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The Physics of Ion Decoupling in Magnetized Plasma Explosions

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Overview: Decoupled Ions in HANES

When a finite pulse of plasma expands into a magnetized background plasma, MHD predicts the pulse expel background plasma and its B-field—i.e. cause a magnetic “bubble”.

The expanding plasma is confined within the bubble, later to escape down the B-field lines. MHD suggests that the debris energy goes to expelling the B-field from the bubble volume and kinetic energy of the displaced background.

For HANEs, this is far from the complete story.

For many realistic HANE regimes, the long mean-free-path for collisions necessitates a Kinetic Ion Simulation Model (KISM). The most obvious effect is that the debris plasma can decouple and slip through the background plasma.

The implications are:
1) the magnetic bubble is not as large as expected and
2) the debris is no longer confined within the magnetic bubble.
Traditional MHD Modeling of HANES Misses Important Physics

Consider a HANE-relevant debris pulse into the ambient ionospheric density at 400 km altitude. For typical densities (here 3e5 O⁺¹ ions/cm³), a STARFISH relevant explosion produces magnetic bubbles such as these.

Contours of magnetic field

“Near” MHD limit

Finite plasma-density inside

Earth not to scale

Kinetic limit

Contours of magnetic field

Nearly-zero plasma-density inside

Time=.3

Just U⁺¹ Debris & O⁺¹ background ionosphere

Time=.1

Today we focus on the early-time coupling of debris ions to the background plasma

ambient densities, charge state +1

Introducing More Realism reveals important non-MHD behavior

Parameter changes towards more realistic physics lead to interesting changes in coupling of the debris to the ionosphere.

Ion debris decoupling is very sensitive to the charge states and drives a requirement for improved atomic physics.
Initial debris configuration:
\(v_r=2\times10^8\) cm/sec \(v_\theta=0\).
Homogeneous, 20 km radius debris "puff"
Here is the same physics with added background for a typical coupled & decoupled case.
A run with U slipping through O reveals what’s important for decoupling.

U⁺ moves out in spite of negative Eᵣ

And the criterion for decoupling is...

\[ \left( \frac{\pi}{2} - 1 \right) \frac{Z_D n_D}{Z_B n_B} < 1 \]

Essential piece is positive Eₒ due to debris outflow in quasi-neutral equations.

It is just this Eₒ that gives the ExB drift at the Alfvén speed.
We developed a simple criterion for when Kinetic Ion Models are essential for modeling HANE

Debris “decouples” depending on background electron density

Higher debris charge states couple more strongly

\[ Z_B n_B = \left( \frac{\pi}{2} - 1 \right) Z_D n_D \]

Just U\(^+\) Debris & O\(^+\) background ionosphere

Time=.1

\( Z_{\text{debris}} = 1 \)

ambient

\( Z_{\text{debris}} = 1 \)

100x ambient

\( Z_{\text{debris}} = +2 \)

Traditional modeling

Flash ionization

More realistic debris charge state

Ion debris decoupling is very sensitive to the charge states and drives a requirement for improved atomic physics
Consider the fields generated by the debris in this quasi-neutral, collisionless plasma

Start with the electron momentum equation

\[ m_e n_e D_t \vec{u}_e = e n_e \vec{E} + \nabla P_e + \vec{J}_e \times \vec{B} / c \]

In the zero electron mass limit, we solve for \( \vec{E} \)

\[ \vec{E} = \frac{\nabla P_e}{e n_e} - \frac{\vec{J}_e \times \vec{B}}{e n_c} \]

Assuming quasi-neutrality and using the Darwin limit of Ampere’s law,

\[ c \nabla \times \vec{B} = 4\pi (\vec{J}_i + \vec{J}_e) \]

the expression for \( \vec{E} \) becomes

\[ \vec{E} = \frac{\nabla P_e - \vec{J}_e \times \vec{B}}{e n_e c} \quad \vec{J}_e = \frac{c}{4\pi} \nabla \times \vec{B} - \sum_{\text{species}} Z_j n_j \Rightarrow \vec{E} = \frac{\nabla P_e - \left( \frac{c}{4\pi} \nabla \times \vec{B} - \sum_{\text{species}} Z_j n_j \right) \times \vec{B}}{e \sum_{\text{species}} Z_j n_j} \]

For this simple 1-D case, early in time

\[ \nabla P_e \sim 0 \quad \nabla \times \vec{B} \sim 0 \]

so what matters is

\[ E_\theta = \frac{Z_D n_D u_{Dr}}{c(Z_D n_D + Z_B n_B)} B_z \quad E_r = -\frac{Z_B n_B u_{Bl}}{c(Z_D n_D + Z_B n_B)} B_z \]

The debris generates electric fields as it passes through the background, however not in the “obvious” directions.

Linearize:

\[ u_{Dr} \sim 0 \quad u_{Bl} \sim 0 \]
Consider how a background ion responds to these fields as the debris passes through the plasma.

\[ E_\theta = \frac{Z_D n_D u_{Dr}}{c(Z_D n_D + Z_B n_B)} B_z \]

\[ E_r = -\frac{Z_B n_B u_{B\theta}}{c(Z_D n_D + Z_B n_B)} B_z \]

With just the azimuthal field.
The background will be left behind if it acquires too little “speed”

To coupling, the background ions must acquire enough velocity to remain in front of the debris.

The secret of the decoupling lies in the
1) magnitude of $E_\theta$
2) time it spends in this field

$$E_\theta = \frac{Z_D n_D u_{Dr}}{c(Z_D n_D + Z_B n_B)} B_z$$

In this simple case, this speed must be acquired in roughly the first $1/4$ of a gyro-period. Combining...

$$\frac{\Delta u_{Bimpulse \theta}}{\Delta t} = \frac{Z_B e}{m_B} E_\theta \quad \Delta t = \frac{\pi}{2 \omega_{cB}}$$

$$\Delta u_{Bimpulse \theta} = \frac{\pi c E_\theta}{2 B_z} = \frac{\pi c}{2 B_z} \frac{Z_D n_D u_{Dr}}{c(Z_D n_D + Z_B n_B)} B_z$$

$$= \frac{\pi}{2} \frac{Z_D n_D u_{Dr}}{Z_D n_D + Z_B n_B} u_{Dr}$$

$$\frac{\Delta u_{Bimpulse \theta}}{u_{Dr}} = \frac{\pi}{2} \frac{Z_D n_D}{2 (Z_D n_D + Z_B n_B)}$$

so the debris is DEcoupled if

$$\frac{\Delta u_{Bimpulse \theta}}{u_{Dr}} = \frac{\pi}{2} \frac{Z_D n_D}{Z_B n_B} \equiv \alpha_{dc} < 1$$
Another way to look at this...
Remember the $u_{B\theta}$ assumption

Look at the ratio $\frac{E_r}{E_\theta}$

$$E_r = -\frac{Z_B n_B u_{B\theta}}{c(Z_D n_D + Z_B n_B)} B_z$$

$$E_\theta = \frac{Z_D n_D u_{Dr}}{c(Z_D n_D + Z_B n_B)} B_z$$

$$\Delta u_{B\text{impulse}} = \frac{\pi}{2} \frac{Z_D n_D}{(Z_D n_D + Z_B n_B)}$$

$$\frac{E_r}{E_\theta} = \frac{\pi Z_B n_B}{2 (Z_D n_D + Z_B n_B)}$$

A series of runs with increasing $n_B$ shows

<table>
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<tr>
<th>$n_B$</th>
<th>$\frac{Z_B n_B}{Z_D n_D}$</th>
<th>$\frac{Z_B n_B}{Z_D n_D}$</th>
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<td>1.2</td>
</tr>
<tr>
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</tr>
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<td></td>
</tr>
<tr>
<td>7e6</td>
<td></td>
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</tr>
</tbody>
</table>

Coupled

$$\frac{E_r}{E_\theta} = \frac{\pi}{2} \frac{5}{4 + 5} = -0.555 \cdots \frac{\pi}{2}$$

Decoupled

$$\frac{E_r}{E_\theta} = -\frac{\pi}{2} \frac{7}{2 + 4.7} = -0.636 \cdots \frac{\pi}{2}$$

Note that $\frac{E_r}{E_\theta} = 1$ at $\frac{Z_B n_B}{Z_D n_D} = 1.7519\ldots$
Simple formula would suggest that the product $Z_B n_B$ is all that matters... However...

$$Z_B n_B > \left( \frac{\pi}{2} - 1 \right) Z_D n_D$$

- Decoupled
  - $Z_B = +1$, $n_B = 3e6$

- Coupled
  - $Z_B = +2$, $n_B = 1.5e6$
  - $Z_B = +3$, $n_B = 1e6$
Higher charge state $Z_B$ (smaller gyro-radius), enhances debris coupling

$Z_D=+1 \ nD=4\times10^6$

Use these fields

$$E_r = -\frac{Z_B n_B u_{Dr}}{c(Z_B n_D + Z_B n_B)} B_z$$

$$E_\theta = \frac{Z_B n_B u_{Dr}}{c(Z_B n_D + Z_B n_B)} B_z$$

but finite debris pulse length

$Z_B=+1 \ nB=3\times10^6$

Always on

Sharp pulse turn on

50 km pulse

Decoupled

$Z_B=+3 \ nB=1\times10^6$

Always on

Sharp pulse turn on

50 km pulse

Coupled
Enhanced coupling leads to another threshold that plays in debris coupling

In addition to the electron density threshold

$$Z_B n_B > \left( \frac{\pi}{2} - 1 \right) Z_D n_D$$

DEcoupling requires the gyro-radius to be BIGGER than the pulse length or

$$r_{B \text{armor}} > \delta_{\text{pulse}}$$

$$r_{B \text{armor}} = \frac{\Delta u_{B \text{impulse}}}{\omega_{Bc}} \quad \omega_{Bc} = \frac{Z_B e B_z}{c m_B}$$

$$r_{B \text{armor}} = \frac{c m_B}{2 e B_z} \frac{Z_D n_D u_{Dr}}{(Z_D n_D + Z_B n_B)} \frac{1}{Z_B} \delta_{\text{pulse}} > \delta_{\text{pulse}}$$

$$\frac{Z_D n_D}{(Z_D n_D + Z_B n_B)} > \omega_B \delta_{\text{pulse}} u_{Dr}$$

that shows the observed additional dependence on background charge state.
Summary: Ion Decoupling in Magnetized Plasma Explosions

Super Alfvénic debris HANE expansions into ionosphere have been shown computationally to decouple from the ionosphere.

Simple, linear arguments have been developed that suggest threshold conditions required for decoupling (or non-fluid-like behavior) to occur.

\[
\frac{\pi}{2} \frac{Z_D n_D}{(Z_D n_D + Z_B n_B)} < 1
\]

\[
\frac{cm_B}{2eB_z} \frac{Z_D n_D u_{Dr}}{(Z_D n_D + Z_B n_B)} > Z_B \delta_{pulse}
\]

This decoupling has interesting implications for both EMP and belt pumping.

Reconsideration of the STARFISH event suggest that these threshold conditions are relevant, and strongly dependent on the initial parameters of the HANE event.

Take away concepts:

Even in linear analysis, finite gyro-radii effect matter.

Threshold seem to apply species by species

(the species in question is the “debris”, all others are part of the “background”)

MHD/Fluid codes will not see these effects