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Quark mass variation constraints from Big Bang nucleosynthesis

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We study the impact on the primordial abundances of light elements created of a variation of the quark masses at the time of Big Bang nucleosynthesis (BBN). In order to navigate through the particle and nuclear physics required to connect quark masses to binding energies and reaction rates in a model-independent way we use lattice QCD data and an hierarchy of effective field theories. We find that the measured ^{4}He abundances put a bound of $\delta - 1\% \lesssim m_q/m_q \lesssim 0.7\%$. The effect of quark mass variations on the deuterium abundances can be largely compensated by changes of the baryon-to-photon ratio η. Including the bounds on the variation of η coming from WMAP results and some additional assumptions narrows the range of allowed values of $\delta m_q/m_q$ somewhat.

I. INTRODUCTION

In theories of physics beyond the standard model the standard model parameters appear not as fundamental constants but as derived quantities. In many of those theories the possibility then arises that the values of the standard model "constants" can vary over time [1]. It is then important to understand which constraints the successes of standard cosmology – which assumes time independent constants – poses on this purported time variation. A natural place to look for strong sensitivity to fundamental constants variation is Big Bang nucleosynthesis (BBN) since it satisfies two important criteria. First, BBN happened very early in the Universe's history, mostly when the Universe was between 3 seconds and 3 minutes old. Second, not only is standard BBN understood at a few percent level but it is very sensitive to microscopic parameters like nuclei binding energies and reaction rates that are, themselves, very sensitive to the certain standard model constants. It is no surprise then that BBN has been used in the past to study the variation of fundamental constants [2]. The purpose of the present paper is to explore the BBN constraints on the variation of the mass of the two lightest quarks.

The binding of nucleons into light nuclei during BBN proceeds through a number of reactions, some in equilibrium with the expansion of the Universe, some not. After the weak reactions like $p + e^- \leftrightarrow n + \bar{\nu}$ are not longer in equilibrium, the ratio of neutron to protons decreases due to neutron β-decay. If the formation of light nuclei occurred in equilibrium, the most bound nuclei (among the light ones this is ^{4}He) would form earlier and more abundantly. The formation of ^{4}He can, however, only occur after ^{2}H , ^{3}He and ^{3}H are formed, since multinucleon fusion reactions are essentially impossible at the low densities prevailing then. Their number is small on the account that their binding is small and it is not energetically favorable for them to exist form until the temperature is low enough to be comparable to their binding energies. So the beginning of nucleosynthesis is delayed by the shallowness of the deuteron binding. Since this shallowness is a product of delicate cancellations between kinetic and potential energies, the binding of the deuteron is an obvious place where a small change in quark masses can significantly alter the primordial abundances. Notice that the rate for the reaction $n+p \leftrightarrow d+\gamma$ is not small; it is sufficient to keep the deuteron number in thermal/chemical equilibrium. It is the equilibrium deuteron number that is too small for them to collide and be assembled in larger nuclei. After the deuteron number grows enough, the reactions leading to the formation of ^{4}He proceed quickly and essentially all the neutrons present in the beginning of BBN are assembled into ^{4}He nuclei. The timing where this assembly starts (determined, among other things by the deuteron binding) is crucial as the neutron numbers are fast decreasing due to neutron β-decay. Small amounts of ^{2}H , ^{3}He and ^{3}H are left out of this process. Their numbers depend crucially on chemical non-equilibrium physics and the rates of the reactions, including the initial one $n + p \rightarrow d + \gamma$. Current observation are not useful in measuring reliably the primordial abundance of ^{3}He and ^{3}H . However, the abundance of ^{2}H and, specialy ^{4}He , are well measured and put a significant constraint on any change f the standard BBN scenario.

A number of authors have previously considered the effect of a changing quark masses in the BBN predicted abundances [3][4] [5] [6][7][8][9] [10][11][12][13][14] [15][16][17]. The main difficulty to be surmounted is that the quark mass dependence of binding energies, reaction cross sections and decay rates that are input into BBN models are difficult to determine. For instance, modern nuclear potentials can describe very well nucleon-nucleon phase shifts. They can also be used to compute binding energies with enough precision (with the help of phenomenologically motivated three-nucleon forces fit to some observables) and cross section for few-nucleon reactions. These potentials are, however, tuned to data obtained from experiment where the quark mass has its current value. What is usually done in estimating the effect of quark mass variation is to change the parameters in these models where this dependence is easy to track. For instance, the range of nuclear forces, given by $1/m_\pi$ can be changed through the relation $m_\pi^2 \sim m_q$. But the long distance part of the potential, sensitive to this range, is actually a small part of the nucleon-nucleon interaction. The medium and short range parts also have a quark mass dependence and, while this dependence is likely to be milder, its effect on the overall nucleon-nucleon interactions is still large due to fine-tuned cancellations that are responsible for, among other things, the shallowness of the deuteron. It is our goal in this paper to avoid as much as possible this kind of modeling of nucleon and nuclear forces properties and stick to what is known about QCD/nuclear physics through more general arguments. In particular we will use effective field theories (and some lattice QCD data) to connect that change in quark masses to the inputs in the BBN codes.

At momentum scales Q below $\Lambda_{QCD} \approx 1$ GeV, the relevant degrees of freedom in QCD are hadrons, not quarks and gluons. Effective field theories for this momentum range (chiral perturbation theory) were developed for the meson, one and many-nucleon sectors. They are able to predict physical observables as an expansion on the small parameters $Q/\Lambda_{QCD}, m_\pi/\Lambda_{QCD}$ taking as inputs a few "low energy constants", like pion decay constants and the nucleon mass in the chiral ($m_\pi = 0$) limit, which are obtained from analyses of experimental results. They predict, for instance, the dependence of nucleon masses on the value of the quark masses. It turns out that this particular change is very small and can be neglected, except for its effect on the phase space for the neutron decay and related weak process (see below). Lattice QCD calculations reinforce the believe in a small quark mass dependence of nucleon masses [18] [19]. Chiral perturbation theory for few-nucleon systems (referred to here as χEFT) is in a less developed phase. First, there are conceptual issues that preclude a reliable prediction of the quark mass dependence of observables [20]. Second, it has not been used extensively in multi-nucleon systems and many reactions involving photons. To bypass this difficulty we will use another effective theory for nuclear systems. One can construct another effective theory (referred to here as "pionless EFT") valid only for smaller momenta Q, much smaller than the pion mass m_π. In this theory all particles, including pions, are integrated out and only nucleons (and photons, neutrinos) are kept as explicit degrees of freedom. This theory can make non-trivial predictions because some bound states (^{2}H , ^{3}He , ^{4}He) are loosely bounded and the typical momenta Q of their constituents is significantly below $\sim m_\pi$, right within the regime of validity of the pionless EFT. The shallowness of these bound states is related to the fine tuning in the s-wave twonucleon scattering. In fact, the scattering length in the two spin channels $^1S_0$ and $^3S_1$ ($a_s \approx -22$ fm and $a_t \approx 5.4$ fm) are unnaturally large, much larger than the naive expectation $\approx 1/m_\pi = 1.4$ fm. The pionless EFT is very successful in predicting observables in the three-nucleon sector and there is indication that the same is true in the four-nucleon sector [21, 22]. Since the α-particle is the most bound of the light nuclei, success of the pionless EFT in the α-particle probably means that the theory can be useful in studying larger nuclei. Since the pionless EFT makes no use of the QCD chiral symmetry, it is incapable, however, to directly predict the quark mass dependence of observables. But the parameters of the pionless EFT, at the lowest orders in the low energy expansion, are the threshold nucleon-nucleon scattering parameters like scattering lengths, effective ranges, etc.. These few parameters have been studied using the χEFT and we can use them to predict their variation with quark masses. In addition, some lattice QCD results confirm and reinforce the χEFT predictions for scattering length dependences on quark masses. We will use these χEFT results as input parameters for the pionless EFT. This allows us to obtain estimates for the quark mass dependence of nuclear properties relevant to BBN. We will then use this information in combination with a standard BBN code to compute the light elements abundances in order to constraint the values of the quark masses during the Universe first minutes. Our strategy of combining these two types of effective theory is summarized in Fig. 1. We will now describe the stages of our calculation.

A. Scattering length dependence on quark masses

Different verions of χEFT have been used by different authors to study the quark mass variation of nucleon-nucleon S-wave scattering lengths. The results depend on the spin channel. In the spin singlet $^1S_0$ channel and at leading order (LO) on the m_π/Λ_{QCD} expansion, the calculation of the quark mass dependence of the scattering length in the version of χEFT used in [23] requires as inputs the chiral limit values of the axial charge of the nucleon g_A, the decay constant of the pion f_π, the nucleon mass M, the pion mass m_π and the coefficients of a two-nucleon contact term C_s^0, fit to the physical scattering lengths. Only the value of these quantities at the physical value of quark masses is precisely known but the difference between them and their chiral limit values is a higher order effect that can be neglected in a next-to-leading order (NLO) calculation. At NLO a new constant D_s^2 appears, the coefficient of a two-nucleon operator with no derivatives but one quark mass insertion, as well as other constants contributing to the quark mass dependence of f_π, g_A and M. The value of D_s^2 is difficult to disentangle from C_s^0 as both contribute equally to nucleon-nucleon scattering at the physical value of the quark masses. They give, however, different extrapola-

$$g_A, f_\pi, M, \cdots \underset{\chi EFT}{\Longrightarrow} a_2, a_t, \cdots \underset{\text{pionless EFT}}{\Longrightarrow} \begin{array}{c}\text{binding energies}\\ \text{reactivities}\end{array} \underset{\text{BBN code}}{\Longrightarrow} \text{abundances}$$

FIG. 1.

tions to other values of quark masses. They can be disentagled only through a study of processes like deuteron-pion scattering or by the use of lattice QCD data (see below). The strategy used in dealing with the lack of knowledge of the value of D_s^2 is to estimate it using naive dimensional analysis arguments. It seems that the difference in the power counting schemes used in [23] and [24] have little impact on the final numbers and the discrepancy between them can be explained by the different assumptions about the reasonable range of values for D_s^2. We will use the calculation described in [24] as those authors computed the deuteron binding, not only the scattering lengths, and the deuteron biding will be the most important ingredient in the BBN calculation.

For a small variation of the quark mass we can read off the figure 11 in [24] the slope (we use the more conservative estimate where the change of the axial constant g_A with quark masses, parametrized by $\bar{d}_{16}$ is included):

$$\frac{m_q}{B_2}\frac{\delta B_2}{\delta m_q} = \frac{m_\pi}{2B_2}\frac{\delta B_2}{\delta m_\pi} = \frac{m_\pi}{2B_2}\,(-0.085 \pm 0.027), \quad (1.1)$$

where m_q is the average mass of the up and down quarks and we made use of the relation $m_\pi^2 \sim m_q$. Similarly, we use figure 12 in [24] to extract the variation of the spin singlet ${}^1S_0$ channel scattering length to find

$$\frac{\delta a_s}{\delta m_\pi} = \frac{2m_q}{m_\pi}\frac{\delta a_s}{\delta m_q} = (-1.4 \pm 1.4)\frac{\text{fm}}{\text{MeV}}. \quad (1.2)$$

Notice that a vanishing a_s variation is consistent with these extrapolations, a feature also seen in the extrapolation in [23]. If a_s were the only parameter determining the change of abundances due to varying quark masses, BBN would impose no constraint on possible quark mass variations

Unquenched lattice QCD calculations of nucleon-nucleon scattering lengths have appeared in the last few years. They are still performed at higher values of quark masses, too high for the effective theory approach to be valid, so they are of limited value for our purposes. Despite that, an attempt was made in [25] to use χEFT to find the quark mass dependence on scattering lengths by interpolating the lowest pion mass lattice data and the known experimental value of the scattering lengths at the physical point. The deuteron binding energy is not measured in the lattice. However, it is related, at leading order in the effective theory, to the triplet scattering length that is measured. Using the extrapolation in [25] and the leading order relation $B_2 = 1/(Ma_t^2)$ we find

$$\frac{m_q}{B_2}\frac{dB_2}{dm_\pi} = -0.14 \pm 0.13, \quad (1.3)$$

in agreement with eq. (1.1). In the extrapolation done in [25] another branch of allowed values of dB_2/dm_π appears. This additional band is excluded from the purely EFT extrapolations in [23] and [24] and will be disregarded in this paper.

The allowed values for the a_s quark mass dependence extracted from the extrapolation in [25] , namely

$$\frac{da_s}{dm_\pi} = (-0.75 \pm 1.0)\frac{\text{fm}}{\text{MeV}} \quad (1.4)$$

are consistent with the ones above but are too loose to put any relevant constraint.

The remaining inputs of the pionless EFT like three-nucleon forces, effective ranges, nucleon magnetic moments, etc., are not fine-tuned and vary much less drastically with the quark masses. Their contribution to the cross sections is also suppressed compared to B_2 and a_s. In the present paper we will take them to be independent of the quark masses.

B. Binding energies, reactivities and lifetimes

We have used the pionless EFT to estimate the quark mass variation of four quantities: the binding energies of the deuteron, ^{3}H , ^{3}He , ^{4}He and the reactivity of the process $n + p \to d + \gamma$. The binding energies of larger nuclei, like ^{7}Li , are important only for the abundances for these larger nuclei. As it is not presently possible to have a reliable estimate on the quark mass variation of these binding energies we will keep them fixed and concentrate on the abundances for the lighter nuclei ^{2}H and ^{4}He , confident that they won't be significantly affected by the binding of $A > 4$ nuclei. We also only include the variation of the reactivity of deuteron-neutron capture as this is the reaction that starts off BBN and is more likely to have an impact on abundances (but, as we will see below, this impact is minimal). The binding energy of the deuteron is taken to be given by eq. (1.1).

The calculation of three-nucleon and four-nucleon properties in the pionless EFT requires as inputs the singlet and triplet scattering lengths as well as one three-body observable, usually taken to be the three-nucleon binding energy. The three-body binding energy can be traded by the value of a three-body force counterterm. The tree-body force is also not fine tuned and has, therefore, only a small variation with the quark masses that we will consequently neglect. Changing the two-body input while keeping the three-body counterterm fixed provides then the scattering length dependence of the three-nucleon system. In other words, the binding energies of the ^{3}He , ^{3}H and ^{4}He nuclei are estimated by

$$\frac{m_q}{B_i}\frac{dB_i}{dm_q} = \frac{m_q}{B_i}\left(\frac{da_s}{dm_q}\frac{dB_i}{da_s} + \frac{dB_2}{dm_q}\frac{dB_i}{dB_2}\right) \quad (1.5)$$

where B_i stands for the binding energy of one of ^{3}He , ^{3}H or ^{4}He . The values of the derivatives appearing in eq. (1.5) were computed using the pionless EFT:

$$\frac{a_s}{B_3}\frac{dB_3}{da_s} = 0.12, \qquad \frac{B_2}{B_3}\frac{dB_4}{dB_3} = 1.41,$$
$$\frac{a_s}{B_4}\frac{dB_4}{da_s} = 0.037, \qquad \frac{B_2}{B_4}\frac{dB_4}{dB_2} = 0.74, \quad (1.6)$$
$$(1.7)$$

where B_4 is the ^{4}He binding and B_3 the ^{3}H or ^{3}He binding energy. The weak dependence on a_s is easily understood when one notices that the typical momenta in these bound states is of order $\sqrt{MB_i}$, which is much larger than $1/a_s$. The dependence of B_i on a_s is a function of the dimensionless parameter $\sim \sqrt{MB_i}a_s \ll 1$ and, on a first approximation, we can take this parameter to vanish.

In order to account for the theoretical uncertainty in the EFT calculation we assign an additional 10% random variation to the bindings of ^{3}He and ^{3}H (computed at NLO in EFT) and a 30% variation on the value of the ^{4}He binding (computed at LO only), as will be shown more explicitly below.

The reaction $n + p \leftrightarrow d + \gamma$ was extensively analyzed in [26] using a NNNNLO calculation in pionless EFT. The inputs at this order are the scattering length a_s, the deuteron biding and effective range parameters, magnetic moments and one two-nucleon-one-photon term fixed by experimental value of cold capture. We use the variation of B_2 and a_s given in eqs. (1.1) and (1.2) to compute, with the help of the explicit formula in [26], the *relative* change in the reactivity as a function of the temperature and used that as an input of the BBN code. In [5] it was argued that the reactivity $\langle\sigma v\rangle$ scales as $\sim B_2^{5/2}a_s^2$. We verified with the explicit formula from [26] that the scaling with $B_2^{5/2}$ is indeed very well satisfied but the scaling with a_s^2 doesn't work so well.

Finally we discuss how quark mass changes affect the neutron lifetime as well as the rates of other one-baryon weak reactions such as $p + e^- \leftrightarrow n + \nu$. This influence comes through a modified value for the axial charge g_A and the neutron and proton masses, changing the phase space. In fact, the neutron width is given by [27]

$$\Gamma = \frac{(G_F\cos\theta_c)^2}{2\pi^3}m_e^5(1+3g_A^2)f\left(\frac{\Delta}{m_e}\right), \quad (1.8)$$

where Δ and m_e are the mass splitting between neutron and proton and the electron mass, $g_A \approx 1.26$ is the nucleon axial decay constant, G_F the Fermi constant and θ_c the Cabibbo angle. $f(\Delta/m_e)$ is the function

$$f(w_0) = \int_1^{w_0} dww\sqrt{w^2-1}(w_0-w)^2\frac{2\pi\alpha}{\sqrt{w^2-1}}\frac{1}{1-e^{-\frac{2\pi\alpha}{\sqrt{w^2-1}}}} \quad (1.9)$$

which describes the phase space and the Coulomb repulsion. The variation of Γ with the quark masses is given then by

$$\frac{m_q}{\Gamma}\frac{d\Gamma}{dm_q} = \frac{m_q}{f\left(\frac{\Delta}{m_e}\right)}\frac{df\left(\frac{\Delta}{m_e}\right)}{dm_q} + \frac{m_q}{1+3g_A^2}\frac{d(1+3g_A^2)}{dm_q}. \quad (1.10)$$

The dependence of g_A with the quark mass is given, at NLO in chiral perturbation theory, by [28]

$$g_A = g_A^0\left[1 - \frac{9g_A^2m_\pi^2}{32\pi^2F^2}\log(\frac{m_\pi}{\Lambda}) + \frac{(g_A^2-4)m_\pi^2}{32\pi^2F^2}\log(\frac{m_\pi}{\Lambda'})\right], \quad (1.11)$$

where g_A^0 is the chiral value of g_A, $F \approx 93$ MeV and Λ, Λ' are constants of order 1 GeV dependent on the Gasser-Leutwyler coefficients. Numericaly we find

$$\frac{m_q}{1+3g_A^2}\frac{d(1+3g_A^2)}{dm_q} = \frac{1}{2}\frac{m_\pi}{1+3g_A^2}\frac{d(1+3g_A^2)}{dm_\pi} \approx 0.2. \quad (1.12)$$

The variation of the phase space $f(\Delta/m_e)$ with the quark mass can be estimate as

$$\frac{m_q}{f(\frac{\Delta}{m_e})}\frac{f(\frac{\Delta}{m_e})}{dm_q} = \frac{m_\pi}{2f(\frac{\Delta}{m_e})}\frac{df(\frac{\Delta}{m_e})}{dm_\pi} \quad (1.13)$$
$$= \frac{m_\pi}{2f(w_0)}\frac{df(w_0)}{dw_0}|_{w_0=\frac{\Delta}{m_e}}\frac{d\Delta/m_e}{dm_\pi}$$

The value of $f(w_0)$ and its derivative at $w_0 = \Delta/m_e$ can be found numerically to be 1.64 and 4.25, respectively. The variation of Δ/m_e with m_q can be estimated by splitting Δ into a strong interaction component Δ_s proportional to the up and down quark mass difference (and, consequently, to the value of m_q) and an electromagnetic piece $\Delta_{e.m.}$, largely independent of m_q. Unfortunately, the electromagnetic part is due to short distance effects and cannot be directly computed in a very reliable way. The best handle we have on its value comes from determining the up and down quark mass ratio using chiral perturbation theory and the meson spectrum and using this value, together with the best estimate of the nucleon $\sigma-$term, to extract Δ_s. The value obtained for Δ_s this way is consistent with the obtained from lattice QCD calculation [29].The difference between this value of Δ_s and the measured value of the neutron-proton mass splitting gives $\Delta_{e.m.} = -0.76 \pm 0.30$ [30].

Chiral perturbation theory predicts a quark mass dependence of Δ_s of the form $\Delta_s = Am_\pi^2(m_d-m_u)/(m_d+$

m_u), a formula valid up to NLO as the leading loop (NLO) contribution to the nucleon mass cancels between the neutron and proton.

We then have

$$\frac{m_q}{f(\frac{\Delta}{m_e})}\frac{f(\frac{\Delta}{m_e})}{dm_q} = \frac{1}{f(w_0)}\frac{df(w_0)}{dw_0}|_{w_0=\frac{\Delta}{m_e}}\frac{m_\pi}{2m_e}A\frac{m_d-m_u}{m_d+m_u}2m_\pi$$
$$= \frac{1}{f(w_0)}\frac{df(w_0)}{dw_0}|_{w_0=\frac{\Delta}{m_e}}\frac{\Delta_s}{m_e}$$
$$\approx 10.4 \pm 1.5. \tag{1.14}$$

Notice that we are taking both the up and down mass to vary while keeping the ratio m_d/m_u fixed. The dependence in Eq. (1.14) dominates over Eq. (1.12) we finally find

$$\frac{m_q}{\Gamma}\frac{d\Gamma}{dm_q} = 10.6 \pm 1.5. \tag{1.15}$$

The quark mass variation of the neutron lifetime is relevant for our calculation. In order to see that, let us remember that the neutron number, after the weak interaction are decoupled, decreases until BBN starts at $t \approx 168$ s. The suppression factor in standard BBN is thus $e^{-168/885} \approx 0.827$. A 5% increase of quark masses (the range considered in this paper) would lead, according to Eq. (1.14) to an increase of about 50% in the neutron lifetime and the suppression factor would change to $e^{-252/885} \approx 0.752$, leading to a ^{4}He abundance change of about 2% in absolute terms.

The rate of other weak reaction changes in a similar manner. The phase space integrals are more involved and are, in BBN codes, computed "on the fly" taking the ratio $Q = \Delta/m_e$ as an input. We calculated the variation of Q as

$$\frac{m_q}{Q}\frac{dQ}{dm_q} = \frac{m_\pi}{2\Delta}\frac{d\Delta}{dm_\pi} = \frac{\Delta_s}{\Delta}$$
$$\approx 1.59 \pm 0.23. \tag{1.16}$$

II. RESULTS

In order to deal with the highly non-linear dependence of the final abundances on the quark masses and, at same time, to include estimates of theoretical errors, we used a stochastic procedure. More specifically, for a given quark mass variation $\Delta m_q/m_q$, we need to specify the binding energies of ^{2}H , ^{3}H , ^{3}He , ^{4}He , the reactivity R for $n + p \leftrightarrow d + \gamma$, the neutron lifetime τ and the phase space parameter Q. All the other BBN parameters are kept at their present values.

We randomly generated a set of 300 values of scattering lengths a_s, deuteron bindings B_2 with a gaussian distribution with mean value and standard deviation given by

$$\bar{X} = \left[1 + \frac{1}{2}\left(\frac{m_q}{X}\frac{dX}{dm_q}|_+ + \frac{m_q}{X}\frac{dX}{dm_q}|_-\right)\frac{\Delta m_q}{m_q}\right]X^{\rm phys},$$
$$\sigma_X = \left[1 + \frac{1}{2}\left(\frac{m_q}{X}\frac{dX}{dm_q}|_+ - \frac{m_q}{X}\frac{dX}{dm_q}|_-\right)\frac{\Delta m_q}{m_q}\right]X^{\rm phys} \tag{2.1}$$

where X stands for either a_s or B_2 and the "+" and "-" subscripts refer to the higher and lower values of dX/dm_q allowed by Eqs. (1.1) (1.2) . The variations of a_s and B_2 are assumed to be uncorrelated. From the ensemble of a_s, B_2 obtained as above we compute a corresponding ensemble of binding energies using Eq. (1.5) and added to the result a random correction with standard deviation of 10% (for ^{3}H and ^{3}He) or 30% (for ^{4}He) in order to take theoretical errors into account, as discussed in the previous section. In other words, the binding energies of ^{3}H , ^{3}He and ^{4}He are given by

$$\frac{B_i}{B^{\rm phys}} = \left[1 + (1+t_i\xi_i)\left(\frac{a_s}{B_i}\frac{dB_i}{da_s} + \frac{B_2}{B_i}\frac{dB_i}{dB_2}\right)(a_s - a_s^{\rm phys}))\right], \tag{2.2}$$

where i indexes the three nuclei ^{3}H , ^{3}He and ^{4}He , the superscript "phys" stands for the present, experimental values of the quantity, ξ_i are gaussian random variables with central value 0 and standard deviation equal to 1and t_i is teh theoretical error of the extrapolation equal to 0.1 (for ^{3}H and ^{3}He) and 0.3 (for ^{4}He).

Similarly, the reactivity $r(T)$ of the $n + p \to d + \gamma$ reaction was computed as a function of the temperature for the ensemble of a_s, B_2 values determined by eq. 2.1 using the explicit expression for the cross section from [26]. This calculation is of very high order in the low energy expansion and its very small theoretical errors were neglected

We also generated, for each value of $\Delta m_q/m_q$, a set of 300 random values of τ and Q whose distribution reflected the discussion of the previous section. More specifically, these values were generated through the formula

$$\frac{1}{\tau} = \frac{1}{\tau^{\rm phys}}\left[1 + (10.6 + 1.5\xi)\frac{\Delta m_q}{m_q}\right],$$
$$Q = Q^{\rm phys}\left[1 + (1.59 + 0.23\xi)\frac{\Delta m_q}{m_q}\right], \tag{2.3}$$

where ξ is a gaussian random variable with central value 0 and standard deviation 1. Notice that this ξ is independent of the ξ_i used in the determination of the binding energies but the same ξ is used in both τ and Q since the leading theoretical uncertainties on both quantities stem from the same rough determination of the σ-term.

For a given value of $\Delta m_q/m_q$, a set of values for $B_2, B_{^3H}, B_{^3He}, B_4, r(T)$ was paired one of set of τ and Q values and used in a standard BBN code. The BBN code we used in our analysis is based on Refs. [31, 32]

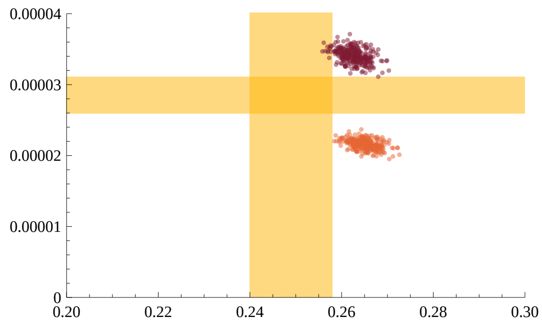

FIG. 2. The yellow bands show the (1-σ allowed abundances for ^{4}He and ^{2}H . The two clouds show the result of 300 simulations, both with $\Delta m_q/m_q$ = -1 % but two different values of η_{10}. The lower cloud (ochre online) is the result of taking $\eta_{10} = 6.23$ and the upper cloud (burgundy online) the value $\eta_{10} = 4.60$. There is very little change on the ^{4}He yield but the deuterium yield changes enough to render the deuterium abundance useless in putting a constraint on $\Delta m_q/m_q$.

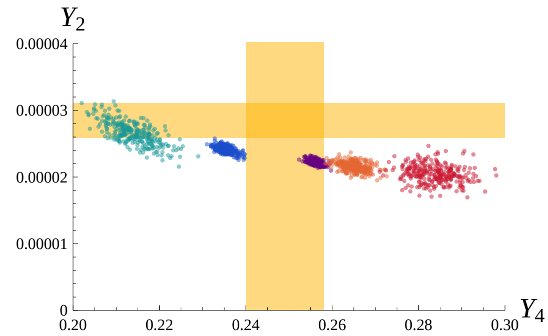

FIG. 3. The yellow bands show the (1-σ allowed abundances for ^{4}He and ^{2}H . The five clouds show the result of 300 simulations at each one of the values (from left to right): $\Delta m_q/m_q$ = 2% (green online), $\Delta m_q/m_q$ = 0.7% (blue online), $\Delta m_q/m_q = -0.5\%$ (purple online), $\Delta m_q/m_q = -1\%$ (ochre online), and $\Delta m_q/m_q = -2\%$ (red online).

and is publicly available [33]. The code was modified to accept temperature-dependent variations in the reactivity corresponding to the $n + p \to d + \gamma$ reaction and the rate of weak interaction processes was changed according to eq. (1.15) and eq. (1.16). The Q-values of all BBN reactions with ^{2}H , ^{3}H , ^{3}He , and ^{4}He as either parent or daughter products of reactions were allowed to vary in accordance with the changes in binding energies of these nuclei. The baryon-to-photon ratio η was changed over a range discussed below. Otherwise, the standard input parameters were used in our BBN simulations.

The main feature seen in the simulations is that a variation in η shifts the deuterium abundance but has little effect on the ^{4}He yields (see fig. (2)). A larger value of η implies in a larger baryon density, a more complete burning of the neutrons into ^{4}He nuclei and a smaller deuterium abundance. As a consequence, in the absence of a restriction on the value of η from other considerations, the deuterium abundance does not put any constraint on the range of allowed quark masses variations.

Additional constraints on the value of η comes from studies of large-scale structure of the Universe. The actual numerical value of the constraints, however, depends on assumptions made in these analyses, including assumptions on the initial spectrum of fluctuations. For instance, the lower range of the determination of $\eta_{10} = 4.79 \pm 0.019$ in [34] and the central value of the determination of $\eta_{10} = 6.23 \pm 0.17$ in [35],are shown, for $\Delta m_q/m_q$ in fig. (2). A similar plot results for other values of $\Delta m_q/m_q$. Consequently, any reasonable change in the deuterium abundance can be accommodated by a change in the value of η_{10}. If we restrict ourselves to the much narrower range $\eta_{10} = 6.23 \pm 0.17$ [35], the deuterium abundances can play a role. However, the values in the range $\eta_{10} = 6.23 \pm 0.17$ are in tension witht he observed deuterium abundances. BBN, by itself, prefers the slightly lower range $5.1 < \eta_{10} < 6.5$, at the 95% confidence level [36]. Thus, even with the current physical values of m_q, the predicted deuterium abundance lies just outside the $1-\sigma$ band, making difficult to distinguish allowed and forbidden values of m_q based on Y_2. For now, let us disregard the deuterium abundances and look at how the ^{4}He abundances change with the quark masses.

In fig. (3) we show the result of changing the quark masses by five values: $2\%, 0.7\%, -0.5\%, -1\%$ and -2%, all corresponding to $\eta_{10} = 6.23$. Each one of these values of $\Delta m_q/m_q$ is represented by a cloud of points in the $Y_4 \times Y_2$ plane. The spread between the 300 points in each cloud accounts for the theoretical uncertainties in the extrapolation of the parameter inputs as described by eqs. (2.2) and (2.3). The tendency is for a smaller Y_4 for larger values of m_q. Two main mechanisms account for this general trend. First, large values of m_q imply in larger values of Δ, larger phase space for neutron decay and shorter neutron lifetime. Consequently, more neutrons decay by the time BBN starts the assembly of ^{4}He and smaller ^{4}He yield. In addition, eq. (1.1) shows that a larger m_q implies into a smaller B_2. The deuteron, being less bound, takes longer to form, delaying the onset of ^{4}He formation, and giving even more time for the neutrons to decay, reducing further the ^{4}He yield. There is also a weak tendency to have smaller Y_2 for smaller m_q, a trend not so easily explained.

Based on the data shown on fig. (3) we put a bound on the allowed values of quark mass changes at

$$-1\% \lesssim \frac{\Delta m_q}{m_q} \lesssim 0.7\%, \tag{2.4}$$

which is the main result of this paper. We will refrain from assigning a numerical value to the uncertainty in this estimate. An attempt in this direction would require us to assign a precise statistical meaning to our theoretical uncertainties. While there are reasons to take these uncertainties seriously at the qualitative level and we believe them to be superior to the model calculations used previously, there is still some amount of judgement that needs to be exercised that cannot be easily captured in precise statistical terms.

III. CONCLUSION

We estimated the abundances of ^{2}H and ^{4}He produces in the standard BBN scenario under the assumption that the light quark masses were shifted at the BBN time from present values. In order to perform this calculation we used input from several kinds of effective field theories as well as lattice QCD results to connect the quark mass variation to the relevant nuclear physics. We found that a variation beyond the $-1\% \lesssim \frac{\Delta m_q}{m_q} \lesssim 0.7\%$ range to be likely inconsistent with the observed abundances.

Two of the BBN parameters played the larger role in changing the light element yields: the deuteron biding energy (and the ^{3}H , ^{3}He and ^{4}He bidings, strongly correlated to ^{2}H biding) and the neutron lifetime. The neutron lifetime dependence on the quark mass values is well constrained by theory. The variation of the deuteron binding is, however, much more loosely constrained and several venues of further progress are clearly visible. Lattice QCD calculations of nucleon-nucleon interactions, even if performed at unphysical values of m_q, would go along way in narrowing these constraints. As long as they are performed with quark masses low enough to be within the region of validity of the chiral nuclear EFT they can nail down the value of parameters of the EFT necessary for the extrapolation of the deuteron binding. The bindings of ^{3}H , ^{3}He and, specially ^{4}He , can and should also be computed in the pionless nuclear EFT to higher orders so the theoretical uncertainty associated with them decreases. Finally, a better understanding of the quark mass variation of other threshold parameters like effective ranges, ... , would also allow for more precisely constrained calculation of the biding energy on the larger nuclei.

Since we are not presently able to obtain reliable values for the ^7Li binding energies, the ^7Li abundances we compute are not very meaningful and were not used in putting constraints in the quark mass variations. Future advances in the nuclear pionless effective theory may change this and allow us to address the "Lithium problem" as a signal of quark mass variation.

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