Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

RECEIVED
LAWRENCE
BERKELEY LABORATORY

DEC 1 1980

Submitted for publication

A CONTINUOUS HIGH POWER BEAM DUMP OF THE HOT-DOG-COOKER TYPE

Kiyoshi Yoshikawa

March 1980

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 6782.

LBL-10643 C.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

A CONTINUOUS HIGH POWER BEAM DUMP
OF THE HOT-DOG-COOKER TYPE*

Kiyoshi Yoshikawa⁺

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

March 1980

ABSTRACT

A beam dump with partially rotating water-cooled tube arrays is proposed and studied for the dump of continuous high power density unneutralized ion beams out of the neutral beam injectors. Analyses were made of both steady and transient heat transfer characteristics.

*Work supported by the Fusion Energy Division of the U.S. Department of Energy under Contract No. W-7405-ENG-48.

⁺Permanent address: Institute of Atomic Energy, Kyoto University, Uji, Kyoto 606, Japan.

1. INTRODUCTION

For continuous operation of a high power density beam dump, neither inertially pre-cooled plate beam dump nor a water-cooled one will work successfully. A beam dump, consisting of tube array with swirl flow inside (swirl tubes), was proposed and experiments were conducted in pulsed operation at ORNL with PLT NBI; successful results were obtained up to several kw/cm^2 depending on the pulse length.¹ For higher power density ion beams of 10 kw/cm^2 or more, however, this beam dump will no longer be applicable either in pulsed or continuous operation. This is because the tubes with swirl flows are fixed and, hence, the maximum power density which irradiates the normal point of the tube surface will limit the duration time of the tube due to the highest temperature evolution there. The numerical calculations for swirl tubes in the non-boiling regime² show the appreciable non-uniform temperature profiles in the azimuthal direction, which means that only the small portion of the inner surface of the tube, having a large temperature difference from the water temperature, can contribute to efficient heat transfer, leading to inefficient overall heat transfer performance. If, however, the tubes were allowed to rotate, the heat transfer capability would be enhanced because of the increased surface area for heat transfer, provided that the period of rotation is small enough compared with the characteristic time of heat diffusion. The concept of rotating tubes in the vacuum is always associated with problems of vacuum sealings and water sealings in the atmosphere in which they are to be used, and these are not trivial problems. Fortunately, in the RTNS (Rotating Target Neutron Source) at LLL, two kinds of sealing are provided successfully and prove to work well.³ Thus, the

problems concerning sealings are expected to be solved by modifying the concept RTNS actually has. In this study the possibility of heat transfer in the nucleate boiling regime is examined.

2. HEAT TRANSFER CHARACTERISTICS OF THE ROTATING TUBE WITH INFINITE ANGULAR VELOCITY

(2-1) Effects of Rotation of the Tube

If the swirl tube is rotated rapidly, i.e., the characteristic time of heat diffusion is longer than the period of rotation, the point fixed on the tube surface, P, in Fig. 1 (where the tubes are arrayed parallel to the z-axis and the beam direction coincides with the x-axis) will receive heat flux (Fig. 2) almost uniformly with respect to time. In this case heat transfer may be assumed to occur uniformly over the entire inner surface of the swirl tube.

Since the point P will have heat flux described by

$$q(z,t) = q_o(z) \sin(\omega t + \epsilon), \quad 2n\pi \leq \omega t + \epsilon \leq 2n\pi + \theta_o \quad (1)$$

where $q_o(z)$, ω , and ϵ are, respectively, the heat flux to the surface normal to the beam line, the angular velocity, and the initial phase of the point P, the total heat which the unit area, including the point P, will receive per unit time, i.e., average heat flux, is thus written as

$$\begin{aligned} \bar{q}(z) &= (\omega/2\pi) \int_{-\epsilon/\omega}^{(\pi-\epsilon)/\omega} q(z,t) dt = q_o(z) (1 + \sin\psi_o)/2\pi \\ &= q_o(z) (\ell \sin\phi_o)/(2\pi b) = q_o(z) \cdot g \end{aligned} \quad (2)$$

where ϕ_0 , ψ_0 , ℓ , and b are the tilting angle of the tube array, the shadowing angle, the pitch of the neighboring tubes, and the outer radius of the tube, respectively, with the relation

$$\sin\psi_0 = \frac{(\ell\sin\phi_0 - b)}{b}, \quad \ell \geq 2b. \quad (3)$$

In Fig. 3 the g factor is shown for tilting angles, ϕ_0 , where the maximum straight line corresponds to no shadowing case, i.e., $\ell\sin\phi_0/2b = 1$, and the minimal line to the case of $\ell=2b$, i.e., no space between neighboring tubes. It is seen from the figure that for $\phi_0 = 30^\circ$, for example, the average heat flux can be reduced as much as 16% of $q_0(z)$ when $\ell=2b$, and even without shadowing effects it can be reduced to 32% of $q_0(z)$, regardless of the tilting angles.

(2-2) Heat Transfer Characteristics and Pressure Drop in the Non-Boiling Regime

In the pipe receiving non-uniform heat flux along the axis, there may exist two types of flow along the pipe if some conditions are fulfilled, i.e., single-phase flow (non-boiling regime) and two-phase flow (nucleate boiling regime). In this case, it is most likely that until a certain distance from the entrance, heat transfer will be made in the non-boiling regime, and the heat transfer coefficient for the swirl tube in the non-boiling regime, h_s , is given experimentally by⁴

$$\frac{h_s}{h_a} = 2.26 y^{-0.248} \quad (4)$$

where y and h_a are, respectively, the tape twist ratio defined as the inner diameter divided by 180° twist length, and the heat transfer

coefficient for empty tubes without swirl flow expressed as

$$\frac{h_a(2a)}{K_c} = N_u = 0.023 R_e^{0.8} P_r^{0.4} \quad (5)$$

In Eq. (5), a and K_c are, respectively, the inner radius of the tube and the thermal conductivity of the cooling water, and P_r and R_e are, respectively, the Prandtl number and Reynolds number, defined by

$$P_r = \frac{c_p \mu}{K_c} \quad , \quad (6)$$

$$R_e = \frac{2G}{\pi a \mu} \quad , \quad (7)$$

where c_p and μ are the specific heat capacity of the water and the viscosity of the water expressed approximately over the wide range of the pressure as (T_b in $^{\circ}\text{C}$)

$$\mu = 1.2703 \times 10^{-3} \exp(-0.0156 T_b) \quad . \quad [\text{Pa.s}] \quad (8)$$

The friction factor for the swirl tube in the non-boiling regime, f_s , is obtained as⁴

$$\frac{f_s}{f_a} = 2.75 y^{-0.406} \quad (9)$$

where f_a is again the friction factor for the tube without swirl flow written as

$$f_a = 0.184 R_e^{-0.2} \left(\frac{\mu_b}{\mu_w} \right)^{-0.35 D_h / 2a} \quad , \quad (10)$$

In Eq. (10), μ_b and μ_w are the viscosity of the water at the bulk temperature

and at the inner wall, respectively, and D_h is a hydraulic diameter defined in terms of the thickness of the twisted ribbon, t_r ($\ll a$), by

$$D_h = \frac{4(\pi a^2 - 2at_r)}{2\pi a + 4a - 2t_r} \cong \frac{\pi}{\pi + 2} (2a) \quad . \quad (11)$$

(2-3) Heat Transfer Characteristics and Pressure Drop
in the Nucleate Boiling Regime

In the nucleate boiling regime, heat flux was given experimentally for the swirl tube with $y = 2.48$ to be¹

$$q_{N.B.} = 7.81 \times 10^2 (\Delta T_{sat})^{2.854} , \quad [W/m^2] \quad (12)$$

and as to burnout heat flux for the swirl flow, the following experimental expression was obtained⁵ (a in m)

$$q_{B.O.} = 6.637 \times 10^5 (G/\rho a^2) [4 + (\pi/y)^2]^{1/2} (L_h/2a)^{-1/3} , [W/m^2] \quad (13)$$

where L_h and ΔT_{sat} are the heating length over which the pipe is irradiated, and the difference between the inner wall temperature of the tube, T_w , and the saturated temperature of the water, T_{sat} , in $^{\circ}C$, and G and ρ are the mass flow rate of the water in kg/s and the density of the water in kg/m^3 , respectively.

With respect to the friction factor in this regime, no reliable expression seems to exist, and extrapolation of Eq. (9) will give a reasonable evaluation.

(2-4) The Minimal Heat Flux for the Nucleate Boiling

Since the expression of the minimal heat flux required for the onset of the nucleate boiling for the swirl tube is not available, we apply the following Bergles-Rohsenow⁶ expression, written as

$$q_i = 1.799 \times 10^{-3} p^{1.156} \left(\Delta T_{\text{sat}} \times \frac{9}{5} \right)^{3.0097/p^{0.234}}, \quad [\text{W/m}^2] \quad (14)$$

where p is the static pressure in Pa, and q_i is shown in Fig. 4 in the pressure range of up to 7×10^6 Pa (1015 psi) for various ΔT_{sat} .

From Eqs. (12) and (14), the condition under which the nucleate boiling occurs is written as

$$q_{\text{N.B.}} \geq q_i \quad \text{for} \quad y = 2.48 \quad . \quad (15)$$

Equation (15), where both terms are equal, gives the relationship between ΔT_{sat} and p , as shown in Fig. 5. The figure indicates that the nucleate boiling is unlikely to occur in the swirl tube under the conditions of our interest except during the excursion to burnout or at the tube exit where the static pressure is small enough to allow nucleate boiling. In the following, therefore, only single-phase (non-boiling) heat transfer is to be considered.

(2-5) Steady State Heat Transfer Characteristics

When the tube rotates very fast, heat transfer can be treated in the steady state, and only radial heat transfer may be assumed to occur if the temperature variation in the axial direction is not large. Since the heat flux has a profile along the water path, the bulk water temperature is a function of the distance z from the entrance of the tube.

The heat transfer equations for cooling water and the tube are

$$Gc_p \frac{dT_b(z)}{dz} = 2\pi b \bar{q}(z) \quad (16)$$

$$\frac{d}{dr} \left[\frac{r dT(r, z)}{dr} \right] = 0 \quad (17)$$

with boundary conditions of

$$z = 0 \quad ; \quad T_b = T_b(0) \quad (18)$$

$$r = a \quad ; \quad K_t \frac{dT}{dr} = h_s (T_w - T_b) \quad (19)$$

$$r = b \quad ; \quad K_t \frac{dT}{dr} = \bar{q}(z) \quad (20)$$

where $T_b(z)$, $T(r, z)$, $T_b(0)$, and K_t are the bulk temperature of the water, the temperature of the tube, the entrance bulk temperature of the water, and the thermal conductivity of the tube, respectively.

Equations (16) and (17) can be integrated to yield, respectively,

$$T_b(z) = \frac{2\pi b}{Gc_p} \cdot \int_0^z \bar{q}(z) dz + T_b(0) \quad (21)$$

$$T(r, z) = T_b(z) + \frac{b \bar{q}(z)}{ah_s} + \left[\frac{b \bar{q}(z)}{K_t} \right] \ln \left(\frac{r}{a} \right) \quad (22)$$

The heat which the tube receives in the distance z per unit time, $Q(z)$, and the static pressure in the tube at the distance z , $p(z)$, are written as

$$Q(z) = 2\pi b \int_0^z \bar{q}(z) dz \quad (23)$$

$$p(z) = p(-z_0) - \left[\frac{G^2}{4\pi^2 \rho a^5} \right] \int_{-z_0}^z f_s(R_e) dz \quad (24)$$

$$z_0 = \frac{L - L_h}{2} \quad (25)$$

The total heat per entire tube per unit time, Q_t , the pressure drop between the entrance and exit of the tube with the length of L , Δp , and pumping work per tube, W_p , are thus written as

$$Q_t = 2\pi b \int_0^{L_h} \bar{q}(z) dz \quad (26)$$

$$\Delta p = \left[\frac{G^2}{(4\pi^2 \rho a^5)} \right] \int_{-z_0}^L f_s(R_e) dz \quad (27)$$

$$W_p = \Delta p \frac{G}{\rho} \quad (28)$$

where L , $p(-z_0)$, ρ , and $f_s(R_e)$ are the total length of the tube, the pressure at the entrance, the density of the water, and the friction factor expressed by Eq. (9), respectively.

(2-6) Yielding Stress Condition for the Tube

From the stress point of view, the following condition is imposed on the entrance pressure in terms of the ultimate tensile strength, σ_t , for thin tubes as

$$p(-z_0) < \frac{2(b-a)\sigma_t}{a+b} . \quad (29)$$

(2-7) Ion Beam Profiles on the Beam Dump

In order to proceed with numerical calculations, the ion beam intensity profiles must be ascertained. The performance characteristics of several major neutral beam injectors of our interest are tabulated in Table 1. The profiles of the unneutralized ion beam intensity depend on various parameters, such as the distance from the deflection magnet, the type of focusing, and the beam divergence. We therefore assume Gaussian profiles to be expressed by

$$q_o(z) = q_{\max} \exp[-2n^2(z/L_h - \frac{1}{2})^2] , \quad (0 \leq z \leq L_h). \quad (30)$$

The number n in Eq. (31) determines the profile of the ion beam intensity along the pipe as is shown in Fig. 6, and the profile in the y direction is not considered here.

With Eqs. (2), (23) and (30), $Q(z)$ is expressed to be

$$\begin{aligned} Q(z) &= (L_h \ell \sin \phi_o q_{\max}) Q'_n(z) \\ &= (L_h \ell \sin \phi_o q_{\max}) \int_0^{z/L_h} \exp[-2n^2(t - 0.5)^2] dt , \end{aligned} \quad (31)$$

and thus the bulk temperature of the water, $T_b(z)$, is immediately written as

$$T_b(z) = T_b(0) + \frac{Q(z)}{Gc_p} = \frac{L_h \ell \sin \phi_o q_{\max}}{Gc_p} Q'_n(z) + T_b(0) . \quad (32)$$

In Fig. 7, $Q'_n(z)$ for $n = 3, 5$ and 10 is shown. It is to be noted that $Q'_n(L_h)$ with larger n can be well approximated by

$$Q'_n(L_h) = \frac{(\pi/2)^{\frac{1}{2}}}{n}, \quad n \geq 3 \quad (33)$$

TABLE 1. Performance characteristics of several neutral beam injectors.

	MFTF A	MFTF B	TFTR
Ion Source $\left\{ \begin{array}{l} E_b \text{ [keV]} \\ I_b \text{ [A]} \\ P_b \text{ [MW]} \\ F_{\infty}^+ \end{array} \right.$	80	150	120
	80	65	65
	6.4	9.75	7.8
	0.3	>0.5	0.5
Focusing	focused	focused	unfocused
Distance of peak from ion source [m]	3	?	monotonically decreasing
Beam cross section at 11m from ion source [cm ²]	60 × 120	?	≅ 60 × 120
Distribution	Gaussian	?	?
Maximum heat flux [kW/cm ²]	8	?	8
Average heat flux [W/cm ²]	270	?	540

(2-8) Numerical Results in Steady State

Since the heat flux always appears as $\bar{q}$, the numerical results for the same $\bar{q}(L_h/2)$, while other parameters are the same, gives the same performances. From Eqs. (2) and (30), $\bar{q}(L_h/2)$ is written as

$$\bar{q}(L_h/2) = q_{\max} \cdot g = q_{\max} \cdot \frac{\ell \sin \phi_0}{2\pi b} \leq \frac{q_{\max}}{\pi}, \quad (34)$$

therefore, the results for

$$q_{\max} = 10 \text{ kW/cm}^2, \quad \phi_0 = 30^\circ, \quad \ell = 3b,$$

$$\text{i.e., } \bar{q}(L_h/2) = 2.387 \text{ [kW/cm}^2\text{]},$$

for example, are immediately applicable to the case satisfying

$$q_{\max} \cdot \frac{\ell \sin \phi_0}{2\pi b} = 2.387 \text{ [kW/cm}^2\text{]}, \quad (35)$$

with other parameters being the same. In Fig. 8 the relationship described by Eq. (34) is shown for various tilting angles, ϕ_0 , and their maximums restrained by the ratio $\ell/2b$ are also shown in order that no beam can penetrate the beam dump.

Steady state temperature distributions as well as pressure and heat transfer coefficients along the axis are shown in Figs. 9, 10 and 11 for various n numbers for the conditions written in Table 2. In the calculations, ΔT_b are given approximately so that maximum surface temperature, $(T_s)_{\max}$, does not exceed 600°C . The limitation of this temperature needs to be examined further from the viewpoint of thermal and mechanical stress. The results indicate that the surface

TABLE 2: Temperature, pressure, and heat transfer coefficients along the swirl-tube axis in steady state for $n = 3$, 5, and 10.

Tube: Copper , $a = 0.35 \text{ cm}$, $b = 0.50 \text{ cm}$, $\ell = 1.50 \text{ cm}$ $L = 1 \text{ m}$, $L_h = 60 \text{ cm}$, $\bar{q}(L_h/2) = 2.387 \text{ kW/cm}^2$, $q_{\max} = 10 \text{ kW/cm}^2$ $p(-z_0) = 5 \times 10^5 \text{ Pa}$, $\phi_0 = 30^\circ$, $T_b(-z_0) = 20^\circ\text{C}$.			
n	3	5	10
$\Delta T_b \text{ [}^\circ\text{C]}$	80	60	30
$Q_t \text{ [kW/tube]}$	187.5	112.8	56.4
$G \text{ [kg/s]}$	0.558	0.448	0.448
$U \text{ [m/s]}$	14.50	11.63	11.63
$(T_s)_{\max} \text{ [}^\circ\text{C]}$	486.6	559.8	569.7
$(T_w)_{\max} \text{ [}^\circ\text{C]}$	371.5	444.7	454.7
$h_s \text{ [W/cm}^2\text{ deg]}$	9.78 ~ 15.06	8.21 ~ 10.64	8.21 ~ 8.64
$\Delta p \text{ [Pa]}$	4.13×10^5	3.04×10^5	3.39×10^5
$W_p \text{ [W]}$	230.3	135.9	151.6
Figure	Fig. 9	Fig. 10	Fig. 11

temperature can be maintained below 600°C without extreme pressure drop (thus pumping power) in steady states. For $n=10$, the axial temperature gradient is large enough that the assumption of no axial heat conduction through the tube no longer holds and the temperature will be over-estimated (Fig. 11). The above results show that the heat transfer occurs entirely in the single-phase heat transfer region.

3. TRANSIENT HEAT TRANSFER CHARACTERISTICS

In the following, transient heat transfer in the swirl tubes is discussed. When the thickness of the tube is small enough compared with the inner diameter, the shell model can be applicable to that case; otherwise, straightforward analysis is required.

(3-1) Transient Heat Transfer Characteristics of the Shell

As was assumed in Chapter 2, when the heat conduction in the direction of the axis can be ignored, the equation of heat transfer at the point P, in the case of $\epsilon=0$ for thin tubes, is expressed as (Fig. 12),

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}, \quad (36)$$

where κ is the thermal diffusivity of the tube and is assumed to be constant. The boundary conditions in the case of no shadowing, i.e., $\ell \sin \phi_0 = 2b$, are

$$t = 0 ; \quad T(r,0) = T_b(\text{const}), \quad (37)$$

$$\epsilon = 0, \quad (38)$$

$$x = 0 ; \quad -K_t \frac{\partial T}{\partial x} = q_0 \frac{(\sin \omega t + |\sin \omega t|)}{2} + q_1 \quad (39)$$

$$x = b-a = \delta \quad ; \quad -K_t \frac{\partial T}{\partial x} = h_s (T_w - T_b) \quad , \quad (40)$$

where q_1 , the constant heat flux, is included in order to examine the effects of constant heat flux on the transient heat transfer characteristics, and the bulk water temperature, T_b , is assumed to remain constant during the evolution of the temperature rise. This assumption will be reasonable since the temperature rise of the water will be restricted below 80°C at most as seen in Chapter 2, and this variation may be ignored compared with the temperature rise within the tube.

By applying Laplace transformation, the temperature can be expressed as

$$T(x,t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{T}(\lambda) \exp(\lambda t) d\lambda \quad , \quad (41)$$

where

$$\bar{T}(\lambda) = \frac{f_s(\lambda)}{K_t \Delta_s(\lambda)} \left\{ \frac{q_0}{\pi \lambda} + \frac{q_0 \omega}{2(\lambda^2 + \omega^2)} - \frac{2q_0}{\pi} \sum_{m=1}^{\infty} \frac{1}{4m^2 - 1} \cdot \frac{\lambda}{\lambda^2 + (2m\omega)^2} + \frac{q_1}{\lambda} \right\} + \frac{T_b}{\lambda} \quad . \quad (42)$$

Equation (42) indicates that when ω goes to infinity, the heat flux involving q_0 becomes equivalent to the constant heat flux of the magnitude of q_0/π in the case of $\ell \sin \phi_0 = 2b$. In Eq. (42), $f_s(\lambda)$ and $\Delta_s(\lambda)$ are both single-valued functions of λ expressed as

$$f_s(\lambda) = \cosh[(\delta-x)\mu] + \left[\frac{h_s}{K_t \mu} \right] \sinh[(\delta-x)\mu] \quad , \quad (43)$$

$$\Delta_s(\lambda) = \left(\frac{h_s}{K_t} \right) \cosh(\delta\mu) + \mu \sinh(\delta\mu) \quad , \quad (44)$$

$$\mu^2 = \frac{\lambda}{K} \quad (45)$$

The integral of Eq. (41) then yields

$$\begin{aligned} T(x', t) = & T_b + \left(\frac{q_0}{h_s} \right) \left(\frac{[1 + c(1-x')]}{\pi} + \left[\frac{F_s(x', \omega)}{2} \right] \sin(\omega t + \zeta_{1/2}) \right. \\ & - \left(\frac{2}{\pi} \right) \sum_{m=1}^{\infty} \left[\frac{F_s(x', 2m\omega)}{4m^2 - 1} \right] \cos(2m\omega t + \zeta_m) \\ & - \sum_{n=0}^{\infty} A_n \exp(-\kappa \alpha_n^2 t) \sin[(1-x') \delta \alpha_n + \gamma_n] \\ & \times \left\{ \left(\frac{2}{\pi} \right) - \frac{\Omega}{1 + \Omega^2} - \left(\frac{4}{\pi} \right) \sum_{m=1}^{\infty} [(4m^2 - 1)(1 + 4m^2 \Omega^2)]^{-1} \right\} \\ & \left(+ \frac{q_1}{h_s} \right) \left([1 + c(1-x')] - 2 \sum_{n=0}^{\infty} A_n \exp(-\kappa \alpha_n^2 t) \sin[(1-x') \delta \alpha_n + \gamma_n] \right), \end{aligned} \quad (46)$$

where

$$x' = \frac{x}{\delta}, \quad (47)$$

$$c = \frac{h_s \delta}{K_t}, \quad (48)$$

$$\Omega = \frac{\omega}{(\kappa \alpha_n^2)}, \quad (49)$$

$$F_s(x', 2m\omega) = \left(\frac{u_s^2(x', 2m\omega) + v_s^2(x', 2m\omega)}{r_s^2(2m\omega) + s_s^2(2m\omega)} \right)^{1/2}, \quad (50)$$

$$H_s(x', 2m\omega) = \frac{r_s(2m\omega)v_s(x', 2m\omega) - s_s(2m\omega)u_s(x', 2m\omega)}{r_s(2m\omega)u_s(x', 2m\omega) + s_s(2m\omega)v_s(x', 2m\omega)}, \quad (51)$$

$$u_s(x', \omega) = \cosh(X') \cos(X') + \left[\frac{c}{2p} \right] \{ \sinh(X') \cos(X') + \cosh(X') \sin(X') \}, \quad (52)$$

$$v_s(x', \omega) = \sinh(X') \sin(X') + \left[\frac{c}{2p} \right] \{ \cosh(X') \sin(X') - \sinh(X') \cos(X') \}, \quad (53)$$

$$r_s(\omega) = \cosh(p) \cos(p) + \left(\frac{p}{c} \right) \{ \sinh(p) \cos(p) - \cosh(p) \sin(p) \}, \quad (54)$$

$$s_s(\omega) = \sinh(p) \sin(p) + \left(\frac{p}{c} \right) \{ \sinh(p) \cos(p) + \cosh(p) \sin(p) \}, \quad (55)$$

$$p = \delta \left[\frac{\omega}{2\kappa} \right]^{\frac{1}{2}}, \quad (56)$$

$$X' = p(1 - x') \quad , \quad (57)$$

$$A_n = \left[\frac{1}{\sin(\delta\alpha_n + \xi_n)} \right] \left[\frac{c}{(\delta\alpha_n)^2} \right] \left\{ \frac{c^2 + (\delta\alpha_n)^2}{(1+c)^2 + (\delta\alpha_n)^2} \right\}^{\frac{1}{2}} \\ = \frac{\tan(\delta\alpha_n)}{\sin(\delta\alpha_n + \xi_n) [(\delta\alpha_n)^2 + \delta\alpha_n \sin(2\delta\alpha_n) + \cos^2(\delta\alpha_n)]^{\frac{1}{2}}}, \quad (58)$$

$$\tan(\gamma_n) = \frac{\delta\alpha_n}{c} \quad , \quad (59)$$

$$\tan(\zeta_n) = \frac{\delta\alpha_n}{(1+c)} \quad , \quad (60)$$

$$\tan(\zeta_m) = H(x', 2m\omega) \quad , \quad (61)$$

and α_n 's are the roots of the following equation written by

$$(\delta\alpha_n) \tan(\delta\alpha_n) = c \quad , \quad (n = 0, 1, 2, \dots). \quad (62)$$

It is found from Eq. (46) that the temperature $T(r, t) - T_b$ in the steady

state is proportional to the heat flux and inversely proportional to the heat transfer coefficients, and also the entire heat transfer performance characteristics are determined by the following parameters:

$$\frac{q_0}{h_s} , \quad \frac{q_1}{h_s} , \quad T_b , \quad \omega , \quad \frac{h_s \delta}{K_t} (=c) , \quad \delta \left(\frac{\omega}{2K} \right)^{\frac{1}{2}} (=p) , \quad \delta \alpha_n . \quad (63)$$

At the surface ($x=0$), $F_s(0,\omega)$, and $H_s(0,\omega)$ are reduced to

$$F_s(0,\omega) = \sqrt{2B} \left\{ \frac{2B^2 [\cosh(2p) - \cos(2p)] + 2B [\sinh(2p) + \sin(2p)] + \cosh(2p) + \cos(2p)}{2B^2 [\cosh(2p) + \cos(2p)] + 2B [\sinh(2p) - \sin(2p)] + \cosh(2p) - \cos(2p)} \right\}^{\frac{1}{2}} \quad (64)$$

$$H_s(0,\omega) = \frac{2B^2 [\sin(2p) - \sinh(2p)] + 2B [\cos(2p) - \cosh(2p)] - \sin(2p) - \sinh(2p)}{2B^2 [\sin(2p) + \sinh(2p)] + 2B [\cos(2p) + \cosh(2p)] - \sin(2p) + \sinh(2p)} \quad (65)$$

and at the inner wall ($x=\delta$), they are reduced to be

$$F_s(1,\omega) = 2B \left\{ 2B^2 [\cosh(2p) + \cos(2p)] + 2B [\sinh(2p) - \sin(2p)] + \cosh(2p) - \cos(2p) \right\}^{-\frac{1}{2}} \quad (66)$$

$$H_s(1,\omega) = - \frac{2B \sinh(p) \sin(p) + \sinh(p) \cos(p) + \cosh(p) \sin(p)}{2B \cosh(p) \cos(p) + \sinh(p) \cos(p) - \cosh(p) \sin(p)} , \quad (67)$$

where

$$B = \frac{c}{2p} . \quad (68)$$

(3-2) Transient Heat Transfer of the Tube

When the tube is not thin enough, the shell approximation is no longer applicable, and the equation of heat transfer is expressed as

$$\frac{\partial T}{\partial t} = \left(\frac{\kappa}{r}\right) \frac{\partial \left[r \left(\frac{\partial T}{\partial r} \right) \right]}{\partial r}, \quad (69)$$

with the same boundary conditions as Eqs. (37), (38), (39) and (40), where thermal conductivity, K_t , the density, ρ_t , and the specific heat capacity c_p , of the tube are assumed constant, thus, constant thermal diffusivity, κ .

Application of Laplace transformation to these equations then gives

$$\begin{aligned} T(r, t) &= T_b + \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{T}(\lambda) \exp(\lambda t) d\lambda \\ &= T_b + \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{f_t(\lambda) Q(\lambda, \omega) \exp(\lambda t) d\lambda}{[K_t \Delta_t(\lambda)]}, \end{aligned} \quad (70)$$

where $f_t(\lambda)$ and $\Delta_t(\lambda)$ are single-valued functions of λ expressed in terms of modified Bessel functions as

$$f_t(\lambda) = \left[\left(\frac{h_s}{K_t} \right) K_0(\mu a) + \mu K_1(\mu a) \right] I_0(\mu r) - \left[\left(\frac{h_s}{K_t} \right) I_0(\mu a) - \mu I_1(\mu a) \right] K_0(\mu r), \quad (71)$$

$$\Delta_t(\lambda) = \left[\left(\frac{h_s}{K_t} \right) K_0(\mu a) + \mu K_1(\mu a) \right] \mu I_1(\mu b) + \left[\left(\frac{h_s}{K_t} \right) I_0(\mu a) - \mu I_1(\mu a) \right] \mu K_1(\mu b), \quad (72)$$

and

$$Q(\lambda, \omega) = q_0 \left\{ \frac{1}{\pi \lambda} + \frac{\omega}{2(\lambda^2 + \omega^2)} - \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{1}{4m^2 - 1} \cdot \frac{\lambda}{\lambda^2 + (2m\omega)^2} \right\} + \frac{q_1}{\lambda}. \quad (73)$$

Since $\bar{T}(\lambda)$ is a single-valued function of λ and has no branch, integration of Eq. (70) can be carried out by calculating residues for $\lambda = 0, \pm i\omega, \pm i2m\omega$, and $\Delta(\lambda) = 0$ as

$$\begin{aligned}
 T(r, t) = & T_b + \frac{q_o}{h_s} \left(\frac{\left[\left(\frac{h_s b}{K_t} \right) \ln \left(\frac{r}{a} \right) + \frac{b}{a} \right]}{\pi} + \left[\frac{F_t(r, \omega)}{2} \right] \sin(\omega t + \eta_{1/2}) \right. \\
 & - \left(\frac{2}{\pi} \right) \sum_{m=1}^{\infty} \frac{1}{4m^2 - 1} F_t(r, 2m\omega) \cos(2m\omega t + \eta_m) - \sum_{n=0}^{\infty} B_n \exp(-\kappa \beta_n^2 t) G_n(r) \\
 & \times \left\{ \frac{1}{\pi} - \frac{\Omega_n/2}{1 + \Omega_n^2} - \left(\frac{2}{\pi} \right) \sum_{m=1}^{\infty} \left[(4m^2 - 1) (1 + 4m^2 \Omega_n^2) \right]^{-1} \right\} \Bigg) \\
 & + \left(\frac{q_1}{h_s} \right) \left(\left[\left(\frac{h_s b}{K_t} \right) \ln \left(\frac{r}{a} \right) + \frac{b}{a} \right] - \sum_{n=0}^{\infty} B_n \exp(-\kappa \beta_n^2 t) G_n(r) \right), \quad (74)
 \end{aligned}$$

where

$$F_t(r, 2m\omega) = \frac{1}{C} \left(\frac{u_t^2(r, 2m\omega) + v_t^2(r, 2m\omega)}{r_t^2(2m\omega) + s_t^2(2m\omega)} \right)^{\frac{1}{2}}, \quad (75)$$

$$\tan(\eta_m) = H_t(r, 2m\omega) = \frac{v_t(r, 2m\omega)r_t(2m\omega) - u_t(r, 2m\omega)s_t(2m\omega)}{u_t(r, 2m\omega)r_t(2m\omega) + v_t(r, 2m\omega)s_t(2m\omega)}, \quad (76)$$

$$\begin{aligned}
 u_t(r, \omega) = & \{ \ker(\sigma a) - C[\ker_1(\sigma a) + \operatorname{kei}_1(\sigma a)] \} \operatorname{ber}(\sigma r) \\
 & - \{ \operatorname{kei}(\sigma a) + C[\ker_1(\sigma a) - \operatorname{kei}_1(\sigma a)] \} \operatorname{bei}(\sigma r) \\
 & - \{ \operatorname{ber}(\sigma a) - C[\operatorname{ber}_1(\sigma a) + \operatorname{bei}_1(\sigma a)] \} \ker(\sigma r) \\
 & + \{ \operatorname{bei}(\sigma a) + C[\operatorname{ber}_1(\sigma a) - \operatorname{bei}_1(\sigma a)] \} \operatorname{kei}(\sigma r), \quad (77)
 \end{aligned}$$

$$\begin{aligned}
 v_t(r, \omega) = & \{ \ker(\sigma a) - C[\ker_1(\sigma a) + \text{kei}_1(\sigma a)] \} \text{bei}(\sigma r) \\
 & + \{ \text{kei}(\sigma a) + C[\ker_1(\sigma a) - \text{kei}_1(\sigma a)] \} \text{ber}(\sigma r) \\
 & - \{ \text{ber}(\sigma a) - C[\text{ber}_1(\sigma a) + \text{bei}_1(\sigma a)] \} \text{kei}(\sigma r) \\
 & - \{ \text{bei}(\sigma a) + C[\text{ber}_1(\sigma a) - \text{bei}_1(\sigma a)] \} \ker(\sigma r) \quad , \quad (78)
 \end{aligned}$$

$$\begin{aligned}
 r_t(\omega) = & \ker(\sigma a)[\text{ber}_1(\sigma b) + \text{bei}_1(\sigma b)] - \ker_1(\sigma b)[\text{ber}(\sigma a) + \text{bei}(\sigma a)] \\
 & + \text{kei}(\sigma a)[\text{ber}_1(\sigma b) - \text{bei}_1(\sigma b)] - \text{kei}_1(\sigma b)[\text{ber}(\sigma a) - \text{bei}(\sigma a)] \\
 & - 2C\{\ker_1(\sigma a)\text{bei}_1(\sigma b) - \ker_1(\sigma b)\text{bei}_1(\sigma a) \\
 & + \text{kei}_1(\sigma a)\text{ber}_1(\sigma b) - \text{kei}_1(\sigma b)\text{ber}_1(\sigma a)\} \quad , \quad (79)
 \end{aligned}$$

$$\begin{aligned}
 s_t(\omega) = & -\ker(\sigma a)[\text{ber}_1(\sigma b) - \text{bei}_1(\sigma b)] + \ker_1(\sigma b)[\text{ber}(\sigma a) - \text{bei}(\sigma a)] \\
 & + \text{kei}(\sigma a)[\text{ber}_1(\sigma b) + \text{bei}_1(\sigma b)] - \text{kei}_1(\sigma b)[\text{ber}(\sigma a) + \text{bei}(\sigma a)] \\
 & + 2C\{\ker_1(\sigma a)\text{ber}_1(\sigma b) - \ker_1(\sigma b)\text{ber}_1(\sigma a) \\
 & - \text{kei}_1(\sigma a)\text{bei}_1(\sigma b) + \text{kei}_1(\sigma b)\text{bei}_1(\sigma a)\} \quad , \quad (80)
 \end{aligned}$$

$$\sigma = \left[\frac{\omega}{\kappa} \right]^{\frac{1}{2}} \quad , \quad (81)$$

$$C = \frac{\sigma K_t}{\sqrt{2}h_s} = \frac{1}{2B} \quad , \quad (82)$$

$$d_n = \frac{h_s}{K_t \beta_n} \quad , \quad (83)$$

$$\Omega_n = \frac{\omega}{\kappa \beta_n^2} \quad , \quad (84)$$

$$B_n = \left(\pi d_n \right) \cdot \frac{J_1(b\beta_n) [J_1(a\beta_n) + d_n J_0(a\beta_n)]}{[J_1(a\beta_n) + d_n J_0(a\beta_n)]^2 - (1 + d_n^2) J_1^2(b\beta_n)} , \quad (85)$$

$$\begin{aligned} G_n(r) &= d_n [J_0(a\beta_n) Y_0(r\beta_n) - Y_0(a\beta_n) J_0(r\beta_n)] \\ &+ J_1(a\beta_n) Y_0(r\beta_n) - Y_1(a\beta_n) J_0(r\beta_n) . \end{aligned} \quad (86)$$

In the above equations, β_n 's are the roots of the following equation, written as

$$\begin{aligned} [d_n J_0(a\beta_n) + J_1(a\beta_n)] Y_1(b\beta_n) - [d_n Y_0(a\beta_n) + Y_1(a\beta_n)] J_1(b\beta_n) &= 0 \\ n &= 0, 1, 2, \dots , \end{aligned} \quad (87)$$

and $\text{ber}(x)$, $\text{bei}(x)$, $\text{ker}(x)$, and $\text{kei}(x)$ are Kelvin functions defined by

$$\begin{aligned} I_0(x \sqrt{\pm i}) &= \text{ber}(x) \pm i \text{bei}(x) , \\ I_1(x \sqrt{\pm i}) &= \mp i \text{ber}_1(x) + \text{bei}_1(x) , \\ K_0(x \sqrt{\pm i}) &= \text{ker}(x) \pm i \text{kei}(x) , \\ K_1(x \sqrt{\pm i}) &= \mp i \text{ker}_1(x) - \text{kei}_1(x) . \end{aligned} \quad (88)$$

(3-3) Numerical Results in Transient Behavior

Calculations were made for the copper tube with thickness ($\delta = 1$ mm) by using shell approximation for $h_s = 5$ and $10 \text{ W/cm}^2 \cdot \text{deg}$ in case of no-shadowing. In Figs. 13 and 14 the non-dimensional surface temperatures, Θ , defined by

$$\Theta = \frac{T(0,t) - T_b}{\frac{q_0}{h_s}} \quad (89)$$

are shown for various rotating speeds in case of $h_s = 5$ and $10 \text{ W/cm}^2 \cdot \text{deg}$, respectively. The calculations were interrupted when the transient part became less than 0.5% of the steady part. Since the time constant may be expressed as¹

$$\tau_{th} = \rho c_p \delta^2 \left(\frac{1}{K_t} + \frac{1}{h_s \delta} \right) \quad , \quad (90)$$

τ_{th} is calculated to be 95 and 53 ms, respectively, for $h_s = 5$ and $10 \text{ W/cm}^2 \cdot \text{deg}$. Those figures show that in approximately twice the τ_{th} , the surface temperature reaches 99.5% of the steady state value independent of the rotation speed. As was predicted, the oscillating magnitude of Θ is found to be smaller the greater the rotating speed of the tube. It is to be noted that the amplitude of the actual surface temperature for $h_s = 5 \text{ W/cm}^2 \cdot \text{deg}$ is twice as large as that for $h_s = 10 \text{ W/cm}^2 \cdot \text{deg}$ for a fixed heat flux (see Eq.(89)). Steady-state surface temperature evolution is shown in Figs. 15 and 16 for $h_s = 5$ and $10 \text{ W/cm}^2 \cdot \text{deg}$, respectively, and the maximum and minimum temperatures are shown in Fig. 17 as a function of rotating speeds.

With increasing rotation speed of the tube, the amplitude of the oscillating temperature decreases, and for infinite ω , Θ is reduced, from Eq. (22), to be

$$\Theta(\omega \rightarrow \infty) = \lim_{\omega \rightarrow \infty} \frac{T(b, L_h/2) - T_b}{q_0/h_s} = \frac{1}{\pi} \left(1 + \frac{h_s \delta}{K_t} \right). \quad (91)$$

From the practical point of view, the rotation speed will require more than 1000 rpm for efficient suppression of the surface temperature increase.

IV. CONCLUSION

It is theoretically shown that the effective heat flux to the rotating tube is reduced by a factor of less than π when the rotating speed goes to infinity, and in this case the surface temperature of the tube can be suppressed below 600°C without extreme pressure drop or mass velocity even for 10 kW/cm² peak heat flux in Gaussian distribution.

In the practical sense, the rotating speed will require more than 1000 rpm for the efficient suppression of the surface temperature rise, and it is shown that about twice the time constant of the shell is necessary for the system to attain the steady state.

Acknowledgments

The author thanks R.V. Pyle for offering the opportunity for this study at Lawrence Berkeley Laboratory, and other members of the Lawrence Berkeley Laboratory Neutral-Beam group, especially W.S. Cooper, J.A. Paterson, and J.M. Haughian, for their useful discussions and support of this study. Discussions with R.W. Moir, R.R. Stone, and C.M. Logan, Lawrence Livermore Laboratory, and M.A. Hoffman, University of California, Davis, were also much appreciated. Finally, the author wishes to express his gratitude to J. Kim, Oak Ridge National Laboratory, for his valuable discussions and advice.

REFERENCES

1. J.Kim, R.C.Davis, W.R.Gambill and H.H.Haselton, "A heat transfer study of water-cooled swirl tubes for neutral beam targets," in Proceedings of 7th Symposium on Engineering Problems in Fusion Research, Knoxville (1977).
2. J.Kim, R.C.Davis, H.H.Haselton and C.C.Queen, "Design considerations of calorimeter-target for high power neutral beam injectors," in Proceedings of 6th Symposium on Engineering Problems in Fusion Research, San Diego (1975).
3. R.Booth and C.M.Logan, "High speed rotating vacuum seal," Nucl. Instr. & Methods 145, 471 (1977).
4. R.F.Lopina and A.E.Bergles, "Heat transfer and pressure drop in tape-generated swirl flow of single-phase water," Trans. of ASME, Series C, J. Heat Transfer 91, 434 (1967).
5. W.R.Gambill, R.D.Bundy and R.W.Wansbrough, "Heat transfer, burnout and pressure drop for water in swirl flow through tubes with internal twisted tapes," Chem. Eng. Prog. Symp. Ser. 57, No. 32, 127 (1961).
6. A.E.Bergles and W.M.Rohsenow, Trans. AMSE, Series C, 86, No. 3, 365 (1964-68).

Figure Captions

- Fig. 1 Schematic of a beam dump of hot-dog-cooker type in the central region.
- Fig. 2 Heat flux evolution at the fixed point on the rotating tube under no-shadowing condition.
- Fig. 3 g (reduction) factor vs. tilting angle Φ_0 under full and no-shadowing conditions.
- Fig. 4 Minimal heat flux for the onset of nucleate boiling in the tube.
- Fig. 5 Conditions for nucleate boiling.
- Fig. 6 Assumed Gaussian profiles of the beam.
- Fig. 7 Beam profile factor, $Q'_n(z)$.
- Fig. 8 Effective heat flux on the tube vs. $q_{\max}(\ell/2b)$ for various tilting angles Φ_0 .
- Fig. 9 Distributions of temperature, pressure, and heat transfer coefficient along the swirl tube in steady state for $n = 3$.
- Fig. 10 Distributions of temperature, pressure, and heat transfer coefficient along the swirl tube in steady state for $n = 5$.
- Fig. 11 Distributions of temperature, pressure, and heat transfer coefficient along the swirl tube in steady state for $n = 10$.
- Fig. 12 Geometry of slab plate.
- Fig. 13 Temperature evolution of the slab surface for $h_g = 5 \text{ W/cm}^2 \text{ deg}$.
The temperature was drawn until 99.5% of the steady state value.
- Fig. 14 Temperature evolution of the slab surface for $h_g = 10 \text{ W/cm}^2 \text{ deg}$.

Fig. 15 Steady state temperature evolution of the slab surface for

$$h_s = 5 \text{ W/cm}^2\text{deg.}$$

Fig. 16 Steady state temperature evolution of the slab surface for

$$h_s = 10 \text{ W/cm}^2\text{deg.}$$

Fig. 17 Maximum and minimum Θ vs. rotation speed for $h_s = 5$ and

$$10 \text{ W/cm}^2\text{deg.}$$

XBL 803-8810

Fig. 1

Fig. 2

XBL 803-8813

XBL 803-8814

Fig. 3

XBL 803-8815

Fig. 4

XBL 803-8816

Fig. 5

XBL 803-8817

Fig. 6

XBL 803-8818

Fig. 7

XBL 803-8819

Fig. 8

XBL 803-8820

Fig. 9

XBL 803-8821

Fig. 10

XBL 803-8822

Fig. 11

XBL 803-8823

Fig. 12

XBL 803-8824

Fig. 13

XBL 803-8825

Fig. 14

XBL 803-8826

Fig. 15

XBL 803-8827

Fig. 16

XBL 803-8828

Fig. 17