STATISTICAL ANALYSIS AND MODELING
OF GAP CONDUCTANCE DATA
FOR REACTOR FUEL RODS
CONTAINING UO_2 PELLETS

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May 1974

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INTRODUCTION

The heat conductance across the fuel-cladding gap in reactor fuel rods is an important parameter because it can greatly influence the stored energy in the fuel and thus affect the outcome of accident analyses. The center-line temperature of a rod operating at a given power can vary several hundred degrees centigrade depending on the value of the gap conductance. Gap conductance, h_{GAP} , is a function of the operating ("hot") gap size, the gas composition in the gap, the temperature in the gap and on its boundaries, and the extent and pressure of fuel-cladding interaction. All of the foregoing are in turn dependent on temperature and dependent on the fabrication parameters and operating history of the rod. The estimation of h_{GAP} is a formidable task because of the variability of these parameters. In this study, h_{GAP} values are inferred by matching a measured temperature in the fuel (at a given power and coolant temperature) with assumed thermal conductivity of the fuel and clad. The required data are clear from the defining equation for h_{GAP} :

$$h_{\text{GAP}} = \frac{\text{heat flux across gap}}{\Delta T \text{ across gap}} = \frac{\text{Linear power}}{\pi \times \text{fuel OD} \times \left[\left(\begin{smallmatrix} \text{fuel surf.} \\ \text{temp} \end{smallmatrix} \right) - \left(\begin{smallmatrix} \text{clad ID} \\ \text{temp} \end{smallmatrix} \right) \right]}$$

$$= \frac{q}{\pi a (T_{\text{FS}} - T_{\text{CI}})} \quad (\text{See Fig. 1, which indicates a typical situation})$$

Local linear power, q , is usually given or can be estimated from measurements made during normal operation. The same is true for the cladding

FIGURE 1. Idealized Radial Temperature Profile

inner temperature, T_{CI} . The fuel surface temperature, T_{FS} , can be estimated from the local power and any measurement of the temperature in the fuel, as described in Part I.

The purpose of this study was threefold:

To examine a large body of in-reactor data for UO_2 pellet fuel where internal temperatures have been measured, and to estimate T_{FS} , T_{CI} , and thus h_{GAP} under a consistent set of assumptions. Part I of this report details the methods and assumptions used.

To estimate the variance (due to experimental uncertainties) in the inferred h_{GAP} values. (The uncertainties and the resulting variance are discussed in Part II.)

To attempt a correlation the h_{GAP} values with design parameters and operating conditions (e.g., with cold diametral gap and local linear power). The various correlations we developed are discussed in Part III.

SUMMARY AND CONCLUSIONS

Approximately 430 data points obtained from 16 sources were used to estimate fuel rod gap conductances with a consistent set of assumptions. The assumptions and methods used are set forth in explicit detail to facilitate repetition of the gap conductance calculations under a different set of assumptions.

The experimental error in parameters used to infer gap conductance was estimated from reported variances and reasonable uncertainties based on a number of irradiation experiments. A random number generator was used to select random samples from the parameters, which were assumed to have normal distribution. Three parameters dominate the estimation of gap conductance: (1) linear power; (2) internal fuel temperature; and (3) UO_2 thermal conductivity. Reasonable uncertainties chosen for the input parameters led to quite high uncertainties in h_{GAP} values; this is confirmed when one attempts to fit the data to simple models.

We developed a correlation to provide a means of estimating gap conductance for helium-filled gaps. This was done in two stages. The first stage consisted of fitting "open" gap data to three proposed correlations with resulting correlation coefficients of 0.77 to 0.83 and a residual standard deviation of $0.12 \text{ W/cm}^2\text{-}^\circ\text{C}$. The second stage involved fitting the residual between the open gap model and data points from gaps approaching closure. The correlation coefficient was 0.737 but the residual standard deviation increased to $0.25 \text{ W/cm}^2\text{-}^\circ\text{C}$.

The correlations are combined into one model that will provide an estimate of gap conductance as a function of cladding ID temperature, linear heat rating, and gap-to-pellet diameter ratio. The model does provide a best estimate and upper bound for gap conductance, but should not be used beyond the stipulated range of conditions nor for operating times other than beginning of life. A sample curve from the model is presented on page 17.

PART I - THE METHOD AND ASSUMPTIONS USED TO INFER h_{GAP} VALUES

I.A - DESCRIPTION OF THE METHOD

As shown in Appendix C, an integral equation can be developed which will yield T_{FS} from a knowledge of the local linear power and a measured internal temperature at some plane in the fuel. This equation has the familiar form:

$$\int_{T_{\text{FS}}}^{T_1} K(T,D) dT = \int_{r_1}^a \frac{1}{r} \int_0^r Q(r') r' dr' dr = \frac{q}{4\pi} F(r_1, \text{flux}) \quad (1)$$

where: $K(T,D)$ = the thermal conductivity of UO_2 as a function of temperature T and fuel density D

q = local linear power (W/cm)

a = pellet radius (cm)

T_1 = measured temperature at $r_1 < a$ (see Figure 1), °C

$Q(r')$ = volumetric heat generation rate in the fuel (W/cm^3)

$F(r_1, \text{flux})$ is a factor related to flux depression in the fuel and to the position of r_1 . For $r_1 = 0$ and no flux depression, $F = 1.0$

Given equation (1), a method for estimating h_{GAP} becomes clear:

- 1) Choose a form for $K(T, D)$ and integrate it with respect to T as indicated (from T_{FS} to T_1)
- 2) Choose a form for $Q(r')$ and perform the indicated double integration.
- 3) Set the two results equal and solve for T_{FS} , probably by iteration.
- 4) Find a value for T_{CI} and calculate h_{GAP} .

I.B - ASSUMPTIONS USED

- 1) For the thermal conductivity of UO_2 $K(T, D)$ we chose the Lyons thermal conductivity equation of the form

$$K(T, D) = F(D) \left(\frac{38.24}{129.4 + T} + 6.1256 \times 10^{-13} T^3 \right)$$

where T is in degrees Kelvin, and

$$F(D) = \text{Maxwell-Eucken Coeff.} = \frac{1.025}{0.95} \left(\frac{D}{1 + (1-D)0.5} \right)$$

(D is in fraction of theoretical density)

- 2) For heat generation rate $Q(r')$, we chose the approximate solution $Q_0 I_0(\kappa r')$ which assumes the heat generation is uniformly proportional to thermal neutron flux. I_0 is the modified Bessel function of the first kind of zero order, and κ is the inverse diffusion length for thermal neutrons in the fuel. Values for κ were chosen from curves in Reference 1, and were corrected for moderator temperature by hand, in accordance with a method outlined in the same document. Results from this method agreed fairly well with a THERMOS calculation at 5%

enrichment. In most data examined, the enrichment was below 6%, and can be seen from Figure 2, the uncertainty in κ plays a minor role in the overall uncertainty of $\int KdT$ and hence of T_{FS} .

- 3) The temperature at the cladding ID, T_{CI} , was calculated according to the standard equation

$$T_{CI} = T_{COOLANT} + \Delta T_{FILM} + T_{CLAD}$$

where ΔT_{FILM} is given by the Jens-Lottes equation and

$$\Delta T_{CLAD} = \frac{q}{2\pi \bar{K}_C} \ln \frac{C}{b} \text{ as described in Appendix C.}$$

- 4) Where temperatures were determined by microstructure, the following temperatures were assigned:

Temperature of UO_2 melting radius - 2790°C

Temperature for columnar grain growth boundary - 1700°C

Temperature for equiaxed grain growth boundary - 1400°C

The latter two values were chosen on the basis of References 2 and 3, plus our own observation of a consistent 300°C difference between equiaxed and columnar grain growth boundaries when the Lyons equation was used to develop the temperature profiles.

The following approach was used for every data point:

- 1) Solve the following equation for T_{FS}

$$F(D) \left[38.24 \ln \left(\frac{T_1 + 129.4}{T_{FS} + 129.4} \right) + 1.5314 \times 10^{-13} (T_1^4 - T_{FS}^4) \right] \\ = \frac{q}{4\pi} F(\kappa r_1)$$

where appropriate forms of $F(\kappa, r_1)$ are tabulated in Reference 19.

- 2) Find T_{CI} as outlined in Appendix C.
3) Calculate h_{GAP} .

FIGURE 2. $F(0, \kappa, a)$ Plotted Versus κ

A short program, BESSEL, was written to accomplish this. A description and listing of the program appear in Appendix A. Notice that the method does not use gap size or the nature of the fill gas in computing h_{GAP} . The program uses the coolant temperature, the heat flux, and an internal temperature, and finds an h_{GAP} value to match the three, under the assumptions listed above.

I.C - LISTING OF DATA EXAMINED

Table 1 contains a listing of the open literature documents examined in the preparation of this report. The documents are grouped according to the type of fill gas and method of temperature measurement used. A listing of the inferred h_{GAP} values from the documents (arranged in the same order as Table 1) appears in Appendix B.

TABLE 1. Listing of Documents Examined

Document	Method of Temperature Measurement(a)	Fill Gas	Range of G/D, % (b)	Range of q, W/cm (kW/ft)	Total Number of Data Points Used
HPR-80	TC	He	0.4-1.3	100-500(3-15)	24
HPR-107	TC	He	1.5	100-600(3-18)	70
AE-318	TC	He	0.4-1.03	100-600(3-18)	40
Testa et al. (c)	TC	He	1.2-2.4	100-500(3-15)	40
RISO-269	TC	He	0.5	100-300(3-9)	3
CNEN (d)	TC	He	1.2	100-500(3-15)	6
CEA-R-2586	TC	He	0.2	200-430(6-13)	24
CEA-R-3588	MS	He	0.7-2.4	450-650(13-20)	11
CVNA-142	MS	He and Ar	1.4-5.9	360-785(11-24)	11
AECL-1676	MS	He	0.5	600-1200(18-36)	6
BelgoNucleaire (e)	MS	He	2.1	400-725(12-22)	33
Lepsky et al. (f)	MS	He	2.2	600-1200(18-36)	6
HPR-129	TC	He + Fiss. Gas	1.5	100-800(3-24)	60
WAPD-228	TC	Xe+Kr	0.7-3.8	100-250(3-7.5)	70
AECL-2588	MS	Ar	0.8-5.2	500-725(15-24)	21
AECL-2662	MS	Ar	0.5	500-725(15-24)	10

(a) "TC" indicates temperature measurements by thermocouples, "MS" indicates temperature measurement by microstructural features.

(b) G/D is the ratio of diametral gap to pellet diameter.

(c) Reference 4

(d) References 5 and 6

(e) Reference 7

(f) Reference 8

PART II - ESTIMATION OF THE SCATTER OF INFERRED h_{GAP} VALUES

II.A - SENSITIVITY OF T_{FS} TO INPUT PARAMETERS

The input parameters that go into the calculation of h_{GAP} values are listed below in Table 2.

Table 2. Input Parameters

<u>Parameter</u>	<u>Symbol</u>	<u>Units, metric (engineering)</u>
Local Linear Power	q	W/cm (kW/ft)
UO ₂ Thermal Conductivity	K	W/cm-°C (Btu/hr-ft-°F)
Measurement internal temperature	T ₁	°C (°F)
Location of T ₁	r ₁	cm (--)
Inverse Diffusion Length	κ	cm ⁻¹ (--)
Pellet ID	a	cm
Clad OD	b	cm
Clad ID	c	cm
Clad Thermal Conductivity	K _C	Same as UO ₂ thermal conductivity
Coolant Temperature	T _{Cool}	°C (°F)
Coolant Pressure	P	bars (psi)

Of these, only the first five are believed to vary significantly; or it might be better said that the variation of the first five parameters far outweigh the effect of usual variation of all the others. Now consider again the equation for h_{GAP} :

$$h_{GAP} = \frac{q}{2\pi a (T_{FS} - T_{CI})}$$

By comparison with the uncertainty in q and in T_{FS} , the uncertainty in T_{CI} is quite small. Thus a study of the sensitivity of T_{FS} to the first five parameters will be quite revealing as to which contributes most to the uncertainty in h_{GAP} . The results of such a study are presented in Figure 3. This figure presents a reliable picture throughout the range of interest (5-20 kW/ft, G/D = 1 to 4%).

FIGURE 3. Sensitivity of T_{FS} to Input Parameters

As one might expect, q , K , and T_1 dominate as the input variables upon which T_{FS} and thus h_{GAP} depend. This point should be kept in mind as one examines the confidence limits placed on all five significant parameters (part II.B).

II.B ASSUMED DISTRIBUTION OF INPUT PARAMETERS, SAMPLING TECHNIQUE, AND RESULTS

The five significant input parameters were assumed to have normal distributions about the reported values, with the 2σ limits (95% confidence limits) taken to be as follows:

q $\pm 10\%$ rel.

K $\pm 2\%$ rel. (95% Confidence limit; 95% prediction limit is 10-20%)

T_1 $\pm 2\%$ rel. (Thermocouple measurements)

$\pm 160^\circ\text{C}$ (grain growth boundary measurements) (See note, pg. 21)

$\pm 50^\circ\text{C}$ (UO_2 melting)

r_1 ± 0.02 cm. (MS measurements only)

κ $\pm 10\%$ rel.

By use of a random number generator (see Appendix D), simultaneous random samples were drawn from these distributions, for each of the situations noted in Table 3. Sample sets ranged from 25 to 50 samples, and replicate sets were taken as a check on the generator.

TABLE 3. Situations for Random Samples

<u>Group</u>	<u>Method of Temperature Measurement</u>	<u>Size of mean h_{GAP} Value ($\text{W}/\text{cm}^2 - ^\circ\text{C}$)</u>	<u>Number of Sets</u>
1	Thermocouple	0.25-0.5	3
2	Thermocouple	0.5-0.9	1
3	Thermocouple	above 1.0	1
4	MS (grain boundary)	0.1-0.35	6
5	MS (grain boundary)	0.4-0.9	5
6	MS (grain boundary)	0.9 and higher	4
7	MS (melt radius)	0.15-0.5	1

These samples were processed through BESSEL to produce 25 to 50 h_{GAP} values per set, and these sets were used to calculate an estimate of σ_h according to the standard formula,

$$s_h^2 = \sum_{i=1}^n \frac{(h_i - \bar{h})^2}{n-1}$$

The results of these samples are summarized in Table 4. As might be expected, the variation is about equal to the combined variation of T_1 and q , for small h_{GAP} values. For large h_{GAP} values, the variation is much larger, owing to the increased effect of the (invariant) T_{FS} uncertainty on the small ΔT_{GAP} . This is shown graphically in Figure 4.

TABLE 4. Results of Random Samples

Group	Mean h_{GAP} Values $\text{W/cm}^2 - ^\circ\text{C}$	Estimated Standard Deviation $s_h, (\text{W/cm}^2 - ^\circ\text{C})$
1	0.36	0.09
	0.43	0.08
	0.44	0.08
2	0.95	0.42
3	1.68	2.0
4	0.13	0.02
	0.33	0.075
	0.25	0.053
	0.34	0.08
	0.31	0.06
	0.35	0.06
5	0.41	0.12
	0.47	0.14
	0.79	0.22
	0.59	0.30
	0.89	0.88
6	1.04	0.49
	1.48	1.17
	2.1	2.4
	2.8	7.0
7	0.16	0.03

With such large scatter probable in the data, the prediction interval about the regression curves will be very broad.

FIGURE 4. Qualitative Explanation of Scatter in h_{GAP} Values

PART III - PROPOSED CORRELATIONS OF THE INFERRED h_{GAP} VALUES

A standard regression analysis code, MARTHA, was used to fit various models to the data listed in Appendix B. In using this code we defined independent variables $X_i = F_i(q, G/D)$ and fit the data to the model $h_{GAP} = Y = \sum_{i=1}^n C_i X_i$. The code finds the regression coefficients C_i which minimize the sum of squares of deviations between the model and the data points. For a more thorough discussion, see Appendix D.

Most of the data is related to low burnup rods with helium filled gaps, and this group of data was modeled first. For one document with significant thermocouple data at slight burnup, (HPR-129), we used the same model but determined separate regression coefficients. A similar approach was taken with WAPD-228, which consisted of low power rods with simulated fission gas mixtures replacing the helium in the gap.

III.A HELIUM-FILLED GAPS (BEGINNING-OF-LIFE CONDITIONS)

Consider Figure 5, which shows a typical set of data points. The definite structure seen here was repeated in most data sets; i.e., a linear increase of h_{GAP} with q at low power followed by a nonlinear rise into the region of indeterminacy ($h_{GAP} > 1.5 \text{ W/cm}^2/\text{°C}$) at higher powers. A scan of the contributing data sets (HPR-80, HPR-107, CVNA-142, CNEN, and CEA-R-3588) will also reveal an inverse dependence of h_{GAP} on G/D , the gap-to-pellet diameter ratio. We divided the problem of modeling this structure into two parts:

- (1) Modeling the linear rise at low temperatures
- (2) Accounting for the residual between the linear model and the h_{GAP} values at higher powers.

"Open" Gaps

The domain of linear rise with q was assumed to correspond to open gap conditions, with the nonlinear rise a consequence of gap closure. As has been emphasized before, the actual condition of the gap does not influence the inference of h_{GAP} values from the data, and this nomenclature is

FIGURE 5. Typical Data

merely a convenient classification of the two regimes. For the "open" gaps, we have approximately 100 data points, which have been correlated in the following ways:

TABLE 5. Correlations for "Open" Helium Gaps
(Standard Deviations Appear in Parentheses)

Correlation Number	Equations (for h_{GAP} in $\text{W}/\text{cm}^2\text{-}^\circ\text{C}$)	Correlation Coefficient	Residual Standard Deviation
(1)	$T_{\text{GAP}} = T_{\text{Cool}} + 273 + \frac{(\pm 0.02)}{0.4642q} F$ $(1.0 + 0.08G/D) = F$ $h_{\text{GAP}} = \frac{(\pm 0.034)}{0.146} - \frac{(\pm 0.21)}{2.125} \frac{(\pm 0.0088)}{G/D} + \frac{0.1001 \sqrt{T_{\text{GAP}}}}{G/D}$	0.77	0.1175
(2)	$T_{\text{GAP}} = T_{\text{CI}} + 273 + \frac{(\pm 0.0088)}{0.2723q} F$ $(1.0 + 0.08G/D) = F$ $h_{\text{GAP}} = 0.0321 + \frac{(\pm 0.037)}{0.1167} \frac{(\pm 0.0102)}{\sqrt{T_{\text{GAP}}}} - \frac{(\pm 0.23)}{2.36} \frac{1}{G/D}$	0.78	0.1160
(3)	$h_{\text{GAP}} = \frac{(\pm 0.032)}{0.0657} + \frac{(\pm 0.048)}{0.1775} \frac{(\pm 0.000100)}{G/D} + \frac{0.00137q}{G/D}$	0.829	0.1039

In (1) and (2) above, T_{GAP} is the average temperature in the gap in degrees Kelvin. The dependence on $\sqrt{T_{\text{GAP}}}$ was chosen because the conductivity of helium is dependent on the square root of its absolute temperature, and conduction through the gas is known to be the dominant heat transfer mechanism in open gaps. Because all the data sets considered involved rods clad with Zircaloy of approximately equal thickness, correlations (1) through (3) predict very nearly the same h_{GAP} values with very nearly the same correlation coefficient. But correlation (2) is probably a more reliable formulation, since it is keyed to the cladding ID temperature, the calculation of which will account for variations in cladding type and thickness.

Correlation (2) appears valid for heat ratings from 100 to 300 W/cm^2 (3 to 9 kW/ft^2) and G/D in the range of 1 to 4%. A plot of the predicted h_{GAP} values and the 95% confidence and prediction intervals is shown in Figure 6. (See Appendix D for an explanation of the two intervals.)

FIGURE 6. Correlation for Helium-Filled Gaps
(G/D = 1.5%)

"Closing" Gaps

The nonlinear rise in h_{GAP} with power was modelled by assuming that the residual between the data in this region and the linear model, $h - \hat{h}$, could be described by

$$h - \hat{h} = y = A(B/1+B)$$

$$\text{where } B = \exp [\alpha + \gamma (G/D) + \beta(q)]$$

and A is a regression coefficient.

(It can be shown that $0 < y < 1$, and $y = 0.5A$ for $q = q_{.5} = \frac{\alpha + \gamma (G/D)}{\beta}$. Also note $z = \ln \frac{y}{1+y} = \alpha + \gamma (G/D) + \beta(q)$.)

Plots of z versus q for various data sets with fixed G/D revealed a good linear fit, with β in the range 0.02 to 0.03. However, $q_{.5}$ did not seem to be a predictable function of G/D . And, when linear regression is applied to fit α , β , and γ simultaneously, β is on the order of .005, in essence "smearing" the rise over a wide span in q . Because the plots confirmed that β for any individual data set was much higher, z was again fit to $\alpha + \gamma (G/D) + \beta(q)$ with β fixed at trial values of 0.02, 0.025 and 0.035. The resulting values of α , β , and γ were then used in the equation $y = A (B/1 + B)$ to give a model which was fit to the residuals to yield estimates of A. Similar values of correlation coefficient (~ 0.75) and residual standard deviation (~ 0.26) occurred for all three runs.

We prefer the correlation at $\beta = 0.035$ (for which $\gamma = -6.3$, $\alpha = -7.0$) because it simultaneously gives (1) the most reasonable value for the span of q for the rise of y from 0 to A, and (2) slightly conservative (high) estimates of $q_{.5}$, the linear power at mid-rise.

We prefer this two-step approach over fitting the total model to the total data. When the latter is attempted, the regression slope for the linear portion is too steep, as it tries to compensate for the scatter in the gaps that are closing.

Note that the total model ($h_{\text{Gap}} = H_{\text{open}} + h_{\text{closing}}$) is valid only in the range $G/D = 1-4\%$, $q = 100-600 \text{ W/cm}$.

The total model is pieced together in Figure 6, again for the typical value $G/D = 1.5\%$. The equations involved appear in Figure 6 and are repeated here:

$$h_{\text{GAP}} = \frac{0.1167 \sqrt{T_{\text{GAP}}} - 2.36}{G/D} + \frac{(\pm 0.044)}{0.6391 \left(\frac{B}{1+B} \right)} + 0.0321$$

where: $T_{\text{GAP}} = T_{\text{CI}} + 273 + 0.2723 (1 + 0.08G/D)q$

$$B = \exp (-7.0 - 6.3 G/D + 0.035q)$$

III.B EFFECTS OF LIMITED BURNUP (HPR-129 DATA)

An interesting comparison can be made between the data in HPR-107 and HPR-129. Both documents describe the same set of rods, the former for rods very early in life (first 2 days) and the latter after an exposure of 4 to 6 GWD/MTU. The data from HPR-107 are fit adequately by the helium gap model just described. The same model was fit to the HPR-129 data to get new regression coefficients. The results were as follows:

$$\text{For "open" gaps: } h_{\text{GAP}} = 0.1398 + \frac{(\pm 0.04)}{0.001043q} \frac{(\pm 0.00022)}{G/D}$$

Correlation coefficient = 0.72

Residual standard deviation = 0.0707

$$\text{For closing gaps: } h_{\text{GAP}} = 0.12 + \frac{0.001043q}{G/D} + \frac{(\pm 0.068)}{0.4569 \left(\frac{B}{1+B} \right)}$$

Correlation coefficient = 0.736

Residual standard deviation = 0.172

These results are graphed in Figure 7, along with the helium gap model for comparison.

III.C EFFECT OF SIMULATED FISSION GAS (WAPD-228 DATA)

The low power, low burnup data from WAPD-228 is interesting in that a simulated fission gas mixture (three parts xenon, one part krypton) was substituted for helium in some rods. Some 70 points were selected from

FIGURE 7. Correlation for HPR-129 Data

this document, with $G/D = 0.7, 1.0$ and 3.8 (2.5, 3.5 and 13 mils). Linear regression on $\frac{q}{G/D}$ was tried, with the following results:

$$h_{\text{GAP}} = 0.1129 + \frac{0.1401}{G/D} + \frac{0.00069q}{G/D}$$

(± 0.02) (± 0.043) (± 0.00024)
 Correlation coefficient = 0.868
 Residual standard deviation = 0.070

This model is plotted in Figure 8, along with the helium gap model for comparison.

FIGURE 8. Correlation of WAPD-228 Data

Note: The 2σ limit on grain growth boundary temperature was estimated from the scatter in temperatures that had to be assigned to measured grain growth boundaries in order to predict consistent surface temperatures via the BESSEL calculation (12 boundary measurement points were examined).

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ACKNOWLEDGMENTS

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APPENDIX A

DESCRIPTION AND LISTING OF THE PROGRAM BESSEL

APPENDIX A

DESCRIPTION AND LISTING OF THE PROGRAM BESSEL

As indicated in Part I, BESSEL is a short FORTRAN program which calculates T_{FS} , T_{CI} and h_{GAP} from the following input:

Variable	Symbol Used in Text	Variable Name in BESSEL	Units for Input
Linear power	q	W	W/cm
Internal temperature	T_1	TCL	°C
1st Coefficient in Lyons Thermal Conductivity Equation	-	RN1	W/cm-°C
Inverse diffusion length	κ	RKAP	cm ⁻¹
Location of internal temperature (diametral measure)	r_1	DR	cm
Pellet ID	-	DI	cm
Pellet OD	2a	DØ	cm
Clad ID	b	DCI	cm
Clad OD	c	DCØ	cm
Coolant temperature	$T_{coolant}$	TCØØL	°C
Clad conductivity	K_C	C	W/cm ² -°C
Coolant pressure	P	P	bars
Number of cases	-	KW	-

The central problem is to solve the following equation for T_{FS}

$$A \left[\ln \left(\frac{T_1 + B}{T_{FS} + B} \right) \right] + C (T_1^4 - T_{FS}^4) = \frac{q}{4\pi} F(r, \kappa, a)$$

where T is in °Kelvin and A, B, and C are given in Part I.

BESSEL branches to the appropriate form of $F(r_1, \kappa, a)$ as tabulated by Robertson (Reference 19) and calculates its value. It then iterates to find T_{FS} in the following way. Since the first term in (1) is always an order of magnitude larger than the second term, the second term is dropped and the equation solved for a first estimate of T_{FS} . This estimate is used to calculate a value for the second term, which is then reinserted into the equation and a second estimate is found for T_{FS} , etc. Convergence is rapid and three to ten iterations bring the two sides of (1) into agreement within 0.01 W/cm, which corresponds to 1°C in T_{FS} .

T_{CI} is found by the method outlined in Appendix C; basically a stable value for the temperature at the center of the clad is found by successive adjustments of $\bar{K}_C$ in the equation

$$\Delta T_{CLAD} = \frac{q}{2\pi a \bar{K}_C} \ln \left(\frac{C}{b} \right)$$

Then T_{CI} is found from $T_{Cool} + \Delta T_{FILM} + \Delta T_{CLAD}$.

BESELNG, T100, P2
 ACCOUNT(PW=BCC008,UN=BCC008)
 JORNAME(PUNCH=PUNCH)
 DISPOSF(PUNCH,*PU=C)
 ATTACH(LIBR,MATH SCIENCES)
 LIBRARY(LIBR)
 FTN.
 LGO.
 JORNAME(PUNCH=PUNCH)

DD LANNING 3000 AREA 9462764 PSE

```

PROGRAM BESSEL (INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT)
  DIMENSION CST(11), C(23), RN1(100), DF(100), RN2(100), R(100)
  DIMENSION TCL(100), TI(100), CONST(100), XI(100), XO(100)
  DIMENSION TERM2(100), AKDT(100), H(100), W(100), TS(100)
  DIMENSION RKAP(100)
  DIMENSION DR(100)
  DIMENSION DLTGP(100)
  DIMENSION FI(2), FK(3), RI0I(100), RK0I(100), RI1I(100)
  DIMENSION RK1I(100), RI0O(100), RK0O(100), RI1O(100), RK1O(100)

C
C   THE FOLLOWING IS SCOTT THERMAL CONDUCTIVITY DATA
C   FOR ZR-2, IN UNITS OF BTU/HR-FT-DEG. F (REF. WCAP-3629-41)
C   THE DATA IS GIVEN IN STEPS OF 50.DEG.F, BEGINNING WITH 100.DEG.F
C   AND ENDING WITH 950.DEG.F
C
  X 8.76,8.94,9.15,9.37,9.60,9.82,10.05,10.27,10.50,10.72/
  DATA (C(I), I = 1,18)/7.34,7.52,7.70,7.88,8.05,8.23,8.41,8.58,

C
C   READ IN DATA
C
1 READ(5,80) DI, DO, DENS, RMODE, KW
  IF(KW.LE.0) GO TO 999
  READ(5,90) DCO, DCI, P, TCOOL
  WRITE(6,70)
  WRITE(6,50)
  WRITE(6,80) DI, DO, DENS, RMODE, KW
  WRITE(6,90) DCO, DCI, P, TCOOL
  WRITE(6,50)
  DO 9000 L = 1,KW
    READ(5,110) W(L), TCL(L), DR(L), RKAP(L),RN1(L)

C
C   RN1 AND RN2 ARE THE COEFFICIENTS IN THE LYONS THERMAL CONDUCTIVITY
C   EQUATION. RN2 IS CALCULATED FROM RN1 UNDER THE CONSTRAINT
C   THAT INTEGRAL KDT EQUALS 93. W/CM. (REF. GEAP-4624)
C
  D1 = 44.90407
  D2 = 1.06244E13
  RN2(L) = (D1 - RN1(L))/(4.0*D2)
  
```

```

C      SET UP AGRUMENTS FOR THE BESSEL FUNCTIONS
C
      R(L) = RKAP(L)*DR(L)/2.0
      XI(L) = RKAP(L)*DI/2.0
      XO(L) = RKAP(L)*DO/2.0
9000  CONTINUE
C
C      SET CONSTANTS NEEDED IN CALCULATIONS OF TCI.
      FLMFAC = 7.94*(EXP(-P/62.0))
      DLN = ALOG(DCO/DCI)
      PI = 3.141592
C
      DO 300 I = 1,KW
C
C      BRANCH TO PROPER CALC. OF THE ROBERTSON INTEGRAL. (REF.CRFD-835)
      IF (RMODE.LT.1.0) GO TO 2000
      IF (DI.EQ.0.0) GO TO 3000
C
C      CALC. ROBERTSON INTEGRAL FOR HOLLOW CYLINDER GIVEN TEMP. AT DR.
      CALL BESNIS(XO(I),2,FI)
      CALL BESNKS(XO(I),3,FK)
C      BESNIS AND BESNKS ARE CANNED SUBROUTINES FOR BESSEL
C      FUNCTION CALCULATION.
      RI00(I) = FI(1)
      RI10(I) = FI(2)
      RK00(I) = FK(1)
      RK10(I) = FK(2)
      CALL BESNIS(XI(I),2,FI)
      CALL BESNKS(XI(I),3,FK)
      RI01(I) = FI(1)
      RI11(I) = FI(2)
      RK01(I) = FK(1)
      RK11(I) = FK(2)
      CALL BESNIS(R(I),2,FI)
      CALL BESNKS(R(I),3,FK)
      RNUM=RI11(I)*(RK00(I)-FK(1)) + RK11(I)*(RI00(I)-FI(1))
      DENOM=0.5*XO(I)*(RI10(I)*RK11(I)-RI11(I)*RK10(I))
      GO TO 5000
C
C      CACLC. ROBERTSON INTEGRAL FOR SOLID CYLINDER GIVEN TEMP. AT DR.
C
3000  CALL BESNIS(XO(I),2,FI)
      RI00(I) = FI(1)
      RI10(I) = FI(2)
      CALL BESNIS(R(I),2,FI)
      RNUM = RI00(I) - FI(1)
      DENOM = 0.5*XO(I)*RI10(I)
      GO TO 5000
2000  IF (DI.EQ.0.0) GO TO 4000

```

```

C
C  CALC. ROBERTSON INTEGRAL FOR HOLLOW CYL. GIVEN TEMP. AT DI.
C
    CALL BESNIS(XO(I),2,FI)
    CALL BESNKS(XO(I),3,FK)
    RI00(I) = FI(1)
    RI10(I) = FI(2)
    RK00(I) = FK(1)
    RK10(I) = FK(2)
    CALL BESNIS(XI(I),2,FI)
    CALL BESNKS(XI(I),3,FK)
    RI0I(I) = FI(1)
    RI1I(I) = FI(2)
    RK0I(I) = FK(1)
    RK1I(I) = FK(2)
    DENOM=0.5*XI(I)*XO(I)*(RI10(I)*RK1I(I)-RI1I(I)*RK10(I))
    RNUM=XI(I)*(RI1I(I)*RK00(I)+RI00(I)*RK1I(I)) - 1.0
    GO TO 5000
C
C  CALC. ROBERTSON INTEGRAL FOR SOLID CYL. GIVEN TEMP. AT DI = CENTER
C
4000 CALL BESNIS(XO(I),2,FI)
    DENOM = 0.5*XO(I)*FI(2)
    RNUM = FI(1) - 1.0
C
C  CALC. Q/4*PI * F(R1,KAPPA)
C
5000 CONST(I) = (RNUM/DENOM)*W(I)*0.0795775
C
C  ITERATE ON LYONS INTEGRAL TO FIND FUEL SURF TEMP., TS
    TK = TCL(I) + 273.1
    D = 1.07895*(DENS/(1.0 + (1.0-DENS)*0.5))
    CNST = CONST(I)/(RN1(I)*D)
    TKST = ((TK + 129.4)*EXP(-CNST)) - 129.4
    TERM2(1) = RN2(I)*(TK**4 - TKST**4)
    DO 400 K = 1,100
    CNST = (CONST(I)/D - TERM2(K))/RN1(I)
    TKST = ((TK + 129.4)*EXP(-CNST)) - 129.4
    TERM1 =RN1(I)*ALOG((TK + 129.4)/(TKST +129.4))
    TERM2(K+1) = RN2(I)*(TK**4 - TKST**4)
    CHECK = D*(TERM1 + TERM2(K+1))
    DEL = CONST(I) - CHECK
    DELA = ABS(DEL)
    IF (DELA.LT. 0.01) GO TO 1000
400 CONTINUE
1000 AKDT(I) = CHECK
    TS(I) = TKST - 273.2
300 CONTINUE
C

```

```

C  CALC. TEMP. AT CLAD ID.
    DO 900 L = 1,KW
      SHF = W(L)/PI
C  CALCULATE TEMP. DROP ACROSS THE FILM
      A = 0.25*(ALOG(SHF/DCO))
      DELT = FLMFAC*(EXP(A))
C
      TCO = TCOOL + DELT
      TCO = TCO*1.8 + 32.0
      KT = (TCO/50.0) - 1.0
      IF(KT.GT.17) GO TO 7000
      A = (C(KT+1) - C(KT))/50.0
      B = C(KT) - A*KT*50.0
      CST(1) = (A*TCO + B)
C  ITERATE TO CONVERGENCE BY SUCCESSIVE INTERPOLATIONS OF THE
C  CLAD THERMAL CONDUCTIVITY.
      DO 2200 N = 1,10
        DEL = (SHF/(2.0*CST(N)))*DLN*104.05
        TCI1 = TCO + DEL
        KT = ((TCO + TCI1)/100.0) - 1.0
        TAV = (TCO + TCI1)/2.0
        IF(KT.GT.17) GO TO 7000
        A = (C(KT+1) - C(KT))/50.0
        B = C(KT) - A*KT*50.0
        CS = (A*TAV + B)
        DEL = (SHF/(2.0*CS))*DLN*104.05
        TCI2 = TCO + DEL
        CHK = TCI1 - TCI2
        IF(ABS(CHK).LT.1.0) GO TO 6000
        TAV = (TAV + TCI2 - DEL/2.0)/2.0
        KT = (TAV/50.0) - 1.0
        A = (C(KT+1) - C(KT))/50.0
        B = C(KT) - A*KT*50.0
        CST(N+1) = (A*TAV + B)
2200  CONTINUE
        GO TO 6000
7000  CS = C(18)
        DEL = (SHF/2.0*CS)*DLN*104.05
        TI(L) = TCO + DEL
        TI(L) = (TI(L) - 32.0)/1.8
        GO TO 900
6000  TI(L) = TCI2
        TI(L) = (TI(L) - 32.0)/1.8
900   CONTINUE
C
C  CALCULATE THE GAP CONDUCTANCE
C
      DO 200 I = 1,KW
        DLTGP(I) = TS(I) - TI(I)

```

```

      H(I) = W(I)/(PI*DO*DLTGP(I))
200  CONTINUE
C
      WRITE(6,10)
      WRITE(6,50)
      WRITE(6,20)
      WRITE(6,40)
      WRITE(6,50)
      DO 500 L = 1, KW
      WRITE(6,30) L, W(L), RKAP(L), CONST(L), TI(L), H(L), TS(L)
      PUNCH 60, W(L), TCL(L), DLTGP(L), RN1(L), TS(L), H(L), L
500  CONTINUE
      GO TO 1
999  STOP
10  FORMAT (50X, 'SUMMARY OF RESULTS')
20  FORMAT(15X, 'HEAT LOAD', 12X, 'KAPPA', 10X, ' LYONS ',
X 10X, 'CLAD ID TEMP.', 10X, ' H ', 10X, 'FUEL SURF. ')
30  FORMAT(2X, I3, 8X, F10.4, 4(10X, F10.4), 8X, F8.2)
40  FORMAT(15X, '(W/CM)', 27X, 'INTEGRAL, W/CM', 12X, 'DEG. C',
X 10X, '(W/SQ.CM-DEG.C)', 6X, 'TEMP(DEG.C)')
50  FORMAT ( )
60  FORMAT(6F10.4, 2X, I3)
70  FORMAT(3X, 'INPUT DATA')
80  FORMAT(4F10.4, 10X, I13)
90  FORMAT(4F10.4)
110 FORMAT(5F10.4)
      END

```

APPENDIX B

LISTING OF INFERRED h_{GAP} VALUES

APPENDIX B

LISTING OF INFERRED h_{GAP} VALUES

The following is a listing of BESSEL input parameters and inferred h_{GAP} values. The references are arranged in the same order as in Table 1, that is:

- (1) Helium filled gaps; thermocouple measurements
- (2) Helium filled gaps; microstructure measurements
- (3) Non-helium-filled gaps; thermocouple measurements
- (4) Non-helium filled gaps; microstructure measurements.

Explanation of abbreviations and symbols used:

- TC - thermocouple
- MS - microstructure
- Dgg - diametral boundary of equiaxed grain growth
- Dcg - diametral boundary of columnar grain growth
- * - indicates h value was too large and uncertain to be used in correlation
- - indicates h value was extremely large or negative, and therefore not recorded.

Listing of Inferred h_{GAP} - Values

Document HPR-80

Reference No. 15

Fill gas Helium

Method of temp. measurement Thermocouples at fuel center

General Input Information:

Fuel Enrichment 5.0%

Moderator temp. (°C) ~230°C

κ value used 1.6

Fuel Density 96 and 98% TD

Fuel OD (cm) 1.254 and 1.243

Fuel ID (cm) 0.12

Clad Material Zircaloy-2

Clad OD (cm) 1.37

Clad ID (cm) 1.259

Coolant Temp. (°C) 230°C

Coolant Pressure (bars) 28.

Comments Values of centerline temp. were interpolated from the listing
 on p. 38. This was felt to be more accurate than reading the points
 off the Figures 6 and 7.

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
100	2.0	0.4	1.02	0.74	Rod HBB-96% TD
200			1.06	0.86	
300			1.16	1.01	
400			1.45	1.18	
500			2.04	1.24	
100	6.5	1.3	0.37	0.33	Rod HBC-96% TD
125			0.36		
150			0.36		
175			0.37		
200			0.39	0.36	
225			0.40		
250			0.41		
275			0.43		
300			0.45	0.42	
325			0.48		
350			0.51		
375			0.56		
400			0.60	0.51	
425			0.67		
450			0.74		
475			0.82		
500			0.91	0.61	
525			1.03		
550			1.18		
575			1.35		
600			1.56	0.76	

Listing of Inferred h_{GAP} - Values

Document HPR-107

Reference No. 11

Fill gas Helium

Method of temp. measurement Thermocouples at fuel centerline

General Input Information:

Fuel Enrichment 6.0

Moderator temp. (°C) 240

κ value used 1.9

Fuel Density 97%

Fuel OD (cm) 1.3845

Fuel ID (cm) 0.175

Clad Material Zircaloy-2

Clad OD (cm) 1.559

Clad ID (cm) 1.406

Coolant Temp. (°C) 240°C

Coolant Pressure (bars) 28.0

Comments These values represent the beginning-of-life for element IFA-117;
double values represent up-ramp/down-ramp measurements. (Average values
were used in the correlations)

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
50	8.3	1.5	0.29		Pin 1
75			0.27		
100			0.27		
125			0.28		
150			0.29		
175			0.30		
200			0.32		
225			0.33/.33		
250			0.34/.43		
275			0.38/.47		
300			0.42/0.52		
325			0.45/0.55		
350			0.51/0.58		
375			0.58/0.62		
400			0.66		
425			0.74		
450			0.83		
475			0.95		
500			1.09		
535	↓	↓	1.36		↓

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
50	8.3	1.5	0.29		Rod 3
75			0.29		
100			0.30		
125			0.32		
150			0.34		
175			0.36		
200			0.38		
225			0.41		
250			0.44		
275			0.47		
300			0.51		
325			0.55		
350			0.59		
375			0.65		
400			0.70		
425			0.77		
450			0.84		
475			0.93		
500			1.03		
535			1.19		

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² °C	h_{GAP} Reported W/cm ² °C	Comments
50	8.3	1.5	0.21		Rod 5
75			0.22		
100			0.23		
125			0.20		
150			0.26		
175			0.28		
200			0.29		
225			0.32		
250			0.36		
275			0.40		
300			0.45		
325			0.51		
350			0.57		
375			0.64		
400			0.72		
425			0.82		
450			0.94		
475			1.07		
500			1.26		

Listing of Inferred h_{GAP} - Values

Document AE-318

Reference No. 13

Fill gas Helium

Method of temp. measurement Thermocouples at Fuel Centerline

General Input Information:

Fuel Enrichment 1.5 (IFA-3), 6.0 (IFA-4), 2.5 (IFA-10)

Moderator temp. (°C) 230

κ value used 0.9, 1.0, 2.0 resp.

Fuel Density 96% TD

Fuel OD (cm) 1.258 (3,4) 1,247 (10)

Fuel ID (cm) 0.11

Clad Material Zircaloy-2

Clad OD (cm) 1.428 (3,4) 1.38 (10)

Clad ID (cm) 1.282 (3,4) 1.26 (10)

Coolant Temp. (°C) 230°C

Coolant Pressure (bars) 28.0 assumed

Comments _____

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
117	9	1.9	0.36		IFA-4
134			0.28		
154			0.22		
174			0.24		
209			0.34		
220			0.22		
256			0.24		
291			0.28		
322			0.22		
375			0.26		
406			0.32		
439			0.32		
475			0.40		
488			0.55		
503			0.66		
564			1.47		
581			1.35		
619			1.46		✓
75.4			0.20		IFA-3
139.4			0.35		
153.2			0.27		
166			0.28		
173			0.62		
209			0.42		
252			0.69		
300			0.60		

[illegible]

Listing of Inferred h_{GAP} - Values

Document Testa et al.

Reference No. 4

Fill gas Helium

Method of temp. measurement Thermocouples at fuel centerline

General Input Information:

Fuel Enrichment 6.0

Moderator temp. (°C) 230°C

κ value used 1.9

Fuel Density 94% TD

Fuel OD (cm) 1.245-1.261

Fuel ID (cm) 0.16

Clad Material Zircaloy-2

Clad OD (cm) 1.416

Clad ID (cm) 1.276

Coolant Temp. (°C) 230°

Coolant Pressure (bars) 28.0

Comments Only startup data was used in correlation; other data is
recorded for IFA-130.

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
100	9.0	1.8	0.18		IFA-130 (Startup)
150			0.24		
200			0.30		
250			0.38		
300			0.47		
350			0.58		
400			0.73		
450			0.93		
500			1.22		
					IFA-130 (All data)
150	9.0	1.8	0.33		
145			0.32		
155			0.34		
215			0.63		
225			0.59		
240			0.70		
362			0.87		
380			0.94		
390			1.28		
412			1.89		
440			2.24		
445			1.62		
485			1.57		
530			1.69		
562			2.60		

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
100	12.2	2.5	0.21		IFA-131
150			0.24		Startup
200			0.27		
250			0.31		
300			0.36		
350			0.42		
400			0.47		
450			0.55		
500	↓	↓	0.63		
100	9.0	1.8	0.31		
150			0.35		
200			0.40		
250			0.48		
300			0.58		
350			0.69		
400			0.83		
450			1.02		
500	↓	↓	1.28		↓

[illegible]

Listing of Inferred h_{GAP} - Values

Document RISO-269

Reference No. 16

Fill gas Helium

Method of temp. measurement Thermocouples at fuel center and on the clad surface.

General Input Information:

Fuel Enrichment 1.35

Moderator temp. ($^{\circ}\text{C}$) $\sim 100^{\circ}\text{C}$

κ value used 1.0

Fuel Density 95.7% TD

Fuel OD (cm) 1.75

Fuel ID (cm) 0.16

Clad Material Stainless Steel

Clad OD (cm) 1.96

Clad ID (cm) 1.76

Coolant Temp. ($^{\circ}\text{C}$) T_{cs} fixed at clad T.C.

Coolant Pressure (bars) reading.

Comments _____

[illegible]

Listing of Inferred h_{GAP} - Values

Document CNEN Article

Reference No. 5, 6

Fill gas Helium

Method of temp. measurement Thermocouples at fuel center and near fuel surface; also on clad surface

General Input Information:

Fuel Enrichment 3.65% Pu(PP1), 7.9% U-235(A1)

Moderator temp. (°C) ~100

κ value used 1.65, 2.2

Fuel Density 94% TD

Fuel OD (cm) 1.29, 1.58

Fuel ID (cm) 0.05

Clad Material Zircaloy-2

Clad OD (cm) 1.506 (PP1), 1.78 (A1)

Clad ID (cm) 1.318, 1.60

Coolant Temp. (°C) T_{cs} fixed at thermocouple reading

Coolant Pressure (bars)

Comments

Listing of Inferred h_{GAP} - Values

Document CEA-R-2586

Reference No. 20

Fill gas Helium

Method of temp. measurement Thermocouples at the fuel center and
near the edge; and in the middle of
the thick cladding.

General Input Information:

Fuel Enrichment 1.67

Moderator temp. ($^{\circ}\text{C}$) ~ 100

κ value used ~ 1.0

Fuel Density 95.6% TD

Fuel OD (cm) 1.766

Fuel ID (cm) 0.16

Clad Material Zircaloy-2

Clad OD (cm) 3.1

Clad ID (cm) 1.77

Coolant Temp. ($^{\circ}\text{C}$) T_{cs} fixed at clad TC reading
located at Diameter = 1.97 cm

Coolant Pressure (bars) These readings are

listed as t_2 in Tables III and IV of the
document

Comments _____

[illegible]

Listing of Inferred h_{GAP} - Values

Document CEA-R-3358

Reference No. 14

Fill gas Helium

Method of temp. measurement Columnar and equiaxed grain growth, T.C.
in clad, which is quite thick.

General Input Information:

Fuel Enrichment 2.98%

Moderator temp. ($^{\circ}\text{C}$) ~ 50

κ value used 1.4

Fuel Density 95-96% TD

Fuel OD (cm) 1.298-1.305

Fuel ID (cm) 0.0

Clad Material Zircaloy-2

Clad OD (cm) 1.66 (TC position)

Clad ID (cm) 1.308-1.335

Coolant Temp. ($^{\circ}\text{C}$) T_{CS} fixed at TC readings given in Table VII.

Coolant Pressure (bars)

Comments The small h_{GAP} values have been raised 5% to account for effect of fission gas.

The linear power was calc. from $\bar{Q}$, the two-day average listed in Table VII.

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
					4112 Series
497	3.93	0.8		3.3-0.9	BE1, Dgg = 0.562 cm
555	3.54	0.7	5.*	2.5-0.7	BE2, Dgg = 0.75
555	↓	↓	--*	↓	BE2 Dcg = 0.362
650	3.93	0.8	--*	∞-2	AE1 Dgg = 0.808
650	↓	↓	--*	↓	AE1 Dcg = 0.56
601	3.54	0.7	--*	∞-1	AE2 Dcg = 0.799
↓	↓	↓	--*	↓	AE2 Dcg = 0.546
					4113 Series
496	11.8	2.3	0.83	0.7-0.4	BE1 Dgg = 0.752
↓			0.69	↓	BE1 Dcg = 0.54
521			0.44	0.45-0.27	BE2 Dgg = 0.938
↓			0.51	↓	BE2 Dcg = 0.718
591			0.59	0.59-0.33	AE1 Dgg = 0.99
↓			0.59	↓	↓ Dcg = 0.844
535			0.44	0.42-0.27	AE2 Dgg = 0.966
↓			0.54	↓	↓ Dcg = 0.742
					4110 Series
480	4.72	0.9	0.75	0.74-0.40	BE1, Dgg = 0.732
565			3.3*	3.3-0.9	BE2, Dgg = 0.78
610			4.3*	1.7-0.5	AE1, Dgg = 0.85
581	↓		1.02	1.8-0.6	AE2, Dgg = 0.90
		↓			

[illegible]

Listing of Inferred h_{GAP} - Values

Document CVNA-142

Reference No. 12

Fill gas Helium (40 hr irradiation) and Argon

Method of temp. measurement Equiaxed and columnar grain growth
T.C. measurement in sodium annulus

General Input Information:

Fuel Enrichment 2.6 (R-1, R-2), 3.8 (R-4), 5.2 (R-6, R-8), 7.6 (R-11)

Moderator temp. (°C) 48

κ value used 1.35, 1.75, 2.1, 2.8, resp.

Fuel Density 94% TD

Fuel OD (cm) 1.092

Fuel ID (cm) 0.0

Clad Material Zircaloy-2

Clad OD (cm) 1.285-1.31

Clad ID (cm) 1.108- 1.155

Coolant Temp. (°C) T_{cs} fixed at calculated
values, page 62

Coolant Pressure (bars)

Comments Small h_{GAP} values are corrected upward by 5% to account
for effect of fission gas. Values of D_{gg} and D_{cg} were back-calculated
from $\int_{T_s}^{T_{gg}} K dt$ values

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	Fill Gas	Comments
<u>R-1 Series</u>					
360	12.5	2.9	0.39	He	Rod 2, Dgg = 0.17 cm
360	25.5	5.9	0.25	He	Rod 3 Dgg = 0.58
<u>R-2 Series</u>					
360	6.-	1.4	0.37	Argon	Rod 2 Dgg = 0.26
360	25.5	5.9	0.10	Argon	Rod 3 Dgg = 1.01
360	25.5	5.9	0.10	Argon	Rod 3 Dcg = 0.83
<u>R-4 Series</u>					
525	6.0	1.4	2.2*	He	Rod 1 Dgg = 0.42
525	12.5	2.9	0.39	He	Rod 2 Dgg = 0.76
↓	↓	↓	0.39	He	Rod 2 Dcg = 0.59
↓	25.5	5.9	0.29	He	Rod 3 Dgg = 0.85
↓	↓	↓	0.27	He	Rod 3 Dcg = 0.73
<u>R-6 Series</u>					
590	6.4	1.5	4.6*	He	Rod 1 Dgg = 0.53
↓	12.5	2.9	0.98	↓	Rod 2 Dgg = 0.66
↓	↓	↓	1.02	↓	Rod 2 Dcg = 0.46
↓	25.5	5.9	6.34	↓	Rod 3 Dgg = 0.87
↓	↓	↓	0.36	↓	Rod 3 Dcg = 0.73
<u>R-8 Series</u>					
590	12.5	2.9	1.0	He	Rod 1 Dgg = 0.66
↓	↓	↓	1.0	↓	Rod 1 Dcg = 0.46
↓	↓	↓	0.24	Ar	Rod 2,3 Dgg = 0.98
↓	↓	↓	↓	Ar(0.5 atm)	Rod 2,3 Dcg = 0.86

Listing of Inferred h_{GAP} - Values

Document AECL-1676

Reference No. 17

Fill gas Helium

Method of temp. measurement Equiaxed and columnar grain growth

General Input Information:

Fuel Enrichment 4.3%

Moderator temp. (°C) ~200

κ value used 1.5

Fuel Density 96.7% TD

Fuel OD (cm) 1.9

Fuel ID (cm) 0.0

Clad Material Zircaloy

Clad OD (cm) 2.06

Clad ID (cm) 1.91

Coolant Temp. (°C) 250°C

Coolant Pressure (bars) 96.

Comments _____

[illegible]

Listing of Inferred h_{GAP} - Values

Document BELGONUCLEAIRE

Reference No. 7

Fill gas Helium

Method of temp. measurement Columnar and equiaxed grain growth

General Input Information:

Fuel Enrichment

Moderator temp. (°C) 100

κ value used 1.3

Fuel Density 94% TD

Fuel OD (cm) 0.742

Fuel ID (cm) 0.0

Clad Material Zircaloy-2

Clad OD (cm) 0.87

Clad ID (cm) 0.758

Coolant Temp. (°C) ~100

Coolant Pressure (bars) 3.4

Comments Double values at same heat loading represent, first, equiaxed grain growth measurement, then columnar grain growth measurement.

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
700	6.3	2.1	1.03		ELP2 Sec. 1
700			1.12		↓
725			1.06		Sec. 2
725			0.94		↓
700			0.80		Sec. 3
700			0.77		↓
425			0.49		ELP3 Sec. 1
440			0.46		Sec. 2
440			0.46		↓
425			0.43		Sec. 3
438			0.54		ELP4 Sec. 1
454			0.56		Sec. 2
438			0.52		↓
480			0.86		Sec. 3
500			0.98		ELP5 Sec. 1
480			0.86		Sec. 2
535			0.55		↓
↓			0.55		Sec. 3
554			0.58		ELP6 Sec. 1
↓			0.59		↓
535			0.58		Sec. 2
↓			0.63		↓
					Sec. 3
					↓

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
465	6.3	2.1	0.60		ELP7 Sec. 1
480			0.46		Sec. 2
↓			0.58		↓
465			0.53		Sec. 3
↓			0.61		↓
642			0.90		ELP8 Sec. 1
↓			0.91		↓
665			0.87		Sec. 2
↓			0.91		↓
642			1.86*		Sec. 3
↓			1.37		↓
530			0.69		ELP9 Sec. 1
↓			0.71		↓
550			0.70		Sec. 2
↓			0.69		↓
530			0.86		Sec. 3
↓			0.83		↓
530			0.63		ELP10 Sec. 1
↓			0.68		↓
550			0.56		Sec. 2
↓			0.58		↓
530			0.96		Sec. 3
↓			0.77		↓

Listing of Inferred h_{GAP} - Values

Document Lepsky, et al.

Reference No. 8

Fill gas Helium

Method of temp. measurement Columnar grain growth and fuel melting boundary

General Input Information:

Fuel Enrichment 10%

Moderator temp. ($^{\circ}\text{C}$) 240

κ value used 2.5

Fuel Density 97% TD

Fuel OD (cm) 1.37

Fuel ID (cm) 0.22

Clad Material Zircaloy-2

Clad OD (cm) 1.55

Clad ID (cm) 1.40

Coolant Temp. ($^{\circ}\text{C}$) 240

Coolant Pressure (bars) 28.

Comments These values are representative of IFA-132

Listing of Inferred h_{GAP} - Values

Document HPR-129

Reference No. 9

Fill gas Helium and fill gas

Method of temp. measurement Thermocouples in fuel center

General Input Information:

Fuel Enrichment 6.0

Moderator temp. (°C) 240

κ value used 1.9

Fuel Density 97% TD

Fuel OD (cm) 1.3845

Fuel ID (cm) 0.175

Clad Material Zircaloy-2

Clad OD (cm) 1.245-1,

Clad ID (cm)

Coolant Temp. (°C) 240

Coolant Pressure (bars) 28.

Comments These values represent element IFA-117 with a burnup of
 ~6,000 MWD/tonne U average.

[illegible]

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
178	8.3	1.5	0.17		Rod 6
194			0.18		
242			0.26/0.21		
344			0.31		
360			0.35		
414			0.40		
445			0.46		
469			0.51		
500			0.56		
523			0.63		
547			0.71		
581			0.87		
589			0.83		
631			1.29		
688			2.04		
703			2.13*		
745			4.4/3.3*		
800			-/10.5*		↓

[illegible]

Listing of Inferred h_{GAP} - Values

Document WAPD-228

Reference No. 10

Fill gas 3 parts xenon, 1 part krypton

Method of temp. measurement Thermocouples in cladding and at fuel center.

General Input Information:

Fuel Enrichment Natural

Moderator temp. ($^{\circ}\text{C}$) $\sim 70^{\circ}\text{C}$? (Not clear in the document)

κ value used 1.0

Fuel Density $\sim 94\%$ TD

Fuel OD (cm) 0.87-0.91

Fuel ID (cm) 0.17

Clad Material Stainless Steel

Clad OD (cm) 0.91-0.92

Clad ID (cm) " "

Coolant Temp. ($^{\circ}\text{C}$) T_{ci} fixed at T_{Bore}
temperature listed

Coolant Pressure (bars)

Comments

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
228	3.5	1.0	.35	.34	22 - 15
230			.34	.33	
227			.42	.40	
225			.51	.51	
223			.54	.49	
103	13.	3.8	.11	.11	22 - 7
113			.13	.13	
138			.15	.16	
148			.17	.18	
158			.17	.18	
181			.23	.24	
181			.24	.25	
180			.23	.24	
181			.22	.23	
181			.22	.22	
181			.21	.22	
181			.20	.21	
181			.20	.20	
182			.19	.20	
181			.19	.20	
180			.18	.19	
182			.18	.19	
181			.19	.20	

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² - °C	h_{GAP} Reported W/cm ² - °C	Comments
171	2.5	0.7	.47	.43	22 - 14
177			.61	.50	
178			.60	.50	
178			.60	.50	
180			.60	.50	
108	2.5	1.0	.15	.15	22 - 15
161			.31	.30	
187			.42	.39	
190			.46	.42	
210			.53	.48	
235			.62	.55	
240			.45	.42	
236			.38	.37	
234			.35	.34	
237			.35	.34	
237			.35	.33	
236			.34	.32	
234			.33	.32	
236			.33	.33	
234			.33	.35	
236			.33	.34	
236			.33	.33	
233			.33	.33	

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
162	2.5	0.7	0.44	0.41	22 - 14
160			.48	.41	
170			.50	.43	
168			.46	.40	
165			.49	.42	
168			.45	.39	
167			.45	.38	
163			.42	.39	
167			.43	.38	
162			.45	.39	
161			.44	.38	
160			.39	.35	
112			.39	.35	
136			.47	.41	
141			.54	.46	
144			.44	.38	
159			.55	.46	
179			.56	.47	
173			.56	.47	
154			.39	.35	
160			.41	.36	
154			.42	.37	
173			.50	.42	
172	↓	↓	.50	.41	↓

Listing of Inferred h_{GAP} - Values

Document AECL-2588

Reference No. 21

Fill gas Argon

Method of temp. measurement Columnar grain growth and melt radii

General Input Information:

Fuel Enrichment 1.5, 2.5 (Rabbits), 1.5 (PWL)

Moderator temp. (°C) ~60

κ value used 0.9, 1.17

Fuel Density ~96% TD

Fuel OD (cm) 1.6 (Rabbits), 1.7-1.8 (PWL)

Fuel ID (cm) 0.0

Clad Material SS (Rabbits), Zircaloy-2 (PWL)

Clad OD (cm) ~1.8 (Rabbits), ~2.0 (PWL)

Clad ID (cm) 1.7, 1.8

Coolant Temp. (°C) 60° (Rabbits), 250° (PWL)

Coolant Pressure (bars) 7, 96.

Comments _____

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
706	2 0	3.1	0.19	0.14	Rabbits, E-17
705	↓	↓	0.18	0.13	21
696	↓	↓	0.19	0.14	23
696	2 6	3.1	0.18	0.13	18
657	↓	↓	0.16	0.12	26
657	↓	↓	0.16	-	25
647	3 3	5.2	0.13	0.10	20
675	↓	↓	0.14	0.11	24
642	↓	↓	0.13	0.11	28
572	11	1.56	0.24	0.20	PWL FAO
674	↓	↓	0.34	0.27	FAP
674	10	1.4	0.36	0.28	FAR
574	↓	↓	0.26	0.21	FAS
722	↓	↓	0.35	0.30	FAN
611	25	3.6	0.15	0.13	FAT
674	↓	↓	0.18	0.15	FAW
611	↓	↓	0.17	0.15	FAX
511	↓	↓	0.15	0.13	FAY
722	↓	↓	0.21	0.18	FAZ
765	6	0.8	1.08	1.25	FAE
965	↓	↓	1.13	1.40	FAJ

Listing of Inferred h_{GAP} - Values

Document AECL-2662

Reference No. 18

Fill gas Argon and xenon

Method of temp. measurement Grain growth boundaries

General Input Information:

Fuel Enrichment 2.4%

Moderator temp. (°C) 210.

κ value used 1.0

Fuel Density 93.4% TD

Fuel OD (cm) 1.94

Fuel ID (cm) 0.0

Clad Material Stainless Steel

Clad OD (cm) 2.042

Clad ID (cm) 1.952

Coolant Temp. (°C) 210

Coolant Pressure (bars) 68.6

Comments Last 3 rods are xenon filled

Local Linear Power W/cm	gap size, mils	G/D, %	h_{GAP} Inferred W/cm ² -°C	h_{GAP} Reported W/cm ² -°C	Comments
495	4.7	0.6	0.22		Rod LFB Dgg = 1.24
↓			0.20		↓ Dcg = 0.946
680			0.33		Rod LFD Dgg = 1.44
↓			0.28		↓ Dcg = 1.317
692			0.42		Rod LFH Dgg = 1.36
↓			0.41		↓ Dcg = 1.16
698			0.76		Rod LFM Dgg = 1.18
↓			0.52		↓ Dcg = 1.04
729			0.44		Rod LFT Dgg = 1.42
↓			0.56		↓ Dcg = 1.28
753			0.52		Rod LFX Dgg = 1.387
↓			0.40		↓ Dcg = 1.253
757			0.92		Rod LFW Dgg = 1.24
↓			0.64		↓ Dcg = 1.11
692			0.33		Rod LFE Dgg = 1.46
↓			0.30		↓ Dcg = 1.31
713			0.44		Rod LFK Dgg = 1.38
↓			0.38		↓ Dcg = 1.23
718			0.45		Rod LFS Dgg = 1.35
↓			0.36		↓ Dcg = 1.224

APPENDIX C

HEAT TRANSFER EQUATIONS

APPENDIX C

HEAT TRANSFER EQUATIONS

The general heat transfer equation, which applies both in the fuel and in the clad, is

$$-\nabla \cdot [K(T) \nabla T] = Q(r, \phi, Z)$$

Assuming radial outward flow of heat (i.e. that the gradient ∇T is dominant in the radial direction) this equation reduces to a one-dimensional form for cylindrical geometry:

$$(1) \quad -\frac{1}{r} \frac{d}{dr} \left[K(r) r \frac{dT}{dr} \right] = Q(r)$$

If it is assumed that $Q(r) = 0$ in the clad, then the heat transfer equation in this region reduces to

$$-K(T) \frac{dT}{dr} = \frac{C}{r} = \frac{q}{2\pi r} = \text{heat flux at } r$$

Since the clad is comparatively thin, it will suffice to evaluate K_C at the cladding OD first; this temperature is given by the coolant temperature plus the ΔT across the film, given by the Jens Lottes equation,

$$\Delta T_{\text{FILM}} = 7.94 e^{-P/62} \left(\frac{q}{\pi d} \right)^{0.25}$$

One can then calculate $\Delta T_{\text{CLAD}} = \frac{q}{\pi d K_C (T_{\text{CS}})} \ln \left(\frac{C}{b} \right)$

Next, evaluate $\bar{K}_C = K_C \left(\frac{T_{\text{CS}} + \Delta T_{\text{CLAD}}}{2} \right)$ and recalculate ΔT_{CLAD} .

Then let $\bar{T} = (T_{\text{CS}} + T_{\text{CI}})/2$, evaluate $K_C(\bar{T})$, and recalculate ΔT_{CLAD} . Repeat this procedure until successive values of T_{CI} agree within the desired convergence criterion.

In the fuel $Q(r) \neq 0$, and one must formulate the following integral:
First multiply both sides of (1) by $2\pi r dr$. Then integrate from $r' = 0$ to $r' = r$.

$$- 2\pi \ell \int_0^r \frac{d}{dr'} \left[K(T) r' \frac{dT}{dr'} \right] dr' = 2\pi \ell \int_0^r r' Q(r') dr'$$

The result is

$$(2) \quad - 2\pi \ell K(T) r \frac{dT}{dr} = 2\pi \ell \int_0^r r' Q(r') dr' = \text{heat produced within radius } r$$

If both sides of (2) are divided by $2\pi r \ell$, we have that

$$- K(T) \frac{dT}{dr} = \frac{\text{heat produced within } r}{\text{cylindrical area at } r} = \text{heat flux at } r$$

If variables are separated, we have

$$(3) \quad \int_{T_{FS}}^{T_1} K(T) dT = \int_{r_1}^a \frac{dr}{r} \left[\int_0^r Q(r') r' dr' \right]$$

where the two sides have been integrated over corresponding intervals.

If the pellet is annular and not solid the lower limit on the inner integral is r_i , the inner radius. In any case Robertson⁽¹⁹⁾ shows that (3) can be reduced to

$$\int_{T_{FS}}^{T_1} K(T) dT = \frac{q}{4\pi} F(r_1, \kappa, r_i, a)$$

APPENDIX D

STATISTICAL TECHNIQUES AND EQUATIONS

APPENDIX D

STATISTICAL TECHNIQUES AND EQUATIONS

Use of a Random Number Generator to Sample From an Assumed Distribution

As described in Part II, each of five input variables was assigned a mean (reported) value and a standard deviation, and assumed to have normal distribution. The problem is to select a representative simultaneous sample from these assumed distributions.

Consider for example $T_1 = U_{T_1} \pm Y\sigma_{T_1}$

where U_{T_1} = the reported value for T_1

Y = normal random variable with 0 mean and
unit standard deviation

A set of n random choices of Y will provide an acceptable sample of T_1 values, provided n is large enough. We shall return to the proper size for n in a moment, but first let us see how to develop the random variable Y .

If X_1 and X_2 are random variables distributed with uniform probability on the interval 0 to 1, then it can be shown that^(a)

$$Y = \sqrt{-2\ln X_1} \sin(2\pi X_2)$$

is distributed normally with 0 mean and unit standard deviation. Since Y can also be found as

$$Y = \sqrt{-2\ln X_1} \cos(2\pi X_2),$$

(a) See A. Roterberg, "A New Pseudo-Random Number Generator," J. ACM 7, 1 (1960) pp. 75-77

it is possible to calculate two independent Y variables for each pair of $X_1 X_2$ selected which cuts the (Monte Carlo) selection process in half with respect to computer time.

If h_{GAP} is normally distributed, a sample size $n = 25$ would surely be large enough to estimate σ_h . Histograms of samples for small mean h_{GAP} values show that h_{GAP} is fairly normally distributed in these cases. But when h_{GAP} increases above $0.5 \text{ W/cm}^2 - ^\circ\text{C}$, the distribution of h_{GAP} becomes increasingly skewed. At large values of h_{GAP} (above 1.0) it is more profitable to examine ΔT_{GAP} , the distribution of which remains symmetric for small and large mean values.

Linear Regression - Estimation of the Coefficients

As indicated in Part III, the only regression work in this report is linear, which involves fitting the model

$$\hat{h} = A_0 + A_1 X_1 + A_2 X_2 + \dots + A_{p-1} X_{p-1}$$

to the data. The regression coefficients $A_0, A_1, \dots, A_{p-1}$ are chosen to minimize the sum of squared deviations from the model, i.e., to minimize

$$\sum_{i=1}^N (h_i - \hat{h})^2 = \sum_{i=1}^N e_i^2$$

In matrix form,

$$\underline{h} = \underline{\hat{h}} + \underline{e} = \underline{XA} + \underline{e}$$

where $\underline{h} = N \times 1$ vector of data points

$\underline{A} = p \times 1$ vector of regression coefficients

$\underline{X} = N \times p$ matrix, the first column of which is weights,
the second column is X_{1i} corresponding to h_i , etc.

$\underline{e} = N \times 1$ vector of deviations

Now $\sum_{i=1}^N e_i^2 = \underline{e}'\underline{e}$, and it is easily shown that^(a) the minimization of $\underline{e}'\underline{e}$ yields the so-called normal equation for $\underline{A}$:

$$\underline{A} = (\underline{X}'\underline{X})^{-1} \underline{X}'\underline{h}$$

The program MARTHA simply finds the inverse of the $p \times p$ $\underline{X}'\underline{X}$ matrix and calculates $\underline{A}$ as indicated.

Calculation of Confidence Intervals and Prediction Intervals

The so-called residual sum of squares SS_{res} is the difference between the deviations (squared) of the h_i from their mean, $\bar{h}$, and the deviations of $\hat{h}_i$ from $\bar{h}$. That is,

$$\begin{aligned} SS_{res} &= \sum_{i=1}^N (h_i - \bar{h})^2 - \sum_{i=1}^N (\hat{h}_i - \bar{h})^2 = \sum_{i=1}^N (h_i)^2 - N(\bar{h})^2 \\ &\quad - \sum_{i=1}^N (\hat{h}_i)^2 - N(\bar{h})^2 \end{aligned}$$

Thus in matrix form,

$$\begin{aligned} SS_{res} &= \underline{h}'\underline{h} - \underline{\hat{h}}'\underline{\hat{h}} = \underline{h}'\underline{h} - (\underline{X}\underline{A})'(\underline{X}\underline{A}) \\ &= \underline{h}'\underline{h} - (\underline{A}'\underline{X}')[\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{h}] = \underline{h}'\underline{h} - \underline{A}'\underline{X}'\underline{h} \end{aligned}$$

The term SS_{res} is calculated in MARTHA by the above equation. The residual variance is just

$$\frac{SS_{res}}{N-2} = s_{res}^2$$

The "variance of predicted h " is calculated as

$$\hat{s}_{h_i}^2 = s_{res}^2 \left[\underline{X}_i'(\underline{X}_i'\underline{X}_i)^{-1}\underline{X}_i \right] \text{ where } \underline{X}_i = \underline{X} \text{ at data point } i$$

(a) See Draper and Smith, Applied Regression Analysis (Wiley Series, 1966) Chapter 2.

An approximate 95% confidence band about the regression line is given by $\hat{h}_i \pm 2 S_{\hat{h}_i}$ at X_i .

On the other hand the 95% prediction interval is given by $\hat{h}_i \pm 2 S_i$ where S_i given by

$$S_i^2 = S_{\text{res}}^2 + S_{\hat{h}_i}^2.$$

When calculating an "upper bound" (i.e., a one-sided 95% confidence interval) one uses the relation

$$h_{\text{upper bound}} = \hat{h} + 1.65 S_i$$

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