, Multi-Region Neutron Diffusion Theory in One Space Dimension, HW-7155. General Electric Company, Richland, Washington, November 1961.

TEMPERATURE COEFFICIENT STUDIES FOR $^{233}\text{UO}_2$ - $^{232}\text{ThO}_2$ LATTICES

C. R. Richey

A question remaining in the development of high temperature graphite con-

verter reactors is the small net temperature coefficients for ^{233}U - ^{232}Th fuels. Error limits in any current calculation permit the possibility of positive coefficients with certain lattice configurations at high temperature.

The initial experiments proposed for the HTLTR with ^{233}U - ^{232}Th are, therefore, important in defining error limits on calculational techniques.

As a first attempt to compute temperature coefficients for ^{233}U - ^{232}Th fuels, calculations have been made for a 7 1/2 in. graphite lattice having a 1.0 in. diam ThO_2 rod (1.5 wt% ^{233}U). The calculations were performed with the GAMTEC II-THERMOS code combination over the temperature range 0 to 1000 °C. Principal characteristics of the calculations were the use of Park's kernels for thermal neutron scattering with graphite, and NR and NRIA treatment of resonance absorption in ^{232}Th over the entire temperature range. The coefficient $\frac{1}{k} \frac{dk_{\infty}}{dT}$ (ignoring thermal expansion) remained positive but decreased from 6×10^{-5} to $3 \times 10^{-5}/^{\circ}\text{C}$ over the range of increasing temperature. The fuel coefficient, $\frac{1}{\rho_0} \frac{d\rho}{dT}$, due primarily to Doppler effect in ^{232}Th , decreased in magnitude with increasing temperature from -1.5×10^{-5} to $-7.7 \times 10^{-6}/^{\circ}\text{C}$; the corresponding moderator coefficient, $\frac{1}{\eta_0 f_0} \frac{d\eta f}{dT}$, decreased from $+8.6 \times 10^{-5}$ to $3.9 \times 10^{-5}/^{\circ}\text{C}$.

Similar calculations were also made for a 2.86 in. diam quasihomogeneous fuel rod, i.e., ~50% by volume graphite fuel rods admixed with ThO_2 (1.5 wt% ^{233}U). When thermal expansion was ignored, the net coefficient $\frac{1}{k_{\infty}} \frac{dk_{\infty}}{dT}$ remained positive but increased from +0 to $1.4 \times 10^{-5}/^{\circ}\text{C}$ over the range of increasing temperature. The fuel coefficient, $\frac{1}{\rho_0} \frac{d\rho}{dT}$, decreased in magnitude with increasing temperature

from -3.4 to $-1.5 \times 10^{-5}/^{\circ}\text{C}$, and the corresponding moderator coefficient, $\frac{1}{\eta_0 f_0} \frac{d\eta f}{dT}$, decreased from $+3.7$ to $3.1 \times 10^{-5}/^{\circ}\text{C}$.

Measurements of effective resonance integrals and Doppler coefficients for a number of ThO_2 rods were recently reported by Hardy and Palowitch.⁽¹⁾ Comparisons with calculated resonance integrals using the data and techniques available in the GAMTEC II code gave computed resonance integrals that were 10 to 20% smaller than the experimental values. On the other hand, application of Nordheim's integration scheme⁽²⁾ gave values that were in quantitative agreement with a Monte Carlo analysis of the Hardy and Palowitch experiments.⁽³⁾ Sehgal used data for the resolved resonance recommended by Stehn, et al.,⁽⁴⁾ and the unresolved contributions were computed using both S and P wave capture. A similar set of data has been incorporated into the GAMTEC II library. P wave parameters are included under a separate isotope identification, thus eliminating extensive modifications of the GAMTEC II code. The Nordheim integration scheme has been programmed into the GAMTEC II code and is currently undergoing debugging.

References

1. J. Hardy Jr. and B. L. Palowitch. "The Effective Resonance Integral and Doppler Coefficients of Thorium Oxide Rods," *Nucl. Sci. and Eng.*, vol. 29, pp. 111-121. July 1967.
2. L. W. Nordheim. *Theory of Resonance Adsorbtion*, GA 638 revised, John J. Hopkins Laboratory for Pure and Applied Science, San Diego, California.

3. Bal Raj Sehgal. "Monte Carlo Calculations of Resonance Integrals of Thorium-232," *Nuc. Sci. and Eng.*, vol. 27, pp. 95-103. January 1967.
4. J. R. Stehn, et al. *Neutron Cross Sections, Z = 88 to 98.* BNL-325, second edition, supplement. Brookhaven National Laboratory, Upton, New York. February 1967.

DOPPLER COEFFICIENT MEASUREMENTS IN THE HTLTR

C. R. Richey

A simple and direct method for measuring the Doppler coefficients, $\frac{1}{T} \frac{dI}{dT}$, for fertile absorbers in the HTLTR has been devised. It is a variation on several other experimental techniques already in use.⁽¹⁻³⁾ Advantages in the method are that no cadmium shield⁽¹⁾ is required, nor is separate heating and insulation^(2,3) for the test sample necessary. In addition to the k_{∞} measurements currently scheduled in the HTLTR, the technique requires an additional measurement of k_{∞} for a fuel sample similar to the reactor fuel, but whose fertile absorber has been replaced by a $\frac{1}{v}$ absorber which just compensates for the $\frac{1}{v}$ absorption in the fertile absorber. The bases for the procedure are as follows:

The infinite multiplication constant for a reactor containing a resonance absorbing fertile isotope can be expressed by the well known, four-factor formula,

$$k_{\infty} = \eta \epsilon f p$$

Ignoring temperature dependent density effects, the gross temperature coefficient for the test unit lattice cell is

$$\frac{1}{k_{\infty}} \frac{dk_{\infty}}{dT} = \frac{1}{p} \frac{dp}{dT} + \frac{1}{f} \frac{df}{dT} + \frac{1}{\eta} \frac{d\eta}{dT} \quad (1)$$

A corresponding definition of k_{∞} for the $\frac{1}{v}$ compensated test sample is

$$k'_{\infty} = \eta f$$

and

$$\frac{1}{k'_{\infty}} \frac{dk'_{\infty}}{dT} = \frac{1}{f} \frac{df}{dT} + \frac{1}{\eta} \frac{d\eta}{dT} \quad (2)$$

Substitution of Equation (2) into Equation (1) reduces to an expression for the Doppler coefficient of the reactor fuel:

$$\frac{1}{p} \frac{dp}{dT} = \frac{1}{k_{\infty}} \frac{dk_{\infty}}{dT} - \frac{1}{k'_{\infty}} \frac{dk'_{\infty}}{dT} \quad (3)$$

The coefficients on the righthand side of Equation (3) can be measured in HTLTR experiments.

The resonance escape probability p is related to the non- $\frac{1}{v}$ capture in the fertile absorber by the usual expression

$$p = \text{EXP} \left[-\frac{N_O I}{\xi \Sigma_S} \right].$$

Therefore, Equation (3) can be reduced to obtain the Doppler coefficient expressed in terms of the effective resonance integral

$$\frac{1}{I} \frac{dI}{dT} = \frac{1}{\ln p} \left[\frac{1}{k_{\infty}} \frac{dk_{\infty}}{dT} - \frac{1}{k'_{\infty}} \frac{dk'_{\infty}}{dT} \right].$$

Thus, the Doppler coefficient is proportional to the difference in the gross measured coefficients for a test fuel piece and a similar test piece containing the $\frac{1}{v}$ compensating absorber.

References

1. E. Creutz, H. Jupnik, and E. P. Wigner. "Effect of Temperature on Resonance Absorption of Neutrons by Spheres of Uranium Oxide," Journal of Applied Physics, vol. 26, No. 3, March 1955.
2. R. I. Smith. "The Variation of k_{∞} with Fuel Temperature of a Uranium Fuel Element," Nucl. Sci. and Eng., vol. 21, pp. 481-489, 1965.
3. J. R. Brown, and J. B. Sampson. Measured Doppler Coefficient of Thorium Dispersed in Graphite, GA-3422, February, 1963.

4.0 FAST REACTORSANALYSIS OF SPLIT-CONE FTR NEUTRONICS

L. L. Maas

Analysis of the neutronics characteristics of the FTR split-cone configuration has been started. Major emphasis has been placed on analyzing the effects of the longitudinal gap and the conical tilt angle.

Effect of Longitudinal Gap on FTR Neutronics

Several possible configurations for the FTR split core concept have been investigated. Figure 4.1 shows the X-Y geometrical representation that was used for the two-dimensional transport theory calculations. Reactor compositions representative of those at the axial core midplane were utilized to average cross sections for each region in the reactor. The cross section data was generated by the FCC-IV code.

The five closed loops in the central gap region were assumed to contain a representative FBR fuel ($^{238}\text{U}/^{239}\text{Pu} = 5/1$). The closed loops and associated sodium and structure were homogenized into the center gap region.

Table 4.I gives the compositions used for each region in the calculations. The fuel contains 85/15 wt% $^{239}\text{Pu}/^{240}\text{Pu}$. The center gap and reflector compositions were fixed for all cases; the fraction of PuO_2 and

TABLE 4.I. Reactor Compositions

	Center Gap	Driver	Reflector
vol% Fuel	6.5	28.0	-
vol% Void	1.0	5.0	-
vol% 304 SS	18.0	25.0	-
vol% Na	74.5	42.0	20.0
vol% Inconel	-	-	80.0

UO_2 in the driver fuel was adjusted for criticality ($k_{\text{eff}} = 1.04$).

Figure 4.2 is a plot of the total flux as a function of position in the Y-direction (see Figure 4.1) for 50, 150, and 250 liter central gaps with an 800 liter driver region. The flux dips in the center gap region because the fuel concentration there is much less than in the driver region.

The center point flux versus the center gap volume for 600, 800, and 1000 liter driver regions is shown in Figure 4.3. A power level of 400 MW is assumed for all cases. Note that a center point flux of 0.7×10^{16} n/cm²-sec might be attained by a 600 liter driver region with ~200 liter center gap, an 800 liter driver and ~125 liter center gap, or a 1000 liter driver with ~85 liter center gap.

Figure 4.4 shows the variation in the ^{239}Pu mass as a function of center gap volume for three driver volumes.

FIGURE 4.1. FTR Split Core Configuration

FIGURE 4.2. Total Flux Versus Transverse Position

FIGURE 4.3. Center Point Total Flux Versus Central Gap Volume

FIGURE 4.4. Total ^{239}Pu Critical Mass Versus Central Gap Volume

The total ^{239}Pu mass includes both the center gap region and the driver region.

Effect of Conical Tilt on FTR Neutronics

In order to assess the effect of the conical core tilt on the neutronics calculations, an r-z representation of the FTR was used.

A schematic of the calculational model is shown in Figure 4.5. In this model a 100 liter central region representing the central closed test loops is surrounded by an 800 liter driver region. Between the driver region and the radial reflector is a 3.64 cm thick annular region of sodium and stainless steel. The core region was divided into five sections, all of which had the same fuel composition ($\text{PuO}_2\text{-}^{238}\text{UO}_2$) and the same outer radius for the base r-z calculation. In the second case, the outer radius of the two top core regions was increased and the bottom two decreased to represent a 5.25 degree tilt angle. This is approximately the maximum angle for an outside driver element in the FTR split core concept. The amount of fuel and stainless steel was maintained in all of the driver regions even though the volumes changed.

The relative total flux and fission rate as a function of axial position for the vertical core (r-z) and for the tilted conical core have been compared and show little change. There was a +0.001 change in k_{eff} in going from

the vertical (r-z) core to the tilted core. On the basis of these calculations, it appears that a vertical (r-z) representation of the present FTR core is adequate for neutronics calculations.

FIGURE 4.5. Schematic of FTR Split Core in R-Z Geometry

CONSIDERATIONS ON THE FTR SAFETY ROD CONTROL SYSTEM

A. E. Waltar

Major parameters of the safety rod control system for the FTR yet to be fixed include:

- 1) The downward accelerating force on the rods (G-values),
- 2) The total time delay, T, between the start of an accident and the release of the rods, and
- 3) The initial (cocked) position of the rods.

Initial studies^(1,2) concerning the shutdown capabilities of the FTR safety rod control system have been directed towards achieving simple, closed-form expressions for the first two items. The achievement of such closed-form expressions, however, necessarily involves a good deal of approximation. The present study, therefore, was made to look more exactly at reactivity inputs associated with various rod release times and G-values, and to consider the effect of Item 3 above. By using the K-III kinetics code,⁽³⁾ the transients ensuing from a given set of parameters can be followed until peak fuel temperatures are reached.

A PNL-II worth curve for a 2% rod (6.67%) was established by slightly modifying a perturbation code run to account for flux depression. The critical part of the curve near the top face of the core was checked at three points by 2DF runs (using 5 energy groups), and, as can be seen from Figure 4.6, the agreement indicates that the assumed worth curve is quite adequate for the present study.

Three initial (cocked) positions for the control rod were considered:

CASE I: Tip of Rod initially 10 cm ABOVE Core Top.

CASE II: Tip of Rod initially EVEN with Core Top.

CASE III: Tip of Rod initially 10 cm BELOW Core Top.

For Case I, rod accelerations of 0.5G, 1.0G, 2.0G, and 3.0G were included, although only a 1.0G rod was considered for Cases II and III. Reactivity versus time curves were then graphically generated by folding the worth curve (Figure 4.6) into a standard acceleration curve.

External ramp rates of 2 and 4\$/sec were chosen to characterize the reactivity inputs which could conceivably arise in a severe reactor accident. Total delay times of 100 msec, 200 msec, and 400 msec were selected to span a reasonable range of interest. Then, assuming a total safety rod shutdown capacity of 20\$, net reactivity inputs for the K-III code were determined by subtracting the delayed Δk of three control rods from the external reactivity ramps. All cases were run until the net reactivity became negative and the fuel centerline temperature reached its maximum.

Figure 4.7 contains the principal results. The bottom cluster of curves corresponds to an initiating accident of 2\$/sec and the upper cluster results from an accident of 4\$/sec. Each curve is labeled as to the G-value of the rods and the number in parenthesis indicates the total time delay, T. The broad horizontal line at about 2300 °K is included to provide some feeling as to when melting might begin somewhere

FIGURE 4.6. Assumed Safety Rod Control Worth

FIGURE 4.7. Results for 2\$/sec and 4\$/sec Accident Conditions

in the core. However, extreme caution should be exercised when attempting to use this limit in any absolute sense. The K-III kinetics code used for this analysis will handle only one dimension for an average pin. Hence, the temperatures plotted correspond to the centerline temperature of an average pin (averaged radially and axially). The 300 °K temperature rise indicated in Figure 4.7 is currently only an estimate of the average pin centerline temperature that corresponds to a melting temperature at the hottest point in the reactor. The determination of this absolute value must await an analysis from a 2-D kinetics code.

In any event, some general observations can immediately be made. Clearly the most effective parameter in the safety rod system that contributes to a successful arrest of an accident is the total delay time, T . For T short enough, even rods which fall less than 1-G (free fall with viscous damping) are quite adequate for the accidents considered. However, if total delay times of the order of approximately 200 msec are unavoidable, then certainly higher G rods are desirable. Most of the curves represent Case I, i.e., the rods initially positioned with the tips 10 cm above the core top. The effect of moving the rods down to the edge and then 10 cm into the core are illustrated in the center of the figure (4\$ ramp, 1-G rods, dropped 200 msec following initiation of the accident). The time to peak temperatures and the magnitude of the peak temperatures achieved are lower for Cases II and III. This is directly the result of placing the rods in a position such that they are moving up a steeper

part of the worth curve when dropped. Although not indicated on the graph, it might be pointed out that total rod movement (to peak temperature) was, in all cases, quite small. The maximum rod movement in any of the situations considered was only about 20 cm. This could be important from the standpoint of using short cocking springs, etc.

Thus far, only the total time, T , from the start of the accident to dropping of the rods, has been considered. This time, however, is comprised mainly of two segments: the time required to reach scram conditions, t_s , and the electronic delay time, t_d , required to then release the rods. Hence, with an over-power factor of 1.2 (condition necessary for detection of the accident) and a ramp rate of 4\$/sec, the actual electronic delay time, t_d , allowable for the curve in Figure 4.7 labeled 2.0G (200) is only 158 msec.

Another parameter pertinent to the total transient description, although not directly a part of the safety rod system, is the reactor Doppler coefficient. All studies illustrated in Figure 4.7 assumed a nominal Doppler coefficient, K_D , of -0.004. To note the effect of changing the feedback reactivity due to Doppler broadening, two cases were rerun using a K_D of -0.002 and -0.008. The runs selected were the 1.0G(200) case from the 4\$ ramp and the 0.5G(400) case from the 2\$ ramp. These were chosen since they represented cases slightly above failure (assuming the upper fuel temperature limit to be valid). Figure 4.8 contains these results. Although the total time to peak temperature is not appreciably altered for different

FIGURE 4.8. Effect of Changing the Doppler Coefficient

values of K_D , it is clear that increasing the magnitude of the Doppler coefficient reduces the peak temperatures attained.

References

1. D. E. Simpson. *Unpublished Data.*
Pacific Northwest Laboratory,
Richland, Washington, February, 1967.
2. J. W. Hagan. *Unpublished Data.*
Pacific Northwest Laboratory,
Richland, Washington, February, 1967.
3. W. W. Little, Jr. *K-III, A Three*
Region Kinetics Code in FORTRAN-IV,
BNWL-252. Pacific Northwest
Laboratory, Richland, Washington,
June, 1966.

ANALYSIS OF FAST CRITICAL EXPERIMENTS

As part of a continuing effort to check the calculational procedures used for FTR nuclear design, a series of theory-experiment correlation studies have been carried out.

Analysis of ZPR Fission Rates and Doppler Coefficients -

L. L. Maas

Radial and axial fission rate distribution calculations for ZPR-3, Assembly 47 (SEFOR) and Doppler coefficient calculations for Assemblies 47 and 48 (P-1) were completed. Assemblies 47 and 48 have been selected because they are similar, in many ways, to the FTR.

For the calculation of fission rate distributions shown in Figures 4.9-4.12, transport theory calculations for regular and adjoint fluxes in 26 energy groups were performed in the radial and axial dimensions. The ^{238}U fission distributions shown in Figures 4.11 and 4.12 exhibit good agreement between measured and calculated fission rate distributions. The calculated ^{239}Pu fission rate, however, does not agree well with experiment. The source of this discrepancy is being studied.

Table 4.II gives the results of the Doppler coefficient calculations for SEFOR and P-1. For these calculations the fast cross section calculation code FCC was used to generate 26 energy group cross sections at 300 °K and 2100 °K. These were then used in a transport theory calculation to obtain $\left. \frac{dk}{dT} \right|_{300}^{2100}$ from which the δk for the

measured temperature change was calculated. Corrections for crystalline binding are included in the calculated values.

TABLE 4.II. Doppler Coefficients in ZPR-III Assemblies 47 and 48

	Assembly 47 (SEFOR) $\left(\frac{dk}{dT} / \text{kg } ^{238}\text{U} \right)$	Assembly 48 (P-1) $\frac{\delta k}{k}$
Calculated	-2.38×10^{-5}	-4.96×10^{-6}
Measured	-2.17×10^{-5}	-4.89×10^{-6}

Analysis of ZPR Spectrum Measurements

R. W. Hardie

The central neutron spectra in two ZPR critical assembly cores were calculated and compared with recent measurements by E. F. Bennett of ANL using a proton-recoil proportional counter. All calculations were done with the FCC-IV fundamental mode code using a modified version of the Russian cross section set.

Figure 4.13 shows the calculated and experimental spectra for ZPR-3 Assembly 48, a 420 liter ^{239}Pu fueled core. The spectra in ZPR-6 Assembly 42, an 1100 liter ^{235}U fueled core, was measured twice because of improvements in the technique. The first measurement is shown in Figure 4.14 and the second in Figure 4.15.

In general, the agreement between calculation and measurement is quite good, except in the low keV region where the experiment is known to have larger errors.

FIGURE 4.9. ^{239}Pu Radial Fission Rate in SEFOR

FIGURE 4.10. ^{239}Pu Axial Fission Rate in SEFOR

FIGURE 4.11. ^{238}U Radial Fission Rate in SEFOR

FIGURE 4.12. ^{238}U Axial Fission Rate in SEFOR

FIGURE 4.13. Calculated and Experimental Spectra for ZPR-3 Assembly 48

FIGURE 4.14. First Measurement of Spectra for ZPR-6 Assembly 42

FIGURE 4.15. Second Measurement of Spectra for ZPR-6 Assembly 42

FTR PHASE A CRITICAL EXPERIMENTS: ANALYSIS

R. A. Bennett and S. L. Engstrom

Reflector Control Rod Worth Calculations

The 3-D, 2 group, diffusion code WHIRLAWAY has been employed to assess axial reflector effects on the rod worth of two reflector rods located in positions P3 and P23 of ZPR-3, Assembly 48A.

To obtain the rod worths, 3-D calculations for the rod-in and rod-out

case were performed and compared with comparable 2-D, WHIRLAWAY runs. In the 2-D runs, the axial buckling for the rod-out case was adjusted to give the same (approximately) k_{eff} as the 3-D rod-out case. The same axial buckling was then used in the 2-D rod-in WHIRLAWAY case.

In Table 4.III the results of these calculations are shown. In addition, more elaborate 2DF five group transport calculations were also performed, and the results are tabulated.

TABLE 4.III. Comparison of Rod Calculations

Case Description	Code	k_{eff}	Rod Worth, % $\Delta k/k$
Core 48A no Control Rods	2DF	0.98294	
Core 48A with Control Rods	2DF	0.97474	0.834
Core 48A no Control Rods	3-D WHIRLAWAY	0.98963	
Core 48A with Control Rods	3-D WHIRLAWAY	0.98024	0.949
Core 48A no Control Rods	2-D WHIRLAWAY	0.98160	
Core 48A with Control Rods	2-D WHIRLAWAY	0.97264	0.913

If one were to adjust the 2-D transport worth of 0.834% $\Delta k/k$ by means of the 3-D result, the predicted rod worth would be $\sim 0.866\%$ $\Delta k/k$. This is about 5% higher than the reported measured value of 0.821% $\Delta k/k$. One possible source of this discrepancy is an overestimate in the ^{10}B absorption cross section values used in these calculations. This discrepancy is being evaluated.

Spatial Reaction Rate Calculations

Previous calculations of ^{10}B spatial reaction rates have shown fairly large discrepancies in comparison with experimental results. Additional calculations are now under way to resolve these discrepancies.

The ^{239}Pu fission rate calculations are in substantial agreement. This suggests that the calculated flux spectra are reasonably correct. Further, the ^{239}Pu fission rate, like the ^{10}B absorption rate, has a significant contribution from the thermal and near thermal energy groups. This further confirms that if the calculated ^{239}Pu fission rate is correct, then the entire calculated flux spectrum is correct.

In searching for other reasons in the discrepancy, a reevaluation of the Bondarenko ^{10}B absorption cross sections is being made. Use of more recent data yields a somewhat better agreement with experiment.

FTR PHASE-B CRITICAL EXPERIMENTS - TECHNICAL BASES

R. A. Bennett

Uncertainties in the calculated values of nuclear parameters of moderately sized fast reactors and the effect of these uncertainties on the

nuclear design of the FTR are being assessed in order to provide technical justification for the performance of FTR critical experiments. Evaluations of the uncertainties in the FTR Doppler coefficient are presently in progress.

The magnitude of the negative Doppler coefficient is an important consideration in the safety analysis of the FTR. Since the FTR is relatively small, and since the split-cone driver fuel arrangement leads to rather unfavorable fuel distributions, a relatively high enrichment fuel results and the Doppler coefficient tends to be small. Suppose that for some typical fertile-to-fissile fuel ratios in the FTR, Tdk/dT is about -0.004. The real FTR Doppler coefficient may be less than this value, since it can be affected adversely by test fuel and control rod patterns.

To illustrate these effects a series of calculations were performed on an 800 liter zoned FTR core for a variety of control rod and test fuel patterns.

For the clean initial rod-free case the Tdk/dT value was established at -0.004. This is obviously an idealized reactor model which, because of the absence of rods, has a $k_{eff} > 1$. Assuming that peripheral shim rods will be used to control the reactor, $k_{eff} = 1$ was established by inserting a poison ring between the core and reflector. This model now represents the reactor in the initial critical condition. Recalculation of the Doppler coefficient for this case resulted in a value of -0.0034 for Tdk/dT . In general, test elements would tend to be less reactive than driver fuel, and consequently, such low reactivity, high ^{238}U elements would improve (increase) the Doppler coefficient. However, it is possible that high reactivity test elements might be introduced, and the in-core test shim rods are provided to compen-

sate for this type of test. This situation was simulated in another calculation where test elements (with a composition similar to that of Zone II drivers) were introduced into Zone I. The reactivity increase was about 0.018 Δk , and criticality was re-established by inserting an in-core rod bank located in Zone II. Again, the Doppler coefficient was recalculated for this reactor model, and, as expected, a sharp decrease in Tdk/dT to a value of -0.0028 was observed. In Table 4.IV, these three calculations are recapitulated.

TABLE 4.IV. Doppler Tdk/dT as a Function of Core Configuration

	Tdk/dT
Rod-Free Core	-0.004
Refl. Rods Inserted	-0.0034
Test Fuel, In-Core and Refl. Rods Inserted	-0.0028

At the end of life, the control rods would, of course, be further withdrawn, to compensate for driver fuel depletion and the presence of fission products. It is expected that under these conditions a further decrease in the Doppler coefficient would occur. In addition, if we now postulate errors of $\pm 25\%$ in the Doppler coefficient calculation, the Tdk/dT value might be as low as -0.002. This is the value of Tdk/dT that one might want to use in reactor safeguards and dynamics studies.

Verification of the FTR kinetics parameters is an important objective of the Critical Experiment Program.

FTR PHASE B CRITICAL EXPERIMENTS- EXPERIMENT PLANNING

R. A. Bennett

A meeting of ANL and PNL personnel was held on May 2-3, 1967 to discuss the rationale and general plans for the FTR Phase B Critical Experiments Program.

Discussions were held among the participants concerning the salient features of the split-core FTR reference design, the physics and safe-guards problems of this design, and the nature of the critical experiments to be performed in support of FTR nuclear design.

The gross asymmetry introduced by the vertical test zone in the split-core FTR design appears to be at the root of the major physics problem of this design. The geometric features of the core require a full 3-D analysis. At the present time no adequate 3-D transport codes exist and, hence, a complete, rigorous, nuclear analysis of the FTR core is not possible. The only available analytical approach to the problem involves sequential 2-D analyses, or possibly synthesis methods. Both of these approaches involve many approximations and, therefore, extensive verification critical experiments are essential.

It was proposed that the forthcoming Phase B experiments be performed in the following cores:

- I. Uniform Right Cylinder
- II. Uniform Split Cylinder
- III. Uniform Cone (simulated by stacked disks)

IV. Nonuniform Split Cone

V. Nonuniform Cone (simulated by stacked disks).

Since the split-cylinder geometry contains the salient FTR gap feature, and at the same time affords the opportunity to do at least some meaningful calculations, most of the detailed experiments are expected to be carried out for this geometry. A considerably less extensive series of measurements is proposed for the other core geometries.

The following specific experiments were proposed for the respective cores:

Core I - Uniform Right Cylinder Core

- At least two Doppler coefficient measurements at two different core locations
- Sodium void effects, via removal of large sodium volumes at select core locations
- Spatial reaction rate measurements
- Spatial reactivity coefficient measurements of core materials
- Rossi alpha measurements
- Spectrum measurements

Core II - Uniform Split Cylinder

- Doppler coefficient measurements at two different core locations
- Measurements to test the effect of gap size, shape and composition on core neutronics
- Sodium void effects, via removal of large sodium volumes at selected core locations
- Spatial reaction rate measurements
- Spatial reactivity coefficient measurements of core materials
- Rossi alpha measurements

Core III - Uniform Cone

- Critical mass determination for at least two different combinations of disk sizes

Core IV - Nonuniform Split Cone

- Experiments similar to those for Core II
- Core V - Nonuniform Cone
- Critical mass determination

FTR SHIELD STUDIES

C. A. Mansius and W. L. Bunch

The one dimension, removal-diffusion code MAC has been used to estimate the neutron and gamma flux distributions in the vicinity of a split conical core arrangement based on the concept shown in Figure 4.16. To carry out the calculations, it was necessary to approximate the various regions by homogeneous slab shields. Results of the calculations are as follows.

Heat Generation Rates

Nuclear heat generation rates in reactor components external to the core were calculated from the MAC gamma distributions using the relation:

$$H = K \phi(E) \mu(E)$$

where

H = heat generation rate in W/cm^3 ,

$K = 1.6 \times 10^{-13} \text{ W-sec/MeV}$,

$\phi(E)$ = energy dependent gamma flux,
 $\text{MeV cm}^2\text{-sec}$

$\mu(E)$ = energy absorption coefficient,
 cm^{-1} .

Heat generation in structural components is shown in Figures 4.17 and 4.18. Figure 4.17 presents heat generation in the shroud (reactor core vessel) and in the reactor vessel. Figure 4.18 presents heat generation in the lower tubesheet. Although these values should be conservative because the one-dimensional code does not include geometric effects, the uncertainty in the calculations is not known.

"Damage" Fluxes

Neutron damage to structural materials is in most cases caused by the higher energy neutrons. An estimate of neutron damage flux intensities incident on various reactor components is presented in Table 4.V, for regions located radially out from the core and axially below the core. The range in values is indicative of the coarseness of the spectral calculations in the vicinity of the assumed 0.1 MeV cutoff energy and is not intended as an estimate of the uncertainty of the calculations.

FIGURE 4.16. Conceptual Arrangement of Split Conical Core

FIGURE 4.17. Heat Generation Rates in Shroud and Vessel

FIGURE 4.18. Heat Generation in Tube Sheet

TABLE 4.V. Damage Flux Incident on Reactor Components

<u>Region</u>	<u>Distance from Core, cm</u>	<u>Incident Damage Flux, $E \geq 0.1$ MeV</u>
<u>A. Radially (from core)</u>		
"Slant" reflector	0	$2.6-3.5 \times 10^{15}$
"Transition" reflector	21.6	$1.2-1.7 \times 10^{15}$
"Vertical" reflector	59.7	$3.5-6.2 \times 10^{13}$
Shroud vessel	97.8	$0.6-1.2 \times 10^{12}$
Sodium annulus	102.9	$3.2-6.1 \times 10^{11}$
Reactor vessel	163.8	$1.5-8.1 \times 10^{10}$
"Blast" shield	170.8	$0.6-2.2 \times 10^{10}$
Insulation	183.5	$0.8-1.8 \times 10^9$
Concrete	214.1	$0.6-1.4 \times 10^9$
<u>B. Axially (below core)</u>		
"Pin" reflectors	0	$2.2-3.2 \times 10^{15}$
End of pins	20.3	$0.6-1.0 \times 10^{15}$
Tubesheet	137.2	$1.2-3.7 \times 10^{12}$
Inlet plenum	152.4	$2.4-4.9 \times 10^{11}$
Refractory clad	212.4	$1.3-6.2 \times 10^{10}$
Refractory material	214.9	$0.8-3.3 \times 10^{10}$
Sodium pool	253.0	$0.8-3.3 \times 10^8$
Reactor vessel	289.5	$0.3-1.1 \times 10^8$
Neutron shield	297.2	$1.0-2.4 \times 10^7$

Reactor Instrument System Sensor
Environment

An estimate was made of the radiation environment that might exist in the FTR radial shield at a potential location for a neutron sensing element in the reactor instrument system. Calculations for full power operation were based directly on the MAC results. Transient results following a shutdown are based on the fission events by delayed neutrons after a 20 milli-k scram. The equilibrium shutdown level will be established by the production rate of neutrons in

the core from (α, n) events and spontaneous fission together with the shutdown multiplication factor. Results are presented in Table 4.VI.

The gamma intensity at the sensor location will be sustained following shutdown of the reactor by the radioactivity of the sodium: about 10^4 R/hr from ^{24}Na and about 1 R/hr from ^{22}Na . The intensity from ^{24}Na would decay with a half life of 15 hr. Thus, two days after shutdown the intensity would be about 10^3 R/hr, four days after shutdown it would be about 10^2 R/hr, etc. down to the residual intensity

*TABLE 4.VI. Neutron and Gamma Intensity at Sensor Location
Due to Reactor Fission Events*

<u>Time After Shutdown, min</u>	<u>Thermal Neutron Flux, n/cm²-sec</u>	<u>Gamma Intensity, R/hr</u>
0 (equilibrium operation)	10^{10}	10^5
1	10^8	10^3
4	10^7	10^2
7	10^6	10^1
10	10^5	10^0
25 (equilibrium shutdown)	10^0	10^{-5}

of about 1 R/hr from the ^{22}Na . In addition, the gamma intensity would probably be sustained during shutdowns by corrosion and fission products in the sodium and by activated structural and shield components in the vicinity of the sensor.

No estimate was made of the magnitude of these sources. Fission product activity from within the core would not contribute to the gamma intensity at the sensor because of attenuation by the thick reflector.

LARGE COMPONENT CASK SHIELDING REQUIREMENTS

W. L. Bunch

An intermediate heat exchanger (IHx) is potentially the most radioactive source that will be handled in the proposed large component cask for the FFTF. The cask will need to be several feet in diameter and tens of feet tall. Total weight of the cask will be strongly dependent on the shielding that is provided. Therefore, a study was made to estimate requirements for potential sources of radiation within an IHx.

The IHx will be located in a region where activation will be negligible.

The three sources of radiation that will be associated with an IHx are:

(1) radioactive sodium adhering to the surfaces of the drained system, (2) corrosion products either plated on or adhering to the IHx wall surfaces, and (3) fission products retained in the IHx.

For the study it was assumed that four primary loops would be used in the FTR, with each IHx containing $3.26 \times 10^6 \text{ cm}^2$ in contact with primary sodium. The bundle of tubes would form a cylindrical array about 4 ft in diam and 16 ft long. When drained, it was assumed that 0.017 lb of sodium would adhere to each square foot of surface area, leaving a total of

2310 Ci of ^{24}Na and 0.86 Ci of ^{22}Na (based on specific activities of 0.08 Ci/cm^3 and $30 \text{ } \mu\text{Ci/cm}^3$, respectively). The ^{24}Na decays with a half life of 15 hr. Thus, removal of the IHX could be delayed to permit decay of this isotope. Calculations using the computer code ISOSHLTD indicate that less than 1/4 in. of lead shielding would be required to reduce the gamma intensity from ^{22}Na to 100 mR/hr outside the handling cask.

It was assumed that the corrosion products from the core would be plated on the cold surfaces of the heat exchanger and would not be drained with the sodium. It has been estimated previously that 118 cm^3 of fuel cladding will be removed from the core each operating month. Dominant activity of the corrosion products is calculated to be from ^{58}Co , which has a half life of 71 days. After about a year of operation, the equilibrium activity of ^{58}Co in an IHX is calculated to be 3400 Ci. ISOSHLTD calculations indicate that about 3 in. of lead shielding or equivalent would be required to reduce

the intensity outside the cask to 100 mR/hr.

The major source of gamma radiation to be shielded by the large component cask will probably be associated with fission products plated to the IHX walls. Although the fuel is to be clad, cladding failures will probably occur. For this study, a cladding failure was assumed that released all of the fission products from fuel generating 6 MW for 200 consecutive days. After dissipating uniformly throughout the sodium (rather than channeling into a single loop), it was assumed the halogens, noble gases, and volatile solids entered the cover gas system and the rest of the fission products plated to the IHX walls. The computer code RIBD was used to generate the fission product inventory and ISOSHLTD was used to estimate the gamma intensity outside the handling cask from the deposited fission products. It was found that about eight inches of lead shielding or equivalent would be required to reduce the intensity to 100 mR/hr outside the cask within a month after the assumed cladding failure.

CRITICALITY SAFETY REVIEWS

C. L. Brown

Experimental Measurements

The type and number of FFTF driver fuel pins that will be needed for the first series of criticality safety experiments have been estimated.

Although these pins cannot be made available for several months, it may be possible to accumulate enough reject pins for a preliminary experiment. It is planned to measure material bucklings and critical masses at about six lattice spacings, both in light water and in water containing boron. These measurements are needed

to confirm theoretical criticality calculations upon which criticality safety in the FFTF will be based. The closest check point now available is at 6.6 wt% $\text{PuO}_2\text{-UO}_2$ pins (Saxton fuel). The pins desired for the experiments are:

Enrichment:	30-35 wt% $\text{PuO}_2\text{-UO}_2$
Diameter:	0.20-0.25 in.
Length:	30-36 in.
Density:	$\sim 90\%$ TD
^{240}Pu	$\sim 10\%$
Number:	~ 230

The pins will be obtained on a loan basis for about six weeks and will not be altered or changed in any way during the experiments.

Radiometallurgy

The underwater storage rack for irradiated FFTF fuel pins in 327 Building has been redesigned. Criticality safety for the revised rack will be based on a safe mass in an overmoderated array, rather than on safe mass per unit. The storage cans for the pins will be 5/8 in. diam, and the cans will be spaced to give an effective water-to-fuel volume ratio of not less than 80. At this ratio the 200 pin capacity of the rack will represent less than one half of a critical mass.

System 42 - Nonirradiated Fuel and Material

A preliminary review of this system indicates that no unusual criticality control problems will be encountered. Storage and handling limits can be based on safe mass or safe geometry,

whichever is the more efficient. A quantitative assessment cannot be made, however, until equipment and operations are defined. The use of a "guaranteed dry" storage vault for fuel pins appears feasible. It is interesting to note that since many of the FFTF driver and test pins will contain recycled fuel, batch limits in handling and storage may be limited more by radiation dose rates than by criticality safety.

System 72 - Underwater Examination

A major question to be answered is whether or not boron poisoning will be required in the basin water for criticality control. At present, poison is specified because at optimum water moderation the pins in a subassembly represent over two critical masses, and at present, geometry control cannot be assured. Before the final decision can be made, information is needed on the following:

- The storage arrangement of FFTF fuel in the basin
- The fuel handling procedures to be used in the basin
- The method of fuel disassembly to be used.

It would also be helpful to have experimental criticality data for the FFTF fuel pins in water to confirm the critical mass calculations. However, these data will probably not be available until after Title I and Title II design are complete. The chances are very good that, with proper equipment design and fuel handling procedures, the boron poison will not be required.

System 81 - Sodium Receiving and Processing System

The potential for criticality in the cold traps will depend on the size and shape of the traps and the maximum amount of fuel that could credibly reach the traps. It is expected that the high critical mass in an unmoderated system (>100 kg) and the non-optimum geometry of the traps will preclude a criticality hazard. Further reviews will be made at later stages of design.

Shipping Cask

The design of the FFTF shipping cask will present several challenging design problems because of the high radiation and heat output of the

irradiated FFTF fuel. Compared to the design of casks for irradiated fuel from thermal reactors, the FFTF cask will require considerable study to obtain reasonable shipping economy. Problems will concern the amount of shielding, the method of cooling, the safe arrangement of fuel, and how to pass the new performance criteria given in Federal regulations. The nature of the FFTF fuel indicates that the cask will have a higher than usual weight because of more shielding and a lower than usual payload because of cooling and criticality problems. Both of these criteria are in the direction of a higher than usual shipping cost. The question of one large cask versus two or three small casks should also be evaluated.

5.0 CRITICAL MASS PHYSICS

PULSE SHAPE DISCRIMINATION

G. M. Hess

Preliminary Rossi alpha and P_0 measurements on small plutonium systems have yielded inconsistent results. The conditional probabilities sought in the Rossi alpha experiment were all essentially equal, and the variance-to-mean ratios calculated from the P_0 data were inconsistent with the theoretical model.

It was felt that the reason for these discrepancies was the large gamma to neutron detection ratio. Consequently, a method of discriminating between neutron and gamma pulses from an organic scintillator was desired. Such a method exists⁽¹⁾ and is feasible.⁽²⁾ With slight modification for the present situation, the method referred to above was successfully attempted. A block diagram is shown in the Figure 5.1.

The pulse shapes resulting from a gamma and a neutron detection in an organic scintillator differ primarily in fall time. If a dynode signal, off an unsaturated dynode, is fed into an

integrating preamplifier and is followed by double delay line amplification, the zero crossover of the resulting bipolar pulse is a measure of the integral of the initial pulse. Neutron pulses, having a slower decay rate, cross further in time than do the gamma pulses. Thus, if the time between the anode signal, which contains accurate timing information, and the zero crossover point of a pulse is measured, the neutron pulses may be separated from the gamma pulses.

Preliminary experiments indicate that in fields with a gamma-to-neutron detection ratio as high as 80, the separation is such that at least 95% of the indicated neutron counts are indeed neutrons.

Further Rossi alpha and P_0 measurements are planned upon refinement of technique. The system will also be used in pulse neutron and reactor noise experiments.

References

1. *ORTEC Catalog 1000*, p. 20
2. E. McCauley. *University of Washington*. (Private Communication)

5.2

BNWL-534

FIGURE 5.1. Block Diagram of Pulse Shape Gamma Discrimination System

MINIMUM CRITICAL ^{235}U ENRICHMENT OF HOMOGENEOUS HYDROGENOUS URANYL NITRATE SYSTEMS

S. R. Bierman and G. M. Hess

Introduction

In the recovery of uranium from spent fuel elements and uranium bearing wastes, the uranium is converted to the nitrate form by dissolution in a nitric acid solution. It remains in this chemical form throughout many of the processing and handling steps associated with its recovery. Consequently, a firm knowledge is needed of the minimum ^{235}U enrichment for which a homogeneous uranyl nitrate system can be made critical. This minimum critical enrichment is the enrichment required to obtain an infinite neutron multiplication factor, k_{∞} , of unity under conditions of optimum moderation. Below this enrichment, criticality would not be possible, and processing restrictions for nuclear safety would not have to be imposed. Also, this enrichment value and the H/U ratio of the homogeneous mixture that gives a maximum k_{∞} of unity provide a fundamental point for comparison with reactor theory.

It is also desirable, for checking calculational models and for establishing nuclear safety guidelines, to know over what range of moderation criticality is possible for a given enrichment. This is particularly true for low enriched uranium systems, since at low enrichments criticality is strongly dependent on both neutron moderation and ^{235}U enrichment.

An experimental program to provide the above information was initiated and completed in the Physical Constants Test Reactor (PCTR). The results and interpretation of these measurements and correlated data from some previous PCTR measurements are presented herein.

Previous experiments⁽¹⁾ performed in the PCTR have shown the minimum critical ^{235}U enrichment for homogeneous mixtures of UO_3 in water to be 1.034 ± 0.010 wt%. Additional experiments⁽²⁾ with 3.04 wt% enriched UO_3 and uranyl nitrate, $\text{UO}_2(\text{NO}_3)_2$, hydrogenous mixtures established that the presence of the nitrate radical had a pronounced effect on criticality. The 3.04 wt% enriched UO_3 had a maximum k_{∞} value of 1.350 ± 0.013 as compared to 1.145 ± 0.01 for the 3.04 wt% enriched uranyl nitrate. Subsequent calculations^(3,4) indicated that the minimum critical enrichment for homogeneous hydrogenous uranyl nitrate mixtures was about 2 wt%. Consequently, experiments were planned for ^{235}U enrichments slightly above this value.

Experimental Measurements

Measurements were made on uranyl nitrate mixtures having ^{235}U enrichments of 2.14 ± 0.005 wt% and 2.26 ± 0.005 wt% and H/U ratios over the range of about 3 to 12. In this region of moderation uranyl nitrate is in the crystalline form. Consequently, 1/8 in. diam spheres of linear polyethylene (Eastman Kodak "Tenite") having a density of 0.916 g/cm^2 and an H:C atomic ratio of 2 were used to vary the hydrogen content of the uranyl

nitrate. Previous measurements⁽²⁾ had established that polyethylene was an acceptable replacement for water as a moderator in these systems.

The preparation of the uranyl nitrate material, the blending, the loading and unloading of the experimental vessels, and the sampling of the material used in these experiments was carried out by the Chemical Development Section of Battelle-Northwest's Chemistry Department. A detailed description of the material preparation is covered in Reference 5.

Since the stable condition for uranyl nitrate is as the hexahydrate, it was necessary to remove some of the waters of hydration to achieve H/U atomic ratios of less than 12. This was accomplished by temperature control during the crystallization step to obtain a dihydrate. An end product having on the average 1.72 molecules of water per molecule of uranyl nitrate was achieved. This relatively anhydrous salt is deliquescent. Consequently, water analyses were made periodically during the course of the experiments to determine the amount of water being picked up by the uranyl nitrate. Based on the degree of hydration at the time of blending, sufficient polyethylene was added to each mixture to achieve the desired H/U ratio.

For making the measurements in the PCTR, each mixture was contained in a 21 in. cube composed of essentially nine 7 x 7 x 21 in. aluminum cans. The center 7 x 7 x 21 in. can consisted actually of three 7 in. cubes, the center one being the center test cell over which changes in neutron multiplication were observed. A typical

loading of the PCTR, for these experiments, is shown in Figure 5.2. A better perspective of the two types of cans making up the 21 in. cube can be obtained from Figure 5.3.

The usual criteria of a constant cadmium ratio⁽⁶⁾ was used to establish that a characteristic flux existed across the test material in each experiment. Gold foils were used to determine cadmium ratios in both the radial and the axial directions across the entire 21 in. cube and on the face of the PCTR graphite opposite these traverses. The foil traverse rods and their relative positions in the material are shown in Figure 5.3

The reactivity response of the PCTR to 13 different unpoisoned H/U mixtures was observed during the course of the experiments to establish the reactivity worth of each mixture relative to k_{∞} of unity. The results of these measurements are shown in Figure 5.4 as a function of the H/U ratio and ^{235}U enrichment.

With the high density (2.75 g/cm^2 as compared to 2.802 g/cm^2 theoretical density for UNH) obtained in the preparation of the uranyl nitrate material, it was possible to make accurate reactivity measurements on test cells having H/U ratios differing from the rest of the material by up to 1 1/2 units. Reactivity measurements thus made were within the reproducibility of the measurements made when the center test cell and the rest of the mixture were identical in composition.

Except as noted in Figure 5.4 the material in the test cell and the rest of the 21 in. cube was the same for

*FIGURE 5.2. Uranyl Nitrate Assembly
Positioned in the PCTR Cavity*

FIGURE 5.3. Uranyl Nitrate Containers and Foil Holders

FIGURE 5.4. Reactivity Response of
PCTR Unpoisoned Uranyl Nitrate

each of the 2.14 wt% enriched mixtures. To help establish an upper bound on the 2.14 wt% enrichment curve shown in Figure 5.4, measurements were made in the 7.24 H/U mixture with test cells having H/U atomic ratios of 8.08 and 9.25 in addition to the test cell hav-

ing a 7.24 H/U ratio. Since both the 8.08 and 9.25 H/U test cells are rich in hydrogen, relative to the under-moderated 7.24 H/U spectrum, the reactivity response of the PCTR to these two test cells would be expected to be greater than if each test cell

were in a more characteristic neutron spectrum. Thus, the reactivity curve, as a function of H/U, should lie somewhere below the observed values for these two test cells.

For the 2.26 wt% enrichment measurements, two different mixtures having H/U atomic ratios of 11.2 and 8.25 were used with test cells having H/U ratios as indicated in Figure 5.4.

Results and Interpretations

Based on the results shown in Figure 5.4, optimum neutron moderation for 2.14 wt% ^{235}U enriched uranyl nitrate occurs at an H/U atomic ratio of about 8.0 ± 1.0 , and for 2.26 wt% ^{235}U enriched uranyl nitrate at about 9.3 ± 0.5 . These values are in line with the optimum H/U of about 10.5 observed in the 3.04 wt% enriched uranyl nitrate experiments.⁽²⁾

As can be seen in Figure 5.4, the 2.14 wt% enriched material has an excess reactivity of only 1.2 cents above a k_{∞} of unity at optimum neutron moderation and thus is very near the minimum critical enrichment. A linear extrapolation of the maximum excess reactivity as a function of ^{235}U enrichment is shown in Figure 5.5 and yields a minimum critical enrichment of 2.104 ± 0.01 wt%. When treated in the same manner, calculations by Ridgway,⁽⁴⁾ using the GAMTEC-II Computer Code,⁽⁷⁾ were found to agree quite well with this value (Figure 5.4).

In addition to the reactivity response of the PCTR to each of the mixtures shown in Figure 5.4, reactivity measurements were also made on each material shown in Table 5.1 to deter-

mine the amount of neutron absorbing material, in the form of borated polyethylene, required to achieve a null reactivity between the test cell void and the test cell filled with the material. Measurements on each mixture were made with at least two different boron concentrations in addition to the zero boron case. From a least squares fit of these data, the amount of boron required for the null reactivity condition can be predicted, as shown in Figure 5.6 for a typical set of measurements. The B/ ^{235}U atomic ratio required to achieve a null reactivity condition for each material is shown in Table 5.1. Based on this data and the expression for k_{∞} of a homogeneous medium,⁽²⁾

$$k_{\infty} = \frac{\nu \Sigma_f}{\Sigma_a} \cdot \frac{\sum_i (\Sigma_a \nu)_i}{\sum_i (\Sigma_a \nu)_i} \cdot \frac{\epsilon p}{\epsilon' p'}$$

k_{∞} values were calculated and are shown in Table 5.1 for each material. The cross section data used in applying this equation to the experimental data were obtained using the GAMTEC-II Code.⁽⁷⁾ Values of k_{∞} are also shown in Table 5.1, reflecting calculated delta k corrections for small amounts of Cu, Zr, and Fe contaminants present in each mixture. Both GAMTEC-II and the transport theory code, DTF IV,⁽⁸⁾ were used to make the delta k corrections. These corrected values of k_{∞} are shown graphically in Figure 5.7 as a function of H/U. Values of k_{∞} obtained by Neeley⁽²⁾ for the 3.04 wt% enriched

FIGURE 5.5. Maximum Excess Reactivity of Uranyl Nitrate as a Function of ^{235}U Enrichment

FIGURE 5.6. Typical Null Reactivity Poison Determination

TABLE 5.1. PCTR Measured k_{∞} for Homogeneous Uranyl Nitrate

Enrichment, wt% ^{235}U	Neutron Moderation, H/U atomic ratio	Neutron Absorber for k_{∞} of Unity, B/ ^{235}U atomic ratio	Infinite Neutron Multiplication Factor	
			Experimental Mixtures	Corrected ^(b)
2.14	6.2 ^(a)	0.0	1.00 ^(a)	1.003
2.14 $\pm$ 0.005	6.36 $\pm$ 0.14	0.0025 $\pm$ 0.0012	1.0019 $\pm$ 0.0009	1.0051 $\pm$ 0.0009
2.14 $\pm$ 0.005	6.36 $\pm$ 0.14	0.0024 $\pm$ 0.0018	1.0017 $\pm$ 0.0013	1.0049 $\pm$ 0.0009
2.14 $\pm$ 0.005	7.17 $\pm$ 0.17	0.0060 $\pm$ 0.0012	1.0045 $\pm$ 0.0009	1.010 $\pm$ 0.0009
2.14 $\pm$ 0.005	8.46 $\pm$ 0.15	0.0133 $\pm$ 0.0025	1.0096 $\pm$ 0.0003	1.0129 $\pm$ 0.0003
2.14 $\pm$ 0.005	8.46 $\pm$ 0.15	0.0138 $\pm$ 0.0009	1.0098 $\pm$ 0.0003	1.0129 $\pm$ 0.0003
2.14 $\pm$ 0.005	10.40 $\pm$ 0.17	-0.0055 $\pm$ 0.0107	0.9957 $\pm$ 0.0086	1.0005 $\pm$ 0.0086
2.14 $\pm$ 0.005	10.40 $\pm$ 0.17	-0.0059 $\pm$ 0.011	0.9954 $\pm$ 0.0086	1.0002 $\pm$ 0.0086
2.14 $\pm$ 0.005	10.36 $\pm$ 0.10	-0.0060 $\pm$ 0.011	0.9953 $\pm$ 0.0086	1.0001 $\pm$ 0.0086
2.14	9.75 ^(a)	0.0	1.00 ^(a)	1.004
2.26 $\pm$ 0.005	8.25 $\pm$ 0.19	0.0479 $\pm$ 0.0004	1.0356 $\pm$ 0.0003	1.0393 $\pm$ 0.0003
2.26 $\pm$ 0.005	11.20 $\pm$ 0.19	0.0418 $\pm$ 0.0014	1.0294 $\pm$ 0.0003	1.0326 $\pm$ 0.0003

(a) Value taken from Figure 5.4

(b) Corrected for Fe, Cu, and Zr containments

FIGURE 5.7. k_{∞} for Low ^{235}U Enriched Uranyl Nitrate Systems

uranyl nitrate are also shown in Figure 5.7

The maximum k_{∞} for the 2.14 wt% enriched uranyl nitrate was determined to be 1.0130 ± 0.0003 and about 1.04 for the 2.26 wt% enrichment. Since it is not necessary to establish k_{∞} as a function of composition to determine the minimum critical enrichment of a system, the expense in defining k_{∞} for enrichments only 0.12 percentage points apart wasn't felt to be justified. Consequently, only two experimental determinations of k_{∞} were made for the 2.26 wt% enrichment. The dashed line shown in Figure 5.7 is an approximation.

Another parameter of some use in checking computational models and in criticality control is the amount of neutron absorbing material required to reduce k_{∞} of a given material to unity. This information is readily available from the poison measurements and is presented in Figure 5.8 as a function of the H/U atomic ratio of 2.14 wt% ^{235}U enriched homogeneous uranyl nitrate.

Conclusions

The minimum ^{235}U enrichment required for criticality in homogeneous uranyl

FIGURE 5.8. Boron Required to Poison a 2.14 wt% ^{235}U Enriched Homogeneous Uranyl Nitrate System to a k_{∞} of Unity

nitrate systems is approximately twice that required for UO_3 systems. Consequently, the existence of the nitrate radical in a uranium system is of considerable importance when establishing criticality safety guides. Although the experimental data presented in this paper establishes this minimum critical enrichment for homogeneous uranyl nitrate at 2.104 ± 0.010 wt%, a lower limit of 2.07 wt% should be used in setting criticality safety guidelines. This lower enrichment provides a 99% confidence limit desirable for criticality safety considerations.

Optimum neutron moderation for 2.14, 2.26, and 3.04 wt% enriched uranyl nitrate homogeneous systems occurs at H/U atomic ratios of 8.0 ± 1.0 , 9.3 ± 0.5 , and 10.5 respectively, with maximum k_∞ values of 1.013, 1.04, and 1.145. For the 2.14 wt% enrichment, k_∞ values greater than unit occur for H/U atomic ratios between 6 and 10 1/2. For the 2.26 wt%, this range increases to between 4 and 15, and for the 3.04 wt% enrichment to between 2 and 31.

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References

1. V. I. Neeley and H. E. Handler. Measurement of Multiplication Constant for Slightly Enriched Homogeneous UO_3 -Water Mixtures and Minimum Enrichment for Criticality, HW-70310. August 21, 1961.
2. V. I. Neeley, J. A. Berberet, and R. H. Masterson. k_∞ of Three Weight Per Cent UO_3 and $\text{UO}_2(\text{NO}_3)_2$ Hydrogenous Systems, HW-66882. September 1961.
3. K. R. Ridgway. "Calculated Nuclear Safety Parameters of Low Enriched Uranium," Transactions, American Nuclear Society, Vol. 9, No. 1, pp. 134-135. June 1966.
4. K. R. Ridgway. Calculated Critical Parameters of Low Enrichment UNH and UO_3 - H_2O Mixtures, ISO-174, February 1, 1966.
5. M. R. Schwab. Informal Criticality Safety Analysis Report for UND Project: Parts 1 and 2, BNWL-CC-1427 Pt. 1. Pacific Northwest Laboratory, Richland, Washington. June 23, 1967.
6. R. E. Heineman. "Experience in the Use of the Physical Constants Testing Reactor," Peaceful Uses of Atomic Energy, Vol. 12, p. 650. 1958.
7. L. L. Carter, C. R. Richey, and C. E. Hughey. GAMTEC-II: A Code for Generating Consistent Multi-group Constants Utilized in Diffusion and Transport Theory Calculations, BNWL-35. Pacific Northwest Laboratory, Richland, Washington. March 1965.
8. G. B. Carlson, et al. DTF Users Manual, United Nuclear Corporation, White Plains, New York. 1963.

6.0 NEUTRON PHYSICSHIGH-RESOLUTION MONOCHROMATIZATION OF
NEUTRONS BY MULTIPLE BRAGG REFLECTION

D. A. Kottwitz

Introduction

Several exploratory experimental measurements have been made relative to a new scheme⁽¹⁾ for producing semi-parallel beams of highly monochromatic slow neutrons at fixed wavelengths. In Reference 1 it is shown that at calculable crystal orientations certain "forbidden" Bragg reflections in diamond-structure crystals, e.g., germanium, and in hexagonal close-packed crystals, e.g., beryllium, may be simulated by multiple Bragg reflection (MBR) so that high resolution beams are obtained with relatively coarse collimation.

Such beams may have important uses in slow neutron research. However, in order to employ them in an optimum manner, it is necessary to study experimentally the various factors which influence beam intensity, resolution, and purity. Preliminary studies of this type employing the Hanford triple-axis crystal spectrometer⁽²⁾ have been done on beryllium and germanium crystals.

Experimental Method

The methods of double crystal spectrometry were used to study several MBR peaks. The crystal under study was mounted at the third axis and the auxiliary crystal at the first axis, or

vice versa; the second axis was vacant and the second arm in its zero position. The most significant collimation (0.09° Soller slits) was between the two crystals. Both "parallel" and "anti-parallel" arrangements of the crystals were used to obtain several types of arm and crystal "rocking curves."

Since multiple Bragg reflections are weak in comparison with ordinary single reflections, it was usually necessary to deal with substantial contamination from several orders of diffracted neutrons which accompany the desired MBR. Quantitative measurements of the various orders were made by means of combinations of resonance filters including gadolinium, cadmium, erbium, and iridium. A least-squares solution to the resulting over-determined set of linear equations was then obtained with computer program LEARN.⁽³⁾

Measurements on Beryllium

The initial experiments were done on a beryllium monochromator crystal having a mosaic of about 1/4°. MBR simulations of the forbidden Be(0003) and Be(0001) reflections were studied by means of much narrower Cu(200) and Ge(111) reflections. Unfortunately the beryllium crystal (mounted at the 1st axis when the experiments were started) was oriented in such a way that the high-resolution "operating points"⁽¹⁾ were not readily available. Nevertheless, because of a question⁽¹⁾ concerning the existence of a significant

amount of ordinary single Bragg reflection from Be(0001) which might swamp MBR's, several MBR's and their immediate neighborhoods were studied at orientations away from operating points. Intensities measured at these orientations are representative of the intensities at orientations corresponding to operating points since large differences in intensity are not expected for the different orientations.

As shown in Table 6.1, measurements were made on four different beryllium MBR peaks, two of which were composites unresolvable on account of the relatively large crystal mosaic. Column 4 contains the lower limits for the ratio of the MBR intensity to the intrinsic single Bragg intensity, and Column 5 has the ratio of the MBR intensity to the intensity of an allowed "reference" reflection in the same crystal, in this case, Be(0002). The precise numerical values are not particularly significant, since they are affected by factors such

as collimation and the quality of the auxiliary crystal. However, they do support two conclusions: (1) the intrinsic single Bragg intensity of the forbidden Be(0001) and Be(0003) reflections is a minor source of contamination for high-resolution monochromatization and, (2) the MBR peaks are roughly 1/50 to 1/10 as intense as ordinary Bragg peaks in this mosaic crystal.

Measurements on Germanium

Similar measurements were made on a relatively perfect germanium crystal (mosaic unknown, but probably less than 0.01°) at high-resolution operating points. In this case the limiting factor on resolution was the artificially induced mosaic of about 0.03° in the auxiliary germanium crystal. As shown in Table 1, measurements were made on two simulations of the forbidden Ge(200) reflection. On account of the

TABLE 6.1. *Intensity Results for Forbidden Reflections Simulated by Multiple Bragg Reflections*

<u>Simulated Reflection</u>	<u>Energy, eV</u>	<u>Secondary Reflection</u>	<u>$I_{\text{MBR}}/I_{\text{SBR}}$</u>	<u>$I_{\text{MBR}}/I_{\text{Ref}}$</u>	<u>Auxiliary Crystal Reflection</u>
Ge(200)	0.0412	533, 333	>67	1/5	Ge(111)
	0.0385	355, 155	>16	1/18	Ge(111)
Be(0003)	{0.117}	{4041 + 3 others}	>500	1/62	Cu(200)
	{0.115}	{5230 + 3 others}			
	{0.117}	{4041 + 3 others}	>380 $\pm$ 130	1/28	Ge(111)
	{0.115}	{5230 + 3 others}			
	0.108	5231 + 3 others	>260 $\pm$ 90	1/50	Ge(111)
Be(0001)	0.0464	3121 + 3 others	>480	1/17	Ge(111)
	{0.0395}	{3120 + 3 others}	>730	1/11	Ge(111)
	{0.0386}	{2021 + 3 others}			

relative perfection of the specimen germanium crystal, counting rates were much lower than for beryllium and statistical uncertainties higher. Thus the limits on relative intrinsic single Bragg intensity in Column 3 are higher, even though no positive evidence of intrinsic reflection was found. In Column 4 the MBR intensity is compared to the intensity of the allowed reflection Ge(400); one of the MBR peaks is surprisingly intense, amounting to 1/5 of the allowed Ge(400) reflection.

Direct measurements of the effective Bragg angle were also made for these two Ge(200) simulations by rotating the specimen crystal between the "parallel" and "antiparallel" reflecting positions. When corrected for a slight twisting motion in the spectrometer, the results agreed with the calculated values to within 0.01° .

Computer Programming

A second computer program, DBSKH1, has been written to locate the crystal

orientations suitable for high-resolution monochromatization by hexagonal close-packed crystals, such as beryllium, magnesium and zinc. It is a generalization of the simpler program DBSK3⁽¹⁾ for cubic crystals. Debugging the new program has not yet been completed.

References

1. D. A. Kottwitz. "High-Resolution Monochromatization of Neutrons and X-rays by Multiple Bragg Reflection," Acta Crystallographica. (In Press)
2. D. A. Kottwitz and B. R. Leonard, Jr. "The Scattering Law of Room Temperature Light Water," Inelastic Scattering of Neutrons in Solids and Liquids (International Atomic Energy Agency, Vienna, 1963), Vol. 1, pp. 359-471.
3. B. H. Duane. "Machine-Analysis Abstract--Maximum Likelihood Non-linear Vector Fields--Battelle Northwest Program LIKELY," Physics Research Quarterly Report, January, February, March, 1966, BNWL-284. Pacific Northwest Laboratory, Richland, Washington.

7.0 THEORETICAL PHYSICSRELATIVISTIC ION IN ELECTROMAGNETIC
WAVE (BATTELLE-NORTHWEST PROGRAM ORBIT)

B. H. Duane

Introduction

Battelle-Northwest program Orbit constructs and plots eight-dimensional phase-space orbits for a relativistic charged particle driven by an electromagnetic plane wave.

Equations of Motion

Consider the relativistic motion of a charged particle driven by an electromagnetic plane wave, as illustrated in Figure 7.1. In the four-dimensional space-time of special relativity⁽¹⁾ spanned by the line element

$$\begin{aligned} ds^2 &= dx^2 + dy^2 + dz^2 + (d\tau c)^2 \\ &= -c^2 d\tau^2, \end{aligned} \quad (1)$$

a charge q having rest energy mc^2 is at four-position $\underline{X} = (x \ y \ z \ i\tau c)$ with four-momentum $\underline{P} = (p_x p_y p_z \ iE/c)$ = $m \ d\underline{X}/d\tau$ at proper time τ . The electromagnetic potential-momentum field

$$\underline{A} \cos(\underline{K} \cdot \underline{X}) \quad (2)$$

has propagation vector $\underline{K} = (0 \ 0 \ k \ i\omega/c)$, amplitude $\underline{A} = (A \ 0 \ 0 \ 0)$, and phase $\underline{K} \cdot \underline{X} = kz - \omega\tau$. The propagation vector has null magnitude and is perpendicular to the wave amplitude.

$$\begin{aligned} \underline{K} \cdot \underline{K} &= 0 = k^2 - \omega^2/c^2, \text{ and} \\ \underline{K} \cdot \underline{A} &= 0. \end{aligned} \quad (3)$$

The magnetic force field [$B_y = kA \sin(\omega\tau - kz)$] and the electric force field [$E_x = \omega A \sin(\omega\tau - kz)$] appear respectively as the space-like ($\underline{B}$) and time-like ($\underline{E}/ic$) components of the potential-momentum curl,

$$(\underline{A} \underline{K} - \underline{K} \underline{A}) \sin(\underline{K} \cdot \underline{X}). \quad (4)$$

In this representation, Einstein's equations of motion take the form

$$\begin{aligned} d\underline{P}/d\tau &= (q/m)(\underline{A} \underline{K} - \underline{K} \underline{A}) \\ &\quad \cdot \underline{P} \sin(\underline{K} \cdot \underline{X}). \end{aligned} \quad (5)$$

First Integrals

Contracting this equation of motion upon the propagation vector $\underline{K}$ with the help of Equation (3) yields the phase integral

$$\begin{aligned} \underline{K} \cdot d\underline{P}/d\tau &= 0, \text{ or } \underline{K} \cdot \underline{P} = \underline{K} \cdot \underline{P}_0, \text{ or} \\ \underline{K} \cdot \underline{X} &= \underline{K} \cdot \underline{X}_0 + \underline{K} \cdot \underline{P}_0 \tau/m. \end{aligned} \quad (6)$$

With the help of this phase integral, the equations of motion (5) now integrate by inspection to the matrix-exponential solution

$$\begin{aligned} \underline{P} &= \{ \exp[(q/\underline{K} \cdot \underline{P}_0)(\underline{K} \underline{A} - \underline{A} \underline{K}) \\ &\quad \cos(\underline{K} \cdot \underline{X}) |_0^\tau] \} \cdot \underline{P}_0. \end{aligned} \quad (7)$$

$$\underline{X} = (x \ y \ z \ ict), \quad \underline{P} = (p_x \ p_y \ p_z \ iE/c), \quad \underline{A} = (A \ 0 \ 0 \ 0), \quad \underline{K} = (0 \ 0 \ k \ i\omega/c)$$

$$ds^2 = dx^2 + dy^2 + dz^2 + (dict)^2 = -c^2 d\tau^2$$

$$d\underline{P}/d\tau = (q/m)(\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot \underline{P} \ f'(\underline{K} \cdot \underline{X})$$

Matrix-Exponential Solution

$$\underline{K} \cdot \underline{X} = \underline{K} \cdot \underline{X}_0 + \underline{K} \cdot \underline{P}_0 \tau/m$$

$$\underline{P} = \{ \exp[(q/\underline{K} \cdot \underline{P}_0)(\underline{K} \underline{A} - \underline{A} \underline{K}) f(\underline{K} \cdot \underline{X})] \big|_0^\tau \} \cdot \underline{P}_0$$

$$\underline{X} = \underline{X}_0 + \int_0^\tau \underline{P}(\tau') d\tau'/m$$

FIGURE 7.1. Ion in Electromagnetic Wave

The matrix power series defining the matrix exponential function terminates here with its second-order term, since from Equation (3)

$$\begin{aligned} (\underline{K} \underline{A} - \underline{A} \underline{K})^2 &= (\underline{K} \underline{A} - \underline{A} \underline{K}) \\ &\quad \cdot (\underline{K} \underline{A} - \underline{A} \underline{K}) \\ &= \underline{K} \underline{A}^2 \underline{K}, \end{aligned}$$

so

$$\begin{aligned} (\underline{K} \underline{A} - \underline{A} \underline{K})^3 &= -(\underline{K} \underline{A}^2 \underline{K}) \cdot (\underline{K} \underline{A} - \underline{A} \underline{K}) \\ &= 0. \end{aligned} \quad (8)$$

The complete first integral, Equation (7), thus simplifies to its low order terms,

$$\begin{aligned} \underline{P} &= \underline{P}_0 + (q/\underline{K} \cdot \underline{P}_0) (\underline{K} \underline{A} - \underline{A} \underline{K}) \\ &\quad \cdot \underline{P}_0 \cos(\underline{K} \cdot \underline{X}) \Big|_0^\tau \\ &\quad - (1/2) [(qA/\underline{K} \cdot \underline{P}_0) \\ &\quad \cos(\underline{K} \cdot \underline{X}) \Big|_0^\tau]^2 \underline{K} \underline{K} \cdot \underline{P}_0. \end{aligned} \quad (9)$$

Translated back into three-space language and arranged as a numeric computation sequence, Equations (6) and (9) become

$$\begin{aligned} p_x/mc &= p_x^0/mc - (qA/mc) \\ &\quad \{ \cos[(E^0/mc^2 - p_z^0/mc) \omega \tau \\ &\quad + \omega t_0 - kz_0] \}_0^\tau, \end{aligned} \quad (10)$$

$$p_y/mc = p_y^0/mc, \quad (11)$$

$$\begin{aligned} p_z/mc &= p_z^0/mc + (1/2) (p_x/mc \\ &\quad - p_x^0/mc) (p_x/mc \\ &\quad + p_x^0/mc) / (E^0/mc^2 \\ &\quad - p_z^0/mc), \end{aligned} \quad (12)$$

$$\begin{aligned} E/mc^2 &= E^0/mc^2 + (p_z/mc \\ &\quad - p_z^0/mc). \end{aligned} \quad (13)$$

Program Orbit constructs the first integrals as a function of proper time τ by direct evaluation of Equations (10-14).

Second Integrals

Multiplying the momentum integral, Equation (9), by $d\tau/m$ and integrating with the help of the phase integral, Equation (6), now gives for the orbit equations,

$$\begin{aligned} \underline{X} &= \underline{X}_0 + \underline{P}_0 \tau/m + \{q/(\underline{K} \cdot \underline{P}_0)^2\} \\ &\quad (\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot \underline{P}_0 [\sin(\underline{K} \cdot \underline{X}) \\ &\quad - \underline{K} \cdot \underline{X} \cos(\underline{K} \cdot \underline{X}_0)]_0^\tau \\ &\quad - (1/2) (qA/\underline{K} \cdot \underline{P}_0)^2 \{ \underline{K} \cdot \underline{X} [1/2 \\ &\quad + \cos^2(\underline{K} \cdot \underline{X}_0)] \\ &\quad - 2 \sin(\underline{K} \cdot \underline{X}) \cos(\underline{K} \cdot \underline{X}_0) \\ &\quad + (1/4) \sin(2\underline{K} \cdot \underline{X}) \}_0^\tau \underline{K}. \end{aligned} \quad (14)$$

Translated back into three-space language and arranged as a numeric computation sequence, these orbit equations become

$$\omega_0 = (E^0/mc^2 - p_z^0/mc) \omega, \quad (15)$$

$$\begin{aligned} kx &= kx_0 + (p_x^0/mc) \omega \tau \\ &\quad + (qA/mc) (\omega/\omega_0) [\omega_0 \tau \cos(\omega t_0 \\ &\quad - kz_0) - \sin(\omega_0 \tau + \omega t_0 \\ &\quad - kz_0)]_0^\tau, \end{aligned} \quad (16)$$

$$ky = ky_0 + (p_y^0/mc) \omega \tau, \quad (17)$$

$$\begin{aligned} kz &= kz_0 + (p_z^0/mc) \omega \tau \\ &\quad + (qA/mc) (\omega/\omega_0)^2 (p_x^0/mc) \\ &\quad \{ \omega_0 \tau \cos(\omega t_0 - kz_0) \} \end{aligned}$$

$$\begin{aligned}
& - \sin(\omega_0 \tau + \omega t_0 - kz_0)]_0^\tau \\
& + (1/2) (qA/mc)^2 (\omega/\omega_0)^2 \{ \omega_0 \tau [1/2 \\
& + \cos^2(\omega t_0 - kz_0)] \\
& + (1/4) \sin 2(\omega_0 \tau + \omega t_0 - kz_0) \\
& - 2 \sin(\omega_0 \tau + \omega t_0 - kz_0) \\
& \cos(\omega t_0 - kz_0)]_0^\tau, \quad (18)
\end{aligned}$$

$$\omega t = \omega t_0 + \omega_0 \tau + kz - kz_0. \quad (19)$$

Program Orbit constructs the second integrals as a function of proper time τ by direct evaluation of Equations (15-19) (divided by 2π to simplify phase recognition).

Bremsstrahlung Integral

An accelerated charge radiates energy from its rest frame at the proper time rate⁽²⁾

$$dE_Y/d\tau = (\mu_0 c/6\pi) (qd\underline{P}/mcd\tau)^2, \quad (20)$$

where $\mu_0 c$ is the impedance of free space (377 ohms). Making use of Equations (5), (6), and (8) reduces this expression to

$$\begin{aligned}
dE_Y/mc^2 &= (\mu_0 q^2/6\pi m) (q/mc)^2 (\underline{P}/mc) \\
&\cdot (\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot (\underline{A} \underline{K} - \underline{K} \underline{A}) \\
&\cdot (\underline{P}/m) \sin^2(\underline{K} \cdot \underline{X}) d\tau, \\
&= (\mu_0 q^2/6\pi m) (qA/mc)^2 (\underline{P}/mc) \\
&\cdot \underline{K} \underline{K} \cdot (\underline{P}/m) \sin^2(\underline{K} \cdot \underline{X}) d\tau, \\
&= (\mu_0 q^2/6\pi m) (qA/mc)^2 \\
&(\underline{K} \cdot \underline{P}_0/mc) \sin^2(\underline{K} \cdot \underline{X}) \\
&d(\underline{K} \cdot \underline{X}). \quad (21)
\end{aligned}$$

The radiative energy E_Y thus integrates to

$$\begin{aligned}
E_Y/mc^2 &= (\mu_0 q^2/12\pi m) (qA/mc)^2 \\
&(\underline{K} \cdot \underline{P}_0/mc) [\underline{K} \cdot \underline{X} \\
&- (1/2) \sin(2 \underline{K} \cdot \underline{X})]_0^\tau. \quad (22)
\end{aligned}$$

Translated back into three space language and arranged for numeric evaluation as a function of proper time τ , this integrated "Beschleunigungsstrahlung" energy becomes

$$\begin{aligned}
E_Y/mc^2 &= (kr_0/3) (qA/mc)^2 (\omega_0/\omega) [\omega_0 \tau \\
&- (1/2) \sin 2(\omega_0 \tau \\
&+ \omega t_0 - kz_0)]_0^\tau, \quad (23)
\end{aligned}$$

where $r_0 = \mu_0 q^2/4\pi m$ is the "charge radius" defined classically by equating its rest energy mc^2 to its electrostatic energy $q^2/4\pi\epsilon_0 r_0$.

Pulsed Wave Trains

The preceding matrix-exponential analytic solution for the relativistic motion of an ion driven by an electromagnetic wave evolved as a recent spin-off of some 25 years of mathematical physics work at Hanford on a far more intricate problem having somewhat similar mathematical structure (nuclear transmutation theory⁽³⁾). Conversely, the domain of applicability of these matrix-exponential ion orbits appears to extend considerably beyond this first application, and certainly includes ions driven by pulsed plane-wave trains of quite arbitrary pulse shape. The legend of Figure 7.1 displays this pulsed wave solution

explicitly, and it can be expected to become available in program Orbit as the need arises.

Computation Logic

The source deck consists of about 250 cards formulated in the machine-independent dialectal intersection of Univac-1107 Fortran-IV-CSC and IBM-7090 Fortran-IV-13, and designed for both modular growth versatility and freedom from obsolescence.

Input consists of the electro-magnetic potential momentum qA/mc , the charge-radius phase $kr_0/2\pi$, the initial position-phase and time-phase of the particle ($kx_0/2\pi$, $ky_0/2\pi$, $kz_0/2\pi$, $\omega t_0/2\pi$), its initial momentum (p_x^0/mc , p_y^0/mc , p_z^0/mc), the proper-time output phase interval $\omega_0 \Delta\tau/2\pi$, the terminal proper-time phase $\omega_0 \tau/2\pi$, and the range and resolution of the graphical output.

Numeric rounding is used throughout the calculations to minimize accrual of truncation bias.

Subroutine Famdr (Floating add-multiply-divide round) performs rounding simply by adding twice the low order part of each arithmetic operation answer to its high order part prior to final truncation. Subroutine Spill writes a line of warning and diagnostic information in the output upon the occurrence of any spill or division by zero.

Subroutine Albert (Einstein) evaluates the momentum integrals, Equations (10), (11), (12), (13); the position integrals, Equations (16), (17), (18), (19); and the radiation integral, Equation (23), all as a function of proper time. This key subroutine has been quite fully optimized with emphasis

upon numeric reliability, unrestricted numeric range, and unrestricted problem size.

Plot Logic

All numeric output is written in duplicate on two separate drum (or disc) files. In a fully developed remote transmission environment, the first drum feeds decimal output to either a desk typewriter or line printer under real-time control of the using analyst. The second drum periodically feeds accrued binary output to the plotter logic under automatic time-lag control to provide cross-plot and plot-overlay versatility.

Subroutine Plot scans the plot drum for all relevant data, assembles eight-dimensional phase-space cross plots, calls upon subroutine Map to transform each data point to Cartesian coordinates via flexible curvilinear coordinate transformation logic, translates and scales the plots to fit the display domain, and feeds fully processed cross plots and plot overlays to the plotter interface. Subroutine Map is preset to an identity transformation but can be overloaded freely by the using analyst to produce polar plots, logarithmic scales, conformal mapping transformations, confocal quadric coordinate plots, or other exotic patterns.

Ion Orbits in Laser Beam

The machine plot in Figure 7.2 illustrates the graphical output versatility of program Orbit. This mosaic-superposition cross plot of the output of 48 successive change cases displays

FIGURE 7.2. Ion Orbits in Laser Beam
[Ions start from rest at (0,0) in random
phase with laser wave flying to right]

the predicted visual appearance of ion orbits in a laser beam. The laser beam is incident from the left of Figure 7.2 upon a cloud of ions centered at the origin (0, 0). The plot shows the statistical distribution of orbits arising from random initial phase conditions. Each penned line traces out an ion orbit. The point symbols cycle through 12 shapes on successive orbits to aid the eye in tracking an ion through its orbit.

The electromagnetic wave hurls the ions first transversely in the direction of the electric force field and then downstream along intricately cusped orbits dependent upon initial phase. Along each orbit the ion reaches maximum energy at the points of inflection and returns to rest at the cusps. The orbits tend generally to zigzag about the maximum energy orbits which appear in Figure 7.2 as the dense black lines of orbit coalescence. The two orbits along which maximum attainable energy are reached are tilted sinusoids which have no

cusps. The dense black lines of high-energy-orbit coalescence show up quite sharply here as the most prominent visible characteristic of the ion beam. The plot leads to the prediction that the ion beam will appear as twin high-energy prongs thrust out of the laser beam in the plane of its electric field and enveloped in some low energy fuzz.

References

1. P. G. Bergamnn. Introduction to the Theory of Relativity, Prentice-Hall, Inc., New York, N. Y. 1948.
2. A. Sommerfeld. Electrodynamics, Academic Press, Inc., New York, N. Y. 1952.
3. B. H. Duane. Time-Variant Isotopic Transmutation (GE-HL Program Alchemy), pp. 4-7, HW-80020, Physics Research Quarterly Report (October-November-December, 1963), General Electric Hanford Laboratories, Richland, Washington.
4. B. H. Duane. Relativistic Ion in Electromagnetic Wave, BNWL-452, Pacific Northwest Laboratory, Richland, Washington. March 17, 1967.

PUBLICATIONS

1. V. O. Uotinen. A "Poison-Free" Loading of UO_2 and Pu-Al in the PRCF with D_2O -Moderator, BNWL-372. Pacific Northwest Laboratory, Richland, Washington. May 1967.
2. O. K. Harling. Compilation of Doubly Differential Cross Sections and the Scattering Law for H_2O and D_2O at 299 °K and for H_2O at 268 °K, BNWL-436. Pacific Northwest Laboratory, Richland, Washington. June 1967.
3. R. A. Bennett, D. F. Newman, C. L. Bennett, A. D. Vaughn, L. C. Davenport, and D. E. Robertson. NPR Mark-II Fuel--PCTR Experiment, BNWL-431. Pacific Northwest Laboratory, Richland, Washington. April 1967.
4. R. E. Schenter. "Damping of Single-Particle Wavefunctions in the Alpha Particle Interior Due to Exchange Effects in Nonlocal Potential", Nuclear Physics, Vol. A94, p. 408. 1967.
5. R. E. Schenter. Effects of UO_2 Crystalline Binding on Doppler Coefficient Calculations for Fast Reactor Systems, BNWL-442. Pacific Northwest Laboratory, Richland, Washington. June 1967.
6. R. W. Hardie and W. W. Little, Jr. PERT-IV, A Two-Dimensional Perturbation Code in FORTRAN IV, BNWL-409. Pacific Northwest Laboratory, Richland, Washington. April 1967.
7. D. R. Marr and E. L. Kelley, Jr. The 1107 Manual for Physics Chain, A Reactor Analysis Code, BNWL-413. Pacific Northwest Laboratory, Richland, Washington. April 1967.

PAPERS ACCEPTED FOR PUBLICATION

1. L. E. Hansen and E. D. Clayton. Criticality of Plutonium Compounds in the Undermoderated Range, BNWL-SA-983. Pacific Northwest Laboratory, Richland, Washington. Accepted for publication in Nuclear Applications to appear in August issue, 1967.
2. D. A. Kottwitz. "High Resolution Monochromatization of Neutrons and X-rays by Multiple Bragg Reflection", in press for publication in the special Ewald Festschrift Issue of Acta Crystallographica.

PAPERS PRESENTED

1. R. C. Lloyd and E. D. Clayton. "Interacting Arrays of Containers with ^{233}U Solution", BNWL-SA-1078, presented at 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967.
2. L. E. Hansen, and E. D. Clayton. "Criticality of Plutonium Compounds in the Undermoderated Range, $H/Pu < 20$ ", BNWL-SA-983, presented at 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967.
3. C. L. Brown, L. E. Hansen, and H. Toffer. "Experimental Buckling Measurements with 2.1 wt% Enriched Uranium Tubes in Light Water", BNWL-SA-109, presented at 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967.
4. C. L. Brown. "Transportation Experience at BNWL Under Revised USAEC and ICC Regulations", BNWL-SA-1140, presented at 8th Annual Meeting of the Institute of Nuclear Materials Management, Washington, D. C., June 19-21, 1967.

5. H. S. Zwibel. "An Angular Synthesis Approximation Applied to Thin Critical Slabs," presented at the 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967.
6. J. K. Long, A. L. Hess, R. L. McVean, P. J. Persiani, A. J. Ulrich (ANL), Q. L. Baird, R. A. Bennett, S. L. Engstrom (PNL). "FFTF Critical Experiments, Control Rod Studies on ZPR-III," presented at 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967.
7. D. H. Thomsen. "Analysis of a UO_2 -19-Rod Cluster Experiment with the RBU Monte Carlo Code," presented at the 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967. Trans. Am. Nuc. Soc., Vol. 10, No. 1, pp. 238.
8. R. I. Smith, J. W. Kutcher, and J. H. Lauby. "Critical Experiments with the UO_2 -2 wt% PuO_2 Batch Core in the PRTR," presented at the 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967. Trans. Am. Nuc. Soc., Vol. 10, No. 1, pp. 185.
9. W. P. Stinson and C. M. Heeb. "Approach-to-Critical Experiments with Phoenix Fuel," presented at the 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967. Trans. Am. Nuc. Soc., Vol. 10, No. 1, pp. 186.
10. V. O. Uotinen and L. D. Williams. "Experiments and Calculations for H_2O Moderated Assemblies Containing UO_2 -2 wt% PuO_2 Fuel Rods," presented at the 13th Annual Meeting of ANS, San Diego, California, June 11-15, 1967. Trans. Am. Nuc. Soc., Vol. 10, No. 1, pp. 186.
11. L. C. Schmid, D. E. Christensen, B. H. Duane, R. C. Liikala and R. P. Matsen. "Experimental and Theoretical Methods Used for Burnup Analysis at Battelle-Northwest," presented at the International Conference on Physics Problems in Thermal Reactor Design, London, England, June 1967.

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