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J. Randrup and S. E. Koonin

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THE DISASSEMBLY OF NUCLEAR MATTER*

J. Randrup

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

and

S. E. Koonin[†]

W. E. Kellogg Radiation Laboratory
California Institute of Technology
Pasadena, California 91125

ABSTRACT

A statistical model is developed for multi-fragment final states in nuclear collisions with bombarding energies $E/A \approx 100$ MeV. A portion of the intermediate system formed is assumed to decay according to the available classical non-relativistic phase space, calculated in a grand canonical ensemble. The model correlates and predicts many experimental observables in terms of three parameters: the available energy per nucleon, the isospin asymmetry, and the effective interaction volume.

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1. INTRODUCTION

The field of intermediate-energy nuclear physics is developing rapidly, with several accelerators throughout the world under construction to deliver nuclear beams with energies from tens to hundreds of MeV per nucleon. Experiments with such beams may create the novel phenomenon of a transient nuclear system with excitation comparable to its binding energy. Such a system can decay through a large number of different final channels and, generally, many fragments will be formed. Indeed, such multi-fragmentation processes are suggested by recent Bevalac data [1].

The fundamental question of how a highly excited nucleus disassembles must therefore be addressed. The detailed dynamical description of the multi-fragmentation process is not yet within reach. Conventional compound-nucleus evaporation theories are not appropriate, since many fragments may be emitted nearly simultaneously. A description of the disassembly would require the specification of how the excitation energy is initially distributed in the system and would have to deal with the problem of how the disassembly develops in space and time. Rather than embarking on such an ambitious task, we seek to develop a simple model which can provide a useful basis upon which to judge both more elaborate calculations and the experimental data. To this end we derive the one-fragment inclusive spectra on the basis of statistical considerations.

The utility of the statistical limit was discussed by Fermi [2] in connection with multi-particle production in high-energy proton-proton collisions. For high-energy nuclear collisions ($E/A \gtrsim 200$ MeV), Mekjian

and others [3] have developed models for composite fragment production which assume thermal and chemical equilibrium within a certain interaction volume. We apply similar ideas to nuclear collisions at intermediate energies ($20 \text{ MeV} \leq E/A \leq 200 \text{ MeV}$) where the excitation energy is comparable to the binding energy. Because our model is based on the phase space available to decays, it is practically identical to assuming an equilibrated source in each collision. However, it is not necessary to argue that equilibrium be reached in any given collision, since a statistical occupation of the phase space at the one-fragment inclusive level can occur as a result of averaging over many separate collision events, each of which can be far from equilibrium throughout. In this respect, ours is a minimal model with little prejudice concerning the collision dynamics.

The formulation of the model is described in Section 2, while in Section 3 we present and discuss some instructive general results. In Section 4 we make contact with some existing data.

2. THE MODEL

We are interested in the multi-fragment disassembly of a highly excited system of nucleons. The most readily accessible observable is the inclusive spectrum of a single fragment type, which is obtained by integrating the decay probability over all other specifications of the final state. Our basic assumption is that this quantity reflects only the available phase space. More precisely, we assume that this statistical limit provides a useful reference calculation of the one-fragment spectra. This simple limit is, of course, less adequate for the calculation of multi-fragment observables, since these are expected to be more sensitive

to the actual disassembly dynamics and, indeed, may be used eventually to probe the details of the collision.

We are thus faced with the task of enumerating the phase space available for the decay process. In an exact treatment, this should be done in a microcanonical ensemble having fixed values of the total energy, mass, momentum, etc. For large systems the problem can be simplified substantially by employing a grand canonical ensemble. The key quantity needed to calculate the statistical properties of the system is then the partition function

$$Z = \sum_f e^{-\beta(E_f - \mu A_f - \nu T_f)} \quad (1)$$

The sum extends over all possible final states which are characterized by the number of fragments produced and their mass numbers, isospin projections, internal excitations, momenta, and positions. The ensemble average of the total energy E_f , the total mass number A_f , and the total isospin projection T_f can be made to be equal to prescribed values E_0 , A_0 , T_0 by adjusting the Lagrange multipliers β , μ , ν appropriately. Here β is the inverse of the ensemble temperature τ and μ and ν are related to the neutron and proton chemical potentials by $\mu = \frac{1}{2}(\mu_n + \mu_p)$ and $\nu = \mu_n - \mu_p$.

The quantity $\ln Z$ is an extensive quantity proportional to A_0 . It is useful to define a corresponding intensive quantity $\omega = (\ln Z)/A_0$. If the interfragment interactions are neglected (see below), so that the total energy E_f is the sum of independent contributions from each of the individual fragments in a given configuration, then ω can be written in terms of contributions from the different fragment species:

$$\omega = \sum_{AT} \omega_{AT}, \quad (2)$$

where AT characterizes the nuclear species with A nucleons and isospin projection $T = \frac{1}{2}(N-Z)$. The multiplicity of the fragment species AT is proportional to ω_{AT} . It is an elementary exercise to verify that

$$\omega_{AT} = \chi \frac{4\pi}{3} r_o^3 \cdot \left(\frac{2\pi m A}{\beta \hbar^2} \right)^{3/2} \cdot z_{AT} \cdot e^{-\beta(V_{AT} - \mu A - \nu T)}. \quad (3)$$

Here the first of the four factors arises from the integration over the fragment position. The result of this integration is an effective volume which expresses the typical size of the system at the time of disassembly. Alternatively, this volume can be thought of as characterizing the A-dependence of the decay probability [2]. We express this effective volume as $\chi \frac{4\pi}{3} r_o^3 A_o$ where $r_o = 1.15$ fm is the nuclear radius constant. The dimensionless quantity χ is a parameter of the model which is expected to be of order unity and can be determined by comparison with experiment.

The second factor in eq. (3) results from the corresponding integration over the fragment momentum, which has a maxwellian distribution at the ensemble temperature τ . The inertial mass of a fragment is equal to A times the nucleon mass m.

The third factor is the intrinsic partition function resulting from the summation over the intrinsic states of the fragment:

$$z_{AT} = \sum_i g_{AT}^{(i)} e^{-\beta \epsilon_{AT}^{(i)}}. \quad (4)$$

Here $\epsilon_{AT}^{(i)}$ is the excitation energy of the i 'th level in the species AT and $g_{AT}^{(i)} = 2j_{AT}^{(i)} + 1$ is its degeneracy. The average fragment excitation energy is given by

$$\bar{\epsilon}_{AT} = \frac{1}{z_{AT}} \sum_i \epsilon_{AT}^{(i)} g_{AT}^{(i)} e^{-\beta \epsilon_{AT}^{(i)}} \quad (5)$$

In the final exponential factor in eq. (3), V_{AT} is the ground-state mass excess.

As noted above, this formulation neglects the fragment-fragment interaction, which is dominated by the electrostatic repulsion. Because of the long range of this interaction, the inter-fragment Coulomb energy contribution to E_f will be largely configuration independent and can be accounted for approximately by assuming that the fragmentation occurs in a configuration-independent electrostatic potential ϕ (identified with one-half of the typical electric potential inside the decaying nucleus). The energy V_{AT} is then increased by the amount $Ze\phi$ or, equivalently, the proton chemical potential μ_p is decreased by $e\phi$. There is thus no physical effect on our calculations.

Once the Lagrange multipliers β , μ , ν and the volume parameter χ have been specified, the statistical properties of the fragment ensemble can be calculated from the partition function Z , or, equivalently, the quantity ω . To ensure that the total energy, nucleon number and isospin projection correspond to their prescribed values E_0 , A_0 and T_0 , the following three relations must be satisfied:

$$\begin{aligned} E_0 &= \langle E \rangle = - \frac{\partial}{\partial \beta} \ln Z \quad , \\ A_0 &= \langle A \rangle = \frac{1}{\beta} \frac{\partial}{\partial \mu} \ln Z \quad , \\ T_0 &= \langle T \rangle = \frac{1}{\beta} \frac{\partial}{\partial \nu} \ln Z \quad . \end{aligned} \quad (6)$$

It is useful to introduce the reduced quantities

$$\epsilon = \frac{\langle E \rangle}{\langle A \rangle} = \frac{E_o}{A_o} \quad (7)$$

$$I = \frac{\langle N - Z \rangle}{\langle A \rangle} = \frac{2T_o}{A_o}$$

which express the available energy per nucleon and the average isospin asymmetry, respectively. Upon dividing the equations (6) by A_o we arrive at the following set of equations:

$$\begin{aligned} \epsilon &= \sum_{AT} \left(\frac{3}{2} \tau + v_{AT} + \bar{\epsilon}_{AT} \right) \omega_{AT} \\ I &= \sum_{AT} A \omega_{AT} \\ I &= \sum_{AT} 2T \omega_{AT} \end{aligned} \quad (8)$$

These equations contain only the intensive quantities ϵ and I . Their solution therefore applies to all systems with the same values of ϵ and I , independently of A_o . This feature greatly simplifies the subsequent discussion.

For specified values of ϵ and I , and the volume parameter χ , we numerically solve the coupled equations (8) for the Lagrange multipliers β , μ , and ν using Newton's method. The various characteristic properties can then be derived from the corresponding intensive quantity ω . Extensive quantities for any given system can be obtained by a proper scaling of the results presented below.

3. CALCULATED RESULTS

We have employed the model described above to explore the gross properties of nuclear multi-fragmentation at intermediate energies.

In the calculations all particle-stable nuclear states with $A \leq 16$ have been included explicitly using the empirical values for V_{AT} and $g_{AT}^{(i)}$ [4]. For heavier fragments we have used the macroscopic formula [5]

$$V_{AT} = (-a_1 A + a_2 A^{2/3}) \left(1 - \kappa \left(\frac{N-Z}{A} \right)^2 \right) + c_3 \frac{Z}{A^{1/3}} - c_4 \frac{Z^2}{A} + Nm_n + Zm_p \quad (9)$$

for the ground-state mass excess. Furthermore, we have assumed that the threshold for particle emission occurs at 8 MeV and used

$$\rho_{AT}(\epsilon) = \frac{6^{1/4}}{12} \frac{g_0}{(g_0 \epsilon)^{5/4}} e^{2 \left(\frac{\pi^2}{6} g_0 \epsilon \right)^{1/2}}, \quad 2 \text{ MeV} < \epsilon < 8 \text{ MeV} \quad (10)$$

with $g_0 = A/50$ MeV, for the density of particle-stable states [6]. Since the light fragments are predominant, the solution is virtually insensitive to how the heavy fragments are treated.

Figure 1 shows the dependence of the ensemble temperature τ on the energy per nucleon ϵ . A number of different values have been considered for the volume parameter χ : 0.1, 0.3, 1.0, 3.0. We expect that the physically relevant χ -values are around unity for the following reason: The partition of the system must occur when the strong interactions have effectively ceased to act; the system has then probably expanded moderately. However, only a portion of the

volume occupied by the system is available to a given fragment because of the presence of other fragments. For example, for a break-up into many equal fragments, which is the dominant partition for a large system, this blocking effect can be estimated to reduce the available volume by a factor of around $e \approx 3$. Since the expansion and the blocking thus counteract each other the relevant effectively available volume may therefore not be far from the original nuclear volume which would correspond to $\chi = 1$. The resulting curves in Fig. 1 are insensitive to the isospin asymmetry parameter I .

The curve corresponding to a gas of free nucleons with the same value of ϵ is shown by the dashed line: $\tau = 2/3 \epsilon$. This limit is obtained formally for $\chi \rightarrow \infty$. For finite values of χ it is advantageous to form composite fragments, the more so the smaller the value of χ . This reduces the number of translational degrees of freedom available for sharing the energy and hence raises the temperature. At moderate excitations this effect is relatively large. In spite of this shift in the temperature, the specific heat capacity $\partial \epsilon / \partial \tau$ fairly quickly approaches the ideal-gas value of $3/2$, indicating that most of the energy added goes into fragment kinetic energy.

In subsequent figures we shall employ the temperature τ rather than ϵ to characterize the fragment ensemble. This is because τ is related directly to such observable quantities as the slope of the fragment momentum distributions, the widths of the isobaric distributions, and the intrinsic nuclear excitation spectra.

The number of fragments of a given species AT is characterized by a Poisson distribution with mean value $A_0 \omega_{AT}$. In Fig. 2 we display

the variation of the overall fragment distribution with the temperature τ . The average fragment size

$$\bar{A} = \sum_{AT} A \omega_{AT} / \sum_{AT} \omega_{AT} \quad (11)$$

is displayed in the upper portion and the lower portion shows the associated dispersion in fragment size,

$$\sigma_A^2 = \sum_{AT} A^2 \omega_{AT} / \sum_{AT} \omega_{AT} - \bar{A}^2 \quad (12)$$

For high temperatures most fragments appear as single nucleons and $\bar{A}$ approaches unity. At lower temperatures the formation of composite fragments becomes more predominant and relatively heavy fragments occur in appreciable abundance. There is a rather strong dependence of the fragment size on the volume parameter χ .

Figure 3 displays two examples of the fragment mass distribution. for $\epsilon = 5$ MeV and 40 MeV, with $\chi = 0.3$ and $I = 0$. The distribution broadens with decreasing excitation. A persistent structure in the mass spectrum arises from the particular occurrence of levels in the different nuclei. For example, nuclei with $A = 10, 11$ emerge as especially abundant because of the relatively many bound levels in these nuclei. In searching for such a structure one should be aware that the present calculation is based on the idealization that only the asymptotic phase space is important. Realistically, the dynamics of the fragmentation is probably such that particle-unstable fragments may be emitted. Their subsequent decay will modify

the results somewhat and tend to wash out this structure. In fact, a perturbative inclusion of unstable fragments indicates that the final observable distributions would be altered appreciably. The mapping of the detailed structure of the elemental and isotopic yields may therefore help determine the dynamics of the fragmentation process.

Figure 4 shows the isobaric distribution for the two different isospin symmetries $I=0$ and $I = 0.2$, at $\epsilon = 5$ MeV and $\chi = 0.3$. It should be noted how the presence of a neutron excess significantly tilts the isobaric populations toward the neutron-rich side. Thus, information on the isobaric distributions may provide a tool for determining the composition of the initial source. The ratio of final free neutron fragments to final free proton fragments is particularly sensitive to the neutron excess in the source. For symmetric sources ($I = 0$) there is generally a slight excess of final free protons over final free neutrons because the composite fragments tend to carry more neutrons than protons, particularly the heavier species which are formed at lower energies. However, even at the low energies the fragments formed can only accommodate a small neutron excess. When $I > 0$, most of the excess neutrons must therefore appear as free neutrons and thus they may out-number the free protons considerably.

This effect is illustrated in Fig. 5 where the ratio of final free neutrons to final free protons is plotted versus τ for the various χ -values considered. For $I=0$ the ratio is close to unity (but somewhat smaller, particularly at small τ). For $I=0.2$, which is representative of a heavy nucleus like Pb, the ratio is substantially above unity, and the more so the smaller the value of χ . For decreasing

temperatures, the ratio grows rapidly, reaching values of ≈ 8 for $\tau \approx 10$ MeV. This behavior is expected from the fact that when the heavier fragments grow more abundant (low τ and small χ) there are less free protons left to match the excess neutrons. In this way the fragmentation process acts as an amplifier of the neutron excess in the source.

4. DISCUSSION

In the previous section we have presented a selection of general results calculated with our simple statistical model. These results possess a number of qualitative features which would be interesting to test experimentally.

The model predicts and correlates a number of quantities which are all amenable to experimental investigation: the relative multiplicity of the various fragment species (element and mass distributions), their momentum distributions, and their intrinsic excitation spectra. The calculated results are given in terms of a few parameters: the specific excitation energy ϵ , the isospin asymmetry I , and the reduced effective volume χ . The values of these quantities depend on the specific dynamical model adopted. Conversely, by analyzing the data in terms of the statistical model one may determine effective values of these characteristic quantities and thus, in turn, obtain constraints on the possible collision dynamics. Therefore, the model should provide a useful basis for interpreting and discussing experiments on processes with many-fragment final states.

At the present time, there is little data on the multi-fragmentation process at intermediate energies. However, certain features of data

taken at somewhat higher energies can be understood within the framework of our model.

For example, in Fig. 1 the temperature T was shown as a function of the available energy per nucleon. As a general consequence of composite fragment formation, this temperature is substantially higher than that expected for a gas of free nucleons with the same energy per nucleon. Price *et al* [7] have measured the inclusive spectra of heavy composite fragments produced in the collisions of Ne with U at $E/A = 400$ MeV and Ar with Au at $E/A = 500$ MeV. They found the data to be well described as isotropic thermal distributions emitted from a moving source. However, the relatively low source velocity and, in particular, the high spectral temperature appeared to the authors to be inconsistent with conventional models. This conclusion relied on a free gas formula to calculate the temperature. However, as we have discussed, when composite fragments are formed the number of translational degrees of freedom is reduced and the temperature is correspondingly increased. As is evident from Fig. 1, this increase is about a factor of 2 in the present case, so that the observations no longer appear fundamentally puzzling.

Similar data have been obtained by Gosset *et al.* [8] for Ne + U at $E/A = 400$ MeV. They find the boron fragments to be described by a temperature of 27 MeV as compared to the 13.5 MeV derived for a gas of free nucleons. Again, this higher value corresponds well with the results displayed in Fig. 1.

As a second example of our model predictions, we consider a recent neutron measurement [9]. By combining this with earlier proton data [10] it has been possible to determine the ratio of neutrons

to protons ejected in collisions of Ne with U at $E/A \approx 350$ MeV. These ejectiles, which are too energetic to arise from ordinary evaporation, typically are in a ratio of about 3:1. This is quite consistent with our present results as displayed in Fig. 5. The relatively abundant composite fragments carry off approximately equally many neutrons and protons, thus amplifying the ratio of free neutrons to protons relative to that in the total system. (For details see the discussion in connection with Fig. 5.) This quantitatively confirms a hypothesis proposed by Stock [11].

These are two examples of how our model can be used to understand semi-quantitatively the observed one-fragment inclusive data. Of course, it makes trivial (uncorrelated) predictions for multi-fragment observables, since it is based on an independent-fragment picture. It is likely that only by studying the simultaneous emission of several fragments will we eventually determine the dynamics of the disassembly process.

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FIGURE CAPTIONS

- Fig. 1. The temperature τ as a function of the energy per nucleon ϵ , for various values of the volume parameter χ . The curves shown correspond to $I=0$, i.e., to sources with equally many neutrons and protons, but calculations with $I=0.2$ give practically identical results. The result for a gas of free nucleons is indicated by the dashed line, $\tau = 2/3 \epsilon$.
- Fig. 2. (a) The average fragment mass number $\bar{A}$ as a function of the temperature τ , for various values of the volume parameter χ . Points corresponding to the same value of the energy per nucleon ϵ are joined by the light curves.
(b) The corresponding width σ_A in the fragment mass number distribution.
- Fig. 3. The distribution of fragment masses for $\epsilon = 5$ MeV and $\epsilon = 40$ MeV, at $\chi=0.3$ and $I=0$. Note that there are no stable fragments with $A=5$.
- Fig. 4. Isobar distributions for different values of the mass number A . The results are for $I=0$ (top) and $I=0.2$ (bottom) at $\epsilon = 5$ MeV and $\chi = 0.3$. Within each group of isobars the charge number increases from left to right.
- Fig. 5. The number of neutron fragments relative to the number of proton fragments, n/p , as a function of the temperature τ , for various values of the volume parameter χ and for isosymmetric systems ($I=0$) and typical neutron excess systems ($I=0.2$).

Fig. 1

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Fig. 2a

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Fig. 2b

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Fig. 3

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Fig. 4

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Fig. 5

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