Final Report
**“Collaborative Research: Experimental and Theoretical Study of the Plasma
Physics of Antihydrogen Generation and Trapping”**

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This report documents progress on studies of the physics of antihydrogen plasmas supported by the Department of Energy. The coordinating PI is Prof. Fajans at UC Berkeley. Prof. Fajans is also the US coordinator for the ALPHA antihydrogen collaboration based at CERN. Profs. Robicheaux and Wurtele are also members of the ALPHA collaboration. The collaboration itself holds meetings every two months, and the US collaborators are in frequent contact with the PI.

The investigators collaborate with each other through participation in weekly ALPHA video meetings, through numerous interactions with the coordinating PI Fajans and other experimenters at CERN, and through the residence of Ordonez’ graduate student at UC Berkeley, where he has been learning to use the beam physics simulation codes, developed at LBNL for heavy ion fusion, and now modified to simulate antihydrogen traps.

1. Experimental Progress

We have made great experimental progress this year, culminating in the likely trapping of six antihydrogen atoms. It is important to note, however, that the antihydrogen events have not yet been absolutely identified as such, and we are not yet making a “discovery” claim.

Some of the areas in which we have made progress are listed below:

Record low temperature lepton temperatures under realistic conditions

Low electron and positron temperatures are necessary to generate trappable antihydrogen. Low electron temperatures are necessary because electrons are used as a bath in which we initially cool the antiprotons. Low positron temperatures are required to increase the antihydrogen formation rate. They are also necessary because, during synthesis, the antiprotons quickly come into thermal equilibrium with the positrons, so the temperature of the antihydrogen which is formed is roughly equal to the positron temperatures.

Before the 2009 season, we were unable to routinely generate plasma whose temperatures were much below several hundred Kelvin, particularly when we were at the unusually low axial magnet field (1T) used during antihydrogen synthesis, when our octupole magnet, used to confine any resultant antihydrogen, was on, and when the lepton densities were high enough to allow quick cooling and synthesis. During the 2009 season, we developed techniques that routinely gave us plasmas, with all the above conditions met, which were in the range of 30 to 100K. Note that these measurements were made with an absolute temperature thermometer based on measuring the evaporation of particles.¹

Record low antiproton temperatures

Antiprotons captured from CERN's Antiproton-Decelerator (AD) are initially cooled in an electron bath. The electrons are removed from the antiprotons by time-of-flight methods, which, if not done very carefully, heat the antiprotons. During the 2009 season, we learned how to better perform this process, and lowered the resulting antiproton temperatures from several 1000K to 200K.

Given that our trap depth is approximately 0.5K, this improvement may not be sufficient. To further lower the temperatures, we developed an evaporative cooling technique, and have achieved temperatures as low as 10K at the expense of losing antiprotons. Typical data is shown in Figure 1.

To measure these low temperatures, we had to develop an appropriate thermometer. We extended a well known technique, based on evaporation of antiprotons out of a shallow confining well, to antiprotons. Typical measurements are shown in Figure 2.

¹In the un-refereed literature The ATRAP collaboration has recently claimed to have achieved substantially lower temperatures. However, their temperature diagnostic, based on plasma modes, measures only relative temperatures. They cannot measure the absolute temperatures of their plasmas. ATRAP has released little information about their measurements, but from what they have released, it appears that their measurements occur in plasmas that are not useful for antihydrogen synthesis; their multipole magnet is off, their axial field may be quite a bit higher than would be used in synthesis, and as they have no control over their plasma radii, their densities are likely low.

A paper reporting these techniques has been submitted to Physical Review Letters.

Figure 1: a) Temperature vs. the on-axis well depth. The error is the combined statistical uncertainty from the temperature fit and an uncertainty associated with the applied potentials (one σ). A model calculation is shown as a line. b) The fraction of antiprotons remaining after evaporative cooling vs. on-axis well depth. The uncertainty on each point is propagated from the counting error (one σ). The initial number of antiprotons was approximately 45,000, in an on-axis well depth of (1484 ± 14) mV.

Figure 2: Integrated antiproton loss vs. applied on-axis well depth, with the integrated antiproton number corrected for the 25% detection efficiency. The antiproton temperature can be measured by fitting an exponential to the curves. (The exponential fit is shown as the solid lines.) As the antiprotons get colder, fewer can be used to determine their temperature. The curves are labeled in decreasing order of the temperatures. The temperatures are: A: 1,040 K B: 325 K C: 57 K D: 23 K E: 19 K F: 9 K

Autoresonant antiproton injection in to a positron plasma

To synthesize antihydrogen, antiprotons must be injected into a positron plasma. As positrons have the opposite charge sign, this injection process can greatly heat the injected antiprotons and the target positron plasma. During the 2009 season, we extended the technique of autoresonance, first explored in Berkeley, to drive antiprotons into the positrons. Autoresonance involves exciting the antiprotons with a chirped frequency drive. The antiprotons are held in a nonlinear well; the antiproton energy will automatically adjust itself so that the antiproton oscillation frequency matches the drive frequency.

Autoresonant injection proves very advantageous; the resultant plasma temperatures have been as low as 200K, many hundreds of Kelvin lower than with other techniques. Further, using autoresonance, we maintain close control of the antiproton energy.

It was not obvious that autoresonance should work for our antiprotons. Autoresonant theory is based on single particle models; our antiprotons are cold enough to be plasmas. Consequently they have strong self fields, which one might think would destroy the coherent oscillations found in autoresonance. Remarkably, we find that these self-fields instead conspire to turn our plasma into an effective single particle, an effect first discovered in simulations within the ALPHA collaboration funded by our previous DOE grant, and lately confirmed by a recent PRL by Barth, et al. We are preparing a paper on autoresonance for submission to PRL. Some data from the manuscript is shown in Figure 3, Figure 4 and Figure 5.

Figure 3: Antiproton axial kinetic energy as a function of the autoresonant drive frequencies (independent axis is vertical.)

Figure 4: Histogram of the antiproton energy after excitation by an autoresonant drive. The smooth curves are the theoretical predictions, while the somewhat irregular curves are the measured data.

Figure 5: Verification of the scaling of the required autoresonant drive strength D with the chirp rate α , verifying the $D=\alpha^{3/4}$, ascaling first discovered at Berkeley.

Probable trapping of antihydrogen

Using our newly developed techniques, we attempted to trap antihydrogen. As discussed above, we mixed the cold antiprotons into the positron plasma using autoresonance. This synthesized antihydrogen with high efficiency. Superimposed on the normal Penning-Malmberg magnetic fields was a magnetic minimum created by octupole and mirror coils. We hoped that some of the antihydrogen would be trapped in the magnetic minimum. However, the magnetic minimum trap depth is on the order of 0.5K, and the antiprotons were, after mixing, at about 200K. Thus, only a very small fraction of the resultant antihydrogen could be trapped. We have not yet incorporated evaporative cooling, or optimized the autoresonant mixing parameters. We hope to be able to lower the temperature of the antiprotons significantly to enhance the probability of trapping.

One second after the positrons and antiprotons were mixed together, we cleared out the remaining charged particles with strong electric fields. This took approximately 150ms. Then we quenched the octupole and mirror coils; the coil currents decayed with an exponential time constant of approximately 9ms. Any trapped antihydrogen would be released as the magnets quench and would then annihilate on the trap walls.

In 211 trapping attempts, six events were positively identified as antihydrogen or antiproton annihilations. (We can discriminate between antiparticles and cosmic rays with a power greater than five sigma.) Unfortunately our detector cannot directly discriminate between antihydrogen and antiprotons. While during the quench there is no longer an electrostatic well to confine the antiprotons longitudinally, antiprotons with high perpendicular energy (and low parallel energy) could be trapped by the mirror magnet fields. Such mirror-trapped antiprotons are our major background, and we went to great lengths to eliminate them.

Extensive simulations at Berkeley and Auburn demonstrated that antiprotons would have to have more than about 25eV of perpendicular energy to survive the strong electric fields that we use to clear them at the end of the mixing cycles. Given that the bulk temperature of the antiprotons is on the order of 200K, or 0.02eV, it is hard to imagine how any antiproton could acquire 25eV of perpendicular energy. Indeed, we have considered many possible sources of high energy antiprotons, including collisions with neutrals, acceleration and collisions of ionized antihydrogen in side wells, and all aspects of our mixing cycles and failed to find a plausible mechanism to generate them. Nonetheless, we cannot utterly rule out the existence of mirror-trapped antiprotons. Consequently, we (Berkeley and Auburn) undertook an extensive study contrasting the behavior of mirror-trapped antiprotons and antihydrogen under the influence of the quenching magnetic fields. The simulations show that there is a clear signature discriminating between the two species. Antihydrogen annihilates over the entire length of the trap, while mirror-trapped antiprotons annihilate near the center of our trap. Antihydrogen begins to annihilate almost immediately after the quench begins, while antiprotons annihilate significantly later. While the evidence from the simulations is quite clear, we have decided not to publish a “discovery” claim until we have performed some additional tests like verifying the simulations with deliberately induced mirror-trapped antiprotons, independent experimental null tests of the trapping mechanism, and establishing enough control over the production rate to increase the trapping frequency. We are preparing a paper on our preliminary results for publication in Physics Letters B.

Figure 6: Spatial and temporal locations of simulated (green) antihydrogen atoms and the six measured (red) antimatter annihilations. The annihilations occur as the antimatter is released from the minimum-B trap during the magnetic field quench. The antihydrogen atoms were launched into the simulations with a distribution thought to be very representative of the actual distribution; 43775 antihydrogen atoms were simulated. Note the measured annihilation locations overlap well with the simulated locations.

Figure 7: Spatial and temporal locations of simulated (green) mirror-trapped antiprotons and the six measured (red) antimatter annihilations. As in Figure 6, the annihilations occur as the antimatter is released from the minimum-B trap during the magnetic field quench. As discussed in the text, we know of no mechanism that will result in mirror-trapped antiprotons surviving into the quench; consequently, the simulations were launched from a highly artificial distribution of high ($>25\text{eV}$) perpendicular energy mirror-trapped antiprotons; this simulation simulated 84040 mirror-trapped antiprotons. A wide range of distributions were studied, and all resulted in similar spatial and temporal locations. Note that the measured annihilation locations are incompatible with the simulated locations, strongly suggesting that the measured events could not be mirror-trapped antiprotons.

Papers published

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2. G. B. Andresen, W. Bertsche, C. C. Bray, E. Butler, C. L. Cesar, S. Chapman, M. Charlton, J. Fajans, M. C. Fujiwara, D. R. Gill, W. N. Hardy, R. S. Hayano, M. E. Hayden, A. J. Humphries, R. Hydomako, L. V. Jørgensen, S. J. Kerrigan, J. Keller, L. Kurchaninov, R. Lambo, N. Madsen, P. Nolan, K. Olchanski, A. Olin, A. Povilus, P. Pusa, F. Robicheaux, E. Sarid, S. Seif El Nasr, D. M. Silveira, J. W. Storey, R. I. Thompson, D. P. van der Werf, J. S. Wurtele, and Y. Yamazaki, Magnetic Multiple Induced Zero-Rotation Frequency Bounce-Resonant Loss in a Penning-Malmberg Trap Used For Antihydrogen Trapping, Phys. Plasmas, **16** 100702, 2009.
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2. Theoretical Progress

Theory research is conducted by Prof. Ordonez (PI for the University of North Texas, Denton), Prof. Robicheaux (PI at Auburn University) and Prof. Wurtele (co-PI at UC Berkeley), and their groups.

Research at Auburn University (Prof. Robicheaux)

Research at Auburn is funded by the NSF, and will not be summarized here.

Research at U. North Texas (Prof. Ordonez)

Collisional scattering between antiprotons and strongly magnetized leptons

The research associated with this topic is being carried out by UNT graduate student, Jose Correa, while in residence at UC Berkeley. Mr. Correa is conducting his doctoral dissertation research under the joint supervision of Profs. Wurtele and Ordonez. One of Mr. Correa's topics of study is collision phenomena between antiprotons and electrons or

positrons under strongly magnetized conditions. An analytical theory of momentum transfer between antiprotons and electrons has been developed, including an expression for the energy loss rate of an antiproton that slows down within an electron plasma. Agreement with prior research reported by Prof. Robicheaux and his students has been reached. The new theory is being extended to include the kinematic effects of velocity distributions and to apply to collisions between antiprotons and positrons. The latter extension is complicated by the formation of quasibound states of antihydrogen. Prior research on effects associated with antiproton collisions is reported in

[*"Antiproton Cross-Field Diffusion in Antihydrogen Production Experiments Due To Anisotropic Binary Interactions," C. A. Ordonez and J. R. Correa, Nuclear Instruments and Methods in Physics Research B 261 (2007) 248-251.]

[*"Positron Cross-Field Transport Due To Quasibound States of Antihydrogen," Y. Ahat, C. E. Correa, and C. A. Ordonez, Nuclear Instruments and Methods in Physics Research B 261 (2007) 252-254.]

Artificially structured boundary for antimatter studies

The possibility of developing an alternative to the nested Penning trap for mixing antiprotons and positrons under conditions for forming antihydrogen provided the initial motivation for studies on this topic. An artificially structured boundary is defined at present as one that produces a spatially periodic static field, such that the spatial period and range of the field is much smaller than the dimensions of a cloud, plasma or beam of charged particles that is next to the boundary. Research on this topic has been overviewed recently in

[*"Drifting Plasma Confinement With a Spatially Periodic Field," C. A. Ordonez, IEEE Transactions on Plasma Science 38 (2010) 388-392.]

The research is intended to provide an increased understanding of the ability to confine and control charged-particle clouds, plasmas and beams. A preliminary discussion of antihydrogen studies that may be possible with an artificially structured boundary is provided in

[*"Charged Particle Reflection From an Artificially Structured Boundary That Produces a Spatially Periodic Magnetostatic Field," C. A. Ordonez, Journal of Applied Physics 106 (2009) 024905.]

Non-rotating non-neutral plasma confinement

A non-neutral plasma within a Penning trap experiences rotation to achieve a radial force balance equilibrium. For antiprotons used in mixing studies within a nested Penning trap, the kinetic energy associated with rotation can be much larger than the maximum energy an antihydrogen atom can have to remain magnetically trapped. It may be possible to trap a dense non-rotating, non-neutral plasma of antiprotons within a modified Kingdon trap that employs an artificially structured boundary.

A Kingdon trap nominally employs a combination of electrostatic and centrifugal forces for confining a non-neutral cloud or plasma of charged particles. Charged particles that are confined within a Kingdon trap typically experience an orbital drift between a pair of cylindrically symmetric coaxial electrodes. The orbital drift provides a radially outward centrifugal force that keeps particles from reaching the inner electrode. The electric field produced by the electrodes provides a radially inward force that keeps the particles from reaching the outer electrode. Axial confinement is also provided electrostatically. Criteria for confining charged particles that have a drifting Maxwellian velocity distribution are reported in

[*"Effective Well of a Static Kingdon Trap," C. A. Ordonez, Physics of Plasmas 15 (2008) 072508.]

It may be possible to reduce or eliminate the orbital drift in a Kingdon trap by employing an artificially structured boundary at the inner electrode. The artificially structured boundary would produce a multipole magnetic field that keeps particles from reaching the inner electrode. The magnetic field is sufficiently short in range to affect the particle trajectories only in close proximity to the inner electrode. The conditions for producing such a magnetic field have been assessed. The results indicate that a magnetic field that is sufficiently short in range to affect the particle trajectories only in close proximity to the inner electrode must be an octupole or higher order field. After a non-rotating, non-neutral antiproton plasma is trapped, it could be evaporatively cooled by slowly reducing the depth of the electric potential well. Afterward, it may be possible to transition operation of the trap to a minimum-B configuration that can simultaneously confine antiprotons and antihydrogen.

Plasma equilibria within nested Penning traps for antihydrogen studies

A computer simulation code "NestedEqV2" that is fast enough for carrying out parameter studies on various schemes for mixing antiprotons and positrons to produce antihydrogen within a nested Penning trap, has been fully documented and thoroughly debugged. The code can provide quick initial assessments of quasi-static mixing schemes, but without considering the effect of an atom-trapping (e.g., octupole) field. The code evaluates static equilibria self-consistently in two dimensions. The code should be helpful for conducting mixing studies similar to those reported in

[*"Two-Species Mixing in a Nested Penning Trap for Antihydrogen Trapping," C. A. Ordonez and D. L. Weathers, Physics of Plasmas 15 (2008) 083504.]

[*"Effect of Positron Space Charge on Operation of an Antihydrogen Trap," C. A. Ordonez, Physical Review E 76 (2007) 017402.]

Time-of-flight antiproton perpendicular temperature diagnostic

A time-of-flight diagnostic might be useful to determine the perpendicular temperature of an antiproton cloud. The following procedure would be used: (1) A few antiprotons would be released suddenly in a short pulse and travel along a diverging magnetic field, as is done when using the ALPHA MCP diagnostic. The release conditions would be

similar to evaporative cooling conditions, such that the parallel energy of the released antiprotons would be much smaller than the average perpendicular energy. During flight, however, perpendicular energy is effectively converted to parallel energy. (2) The time of flight of the antiprotons would be measured and, together with a theoretical model of the conversion of perpendicular energy to parallel energy during flight, would be used to determine the perpendicular temperature of the antiprotons prior to their sudden release. An alternative to step (2) would be to adiabatically extract the antiprotons from the magnetic field using an artificially structured boundary and then measure the energy of the antiprotons. Related research is found in

[*"Charged Particle Transport Through a Periodic Electrostatic Potential Having a Small Spatial Period," C. A. Ordonez, Journal of Applied Physics 104 (2008) 054903.]

[*"Effect of a Periodic Electrostatic Potential on Magnetized Particle Transport," C. A. Ordonez, Physics of Plasmas 15 (2008) 114507.]

Direction of gravitational force on antimatter

The endeavor to study antihydrogen is not unlike the endeavor to study a gaseous Bose-Einstein condensate. Both require the preparation of a system of particles, which transitions into the state to be studied. Both have been pursued by substantial numbers of researchers. The endeavor to study a gaseous Bose-Einstein condensate has, of course, met with spectacular success. Evaporative cooling was a key breakthrough that allowed the study of a gaseous Bose-Einstein condensate. Evaporative cooling may also be a key breakthrough that allows the first studies on the effect of gravity on antihydrogen. The research associated with this topic is aimed at investigating a possible measurement of the direction in which the force of gravity acts on antihydrogen atoms. The following procedure would be used: (1) Antiprotons would be compressed to a radius of about 0.29 mm within a 3 T field. (2) Antiprotons would be evaporatively cooled using techniques developed by the ALPHA collaboration. (3) The antiprotons would be released to travel along a diverging magnetic field, which effectively converts the perpendicular antiproton energy into parallel energy. (4) The antiprotons enter a region where a few of them form antihydrogen atoms. (5) Antihydrogen atoms pass through apertures and travel a sufficient distance for gravity to have a measurable effect on their vertical position. An alternative approach would be to adiabatically extract the antiprotons from the magnetic field using an artificially structured boundary prior to step (4), send the antiprotons into a change in electric potential to reduce the energy of the fraction that is transmitted, and then trap the antiprotons within a Kingdon trap for forming the antihydrogen atoms.

Research at UC Berkeley (Prof. Wurtele)

Statistical Methodologies

Employing a Bayesian framework, we have begun to address two questions of statistical inference important for the ALPHA experiment:

- (a) Estimation of Temperature from Trap-Lowering Protocol

Much circumstantial evidence suggests that the trapped charges are not equilibrating to the temperature of the trap walls maintained by liquid hydrogen near 4K. Information about the energy distribution can be obtained by lowering the potential well and measuring a signal proportional to the number of particles that escape the well. To help obtain optimal estimates of temperature from these measurements, we have developed a Bayesian algorithm that outputs a posterior distribution for the unknown temperature. Currently, we have applied this to a highly simplified case where the particles constitute a one-dimensional ideal plasma moving predominately in an external potential, which is lowered either suddenly (conserving energy) or adiabatically (conserving action except at the separatrix-crossing). Generalization to include transverse spatial effects, self-consistent fields or even particle correlations, and arbitrary rates of change in the external potential may be possible.

(b) Evidence for Anti-Hydrogen Observation

After the last experimental run, producing after suitable cuts a handful of possible antihydrogen events, an obvious question arises: how many of these candidate events were actually from trapped anti-hydrogen, versus unswept anti-protons, or else cosmic rays or other background effects? While of direct interest, this question cannot be directly addressed by orthodox statistics interpreting probabilities invariably as relative frequencies in random experiments, which treats the number of observed antihydrogens as an unknown but ineluctable fact, which cannot be ascribed a probability. However, in a Bayesian approach, probabilities can be assigned directly to the possible classifications of candidate events as anti-hydrogen or not. This approach automatically makes use of any available prior information about the expected numbers of antihydrogen atoms or antiprotons remaining in the trap, as well as the expected number of cosmic rays passing through the trap, any information about the detector efficiencies, and information predicted from Monte Carlo simulations about the different spatio-temporal patterns of reconstructed annihilation events from trapped antihydrogens versus lingering anti-protons, or else false events from background cosmic rays. Such a procedure can lead to the most reliable inferences possible about the possibility of anti-hydrogen detection in the light of very limited numbers of candidate events.

Zero-frequency bounce resonant loss

In many antihydrogen trapping schemes, antiprotons held in a short-well Penning–Malmberg trap are released into a longer well. This process necessarily causes the bounce-averaged rotation frequency of the antiprotons around the trap axis to pass through zero. In the presence of a transverse magnetic multipole, experiments and simulations show that many antiprotons, over 30% in some cases, can be lost to a hitherto unidentified bounce-resonant process when it is close to zero. The resonance can be qualitatively seen in both simple guiding-center codes and in the full Lorenz equations. Initial results have been published in an initial paper [Ref. 2 above]. Parametric studies of

initial conditions for which a particle becomes caught in the resonance and hits the trap wall are being conducted with a new simulation code (Shepard Code).

Figure 7. An antiproton orbit is plotted projected onto the (x,y)-plane. The trap is aligned on the z-axis. The initial conditions are chosen to illustrate the radial drift outwards when the zero-frequency bounce-resonance occurs.