Distribution and occurrence of localized-bursts in two-phase flow through porous media

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ABSTRACT

This study examines the dynamics of two-phase drainage with experiments of air invasion into a translucent water-saturated porous medium, at low injection speeds. Air displaces the water by irregular bursts of motion, suddenly invading small portions of the medium. These periods of activity, followed by dormancy, are similar to descriptions of systems at a self-organized critical point, where a slight disturbance may induce an avalanche of activity. The fractal characteristics of the invading air structure at breakthrough are examined through static (box-counting) calculations of the air mass and through an evaluation of the time-dependent motion of the invading mass; results are compared with prior low-velocity two-phase studies in porous media. Dynamic, power-law scaling for invasion percolation is shown to be well suited to describing the structure of the invading fluid. To examine the applicability of self-organized criticality predictions to the invading fluid movement, a new image analysis procedure was developed to identify the location of individual bursting events during the drainage experiments. The predictions of self-organized criticality, namely the scaling of the occurrence of bursts to the mass of the bursts and a spatio-temporal randomness of different sized bursts, are also examined. Bursts of a wide range of sizes are shown to occur throughout the porous medium, over both time and space. The mass distribution of burst sizes is shown to be well described by self-organized criticality predictions, with an experimentally determined scaling exponent of 1.53.

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1. Introduction

The low saturation of a non-wetting fluid as it invades a porous medium saturated with a wetting fluid is significant in a wide variety of geological and engineering activities. This phenomenon is a primary reason why secondary oil recovery produces a small portion of the in-place oil and why remediation of contaminated subsurface regions is so difficult [1,2]. Additionally, a constraint on the amount of CO₂ that can be stored in deep aquifers during geological sequestration is the motion of the less-viscous CO₂ into brine saturated reservoirs [3,4]. Thus a thorough understanding of two-phase motion in porous media is important. Self-organized criticality has been suggested as one method of describing the sporadic motion of an invading fluid into porous media at low injection rates during drainage processes [5–10].

First described by Bak et al. [11], self-organized criticality (SOC) was introduced as a description of dissipative dynamical systems. Systems that obey SOC are characterized by the organization of an initially complex state of the system into a critical
state, where mass avalanches (or similar sporadic phenomena) occur over a wide range of sizes. This self-organization of the originally complex system into a critical state has been used to describe the behavior of sand piles, forest-fires, earthquakes, evolution, and mass aggregation [11–15], using relatively simple numerical models. These numerical models agree with the predicted behavior of SOC [11,14,15] and have been shown to correspond to the behavior of the physical systems [13,16]. This correspondence has been observed for various scale-invariant relationships, such as the distribution of the size of avalanches in growing piles of granular material, which are linearly related to each other.

Experimental studies of these evolving systems have been less conclusive. The growing sand pile has been frequently studied as an experimental system because of its connection to the first SOC model of Bak et al. [11], who investigated the redistribution of numerical sand grains stacked in a square lattice. Measurements of the occurrence of avalanches in sand piles that have increased to their critical angle of repose (either by tilting a partially filled bin of sand or by slowly growing a pile on a circular disk) have shown fairly good agreement to the predicted scaling form of SOC [16–20]. Even experiments that have not strictly adhered to the SOC scaling behavior have shown a broad distribution of mass avalanches occurring in sporadic fashion [17], as typified by the system at a self-organized critical point. The rice avalanche study of Frette et al. [21], where mass avalanches were measured in several rice piles, is one of the best known of these experiments. By studying piles of rice with different properties (surface roughness, mass, and aspect ratio) Frette et al. [21] demonstrated that when inertial forces play a significant role, a greater deviation from SOC predictions is observed.

Two-phase flow in porous media has been studied experimentally by numerous investigators [22–27]. The injection of a non-wetting fluid into a porous medium initially saturated with a wetting fluid is known as drainage. When the injection is performed at a constant rate the pressure within the invading fluid gradually increases. Restrictions in the pore-throat structure of the porous medium hinder the fluid from advancing linearly. Once the invading fluid pressure exceeds the capillary pressure at the largest restriction (i.e. the lowest throat capillary pressure) the invading fluid will move into the adjacent pore. The interface between fluids readjusts to touch the newly adjacent throats, and quite often the pressure within the invading fluid pressure is large enough to allow fluid movement through one or some of these restrictions as well. Thus, individual bursting events can vary in size from one pore being invaded to many pore bodies being invaded at one time. This localized bursting of the advancing non-wetting fluid into the initially saturated porous medium is the step-wise process of drainage. These “localized bursts” have been called avalanches, Haines jumps, or interface depinnings in the literature.

Relatively few experimental studies have evaluated the distribution of burst sizes that occur when the invading fluid rapidly moves into a region of the porous medium [10,28–30]. The distribution of pore restrictions within a porous medium creates a situation in which bursting events occur sporadically, at various locations along the meniscus, and with varying size. Previous experimental examinations of bursts during drainage in porous media have relied on detailed pressure measurements within the invading and defending fluids [10,28,29]. By recording the amount of time between the relatively large pressures required to advance the invading fluid through the narrow throat restrictions, and relating this to the constant injection rate, the size of individual bursts has been measured. With these pressure-measured bursts it has been shown that the distribution of burst sizes were exponentially decreasing [28,29]. Previous experimental measurements using image analysis techniques to determine avalanche properties during an imbibition process (wetting fluid invasion) were difficult to interpret due to fairly low resolution of the video images and the random structure of the bead pack studied [30].

This pressure measurement technique has also been used to develop modified invasion percolation (IP) models to simulate this behavior [10,28,29]. IP models simulate drainage at the limit of zero injection velocity [7–10,38]. For this zero injection velocity condition capillary forces are dominant and the viscous forces are negligible. The standard IP modeling technique identifies the throat adjacent to the fluid–fluid meniscus with the lowest capillary pressure and advances the invading fluid through that throat. Modifications to this method included a description of the invading fluid pressure, and when the invading fluid moves into a new region the invading fluid pressure decreases due to the increased occupied volume. This treatment of the pressure in the modified IP models has shown that large bursting events in standard IP models break up into smaller bursting events [28,29].

It has been claimed that the spatial and temporal dynamics of this fluid motion obeys SOC relationships [5–10]. This has been evaluated with several numerical models [8–10] but little experimental evidence has been presented to corroborate this claim. The best example of experimental evidence was presented by Aker et al. [10] using the pressure measurement technique. Aker et al. [10] were able to determine a scaling exponent, $\tau^*$, of 1.9 ± 0.1 for the distribution of burst sizes observed in multiple experiments with different fluid properties. This $\tau^*$ value was in good agreement with a modified IP model they developed as well. Other modified IP models have shown lower scaling exponents for the burst size distribution, with Roux and Guyon [8] obtaining $\tau^* \approx 1.527$. The experimental studies described in this paper examine these claims using an image processing procedure to identify the bursts visually.

It has been previously noted that self-organized criticality and two-phase flow in porous media at very low flow rates are fundamentally different [29]. First, two-phase flow in porous media is only critical in a zero-gravity field. To account for this we will discuss only experiments that were performed perpendicular to the direction of gravity. Second, in self-organized critical systems, the disorder is produced as energy is introduced into the system and forms as a function of the random distribution of the energy (i.e. sand grains dropping on a pile). The random structure of the porous medium creates a situation of ‘quenched disorder’ [29], restricting the evolution of the system as mass is added.
For the present study a new porous medium has been constructed that has a wide range of flow properties [31], in order to best approximate a random structure through which the flow can evolve in as many ways as possible. This paper first describes the porous medium model, the details of the experiments of air invasion into the water-saturated flowcell, and the new image analysis technique for identifying burst events. The resulting flow structures are shown to obey fractal relationships, typical of low-velocity drainage in porous media. The distribution of mass avalanche size is compared to the frequency with which they occur, and found to corroborate previous predictions of invasion percolation models. The distribution of mass avalanches as functions of the percent of the medium invaded and the distance from the burst center-of-mass to the injection manifold are then shown to illustrate the temporal and spatial randomness of the burst occurrences.

2. Experimental porous medium and experimental procedure

A flowcell was fabricated from a detailed computer model with the height of the throat/pores varied using stereolithography layered production techniques [31]. This production method was used to produce a flowcell with a wide range of throat areas and properties related to the individual throat resistances [32]. The flowcell was designed with a uniform distribution of 5200 throats within a 10.16 cm square matrix. This matrix is shown in Fig. 1 with cross-sections of the open matrix. The top and bottom of the flowcell in Fig. 1(a) include the injection manifolds. Details of the flowcell construction method and geometry can be found in Ref. [31].

The static contact angle of the water-air-solid interface was measured to be $\theta = 72^\circ \pm 2^\circ$ using goniometry with an OCA 20 video-based contact angle meter (DataPhysics Instruments, Germany). The porosity of the cell, as defined by ratio of the open matrix volume (throats and pores) and the minimum surrounding volume which would completely surround the matrix, was measured as $43 \pm 2.5\%$. The open matrix volume was measured by weighing the flowcell with the throats filled with water, subtracting the weight of the dry flowcell, and determining the volume of water. The minimum surrounding volume was determined from the initial stereolithography model, 10.16 cm by 10.16 cm by 0.8 mm. The permeability of the cell, $k$, was obtained by injections of air at known volumetric flow rates, $Q$, recording the pressure drop across the cell, $\Delta P$, and evaluating $k$ from Darcy’s Law for flow through porous media [33],

$$Q = \frac{kA}{\mu} \frac{\Delta P}{L}. \quad (1)$$

Here $A$ is the cross-sectional area of the flowcell perpendicular to the mean flow, $\mu$ is the viscosity of the fluid, and $L$ is the cell length. The corresponding permeability was found to be 408 Darcy.

The experimental setup included a constant rate syringe pump (KD Scientific KDS 200) that was used to control the injection of air, a CCD camera (NTSC COHU 4915-400/000) to capture the images of fluid distributions, and a pressure transducer (Setra C239) to measure the pressure drop across the flowcell. The acquisition and storage of this data was controlled by a LabView™ module. A schematic of this setup is shown in Fig. 2. All experiments were performed with a horizontal flowcell and air injection into an initially water-saturated flowcell. Percent saturations were measured from the beginning of the injection until the “breakthrough” time, when the air reached the end of the flowcell matrix. The invading fluid structure was not observed to change significantly after breakthrough occurred.
For this study only low-velocity flows were examined, with $Q$ varying from 0.2 to 0.002 ml/min. The capillary number, $N_c$, is the ratio of viscous forces to interfacial tension forces and is defined as,

$$N_c = \frac{\mu_{\text{def}} U}{\sigma \cos \theta}$$

where, $\mu_{\text{def}}$ is the defending fluid (water) viscosity, $\sigma$ is the interfacial tension, and $U$ is the mean velocity through the cell. For $N_c$ calculations the Darcy velocity, $V_{\text{Darcy}}$ [34] was used as the mean velocity in Eq. (2).

$$U \equiv V_{\text{Darcy}} = \frac{Q}{A}$$

The associated $N_c$ for flows studied here are all less than $10^{-5}$.

The captured black-and-white images were post-processed to isolate the invading air mass. To remove glare and lighting inconsistencies an image of the cell prior to air invasion was subtracted from the dynamic invasion images. These merged images were then cropped to include only the square matrix. The images were enhanced to make the invaded regions the brightest, a threshold was applied to isolate the portion of the matrix which had been invaded by air, the grayscale images were converted to 8-bit black-and-white images, scaled to 400 by 400 pixels, and finally these image files were converted to binary data sets. This image processing was performed with batch processing using the GNU Image Manipulation Program (GNU General Public License) and ImageJ (National Institutes of Health, USA). Breakthrough images for three flow rates, for each flow direction, are shown in Fig. 3. The invading air distribution was not observed to change significantly after the air broke through the matrix, for any of the studied flow rates. In Fig. 3, the injection speed decreases from left to right and all images have been oriented so that the flow is from the bottom to the top. Three experiments through each injection sides and at all flow rates were conducted; 18 individual experiments were performed.
3. Fractal flow

The scale-invariant geometry of fractal structures has been identified in numerous ways since Mandelbrot’s seminal work [35]. The non-Euclidian structure of the invaded air mass is apparent in the representative breakthrough images shown in Fig. 3. The use of box-counting methods to determine the fractal dimension \( D_f \) of non-Euclidian geometries is well documented [36] and has been used to describe the mass distribution of two-phase flow structures in porous media studies, both numerical [37] and experimental [25]. By overlaying square grids of different sized boxes on the structure, counting the number of boxes that cover a part of the structure, and relating the length of the box sides to the number of covering boxes on a log–log plot, one may obtain a curve that has a linear region. If the structure is fractal, the slope of this line is \( -D_f \) [36]. By relating the amount of injected fluid in an evolving fractal structure to the linear size of the structure, one can also evaluate the fractal characteristics. Related power-law scaling forms have been used to determine the space-time dependence of saturation profiles for invasion percolation [7]. Within this section, \( D_f \) is determined using box-counting and the structure evolution is shown to be in good agreement with proposed invasion percolation scaling laws.

3.1. Box-counting

Images of the invaded air mass at breakthrough, such as in Fig. 3, were analyzed using the image processing software ImageJ and the built-in fractal analysis features of that program. The log–log plot of the measurement box size versus the number of occupied boxes is shown in Fig. 4. Over the range of boxes measured (from 2 to 32 pixels; 0.05 to 0.8 cm), the invading fluid mass of all the flows studied here does indeed have the straight line relationship expected for fractals. The traditionally accepted value of \( D_f \) for invasion percolation is 1.89 [36]. More recent studies of invasion percolation with trapping (IPwT), the case of zero viscous forces and an incompressible invading fluid, predict a \( D_f \) of 1.825 [38]. A line with this IPwT slope, \(-1.825\), is shown in Fig. 4 and the experimental data are shown to closely follow the IPwT predictions. For each \( N_c \), the \( D_f \) was determined from the sum of the occupied boxes for all three of the \( N_c \) trials. The slope of the occupied boxes against the box length was approximately \(-1.75\), lower than the IPwT predictions. The standard deviation of the slopes obtained from the six different box-counting curves was determined to be 0.036, illustrating the slight amount of variation between the curves shown in Fig. 4. The model’s finite size and the maximum box size that can be used on the flowcell may be factors in the lower than expected \( D_f \) values.

3.2. Power-law scaling

For the constant-rate injection experiments performed, the time of injection is proportional to the mass of the injected fluid. If the invading fluid structure is fractal, a relationship between the amount of injected fluid and its center-of-mass, \( \langle x(t) \rangle \), should exist. This is similar to the relationship between the number of covering boxes and the linear length scale of the boxes used in box-counting. The relationship between time and the fractal dimension can be described as Ref. [7]

\[
t \propto m(t) \propto A_E \times \langle x(t) \rangle^{D_f - 1}
\]  

where \( m(t) \) is the injected mass and \( A_E \) is the entrance area into the flowcell matrix. From this proportionality, the time dependence of the center-of-mass can be described as Ref. [7]

\[
\langle x(t) \rangle = Bt^{1/(D_f - 1)} = Bt^{1+\varepsilon}
\]
where $B$ is a constant and $\varepsilon$ is the scaling exponent. Here $t$ is the percent of the porous medium saturated with invading fluid. This description of time allows us to compare the laboratory data with previous numerical studies [7] and to compare results among flows at different injection rates.

When $\langle x(t) \rangle$ obeys the above relationship, the time dependant saturation profile, $S(x, t)$, should obey the following power-law scaling form [7],

$$S(x, t) = t^{-\varepsilon} \zeta \left( x / t^{1+\varepsilon} \right)$$

where $\zeta$ is an unknown function. Since $D_f = 1.825$ for invasion percolation, the value of the exponent $\varepsilon$ is 0.212.

In order to test the applicability of Eq. (6) to our experimental data, we obtained saturation profiles from the data using an in-house code to analyze the digital images. Results were acquired from all 18 experiments. Profiles from the six flows with the same $N_c$ were averaged to reduce scatter. The percent saturation at 20 equally spaced intervals along the flowcell (approximately every 5 mm) was calculated by averaging the saturation over several rows of throats. This ensured that the reported saturation included throats both perpendicular and parallel to the mean flow direction. These saturation profiles are shown in Fig. 5 for $t'$ values of 5, 10, 15, and 20%. In some experiments, breakthrough occurred when nearly 45% of the medium was saturated. The range from 5% to 20% for the dynamic fractal scaling ensured that all trials would be included and that no individual experiments would skew the results. As can be seen in Fig. 5, these profiles do not uniformly decrease as they invade the medium. This erratic distribution is to be expected when one considers the non-uniform saturation distributions in the breakthrough images (Fig. 3).

Applying the scaling predictions shown in Eq. (6) to the saturation profiles in Fig. 5 should collapse the profiles to a single curve. This is shown in Fig. 6. Although the collapse is not as dramatic as that for numerical studies, which averaged the profiles from much larger systems to reduce noise in the data [7], the scaling prediction does significantly collapse the profiles in Fig. 5.

4. Localized burst distributions

SOC studies of sand piles have defined avalanches as the amount of mass that tumbles down the side of the pile after a single grain is added [9,14–16] or, similarly, the amount of energy dissipated during this event [20]. A handful of previous two-phase flow studies have shown fairly good agreement with this theory [7–10,28–30]. Given the agreement between our experimental results with the predictions of fractal theory, it seems reasonable to ask whether the predictions of SOC will hold true as well.

The high rate of image capturing (one every 0.2 s for $N_c = 9 \times 10^{-6}$, every 20 s for $N_c = 9 \times 10^{-6}$) enabled the motion of the invading mass into individual throats and pores to be identified. Localized mass bursts were identified from the images (after conversion to binary data) using an in-house code. The location of the invading air mass for every image, from initial invasion until breakthrough, was identified and changes in the invading fluid structure were noted. This enabled the location of mass bursts to be determined at each imaging time. When successive images showed that a bursting phenomenon had occurred adjacent to each other the burst masses were combined into a single event. This combining of the mass in successive images is further explained in Fig. 7. The invading air mass at a specific location in the flowcell is shown in Fig. 7(a), at time $T$. In Fig. 7(b), a burst is observed at time $T + \Delta t$ and is shown in a lighter shade. If at time $T + 2 \times (\Delta t)$ another burst contiguous to the first burst occurs, as shown in Fig. 7(c), the bursts will be combined. Additionally, as shown in Fig. 7(d), if a burst occurs at time $T + 2 \times (\Delta t)$ and is not bordering the initial burst, these bursts will be treated as separate events. The mass bursts during the invasion process were identified in this manner.
This identification procedure differs from previously described methods of identifying the occurrence of mass bursts during a two-phase drainage experiment in porous media. Previous authors have described methods of identifying bursting activity using the information on the constant rate of injection and the time between observable pressure fluctuations across the medium [8–10,28,29]. The current procedure is made possible through the use of the translucent two-dimensional cell used for these studies [31] and the image analysis procedure described previously.

In these experiments of air injection into the flowcell saturated with water, the constant injection rate was varied from 0.2 ml/min to 0.002 ml/min, corresponding to the values of \( N_c \) from \( 9 \times 10^{-6} \) to \( 9 \times 10^{-8} \). Only these low \( N_c \) experiments were analyzed. At higher flow rates, individual occurrences of localized bursts were too rapid to be acquired using the image capturing equipment. Additionally, low \( N_c \) flows are expected to better approximate the condition known as invasion percolation, where the viscous forces are negligible, due to the invading velocity approaching a small value [25]. Experiments with \( N_c = 9 \times 10^{-6} \) reached breakthrough in approximately 8 min, while \( N_c = 9 \times 10^{-8} \) experiments were completed in approximately 12 h. Hundreds to thousands of images were recorded at regular intervals for each experimental run.

### 4.1. Mass distribution of localized bursts

As the invading fluid pressure increases, the capillary pressure, \( P_c \), of the largest throat on the perimeter of the invaded mass of air is eventually exceeded and the air moves into that throat. If the pressure is greater than the \( P_c \) of the newly
adjacent throats, the air will move into these regions as well. For invasion percolation (the limiting case of zero viscous forces, velocity $\sim 0$) it has been shown that the size distribution of these bursts and the number of occurrences of bursts obey a scaling relation \[ N(s_a) \propto s_a^{-\tau'} \] (7) where $N(s_a)$ is the number of occurrences of bursts of avalanche size, $s_a$, and $\tau'$ is the scaling exponent. For our experiments $s_a$ was recorded as the number of pixels identified in a burst. It has been predicted for invasion percolation that $\tau' = 1.527$ [8].

Using the methods described, the $s_a$ were determined and plotted against the number of occurrences recorded. We report only bursts that occurred after the first 1% of the medium was invaded and that were greater than 20 pixels in size. Fig. 8 is a log–log plot of $N(s_a)$ and $s_a$ for six trials with $N_c = 9\times 10^{-7}$. The burst size shown in Fig. 8 has been normalized by the average pore size within the flowcell, $s_{\text{pore}} = 9$ pixels. This plot shows the predicted behavior of many more small bursts occurring during the air invasion. The black line, with slope $-1.5272$, shows the best fit to the data, in agreement with the modified invasion percolation model prediction of Roux and Guyon [8].

There is a substantial amount of variability in the largest burst sizes in these experiments, and correspondingly the goodness–fit ($R^2$ value) is only 0.92 for the linear power-law fit shown Fig. 8. The smaller size of the porous medium model, which limited the maximum burst size, is believed to cause some of this variation. Additionally, in the physical experiment the pressure of the invading fluid decreases as the newly invaded pore volume is occupied. This reduces the possibility that a large number of throats can be invaded in one bursting event, as discussed by Furuberg et al. [29]. With these two restrictions to the maximum burst size, the comparison of the fit in Fig. 8 to the predicted invasion percolation scaling exponent is good. We predict that if a larger number of experiments were performed, with a corresponding increase in the number of observed large bursts, the quality of this linear power-law fit would improve.

4.2. Localized bursts as a function of time

In the spatio-temporal model of critical systems described by SOC, avalanches occur at randomly distributed times. Fluctuations in sand pile models have shown this variation over a wide range of time scales [11, 12, 16, 17, 20]. To determine if the burst size within the low-velocity drainage studied in these experiments is independent of time the size of bursts recorded for the six trials with $N_c$ of $9\times 10^{-8}$ were plotted against the non-dimensional time. The non-dimensional time was defined as the ratio of air mass to the mass of air at breakthrough, similar to the time description used in Eq. (4)). Flows from injection side 2 are displayed with diamonds, and injections from side 1 are represented with circles. To remove some of the statistical noise, the burst size data for flows from each side of the flowcell were averaged. As can be seen in Fig. 9, the average burst size throughout the experiments was essentially constant, indicating that the burst size was not dependant upon the time from initial injection. The standard deviation of the averaged values at each non-dimensional time is shown with the vertical error bars in Fig. 9.

Of the 5905 combined bursts recorded for the six trials used to generate the average burst sizes in Fig. 9, fewer than 2% (116 bursts) were observed to have a mass greater than 300 pixels. Of these larger bursts, 44 occurred during injection into side 1 and 72 occurred during injection into side 2. Bursts of this size were observed to occur throughout the injection process, from the dimensionless time of 0.045 until breakthrough. These large bursts had a negligible effect on the average burst size identified during the drainage process, except for a large burst at the non-dimensional time of 0.38 during injection into side 1. As can be seen in Fig. 9, this skewed the results at that time significantly higher. If more trials were averaged...
Fig. 9. Averaged mass burst sizes as a function of time for the six air injections with an associated \( N_c \) of \( 9 \times 10^{-8} \). The non-dimensional time is ratio of the air mass to the mass of air at breakthrough, similar to Eq. (4). Error bars show \( \pm \) one standard deviation.

with these results, this higher value at the non-dimensional time of 0.38 would be expected to reduce to approximately the size of the other averaged burst values.

4.3. Localized burst distributions as a function of location

In the self-organized critical state avalanches can occur over a wide range of sizes, from single grains to large sliding events [17–21]. The distribution of localized bursts within two-phase drainage in porous media should follow similar trends if SOC is an applicable model for this physical situation. In order to test this, the size of the mass bursts observed in the six \( N_c = 9 \times 10^{-8} \) flows were plotted against the maximum distance from the injection side, \( x_{\text{max}} \), they reached. To show that this distribution was independent of the distance from the injection manifold, the data was split into three slices corresponding to bursts with an \( x_{\text{max}} \) in each third of the flowcell. This is shown in Fig. 10 along with the distribution of the combined bursts. As is shown these curves are well described by a power-law distribution with an exponent of \(-1.566\). The data presented here is again the low \( N_c \) flows so as to report results with the lowest possible viscous resistances. Localized mass bursts over a large range of sizes occur throughout the flowcell matrix.

This distribution of burst locations is different from those observed in invasion percolation models with much larger porous domains [7]. In the results for invasion percolation models [7], the largest bursts were observed to occur primarily towards the exit side of the porous model and not evenly distributed throughout the domain as in our experiments. The reasons for this difference might include the small deviation from the \( N_c = 0 \), the invasion percolation limit becoming more important near the outlet, or the different injection conditions between the physical model and the computational model. As noted by Furubeget al. [29], during constant-rate injection experiments the pressure in the invading fluid drops off sharply once throats are invaded. This is due to the limited size of the model; in geological situations the porous medium is substantially larger and this rapid decline of invading fluid pressure due to a single pore (or collection of pores) being invaded is orders of magnitude less. Invasion percolation numerical models increase the pressure at the inlet until a single throat is breached, then readjust the injection pressure to the pressure required to invade the next largest throat adjacent to the meniscus [7,37,38]. With the constant rate injection used in the experimental mode this injection condition is not realized. The modified IP models presented by Aker et al. [10], Måløy et al. [28], and Furubeget al. [29] that account for this rapid decline in invading fluid pressure as a burst occurs seem to best match the observed behavior in our experiments.

5. Discussion

Even with the limited size of the porous medium used in these studies, good agreement with fractal power-law scaling was observed. The static \( D_f \) measurements showed a greater amount of variability, particularly for the lowest \( N_c \) flows studied. For the lowest injection rates irregularity was noted with the final percent saturations and amount of time until breakthrough. These varied from 25% to 45% and 10 to 19 h, respectively. This variation is partially due to the different injection sides and hence the ‘different’ porous media. Substantial variation occurred, however, even during injection into the same side of the medium. For these cases, the breakthrough location was noted to be the same, but different invasion patterns near to the injection manifold appear to have substantially affected the overall distribution of the resultant invading air mass. The possibility of small bubbles of air within the throats was eliminated by close visual examination of the flowcell prior to running each experiment. Possible causes of this variation include the effect of slight vibrations from the syringe
Fig. 10. Distribution of mass bursts that reached different locations along the flowcell. Localized bursts from the six $N_i = 9(10^{-8})$ flows shown.

pump (which was on the same table as the rest of the experiment), vibrations from the ambient surroundings, and/or small changes in temperature occurring during the long experiments.

The variations observed between trials do not seem to have affected the fractal nature of the resulting air mass (Fig. 4) or the dynamic scaling of the saturation profiles (Fig. 6). The distribution of burst sizes is in good agreement to previous numerical invasion percolation predictions (Fig. 8), matching the numerically determined $\tau' = 1.527$ [8]. The small bursts invaded the water-saturated porous medium with a much greater frequency. These bursts were observed to occur throughout the porous medium and during any time after the injection had begun, which was not in good agreement with some previous standard invasion percolation models but is similar to the behavior observed in modified invasion percolation models where the invading fluid pressure variation is accounted for.

Agreement with the behavior predicted by SOC is partially confirmed. Various localized bursts are shown to occur at random times and locations throughout the injection, as shown in Figs. 9 and 10. The observed non-dependence of the burst size on time, as shown in Fig. 9, implies that bursts of approximately equal size will occur during constant-rate injection drainage in porous media. As has been discussed in Crandall et al. [31], the translucent porous medium used in this study has a wide range of throat resistance properties, similar to that observed in naturally occurring geologic porous media. Thus, one would expect to encounter this same behavior in large-scale processes where a non-wetting fluid is injected at constant rate, and sufficiently far enough from the injection source to be appropriately described as capillary-dominated flow. This expectation of the translation of the observed SOC behavior over to field scale studies is further corroborated by the data in Fig. 10, where no substantial difference between the distributions of burst sizes as a function of distance from the injection manifold was determined. Once again, the limited size of these studies does limit the degree to which this theory can be confirmed or disproved. Further studies using a similar imaging procedure and a larger porous medium may show better agreement to the predicted spatial-temporal scale-invariant behavior of SOC.

6. Summary

Low-velocity, two-phase drainage experiments of a non-wetting fluid into a saturated porous medium have been conducted. Using a fairly high image capture rate, freely available imaging software, and in-house analysis code, we have studied the occurrence and distribution of 'localized bursts' to test the agreement of behaviors predicted by fractal theory and SOC. Good agreement with the proposed dynamic power-law scaling of the invaded air mass has been shown, confirming the applicability of invasion percolation (fractal) theories to describe low-velocity, two-phase drainage in porous media. Static $D_f$ measurements of the residual invaded air mass showed significant variability, but adhered to the predicted log–log fractal characteristics.

Localized bursts of a wide range of sizes were shown to occur throughout the injection, in agreement with SOC predictions. Bursts were shown to occur with similar frequency and size regardless of either the time from initial injection or the location of the invading air mass within the porous medium. Additionally, the mass distribution of avalanches was shown to be in good agreement with the scaling exponent predicted by invasion percolation. Previously described numerical models have predicted $\tau' = 1.527$ [8], $\tau' = 1.9 \pm 0.1$ [10], and $\tau' = 1.48 \pm 0.05$ [29]. Earlier experiments have measured $\tau'$ as 1.45 [29] and $\tau' = 1.9 \pm 0.1$ [10] using pressure measurements to determine the burst sizes. Our current image analysis method of determining the burst sizes of air during drainage experiments is in good agreement with this previous body of work, with $\tau' = 1.53$. 
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References