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Programming the New Sextupole Strings in Booster

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The sextupoles in Booster have been reconfigured into a new set of strings to allow for the resonant extraction of beams to the new Booster facility NSRL (National Space Radiation Laboratory). Following is a description of the new strings and their programming for chromaticity adjustment and resonant extraction.

1 The Sextupole Strings

Each superperiod of Booster contains eight sextupoles which are labeled SVX1, SHX2, SVX3, SHX4, SVX5, SHX6, SVX7, and SHX8, where SH and SV denote, respectively, sextupoles located near horizontal and vertical beta maximums, and X refers to superperiod A, B, C, D, E, or F. We shall refer to the SH and SV sextupoles as horizontal and vertical sextupoles respectively. The sextupoles have a main winding consisting of 8 turns, a monitor winding consisting of one turn, and an auxiliary winding consisting of either one or two turns. The main windings are connected together to form four series strings called the Horizontal and Vertical strings and the C and F strings. The Horizontal and Vertical strings are used for chromaticity adjustment; the C and F strings are used to excite the $3Q_H = 13$ resonance for resonant extraction. (Q_H and Q_V are the horizontal and vertical tunes, respectively.) Within each string, all sextupoles are excited with the same polarity.

The vertical string contains all 24 of the vertical sextupoles and remains in its original configuration. The horizontal string used to contain all 24 of the horizontal sextupoles, but now four of these, SHC8, SHF8, SHB4 and SHE4, have been taken out to form the C and F strings. With these four sextupoles removed, there are four “holes” in the horizontal string, each hole having a partner located three superperiods away. This arrangement

of horizontal sextupoles is depicted schematically in **Figure 1**. Here the open red circles show the positions of the four holes; the filled circles show the positions of the remaining 20 horizontal sextupoles.

The horizontal and vertical strings are powered by programmable **monopolar** power supplies that can deliver a maximum current of 300 A at a maximum of 90 V. The polarity of the strings is such that current from the horizontal supply increases the horizontal chromaticity, while current from vertical supply increases the vertical chromaticity. According to the definitions of Ref. [1], this means that current from the horizontal supply puts the sextupoles of the horizontal string in “B” polarity, while current from the vertical supply puts the sextupoles of the vertical string in “A” polarity. The wiring diagrams for the horizontal and vertical strings are shown in **Figures 2** and **3**. Here the letter “T” denotes the magnet lead that is physically stamped with the letter “T”. In accordance with the definitions of Ref. [1], a positive current into the “T” lead puts the magnet in “A” polarity; a negative current into the lead puts the magnet in “B” polarity.

The main windings on sextupoles SHC8, SHF8, SHB4 and SHE4 are connected together to form the C and F strings. The C string contains sextupoles SHC8 and SHE4; the F string contains SHF8 and SHB4. The two strings are depicted by the blue lines in Figure 1. Each string is connected to its own 350 A **bipolar** programmable power supply as shown in **Figure 4**. Here, in accordance with the conventions of Ref. [1], a positive reference at the power supply input produces a positive current out of the terminal labeled “1”. This puts the sextupoles in the strings in “B” polarity. The $Q_H = 4 + 1/3$ resonance is excited by powering the two strings with opposite polarity. Note that in the control system, the power supplies for the C and F strings are called “b-sxr1-ps” and “b-sxr2-ps” respectively.

2 Harmonics Produced by the Holes

The superperiod symmetry of the machine implies that the holes in the horizontal string will produce only even harmonics in azimuthal angle θ around the ring. Although this ensures that the $3Q_H = 13$ and $Q_H + 2Q_V = 13$ resonances (which are excited by harmonic 13) will not be excited, the $3Q_H = 14$ and $Q_H + 2Q_V = 14$ resonances (which are excited by harmonic 14) can be excited. Note, however, that because the four

holes are approximately equally spaced in betatron phase and in azimuth θ , harmonics 2, 6, 10, 14, 18, and so on, are suppressed to some extent. To ensure that the $3Q_H = 14$ and $Q_H + 2Q_V = 14$ resonances are not excited during injection and acceleration, the currents in the four resonant extraction sextupoles will track the current in the horizontal string until extraction time.

3 Calculation of Chromaticities, Resonance Parameters and Currents

Let I_H be the current in the string of 20 horizontal sextupoles.

Let I_V be the current in the string of 24 vertical sextupoles.

Let I_C be the current in the C string.

Let I_F be the current in the F string.

Then the horizontal and vertical chromaticities are given by

$$\xi_H = \xi_H^0 + \frac{S}{B\rho}(K_H I_H + K_V I_V + K_C I_C + K_F I_F) \quad (1)$$

and

$$\xi_V = \xi_V^0 + \frac{S}{B\rho}(L_H I_H + L_V I_V + L_C I_C + L_F I_F) \quad (2)$$

where ξ_H^0 and ξ_V^0 are the bare chromaticities, S is the integrated strength per unit current of each sextupole, and $B\rho$ is the magnetic rigidity. The constants K_H , K_V , K_C , K_F , L_H , L_V , L_C , and L_F are derived in Appendix I. Note that, as defined by (1) and (2), the bare chromaticities are what one would measure with zero current in the four sextupole strings. The values to be assigned to ξ_H^0 and ξ_V^0 are discussed in Appendix II.

It is convenient to parameterize I_C and I_F as

$$I_C = I_H + I_0 + I_R, \quad I_F = I_H + I_0 - I_R \quad (3)$$

where

$$I_0 = -I_H + (I_C + I_F)/2, \quad I_R = (I_C - I_F)/2. \quad (4)$$

The currents I_R and I_0 are responsible respectively for the excitation of the $3Q_H = 13$ resonance and the excitation of sextupole harmonics 0, 4, 8, 12, 16, and so on. (As noted in Section 2, harmonics 2, 6, 10, 14, 18, and

so on, are suppressed to some extent by superperiod symmetry.) This suggests that we define resonance parameters

$$R_0 = \frac{S}{B\rho} K_0 I_0, \quad R = \frac{S}{B\rho} K_R I_R \quad (5)$$

where the constants K_0 and K_R are chosen to give R_0 and R convenient units. Thus, the four physics parameters controlled by the sextupole strings are ξ_H , ξ_V , R_0 , and R .

If the four currents I_H , I_V , I_C , and I_F are specified, then ξ_H , ξ_V , R_0 , and R are calculated according to equations (1), (2), (4), and (5). Conversely, if ξ_H , ξ_V , R_0 , and R are specified, then the four currents can be calculated as follows. Using (3) in (1) and (2), we have

$$\Delta_H = (K_H + K_C + K_F)I_H + K_V I_V + \Gamma_H \quad (6)$$

and

$$\Delta_V = (L_H + L_C + L_F)I_H + L_V I_V + \Gamma_V \quad (7)$$

where

$$\Delta_H = \frac{B\rho}{S}(\xi_H - \xi_H^0), \quad \Delta_V = \frac{B\rho}{S}(\xi_V - \xi_V^0), \quad (8)$$

$$\Gamma_H = (K_C + K_F)I_0 + (K_C - K_F)I_R, \quad (9)$$

and

$$\Gamma_V = (L_C + L_F)I_0 + (L_C - L_F)I_R. \quad (10)$$

The currents

$$I_0 = \left(\frac{B\rho}{SK_0} \right) R_0, \quad I_R = \left(\frac{B\rho}{SK_R} \right) R \quad (11)$$

are obtained from equation (5). Using these in (9) and (10), one obtains Γ_H and Γ_V in terms of R_0 and R . Equations (6) and (7) then may be solved to obtain I_H and I_V in terms of $\Delta_H - \Gamma_H$ and $\Delta_V - \Gamma_V$. One finds

$$I_H = L_V \left\{ \frac{\Delta_H - \Gamma_H}{D} \right\} - K_V \left\{ \frac{\Delta_V - \Gamma_V}{D} \right\} \quad (12)$$

$$I_V = -(L_H + L_C + L_F) \left\{ \frac{\Delta_H - \Gamma_H}{D} \right\} + (K_H + K_C + K_F) \left\{ \frac{\Delta_V - \Gamma_V}{D} \right\} \quad (13)$$

where

$$D = (K_H + K_C + K_F)L_V - (L_H + L_C + L_F)K_V. \quad (14)$$

Finally, using in (3) the values of I_0 , I_R and I_H obtained from (11) and (12), one obtains currents I_C and I_F . Thus, given ξ_H , ξ_V , R_0 and R , currents I_H , I_V , I_C and I_F are calculated according to equations (6–14) and (3).

4 Chromaticities and Currents in the Original Chrom Control Program

The chromaticities and currents are programmed in the Booster Chrom Control part of the Optics Control application. The original Chrom Control program dealt with just two sextupole strings; the vertical string and the original horizontal string consisting of all 24 horizontal sextupoles. The chromaticities in this case are given by equations (1) and (2) with

$$I_C = I_F = I_H. \quad (15)$$

One then has

$$\xi_H = \xi_H^0 + \frac{S}{B\rho} \{(K_H + K_C + K_F)I_H + K_V I_V\} \quad (16)$$

$$\xi_V = \xi_V^0 + \frac{S}{B\rho} \{(L_H + L_C + L_F)I_H + L_V I_V\}. \quad (17)$$

These equations and their inverse give the chromaticities and currents as calculated by the original program.

5 Formulae for the New Chrom Control Program

The new Chrom Control program is a version of the original one with modifications to include the new sextupole strings and the resonance parameters. Both the new and original programs deal with the normalized chromaticities

$$C_H = \frac{1}{Q_H} \xi_H, \quad C_H^0 = \frac{1}{Q_H} \xi_H^0, \quad C_V = \frac{1}{Q_V} \xi_V, \quad C_V^0 = \frac{1}{Q_V} \xi_V^0 \quad (18)$$

where Q_H and Q_V are the horizontal and vertical tunes. Both programs use parameters

$$A_{11} = \frac{1}{Q_H} \left(\frac{S}{B_0\rho} \right) (K_H + K_C + K_F), \quad A_{12} = \frac{1}{Q_H} \left(\frac{S}{B_0\rho} \right) K_V \quad (19)$$

$$A_{21} = \frac{1}{Q_V} \left(\frac{S}{B_0\rho} \right) (L_H + L_C + L_F), \quad A_{22} = \frac{1}{Q_V} \left(\frac{S}{B_0\rho} \right) L_V \quad (20)$$

which are specified by the user as functions of the magnetic field $B = B\rho/\rho$. Here $\rho = 13.8656$ m is the nominal radius of curvature in the

Booster dipoles. The field B_0 is a reference field chosen to be 0.1563 T. The values of A_{11} , A_{12} , A_{21} and A_{22} are given in Appendix III.

The new program requires additional parameters

$$R_{11} = \frac{1}{Q_H} \left(\frac{S}{B_0 \rho} \right) (K_C + K_F), \quad R_{12} = \frac{1}{Q_H} \left(\frac{S}{B_0 \rho} \right) (K_C - K_F) \quad (21)$$

$$R_{21} = \frac{1}{Q_V} \left(\frac{S}{B_0 \rho} \right) (L_C + L_F), \quad R_{22} = \frac{1}{Q_V} \left(\frac{S}{B_0 \rho} \right) (L_C - L_F) \quad (22)$$

and

$$A_0 = \left(\frac{S}{B_0 \rho} \right) K_0, \quad A_R = \left(\frac{S}{B_0 \rho} \right) K_R. \quad (23)$$

These are also specified by the user as functions of the field B . The values of R_{11} , R_{12} , R_{21} and R_{22} are given in Appendix III. The values of A_0 and A_R are to be chosen to give R_0 and R convenient units.

Note that in equations (18–22), Q_H and Q_V are neither the measured tunes nor are they the tunes specified in the Booster Tune Control part of the Optics Control application. Instead, they are fixed at the values $Q_H = 4.82$ and $Q_V = 4.83$. Thus, to recover the values of ξ_H and ξ_V , one has to multiply C_H and C_V by these fixed values. (One could avoid having to do this by simply dealing with ξ_H and ξ_V instead of the normalized chromaticities C_H and C_V .)

In terms of the user specified parameters, equations (1), (2) and (5) become

$$C_H = C_H^0 + \frac{B_0}{B} \{A_{11}I_H + A_{12}I_V + R_{11}I_0 + R_{12}I_R\} \quad (24)$$

$$C_V = C_V^0 + \frac{B_0}{B} \{A_{21}I_H + A_{22}I_V + R_{21}I_0 + R_{22}I_R\} \quad (25)$$

$$R_0 = \frac{B_0}{B} A_0 I_0, \quad R = \frac{B_0}{B} A_R I_R \quad (26)$$

where

$$I_0 = -I_H + (I_C + I_F)/2, \quad I_R = (I_C - I_F)/2. \quad (27)$$

These equations give the normalized chromaticities and resonance parameters in terms of currents I_H , I_V , I_C and I_F . Their inverse gives the currents in terms of the chromaticities and resonance parameters. The

formulae for going back and forth between chromaticities, resonance parameters, and currents are summarized in the following two subsections. The user interface for programming C_H , C_V , R_0 , R , I_H , I_V , I_C and I_F is shown schematically in **Figure 5**.

5.1 Formulae for Going from Currents to Chromaticities and Resonance Parameters

Here one computes

$$I_0 = -I_H + (I_C + I_F)/2, \quad I_R = (I_C - I_F)/2 \quad (28)$$

and then

$$C_H = C_H^0 + \frac{B_0}{B} \{A_{11}I_H + A_{12}I_V + R_{11}I_0 + R_{12}I_R\} \quad (29)$$

$$C_V = C_V^0 + \frac{B_0}{B} \{A_{21}I_H + A_{22}I_V + R_{21}I_0 + R_{22}I_R\} \quad (30)$$

$$R_0 = \frac{B_0}{B} A_0 I_0, \quad R = \frac{B_0}{B} A_R I_R. \quad (31)$$

5.2 Formulae for Going from Chromaticities and Resonance Parameters to Currents

Here one computes

$$I_0 = \frac{1}{A_0} \frac{B}{B_0} R_0, \quad I_R = \frac{1}{A_R} \frac{B}{B_0} R \quad (32)$$

$$D_H = C_H - C_H^0 - \frac{B_0}{B} \{R_{11}I_0 + R_{12}I_R\} \quad (33)$$

$$D_V = C_V - C_V^0 - \frac{B_0}{B} \{R_{21}I_0 + R_{22}I_R\}. \quad (34)$$

Equations (24) and (25) then give

$$\frac{B_0}{B} \{A_{11}I_H + A_{12}I_V\} = D_H \quad (35)$$

$$\frac{B_0}{B} \{A_{21}I_H + A_{22}I_V\} = D_V. \quad (36)$$

These may be inverted to give

$$I_H = \frac{1}{D} \{A_{22}D_H - A_{12}D_V\} \frac{B}{B_0} \quad (37)$$

$$I_V = \frac{1}{D} \{-A_{21}D_H + A_{11}D_V\} \frac{B}{B_0} \quad (38)$$

where

$$D = A_{11}A_{22} - A_{12}A_{21}. \quad (39)$$

Finally, one computes

$$I_C = I_H + I_0 + I_R, \quad I_F = I_H + I_0 - I_R. \quad (40)$$

Note that when $R = R_0 = 0$, one has $I_R = I_0 = 0$ and $I_C = I_F = I_H$. The currents in the C and F strings must then track the current in the horizontal string. This is the normal mode of operation when the $3Q_H = 13$ resonance is not being excited for resonant extraction. When R is nonzero (as it must be during resonant extraction), one would generally like to keep $R_0 = 0$, but this may require that either I_C or I_F be larger than the available current. In such situations, R_0 must be adjusted to some nonzero value to keep the currents within the bounds of what is available. **Figure 6** shows how the currents might be programmed for a typical NSRL magnet cycle.

6 Appendix I

The horizontal and vertical chromaticities in Booster are given by [2, 3]

$$\xi_H = \xi_H^0 + \frac{1}{4\pi} \frac{S}{B\rho} \sum_j \beta_H^j D_j I_j \quad (41)$$

and

$$\xi_V = \xi_V^0 - \frac{1}{4\pi} \frac{S}{B\rho} \sum_j \beta_V^j D_j I_j \quad (42)$$

where the sums are taken over all 48 sextupoles. Here ξ_H^0 and ξ_V^0 are the bare chromaticities and $B\rho$ is the magnetic rigidity. β_H^j , β_V^j , and D_j are

respectively the horizontal and vertical beta functions and the periodic dispersion at the j th sextupole. I_j is the current in the j th sextupole. The integrated strength per unit current of each sextupole is [4]

$$S = |B_2 L| = 2 \times 6.566 \times 10^{-3} \text{ (T/m)/A} \quad (43)$$

where $B = B_2 x^2/2$ is the magnetic field (per unit current) on the midplane of the sextupole, x is the horizontal coordinate on the midplane, and L is the magnetic length.

Let us now define

$$K_H = \frac{1}{4\pi} \sum_h \beta_H^h D_h, \quad K_V = -\frac{1}{4\pi} \sum_v \beta_H^v D_v, \quad (44)$$

$$K_C = \frac{1}{4\pi} \sum_c \beta_H^c D_c, \quad K_F = \frac{1}{4\pi} \sum_f \beta_H^f D_f, \quad (45)$$

$$L_H = -\frac{1}{4\pi} \sum_h \beta_V^h D_h, \quad L_V = \frac{1}{4\pi} \sum_v \beta_V^v D_v, \quad (46)$$

and

$$L_C = -\frac{1}{4\pi} \sum_c \beta_V^c D_c, \quad L_F = -\frac{1}{4\pi} \sum_f \beta_V^f D_f \quad (47)$$

where the sum over h is the sum over all horizontal sextupoles except SHC8, SHE4, SHF8, and SHB4; the sum over v is the sum over all vertical sextupoles; the sum over c is the sum over sextupoles SHC8 and SHE4; and the sum over f is the sum over sextupoles SHF8 and SHB4. Equations (41) and (42) then become

$$\xi_H = \xi_H^0 + \frac{S}{B\rho} (K_H I_H + K_V I_V + K_C I_C + K_F I_F) \quad (48)$$

and

$$\xi_V = \xi_V^0 + \frac{S}{B\rho} (L_H I_H + L_V I_V + L_C I_C + L_F I_F) \quad (49)$$

where the four currents I_H , I_V , I_C , and I_F are defined in Sections 1 and 3. Note that the signs of K_H , K_C , K_F , L_H , L_C , L_F are such that positive currents I_H , I_C , I_F increase the horizontal chromaticity and decrease the vertical chromaticity. The signs of K_V and L_V are such that positive current I_V increases the vertical chromaticity and decreases the horizontal chromaticity.

The sums in (44–47) are readily evaluated using the MAD code. One finds

$$K_H = \frac{472.86}{4\pi}, \quad K_V = -\frac{122.90}{4\pi}, \quad K_C = K_F = \frac{47.267}{4\pi} \quad (50)$$

$$L_H = -\frac{162.03}{4\pi}, \quad L_V = \frac{355.38}{4\pi}, \quad L_C = L_F = -\frac{16.067}{4\pi} \quad (51)$$

and

$$K_H + K_C + K_F = \frac{567.39}{4\pi}, \quad L_H + L_C + L_F = -\frac{194.16}{4\pi} \quad (52)$$

where the units are meters squared. Here the horizontal and vertical tunes were taken to be $Q_H = 4.82$ and $Q_V = 4.83$. Note that it follows from superperiod symmetry that

$$K_C = K_F, \quad L_C = L_F. \quad (53)$$

Since the constants defined in (44–47) depend on the beta functions and dispersion, they will depend on the machine tunes. This dependence could be put into the Chrom Control program by means of a lookup table which gives the values of the constants for various tunes.

7 Appendix II

As defined by (1) and (2), the bare chromaticities, ξ_H^0 and ξ_V^0 , are what one would measure with zero current in the four sextupole strings. There are several contributions to ξ_H^0 and ξ_V^0 which may be expressed by writing

$$\xi_H^0 = \xi_H^N + \frac{1}{B\rho} (A_H + \dot{B} B_H) + E_H \quad (54)$$

and

$$\xi_V^0 = \xi_V^N + \frac{1}{B\rho} (A_V + \dot{B} B_V) + E_V. \quad (55)$$

Here ξ_H^N and ξ_V^N are the so called natural chromaticities [2, 3] due to the intrinsic quadrupole structure of the lattice. (They do not depend on sextupole fields.) For $Q_H = 4.82$ and $Q_V = 4.83$, the MAD code gives

$$\xi_H^N = -4.941, \quad \xi_V^N = -5.258. \quad (56)$$

The A_H and A_V terms are due to remanent sextupole fields. The B_H and B_V terms are due to sextupole fields that are proportional to $\dot{B} = dB/dt$.

The values of A_H , A_V , B_H , and B_V must be determined from measurements. The E_H and E_V terms are due to sextupole fields in the Booster dipoles that scale with the magnet excitation. For given tunes, E_H and E_V are assumed to be constant until the dipoles begin to saturate. For $Q_H = 4.82$ and $Q_V = 4.83$, the MAD code gives (with no dipole saturation)

$$E_H = -2.342, \quad E_V = +2.194 \quad (57)$$

Thus we have (with no dipole saturation)

$$\xi_H^0 = -7.283 + \frac{1}{B\rho} (A_H + \dot{B} B_H) \quad (58)$$

and

$$\xi_V^0 = -3.064 + \frac{1}{B\rho} (A_V + \dot{B} B_V). \quad (59)$$

In the present Chrom Control program, the normalized bare chromaticities

$$C_H^0 = \frac{1}{Q_H} \xi_H^0, \quad C_V^0 = \frac{1}{Q_V} \xi_V^0 \quad (60)$$

are specified by the user as functions of the magnetic field B .

8 Appendix III

Using (50–52) in (19–22), one finds

$$A_{11} = 0.0568, \quad A_{12} = -0.0123, \quad A_{21} = -0.0194, \quad A_{22} = 0.0355 \quad (61)$$

$$R_{11} = 0.00946, \quad R_{12} = 0.0, \quad R_{21} = -0.00321, \quad R_{22} = 0.0. \quad (62)$$

Here the units are A^{-1} and we have used $Q_H = 4.82$, $Q_V = 4.83$, $B_0 = 0.1563 \text{ T}$, $\rho = 13.8656 \text{ m}$, and $S = 2 \times 6.566 \times 10^{-3} \text{ (T/m)/A}$. In the present Chrom Control program, the archived values of A_{11} , A_{12} , A_{21} , A_{22} are

$$A_{11} = 0.059, \quad A_{12} = -0.013, \quad A_{21} = -0.019, \quad A_{22} = 0.036. \quad (63)$$

These are in good agreement with the calculated values.

Note that even though A_{11} , A_{12} , A_{21} , A_{22} , R_{11} , R_{12} , R_{21} , R_{22} , A_0 and A_R can be programmed as arbitrary functions of B in the Chrom Control program, they are simply programmed to be constant functions of B .

Figure 1: Horizontal Sextupoles in the Booster Ring. The open circles show the positions of the four sextupoles used for resonant extraction. The filled circles show the positions of the remaining 20 horizontal sextupoles.

Figure 2: Wiring Diagram for the Horizontal Sextupole String. Here, in accordance with the definitions of Ref. [1], the monopolar power supply puts the sextupoles in “B” polarity. This increases the horizontal chromaticity.

Figure 3: Wiring Diagram for the Vertical Sextupole String. Here, in accordance with the definitions of Ref. [1], the monopolar power supply puts the sextupoles in “A” polarity. This increases the vertical chromaticity.

Figure 4: Wiring Diagram for the C and F Strings. In accordance with the conventions of Ref. [1], a positive reference at the power supply input produces a positive current out of the terminal labeled “1”. This puts the sextupoles in the strings in “B” polarity. The $Q_H = 4 + 1/3$ resonance is excited by powering the two strings with opposite polarity. Note that in the control system, power supplies C and F are called “b-sxr1-ps” and “b-sxr2-ps” respectively.

Figure 5: New User Interface

Figure 6: Actual user interface showing how currents I_H , I_V , I_C , I_F and parameters C_H , C_V , R , R_0 might be programmed for a typical NSRL magnetic cycle.

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