DØ CC CRYOSTAT TEST COOLDOWN - COOLDOWN TIME

J.D. FUERST

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Summary:
The DØ CC Cryostat is to be cold tested with LN₂. Calculations show that the time required for the 12.5 ton stainless steel inner vessel to reach equilibrium is around 5 hours if the vessel is cooled by introducing liquid into a nozzle at the bottom. The heat transfer calculations contain many assumptions, however. As a result, the vessel will be cooled by spraying LN₂ through a nozzle at the vessel top, providing as fast a cooldown as desired.

Discussion:
The cooldown was modeled as if the vessel at room temperature suddenly contained 3000 gal. (50") of LN₂ pooled in the bottom. As the calculations that follow show, conduction proved to be an insignificant mode of heat transfer (hundreds of watts). Inaccuracies include the assumption of one dimensional heat flow through a uniform cross section.

Radiation was treated with the assumption of a constant average temperature difference between vessel wall and cold sink, giving a heat transfer rate of thousands of watts.

Convection was modeled after a vertical plate geometry under free convection and turbulent flow. Given these assumptions and an average (constant) temperature difference, the convection heat transfer coefficient was calculated to be 12 W/m²K. This indicated a heat transfer rate of about 150,000 watts - by far the dominant mode of heat transfer.

With these rates and the known amount of heat in the vessel (mass times enthalpy change from 300 K to 80 K), the cooldown time is readily calculated.

The end of this note contains more detailed calculations by T.J. Peterson considering convection only (a reasonable assumption) and taking into account the decaying nature of the temperature difference between vessel wall and gas. This calculation provides the 5 hour cooldown estimate.
Conclusion:

Although calculations of the bottom-fill cooldown method indicate a reasonable cooldown time, the assumption of uniform gas temperature (absence of stratification) is vital to the analysis and in fact may not be valid. Initially, as liquid is introduced into the bottom of the vessel, it will boil rapidly creating large amounts of cold gas which then cool the walls above. As the vessel bottom cools and LN₂ begins to pool, however, the boiloff rate could decrease significantly. Thus the cold gas assumed in the free convection calculations is not generated. For this reason and in the interest of a speedy cooldown it has been decided to fill the vessel by spraying LN₂ in through a nozzle in the vessel top.

JP/ys
COOLDOWN TIME

- consider conduction, convection, radiation.

Conduction:

\[ \dot{Q} = K \frac{A}{L} \]

where

- \( K \): thermal conductivity integral for steel, from 300 to 100 K = 2.532 W/m

- \( A \): cross-sectional heat transfer area. Although this value changes as the "cold wave" progresses up the vessel walls, a value determined at the vessel "equator" will be used (circumference \times wall thickness) = \( \pi \times (488 \text{ cm} + 2 \times 305 \text{ cm}) \times (1.59 \text{ cm}) = 2522 \text{ cm}^2 \)

- \( L \): length of conduction path up side = 500 cm

Hence \( \dot{Q} = (2.532 \text{ W/m}) \times (500 \text{ cm}) \times (2522 \text{ cm}^2) / 500 \text{ cm} = 12.8 \text{ W} \)

Convection:

As a crude model assume a vertical plate geometry under free convection, turbulent flow. Let characteristic length of geometry "L" be equal to the length of arc from liquid edge to top of vessel (500 cm).

1st assume \( \Delta T \) from 300 to 100 K, then 300 to 200 K

So \( \bar{h} = \frac{Nu \cdot k}{L} \) and \( Nu = \left( \frac{0.825 + \frac{0.387 \cdot Re^{1/6}}{1 + (0.492/Pr)^{9/16}}}{1} \right)^{2} \)

where \( Pr \): Rayleigh number = \( Gr \cdot Pr = \frac{g \beta (T_{w} - T_{l}) L^{3}}{\nu \alpha} \)

- \( \beta \): volumetric thermal exp. coeff. \( \approx \frac{1}{\nu} = 0.01 \text{ K}^{-1} \)

- \( q = 9.81 \text{ m/s} \)

- \( \nu = 2 \times 10^{-6} \text{ m}^2/\text{s} \)

- \( \alpha = 2.6 \times 10^{-6} \text{ m}^2/\text{s} \)

- \( Pr = 0.768 \)

- \( L = 5 \text{ m} \)

- \( k = 9.58 \times 10^{-3} \text{ W/m} \text{K} \)
Then \( \text{Ra} = \left( \frac{9.81 \times 0.01 \times 200 \times 5^3 \text{m}^2}{2 \times 10^{-6} \text{m}^4 \times 2.6 \times 10^{-2} \text{m}^2} \right) = 4.72 \times 10^{14} \)

and \( \text{Nu} = [0.825 + \frac{10797}{1.16}]^2 = 8439 \)

Finally \( h = \frac{(8439 \times 0.00958 \text{W m}^{-2} \text{K})}{5 \text{m}} = 16.17 \text{ W m}^{-2} \text{K} \)

if \( \Delta T \) from 300 to 200, then

\( V = 7.66 \times 10^{-6} \text{ m s}^{-1} \)
\( \alpha = 1.04 \times 10^{-6} \text{ m s}^{-1} \)
\( P = 0.736 \)
\( k = 183 \times 10^{-3} \text{ W m}^{-1} \text{K}^{-1} \)
\( \beta = 0.005 \text{ K}^{-1} \)

\( \text{Ra} = 7.71 \times 10^{12} \), \( \text{Nu} = 2.166 \), \( h = 7.93 \text{ W m}^{-2} \text{K} \)

I will assume a constant average value of 12 \( \text{ W m}^{-2} \text{K} \) for simplicity.

Heat transfer \( Q = h \Delta T \) where

\( h = 12 \text{ W m}^{-2} \text{K} \)
\( \Delta T = 150 \text{K} \) (avg.)
\( A = \) inside surface area of warm part of vessel = cylinder area + end wall area
  \( = \pi d L + 2\pi r^2 \)
  \( = \pi \left[ (48 \text{ cm}) \times (305 \text{ cm}) + 2(2.44 \text{ cm})^2 \right] \)
  \( = 840000 \text{ cm}^2 = 84 \text{ m}^2 \)

\( Q = (12 \text{ W m}^{-2} \text{K}) \times (84 \text{ m}^2 \times 150 \text{K}) = 151200 \text{ W} \)

Radiation:

\( Q = e\sigma A(T_0^4 - T_a^4) \) where

\( e = \) emissivity = 0.22
\( \sigma = \) Stefan-Boltzmann constant = \( 5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4} \)
\( T_0 = 300 \text{K} \)
\( T_a = 150 \text{K} \) (avg.)
\( A = \) same as for convection = 84 \text{ m}^2

so \( Q = (0.22) \times (5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4}) \times (7.57 \times 10^4 \text{K}^4)(84 \text{ m}^2) \)

\( = 7960 \text{ W} \)
\[ Q_{\text{tot}} = Q_{\text{cond.}} + Q_{\text{conv.}} + Q_{\text{rad}} \]
\[ = 120W + 151200W + 7960W \quad \text{(convection dominates)} \]
\[ = 159300W \]

*Joules in the steel:

12.5 tons @ 73.54%g enthalpy gives

\[ \frac{12.5\text{ tons}}{\text{t}} \times \frac{2000\text{kJ}}{\text{t}} \times \frac{1454144\text{J}}{1000\text{kJ}} \times \frac{1}{0.035} \times (73.54\%g) = 833.9 \times 10^6 J \]

Cooldown time is then \( \frac{\text{Joules}}{\text{ht. trans. rate}} \)

\[ = \frac{833.9 \times 10^6 J}{1593007.5} = 5.24 \times 10^3 \text{ sec} = 1\frac{1}{2} \text{ hours} \]

If the convection heat transfer coefficient were 10 times less than calculated (12 instead of 12), \( Q_{\text{conv.}} \) would be 15120W and cooldown time would be 10 hours.

(\( h \) for a \( \Delta T \) from 200 to 100K = 12.9 \( \frac{W}{m^2K} \))
Surface area inside (neglect liquid pool): (4 hole)

\[ A_{\text{cyl}} + A_{\text{end}} + A_{\text{end}} \]

\[ 2\pi r l + 2\pi r^2 \]

\[ 2\pi \left[ (244\text{ cm})(305\text{ cm}) + (244\text{ cm})^2 \right] = 840000\text{ cm}^2 = 84\text{ m}^2 \]

Assume effects of liquid pool (remove surf. area) and hole (add surf. area) more or less cancel out.
\[ E_{\text{out}} = \mathcal{E}_{\text{stored}} \]
\[ E_{\text{out}} = hA(T_{\text{wall}} - T_{\text{g}}) \]
\[ \mathcal{E}_{\text{stored}} = mc \frac{dT_w}{dt} \]
\[ g \frac{3}{2} \frac{k}{\gamma} \]
\[ -hA(T_w - T_g) = mc \frac{dT_w}{dt} \]
\[ = mc \frac{d(T_w - T_g)}{dt} \]
\[ -\frac{hA}{mc} \frac{dT_w}{dt} = \frac{d(T_w - T_g)}{T_w - T_g} \]
\[ -\frac{hA}{mc} t + \text{Const} = \ln(T_w - T_g) \]
\[ \text{Const} e^{-\frac{hA}{mc} t} = T_w - T_g \]
\[ t = 0 \Rightarrow T_w - T_g = \frac{2}{3} T_{w_i} - T_g \]
\[ \text{Const} = T_{w_i} - T_g \]
\[ (T_{w_i} - T_g) e^{-\frac{hA}{mc} t} = T_w - T_g \]
\( h = 12 \text{ W/m}^2\text{K} \)
\( A = 8.4 \text{ m}^2 \)
\[ mc = (12.5 \text{ ton}) \left( \frac{2000 \text{ lb}}{\text{ ton}} \right) \left( \frac{14.594 \text{ kg}}{32.174 \text{ lb}} \right) \left( \frac{500 \text{ J}}{\text{ kg K}} \right) = 5.67 \times 10^6 \text{ J/K} \]
\[
\frac{hA}{mc} = \frac{12 \times 84}{5.67 \times 10^6} \text{ s}^{-1} = 1.78 \times 10^{-4} \text{ s}^{-1}
\]
\[
\frac{T_w - T_g}{T_w - T_i} = e^{\frac{(-1.78 \times 10^{-4})t}{x}}
\]

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\[ t = -5.62 \times 10^3 \ln \left( \frac{T_w - T_g}{T_w - T_i} \right) \]
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Vessel will operate at ~ 88 K

LN₂ is at about 78 K
so want to reach \( \frac{T_w}{T_{wi}} = \frac{88}{78} \) = .05

Takes ~ 5 hrs

Thermal contraction of SS₄₆
is 91% complete at 90 K
93% complete at 80 K