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**Transforming How Climate System Models are Used:
A Global, Multi-Resolution Approach
to Regional Ocean Modeling**

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We review the results obtained under grant support. Details are given in the publications listed at the end of the review. We also provide lists of the personnel funded by the grant and of other collaborators on grant-related research and of the talks delivered, also under grant related research. We collaborated closely with geophysicists at the Los Alamos National Laboratory and the National Center for Atmospheric Research; especially noteworthy is our collaboration with Todd Ringler of LANL who was an active partner on much of our work.

1. Scale-Invariant Formulations of the Anticipated Potential Vorticity Method

The long-term success of climate models that operate on multiresolution grids depends on access to subgrid parameterizations that act appropriately across a wide range of spatial and temporal scales. As the first step in a series of efforts to obtain such scale-aware subgrid parameterizations, we focused on the anticipated potential vorticity method (APVM) on a sequence of quasi-uniform unstructured grids with varying resolutions. Through a scale analysis technique and phenomenological theories for two-dimensional turbulent flows, we derived a new formulation of the APVM that depends on a single parameter that is formally independent of the time-step size, the grid resolution, and the flow itself. Results of numerical experiments with this new formulation demonstrated that the optimal parameter of the new APVM formulation is invariant with respect to the time-step size, is insensitive to the flows, and is only weakly dependent on the grid resolution.

We then extended our new scale-invariant APVM from the quasi-uniform unstructured grids grid to multiresolution unstructured grids. Comparison was made to the original, nonscale-aware formulation of the APVM. Numerical experiments were performed using the shallow-water standard test case 5. The scale awareness of the new formulation was demonstrated by the following observations: (i) the range of optimal values for the single parameter of the new formulation is much less sensitive to grid resolution than that of the original formulation; (ii) within the optimal parameter range, the new formulation is able to maintain proper dissipation across scales and is thus able to produce better results in terms of errors in the potential enstrophy spectrum curves; and (iii) the new formulation is robust in that a single optimal parameter obtained for a specific grid can be safely used on other grids as well.

For the shallow water test case 5, plots of the potential enstrophy spectrum curve with the single parameter in the new APVM set to $\alpha = .0010, .0020, .0030, .0040, .0050$. The simulations are conducted with the APVM in the new formulation and on the 40962-cell variable-resolution grid with coarse-to-fine grid ratio of 16. The plots show the robustness of the new method with respect to changes in the single parameter in the model over a factor of 3.

For the shallow water test case 5, plots of the potential enstrophy spectrum on 40962-cell variable-resolution grids with coarse-to-fine grid ratios of 2, 4, 8, and 16 and on 163842-cell variable-resolution grids with coarse-to-fine grid ratios of 1 and 2. The simulations are conducted with the APVM in the new formulation with the single set at $\alpha = 0.002$. The plots show the robustness of the new method with respect to grid resolution changes and grid refinement changes.

2. A Co-Volume Scheme for the Rotating Shallow Water Equations on Conforming Non-Orthogonal Grids

A co-volume scheme was introduced for the rotating shallow water equations for which both velocity components are specified on cell edges and the thickness variables evolve on both the primary and the dual cell centers. The scheme applies to generic, conforming and nonorthogonal staggered grids, including the widely used latlon quadrilateral grids and DelaunayVoronoi tessellations. The scheme can be viewed either as coupled C-grid schemes on the primary and dual meshes, or

as a generalization of the traditional E-grid scheme on a new non-overlapping grid. Linear dispersive wave analysis shows that the dispersive relations resolved by either the primary or the dual mesh of a uniform quadrilateral staggered grid are the same as those of the Z-grid scheme. The total wavenumber space resolved by the staggered grid is twice as large, on which the co-volume behaves exactly like the E-grid scheme. On a uniform hexagon-triangular staggered grid, the co-volume scheme has two steady modes and two inertial-gravity modes on the hexagonal mesh, and one steady mode, two inertial gravity modes, and two spurious modes on the triangular mesh. On the wavenumber space resolved by either the hexagonal or the triangular mesh, the inertial-gravity wave modes remain positive and largely monotone. For the nonlinear shallow water equations, the co-volume scheme is shown to preserve the potential vorticity dynamics and the total energy exactly. Numerical results were obtained to corroborate and supplement the analyses.

Top: The new co-volume scheme can be viewed as a C-grid scheme on the primary mesh (left) coupled with a C-grid scheme on the dual mesh (right). Bottom: The new co-volume scheme on the quadrilateral staggered grid (solid and dashed lines) can also be viewed as an E-grid scheme on the mesh (dotted lines) connecting the velocity points.

Top row: Convergence of the L^2 (left) and L^∞ (right) errors in various variables for the new co-volume scheme applied to shallow water test case 5. h : thickness; u : normal velocity components w.r.t. primary cells; v : tangential velocity components; pv : potential vorticity. Resolutions range from 480km to 30km. The plots show the good accuracy properties of the new scheme. Bottom row: For the new co-volume scheme, the mass double time is on the order of 10^{13} years and the total energy (available potential energy + kinetic energy) doubling time is on the order of 10^9 years. This shows the excellent conservation properties of the new scheme.

Snapshot of the relative vorticity (RV) field on day 50 for the new co-volume scheme. Top left: RV on hexagons, no diffusion. Top right: RV on triangles, no diffusion. Bottom left: RV on hexagons, with the ∇^4 diffusion of $1.6e^{14}$. Bottom right: RV on triangles, with the ∇^4 diffusion of $1.6e^{14}$.

3. Exploring a Multiresolution Modeling Approach within the Shallow-Water Equations

The ability to solve the global shallow-water equations with a conforming, variable-resolution mesh was evaluated using standard shallow-water test cases. Whereas the long-term motivation for this study is the creation of a global climate modeling framework capable of resolving different spatial and temporal scales in different regions, the process begins with an analysis of the shallow-water system in order to better understand the strengths and weaknesses of the approach developed herein. The multiresolution meshes are spherical centroidal Voronoi tessellations where a single, user-supplied density function determines the region(s) of fine- and coarse-mesh resolution. The shallow-water system is explored with a suite of meshes ranging from quasi-uniform resolution meshes, where the grid spacing is globally uniform, to highly variable resolution meshes, where the grid spacing varies by a factor of 16 between the fine and coarse regions. The potential vorticity is found to be conserved to within machine precision and the total available energy is conserved to within a time-truncation error. This result holds for the full suite of meshes, ranging from quasi-uniform resolution and highly variable resolution meshes. Based on shallow-water test cases 2 and 5, the primary conclusion of this study is that solution error is controlled primarily by the grid resolution in the coarsest part of the model domain. This conclusion is consistent with results obtained by others. When these variable-resolution meshes are used for the simulation of an unstable zonal jet, the core features of the growing instability are found to be largely unchanged as the variation in the mesh resolution increases. The main differences between the simulations occur outside the region of mesh refinement and these differences are attributed to the additional

truncation error that accompanies increases in grid spacing. Overall, the results demonstrate support for this approach as a path toward multiresolution climate system modeling.

4. Goal-oriented A Posteriori Error Estimation for Finite Volume Methods

A general framework for goal-oriented a posteriori error estimation for finite volume methods was developed. The framework does not rely on recasting finite volume methods as special cases of finite element methods, but instead directly determines error estimators from the discretized finite volume equations. Thus, the framework can be applied to arbitrary finite volume methods. It also provides the proper functional settings to address well-posedness issues for the primal and adjoint problems. Numerical results are presented to illustrate the validity and effectiveness of the a posteriori error estimates and their applicability to adaptive mesh refinement.

Snapshot of the solution of a shallow-water system in one dimension as determined by a finite volume scheme showing two wave packets. The arrows point to the wave travel directions; longer arrow indicate greater speeds, but not proportionally.

The final grids generated by our adaptive mesh refinement strategies for finite volume discretizations of a shallow-water system in one dimension. Left: A single optimal grid is determined over the whole time interval of interest showing how grid refinement only occurs in regions where the wave packets are present or have been present. Right: The better (in the sense of fewer number

of grid points for the same accuracy) final grid generated for the same problem by an adaptive mesh refinement strategy for which an optimal grid is determined by first dividing the time interval of interest into 3 subintervals, showing how our adjoint-based goal-oriented gridding strategy refines and de-refines the grid as the two wave packets move to the left and right.

5. Partial and Spectral-Viscosity Models for Geophysical Flows

Two models based on the hydrostatic primitive equations were proposed and analyzed. The first model is the primitive equations with partial viscosity only, and is oriented towards large-scale wave structures in the ocean and atmosphere. The second model is the viscous primitive equations with spectral eddy viscosity, and is oriented towards turbulent geophysical flows. For both models, the existence and uniqueness of global strong solutions are established. For the second model, the convergence of the solutions to the solutions of the classical primitive equations as eddy viscosity parameters tend to zero is also established.

6. Parallel Algorithm for Spherical Delaunay Triangulations and Spherical Centroidal Voronoi Tessellations

Spherical centroidal Voronoi tessellations (SCVT) are used in many applications in a variety of fields, one being climate modeling. They are a natural choice for spatial discretizations on the Earth, or any spherical surface. The climate modeling community, which has started to make use of SCVTs, is beginning to focus on exa-scale computing for large scale climate simulations. As the data size increases, the efficiency of the grid generator becomes extremely important. Current high resolution simulations on the earth call for a spatial resolution of about 15km. In terms of an SCVT this corresponds to a quasi-uniform SCVT with roughly 2 million Voronoi cells. Computing this grid serially is very expensive and can take on the order of weeks to converge sufficiently for the needs of climate modelers. To compute SCVTs in parallel, we have developed a new algorithm that utilizes existing computational geometry tools such as spherical projections and planar triangulation algorithms along with a novel overlapping domain decomposition approach to do the stitching between the local grids produced by the different processor. As a result, the overall time to convergence is greatly reduced. The new algorithm shows speedup on the order of 4000 when using 42 processors over the well-known STRIPACK package for computing a triangulation used for constructing a SCVT.

7. A finite element, filtered eddy-viscosity method for the NavierStokes equations with large Reynolds number

The direct numerical simulation of the NavierStokes system in turbulent regimes is a formidable task due to the disparate scales that have to be resolved. Turbulence modelin attempts to mitigate this situation by somehow accounting for the effects of smallscale behavior on that at large-scales, without explicitly resolving the small scales. One such approach is to add viscosity to the problem; the Smagorinsky and Ladyzhenskaya models and other eddy-viscosity models are examples of this approach. Unfortunately, this approach usually results in over-dampening at the large scales, i.e., large-scale structures are unphysically smeared out. To overcome this fault of simple eddy-viscosity modeling, filtered eddy-viscosity methods that add artificial viscosity only to the high-frequency modes were developed in the context of spectral methods. We applied the filtered eddy viscosity idea to finite element methods based on hierarchical basis functions. We proved the existence and uniqueness of the finite element approximation and its convergence to solutions of

the NavierStokes system; we also derive error estimates for finite element approximations.

8. Unified Grids for Multiple Domain Multiple Physics Modeling

We considered grid generation for multiple-domain multiple-physics problems in which the single-physics components are applied on disjoint abutting domains. In general, grids that are separately constructed for each domain, as is the case when one uses legacy codes, may have overlaps and gaps and have grid points that do not match along the interface between the domains. Using very simple settings, we illustrated the possible difficulties that can arise when using non-matching grids. Then, based on centroidal Voronoi tessellation gridding strategies, we developed a new algorithm for the unified construction of grids in multiple domains that match perfectly, i.e., have no gaps or overlaps and have only common cell faces where they abut. Grid refinement is allowed for both to resolve boundaries, including interfaces, and also to refine in regions of interest. The use of the matching-grid construction algorithm is illustrated through its application to a simple example as well as to ocean-land gridding on the globe.

9. Well-Posedness Results for a Nonlinear Stokes Problem Arising in Glaciology

We provide well-posedness results for a nonlinear, incompressible Stokes problem that models the flow of ice in glaciers and ice sheets, e.g., Greenland and Antarctica. An important feature of the problem, in addition to the highly nonlinear rheology, are boundary conditions which describe the Coulomb-like friction at the ice-bedrock interface. Results available in the literature for similar nonlinear Stokes problems do not fully account for boundary conditions typical of glaciology. Most well-posedness results previously obtained deal with the elliptic Blatter-Pattyn model that is an approximation of the Stokes model. In this work, we considered the boundary conditions proposed by Schoof and extended his well-posedness results to the Stokes case. Difficulties in the analysis of the Stokes model are generated by the fact that boundary conditions can possibly depend on the normal stress. In this case, led by arguments typical in the linear elasticity context, we proved an existence result for nonlocal friction boundary conditions. Moreover, following Schoof, we proved that Coulomb friction problem can be approximated by differentiable boundary conditions that render numerical simulations more affordable.

Other Project Participants

The following personnel participated in grant projects. The two main collaborators were:

Qingshan Chen (Postdoctoral Associate, Florida State University)

Qingshan Chen was funded by the grant throughout his residence at Florida State University. He is currently a postdoctoral associate at the Los Alamos National Laboratory and will join the faculty of Clemson University as an Assistant Professor.

Todd Ringler (Los Alamos National Laboratory)

Todd Ringler was not funded by the grant.

Additional collaborators, none funded by the grant except as indicated:

John Burkardt (Research Associate, Florida State University)

Qiang Du (Professor, Penn State University)

Michael Duda (National Center for Atmospheric Research)

Douglas Jacobsen (Graduate student, Florida State University) Douglas Jacobsen was partially funded by the grant; he is now postdoctoral associate at the Los Alamos National Laboratory.

Lili Ju (Professor, University of South Carolina)

Mauro Perego (Postdoctoral Associate, Florida State University)

Mauro Perego currently holds a staff position at the Sandia National Laboratories.

Janet Peterson (Professor, Florida State University))

William Skamarock (National Center for Atmospheric Research)

Geoffrey Womelrdorff (Graduate student, Florida State University) Geoffrey Womelrdorff was partially funded by the grant; he is now postdoctoral associate at the Los Alamos National Laboratory.

Xiaoming Wang (Professor, Florida State University)

Papers Prepared Under Grant Sponsorship

1. Q. CHEN, TODD RINGLER, AND M. GUNZBURGER, *A scale-aware anticipated potential vorticity method. Part II: on variable-resolution meshes*, Monthly Weather Review, Vol. 140, pp. 3127-3133, 2012.
2. Q. CHEN, M. GUNZBURGER, AND TODD RINGLER, *A scale-invariant formulation of the anticipated potential vorticity method*, Monthly Weather Review, Vol. 139, pp. 2614-2629, 2011.
3. Q. CHEN, TODD RINGLER, AND M. GUNZBURGER, *A co-volume scheme for the rotating shallow water equations on conforming non-orthogonal grids*, Journal of Computational Physics, Vol. 240, pp. 174-197, 2013.
4. Q. CHEN AND M. GUNZBURGER, *Goal-oriented a posteriori error estimation for finite volume methods*, submitted to Journal on Computational and Applied Mathematics.
5. Q. CHEN, M. GUNZBURGER, AND X. WANG, *Partial and spectral-viscosity models for geophysical flows*, Chinese Annals of Mathematics. Series B, Vol. 31, pp. 579-606, 2010.
6. Q. CHEN, M. GUNZBURGER, AND M. PEREGO, *Well-posedness results for a nonlinear Stokes problem arising in glaciology*, submitted to SIAM Journal on Mathematical Analysis.
7. Q. DU, M. GUNZBURGER, AND L. JU, *Advances in studies and applications of centroidal Voronoi tessellations*, Numerical Mathematics: Theory, Methods and Applications, Vol. 3, pp. 119-142, 2010.
8. L. JU, T. RINGLER, AND M. GUNZBURGER, *Voronoi diagrams and their application to climate and global modeling*, Lecture Notes in Computational Science and Engineering – Numerical Techniques for Global Atmospheric Models, Vol. 80, pp. 313-342, Ed. by P. Lauritzen, C. Jablonowski, M. Taylor, R. Nair, Springer, 2011.
9. T. RINGLER, D. JACOBSEN, M. GUNZBURGER, L. JU, M. DUDA, AND W. SKAMAROCK, *Exploring a multi-resolution modeling approach within the shallow-water equations*, Monthly Weather Review, Vol. 139, pp. 3348-3368, 2011.
10. E. LEE AND M. GUNZBURGER, *A finite element, filtered eddy-viscosity method for the Navier-Stokes equations with large Reynolds number*, Journal of Mathematical Analysis and its Applications, Vol. 385, pp. 384-398, 2012.

11. D. JACOBSEN, M. GUNZBURGER, T. RINGLER, J. BURKARDT, AND J. PETERSON, *Parallel algorithms for planar and spherical Delaunay construction with an application to centroidal Voronoi tessellations*; submitted to Geophysical Modeling and Development.
 12. G. WOMERLDORFF, J. PETERSON, M. GUNZBURGER, AND T. RINGLER, *Unified matching grids for multidomain multiphysics simulations*, submitted to SIAM Journal on Scientific Computing.
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Talks Delivered

- “*New grid and discretization technologies for ocean and ice simulations*” (Poster), Climate Change Prediction Program Meeting, Indianapolis, September 2007. (by M. Gunzburger et. al.)
- “*Grids for ocean and ice modeling*”, Graduate Student Seminar, Florida State University, January 2008. (by M. Gunzburger)
- “*Grids and discretization schemes for ocean and ice modeling*”, The sixth International Conference on Scientific Computing and Applications, Pusan National University, Busan, Korea, June 2008. (by. M. Gunzburger)
- “*A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*”, Colloquium, Florida International University, January 2009. (by M. Gunzburger)
- “*A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*”, Colloquium, George Mason University, January 2009. (by M. Gunzburger)
- “*A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*”, Conference on Energy, Wind and Water: Algorithms for Simulation, Optimization and Control, Auckland, New Zealand, February 2009. (by M. Gunzburger)
- “*A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*”, Colloquium, University of North Carolina at Charlotte, February 2009. (by M. Gunzburger)
- “*A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*”, Colloquium, Oxford University, United Kingdom, March 2009. (by M. Gunzburger)
- “*New grid and discretization technologies for ocean and ice simulations*” (Poster), Climate Change Prediction Program Meeting, Bethesda, April 2009. (by M. Gunzburger)
- “*A multi-resolution method for climate system modeling: application of spherical centroidal Voronoi tessellations*”, Colloquium, Tulane University, April 2009. (by M. Gunzburger.)
- “*Centroidal Voronoi tessellations: algorithms and applications*”, Frontiers of Scientific Computing Distinguished Lecture Series, Louisiana State University, MD, 2010 (by M. Gunzburger)

- “*Centroidal Voronoi tessellations: algorithms and applications*”, Colloquium, Massachusetts Institute of Technology, 2010 (by M. Gunzburger)
 - “*Variable resolution centroidal Voronoi grids for climate simulations*” (Poster), DOE Climate Change Modeling Program Science Team Meeting 2010, Gaithersburg, MD, April 2010 (by M. Gunzburger et. al.)
 - “*Centroidal Voronoi tessellations: algorithms and applications*”, Colloquium, Emory University, 2010 (by M. Gunzburger)
 - “*The science of ice sheets: The mathematical modeling and computational simulation of ice flows*”, University of Oxford, United Kingdom, 2012 (by M. Gunzburger)
 - “*The science of ice sheets: The mathematical modeling and computational simulation of ice flows*”, Rostchild Lecture, Newton Institute for Mathematical Sciences, Cambridge University, United Kingdom, 2012 (by M. Gunzburger)
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