COMPARISON OF METHODS FOR CALCULATING RADIATIVE HEAT TRANSFER *

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Abstract

Various approximations for calculating radiative heat transfer between parallel surfaces are evaluated. This is done by applying the approximations based on total emissivities to a special case of known spectral emissivities, for which exact heat transfer calculations are possible. Comparison of results indicates that the best approximation is obtained by basing the emissivity of the receiving surface primarily on the temperature of the emitter. A specific model is shown to give excellent agreement over a very wide range of values.

Accurate computation of radiative heat transfer in thermionic converters is of great importance for predicting the conversion efficiency and the open-circuit temperature rise.

The diffuse radiation heat transfer rate from a plane emitter E to a parallel collector C is given by

$$q = \int_0^{\infty} [q_E(\lambda) - q_C(\lambda)] d\lambda, \quad (1)$$

where $q(\lambda) d\lambda$, the radiant energy flux in the wavelength interval λ to $\lambda + d\lambda$, leaving each surface is given by the sum of the emitted and reflected radiation:

$$q_E(\lambda) d\lambda = \epsilon_E(\lambda, T_E) \frac{2\pi h c^2 \lambda^{-5} d\lambda}{\exp(hc/\lambda k T_E) - 1} + [1 - \epsilon_E(\lambda, T_E)] q_C(\lambda) d\lambda, \quad (2)$$

$$q_C(\lambda) d\lambda = \epsilon_C(\lambda, T_C) \frac{2\pi h c^2 \lambda^{-5} d\lambda}{\exp(hc/\lambda k T_C) - 1} + [1 - \epsilon_C(\lambda, T_C)] q_E(\lambda) d\lambda. \quad (3)$$

Here h and k are Planck's and Boltzmann's constants, c is the speed of light, and $\epsilon(\lambda, T)$ denotes the hemispheric emissivity at wavelength λ and surface temperature T . Solving Eqs. (2) and (3) for $q_E(\lambda)$ and $q_C(\lambda)$ and inserting the results in Eq. (1), we obtain

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$$q = 2\pi h c^2 \int_0^\infty \left\{ \left[\exp\left(\frac{hc}{\lambda k T_E}\right) - 1 \right]^{-1} - \left[\exp\left(\frac{hc}{\lambda k T_C}\right) - 1 \right]^{-1} \right\} \left[\frac{1}{\epsilon_E(\lambda, T_E)} + \frac{1}{\epsilon_C(\lambda, T_C)} - 1 \right]^{-1} \frac{d\lambda}{\lambda^5} \quad (4)$$

It is now convenient to let $\bar{\epsilon}$ denote an effective emissivity, defined by

$$\bar{\epsilon} = \frac{\int_0^\infty \left\{ \left[\exp\left(\frac{hc}{\lambda k T_E}\right) - 1 \right]^{-1} - \left[\exp\left(\frac{hc}{\lambda k T_C}\right) - 1 \right]^{-1} \right\} \left[\frac{1}{\epsilon_E(\lambda, T_E)} + \frac{1}{\epsilon_C(\lambda, T_C)} - 1 \right]^{-1} \frac{d\lambda}{\lambda^5}}{\int_0^\infty \left\{ \left[\exp\left(\frac{hc}{\lambda k T_E}\right) - 1 \right]^{-1} - \left[\exp\left(\frac{hc}{\lambda k T_C}\right) - 1 \right]^{-1} \right\} \frac{d\lambda}{\lambda^5}} \quad (5)$$

Combining Eqs. (4) and (5) and integrating, we obtain the usual expression

$$q = \sigma \bar{\epsilon} (T_E^4 - T_C^4), \quad (6)$$

where σ is the Stephan-Boltzmann constant defined by

$$\sigma = 2\pi^5 k^4 / 15 c^2 h^3. \quad (7)$$

Equations (5) and (6) permit accurate computation of radiative heat transfer when experimental measurements of emissivity versus temperature and wavelength are available for both surfaces. For most surfaces, however, the only available data are not the spectral emissivity $\epsilon(\lambda, T)$ but the total emissivity $\epsilon(T)$, defined by

$$\epsilon(T) = \frac{\int_0^\infty \left[\exp(hc/\lambda k T) - 1 \right]^{-1} \lambda^{-5} \epsilon(\lambda, T) d\lambda}{\int_0^\infty \left[\exp(hc/\lambda k T) - 1 \right]^{-1} \lambda^{-5} d\lambda} \quad (8)$$

In such cases, it is customary to approximate the effective emissivity in terms of the total emissivities of the two surfaces, using an expression of the form

$$\bar{\epsilon} = \left[\epsilon_E(T) + \epsilon_C(T) - 1 \right]^{-1} \quad (9)$$

The crucial question, to which this paper addresses itself, is what temperatures to use in the arguments of Eq. (9). In general, the options considered include the use of the emitter temperature T_E , the collector temperature T_C and the geometric mean temperature

$$T_G = (T_E T_C)^{\frac{1}{2}}, \quad (10)$$

in various combinations. To determine which recipe yields the most accurate approximation of the actual heat transfer, we shall examine a special case (tungsten emitter and collector) for which sufficient measurements of spectral emissivity are available⁽¹⁾ to permit exact calculations. As shown by Figure 1, the spectral emissivity of tungsten is essentially a linear function of temperature, so that

$$\epsilon(\lambda, T) = a(\lambda) + b(\lambda)T. \quad (11)$$

Figure 2 presents a plot of the coefficients a and b as a function of λ .

To ensure complete consistency between the exact and the approximate calculations, the above set of emissivity data was used throughout. For the exact calculations the effective emissivity was computed from Eq. (5), while for the approximate calculations the total emissivities of the emitter and collector (presented in Figure 3) were computed from Eq. (8). It should be pointed out that the integration range in Eqs. (5) and (8) had to be truncated, because of the lack of experimental data below 0.25 and above 10.0 microns. However, the range covered included all but a small fraction of the radiant energy, since the Planck weighting function drops off rapidly at small and large wavelengths. Moreover, since the integration ranges in the numerator and denominator of Eqs. (5) and (8) are equally truncated, the resultant error in their ratio should be negligible.

Effective emissivities using four different approximations were computed for comparison with the exact solution:

$$\bar{\epsilon}_{\epsilon c} = \{[\epsilon_{\epsilon}(T_{\epsilon})]^{-1} + [\epsilon_c(T_c)]^{-1} - 1\}^{-1} \quad (12a)$$

$$\bar{\epsilon}_{\epsilon} = \{[\epsilon_{\epsilon}(T_{\epsilon})]^{-1} + [\epsilon_c(T_g)]^{-1} - 1\}^{-1} \quad (12b)$$

$$\bar{\epsilon}_{\epsilon \epsilon} = \{[\epsilon_{\epsilon}(T_{\epsilon})]^{-1} + [\epsilon_c(T_g)]^{-1} - 1\}^{-1} \quad (12c)$$

$$\begin{aligned} \bar{\epsilon}_{\epsilon gc} = & \{[\epsilon_{\epsilon}(T_{\epsilon})]^{-1} + [\epsilon_c(T_g)]^{-1} - 1\}^{-1} [1 - (T_c/T_{\epsilon})^4]^{-1} \\ & - \{[\epsilon_{\epsilon}(T_g)]^{-1} + [\epsilon_c(T_c)]^{-1} - 1\}^{-1} [(T_{\epsilon}/T_c)^4 - 1]^{-1}. \end{aligned} \quad (12d)$$

The last of these had been recommended by a recent study⁽²⁾ devoted to the same question as the present paper. That study had been based on a theoretical model rather than experimental data to describe the dependence of emissivity on wavelength.

The results of the present study, illustrated by Figure 4, showed that over the temperature range of interest in thermionic converters ($850 \leq T_c \leq 1150^\circ\text{K}$, $1700 \leq T_e \leq 2400^\circ\text{K}$) the third approximation ($\bar{\epsilon}_{ee}$, Eq. 12c) gives by far the best agreement with the exact solution (Eq. 5). Thus, while Reference 2 is correct in stating that the relatively crude approximation $\bar{\epsilon}_{ec}$ gives values that are 10-15% lower than $\bar{\epsilon}_{eqc}$, the present results indicate that $\bar{\epsilon}_{eqc}$ is still 10-15% below the actual value of the effective emissivity, and that a much better approximation is provided by model $\bar{\epsilon}_{ee}$.

In conclusion, a plausible qualitative argument can be offered for basing the receiver emissivity primarily on the emitter temperature: Spectral emissivity depends rather strongly on wavelength, but only weakly on temperature; in fact, as shown by Figure 1 the direction of the latter dependence reverses with wavelength. The principal reason why the total emissivity of a surface varies with temperature is the latter's effect on the spectral distribution. Therefore, the total emissivities used in calculating the heat transfer should be based not on the temperature of the surface, but rather on that temperature which best characterizes the spectral distribution of the predominant radiation reflected between the electrodes. Because of the fourth-power dependence, the dominant radiation in a thermionic converter is clearly that which originally emanated from the emitter. It is therefore not surprising that basing both emissivities on the emitter temperature yields the better approximation.

A still better approximation can be obtained by giving predominant, but not sole, weight to the temperature of the radiation source, with much lighter weight given to the temperature of the receiver. To determine the best way of doing this, exact heat transfer calculations were carried out over a very wide range of emitter and collector temperatures. Examination of the results indicated that they could be quite well approximated by the purely pragmatic equation:

$$q = \sigma \left[\frac{T_e^4}{\epsilon_e(T_e) + \epsilon_c(.8T_e + .2T_c) - 1} - \frac{T_c^4}{\epsilon_e(.2T_e + .8T_c) + \epsilon_c(T_c) - 1} \right]. \quad (13)$$

The computed results are presented in Table I. For each set of emitter and collector temperatures, the three numbers shown are respectively the exact heat flux (Eq. 4), the approximate heat flux (Eq. 13), and the percent error. As can be seen, this approximation exhibits remarkably good agreement over a thousand-fold variation in heat flux.

		Emitter Temperature, °K											
		3000	2800	2600	2400	2200	2000	1800	1600	1400	1200	1000	800
Collector Temperature, °K	400	81.886 81.199 -0.8	58.944 58.754 -0.3	41.198 41.299 0.2	27.822 28.069 0.9	18.044 18.341 1.6	11.156 11.441 2.6	6.516 6.754 3.7	3.555 3.732 5.0	1.788 1.903 6.4	0.814 0.878 7.9	0.327 0.356 8.8	0.110 0.119 8.6
	600	82.437 81.685 -0.9	59.432 59.158 -0.5	41.615 41.624 0.0	28.163 28.319 0.6	18.309 18.522 1.2	11.350 11.562 1.9	6.646 6.825 2.7	3.632 3.765 3.7	1.822 1.908 4.7	0.819 0.866 5.7	0.314 0.334 6.3	0.087 0.093 6.4
	800	82.865 82.072 -1.0	59.805 59.468 -0.6	41.923 41.859 -0.2	28.401 28.483 0.3	18.478 18.621 0.8	11.452 11.604 1.3	6.689 6.821 2.0	3.627 3.724 2.7	1.780 1.842 3.5	0.752 0.784 4.3	0.232 0.244 4.9	
	1000	83.072 82.262 -1.0	59.964 59.587 -0.6	42.025 41.907 -0.3	28.441 28.465 0.1	18.456 18.543 0.5	11.371 11.474 0.9	6.557 6.648 1.4	3.452 3.520 2.0	1.575 1.616 2.6	0.528 0.545 3.3		
	1200	82.892 82.092 -1.0	59.746 59.351 -0.7	41.760 41.608 -0.4	28.124 28.106 -0.1	18.085 18.130 0.2	10.951 11.015 0.6	6.094 6.153 1.0	2.956 2.998 1.4	1.058 1.078 1.9			
	1400	82.086 81.319 -0.9	58.913 58.521 -0.7	40.890 40.722 -0.4	27.212 27.168 -0.2	17.131 17.146 0.1	9.959 9.994 0.4	5.070 5.103 0.6	1.910 1.929 1.0				
	1600	80.335 79.623 -0.9	57.146 56.776 -0.6	39.099 38.931 -0.4	25.391 25.334 -0.2	15.280 15.276 -0.0	8.080 8.094 0.2	3.171 3.184 0.4					
	1800	77.245 76.604 -0.8	54.052 53.718 -0.6	35.991 35.836 -0.4	22.265 22.206 -0.3	12.135 12.121 -0.1	4.919 4.921 0.0						
	2000	72.350 71.791 -0.8	49.163 48.874 -0.6	31.098 30.964 -0.4	17.363 17.311 -0.3	7.223 7.209 -0.2							
	2200	65.118 64.643 -0.7	41.944 41.706 -0.6	23.881 23.777 -0.4	10.144 10.110 -0.3								
	2400	54.952 54.564 -0.7	31.793 31.613 -0.6	13.737 13.673 -0.5									
	2600	41.195 40.903 -0.7	18.051 17.944 -0.6										
	2800	23.134 22.961 -0.7											

exact solution Eq. 4

approximation Eq. 13

percent error

Table 1. Radiative Heat Transfer in w/cm^2 between two tungsten surfaces

References

- (1) GUBAREFF, G.G., JANSSEN, J.E., and TORBORG, R.H., "Thermal Radiation Properties Survey," Honeywell Research Center, Minneapolis, Minnesota, 1960.
- (2) McCANDLESS, R. J. and HILL, P.R., "Radiation Heat Transfer Calculations," 1967 IEEE Conference Record of the Thermionic Conversion Specialist Conference, p.370.

Fig. 1: Spectral Emissivity of Tungsten

Fig. 2: Curve-Fit Constants $a(\lambda)$ and $b(\lambda)$

Fig. 3: Total Emissivity of Tungsten

Fig. 4: Comparison of Approximations with Exact Solution