Below please find my **Final Technical Report for DE-FG02-05ER15697.**

Many thanks.

Best regards,

Art

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Seismic imaging and inversion beneath an unknown overburden: fundamental theory and realistic model development for testing and evaluation (DE-FG02-05ER15697)

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There are two categories of activities and objectives that were supported by this funding.

Below I describe the major contributions and progress that this funding supported and made possible.

Category one: Direct Seismic Imaging and Inversion at Depth without Subsurface Information

This funding allowed significant progress on locating the separate terms within the inverse scattering series (ISS) that accomplish: (1) image location and (2) image delineation and inversion. Thesis by Simon Shaw, Fang Liu, Kris Innanen, Jingfeng Zhang, Haiyan Zhang, Xu Li and Zhiqiang Wang contributed to one or both of these initiatives. Papers were published and a book published " Seismic Imaging and Inversion: application to linear theory" (2012), Cambridge University Press, and co-authored with Bob Stolt , all have acknowledged this DOE BES support as critical to that advancement and progress. The first field data examples of ISS direct depth imaging without a velocity was published last year in JSE and was chosen as a Recent Advances and Road Ahead presentation at the 2012 SEG International Convention and Conference in Las Vegas. All of these papers and presentations gratefully recognize and acknowledge this DOE BES support.

In the second category, of improving the modeling of data to allow accurate model data to provide confidence and be able to focus on the effectiveness of the processing of that data, three major contributions took place and are newsworthy. The first involves a method to perform the finite difference modeling required in RTM methods without the need for PML boundary conditions, the second is a method to input the source function analytically in finite difference modeling and thereby improving the control of wave modeling amplitude, and the third is a method to model P only waves in a heterogeneous anisotropic medium, with wave theory amplitude and phase fidelity and without modeling shear waves or converted waves. All three of these advances have led to papers and all acknowledge this DOE BES support for making this possible.

INVERSE SCATTERING SERIES DIRECT DEPTH IMAGING WITHOUT THE VELOCITY MODEL: FIRST FIELD DATA EXAMPLES

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ABSTRACT

Weglein, A.B., Liu, F., Li, X., Terenghi, P., Kragh, E., Mayhan, J.D., Wang, Z., Mispel, J., Amundsen, L., Liang, H., Tang, L. and Hsu, S.-Y., 2012. Inverse scattering series direct depth imaging without the velocity model: First field data examples. *Journal of Seismic Exploration*, 21: 1-28.

In Weglein et al. (2010) an update and status report were provided on the progress on the inverse scattering series (ISS) direct depth imaging without the velocity model. In that article, results on synthetics with sufficient realism indicated that field data tests were warranted. This paper documents those first field data tests. These first early tests are encouraging and indicate that ISS direct depth imaging on field data is possible. Each member of a set of three distinct data or algorithmic conditions and requirements are identified and shown to be necessary for inverse scattering direct depth imaging, without a velocity model, to be effective and to produce the accurate structural configuration of reflectors and interfaces in the subsurface. Taken together, that set represents both necessary and sufficient conditions. In addition, for ISS imaging, the CIG flatness condition is a necessary and sufficient indication that an accurate depth image has been reached. The latter property is in contrast to conventional velocity dependent imaging methods where common image gather (CIG) flatness is a necessary but not a sufficient condition that a correct depth image has been achieved. The next steps, and open issues, on the road between viable and providing relevant and differential added value to the seismic tool-box are described and discussed.

KEY WORDS: imaging, migration, inverse scattering series, field data, velocity, CIG flatness.

INTRODUCTION/BACKGROUND

For the purposes of this paper, an accurate depth image means a correct spatial location and configuration of interfaces/reflectors in the earth. All currently applied direct depth imaging methods and indirect imaging concepts firmly believe that depth and velocity are inextricably linked. That cornerstone of all current imaging means that any direct imaging method requires an accurate velocity model to produce an accurate image in depth.

DIRECT AND INDIRECT METHODS FOR IMAGING AND/OR INVERSION ARE NOT EQUIVALENT

It is essential to understand the significance of the term ‘direct’ in ‘direct depth imaging’. Given an accurate velocity model (with an appropriate imaging method that can accurately backpropagate in space, or time, down through the velocity model), all current leading-edge imaging methods (e.g., Kirchhoff, FK, Beam and RTM) are able to directly output the depth (the actual spatial configuration) of reflectors. Within the framework of RTM methods, which assume an accurate velocity model, a recent set of papers (Weglein et al., 2011b, and Weglein et al., 2011c), advanced RTM concepts and methods, to address fundamental issues and shortcomings within current RTM practice, including the removal of the need for Perfectly Matched Layers (PML) and issues associated with that approach.

Indirect imaging methods (e.g., examining move-out trajectories and seeking flat common image gathers (CIG), Common Focus Point (CFP), Common Reflection Surface (CRS) and ‘path integral’ approaches) seek to satisfy a property or condition that an image with an accurate velocity would satisfy. Those properties are necessary conditions, but not sufficient, and hence satisfying the indirect proxy for an adequate velocity model is not equivalent to knowing the velocity and direct depth imaging. Therefore, satisfying these indirect criteria is no guarantee, and can lead to the correct depth or to any one of a set of incorrect depths. The latter truth is rarely (if ever) spoken and even rarer to find mentioned or exemplified in print. Most importantly, these indirect approaches fervently believe that a direct depth imaging method would require and demand a velocity model, and that there is absolutely no way around it, and that depth and velocity are innately linked and coupled on a very basic and fundamental level. That thinking is clear, and 100% correct within the framework of current imaging concepts and methods. However, that conventional mainstream thinking is limited from another broader perspective, and is superseded by the new broader framework for imaging provided by the inverse scattering series (ISS). Amundsen et al. (2005, 2006, 2008) have developed direct inversion methods for 1D acoustic and elastic media. The ISS is the only

direct inversion for both a 1D and a multi-dimensional acoustic, elastic and anelastic earth.

In addition to being direct and applicable for a multi-dimensional earth, the ISS (Weglein et al., 2003) is further unique in allowing for all processing objectives (including multiple removal, depth imaging, target identification, and Q compensation) to be achieved directly and without subsurface information.

In the same ‘direct’ sense that current imaging methods can directly output the spatial configuration of reflectors with a velocity model, ISS imaging algorithms can directly output the correct spatial configuration without the velocity model. It is the only method with that potential and capability.

Within the tool-box of multiple removal methods, the ISS internal multiple method is the only algorithm that predicts the removal of all internal multiples directly, and without subsurface information. The latter claim of ISS multiple removal capability was once highly controversial, incredulous and widely considered impossible. However, now it’s not only understood to be possible, it has been documented and reported to have stand-alone capability in comparison with all other internal multiple methods (see, e.g., Luo et al., 2011). There is one set of unchanged ISS equations that at once communicates that all multiples can be removed, and all primaries can be accurately depth imaged - directly and without any subsurface information, including velocity. If one believes that it is impossible to directly depth image without the velocity using the ISS, then one is forced to also believe that it is impossible to remove internal multiples without subsurface information. They come from the exact same set of equations and logic.

The ISS subseries for direct depth imaging communicates that depth and velocity are not inextricably and fundamentally linked. The ISS provides a new superseding theory that views the current velocity-depth relationship and framework as a special limiting case, as quantum mechanics and relativity view classical physics as limiting and special cases, within a new comprehensive and broader platform and framework. The new broader ISS framework for imaging reduces to current imaging algorithms when the velocity model is adequate, a property that a superseding theory must satisfy, and most amazingly it determines automatically on its own for any particular data set, or portion of a data set (and a given velocity model and migration algorithm), whether the new framework is needed, or whether the current conventional imaging framework and a given velocity model is in any individual case accurate and will suffice. The new imaging framework determines if its services, that is, whether the terms beyond the first term (the first and linear term in ISS imaging corresponds to current linear conventional imaging) are needed and will be called upon, and if it determines a response in the affirmative, then and only then, will it activate the new ISS imaging framework terms and call them into action. That need or

no-need, yes or no ISS imaging decision is made unambiguously and automatically in the first term within the ISS imaging series after the conventional linear image, and a 'no-need' determination not only shuts down the first non-linear term but all subsequent terms in the imaging series at any specific well-located image in the linear conventional migration. How does it know if it's an adequately or an inadequately located image? The guess would be that some criteria is being used by ISS imaging to determine 'wellness' of the image, since we are so oriented to that 'indirect' criteria orientation and religion (e.g., CIG flatness or iterative updating with an objective function and search engine). There is no 'indirect' criteria being employed in any ISS application, rather the directness within the ISS is the driver, and it doesn't provide merely perturbation theory, where some initial estimate is perturbed and updated, but rather purposeful perturbation theory, with 'direct' and 'purposeful' being the key and coupled concepts and the central and essential point. The 'directness' in ISS is what makes each term have a purpose, and identifiable within specific tasks towards inversion, and those terms within the ISS imaging series determine first if their purpose is needed at some location within a conventional migrated image before they act. Direct inversion, provided in the form of a series (perturbation), inexorably leads to purposeful perturbation theory, where each and every term has a unique and specific purpose and role, that can in turn be associated with isolated tasks - within the overall goal of inversion. That's in contrast with indirect methods, e.g., typical iterative linear or other indirect updating and search engine schemes, the latter more often based on mathematics and optimization, on 'all or nothing' thinking, typically settling for the latter. In some circles indirect methods have even defined themselves as the only definition and meaning of inversion as 'datafitting'. The consequences of ignoring the distinction between direct and indirect methods are tremendously significant. From a direct inversion perspective, for determining changes in earth mechanical properties, the so-called 'full wave inversion' methods today are inverting the wrong and fundamentally inadequate P to P data, with wrong algorithms, and with a wrong earth model. Would we today ignore the insights, lessons and the direct solution offered by $x = [-b \pm \sqrt{b^2 - 4ac} / 2a]$ for the quadratic equation, $ax^2 + bx + c = 0$ in favor of minimizing and searching various norms of $\|ax^2 + bx + c\|$? The latter is precisely what we are too often pursuing in the field of 'inversion' for changes in Earth mechanical properties. The indirect methods seem based on lack of hope or awareness of direct methods, and depend on big expensive, fast computers - and therefore have the affectation and imprimatur of being modern, computational and 'scientific', but too frequently are merely old ideas dressed up in abstract, rigorous and obfuscating mathematical language (for details and implications see, e.g., Weglein et al., 2009).

All current leading edge migration methods, such as, Beam, Kirchhoff and RTM, are linear. The ISS direct depth imaging without the velocity algorithm is a non-linear relationship between data and the wavefield at depth.

ISS TASK SPECIFIC SUBSERIES FOR MULTIPLE REMOVAL, DEPTH IMAGING AND DIRECT NON-LINEAR AVO

Each and every term and portion of any term within the ISS is computed directly in terms of data. All tasks associated with inversion (e.g., multiple removal, depth imaging, nonlinear direct AVO, and Q compensation) are each contained within the series. Hence, these individual tasks are each achievable directly in terms of data, and without subsurface information. Every seismic processing objective is carried out as an isolated task subseries of the ISS, and operates without subsurface information, by involving distinct non-linear communication of the recorded seismic data. Only the ISS communicates that all seismic objectives can be achieved in basically the same way that free surface multiples are removed. The free surface and internal multiple removal subseries have not only been shown to be viable but have also demonstrated added value and stand alone capability for predicting the amplitude and phase of multiples (see, e.g., Luo et al., 2011; Matson et al., 1999; Weglein and Dragoset, 2005; Fu et al., 2010), in particular, demonstrated under complex marine and on-shore circumstances. In this paper, we examine for the first time the issue of ISS depth imaging viability on field data. All conventional direct depth imaging methods only require knowledge of the velocity model to determine the spatial locations of reflectors. Hence, the ISS direct depth imaging subseries project began by assuming that only the velocity was variable and unknown. While all processing tasks are realizable directly and without subsurface information, all tasks are not equally simple or easy for the inverse series to achieve. For example, for different task specific subseries: one term eliminates each order of free surface multiple, while one term attenuates each order of internal multiple, furthermore, a series can locate reflectors beneath a small difference between actual and reference velocity, and another more inclusive imaging series can deliver the same structure determination effectiveness for larger contrasts in velocity. To-date the evolution and status of ISS imaging algorithms involves greater inclusiveness of ISS imaging terms and types of terms, and capability, but is not close to all of the ISS terms and capability that reside within the ISS. The issue is when a certain degree or state of ISS imaging capture will provide a tool box contribution with differential added-value in comparison to current imaging methods that require a velocity model.

The initial ISS imaging series (Weglein et al., 2002, Shaw et al., 2004, Innanen, 2004) were labeled LOIS for leading order imaging series. That evolved into a higher order cascaded series in closed form developed by Fang Liu (2006) and labeled as HOIS. All of that early development assumed that the only variable and unknown subsurface property was acoustic P wave velocity.

Figs. 1-8 illustrate the ISS imaging results for an earth in which only velocity varies. The algorithms are described in Liu (2006); Liu et al. (2005);

Zhang et al. (2007). The higher order imaging series (HOIS) methods pioneered in Liu and Weglein (2009) developed for velocity only varying media, are multi-dimensional ISS imaging algorithms that address imaging challenges that also exist in a one dimensional subsurface. Fang Liu's HOIS method was extended to HOIS+LE by Wang and Weglein (2011) to incorporate the imaging challenges addressed by HOIS, and in addition to accommodate certain imaging challenges that exclusively arise in a multi-dimensional laterally varying subsurface. The latter higher order imaging series plus laterally exclusive algorithm uses the acronym HOIS+LE. The progression of that velocity only varying subsurface ISS direct depth imaging capability, without the velocity model (HOIS and HOIS+LE) capability is shown in Figs. 1-8, for the fault shadow zone and pre-salt examples. HOIS and HOIS+LE contain higher order imaging terms for large contrasts and duration of those differences, imaging terms extracted from within the ISS. As mentioned above, these algorithms represent increasing capability, but are not containing all orders or all imaging terms for either a one-dimensional or multi-dimensional subsurface. More capability will be included in future algorithms. However, the HOIS and HOIS+LE algorithms are direct, require no velocity model building and updating, are closed form and both are lightning fast. The cost to run the HOIS and HOIS+LE algorithms is roughly 30% more than the single water speed FK Stolt migration. The single water speed migration is the costliest part of the ISS algorithm. It's essentially free, and that's amazing. The imaging results in Figs. 1-8 would be impressive for a conventional method that spent much time and effort to find the velocity from the data with a velocity analysis and then to image through it, that would take

Fig. 1. The fault shadow zone model. Not to scale.

Fig. 2. The water speed pre-stack FK Stolt migration for the data from the fault shadow model (Liu and Weglein, 2009).

Fig. 3. Fault model - HOIS (Liu and Weglein, 2009).

Fig. 4. Fault model HOIS+LE (Wang and Weglein, 2011).

Fig. 5. Salt model.

Fig. 6. Salt model water speed migration.

Fig. 7. Salt model - HOIS (Liu, 2006).

Fig. 8. Salt model - HOIS+LE (Wang and Weglein, 2011).

orders of magnitude longer to approximate the velocity and to migrate and image. These examples and algorithms exemplify the tremendous potential, promise and power that reside within the ISS for direct depth imaging without a velocity model, although these two specific algorithms (HOIS and HOIS + LE) are merely the tip of the iceberg in terms of capturing ISS imaging terms and capability. They represent progress within an important front (assuming that only velocity varies in the subsurface) in the ISS depth imaging campaign.

Imaging methods that require the velocity use only the phase of the data to determine depth. In contrast, all ISS tasks achieve their goals without subsurface information by using both the amplitude and phase of the events in seismic data.

MODEL TYPE AND ISS DEPTH IMAGING

The distinct ISS free surface and internal multiple attenuation subseries are model-type independent (see Weglein et al., 2003). That means those algorithms are completely unchanged if the earth is assumed to be an acoustic, elastic or anelastic medium. The ISS direct depth imaging subseries project has not reached a stage of development where a model-type independent ISS depth imaging algorithm is available. Therefore, at this time, the issue of ISS depth imaging and the assumed earth model-type needs to be considered in developing, evaluating and applying these techniques.

When we assume an acoustic earth model and, furthermore, that mechanical property changes in the medium are only due to velocity changes (and not density changes), then each reflection corresponds to a change in velocity at the reflector. For the latter earth model, if in addition the ISS imaging algorithm assumes (as we assume in this paper) that the reference velocity everywhere is constant and equal to the wave speed above the first and shallowest reflector, then ISS imaging will require (and the ISS depth imaging automatically provides and arranges) that all reflections above every initially mislocated reflector to be included and involved in the ISS depth imaging algorithm's input and action to correctly place the initially misplaced reference velocity imaged reflector.

However, if the medium allows changes in both velocity and density (as can and often does occur in the real earth), then the situation for ISS depth imaging is considerably more complicated both in theory and practice. To understand what gives rise to this new complexity, in a world where we assume that velocity and density can both vary, we will consider a specific example in such a medium, where the velocity is actually constant, and throughout the volume is equal to the constant reference velocity. The reflections in this example are then only caused by density variations. Then the constant reference velocity migration will accurately locate each reflector in this actual velocity equals reference velocity case. There is no need for ISS imaging beyond the first ISS term corresponding to reference velocity migration. Therefore density only reflections do not enter ISS depth imaging algorithms. The latter statement assumes that the objective of depth imaging is simply to locate reflectors and nothing more. Hence, we conclude that for an earth where both density and velocity can vary, that all reflections shallower than a given mislocated reflector are no longer necessarily involved through ISS imaging in aiding a mislocated deeper reflector. If the shallower reflection corresponds to either a velocity change, or to a velocity and density change, then that reflection enters the assistance package to aid the deeper mislocated reflector, but if the shallower reflection corresponds to only a density change, it doesn't enter that ISS depth imaging aid package. That's one of several new issues for a velocity and density varying earth that doesn't exist in a world where velocity is the only parameter that can vary. Further, if an ISS depth imaging algorithm that was derived from the ISS for an earth model where velocity is the only variable that can change, and the data that is input into the algorithm comes from a model where velocity and density can both vary, then initially well-located reflectors can be moved to an incorrect location and erroneously located images will not be corrected. However, there is no need to despair, because the multi-parameter acoustic or elastic ISS imaging, from the moment the multi-parameter ISS is written down, are immediately aware of this new issue that it needs to address, and automatically removes density only reflections from the ISS imaging algorithms, without knowing or determining the velocity and density configuration in the earth. Hence, the consequence of using all of the information in seismic primary

events (amplitude and phase) in ISS depth imaging without the velocity model, ultimately results in the need for ISS depth imaging to preclude density only reflections. The angle dependence of the amplitude of events is used by ISS imaging to preclude density only reflections.

THE EXCLUSION OF DENSITY ONLY REFLECTIONS IS APPROPRIATE FOR ISS MIGRATION FOR STRUCTURE BUT WOULD BE INCLUDED IF MIGRATION-INVERSION IS THE ISS GOAL

The exclusion of density only reflections would be inappropriate if the isolated task was designed to provide both a depth image plus an angle dependent reflection coefficient at those depth images, the latter for the purposes of migration-inversion. The ISS depth imaging in an acoustic earth where the P-wave velocity V_p and density (and for an elastic earth with P-wave velocity V_p , shear wave velocity V_s and density) can all vary and all are initially (and remain, completely) unknown, was formulated in Weglein et al. (2008) to retain the strength of a velocity only earth with a single imaging output, as a generalized reflectivity, while the exclusion of density only reflections is extracted from the strength of the multi-parameter ISS machinery. The results were summarized in Weglein et al. (2010).

THE IMPACT OF DATA LIMITATIONS ON ISS SUBSERIES

Table 1 summarizes the dependence/sensitivity of different ISS subseries on seismic bandwidth. As the latter table indicates, there is an increased dependency as we progress from the ISS free surface multiple case (where the subseries works one frequency at a time, and has absolutely no concern about bandlimited data) to the depth imaging subseries where the absence of low frequency in the data can have a deleterious effect on the ability of the ISS to move from the original linear incorrect depth image to the correct depth.

Table 1. The degree to which each ISS task specific subseries depends on the temporal frequency content of the data.

Specific subseries	Dependence on temporal frequency content of the data
Free surface multiple	None
Internal multiple	Very mild
Depth imaging	Some

THE CONDITIONS/ISSUES/REQUIREMENTS THAT NEED TO BE ADDRESSED FOR ISS DEPTH IMAGING ALGORITHMS TO BE EFFECTIVE ON FIELD DATA AND TO PROVIDE ACCURATE SPATIAL CONFIGURATIONS OF REFLECTORS IN THE SUBSURFACE

There are several absolute conditions, requirements and issues that need to be satisfied or addressed, respectively - in order for the current approach to ISS direct depth imaging without a velocity model to provide an effective and accurate spatial configuration of reflectors and interfaces in the earth. Those issues are: (1) sensitivity to (and interest in) low frequency/low vertical wavenumbers in the data; (2) assuming the appropriate earth model type and number of spatial dimensions in deriving the ISS imaging algorithm; (3) within a given and appropriate earth model type, the inclusion of sufficient imaging terms from the inverse scattering series, to address all shortcomings and problems from using as the first step a single constant velocity migration. Among items the ISS must accommodate, and address are: (I) how different is the actual velocity from the reference value, and the extent, duration, width of the layer, or over what region is that difference occurring; (II) how many parameters are assumed to be unknown in the appropriate model-type - the larger the number of unknown parameters in the model type, the harder the series has to work (that is, more ISS imaging terms have to be included) for the same contrast and duration of differences, in comparison with how hard the imaging series has to work for a single parameter changing model with comparable difference values and duration. The CIG flatness criteria is a necessary and sufficient condition that these conditions (1) and (3) have been satisfied and with an appropriate model type (condition 2) that the direct ISS imaging has produced the correct depth.

In the examples below, we will isolate and separately examine each of these issues. The order that we will follow is: first assume that there is no low frequency issue, and no model type matching issue between the model used to generate the data and the model used for processing the data, that is, the model behind the ISS imaging algorithm. In that first example, we examine the consequence of including or not including sufficient ISS imaging terms, in the ISS imaging algorithm to match and address the contrast in properties between actual and reference, and the duration of those differences. Fig. 9 shows two different ISS imaging results for a layered model. The first is LOIS (Leading Order Imaging Series, see, e.g., Shaw et al., 2004 extended to the multi-parameter case Weglein et al., 2008) and the second is HOIS (Higher Order Imaging Series, see e.g., Liu et al., 2005 also extended to the multi-parameter case Weglein et al., 2008) where LOIS has fewer types of terms and imaging capability in comparison to HOIS. The fact that HOIS is adequate inclusion of imaging terms (and LOIS is not) for this particular example's contrast and duration of differences between reference and actual velocity, is indicated by HOIS predicting the correct depth and having a flat

Fig. 9. Left: input FK-migrated data in pseudodepth domain. Center: LOIS result. Right: HOIS result. These figures demonstrate that with more capture, i.e., inclusion of more imaging terms, HOIS imaged the reflectors to their correct depth location, whereas LOIS did not.

Fig. 10. (a) shows the acoustic model we are testing for evaluating the dependence of ISS on seismic bandwidth. (b) is the water speed FK Stolt migration, the red lines represent the true location of the reflectors.

CIG (Common Image Gather) where the latter only occurs when the correct depth is predicted. Hence, within a given model type and with adequate low frequency (see Figs. 12-14 for the assumed source signature spectrum, with the inclusion of significant low frequency), for ISS imaging to be effective requires adequate inclusion of imaging terms to match the contrasts and durations between actual and reference properties. The LOIS images have a move-out pattern indicating inadequate capture/inclusion of ISS imaging terms. However, the ISS imaging results with HOIS outputs common image gather flatness at the correct depth and indicates that the latter capture of imaging terms matches the contrasts and duration in the data. CIG flatness indicates adequate capture of ISS imaging terms. Hence, within a given model type and adequate low frequency content, the CIG moving and flatness output is a necessary and sufficient condition that this direct ISS depth imaging is working and the spatial configuration of the image is accurate.

All LOIS terms are also within HOIS, plus additional higher order imaging terms to address larger contrasts and duration than LOIS can accommodate. While HOIS is higher order than LOIS, it is not all orders, even for a 1D earth, nor does it accommodate imaging issues and challenges which exclusively exist in a laterally varying earth, for example, like diffractions (see, e.g., Wang and Weglein, 2011).

The second case: assume that the model-type between data generation and processing, that is, the model behind the ISS imaging algorithm, is a match, and that adequate capture is within the ISS imaging algorithm to address and accommodate the contrasts and duration. Now compare the results with adequate (see Fig. 11(a)) and decimated low frequency data, the latter with a sine squared taper (see Fig. 11(b)).

The results are shown in Fig. 11 where the former has adequate low frequency, and the latter has low frequency decimated. With low frequency tapered the result of ISS imaging is severely damaged, becoming equivalent to the original and erroneous water speed FK Stolt migration (see Fig. 10(b)). In Fig. 11(c) a source signature regularization has been applied which first removes the original wavelet and replaces it by a Gaussian. The source regularization of the low frequency tapered data allows the ISS depth imaging to become as effective as when the low frequency content was originally adequate. The ISS imaging results comparing adequate low frequency data, tapered low frequency data, and source signature regularized data are shown in Figs. 11(a), 11(b) and 11(c), respectively. The tapered data is a more severe and daunting test, than often occurs with field data, where some low frequency is present. The absolute shutdown of ISS imaging in the tapered data case, is a bit too severe a conclusion for field data, where some low and zero frequency in the data brings some small ISS effectiveness, where once again it is significantly enhanced by

Fig. 11. This figure illustrates the imaging result for a velocity varying only earth model. (a) shows ISS imaging with data which has low frequency information. (b) shows ISS imaging with band-limited data. (c) shows the imaging result with the regularization being applied. This ISS imaging bandwidth issue is documented in Shaw (2005).

Fig. 12. Spectrum of data with low frequency.

Fig. 13. Spectrum of data with diminished low frequency.

Fig. 14. Spectrum of regularized data.

the regularization. Hence, within a model type match, and adequate imaging term capture, the moved and flat CIG condition is a necessary and sufficient condition that the critical low frequency issue has been addressed. The high bar and demand on the source regularization in this synthetic example is a very positive and encouraging note and message.

Finally, we examine the case where a source signature regularization is needed and has been applied, and while the particular ISS imaging algorithm has adequate capture for a simpler earth model type, it is inadequate for a more

complicated earth model. In that case the data is generated using the more complicated earth model (more ‘complicated’ means more earth mechanical properties/parameters can vary) but the ISS imaging algorithm corresponds to the simpler model type. We will examine this for the case where the data is generated by a model where velocity and density both vary, but the ISS imaging algorithm assumes that only the velocity varies. That situation is illustrated in Figs. 15 and 16 where: (1) the model, (2) the water speed FK Stolt migration and (3) the velocity only varying ISS imaging algorithm result are shown. The flat CIG result indicates the ISS depth imaging result would produce the accurate depth if the data had come from a velocity only model, and that within the latter model, that effective source regularization and adequate and appropriate capture of ISS imaging terms had taken place. Hence, a flat CIG is a necessary and sufficient condition that source regularization and adequate imaging capability within a model type has been achieved. If the model type is appropriate for the data being imaged then depth will be produced. If the model

Model

$\rho_0 = 1.0 \text{ g / cm}^3$	$v = 1500 \text{ m / s}$	
<hr/>		$Z_1 = 50 \text{ m}$
$\rho_0 = 1.1 \text{ g / cm}^3$	$v = 1650 \text{ m / s}$	
<hr/>		$Z_2 = 80 \text{ m}$
$\rho_0 = 1.2 \text{ g / cm}^3$	$v = 1800 \text{ m / s}$	
<hr/>		$Z_3 = 120 \text{ m}$
$\rho_0 = 1.3 \text{ g / cm}^3$	$v = 2000 \text{ m / s}$	

Fig. 15. An acoustic model with both velocity and density variations.

type used in the imaging is not a match with the model of the data, then CIG flatness would indicate that a critical source regularization is effective and that either the depth is being output or depth in a parallel and less complicated world is being accurately predicted. Flat CIG indicates that the critical source signature regularization is working. If you did not adequately source signature regularize and capture imaging terms for a velocity only world, then images would neither move nor flatten from the water speed migration result.

Under the latter circumstances, and as long as the low vertical wave-number sensitivity is addressed then, if: (1) the data comes from an earth where both velocity and density actually vary, (2) the ISS imaging algorithm, that will be used, was derived assuming that only the velocity in the Earth varies, and (3) the ISS imaging algorithm's ISS capture of imaging terms is adequate to correctly locate reflectors in depth if the data had been derived from a velocity only earth model, then the ISS imaging will produce flat common image gathers at the correct depth for a velocity only varying earth but at the incorrect location for an earth model type where velocity and density are both variable. Hence, if you assume a less complete model than is represented by the actual earth, then the ISS imaging output for the less complete model with flat common image gathers indicates it has addressed: (1) the low vertical wave-number issue and (2) adequate capture to produce the correct depth if the

Fig. 16. (a) shows the input data generated from the geological model in Fig. 15. (b) shows ISS imaging results with velocity-only formulism. Red lines indicate the true depth of the reflectors.

lesser model type were adequate. That is behind the base-line ISS depth imaging testing of field data in this paper, and the conclusion that ISS direct depth imaging is viable. Figs. 15-16 demonstrate this idea, by having the actual model correspond to a velocity and density Earth while the ISS imaging assumes that only the velocity is varying. Note the ISS imaging flat CIG in Fig. 16(b).

There are many other issues that need to be taken into consideration in developing practical ISS depth imaging algorithms. Among these issues are: (1) within an appropriate earth model type, whether the necessary number and types of terms from the inverse series have been included to match the imaging challenge due to the difference between the actual and reference velocity, and the duration of that difference; and (2) whether the density only reflections have been excluded from the ISS depth imaging algorithm. All of these issues need to be addressed to have the ISS depth imaging algorithm produce an accurate depth section. When these requirements are met the ISS image moves until it stops, and when it stops it's there. The move-out becomes flat and the imaging series directly produces a flat common image gather (CIG) at the correct depth. In contrast to all current imaging methods where CIG flatness is a necessary but not a sufficient condition for depth imaging accuracy, the CIG flatness is a by-product of ISS imaging, and a necessary and sufficient indication that depth has been found. It's a direct depth finding machine, and when it stops it is done. With ISS imaging CIG flatness is an indication that a direct method is done, not an indirect proxy for velocity used to find the depth, where for the latter conventional use it is necessary but not sufficient for depth location. Within the current ISS imaging formulation, the overriding requirement and number one issue for field data application of ISS depth imaging is being able to address the sensitivity to missing low frequency components in the data (or more accurately, low vertical wave number). If that low frequency sensitivity is not addressed, then gathering or not gathering appropriate and necessary ISS imaging terms or excluding density only reflections will not matter, and will be of no practical consequence. Hence, addressing the bandwidth issue for ISS imaging is the number one priority, the make or break issue for field data application, viability and delivery of its promise of high impact differential added value. A regularization scheme has been developed in Liu et al. (2010) to directly address that low frequency challenge. A key purpose of this paper is to examine whether this regularization method will allow the ISS imaging algorithms to be effective and work on field data. Therefore, with this first field data examination, we relax all of the other requirements for ISS depth imaging and consider the field data as though it were generated by a velocity only varying earth. Within that parallel world where only velocity varies, the ISS depth imaging will need to address the band-limited nature of field data, and also will require having enough ISS imaging terms (within an acoustic velocity only varying subsurface assumption) to be effective for accurately locating reflectors.

The three requirements or conditions of: (1) source signature regularization, (2) adequate algorithmic ISS capture, and (3) appropriate earth model type between data generation and data processing, taken together represent a set of necessary and sufficient conditions that ISS imaging requires to be effective and working. A flat CIG is a necessary and sufficient condition that indicates ISS depth imaging effectiveness within an appropriate earth model type.

KRISTIN FIELD DATA ISS IMAGING

A similar approach is followed for a CMP gather selected from the Kristin data set (Fig. 17, Majdanski et al., 2010). Fig. 20(a) shows a water-speed migration of the data in Fig. 17, while Fig. 20(b) shows the ISS imaging result after regularization. Event 1 is the water bottom primary, event 2 is the sub-water bottom primary, event 3 is the internal multiple between event 1 and 2 and event 4 is the third primary. Event 4, the third primary has a non-flat move-out with a water speed migration. It turns out that event 1, the water-bottom primary, represents a density change but no velocity change. That was deduced by: (1) assuming that the shear velocity was probably close to zero at the water bottom, and hence an acoustic model would be adequate, (2) examining the angle dependence of the data at the water bottom, (3) the angle independence of that water bottom reflection indicates that density changed but not velocity, and (4) that $\alpha_1 - \beta_1$, is the difference between linear estimates of bulk modulus and density (and corresponds to the linear estimate to the change in acoustic velocity), and is zero when the reflection has no velocity change. In acoustics, if the linear estimate of the change in velocity is zero, then the actual change in velocity is zero. Velocity is the only acoustic parameter that exhibits that property. Fig. 21 shows the water bottom has no change in acoustic velocity.

Fig. 17. The CMP gather tested from Kristin data.

Fig. 18. Source signature regularization analysis. Amplitude spectrum of original wavelet (in red) and the target wavelet we wish to have (in green). We scale the spectrum of the target wavelet [$\exp(-\omega^2/a^2)$, where $a = 80\pi$] to be of the same magnitude as that of the original wavelet for easy comparison.

Fig. 19. Wavelet A(t), cable II (source at depth 7 m and receivers at depth 18 m).

Fig. 20. For the Kristin data test: (a) shows water speed migration. The red line indicates water speed migration image for event 4. (b) shows ISS imaging result. The red line shows ISS image for event 4.

Fig. 21. Kristin data ISS depth imaging result.

Hence, the layer below the water-bottom has the same acoustic velocity as water. Further, the first order internal multiple (event 3) in that first sub-water-bottom layer also has a water-speed move-out. Hence, events 1, 2, and 3 all have flat CIGs with a water-speed FK Stolt migration (Fig. 22). Event 4 has move-out due to a velocity change at the base of the first sub-water-bottom layer. With a regularized ISS depth imaging the result for the image of event 4 is a shifted and flat CIG output. Hence, the ISS depth imaging is working on the very shallow sub-sea-bottom portion of the Kristin data set

within the context of a velocity only varying earth. The shifted ISS image and flat CIG of event 4, the third primary, indicates that bandwidth issues have been addressed, and sufficient capture of ISS imaging terms are within the ISS imaging algorithm. If for this field data set and ISS depth imaging test, either one of these conditions (addressing bandwidth sensitivity and adequate inclusion of ISS imaging terms) were a remaining and outstanding issue, then event 4 would not have moved and produced a flat CIG. The success of this test is thus defined. The next steps are to apply the regularized ISS depth imaging to an acoustic variable velocity and density model for the very shallow and sub-water-bottom reflectors, and a P-wave velocity V_p , shear wave velocity V_s and density varying elastic earth model for the deeper reflectors, to preclude density only reflections, and for outputting actual depth. If the second reflector corresponds to only a velocity change then Fig. 20(b) represents the correct depth of the first, second and third reflectors, the latter corresponding to event 4, directly and without knowing, needing or determining the velocity change across the second reflector.

Fig. 22. This figure summarizes the results of the initial ISS depth imaging tests on the very shallow, near ocean bottom section of the Kristin data.

THE PLAN FORWARD

In the entire line of papers and theses within the history of the ISS imaging project, Weglein et al. (2002); Shaw et al. (2004); Liu (2006); Wang and Weglein (2011) there is an assumption (which for a simple two reflector/single layer example, and a chosen constant reference velocity equal

to the actual velocity above the shallower reflector) that the perturbation or difference between the actual and reference properties, and the linear estimate of that perturbation, are both ‘box-like’ in character. For this simple example, the linear estimate (using the reference velocity to migrate) is a box whose upper surface is at the correct depth and the lower is at the incorrect depth. Within this box-like formulation, defining imaging as locating the edges of the box (reflectors) and hence, structure, the ISS then has to correct the original box into a box whose upper and lower end are both at the correct depth. That change requires the ISS imaging to create a box that takes the mislocated original lower interface and adds a box to bring it to the correct depth. Box construction places a high burden on low vertical wave-numbers. In hindsight, that box-like formulation was not the most strategic, frugal, and well-considered way to correct mislocated images at depth. Constructing boxes when only the location of edges is of interest is paying too high a price for the sought after boundary location output. Staying closer to the data space and away from model space is the key. We plan to recast all ISS imaging methods as we move forward, in terms of moving mislocated data-like ‘spikes’, and that new formulation will provide a set of theoretical and practical benefits for ISS imaging efficacy that are being formulated and examined at this time. We are also examining reducing the burden of capturing adequate ISS imaging terms in an elastic earth, where the P-wave velocity, shear wave velocity and density are variable and unknown and remain unknown. We are exploring the possibility of reducing the number of independent parameters through either empirical relationships, e.g., using Garner-Garner or Greenburg-Castagna, or effective media (e.g., the optical potential). The combination of: (1) moving spikes (images) rather than boxes, and (2) reducing the elastic parameters to one effective parameter will expedite the promise and delivery of, e.g., Figs. 4 and 8 to field data.

The M-OSRP imaging research team (working in cooperation with our sponsors) is engaged in moving from the current news and report that demonstrates field data viability for ISS imaging to providing added value. The ultimate goal is to have ISS imaging match the efficacy that ISS free surface and internal multiple removal have provided for the removal of coherent noise, i.e., multiples (see, e.g., Weglein et al., 2011a), and to extend that capability for extracting information from signal (the collection of all primaries).

CONCLUSIONS

In this paper, we have shown that the ISS depth imaging algorithm can address the most serious practical limitation/challenge field data will place on ISS depth imaging: that is, limitations in seismic bandwidth. With this accomplished, the further steps to extend these tests to variable density and velocity acoustic and elastic media are achievable, and realizing that is within

the sphere of issues we can influence and make happen. The most significant difference and potential obstacle between synthetic data tests and field data for developing and delivering ISS depth imaging has been addressed.

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Stolt and Weglein
Seismic Imaging and Inversion

Seismic Imaging and Inversion

Application of Linear Inverse Theory

Robert H. Stolt and Arthur B. Weglein

Extracting information from seismic data requires knowledge of seismic wave propagation and reflection. The commonly used method involves solving linearly for a reflectivity at every point within the Earth. The resulting reflectivity, however, is not an intrinsic Earth property, and cannot easily be extended to nonlinear processes which might provide a deeper understanding and a more accurate image of the subsurface.

In this book, the authors follow an alternative approach, which invokes inverse scattering theory. By developing the theory of seismic imaging from basic principles, they relate the different models of seismic propagation, reflection, and imaging – thus providing links to reflectivity-based imaging on the one hand, and to nonlinear seismic inversion on the other. Full, three-dimensional algorithms are incorporated for scalar, acoustic, and elastic wave equations.

The comprehensive and physically complete linear imaging foundation developed in this volume presents new results at the leading edge of seismic processing for target location and identification. The book serves as a fundamental guide to seismic imaging principles and algorithms, and their foundation in inverse scattering theory, for today's seismic processing practitioners and researchers. It is a valuable resource for geoscientists wishing to understand the basic principles of seismic imaging, for scientific programmers with an interest in imaging algorithms, and for theoretical physicists and applied mathematicians seeking a deeper understanding of the subject. It will also be of interest to researchers in other related disciplines such as remote sensing, non-destructive evaluation, and medical imaging.

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