How well do we know the neutron structure function?

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We present a detailed analysis of the uncertainty in the neutron $F_{2n}$ structure function extracted from inclusive deuteron and proton deep-inelastic scattering data. The analysis includes experimental uncertainties as well as uncertainties associated with the deuteron wave function, nuclear smearing, and nucleon off-shell corrections. Consistently accounting for the $Q^2$ dependence of the data and calculations, and restricting the nuclear corrections to microscopic models of the deuteron, we find significantly smaller uncertainty in the extracted $F_{2n}/F_{2p}$ ratio than in previous analyses. In addition to yielding an improved extraction of the neutron structure function, this analysis also provides an important baseline that will allow future, model-independent extractions of neutron structure to be used to examine nuclear medium effects in the deuteron.

Because the free neutron is an experimentally impractical scattering target, extracting its structure function, $F_{2n}$, requires cross section data from inclusive deep-inelastic scattering (DIS) measurements on proton and deuteron targets, together with a model describing the smearing produced by the nuclear binding in the deuteron. Previous extractions [1–3] using a variety of models of the deuteron bound state yielded a large range of $F_{2n}/F_{2p}$ values from the same proton and deuteron data, indicating large theoretical uncertainties, particularly at high values of the Bjorken variable $x$ — see Fig. 1. In addition to limiting our ability to extract the neutron structure function, the large spread of results, even among extractions including only traditional nuclear effects such as Fermi motion and binding, has made it difficult to identify a reliable baseline which could be used to search for more “exotic” nuclear effects such as the modification of the nucleon structure function in nuclei or non-nucleonic degrees of freedom.

A more recent extraction of the neutron $F_{2n}$ structure function [4] showed that some of the variation in results can be attributed to inconsistent treatment of kinematics of the data and calculations. In particular, it was vital to properly account for the $Q^2$ dependence of the proton and deuteron data, especially for $x \gtrsim 0.7$ [4, 5].

Accurate information on the $F_{2n}/F_{2p}$ ratio at large $x$ is essential for a number of reasons. These include constraining leading-twist parton distribution functions (PDFs) in the region $x \gtrsim 0.7$, which are an important input for QCD background calculations in searches of physics beyond the standard model at the Tevatron and the LHC, and in neutrino oscillation experiments. The ratio $F_{2n}/F_{2p}$ also provides insight into the nonperturbative quark-gluon dynamics in the nucleon [3, 6]. There are several predictions for $F_{2n}/F_{2p}$ based on symmetry arguments that determine the dominant contributions as $x \to 1$, ranging from $F_{2n}/F_{2p} = 2/3$ for exact spin-flavor SU(6) symmetry, to $3/7$ assuming helicity conservation through hard gluon exchange [7], to $1/4$ when SU(6) symmetry is broken through scalar diquark dominance [8, 9].

In this paper we extend the analysis of Ref. [4] by performing a detailed study of the model dependence of the extraction procedure, systematically assessing the uncertainties arising from the deuteron wave function at short distances, nucleon off-shell effects, and different nuclear smearing models used to compute the nuclear corrections. The goal is to determine the degree to which the $F_{2n}$ neutron structure function can be determined at large $x$, using a consistent treatment of input data sets and realistic models of nuclear effects, and employing a methodology that is transparent and conceptually accessible.

Following Ref. [4], we consider $F_{2d}/F_{2p}$ data from SLAC, BCDMS, and NMC, as compiled in Ref. [1] for $3 < Q^2 < 230$ GeV$^2$, using the full $Q^2$ range to determine the $Q^2$ dependence of the ratio. The extraction of $F_{2n}/F_{2p}$ itself is limited to data in the range $8 < Q^2 < 32$ GeV$^2$, and the results are interpolated to a fixed $Q^2 = Q^2_0 = 16$ GeV$^2$. The interpolation to $Q^2_0$ differs from the simple average over the limited $Q^2$ range by at most 0.7%, with a typical correction of 0.3%.
Most previous extractions of $F_{2n}/F_{2p}$ used of $F_{2d}/F_{2p}$ from the analysis of Ref. [1], in which $Q^2$ varies from 4.7 to 23.6 GeV$^2$ over the $x$ range of the data. However, the extractions treat the $F_{2d}/F_{2p}$ ratios as though they were all at some average $Q^2$ value. Such extractions neglect the $Q^2$ dependence of the nuclear effects, which have been found to be significant at large $x$ [4, 5]. By interpolating all of the source data to a common $Q_0^2$ and extracting $F_{2n}/F_{2p}$ using different models evaluated at the same scale, a more systematic and meaningful assessment of the model dependence can be made.

The extraction of $F_{2n}/F_{2p}$ proceeds by assuming that the deuteron structure function $F_{2d}$ can be expressed as a sum of smeared proton and neutron structure functions, and an additional correction, $\delta F_{2d}$, that goes beyond the convolution approximation, $F_{2d} = F_{2p} + F_{2n} + \delta F_{2d}$, where $F_{2N} = S_N F_{2N}$ is the smeared nucleon structure function ($N = p, n$), and $S_N$ is the smearing ratio. The term $\delta F_{2d}$ includes any corrections, such as relativistic or nucleon off-shell corrections [10, 11], that cannot be expressed as a convolution of a smearing function and the free nucleon structure function. Parametrizing this correction as a ratio to the total deuteron structure function, $\Delta = \frac{\delta F_{2d}}{F_{2d}}$, the $F_{2n}/F_{2p}$ ratio can then be extracted using a modified smearing factor $S_N = S_n/(1 - \Delta)$,

$$
\frac{F_{2n}}{F_{2p}} = \frac{1}{S_N} \left( \frac{F_{2d}}{F_{2p}} - \tilde{S}_p F_{2p} \right).
$$

The neutron to proton ratio can thus be extracted from $F_{2d}/F_{2p}$ and $F_{2p}$ data, and a model of the smearing ratios $S_N$ and the off-shell correction $\Delta$.

To standardize comparisons of the different calculations, all extractions use the same values for $F_{2p}$ and $F_{2n}$ to compute the smearing functions $\tilde{S}_N(x)$. We use the parametrization of the world’s $F_{2p}$ data and the extracted neutron structure function from Ref. [4]. Each calculation yields a slightly different $F_{2n}$, however, the impact of using this modified $F_{2n}$ value to update the calculation of $S_n$ is small compared to the other uncertainties ($F_{2n}/F_{2p}$ changes by less than 0.01 at $x = 0.85$) [12].

The assessment of the model dependence of the extracted $F_{2n}/F_{2p}$ ratio ultimately depends on the choice of nuclear models used in the analysis. The introduction of some degree of bias is therefore inevitable, although we aim to make the selection criteria as objective as possible by restricting ourselves to microscopic calculations involving high-precision $NN$ potentials that give realistic descriptions of the deuteron bound state. Common features of the models surveyed in this analysis include the use of realistic deuteron wave functions which account for nuclear Fermi motion and binding, an exact treatment of finite-$Q^2$ kinematics, and allowance for possible nucleon off-shell corrections in the deuteron [3, 4, 13, 14]. We exclude models that involve extrapolations of nuclear medium modifications observed in structure functions of heavy nuclei to the deuteron [1, 2, 15], which typically invoke a very large nuclear density, contain no $Q^2$ dependence, and effectively assume $S_n = S_p$. Note that this has less impact than one would expect from Fig. 1. If one repeats these analyses taking into account the $Q^2$ dependence of the $F_{2d}/F_{2p}$ data, the ratios are below 0.5 at $x \lesssim 0.6$ for all of the extractions.

The dependence on the choice of deuteron wave functions is illustrated in Fig. 2, which shows the results for $F_{2n}/F_{2p}$ using different modern nonrelativistic (N3LO [16], CD-Bonn [17], AV18 [18]) and relativistic (WJC-1, WJC-2 [19]) deuteron wave functions. The smearing factors for each of the calculations were computed using the weak binding approximation (WBA) model [11, 13], which is derived by expanding the nucleon correlation function in the nucleus in powers of the nucleon momentum $p$ up to order $p^2/M^2$, where $M$ is the nucleon mass. The gray band represents the RMS spread of the $F_{2n}/F_{2p}$ ratios for the five wave functions considered, which is the 1σ band for the wave function uncertainty if we treat each of the $NN$ potentials on an equal footing. The ratio $F_{2n}/F_{2p}$ clearly becomes increasingly sensitive to the choice of deuteron wave function at larger $x$ values, reflecting the larger uncertainty in the $NN$ interaction at short distances or high nucleon momenta in the deuteron.

The dependence of $F_{2n}/F_{2p}$ on the model used for the smearing function (or smearing factor) and off-shell prescription is illustrated in Fig. 3. The curves show the results for the given calculation, averaged over the potentials shown in Fig. 2. For calculations where not all potentials were available, the WBA result was used to extrapolate to the average potential. Note that some smearing calculations, such as those used in Ref. [1], have been omitted as they represent calculations similar to those included here, but with additional numerical approximations. The solid curves in Fig. 3 are the results.
of smearing calculations with on-shell nucleon structure functions, with their spread indicating the uncertainty associated with the smearing function. The WBA calculation (solid black curve) is a modern calculation that makes minimal approximations, and is used as the baseline for showing the results of calculations including nucleon off-shell corrections. The four WBA results (solid black curve and the three dashed curves) indicate the model dependence in the off-shell prescriptions.

To estimate the combined uncertainty, we take the RMS spread of all of the extractions of $F_{2n}/F_{2p}$ shown in Fig. 3 at each $x$ value, indicated by the light gray band. Similar uncertainties are obtained if the model dependence of the smearing function and off-shell contributions are extracted separately and combined in quadrature. The uncertainty associated with the smearing function is essentially negligible up to $x = 0.6$, but is comparable to the off-shell corrections for $x > 0.75$.

Figure 4 shows the combined uncertainty range (gray band) compared to the range of results shown in Fig. 1 (red hatched region). The central result is taken as the global average of the extracted $F_{2n}/F_{2p}$ values obtained in Fig. 3. The individual uncertainties associated with the experimental systematic uncertainties (evaluated in Ref. [4]), the dependence on the deuteron wave function, and the dependence on the smearing function and off-shell effects (labeled “Model Uncertainty” in Fig. 4), are shown separately, as well as the sum of uncertainties added in quadrature.

Our analysis provides a significantly narrower range of results than that evident in Fig. 1 for the full spectrum of models. At $x = 0.85$, for example, the range in Fig. 1 spans $0.2 < F_{2n}/F_{2p} < 0.7$, whereas the present analysis suggests a $1\sigma$ range of $0.18 < F_{2n}/F_{2p} < 0.32$. The tighter bounds are largely due to the exclusion of models involving extrapolation of nuclear medium effects from heavy nuclei, and would appear to exclude the SU(6) predictions of $F_{2n}/F_{2p} \to \frac{2}{3}$, while favoring the lower estimates consistent with the partonic lower limit of $F_{2n}/F_{2p} \to \frac{1}{4}$.

The dependence of the extraction on the choice of nuclear model is further illustrated in Fig. 5, which shows the total uncertainty bands corresponding to two different subsets of results. The solid lines show the range obtained by taking only the on-shell extractions in Fig. 3, which yield $0.16 < F_{2n}/F_{2p} < 0.28$ at $x = 0.85$. While much of the range at large $x$ is below $F_{2n}/F_{2p} = 0.25$, which in the parton model is forbidden by the requirement that PDFs are positive, the results are not inconsistent with $F_{2n}/F_{2p} > 0.25$. This range can be thought of as a baseline for effects beyond the on-shell convolution approximation, and comparison of these results to model-independent extractions of $F_{2n}/F_{2p}$ can be used to isolate off-shell contributions or other more exotic nuclear effects.

On the other hand, most modern quantitative analyses of nuclear structure functions and the nuclear EMC effect require the inclusion of some modification of the nucleon structure function in a nuclear medium [11, 21–23]. Restricting the set to only models that incorporate off-shell effects, one obtains the dashed blue band in Fig. 5. As expected, this leads to a higher range for the neutron to proton ratio, $0.25 < F_{2n}/F_{2p} < 0.36$ at $x = 0.85$. Note that in both cases, the experimental systematics and deuteron wave function dependence is included in
the bands.

At leading order in the strong coupling constant, the nucleon structure functions are given by the charge-squared weighted sum of the $u$ and $d$ quark distributions. In this approximation the extracted value of $F_{2n}/F_{2p}$ is directly related to the ratio of $d$ to $u$ quark distributions,

$$\frac{d}{u} = \frac{4F_{2n}/F_{2p} - 1}{4 - F_{2n}/F_{2p}}.$$  \hspace{1cm} (2)

The resulting $d/u$ ratio is shown in Fig 6, along with the fractional uncertainty (inset), for the full range of models, as well as for the on-shell and off-shell models from Fig. 5. Such an extraction neglects higher-order perturbative QCD corrections, target mass corrections, and higher twist effects. On the other hand, higher twists will cancel in this ratio, unless they differ for the proton and neutron [24], as will target mass corrections to a large extent, although some residual prescription dependence survives at large values of $x$ [25]. Therefore, even though Eq. (2) is approximate, the results in Fig. 6 do serve to illustrate how uncertainties in $F_{2n}/F_{2p}$ propagate to the $d/u$ ratio.

The absolute uncertainty on the $d$ quark distribution is small compared to the overall size of the $u$ quark distribution ($<10\%$ for all $x$ values shown). In contrast, the fractional uncertainty on the $d$ quark PDF is large, and will yield significant uncertainties on quantities sensitive to small relative uncertainties in $d$, such as the PDF inputs for cross section calculations in high-energy collisions. Although the error band in Fig. 6 includes a significant region with $d < 0$ for $x > 0.8$, corresponding to $F_{2n}/F_{2p} < 0.25$, the 1σ bands are never inconsistent with a positive PDF.

While the extracted $d/u$ ratio in Fig. 6 is illustrative of the reduced uncertainty from the restricted range of nuclear corrections considered here, the primary goal of the present work is an extraction of the total $F_{2n}$ rather than a separation of the PDFs from target mass and higher twist contributions. A complementary study which quantified the effects of nuclear corrections on PDFs within a global QCD analysis was recently performed by the CJ Collaboration [20]. The largest impact of the nuclear effects, which were computed using the WBA smearing function with nucleon off-shell corrections from the modified KP model (see Fig. 3), was found for the $d$ quark PDF at large $x$. The uncertainties in the resulting $d/u$ ratio were similar to those in Fig. 6, although with a somewhat larger spread. In particular, the range in Ref. [20] for the neutron to proton ratio at $x = 0.85$ was found to be $0.32 < F_{2n}/F_{2p} < 0.50$ at $Q^2 = 16$ GeV$^2$, where the upper and lower limits were obtained using models with the minimum and maximum nuclear corrections, respectively. This is a larger range than found here, although it represents the full range or results, rather than an estimated 1σ error band. Taking a linear sum of all uncertainties from Fig. 4 yields the range $0.04 < F_{2n}/F_{2p} < 0.42$. The upper limit is in reasonable agreement with the CJ global fit, while the lower limit is significantly smaller. This is because PDFs in global QCD analyses are constrained to be positive a priori, forcing the overall $d/u$ bands (and consequently $F_{2n}/F_{2p}$) to lie higher than those in Fig. 6.

In summary, we have extracted the $F_{2n}/F_{2p}$ structure function ratio from a range of models of the deuteron structure, and estimated the uncertainties associated with the choice of nuclear smearing model, deuteron wave function, nucleon off-shell corrections, and experimental uncertainties. Restricting the analysis to models based on microscopic calculations of deuteron structure, and excluding those that rely on extrapolations from heavier nuclei, we find a range of results that is significantly smaller than in some previous extractions. The general consistency between different microscopic models eval-
uated here provides a reliable baseline for the neutron structure assuming only traditional nuclear effects, so that future model-independent extractions of the neutron structure function [26, 27] can both improve our knowledge of the neutron structure and have the potential to provide signatures for more exotic nuclear effects.

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