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D0 SILICON UPGRADE

THERMALLY INDUCED BOWING IN A 3-CHIP LADDER

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The end of the 3 chip ladder, shown below, consists of silicon mounted on a piece of beryllium which is adhered to the cooling channel. Outboard of the cooling channel is a region of ladder composed primarily of silicon/beryllium. Operation and cooling of the ladder results in a change in temperature from the assembly temperature, which will result in deflections due to the difference in expansion coefficients of the two materials, otherwise known as 'bi-metal' bowing. The goal of this note is to present a design of the beryllium plate on the underside of the ladder which reduces the thermally induced bow to a reasonable deflection.

This region of ladder will see a fairly large temperature gradient during detector operation due to the heat load of the transceivers on the ladder end. Expected temperatures [1] range between 22°C on the ladder end to 9.5°C near the cooling channel for a coolant temperature of 5°C. The coolant temperature may be as low as -5°C [2], so we may estimate a lower limit on the ladder temperatures to be 10°C cooler, ranging from 12°C on the ladder end to -0.5°C near the bulkhead (assumes negligible convection from the ladder surface).

With a ladder assembly temperature of 23°C we may estimate the end deflection of the ladder based on the assumed temperatures during operation by applying Roark [3] equation 6a. Equation 6a describes end deflection for a cantilever beam under application of a uniform temperature change. The equation is modified to account for a uniform temperature gradient along the bi-metal region. The equation is differentiated twice, the assumed temperature dependence is plugged, and the equation is re-integrated twice [4] (see appendix).

\[
deflection = C \Delta T L^2
\]

The constant C is a function of material thicknesses and moduli. The composite region (beryllium and silicon 'bi-metal' region) is between 21.0 and 25.0 mm in length (the HDI design is still in progress) plus the additional 1.975 mm shown in the ladder drawing below. Hence, the composite region is assumed to extend 27 mm beyond the bulkhead ledge. The silicon extends 4 mm beyond the composite region. Deflection of the ladder end, the silicon, is calculated.
It must be noted that the heat transfer FEA model which predicts the assumed temperature gradient was performed assuming the extension of the bi-metal region to be 21 mm, not 25 mm. The temperature distribution will be different than these predicted values for two reasons:

(1) The underside beryllium will extend outboard of the bulkhead reducing the conduction thermal resistance, hence the temperature difference along the silicon.
[1] Paul Ratzmann, et. al., Heat Transfer Analysis in the D0 Silicon Tracker, D0EN work in progress.


(2) The bi-metal extension is expected, currently, to increase from 21 to 25 mm, resulting in an increase in the temperature difference along the silicon, compared to the results of the FEA model.

The two effects offset one-another to some degree. Once the ladder design and the coolant temperature is finalized the FEA model will be re-run to understand the temperature distribution in the ladder. The resulting temperature profile will be re-implemented into the bowing model to determine the final bowing of the ladder.

Silicon is assumed to have a modulus of 131 GPa, and beryllium a modulus of 290 GPa 42 Mpsi. The expansion coefficient of silicon is 2.6 ppm/°C and that of beryllium is assumed to be 13.0 ppm/°C.

Acceptable ladder uncertainties in the radial dimension are on the order of 70 microns [5]. By setting a limit of roughly 50 microns end deflection, the bow of the ladder end is reduced by extending the beryllium underside piece beyond the bulkhead. Represented schematically below, the assumed temperature profile is shown as it corresponds to the ladder. The temperature at the base of the bi-metal region is dependent on the length of the beryllium extension on the underside of the ladder.
Extending the beryllium piece to a length such that the bi-metal region is 14 mm results in a deflection of 52 microns in the ladder end, for the colder ladder temperatures mentioned above (-5°C coolant). The base of the bi-metal strip is at a temperature of roughly 5°C.
{bimetal strip solution for calculation of the bowing of be/silicon}

\[ L_{\text{tot}} = L_{\text{bi}} + a \] (mm)

\[ L_{\text{bi}} = 27 - X \] (mm, extended part beyond bi-metal region)

\[ t_{\text{total}} = 0.675 \] (mm, total thickness)

\[ t_b = 0.3 \] (mm, silicon)

\[ E_b = (131E9)/1E6 \] (N/mm^2, silicon modulus)

\[ L_b = (1/12) * w * t_b^3 \] (mm^4)

\[ t_a = t_{\text{total}} - t_b \] (mm, beryllium effective thickness)

\[ t_{\text{beryllium}} = 0.300 \] (mm, actual thickness of the beryllium)

\[ E_a = (290E9)/1E6 \] (N/mm^2, range of 186000-290000 N/mm^2 from Material Selector)

\[ E_{\text{a Mpsi}} = E_a / 6895 \]

\[ I_a = (1/12) * w * (t_a - t_{\text{glue}})^3 \] (mm^4)

\[ E_{\text{transform}} = E_{\text{tot}} / E_{\text{transform}} \] (N/mm^2)

\[ \alpha_b = 2.6E-6 \] (le)

\[ \alpha_a = 13.0E-6 \] (le)

\[ y_{\text{uniformT}} = 1000 * (\text{constant}) * (((T2 - T0) * (L_{\text{bi}} / 2)) + (T1 - T2) * (L_{\text{bi}} / 2)) \] (microns, bi-metal with uniform \( dT \))

\[ T_0 = 23 \] (°C, assembly temperature)

\[ T_{\text{hot}} = 22 - 10 \] (°C, end of ladder)

\[ T_{\text{cold}} = 9.5 - 10 \] (°C, near cooling channel)

\[ T_{\text{inter}} = (T_{\text{hot}} + T_{\text{cold}}) / 2 \]

\[ T_1 = T_{\text{hot}} \]

\[ m = (T_{\text{hot}} + T_{\text{cold}}) / 27 \]

\[ T_2 = m * X + T_{\text{cold}} \]

\[ \alpha_b = 2.6E-6 \] (le)

\[ \alpha_a = 13.0E-6 \] (le)

\[ y_{\text{max uniform}} = y_{\text{uniformT}} + a * 1000 * \sin(angle_{\text{uniform}}) \] (actual deflection of the end, based on uniform temperature gradient)

\[ \alpha_{\text{a Mpsi}} = E_{\text{a Mpsi}} / 6895 \]

\[ E_{\text{a Mpsi}} = 42.059 \]

Unit Settings: [°C]/[kPa]/[kg]/[radians]

\[ a = 4.000 \]

\[ \alpha_b = 0.000 \]

\[ \alpha_a = 0.000 \]

\[ \text{constant} = -0.000 \]

\[ \text{den} = 3.721 \]

\[ E_{\text{a}} = 290000.000 \]

\[ E_{\text{b}} = 131000.000 \]

\[ I_{a, \text{prime}} = 0.002 \]

\[ I_{\text{transform}} = 0.004 \]

\[ L_{\text{bi}} = 14.000 \]
File: 3 chip bimetal

\[ m = 0.463 \]
\[ T_0 = 23.000 \]
\[ T_2 = 5.519 \]
\[ \text{Thot} = 12.000 \]
\[ t_b = 0.300 \]
\[ t_{\text{glue}} = 0.075 \]
\[ w = 1.000 \]
\[ y_{\text{max uniform}} = 52.052 \]
\[ y_{\text{uniformT}} = 33.996 \]

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\[ \text{num} = -0.000 \]
\[ T_1 = 12.000 \]
\[ T_{\text{cool}} = -0.500 \]
\[ t_a = 0.375 \]
\[ t_{\text{beryllium}} = 0.300 \]
\[ t_{\text{total}} = 0.675 \]
\[ X = 13.000 \]
\[ y_{\text{real2}} = 33.996 \]
From Roark & Young:

\[ y_0 = C (T - T_0) e^{-x} x^2 C = \]

differentiable twice w.r.t. \( x \)

\[ y_0' = 2C (T - T_0) x + (T_1 - T_2) \]

\[ y_0'' = 2C (T - T_0) \]

true uniformly distributed temperature distribution is:

\[ T(L) = T_2 + (T_1 - T_2) \frac{x}{L} \]

where \( T(0) = T_2 \)

\[ T(L) = T_1 \]
\[ \begin{align*}
\frac{\partial^2 y}{\partial x^2} &= 2C \left[ \frac{T_L}{2} + \left( T_1 - T_2 \right) \frac{x}{L} - T_0 \right] \\
\frac{\partial y}{\partial x} &= 2C \left[ (T_2 - T_0) - \left( T_1 - T_2 \right) \frac{x}{L} \right] \\
y &= 2C \left[ (T_2 - T_0) x + \left( T_1 - T_2 \right) \frac{x^2}{2L} \right] \\
y &= \frac{2C}{x} \left[ (T_2 - T_0) \frac{x^2}{2} + \left( T_1 - T_2 \right) \frac{x^3}{6L} \right] \\
y(L) &= 2C \left[ \left( \frac{T_2 - T_0}{2} \right) x + \left( T_1 - T_2 \right) \frac{x^2}{6} \right] \\
n_{\mu} &= \left( T_0 + (T_1 - T_2) \frac{L^2}{6} \right) e^{-\frac{L}{\mu}} \\
T(0) &= T_0 \\
T(L) &= T_1
\end{align*} \]