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Ultrashort Pulse Laser-Triggering of Long Gap High Voltage Switches

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Abstract

Laser-triggered high voltage switches need longer length and lifetime laser plasmas at controlled plasma densities in order to achieve optimal operational parameters (such as short runtime, low temporal jitter, and high voltage hold-off). We have approached the problem by considering ultrashort laser pulses of sub-picosecond duration for the high voltage trigger source. To this end, several laser plasma diagnostic techniques were employed to quantify the ionization and to determine methods for appropriately tailoring the laser plasma for switching purposes. Switching behavior under basic plasma conditions and with improved plasma channels are discussed.

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Nomenclature

CCD Charge Coupled Device (camera)

CVR Current Viewing Resistor

DC Direct Current

FWHM Full Width at Half Maximum

HV High Voltage

IR InfraRed (light)

KDP Potassium Dihydrogen Phosphate

LIB Laser-Induced Breakdown

LSP Large-Scale Plasma

LTGS Laser-Triggered Gas Switch

MPI Multi-Photon Ionization

PIC Particle-In-Cell (code)

PMT Photo Multiplier Tube

PPT Perelomov, Popov, and Terentev

PSI Pounds per Square Inch

PSIA Pounds per Square Inch Absolute

PSIG Pounds per Square Inch Gauge

RMS Root Mean Square

USP Ultra-Short Pulse

UV Ultra-Violet (light)

1 Introduction

Long-gap laser-triggered high voltage discharges have numerous applications from laser wakefield acceleration [1] and super-radiant light sources (both based on extended controlled-density plasmas) to laser-triggered spark gaps used for megavolt switching in pulsed power devices [2, 3] to triggered and guided lightning [4–10]. The problem facing all such efforts is the ability to create long ionized channels with adjustable plasma density and extended plasma lifetimes in order to improve discharge reliability, reduce temporal jitter, and allow for longer gaps and higher hold-off voltages. To overcome these problems, we propose using ultrashort pulse (USP) sub-picosecond lasers to look at switching aspects in long-gap high voltage discharges, with special emphasis on extension of plasma lifetime. Application of ultrashort laser pulses (instead of the nanosecond pulses more commonly used in pulsed power switching) can create the desired extended length plasma channels via nonlinear filamentation processes [11] while having a very prompt plasma creation time which can reduce switch jitters. Based upon these premises, existing work in this area has either focused on discharging the maximum length gaps with a USP laser (thus using the extended plasma channel length to minimize the ambient electric field for a given bias voltage) [12–16] or focused on minimizing the jitter at very small gaps and low voltages [17, 18]. At the extremes, some USP laser-triggered discharge research has shown discharged gaps as large as 3.8 m at about 2 MV [15] while other work has shown root-mean square (RMS) jitters as low as 15 ps for 1 mm gaps at 4.5 kV [18]. Our goal is to pull together various aspects: lower jitters for longer gaps and higher voltages.

To accomplish this, we must consider problems apparent in the existing literature. For example, the hindrance to increasing discharge lengths with USP lasers stems not only from the plasma length but also the plasma lifetime and density. Filaments [11] occur when the USP laser power exceeds the critical power P_{crit} for self-focusing, which is a manifestation of the optical Kerr effect wherein the refractive index of the medium changes with intensity and causes the radial intensity profile of the laser beam to establish a refractive index gradient and thus positively self-lens or self-focus the beam. The self-focusing continues until ionization (usually from multiphoton ionization) occurs significantly, causing a negative lensing from the refractive index contribution of the plasma gradient. The two mechanisms balance in a process called intensity clamping, with the net result that multi-meter scale plasma channels at electron densities of around $10^{16} \text{ e}^-/\text{cm}^3$ can be produced (as compared to the number density of air at 1 atm and 20°C of $2.5 \times 10^{19} \text{ molecules/cm}^3$ [19]). Recombination and electron attachment reduce to the electron density in a filament to less than $10^{13} \text{ e}^-/\text{cm}^3$ within a microsecond or so [13], with this density being considered a threshold level to maintain when trying to guide a streamer [14]. A streamer is a self-maintaining ionizing pulse front which moves along at 10^6 to 10^7 m/s while maintaining electron densities of $\sim 10^{14} \text{ e}^-/\text{cm}^3$ at the streamer tip [20]. Part of the barrier to longer gap discharges is the disappearance of electrons at one end of the gap before the streamer arrives. Put another way, the product of the streamer velocity and the electron lifetime to reach a threshold seed number density must equal or exceed the target discharge gap length. To do this, either higher initial electron densities (which lead to faster streamer velocities) or longer lifetime plasmas are necessary.

Since recombination and attachment processes increase with pressure, similar processes may

come into play in a laser-triggered gas switch (LTGS) which operate at higher pressures (with 4 to 10 atmospheres of either dry air or sulfur hexafluoride (SF_6) being common to increase the hold-off voltage) over a shorter gap. In the case of the Z-Machine 6 MV LTGS [3] (see Figure 1), a converging laser beam at 266 nm enters the switch gap area pressurized to around 70 PSIA. This discharges roughly 1 MV across the trigger section, causing the subsequent cascade section to see a voltage change and associated increase in electric field. With the LTGS system operating at electric fields E and pressures P which constitute E/P values which are 80 to 85% of the value for self-breakdown of the gap, the voltage change associated with switching the trigger section throws the cascade section into self-breakdown conditions. Subsequent self-breakdown discharges propagate down the cascade electrodes to the cathode until the switch is fully conductive and the switch is closed. However, this cascade section self-breakdown is more stochastic and thus inherently has more temporal jitter. If the length of the trigger section could increase with reliable functionality, then the cascade section length could decrease, thus decreasing the jitter. To do this, longer length laser-generated plasmas are necessary in the trigger section.

Based upon these concepts, methods of measuring the plasma lifetime, plasma length, and plasma density will help with the optimization process. In addition, the ionization and recombination dynamics are complex and are sometimes represented with conflicting formulas, constants, and explanations in the literature. This is in part due to the complexity of ionization processes which may occur individually or simultaneously, including:

1. Multiphoton ionization (MPI): A key process in filamentation wherein multiple photons are simultaneously absorbed to bridge the ionization potential of a given molecule. The process is dominant for lower gas pressures, shorter laser pulsewidths, and shorter laser wavelengths.
2. Cascade ionization: Laser ionization mechanism wherein a free electron is accelerated by inverse Bremsstrahlung absorption of the laser light until the electron can impact ionize a neutral molecule, thus creating additional free electrons for the process to iterate upon. The process generally cascades until full ionization and high plasma temperature have been achieved. The process is dominant for longer laser pulsewidths (> 1 ns), longer wavelengths, and higher gas pressures.
3. Tunneling ionization: For the most intense laser pulses, the electric field of the laser becomes large enough to distort the potential wells of the electron bound state in neutral molecules, enough so that the electrons can tunnel free through the well barrier.

To cite a specific problem, MPI is a key process in filamentation, with the process following a model developed by Keldysh [21]. The original paper however cites multiple sample cases which have become distorted in subsequent less rigorous journal articles in the intervening years. Furthermore, most of the literature indicates that MPI is pressure independent [11] while a few outlying references cite a pressure dependence in the process [22]. Keldysh theory and subsequent variants like the PPT theory (so-called because it was developed by Perelomov, Popov, and Terentev [23]) have been benchmarked at very low pressures ($\leq 10^{-7}$ Torr), but (to our knowledge) have not appropriately been validated at the higher pressures ($\geq 10^3$ Torr) that are present in an LTGS system.

As such, the same plasma diagnostic methods used to tailor plasma parameters should be used to quantify the ionization processes involved.

With this in mind, we will first discuss plasma diagnostics and ionization measurements, followed by methods of tailoring the laser-produced plasmas, and finally laser-triggering of gas switches. As modeling is an important part of any experiment, we wished to develop a simulation which could show switch operation from the laser plasma generation to the streamer development and subsequent high voltage breakdown. The USP laser ionization mechanism, as we will show, is full of complex simultaneous effects while the nanosecond-scale laser ionization is a little more manageable. In both cases, it is prudent to start LTGS simulation from a stand-point of the laser plasma providing a seed electron distribution on which the switch can work (rather than considering the complex dynamics of simultaneous laser ionization and electron acceleration in an external electric field). To this end, one modeling effort focused on the laser-plasma generation methods while other focused on the HV switch. These efforts are discussed after the experimental data.

Figure 1. The laser-triggered gas switch in the Z-Accelerator is filled with 70 psia of SF_6 and consists of a laser “trigger section” and a “cascade section”. The laser induced breakdown of the trigger section causes an over-voltage on the cascade section and starts the self-breakdown process along the cascade electrodes. The process takes about 40-50 ns to complete.

2 Plasma Diagnostics and Ionization Measurements

A systematic investigation should first examine ionization dynamics (including ionization by filamentation) through a suite of diagnostics techniques. The literature was examined for appropriate methods of laser plasma detection, with key candidates being electron collection, optical probing (interferometry/shadowgraphy/Schlerlein), spectroscopy, plasma fluorescence, and terahertz (THz) emission and probing. These are summarized here:

- *Electron Collection:* Electron collection uses the laser to create an ionized region between two biased electrodes (either transverse [24–26] or longitudinal [26–29] to the laser beam) which subsequently collect the charge.

Timing Information: The method spatially integrates over the electrode spacing but can provide temporal resolutions at the nanosecond scale depending on the level of charge integration.

Spatial Information: Collection volume is integrated in the transverse dimension to the bias electric field. Longitudinal plasma dimension ambiguity is a problem for the transverse electrode configuration due to edge effects but the length is better defined with longitudinal electron collection. A simultaneous side-on image of the plasma length can also help. Collection volume is also integrated along the bias electric field but, if the electrodes are small relative to the plasma length, then probing of the plasma length is possible.

Data Types: The technique yields the total number of collected charges. To get number density, one must accurately know the detected plasma volume (which can be tricky). Limited information about the plasma lifetime is available, depending upon the integration times used.

Sensitivity: Electron collection can be very sensitive but is limited in precision (due to questions about ionized volume and charge collection region) although the precision can be improved if calibrated via a more precise external method. With calibration and higher bias voltages, the technique can sense electron densities $\leq 10^{14} \text{ e}^-/\text{cm}^3$.

Nuances: Ideally, electrode separation and pressure should be small to avoid collisional effects during electron drift. Voltage should be low such that response is Ohmic and no avalanche breakdown is occurring. Protection of the electrodes with an insulator (used to mask the input laser beam) is necessary to avoid generating laser plasmas from the electrode surfaces, thus contaminating the gas ionization results.

- *Optical Probing:* Optical probing typically involves orthogonally delivering a synchronous lower power beam across the region of interest. This probing can be multichannel simultaneously, meaning either multiple wavelengths can be used or beam splitting of the probe can allow multiple optical probing techniques to be used simultaneously [31].

Timing Information: With USP probe beams [30–33], precise temporal resolution (of the order of the probe pulsewidth) can be achieved. Temporal jitter is negligible if pump and probe have a common source. Since the delay adjustment is optical and is thus based upon physical pathlengths, spatial limitations usually restrict delays to $\leq 10 \text{ ns}$.

Spatial Information: Transverse probing yields two-dimensional data which is spatially resolved ($\geq 5 \mu\text{m}$ typically). Information is integrated along the direction of the probe beam but symmetry conditions may allow data disentanglement.

Data Types: Shadowgraphy can unfold absorption and refractive index (but the unfolds can be difficult). Interferometry can unfold absorption and refractive index (and again the unfolds can be difficult but the literature is more abundant here for aid in analysis). Schleierin detects a change in refractive index (which does not in the end provide the data we seek). Abel inversion for cylindrically symmetric cases is a common unfold mechanism.

Sensitivity: Optical probing is limited in sensitivity to a dynamic range of 2 to 3 orders of magnitude (10^{17} to $10^{19} \text{ e}^-/\text{cm}^3$) in part due to the ability to resolve fractional fringes (for interferometry) and in part due to the bit depth of available cameras.

Nuances: All techniques do not distinguish between index changes due to electron density and gas density changes (i.e. neutral particle gradients such as can be generated by a shock front). Gas density index change contribution is negligible for times below hydrodynamic expansion ($\leq 1 \text{ ns}$)

- *Spectroscopy:* Plasma spectroscopic methods for USP laser-generated plasmas [34–36] collect the plasma self-emission and directs this light into a high resolution spectrometer for analysis.

Timing Information: With a typical high resolution CCD as detector, spectroscopy temporally integrates the data. However, with a streak camera used to temporally resolve the spectrum (which will require significantly higher plasma emission and/or collection efficiency), picosecond-scale resolution can be achieved. Using a time-gated CCD at the spectrometer output, resolutions of a few nanoseconds can be achieved. Similar temporal resolution can be achieved if a photomultiplier tube (PMT) is used at the output and the spectrometer is used as a monochromator.

Spatial Information: Spectroscopy can be spatially resolved in one dimension with an imaging spectrometer or can be spatially integrated with input lenses for higher signal-to-noise. However, with the exception of a time-gated CCD on the spectrometer output, time-resolved methods typically integrate the spatial information.

Data Types: Spectroscopy can yield electron density information based on the broadening of specific plasma emission lines [34, 36] or electron temperatures based on the ratios of the amplitudes of specific plasma emission lines [34, 35].

Sensitivity: Spectroscopy for electron density measurements is limited in sensitivity to a dynamic range of 2 to 3 orders of magnitude (10^{17} to $10^{19} \text{ e}^-/\text{cm}^3$) in part due to limited spectral resolution for common spectrometers and in part due to the bit depth of available cameras. Collection efficiency, spectrometer efficiency, and plasma emissivity also limit sensitivity.

Nuances: Spectroscopy can indicate gas plasma chemistry (which is relevant for subsequent laser pulses which interact with new species).

- *Plasma Fluorescence:* Plasma fluorescence methods [26, 37] collect the plasma self-emission (as with spectroscopy) but use lower loss and lower cost filters in specific spectral regions to

increase the signal strength. In addition, the detection method without a spectrometer can offer different information about the plasmas.

Timing Information: In using a PMT [26], nanosecond timescale resolutions (although spatially integrated) are achievable. Similarly, a time-gated image intensified CCD can offer nanosecond timescale resolutions but with two dimensional spatially resolved image frames.

Spatial Information: Unless using a time-gated image intensified CCD, a CCD with imaging collection optics will offer two dimensional spatially resolved but temporally integrated frames [37].

Data Types: Plasma lifetime and/or plasma density can be obtained in relative sense. However, when calibrated to an absolute measurement method, plasma fluorescence can provide a sensitive absolute measurement of electron density or lifetime.

Sensitivity: When used in conjunction with a highly sensitive detection method such as a PMT or an intensified CCD camera, plasmas of 10^{16} to 10^{19} e^-/cm^3 can be detected. When gains are turned up on such detectors and calibrated filters are used, the sensitivity can be increased by several additional orders of magnitude to as low as 10^{13} e^-/cm^3 [37].

Nuances: Ideally, to provide reliable electron information and not just molecular excited state information, the plasma fluorescence can be used at known spectral bands corresponding to the electron parameters. In air, the emission strength of the 391 nm spectral line associated with N_2^+ has been shown to scale with electron density [38]. For high gain detectors such as PMT's or intensified CCD's, detector linearity can be a concern.

- *THz Emission and Probing:* In the case of filaments, THz self-emission has been observed and scales with electron density [63]. More generally, THz can be used like an optical probe (albeit with different techniques) with high temporal resolution [33,41,42]. Due to the much longer wavelength for THz than traditional optical probes, the level of plasma density to which the probe is sensitive is much lower

Timing Information: The THz light used for probing as well as the filament emission are of picosecond scale or less.

Spatial Information: Spatial resolution at these long wavelengths is difficult with the plasma transverse dimension at 0.1 mm scale, which is small compared to the probe wavelength. However, if the plasma length is long, it can be resolved with 1 to 10 mm scale resolution.

Data Types: The THz probing and emission can be used to determine electron density.

Sensitivity: Optical pump-THz probe methods are not usually single shot but can cover a range of 10^{13} to 10^{17} e^-/cm^3 in a multishot mode [42]. This is consistent with single cycle THz beams with frequencies from DC to around 3 THz. By comparison, filament THz self-emission correlates to electron density and has been calibrated with absolute methods to indicate $\geq 10^{15}$ e^-/cm^3 .

Nuances: The generation and detection methods for THz are more elaborate than traditional optical methods.

Based upon complexity and possible safety issues, THz and electron collection methods were deemed undesirable. This left optical probing, plasma fluorescence, and spectroscopy were initially explored. Eventually, spectroscopy was dropped due to poor sensitivity since only the very brightest of plasmas were detectable by a traditional 16 bit CCD behind the spectrometer. With these kinds of diagnostics in mind, an elaborate setup (see Figure 2) was developed which could be used for measurements of basic ionization information as well as measuring tailored plasma spatiotemporal distributions.

Figure 2. The laser ionization diagnostic layout. The USP laser receives an initial split at a beamsplitter (BS) to allow a weak delay-adjustable (Δt) optical probe at 1054 nm and/or 527 nm to be generated. A subsequent half waveplate ($\lambda/2$) and polarizer P allow the generation of collinear delay-adjustable s and p-polarized USP pulses for laser ionization. A dichroic allows a collinear nanosecond-scale 532 nm laser for ionization in the gas cell. Leakage at the dichroic is used to measure the intensity of the ionizing laser pulses via cameras (C), energy meters (EM), and photodiodes (PD).

2.1 Optical Probes

Interferometry

Interferometry was deemed to be the best initial diagnostic choice for its time-resolved two-dimensional images which could be directly unfolded to yield absolute electron density measurements. In conjunction, leakage light of the ionizing laser pulse (as taken at the final fold dichroic optic before the ionization cell) is used to establish an equivalent image plane of the focal spot at much lower intensities. The same leaked light would also provide the calibrated laser energy and pulsewidth such that the overall focal intensity in the ionization cell should be known. Knowing the focal intensity and the resultant electron density from interferometry should design the key aspects of the ionization process. However, one should question the source of the measured plasma interferogram fringe shifts as well as the integrity of the equivalent focal intensity measurement in order to maintain the validity of the final data analysis.

With respect to the interferometry, at early times, electrons are the main contributor to this refractive index data. However, at later times, hydrodynamic evolution of the plasma shock front creates a varying density of neutral particles which is convoluted with the electron density data. With only one piece of data and two unknowns, an ambiguity is thus created in the interferometric data between the effects of the neutral and electron densities. To deal with this, nanosecond pulsed lasers and continuous wave lasers have been used to provide simultaneous two-color interferometry, providing two pieces of data for two unknowns [43–46].

With regards to the experimental setup, we have set up a femtosecond laser pump-probe experiment where one part of a 500 fs laser pulse with 20 mJ at 10 Hz at 1054 nm creates an atmospheric plasma in a test cell and a 10% leakage part of that pulse for transverse probing purposes. The probe beam is spatially filtered and delayed from -1 ns to >10 ns with respect to the main beam. A thin potassium dihydrogen phosphate (KDP) crystal in the probe beam allows frequency doubling to 527 nm such that the collimated fundamental (at 1 cm diameter beamsize) and second harmonic probe the plasma with negligible delay. This delay is roughly 1 ps and consists of the sum of the delays from the group velocity mismatch in the 3 mm thick KDP doubler and the 1 inch thick fused silica probe beam input window. After probing the plasma, the beam is enlarged by an all reflective beam expander which magnifies the probe region by 3 times for better resolution at a subsequent interferometer camera. This reflective telescope uses longer focal length ($f_1=50$ cm and $f_2=150$ cm) spherical curvature mirrors at shallow incidence angles to mitigate any aberrations (specifically any astigmatism resulting from the off-axis use). Protected silver coatings are used in the telescope and in all subsequent high reflectors to support large bandwidths.

After the beam expansion, the beam goes to an interferometer. Initially, an easy approach was taken via the simple air wedge lateral shearing interferometer [31, 47]. However, for a collimated probe beam input, this interferometer generates two images offset from each other by the wedge angle. For long plasma lengths along the wedge direction, these images can blur together and create convoluted interferograms. As such, a modified Michelson interferometer (similar to [30]) was designed with a hollow roof prism made of silver mirrors in one arm and a flat silver high reflector in the other. For an appropriately placed beam, the roof mirror will allow the test region of the

probe beam to interfere with an unperturbed region of the same beam (i.e. a lateral displacement rather than a lateral shear interferometer). A beamsplitter and compensator plate are necessary in the interferometer in order for the group delays for the two probe beam wavelengths to be the same. A dichroic is used after the interferometer to separate the two different wavelength interferograms. Different color and neutral density filters are used to provide unambiguous interferograms in each vertical portion of the camera.

For a plasma being probed, the interferogram obtained provides total phase shift information $\Delta\phi_{tot}$ which is the sum of phase shift contributions from electrons ($\Delta\phi_e$) and from neutral density particles ($\Delta\phi_0$). Generally speaking, the low mass and high mobility of the laser-generated electrons will lead to early time phase data being dominated by the electron contribution while the higher mass and lower mobility of the neutral particles will dominate at later times (especially with recombination losses decreasing the electron contribution). The generalized phase shift $\Delta\phi$ can be expressed in terms of probe wavelength λ and the line-integrated medium refractive index n along the direction x over length L as:

$$\Delta\phi = \frac{2\pi}{\lambda} \int_0^L (n(x) - 1) dx. \quad (1)$$

For a plasma with electron density N_e , the refractive index n_e due to electrons is expressed [49]:

$$n_e = \sqrt{1 - \frac{\lambda^2 e^2 N_e}{4\pi^2 \epsilon_0 m_e c^2}} = \sqrt{1 - \frac{N_e}{N_c}} \approx 1 - \frac{N_e}{2N_c} = 1 - \frac{\lambda^2 N_e r_e}{2\pi}, \quad (2)$$

where $N_c = (4\pi^2 \epsilon_0 m_e c^2) / (\lambda^2 e^2)$ is the cutoff electron density and the approximation assumes that the refractive index $n_e \sim 1$ such that $(n_e^2 - 1) = 2(n_e - 1)$. The wavelength dependence is broken out of the cut-off density by expressing it in terms of the classical electron radius $r_e = e^2 / (4\pi \epsilon_0 m_e c^2)$ [46] such that $N_c = \pi / (\lambda^2 r_e)$.

Similarly, for a neutral gas of density N_0 , the refractive index due to the neutral gas n_0 is expressed [48] in terms of a modified Cauchy relation:

$$n_0 - 1 = \frac{N_0}{N_{atm}} [A_{atm} \cdot (1 + \frac{B_{atm}}{\lambda^2})] = \frac{N_0}{N_{atm}} \beta, \quad (3)$$

where N_{atm} is the number density at standard atmospheric pressure and A_{atm} and B_{atm} are fitting parameters dependent on the gas type (as measured at standard atmospheric pressure) which can be consolidated into the parameter β (which for air is the Gladstone-Dale constant [43]).

Based upon these refractive indices, the phase shift contributions from electrons ($\Delta\phi_e$) and from neutral density particles ($\Delta\phi_0$) are (respectively):

$$\Delta\phi_e = \frac{2\pi}{\lambda} \int_0^L (n_e(x) - 1)dx = -\lambda r_e \int_0^L N_e(x)dx, \quad (4)$$

$$\Delta\phi_0 = \frac{2\pi}{\lambda} \int_0^L (n_0(x) - 1)dx = \frac{2\pi\beta}{\lambda N_{atm}} \int_0^L N_0(x)dx. \quad (5)$$

When interferograms are recorded at two wavelengths λ_1 and λ_2 , equations 4 and 5 at these two wavelengths can be combined to yield:

$$\int_0^L N_e(x)dx = \frac{\lambda_1 \Delta\phi_{\lambda_1} - \lambda_2 \Delta\phi_{\lambda_2}}{r_e(\lambda_2^2 - \lambda_1^2)}, \quad (6)$$

$$\int_0^L N_0(x)dx = \frac{N_{atm}(\lambda_2 \Delta\phi_{\lambda_1} - \lambda_1 \Delta\phi_{\lambda_2})}{2\pi\beta(\lambda_2/\lambda_1 - \lambda_1/\lambda_2)}. \quad (7)$$

With this derivation in mind, we record approximately synchronous two-color interferograms first and then calculate the phases. To do this, we employ the standard spatial carrier method [50]. This process takes the Fourier transform of the interferogram image, which yields a strong center frequency along with two symmetrically placed side-lobes at the frequency of the interferogram fringes (or the spatial carrier). One of these side-lobes is extracted and shifted back to the center frequency. A subsequent inverse Fourier transform is taken to yield the phase map. One should note that, first of all, many fringes are desirable (at least more than 50 in practice) such that the information in frequency space of the central lobe does not spill over into the information from the side-lobes. Secondly, soft apodization filtering of the frequency side-lobe is necessary to mitigate spatial ringing in the phase image. Finally, in a similar manner, soft apodization filtering of the overall image edges is necessary to mitigate ringing when going from either spatial or frequency images to its Fourier transform analog.

Since phase determination methods wrap the phase to lie within the range of $-\pi$ to π radians, a phase unwrap routine was written to look at the phase discontinuities and reproduce the correct continuous phase. Once unwrapped, the phases at the two wavelengths are used to determine the line-integrated electron and neutral densities as denoted in the left-hand side of Eqs. 6 and 7. To do this in a practical manner, a spatial fiducial is used in each interferogram frame to provide registration for the accurate combination of these two phase maps. The resulting line-integrated number densities incorporate both the value of the number densities and the spatial extent of the region in question. If cylindrical symmetry is assumed, one can disentangle these values with an Abel inversion technique, yielding number density as a function of radial position. Researchers that employ such methods may have extensive problems when plasma symmetries become clearly broken in the line-integrated phase map. As such, an asymmetric Abel inversion technique (which assumes a partial symmetry about the viewing plane) is used here [51,52]. For the two dimensional phase map in the xz plane, this approach takes, about longitudinal beam propagation axis z at a position x_0 , the phase maps $\Delta\phi_{plasma}(x, z)$ in the upper half (from $x \geq x_0$) and lower half (from $x \leq x_0$) and relates them to a subsequently used integration variable η thus:

Figure 3. Interferometry and subsequent asymmetric Abel inversion at a near zero delay.

$$\Delta\phi^+(\eta, z) = \Delta\phi_{plasma}(x - x_0, z) \text{ for } x > x_0, \quad (8)$$

$$\Delta\phi^-(\eta, z) = \Delta\phi_{plasma}(x_0 - x, z) \text{ for } x < x_0. \quad (9)$$

These go into symmetrized and antisymmetrized half maps $\Delta\phi_s$ and $\Delta\phi_a$ respectively:

$$\Delta\phi_s(\eta, z) = \frac{1}{2}(\Delta\phi^+(\eta, z) + \Delta\phi^-(\eta, z)), \quad (10)$$

$$\Delta\phi_a(\eta, z) = \frac{1}{2}(\Delta\phi^+(\eta, z) - \Delta\phi^-(\eta, z)). \quad (11)$$

Using these, two different two dimensional maps $n_{e0}(r, z)$ and $n_{e1}(r, z)$ are developed:

$$n_{e0}(r, z) = \frac{n_c \lambda_p}{\pi^2} \int_r^\infty d\eta \frac{\frac{\partial}{\partial \eta} [\Delta\phi_s(\eta, z)]}{\sqrt{\eta^2 - r^2}}, \quad (12)$$

$$n_{e1}(r, z) = \frac{n_c \lambda_p r}{\pi^2} \int_r^\infty d\eta \frac{\frac{\partial}{\partial \eta} [\frac{\Delta\phi_a(\eta, z)}{\eta}]}{\sqrt{\eta^2 - r^2}}. \quad (13)$$

$$(14)$$

The method assumes that the xz plane of the phase map constitutes a mirror symmetry plane

such that the final three dimensional electron density n_e is expressed:

$$n_e(r, \theta, z) = n_{e0}(r, z) + n_{e1}(r, z)\cos(\theta) \quad (15)$$

Note that, in the event that the data truly is symmetric about the z propagation axis, one gets $\Delta\phi^+(\eta, z) = \Delta\phi^-(\eta, z) = \Delta\phi(\eta, z)$ which causes $\Delta\phi_a(\eta, z) = 0$ and $\Delta\phi_s(\eta, z) = \Delta\phi(\eta, z)$. At that point, the form of $n_e(r, z)$ looks like the standard symmetric Abel inversion seen elsewhere [32].

We first used the technique on a single USP interferogram, an example of which is shown in Figure 3. Using the process described, the unwrapped electron density (at $\theta = 0^\circ$ in the xz plane) measures approximately 50% of full single ionization of all air molecules for a longitudinal position slightly away from the peak ionization. The process shows the applicability of the asymmetric Abel inversion technique, as this region would be difficult to force symmetry conditions upon in a normal Abel inversion approach. Subsequently, we tested the two-color interferogram method but found negligible difference in the measured phases for the two color. This is expected because the USP laser-generated plasma lifetime is only a few nanoseconds at best, meaning that the plasma does not have enough time to hydrodynamically evolve into a measureable neutral particle density gradient while there is still a significant electron density.

Shadowgraphy

In many cases, spatial information about the plasma extent may be adequate, by-passing the need for rigorous interferometric analysis. Such data can be simply extracted by blocking the reference arm of the interferometer to yield a shadowgraph. In this case, the optical probe sees local refractive index and opacity differences from the plasma. Figure 4 compares shadowgraphs of laser-generated plasmas from USP and nanosecond lasers pulses. Note that the nanosecond pulse plasma is extremely opaque and shows signs of beading plasma structure for a fairly significant laser energy. By comparison, the USP plasma is longer and more continuous at a significantly lower laser energy. The shadowgraphs point out another important fact: Attempting to perform interferometry on nanosecond pulse laser-generated plasmas will not work due to the extremely hot dense plasma formed. If the shadowgraph is opaque in the plasma region, the interferogram will show no fringe data in this region either. As such, pursuit of nanosecond laser plasma information subsequently focused upon plasma fluorescence and shadowgraphy techniques.

A more detailed application of shadowgraphy was to examine the plasma length as a function of USP laser energy (see Figure 5). As the energy is turned up, two things occur: 1) The plasma length (as measured by the distribution FWHM) increases and 2) The plasma distribution (as measured by the distribution centroid) extends back towards the focusing lens. This first point goes to the heart of creating tailored plasmas (see section 3). However, one should not be able to turn up the energy infinitely and see the same plasma lengthening improvement. At a peak power of several times the critical power P_{crit} for self-focusing (which are energies unattainable with the system used in this testing), the beam will fragment and begin to generate multiple filaments. The further that the peak laser power is above P_{crit} , the more filaments and associated plasma channels are generated. However, at the lower energies tested, as the plasma lengthens, it should be noted that

Figure 4. Basic shadowgraphy of a laser plasma. In both cases the laser enters from the left side and is focused by an $f=25$ cm lens in ambient air. The probe beam delay was set at the time of peak ionization (taken to be zero delay between pump and probe). The horizontal image length along is 5.5 mm. (a) USP laser ($\lambda=1054$ nm, $\phi=12$ mm FWHM, 14 mJ, $t\sim 500$ fs) plasma. (b) Corresponding plasma for a nanosecond laser pulse ($\lambda=532$ nm, $\phi=8$ mm FWHM, 300 mJ, $t\sim 8$ ns).

farthest distance of the plasma distribution is fairly consistent and remains near the farthest end of the lowest energy laser plasma created near the paraxial focus. As the plasma lengthens, it grows back towards the focusing lens, as one would expect in the presence of self-focusing. Thus, the lengthening of the plasma in one direction leads to a plasma centroid shift in that direction.

One should note that at the lowest energies, the plasma still seemed to be showing some centroid shift below P_{crit} (although it was somewhat ambiguous). However, according to [54], the plasma should exhibit no further centroid shift at peak powers below P_{crit} . The inability to confirm this in our experiment poses a problem in that the goal of the plasma diagnostics is to quantify ionization based on an equivalent intensity measurement to the ionizing laser pulse. If the focal plane and focal spot size varies with energy, the equivalent intensity measurement is invalid. Further, elaboration of the low energy data in Figure 5 proved inconclusive because the evidence of the plasma tends to vanish below 0.5 mJ for both shadowgraphs and interferograms. Above this energy, the effects of both self-focusing and plasma generation appear to be present. This issue prompted a more sensitive measurement technique, which is the time-integrated wavefront measurement discussed in the following section.

Figure 5. Shadowgraphy of a laser plasma. (a) The laser ($\lambda=1054$ nm, $\phi=12$ mm FWHM, $t\sim 500$ fs) enters from the left side, as focused by an $f=25$ cm lens in ambient air. The arrow indicates the nominal paraxial focus. The probe beam delay was set at the time of peak ionization (taken to be zero delay between pump and probe). The horizontal image length along is 5.5 mm. (b) Image analysis of data from (a). Note that the critical power estimated from [53] is shown to indicate where self-focusing should begin.

2.2 Wavefront Sensor Plasma Diagnostics

New approach to measure MPI coefficients and n_2

As shown in the previous section, for energies above 1 mJ, the measurement of MPI coefficients and nonlinear index n_2 cannot be performed since both phenomena are present simultaneously. We propose a new method for measuring n_2 and ionization rates which allows us to detect beam shifts at much lower energy (or power $P \ll P_{crit}$). In our setup (see Fig. 6), a wavefront sensor is used to directly sense a shift in focal length as opposed to indirectly measuring shifts via a secondary effect such as plasma fluorescence [55]. This is critically important because plasma induced focal shifts occur well before plasma fluorescence can be detected.

Figure 6. Setup to measure the self-focus shift using a wavefront sensor (WFS). The blue lines represent the low peak power case while red lines represent powers sufficient for self-focusing.

A short pulse laser at low power $P \ll P_{crit}$ is focused with a focal element $M1$ (focal length $f1$) into a gas ionization cell and re-collimated with a second focal element $M2$ (focal length $f2$). A wavefront sensor (WFS) is placed at a distance d behind $M2$ and a flat wavefront is recorded as a reference (see Fig. 7(a)) at minimum powers. Spherical mirrors were used in the focusing and re-collimation of the laser beam (see $M1$ and $M2$ in Fig. 6) in order to avoid nonlinear phase accrual in the lens material which may compromise data.

As the laser beam power increases, the beam focus will shift by a distance Δ toward $M1$. Correspondingly, the light emanating from the new self-focused position $f1 - \Delta$ or $f2 + \Delta$ will not be collimated any more but rather be imaged a certain distance D behind $M2$. This distance D is just the radius of curvature R of the wavefront (as measured by the wavefront sensor; see Fig. 7(b)) plus the distance d of the sensor to the lens $M2$: $D = R + d$. The focal shift Δ can then be calculated from the thin lens equation:

Figure 7. (a) Flat reference/initial wavefront recorded for the f/120 geometry at a beam energy of 0.3 mJ. The associated “defocus” term was measured at $R_2^0 = -0.005$ waves. (b) Wavefront curvature recorded at 5.4 mJ energy with $R_2^0 = 0.05$ waves “defocus” for the same experimental configuration as (a).

$$\frac{1}{f2} = \frac{1}{f2 + \Delta} + \frac{1}{R + d} \quad \Leftrightarrow \quad \Delta = \frac{1}{\frac{1}{f2} - \frac{1}{R + d}} - f2. \quad (16)$$

The wavefront radius R at position d can be calculated from the measured defocus Zernike term R_2^0 (see Fig. 7(b)). In our case $R_2^0 = 2a^2 - 1$, where a is the pupil radius. For a measured pupil of radius a on the wavefront sensor and a defocus of R_2^0 in waves one calculates:

$$R = \frac{a^2}{4R_2^0\lambda}, \quad (17)$$

where λ is the wavelength of the laser.

Experimental results

A laser beam at $\lambda = 1054$ nm with a pulsewidth of $\tau = 540$ fs FWHM and a spatial first order Gaussian with a $1/e^2$ waist radius of $w_i = 6.2$ mm was focused by spherical concave mirrors of f/120 and f/40. A larger f/# (greater than f/120) was used as well but led to the onset of filamentation at low laser energies. Each data point represents an average of 200 measurements at 10 Hz on the wavefront sensor (Phasics SID-4, see <http://www.phasicscorp.com/>) as well as the energy meter. The peak to valley “defocus” Zernike term R_2^0 was then fitted to the averaged wavefront and converted to focal shift as discussed earlier.

Figure 8 shows the measured focal shift versus laser input energy for the two f/# mentioned earlier. One can see that, depending on f/#, the onset (elbow in the graph) of the focal shift is

Figure 8. Plot of focal shift Δ versus laser input energy. The scatter plots show experimental data for f/120 and f/40 focusing geometries. The solid lines represent theoretical fits based on the pulse propagation equation 19 (see below) and the dashed lines depict the theoretical behavior without ionization present.

different for any given focus geometry. Two main processes contribute to focal shift:

- **Nonlinear self-focusing:** As the laser beam power P becomes comparable to the critical power $P_{crit} = 3.77\lambda / (8\pi n_0 n_2)$, the nonlinear index of refraction n_2 leads to additional focusing in beams with Gaussian (and also other) intensity profiles. For a collimated input beam entering a nonlinear medium and an unchanged amplitude profile during self-focusing, one can write the self-focusing distance z_{sf} due to the nonlinear index n_2 as [56]:

$$z_{sf} = \frac{\pi w_i^2}{\lambda} \left(\frac{P}{P_{crit}} - 1 \right)^{-1/2}, \quad (18)$$

where w_i is the initial beam waist at the entrance of the nonlinear medium. In a focusing geometry, the new effective focal length f_{new} can then be found as a combination of the geometrical focal length f_g through the lens transformation formula: $f_{new} = f_g z_{sf} / (f_g + z_{sf})$.

One can see that this will lead to a shift of the minimum beam waist (focus) toward the input focusing lens.

Based on Eq. 18, a focal shift can only occur at $P \geq P_{cr}$ which would be independent of focus geometry (see dashed lines in Fig. 8). Liu *et al* [54] uses this argument when he identifies this elbow as the onset of $P = P_{cr}$ in order to calculate n_2 . According to measured data in Fig. 8, the elbow is clearly dependent on beam focusing geometry and hence one cannot consider the effects of nonlinear self-focusing alone.

- **Ionization:** It is clear that ionization must occur as the laser beam propagates towards the minimum beam waist. This ionization will be highest around the focal region and will profoundly affect the self-focusing behavior [57]. As the beam collapses towards the focus the electron density increases as a result of ionization due to increased beam intensity. This leads to plasma de-focusing before the geometrical focus is reached. Hence ionization will lead to a focal shift toward the input focal lens as will nonlinear self-focusing due to the nonlinear index n_2 .

It can be seen that one has to consider both processes in order to adequately explain the results presented in Fig. 8. One should note that for f/40, where ionization is dominant, the focal shift begins at lower power but shows smaller shifts at larger powers as compared to the regime where nonlinear self-focusing is dominant (f/120).

Experimental analysis

In order to extract values for both n_2 and the ionization rates, one has to model the beam waist as a function of laser power P and laser induced ionization. We used the semianalytical model employed in [11, 57, 58] and it was shown that it adequately reproduces the beam waist evolution until the minimum waist is reached [57] which is the regime of interest. The laser beam waist $w(z, t)$ as a function of time and propagation distance z can be written as:

$$\frac{\partial^2 w(z, t)}{\partial z^2} = \frac{4}{k^2 w(z, t)^3} \left(1 - \frac{P(t)}{P_{crit}} \right) + \frac{2K}{(K+1)^2 n_0 N_{crit} w(z, t)} \times N_e(z, t), \quad (19)$$

where the first term describes diffraction and self-focusing and the last term describes the defocusing due to ionization. In this equation k is the wavenumber, $P(t)$ is the laser power, $P_{crit} = 3.77\lambda^2/8\pi n_0 n_2$ is the critical power for self-focusing for a Gaussian beam, and n_2 is the nonlinear index of the gas under study (air). K is the ionization order of the multi-photon ionization (MPI) process. For an ionization potential of $W_{O_2} = 12.06$ eV for oxygen and a photon energy of 1.18 eV for a photon at 1054 nm one calculates: $K = \lceil 12.06\text{eV}/1.18\text{eV} \rceil = 11$. n_0 is the index of air and $N_{crit} = \epsilon_0 m_e \omega_l^2 / e^2$ is the critical plasma density for the laser. Here, ϵ_0 is the dielectric constant in vacuum, m_e is the mass of the electron, ω_l is the angular frequency of the laser, and e is the charge of the electron.

Since there exist two data sets and two unknowns (n_2 and $\sigma^{(K)}$), one should be able to find a unique solution that fits the measurement. This is accomplished rather easily since the physics in the f/40 focusing geometry is largely dominated by ionization which sets the limit for $\sigma^{(K)}$. Conversely, self-focusing plays a dominant role in the f/120 focusing geometry which leads to a determination of n_2 . The higher influence of ionization in the f/40 case is evidenced by the fact that the onset of a focal shift (elbow) occurs at much lower powers.

The importance of ionization for the correct estimation of n_2 and $\sigma^{(K)}$ is illustrated by the dashed lines in Fig. 8. They are based on theoretical curves that do not include ionization. As expected, those curves are shifted to the right because the focal shift contribution from the laser plasma is omitted. If only one f/# measurement is considered (as is the case in [54]), one is inclined to identify the elbow in the curve as P_{crit} , leading to an erroneously low P_{crit} (or conversely high n_2).

Figure 9. Ionization rate versus intensity for oxygen and nitrogen. The dashed, vertical green line depicts the transition region between MPI and tunneling ionization, where the Keldysh parameter $\gamma = 1$ for $\lambda = 1054$ nm.

The solid lines in Fig. 8 show the calculated focal shift versus energy for $n_2 = (2.6 \pm 0.2) \times 10^{-23} \text{ m}^2/\text{W}$ and $\sigma^{(11)} = (3 \pm 1.5) \times 10^{-19} \text{ m}^2 \text{W}^{-1} \text{s}^{-1}$. The value for n_2 compares well with other data from the literature. On the other hand, one should be aware that the measured ionization coefficient for the 11th order process is 10 orders of magnitude lower than expected based on a weak field approximation of the Keldysh theory. The reason for this discrepancy is due to the fact that both focusing geometries reach an intensity clamped regime of $I_{cl} = 3.5 \times 10^{18} \text{ W/m}^2$ at which tunneling ionization dominates the ionization process. To illustrate this point, we have simulated ionization rates for different gases using the model by Perelomov, Popov, and Terentev (PPT model) [23].

Figure 9 shows the calculated ionization rate R versus laser intensity for oxygen and nitrogen. Note, that $dN_e/dt = (N_{O_2} - N_e) \times R$. The model was benchmarked against ionization time of flight data conducted with ultra short pulse lasers at pressures $\leq 10^{-7} \text{ Torr}$. One can see from Fig. 9 that the measured ionization rate at the clamped intensity agrees well with the PPT model, including both MPI and tunneling ionization.

Conclusions about the wavefront sensor method

We have presented a novel method for measuring the nonlinear index of gases and their corresponding ionization rates (in this case air/ O_2) under atmospheric pressure at $\lambda = 1054 \text{ nm}$. It has been pointed out that measured focal shifts result from a combination of self-focusing and plasma-defocusing which has been appropriately treated in the semi-analytical model. Based on this model, we have calculated effective MPI coefficients for O_2 in the intensity regime of 10^{18} W/m^2 . The measured coefficient is in excellent agreement with the PPT model. This result has broader implications, because it allows one to safely use the presented PPT curves for modeling efforts on laser propagation in air and other gases.

2.3 Plasma Fluorescence

Plasma fluorescence methods can provide a powerful simple approach to plasma diagnostics. The broadband fluorescence will not provide information about electron density because the emission comes from a variety of pathways. However, filtered regions around 391 nm have been shown correlate to the electron density [37, 38]. A slightly broader transmission region was achieved via a combination of Schott BG39 and UG11 color filters (see Figure 10). In Figure 10a, the temporal duration of the plasma fluorescence is shown to increase with shorter $f/\#$ (short focal length) optics for the USP laser source. In the longer focal length case, the temporal pulsewidth correlates well to the expected PMT response (which is expected since the ionizing pulse is much shorter than the PMT response time and the plasma lifetime for a relatively cold plasma is of similar or shorter timescale). In the case of the shorter focal length lens, the lifetime is significantly longer, indicating a hotter and/or denser plasma with a longer recombination time. Based upon the anticipated 25 times increase in intensity for $f=5$ cm lens compared to the $f=25$ cm one, this observation could be due to an increase in tunneling ionization and/or additional heating from inverse Bremsstrahlung. Note that the PMT exhibits a spurious short in the signal after a few hundred nanoseconds.

Figure 10. Temporal detection of laser-generated ($\lambda=1054$ nm, $\phi=12$ mm FWHM, $t \sim 500$ fs) plasma fluorescence in ambient air. Detection occurred via a Photonis XP2020/UR PMT (1.6 ns risetime) with UG11 (1 mm thick)+BG39 (1 mm thick) filtering (a) Temporal response of PMT for two different focal lengths: $f=5$ cm and $f=25$ cm. (b) Averaged PMT signal amplitude versus laser energy for $f=25$ cm case.

In Figure 10b, the peak signal output voltage of the PMT was averaged and plotted as a function of laser energy. Ideally, for MPI only using 1054 nm (1.18 eV) light, ionization of O_2 (at an ionization potential of 12.1 eV) requires 11 photons and N_2 (at an ionization potential of 15.6 eV) requires 14 photons, meaning that the oxygen will ionize first and dominate the process. If the ionization process only included MPI (without any inverse Bremsstrahlung or tunneling ioniza-

tion contribution), the slope of the data on a logarithmic plot would then be 11. While a slope of 11 is shown in the figure, one might more easily assume that a lower slope of around 7 matches the data better. Based upon our experiences with intensity clamping in the wavefront sensor data, one expects that a contribution from tunneling ionization is present, which lowers the effective slope. To verify this, we take the 100 μJ to 200 μJ range of the plasma fluorescence data in Figure 10b and calculate via Zemax software [39] the focal intensity of the 12 mm FWHM diameter apodized flattop beam focused by the $f = 25$ cm lens, which gives intensities of 3.8×10^{13} W/cm² to 7.6×10^{13} W/cm² (which is well below the region of intensity clamping and large nonlinear Kerr effects). The slope of the PPT curve is taken at the beginning and end of this intensity range. The slopes vary from 7.5 down to 6.8 over this range, agreeing well with the measured data in Figure 10b.

3 Tailored and Enhanced Plasmas

With the tools in hand, the ionization control must be increased, emphasizing the ability to tailor our plasma length, lifetime, and density. We investigated this with several methods of achieving these goals, including sequences of multiphoton ionization (MPI) ultrashort laser pulses and secondary heating pulses (utilizing photodetachment and cascade ionization).

3.1 Plasma Lengthening

The topic of longer plasma channels is critical to generating longer plasma lengths. Historically, researchers have tried using increased an $f/\#$ for the focal optic to extend the depth of focus of the system (while also deleteriously increasing the laser spot size and dropping the intensity), using strongly spherically aberrant focal elements to blur out the longitudinal focal region [59], using axicons [8, 60, 61], using segmented optics with different focal lengths at each segment [61], or using diffractive optical elements engineered to have aspects of the previously described attempts [62]. Another straightforward method in a similar context is to simply twist the lens to induce astigmatism. The slight angle of incidence of the lens breaks the focal symmetry shifting the foci in one axis away from the focus in the other axis, leaving the area of best circular symmetry (the so-called “circle of least confusion”) in between. As the lens angle of incidence increases from 0° , the “circle of least confusion” moves back towards the lens. Note that this method of plasma extension applies to both USP lasers and nanosecond lasers (see Figure 11). The effect of this is illustrated in Figure 11, using a fused silica $f=75$ cm lens to focus a 2.6 J average energy laser at 532 nm in a 4 ns FWHM pulsewidth. As the lens tilts, a physical optics Zemax model (without any nonlinearities or plasma effects) shows the two orthogonal foci shifting away from each other as the average intensity centroid pulls back toward the focal lens. The measured plasma fluorescence matches reasonably at the normal incidence (0° tilt) case but begins to disagree more in amplitude as the lens tilt increases. By the time the 10° case is reached, the plasma peak positions look reasonable but the amplitude of the plasma fluorescence distribution further from the lens begins to drop as the laser light is absorbed and scattered more by the early plasma. Note that the 0° case has been modeled with a Sandia-developed nonlinear optical propagation code (see Section 5) with favorable agreement as well. In comparing that data (see Figure 32) to this data (see Figure 11), note that the calculated electron density length far exceeds the length determined by the plasma fluorescence, which is in agreement with experimental data comparing the bright plasma emission length to electrical conductivity measurements [64].

USP lasers offer certain other options. In our case, with USP lasers, we have already demonstrated plasma channel length increases via the method of increasing the laser energy to the point where multiple filamentation occurs (see Section 2). Other research has extended plasmas via pulse concatenation of multiple pulses of different temporal delay and focal elements [63]. This method can be generalized to any multipulse method wherein each pulse has a different wavefront curvature. A brute force approach to this can be achieved by beam splitting, varying the wavefront curvature of one beam via a telescope, and then recombining the beams before entry into a common focal element. The difference in wavefront curvature on each beam causes each beam to

Figure 11. The effect of astigmatism upon plasma length, as illustrated by 3 different lens tilts or lens angles of incidence. Figures (a)-(c) are calculated based upon a physical optics Zemax model assuming an apodized flattop beam profile of 24 mm FWHM and a nominal 1 GW (4 J/4 ns) peak power at 532 nm. Figures (d)-(f) are the longitudinal averages for 4 measured plasma fluorescence images for the nominal 24 mm FWHM beam with a measured 0.65 GW (2.6 J average/4 ns) peak power at 532 nm. Each image is summed transverse to the laser to account for increased plasma volume prior to the averaging.

focus at a slightly different location, stitching or concatenating the individual laser plasmas into a longer overall plasma length. This method can be applied by either a nanosecond or USP laser but the plasma uniformity will be better with either ultraviolet pulses or USP pulses (based upon the ionization mechanisms). The nanosecond laser demonstration for a 4 ns pulsewidth at 532 nm is shown in Figure 12. In this test, the fixed arm (arm 1) had 0.58 ± 0.03 J, while the adjustable focus arm (arm 2) had 0.52 ± 0.02 J, such that combined beams measured 1.13 ± 0.05 J. With the common $f=75$ cm lens at 0° tilt after recombination of the beams, arm 1 was in position 2 (with the resulting plasma about 1 cm below the upper positive HV electrode) while arm 2 was adjusted to positions A, B, and C in order to create plasmas at 1, 2, and 3 cm below that of arm 1. No measureable delay was present between the arms. The combined plasmas (as shown in Figure 12) were longer but showed no significant improvement in continuity.

Another approach of the same wavefront curvature concept which can apply to either nanosecond or USP lasers is based on chromatic aberration. If multiple harmonics of a laser are to be used, the material dispersion of the lens will create different wavefront curvatures for each wavelength. These separate focal regions generate plasmas which can then concatenate together into a longer

Figure 12. Plasma Fluorescence imaging of nanosecond pulse laser-generated plasma concatenation based upon pulse wavefront curvature.

plasma. As wavelengths in the visible and infrared range tend to exhibit plasma beading and discontinuity, USP laser beam harmonics will show better performance (see Figure 13). Note that longer focal length lenses and higher harmonic orders both show larger plasma separations which may not coalesce (i.e the overall plasma uniformity and continuity is poor). The effects can be reduced by increasing the $f/\#$ of the focal system (via an input beam diameter reduction) such that the depth of focus extends, resulting in extending these distinct plasma channels to the point where they join or coalesce.

A final approach of the wavefront curvature concept which is unique to USP lasers is based on self-focusing. If multiple pulses of different energy are generated, the self-focusing of each pulse will generate different wavefront curvatures and hence different foci for each pulse. As before, these separate focal regions generate plasmas which can then concatenate together into a longer plasma. To test the concept, the USP laser pulse was split into two different energies for two different polarizations (3 mJ for s-polarized and 9 mJ for p-polarized). The orthogonally polarized pulses were recombined at the same time (zero delay). The resulting shadowgraph demonstrates the concept in Figure 14. Note that when combined, the overall length is longer than the combined individual pulses. However, the overall length is nearly identical to the single pulse case shown in the 11 mJ case in Figure 5. While this may at first seem discouraging, it actually is not since the single pulse case will break into multiple unconnected filaments at a few times the critical power. Using pulse combination methods, energy can be distributed amongst multiple pulses to avoid this problem, allowing longer continuous plasma channels in the process.

Note that time delays in these multiple pulse concatenation methods may impact switching applications. Also note that several of these methods can be used in conjunction with each other to increase the overall effect

Figure 13. USP laser-generated plasma concatenation based upon chromatic aberration. The figures have been false colored horizontally to correlate to the ionizing wavelength. (a) Shadowgraph of 1054 nm, 527 nm, and 263.5 nm USP at ~ 12 mJ and ~ 500 fs focused with an $f=7.5$ cm lens (b) Shadowgraphs at various probe delays with an $f=5$ cm lens looking at the coalescence of two plasma channels from 1054 nm and 527 nm pulses.

Figure 14. USP laser-generated plasma concatenation based upon self-focusing. The laser pulses ($\lambda=1054\text{ nm}$, $\phi=12\text{ mm}$ FWHM, $t\sim 500\text{ fs}$) are entering from the left with 3 mJ for the s-polarized pulse and 9 mJ for the p-polarized pulse, as focused by an $f=25\text{ cm}$ lens in ambient air.

3.2 Plasma Lifetime Increases

For a given laser energy budget, effective plasma lifetime can be shown to increase in two ways: multiple laser pulsing in rapid succession so as to re-ionize a volume [65–67] and multiple laser pulsing with one laser providing seed electrons for another longer pulse laser to heat via inverse Bremsstrahlung [68–70]. The latter effect usually involves a both large electron density and temperature increase, with both effects increasing the corresponding recombination time [14]. This approach will be discussed under the next subsection.

The first technique usually uses USP lasers which ionize weakly and have short lifetimes such that the subsequent pulses are not greatly disturbed by the pre-existing plasma. Having said this, the time frame between pulses must be short enough such that the electron density does not fully decay before secondary laser pulses enter the region (lest the effective lifetime not be really increased). We demonstrate this via orthogonal USP pulses generated by the setup in Figure 2. Each USP has 3 mJ energy each and has been focused with an $f=25$ cm lens. The time delay between pulses was adjustable from the simultaneous (zero-delay) case to several nanoseconds. The plasma fluorescence emission is then monitored via a UV-filtered PMT (as was used for the data in Figure 10). The data, shown in Figure 15, shows an increase in fluorescence width and amplitude as the second pulse is added. The additional energy increases the electron number density but also increases the ionized volume, correspondingly raising the overall signal due to the greater total number of recombining molecules. As the delay between pulses increases, the fluorescence width increases. At a 3.8 ns delay, we begin to see the distinct fluorescence peaks of each pulse. Note that the overall $1/e$ width of 14.9 ns exceeds the sum of the two individual widths (3.6 ns) plus the delay, implying that some other effects such as plasma heating by the second pulse may be occurring.

Figure 15. Plasma lifetime enhancement by double USP pulses with variable delay.

3.3 Plasma Density Increases

The concept of used USP lasers with nanosecond heating beams is of growing interest in the literature [68–70]. Essentially, the USP operates at a low level to provide a small seed electron distribution which can initiate laser-induced breakdown (LIB) in air caused by inverse Bremsstrahlung heating from a subsequent nanosecond-scale laser pulse. We tested the concept by making a USP laser at 1054 nm and a nanosecond laser at 532 nm collinear and cotemporal (with the latter meaning that the USP was placed in the center of the 8 ns pulse at 532 nm). The beams are then focused by a common $f=25$ cm lens. Shadowgraphy and temporal parametric fluorescence measurements were then made (see Figure 16).

The first observation is that the USP allows air breakdown at energies far below the 59 mJ LIB threshold required by the nanosecond beam alone. With USP electron seeding, the longer pulse could create LIB in air with only 30 mJ, a significant energy reduction. Note that the ignition of the LIB process has a short incubation time, as evidenced in the two different probe delays for the shadowgraphs in Figure 16b. In addition, the process increases the plasma density and temperature to allow for longer lifetimes (as seen in the plasma fluorescence PMT data in Figure 16a). To compare, the USP beam alone in this configuration has a $1/e$ plasma lifetime pulsewidth of 8.4 ns for 6 mJ and 27 ns for 9 mJ while the nanosecond pulse alone has a $1/e$ pulsewidth of 38 ns for 59 mJ (not shown in the figure). The USP plus nanosecond pulse sequence shows odd behavior in the time domain: At lower nanosecond energies, the dual pulse plasma lifetime is the same or longer than the USP alone while at higher nanosecond laser energies, the lifetime actually reduces. This may be due to a faster rise in plasma density for the higher nanosecond energy case, leading to a plasma which scatters, reflects, and refracts more of the heater beam laser light. This would in turn prevent efficient coupling of the latter portion of the nanosecond heater pulse and thus result in a cooler plasma with a shorter plasma lifetime.

In addition, we should point out that dual pulse plasma using the 532 nm nanosecond heater beam appears to show discrete beading structure in the plasma channel. In self-breakdown laser plasmas, such beading becoming more prominent for longer wavelength light of similar duration. This is due to plasma refraction and absorption issue at these wavelengths, as based upon the plasma gradients which result from the prevalent cascade ionization. One might hope that seeding a cascade ionization process with a uniform initial electron distribution (as one sees with USP MPI plasmas) could lead to a more uniform plasma channel out of the cascade process with a lower energy requirement. However, as shown in the shadowgraphs in Figure 16, beading structures are beginning to appear in the heated plasmas. This implies that USP laser-generated plasma seeding of a cascade ionization process from a nanosecond laser might not lead to the most uniform plasma distributions, which was subsequently pointed out by Polynkin *et al* [70] during the course of our research.

Figure 16. Dual pulse USP plus nanosecond laser plasma generation. (a) Plasma lifetime enhancement by USP+ns laser, as detected by plasma fluorescence PMT method. (b) Shadowgraphy of resulting laser plasma for different cases, with pulses entering from the left. Probe beam delays $\Delta\tau$ are with respect to the time of peak ionization (zero-delay between the ionizing and USP lasers).

4 Laser-Triggered Discharges

4.1 The Laser System

Based upon the size of the high voltage setup, this setup could not be located near the previously used laser system. As such, two other laser sources in a larger work area were considered for the laser triggered discharge work: A developmental optical parametric chirped pulse amplification (OPCPA) USP laser at 1054 nm wavelength and a pulsewidth of 100 fs, and its pump laser source, a 5.5 J, 532 nm laser with a pulsewidth of 4 ns FWHM. Due to time constraints, the work on the short pulse system could not be completed. However, a full system design was performed (and is to be published) and all hardware (including the temporal compressor) are in place. In the following section, we will discuss the performance of the long pulse pump laser source (used in the discharge work) and the current status of the short pulse system.

The OPCPA pump laser

This commercial pump laser is specifically designed for pumping of an OPCPA system. It runs at 2 Hz and generates 5.5 J per pulse at 532 nm, with a 4 ns flattop temporal pulse shape. The initial pulse is sliced out from a 40 mW, 1064 nm single longitudinal wave fiber laser by a Mach-Zehnder intensity modulator driven by an arbitrary waveform generator. The pulse is then amplified to 9.5 J at 1064 nm by a 5 mm diameter preamplifier head, a regenerative amplifier, two 12 mm diameter amplifier heads, two 19 mm diameter amplifier heads, and two 25 mm diameter amplifier heads, and then is finally frequency doubled to yield 5.5 J per pulse at 532 nm. Figure 17a shows a representative pump beam near field at the output surface of the doubling crystal. One can see that a 5th order Gaussian function (of the form $(e^{-x^2})^n$, where n is the super Gaussian coefficient) is a good fit for the horizontal lineout (see Fig. 17b). Correspondingly, the temporal trace (not shown here) shows excellent agreement with a 4th order Gaussian fit ($R^2 = 0.998$).

This laser source was used at an energy of 2.5-2.7 J for the laser triggered discharge work discussed below. Furthermore, the 532 nm output was frequency doubled to 266 nm for a small set of discharge experiments at UV wavelengths. Due to the damage threshold of our optics and the obtainable conversion efficiency, UV energies were used around 600 mJ.

The OPCPA system

The first OPCPA stage consists of two 25 mm long walk-off compensated LBO crystals that are pumped with 100 mJ of 532 nm light. Stage two has two 13 mm long walk-off corrected LBO crystals pumped at 150 mJ of green light. Stages three and four consist of 6 mm and 9 mm thick BBO crystals with pump energies of 400 mJ and 4 J each. Due to the high pump beam energies, careful ghost analysis had to be performed in order to avoid damage on pump beam optics.

Figure 17. OPCPA pump laser performance: (a) Pump laser near field at the plane of the doubling crystal. (b) Horizontal line-out was fitted with a 5th order Gaussian function and shows a FWHM of 25 mm.

The pump system is designed in such a way that the initial beam near field at the plane of the pump laser doubling crystal is spatially filtered and relay imaged onto the crystal face of all four OPA stages to ensure good spatial beam quality. Pump and signal beam sizes have been well matched at the OPA crystal planes and pump intensities have been kept below 800 MW/cm^2 in order to avoid laser damage and parametric fluorescence which will ensure high temporal contrast. In addition, great care was taken to verify that signal and pump beam paths have exactly the same time of flight, again mitigating poor temporal contrast. Furthermore, non-collinear pumping angles are $< 2^\circ$ but $> 0^\circ$ in order to spatially separate signal (1046 nm - 1062 nm) and idler (1066 nm - 1082 nm) beams after amplification while maintaining the high gain of near collinearity. Spatial separation of the idler is required since no dichroic beamsplitters exist that have the necessary extinction and/or rapid transition over only 4 nm bandwidth while having a damage threshold that can withstand our fluences. Lastly, spatial relay filters have been employed for the signal beam between each OPA stage in order to improve beam near field quality and mitigate any residual parametric fluorescence. Figure 18 shows a layout of the overall four stage OPCPA system, including all four OPA pump relay vacuum telescopes.

Modeling for all OPA stages was performed using a modified version of SNLO [71]. The code was expanded to allow up to 10th order Gaussian temporal and spatial input beam shapes as well as an option to parametrically scan through several nonlinear crystal thicknesses. Based on the modeling, one predicts a 4 ns FWHM temporally chirped beam with a spectral bandwidth of 16 nm and an output energy of up to 1.5 J.

Temporal Compressor

For use of a short pulse laser in laser triggered discharge work, one has to consider the accumulation of nonlinear phase and subsequent beam break-up as the short pulse traverses through air

Figure 18. Overview of the new four stage OPCPA system as it integrates into the existing short pulse oscillator, stretcher setup. The pump laser, housed on a $4' \times 8'$ table, is not shown here. Master oscillator and temporal stretcher are located on a $5' \times 12'$ table with remaining table space for diagnostics. The actual OPA stages are placed on the adjacent $5' \times 12'$ optics table with the output leading to the $4' \times 8'$ temporal compressor table (not shown here).

toward the high voltage testbed. This will set a limit for the maximum laser intensity that can safely be used. The so called “B-Integral” is a measure of nonlinear phase and its value should be kept below $\pi/2$ for safe operation.

$$B = \frac{2\pi}{\lambda} n_2 \int_0^L I(l) dl \approx \frac{2\pi}{\lambda} n_2 IL \leq \frac{\pi}{2}, \quad (20)$$

where $\lambda = 1054 \text{ nm}$ is the central wavelength of the laser, $n_2 = 3 \times 10^{-19} \text{ W/cm}^2$ is the nonlinear index of air, $I \leq 10^{11} \text{ W/cm}^2$ is the maximum safe intensity before beam break-up and damage occurs, and $L = 8 \text{ m}$ is resulting distance at which the B-Integral exceeds $\pi/2$. For the grating in question (here $42 \times 21 \text{ cm}^2$ gold at 1480 l/mm), one has to find a balance between maximum beam size (in order to support maximum power) and maximum spectral transmission (in order to generate this power through temporal compression). Figure 19 shows such a configuration in which we have used a beam expander to generate a 73 mm diameter beam which can clear 14 nm (1046 nm to 1060 nm) spectral bandwidth through the compressor. At this beam area of 42 cm^2 , an output power of 4.2 TW ($480 \text{ mJ}/115 \text{ fs}$) can be supported assuming 100 GW/cm^2 as a conservative limit for filamentation in the air compressor. This can easily be generated with the available signal energy of 1.5 J and the bandwidth limited temporal compression of $\leq 150 \text{ fs}$. Note that commercial systems push the intensity limit for air grating compressors to $> 200 \text{ GW/cm}^2$, which could yield up to 10 TW operation.

Figure 19. Zemax model of the temporal compressor layout. The minimum transmitted wavelength of 1046 nm is depicted in blue, the central wavelength of 1053 nm is green, and the maximum transmitted wavelength of 1060 nm is shown in red.

Current Status

All hardware has been assembled, all optics are in place, and all vacuum relay telescopes have been leak checked and are operating below 150 mTorr. The signal and pump beams are aligned through all four OPA stages and spatio-temporal overlap between pump and signal beam has been achieved at OPA stage 1. Pumping of OPA stage 1 is currently underway and initial gain has been observed. The temporal compressor has been built and was carefully aligned using a 1054 nm continuous wave alignment beam. All diagnostic cameras and energy meters are in place and their respective data acquisition software is functional.

To summarize, a 2 Hz OPCPA system has been designed that is capable of 4.2 TW (480 mJ at 115 fs) beam power with no need for a vacuum grating compressor. Every stage in the system has been designed to be insensitive to the input energy (except for stage one). All simulations represent an idealized case and one should expect less performance than is modeled. This has been taken into account when choosing the input energies for each stage. The output spectrum from each stage has been modeled and shows no unusual behavior. System activation is underway with the first OPA stage being pumped right now.

4.2 The High Voltage System

The high voltage system design quickly became highly constrained by safety and lab environment issues. The net issues were as follows:

- *Electrode Choice:* The switch should be like other laser-triggered sections at Sandia.

Result: Unused electrodes from Z20 system were obtained. These reflect the design in Figure 1. However, a slight modification was made to the electrode insert on the side opposite the entrance electrode. In SF₆ LTGS, the argument is made that little beam energy reaches the opposite side such that a blank-off insert is used. Our switch anticipated the use of air (see below) which is not so absorptive. To avoid switch ablation issue impacting the performance, we had a small exit hole drilled in this blank-off insert.

- *Cleanroom:* The switch design should be compatible with a cleanroom work environment since the lasers reside there.

Result: We will use an oil-free design to avoid organic vapors depositing films on the optical systems. SF₆ should be avoided to minimize additional hazards and complexity. This drives the design to larger high voltage stand-off distances for air and to slightly lower voltages.

- *Electrical Safety:* Sandia electrical safety requires either very large (conservative) stand-off distances for exposed high voltage or requires the high voltage to be enclosed.

Result 1: We chose to reduce the energy to less than 10J to reduce safety issues. The resulting HV decay time is less than 1 s, further alleviating some of the typical safety concerns.

Result 2: We chose to enclose the switch for safety, housing it in a roughly 3 X 4 X 7 ft³ surplus Faraday shield enclosure. Additionally, engineered interlocks were required to avoid easy access to the exposed high voltage interior. As such, the Faraday enclosure has a magnetic latch on the door with a timed interlock feeding back to the commercial HV power supply. When entry into the Faraday enclosure is necessary, the power supply is turned down and the power is turned off, allowing the small capacitor bank to quickly discharge to ground. Using an access control panel with timed relay in conjunction with the magnetic door lock, a code is entered to permit entry into the enclosure after 30s, allowing a full bleed down of any residual stored energy in the interim. When the door is opened, a grounding stick can connect to the HV electrode, redundantly verifying safety. Note that the HV power supply is interlocked whenever the Faraday enclosure door is open.

- *Electrode Gap:* The switch design should have similar gaps to that of Z LTGS (or longer).

Result: We will have an adjustable gap separation from a few centimeters to 30 cm. The standard minimum gap is 5.1 cm as compared to 4.6 cm on Z LTGS.

- *Ambient air:* Consider ambient air gap rather than pressurized air or SF₆ to mitigate complexity.

Result: This simplification allows us to eliminate pressure safety hazards associated with establishing a pressurized cell while allowing the basic physics to be explored. Elimination of brittle barrier windows and lenses in close proximity to the switch volume reduces laser damage and debris deposition issues. Finally, the lack of a pressurized vessel cylinder facilitates diagnostics access to the switch volume. Note that, at 5390 ft which is the lab elevation, the atmospheric pressure is 83.1 kPa.

- *E/p Levels:* The switch should have lower switch currents and energies than Z LTGS for safety while having similar E/p values for the switch dynamics in the trigger section.

Result: Glassman dual polarity option 150 kV DC HV power supplies were purchased. As these are oil-free and were designed for air at sea level, the applied supply voltage should not really exceed about 80% of this value or 120 kV, based upon scaling to local ambient pressures. For the voltage divider used in the switch, 86.9% of the applied voltage reaches the biased electrode in the switch. This yields up to 104 kV at the top electrode. Measurements made at 1 bar (100 kPa) at 20° on the breakdown voltage $V_{Breakdown}$ in kV in air in a uniform electric field [72] follow the relation:

$$V_{Breakdown} = 6.72\sqrt{pd} + 24.36(pd), \quad (21)$$

for pressure p in bar and electrode gap d in cm. At 1 bar and 1 cm, the voltage is thus 31.08 kV (for an electric field strength of 31.08 kV/cm and a breakdown E/p of 31.08 kV/(cm·bar)). At our elevation, for 0.831 bar and 1 cm, the voltage from Eq. 21 is thus 26.37 kV (for an electric field strength of 26.37 kV/cm and a breakdown E/p of 31.7 kV/(cm·bar)). We can achieve 104 kV over 5.1 cm which is 20.4 kV/cm or 77% of the calculated air self-breakdown condition. This is similar to the percentage of the self-breakdown E/p used on the Z LTGS.

The end result of these constraints is a fully enclosed, interlocked oil and SF₆-free system with a 100 kV DC air gap, adjustable gap separation up to 30 cm, and easy diagnostic access. The system is depicted in Figure 20. The Glassman HV power supply is commonly grounded to the Faraday cage and the Facility ground. The protection resistor is a 60 MΩ network composed of 30 Ohmite Super MOX resistors (5" long each, 50 MΩ, part MOX96025005FVE). Five of each resistor are bundled in parallel, with 6 such bundles placed in series (for a nominal 32" length). Each of the 6 bundle levels is rated to 72 kV, for a total stand-off voltage rating of 432 kV. Since each resistor handles 16 W, the 30 resistor array handles up to 480 W. The worst case short circuit would be 150 kV/60 MΩ=2.5 mA, for a total Ohmic power of 375 W (which is easily within the array rating). The charging capacitor array consists of 4 doorknob-style capacitors in series (TDK UHV-11A, 1 nF, 50 kV DC rating). Charging resistors (Ohmite Super MOX, 5" long each, 100 MΩ, part MOX96021006FVE) establish a voltage divider and set reliable voltages at each capacitor. Based upon this, each capacitor charges to 21.7% of the power supply voltage, leading to a maximum capacitor voltage of 26.1 kV for 120 kV at the power supply. Since $E = (C \cdot V^2)/2$, each capacitor only stores 0.34 J, totalling 1.36 J for the 4 capacitor array. This is significantly below the 10 J safety value for which Sandia requires higher rigor safety precautions and permissions. The load resistor is a ceramic-style resistor from US Resistor (15" long X 1" ϕ at 300 Ω, rated for 150 kV, 140 W continuous, and 56 kJ). Above the load resistor is a current viewing resistor (CVR) used as

a diagnostic to monitor the current pulse out. The CVR is from T&M Research (A-2-02 in-line CVR, $0.01\ \Omega$ with 1ns rise time and a 16J maximum rating).

In addition to the CVR, primary diagnostics include a 16 bit CCD (which side-on images the plasma and/or discharge through the Faraday cage view port), a photodiode measuring the time-of-arrival of the ionizing laser pulse into the trigger gap, and a remote photodiode collecting plasma light emission through the view port. A timing fiducial to the first photodiode is used to trigger a Tektronix 3.5GHz, 40GS/s oscilloscope looking at all the three temporal signals (the CVR and the two photodiodes). Runtimes are defined by the time from the turn up in the fiducial to the time when the CVR similarly turns up from its baseline. Jitters are taken as the RMS deviation in 20 sample runtimes.

Figure 20. High Voltage switch stand. (a) Schematic. Note that the green paths represent ground. (b) Photograph.

4.3 Nanosecond Pulse Laser-Triggered Discharges

532 nm studies

The HV switch was initially used with the 532 nm OPCPA pump source at 2.63 ± 0.09 J to examine the effects of plasma placement and length upon switch performance (5.08 cm gap at up to 104 kV) with ambient air. At these energies, brightly luminous plasmas are formed with a few ten's of nanoseconds lifetime, meaning that the air is fully singly ionized (i.e. the electron number density equals the normal number density of air molecules) and that the lifetime is of the order of (or longer than) the runtime of the switch. Thus, the only variables to switch performance should be by plasma length, distribution, and placement. To investigate this, we did parametric scans of the focal lens position (which is a fused silica $f=75$ cm plano-convex lens), applied high voltage, and lens tilt. Representative images of the side-on plasma fluorescence before and during discharge are shown in Figure 21. Note that the low capacitor values in the switch charging circuit make the discharge channels less luminous than the laser-generated plasma which initiates the discharge. The discharge images also show very straight guidance where the laser plasma is not luminous, again pointing to lower electron densities which are electrically conductive but not optically emissive. Erratic unguided discharges (as in the far right image of Figure 21b) tended to be near the HV threshold for laser-triggered switching, with the near threshold condition correspondingly having long temporal runtimes and high jitters. Larger gaps of 10.2 cm were successfully triggered but did show this erratic behavior due to the lower electric fields (for the available HV) being too close to threshold conditions. As such, the emphasis remained on the 5 cm gap studies, which correlate to the Z LTGS better anyway.

The basic summary of the parametric scans are presented in Figure 22. Regardless of lens tilt and applied HV, the switch has minimal runtimes when the laser plasma is slightly toward the positive HV electrode from the gap center. Also, as expected, the highest voltages have the shortest runtimes and lowest jitters for all lens positions and cases. As such, we reconsider this data for the highest voltage (104 kV) only in Figure 23. Looking at the data in Figure 23, we see that:

1. The runtime reaches a low of 6.89 ns for the 0° case, with this case being marginally better than the 5° case (and significantly better than the 10° case) at almost all plasma positions within the switch gap.
2. The jitter reaches a low of 350 ps for the 5° case, with this case being better than the 0° case (and significantly better than the 10° case) at all plasma positions within the switch gap. The lowest jitter for the 0° case is 600 ps, underscoring that the 5° case has almost half (or more) of the jitter of the 0° case at all positions. To compare, the 6.1 MV LTGS system used on Z shows around ~ 1 ns of RMS jitter for the trigger section (at only around 1 MV in 4 to 6 bar of SF_6) of the gap when under 20 mJ or 266 nm laser light [3]. Runtimes there are typically in the 10 to 20 ns range.
3. The HV laser-triggered breakdown threshold reaches a low of 27.4 ± 1.3 kV (for a minimum electric field of 5.4 kV/cm) for the 10° case. Note that the plasma centroid shifts down significantly with respect to the lens position, such that the two minimum voltage lens positions

Figure 21. Plasma fluorescence imaging for laser plasmas and triggered discharges. Position 1 for the lens placed a very low intensity laser spark just at the entrance to the top positive HV electrode. At higher intensities, the laser plasma pulled back into the switch gap. Subsequent lens positions are spaced by 1 cm. (a) Laser plasmas at a few representative positions. (b) Corresponding laser-triggered discharges.

for the 10° case correspond to plasmas in the middle of the gap rather than near the HV electrode (see Figure 21a).

The point is that, while jitter, runtime, and HV breakdown threshold generally correlate, there is no absolute relation. Generally as the high voltage lowers, the switch runtime increases and the jitter increases, leading one to assume that the jitter correlates to the runtime. To clarify the apparent discrepancy with the data in Figure 23, we hypothesize that things correlate back to the plasma distribution:

1. Referring back to Figure 11, we see that the peak plasma fluorescence is fairly consistent for all three lens tilts, being marginally higher for the 0° case. We assume that, since the 0° case shows a slightly higher peak laser intensity in the Zemax model, the absolute electron number and volume are slightly larger for this case. If there is a high electron number density or even better if there are more electrons in a larger diameter channel, the plasma conductance is higher, which translates into a faster streamer velocity [8] and a shorter runtime.
2. Similarly, one can see from Figure 11 that the main plasma distribution is slightly wider at the base for the 5° case than either of the other two cases, with the 10° case having the narrowest main peak. A longer continuous electron distribution means that the electrical discharge has less length over which it must stochastically develop into streamer, thus reducing the jitter.
3. When considering the overall plasma distribution to outlying low level plasmas, the 10° case is clearly longer, although it has some gaps between visibly emitting plasma regions. While a discontinuous laser plasma will result in more switch jitter, it allows plasma island hopping for a longer straight discharge length. A longer plasma channel in turn means that the external electric field does not have to be as high to allow electrons to bridge the gap.

To underscore this, we used the concatenation method in Figure 12 in the switch. With arm 1 at position 2 (such that the laser plasma is roughly 1 cm below the upper positive HV electrode) and arm 2 adjusted to positions A, B, and C (with corresponding laser plasma positions 1, 2, and 3 cm below the plasma from arm 1), the associated runtimes and jitters for positions A, B, and C are 10.44 ± 0.31 ns, 10.13 ± 0.22 ns, and 9.54 ± 0.39 ns. As one can see, for the lower average laser energy of 1.13 J in the combined arms 1 and 2 (as compared to the single pulse 532 nm case at 2.63 J average), the runtime is marginally longer than the best single pulse case (see Figure 23a) but the minimum dual pulse jitter is in fact the best observed at 220 ps (for the position 2 + position B case). The dual pulse (for the position 2 + position C case) laser-triggered HV breakdown threshold measured 26.1 kV (for an electric field of 5.1 kV/cm), which is still better than the minimum single pulse threshold at less than half of the laser energy. From the laser fluorescence images in Figure 12, the dual pulse plasma is longer albeit more discontinuous than any of the single pulse plasma cases. Based on our hypothesis above, such a laser-generated plasma will lead to switching with a lower jitter and lower HV threshold, as observed.

Obviously, the best scenario is a long continuous dense plasma for HV switching. But in lieu of this capability, the work shown points to tailored switch performance via tailored laser-generated plasma channels.

Figure 22. Air switch runtimes for 532 nm at various lens tilts and voltages across the gap.

Figure 23. Air switch performance for 532 nm at various lens tilts for 104 kV across the gap.

266 nm studies

The 532 nm laser source was converted to 266 nm via a fourth harmonic crystal (KDP) designed for high efficiency. Of the 2.6 J at 532 nm, up to 0.9 J at 266 nm was generated but the energy quickly degraded. Over the course of 10 minutes, the energy would drop to around 0.4 J and the conversion crystal would be adjusted for optimal performance again. We feel that the heat load from the laser beam was shifting the crystal tuning curve and that a heater may be necessary on the crystal for stable operation. However, we did perform some HV testing with the erratic laser pulse, which would probably have measured 0.6 ± 0.1 J over the experimental time frame. The plasma position (for 0° lens tilt) in the switch gap shifted significantly from the 532 nm case due to chromatic aberration, so position correlation to the previous work would have to be done from the plasma fluorescence images. A sample laser plasma position in the middle of the gap is shown in Figure 24. The visible plasmas look longer and more continuous than their 532 nm peers (at a fraction of the laser energy) but do show the impact of laser energy fluctuation upon the plasma reproducibility. The corresponding switch operation showed very straight guided discharges with a runtime of 9.7 ± 0.32 ns. Differences in runtime as compared to the single pulse 532 nm case could be due to the lower laser energy but could also be due to the tighter UV focus generating a narrow and less conductive plasma channel (especially at the channel ends). The low jitter is consistent with the high plasma channel continuity.

Figure 24. Plasma fluorescence imaging for UV laser plasmas at the same lens position (left two images). Variances in the two images are due to laser energy fluctuations. The far right image shows the corresponding triggered discharge.

5 Nonlinear Electromagnetic Propagation Model

This section describes the laser ionization and propagation modeling effort associated with the LDRD. Over the course of the period of performance we developed a complete scaled model including a wide variety of physical effects, a numerical scheme for implementing a viable simulation of the model and Matlab codes implementing the numerical physics model. We have also developed a fully distributed (via MPI) C++ version of the code running on desktop workstations as well as super-computers such as Sandia's RedSky cluster and have nearly completed an adaptive mesh refinement version of the code capable of simulating a completely new class of propagation problems. Each of these codes has been fully validated against numerous papers from the literature and available theoretical physics models. We are not aware of any codes with results available in literature offering the full range of physical effects available in this newly developed simulation code. Simulations of models of this type involve a host of numerical methods. Two of the features our code possesses allow us to study and understand some of the important effects of this aspect of the simulation. Our linear propagator allows the construction of differential operators with arbitrary order giving us the ability to understand the effect of varying this aspect of the numerical model. We have discovered that broader support for the differential operators improves the fidelity of the numerical model only up to a certain point after which increasing the order actually degrades the performance. Also, we can vary the type of linear system solver used in the linear propagator. In doing so we have discovered that introduction of a matrix preconditioner can affect the computed pulse shapes meaningfully. This implies that care must be given to understand the numerical as well as the mathematical physics in the simulation before drawing conclusions about physical pulse propagation. We also learned that the numerical model of the Raman effect must be of sufficiently high order in order to correctly capture the physics.

We begin with a development of a properly scaled mathematical physics model that includes linear diffraction, group velocity dispersion, and third order dispersion, as well as instantaneous and delayed (Raman) Kerr, linear and non-linear shock, and ionization with avalanche, multi-photon ionization (MPI), multi-photon absorption (MPA) and second order recombination. Also, available are linear and non-linear shock components. Fundamental understanding of the model equations at a basic level is crucial to successful addition of correctly modeled and scaled physical effects. The development beginning with Maxwell's equations for electromagnetic wave propagation is given in section 5.1. In section 5.1 we briefly review the relationship between Maxwell's equations in free-space, introduce models of linear and non-linear refractive index for material interaction and arrive at a full vector description of wave-propagation in non-linear materials with equations (68) and (69). Section 5.1 gives the scalar model (which we use), defines some key parameters of the model and specifies the method for factoring out the fast phase variation or carrier frequency to leave a computationally tractable envelope model. In section 5.1 we discuss the transformation of the dispersion operator from a full convolutional description to a differential approach commonly used in such models. This transform introduces the concepts of group velocity and third order dispersion as approximations to the full model leaving us with a simpler equation for pulse propagation. By introducing a temporally retarded coordinate system traveling at the group velocity of the pulse in section 5.1 we further simplify the requirements of the simulation (the computational grid travels with the pulse) without approximation. Section 5.1 gives the model

description after assumptions about the slowly evolving nature of the pulse envelope. Normalization of the equations in section 5.1 by quantities related to the pulse temporal and spacial extents in a fashion similar to that found in Zozulya [73] gives a scaled code for numerical simulation. Our model is different from Zozulya's in that we include ionization. The ionization model described in section 5.1 gives the ordinary differential equation for the time evolution of the electron density needed for the descriptions of pulse-plasma interactions. As is standard we use the Drude model to relate frequency, collision time and ionization avalanche rate coefficient. However, we have discovered in practice that this model is insufficient for some measured values of the ionization rate giving aphysical (imaginary) collision times. Hence we allow separate specification of these two parameters. We have seen no discussion of this problem in the modeling literature. Finally in section 5.1 we describe the complete simulation model in summary fashion with all the necessary scaling factors, equations, and physical simulation parameters necessary to the model.

After establishing the complete model in section 5.1 we describe briefly the numerical framework for the simulation in section 5.2. Comparison of the model to papers by Mlejnek [74] (for instantaneous Kerr and ionization) and Zozulya [73] (for Raman and shock) in section 5.3 shows very good agreement. Also, in section 5.3 we compare against some available theoretical results such as diffraction and dispersion lengths and non-linear critical power Kerr-diffraction balancing with excellent agreement. In section 5.3 we show a comparison of the model data with an experiment performed in the laboratory here at Sandia.

5.1 Mathematical Physics

In this section we discuss the relationship of the mathematical model to the basic physics relationships of wave propagation.

From Maxwell's Equations to the Wave Equation

We begin with Maxwell's equations in SI units.

$$\nabla \times \mathcal{E} = -\partial_t \mathcal{B} \quad \text{Faraday's Law} \quad (22)$$

$$\nabla \times \mathcal{H} = \mathcal{J} + \partial_t \mathcal{D} \quad \text{Ampere's Law} \quad (23)$$

$$\nabla \cdot \mathcal{D} = \rho \quad \text{Electric Gauss' Law} \quad (24)$$

$$\nabla \cdot \mathcal{B} = 0 \quad \text{Magnetic Gauss' Law} \quad (25)$$

where

$$\mathcal{E} = \text{Electric field strength (volts/meter)} \quad (26)$$

$$\mathcal{H} = \text{Magnetic field strength (amperes/meter)} \quad (27)$$

$$\mathcal{D} = \text{Electric flux density (coulombs/square meter)} \quad (28)$$

$$\mathcal{B} = \text{Magnetic flux density (webers/square meter)} \quad (29)$$

$$\mathcal{J} = \text{Current density (amperes/square meter)} \quad (30)$$

$$\rho = \text{Electric charge density (coulombs/cubic meter)} \quad (31)$$

are generally functions of time and space. In order to solve the equations for a given circumstance we need additional equations relating the field quantities. These constitutive relations specify the relationships between field quantities that are functions of a specific material. In free space we have

$$\mathcal{D} = \epsilon_o \mathcal{E} \quad (32)$$

$$\mathcal{B} = \mu_o \mathcal{H} \quad (33)$$

where

$$\epsilon_o = 8.854 \times 10^{-12} \text{ F/m} \quad (34)$$

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m} \quad (35)$$

It should be noted at this point that some of the most useful papers on non-linear propagation (see for example Brabec and Krausz [75]) use Gaussian units.¹ This must be taken into account when comparing results and determining the values of various parameters used for computation. A complete discussion of these differences may be found in Cohen [76].

In order to develop the constitutive relationships for a dispersive material we general move to the frequency domain

¹In the gaussian unit system the field quantities are related via

$$\nabla \times \mathcal{E} = -\frac{1}{c} \partial_t \mathcal{B} \quad \text{Faraday's Law} \quad (36)$$

$$\nabla \times \mathcal{H} = \frac{4\pi}{c} \mathcal{J} + \frac{1}{c} \partial_t \mathcal{D} \quad \text{Ampere's Law} \quad (37)$$

$$\nabla \cdot \mathcal{D} = 4\pi \rho \quad \text{Electric Gauss' Law} \quad (38)$$

$$\nabla \cdot \mathcal{B} = 0 \quad \text{Magnetic Gauss' Law} \quad (39)$$

with correspondingly different units and free-space constitutive relationships of

$$\mathcal{D} = \mathcal{E} \quad (40)$$

$$\mathcal{B} = \mathcal{H} \quad (41)$$

Derivation of the Wave Equation

Several assumptions are made to be able to simplify this derivation. The first is that the propagation media is nonmagnetic (see Eq. (33)) The optical wave equation is then derived by taking the curl of the Faraday's Law of Induction equation (see Eq. (22)).

$$\nabla \times \nabla \times \mathcal{E} = -\nabla \times \partial_t \mathcal{B} \quad (42)$$

By interchanging the order of the space and time derivatives on the right-hand side we get

$$\nabla \times \nabla \times \mathcal{E} = -\partial_t (\nabla \times \mathcal{B}) \quad (43)$$

The relationship between $\nabla \times \mathcal{B}$ and $\mathcal{D}$ is then derived using Maxwell's equations and the assumptions, giving

$$\nabla \times \mathcal{H} = -\partial_t \mathcal{D} + \mathcal{J} \quad (44)$$

$$\nabla \times \mathcal{B} = \mu_0 \partial_t \mathcal{D} + \mu_0 \mathcal{J} \quad (45)$$

This then can be manipulated into the equations

$$\nabla \times \nabla \times \mathcal{E} = -\mu_0 \partial_t^2 \mathcal{D} - \mu_0 \partial_t \mathcal{J} \quad (46)$$

$$\nabla \times \nabla \times \mathcal{E} + \mu_0 \partial_t^2 \mathcal{D} + \mu_0 \partial_t \mathcal{J} = 0 \quad (47)$$

The material is allowed to be nonlinear

$$\mathcal{D} = \epsilon_0 \mathcal{E} + \mathcal{P} \quad (48)$$

where $\mathcal{P}$ is the polarization density vector. If the polarization $\mathcal{P}$ is broken down into linear and nonlinear segments then

$$\mathcal{P} = \mathcal{P}_{\mathcal{L}} + \mathcal{P}_{\mathcal{NL}} \quad (49)$$

$$\mathcal{P}_{\mathcal{L}} = \chi^{(1)} \epsilon_0 \mathcal{E} \text{ (Where } \chi^{(1)} \text{ is the linear susceptibility)} \quad (50)$$

$$n^2 = 1 + \chi^{(1)} \text{ (Where } n \text{ is the linear index of refraction)} \quad (51)$$

$$\epsilon = (1 + \chi^{(1)}) \epsilon_0 \text{ (Where } \epsilon \text{ is the permittivity)} \quad (52)$$

$$(53)$$

This leads to the nonlinear form of the electric flux density of

$$\mathcal{D} = \epsilon_0 \mathcal{E} + \mathcal{P} = \epsilon_0 \mathcal{E} + \chi^{(1)} \epsilon_0 \mathcal{E} + \mathcal{P}_{\mathcal{NL}} = \epsilon_0 (1 + \chi^{(1)}) \mathcal{E} + \mathcal{P}_{\mathcal{NL}} = \epsilon \mathcal{E} + \mathcal{P}_{\mathcal{NL}} \quad (54)$$

Looking at the linear segment of the nonlinear form of the electric flux density equation it shows

$$\mathcal{D}_{\mathcal{L}} = \epsilon \mathcal{E} \quad (55)$$

This is the form of the equation for the case of an isotropic material (material that is uniform in all directions. If this assumption is not made then ϵ is a dielectric tensor and the equation is written as

$$\mathcal{D}_{\mathcal{L}} = \epsilon \cdot \mathcal{E} \quad (56)$$

For this derivation the isotropic material assumption is used. For a dispersive medium ϵ is frequency dependent. This leads to the equation

$$\mathcal{D}_{\mathcal{L}}(\omega) = \epsilon(\omega)\mathcal{E}(\omega) \quad (57)$$

where

$$\mathcal{D}_{\mathcal{L}}(t) \xrightarrow{\mathcal{F}} \mathcal{D}_{\mathcal{L}}(\omega) \quad \text{and} \quad \mathcal{E}(t) \xrightarrow{\mathcal{F}} \mathcal{E}(\omega) \quad (58)$$

Converting this to the time domain using the inverse Fourier transform gives

$$\mathcal{D}_{\mathcal{L}}(t) = \int_{-\infty}^t \epsilon(t-t')E(t')dt' \quad (59)$$

Using the nonlinear material relationship and replacing $\mathcal{D}$ with $\mathcal{E}$ and $\mathcal{P}$ gives:

$$\nabla \times \nabla \times \mathcal{E} + \mu_0 \partial_t^2 \left(\int_{-\infty}^t \epsilon(t-t')E(t')dt' + \mathcal{P}_{\mathcal{NL}} \right) + \mu_0 \partial_t \mathcal{J} = 0 \quad (60)$$

By replacing μ_0 by $\frac{1}{\epsilon_0 c^2}$ and using the nonlinear material relationship obtained by replacing $\mathcal{D}$ with $\mathcal{E}$ and $\mathcal{P}$ we may write

$$\nabla \times \nabla \times \mathcal{E} + \frac{1}{\epsilon_0 c^2} \partial_t^2 \left(\int_{-\infty}^t \epsilon(t-t')E(t')dt' + \mathcal{P}_{\mathcal{NL}} \right) + \frac{1}{\epsilon_0 c^2} \partial_t \mathcal{J} = 0 \quad (61)$$

This is the most general form of the wave equation for nonlinear optics. There are some cases where this can be simplified. One that is used for this derivation is using the identity

$$\nabla \times \nabla \times \mathcal{E} = \nabla(\nabla \cdot \mathcal{E}) - \nabla^2 \mathcal{E} \quad (62)$$

With no other assumptions the wave equation becomes:

$$-\nabla(\nabla \cdot \mathcal{E}) + \nabla^2 \mathcal{E} - \frac{1}{\epsilon_0 c^2} \partial_t^2 \int_{-\infty}^t \epsilon(t-t')E(t')dt' = \frac{1}{\epsilon_0 c^2} \partial_t^2 \mathcal{P}_{\mathcal{NL}} + \frac{1}{\epsilon_0 c^2} \partial_t \mathcal{J} \quad (63)$$

Given that the relative permittivity is:

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \quad (64)$$

the equation can also be written as:

$$-\nabla(\nabla \cdot \mathcal{E}) + \nabla^2 \mathcal{E} - \frac{1}{c^2} \partial_t^2 \int_{-\infty}^t \epsilon_r(t-t')E(t')dt' = \frac{1}{\epsilon_0 c^2} \partial_t^2 \mathcal{P}_{\mathcal{NL}} + \frac{1}{\epsilon_0 c^2} \partial_t \mathcal{J} \quad (65)$$

or in terms of the refractive index as:

$$-\nabla(\nabla \cdot \mathcal{E}) + \nabla^2 \mathcal{E} - \frac{1}{c^2} \partial_t^2 \int_{-\infty}^t n^2(t-t')\mathcal{E}(t')dt' = \frac{1}{\epsilon_0 c^2} \partial_t^2 \mathcal{P}_{\mathcal{NL}} + \frac{1}{\epsilon_0 c^2} \partial_t \mathcal{J} \quad (66)$$

The majority of papers and books assume that $\nabla(\nabla \cdot \mathcal{E}) \approx 0$. This assumption is sometimes referred to as the slowly varying amplitude approximation. Making it gives:

$$\nabla^2 \mathcal{E} - \frac{1}{c^2} \partial_t^2 \int_{-\infty}^t \epsilon_r(t-t')\mathcal{E}(t')dt' = \frac{1}{\epsilon_0 c^2} \partial_t^2 \mathcal{P}_{\mathcal{NL}} + \frac{1}{\epsilon_0 c^2} \partial_t \mathcal{J} \quad (67)$$

Substituting $(\partial_z^2 + \nabla_\perp^2)$ in for ∇^2 gives:

$$(\partial_z^2 + \nabla_\perp^2)\mathcal{E} - \frac{1}{c^2}\partial_t^2 \int_{-\infty}^t \epsilon_r(t-t')\mathcal{E}(t')dt' = \frac{1}{\epsilon_0 c^2}\partial_t^2 \mathcal{P}_{NL} + \frac{1}{\epsilon_0 c^2}\partial_t \mathcal{J} \quad (68)$$

If ionization is not included ($\mathcal{J} = 0$) then this is the starting equation for formulations like that in Brabec (his formulation uses Gaussian units instead of SI units) [75].

The plasma current density associated with the ionized gas is:

$$\partial_t \mathcal{J} = \frac{e^2}{m}\rho\mathcal{E} - \epsilon_0 c \partial_t \frac{\sum_n \rho_n W_n K_n \hbar \omega_o}{|E|^2} \mathcal{E} \quad (69)$$

where quantities e and m denote electron charge in Coulombs and mass in kilograms, ρ is the free-electron number density (units of inverse cubic length), and n is the number of gas species.

Scalar Model

Here we develop the nonlinear envelope equation found in [75]. It is a scalar model (assumes input polarization is not significantly altered during propagation). We begin with the following wave equation for a dispersive material (SI units).

$$(\partial_z^2 + \nabla_\perp^2)E(\mathbf{r}, t') - \frac{1}{c^2}\partial_t^2 \int_{-\infty}^t dt' \epsilon_r(t-t')E(\mathbf{r}, t) = \frac{1}{\epsilon_0 c^2}(\partial_t^2 P_{nl}(\mathbf{r}, t) + \partial_t \mathcal{J}) \quad (70)$$

We intend to separate the fast oscillation of the wave group from the general envelope shape. We may write the complete electric field as the product of a fast phase term and an envelope. The model we develop here will ultimately be paraxial in nature. We therefore expect the propagation and transverse directions to be handled somewhat differently in the approximations that follow. and thus write the complete electric field component of the disturbance as follows

$$E(\mathbf{r}, t) = A(\mathbf{r}_\perp, z, t)e^{i(\beta_o z - \omega_o t + \psi_o)} + \text{c.c.} \quad (71)$$

where c.c. stands for “the complementary complex conjugate term”. Ultimately we obtain a partial differential equation that moves a description of the pulse envelope at the initial input plane in the transverse and temporal coordinates to a similar description at a new location in the propagation direction z . The phase ψ_o is defined so that the imaginary component of the complex envelope $A(\mathbf{r}_\perp, z, t)$ is zero at the input plane ($z = 0$). The spectral centroid of the wave group (or carrier frequency) is defined to be

$$\omega_o \equiv \frac{\int_o^\infty d\omega \omega |\hat{E}(\omega)|^2}{\int_o^\infty d\omega |\hat{E}(\omega)|^2}, \quad (72)$$

the wave number β_o is defined as

$$\beta_o \equiv \text{Re}[k(\omega_o)] = \frac{\omega_o}{c} n_o, \quad (73)$$

and

$$k(\omega) \equiv \frac{\omega}{c} \sqrt{\epsilon_r(\omega)} \quad (74)$$

As Brabec [75] points out, the wave group must be such that its basic carrier frequency does not depend strongly on the value of the initial fast phase component. After obtaining the envelope distribution at a particular z -coordinate location we may obtain the complete electric field distribution using Eq. (71).

Similarly the induced nonlinear polarization is written as:

$$P_{NL}(\mathbf{r}, t) = B(\mathbf{r}_\perp, z, t) e^{i(\beta_o z - \omega_o t + \psi_o)} + \text{c.c.} \quad (75)$$

Dispersion Operator

Now let us look at the linear dispersion operator

$$L_d E(\mathbf{r}, t) \equiv \frac{1}{c^2} \partial_t^2 \int_{-\infty}^t dt' \epsilon_r(t - t') E(\mathbf{r}, t') \quad (76)$$

It is useful to manipulate this term in the frequency domain. Let

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{f}(\omega) e^{-i\omega t} \quad (77)$$

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} dt f(t) e^{i\omega t} \quad (78)$$

define the Fourier transform. The sign of the exponent in eq. (77) is important because it determines the signs of the terms involving the time derivative later. If the final differential equation is solved numerically in the temporal Fourier domain then the kernel of the discrete Fourier transform (implemented via FFT or Fast Fourier Transform routines) must match the kernel of the forward Fourier transform used to transform the temporal differential operators. With some manipulation we may write the dispersion operator as

$$\hat{D} = \frac{-\alpha_1}{2} \partial_t + \sum_{m=2}^{\infty} \frac{\beta_m + i\alpha_m/2}{m!} (i\partial_t)^m \quad (79)$$

Retarded Temporal Coordinates

We now move to a retarded time coordinate moving with respect to the z -axis at the group velocity β_1^{-1} . We might also view this as a z -axis moving with respect to the time coordinate at the group velocity. Let

$$\tau = t - \beta_1 z \quad (80)$$

$$\zeta = z \quad (81)$$

Use of the dispersion operator, the temporal coordinate transform and removal of the fast phase allows us to arrive at

$$\begin{aligned} & (2i\beta_o\partial_\zeta + \partial_\zeta^2 - 2\beta_1\partial_\zeta\partial_\tau + \nabla_\perp^2)A + (i\alpha_o\beta_o - \frac{\alpha_o^2}{4} - \alpha_o\beta_1\partial_\tau + 2\beta_o\hat{D} + i\alpha_o\hat{D} + 2i\beta_1\partial_\tau\hat{D} + \hat{D}^2)A \\ & = -\frac{\omega_o^2}{\epsilon_0 c^2} \left(1 + \frac{i}{\omega_o}\partial_\tau\right)^2 B + \frac{1}{\epsilon_0 c^2} \left[\frac{e^2}{m}\rho A + i\omega_o\epsilon_0 c \left(1 + \frac{i}{\omega_o}\partial_t\right) \frac{\sum_n \rho_n W_n K_n \hbar \omega_0}{|A|^2} A \right] \end{aligned} \quad (82)$$

Approximations

Assuming that the change in the envelope with respect to time is small compared to the temporal variation of the electric field value in time (the “fast phase” we factored out)

$$|\partial_\tau A| \ll \omega_o |A| \quad (83)$$

We also use the slowly varying nature of the envelope shape with respect to the direction of propagation, or

$$|\partial_\zeta A| \ll |\beta_o A|. \quad (84)$$

Additionally, we assume $\beta_o \approx \omega_o\beta_1$ and $\beta_o \gg 1$ (which means that the group velocity is near c/n_o). For simplicity we define

$$T \equiv \left(1 + \frac{i}{\omega_o}\partial_\tau\right) \quad (85)$$

then (after a good deal of work) we arrive at the following one-way² propagation equation

$$\left[\left(\partial_\zeta + \frac{\alpha_o}{2} - i\hat{D} \right) A - i \frac{\beta_o}{2\epsilon_0 n_o^2} T B \right] + T^{-1} \frac{1}{2i\beta_o} \nabla_\perp^2 A = -T^{-1} \frac{i}{2\epsilon_0 \omega_o n_o c} \left[\frac{e^2}{m} \rho A + i\omega_o \epsilon_0 c T \frac{\sum_n \rho_n W_n K_n \hbar \omega_0}{|A|^2} A \right] \quad (86)$$

Leaving out the plasma current density terms on the right hand side of equation (86) we may write

$$\partial_\zeta A = -\frac{\alpha_o}{2} A + i\hat{D}A + \frac{i}{2\beta_o} \left(1 + \frac{i}{\omega_o}\partial_\tau\right)^{-1} \nabla_\perp^2 A + i \frac{\beta_o}{2\epsilon_0 n_o^2} \left(1 + \frac{i}{\omega_o}\partial_\tau\right) B \quad (87)$$

Eq. 87 is given as Eq. 6 in [75] (Brabec assumes the plasma current density is zero in his derivation).

The term B on the right hand side of Eq. (87) is really a non-linear constitutive relationship for the medium in which the wave is propagating. If the material has only the nearly instantaneous Kerr non-linearity we may write

$$B = 2\epsilon_0 n_o n_2 |A|^2 A = N_{nl} |A|^2 A \quad (88)$$

²The term “one-way” refers to the absence of the second order derivative with respect to the propagation direction. This eliminates the ability to describe waves reflected from discontinuities in material parameters.

where the units of the envelope A have been normalized to the square root of intensity. Also, the coefficients are found such that B corresponds to the non-linear portion of the polarization (see section 5.1 or Couaron [11] page 75, equation 38). If we include the delayed Kerr or Raman response we write

$$B = N_{nl} \left(|A(t)|^2 (1 - \alpha) + \alpha \int_{-\infty}^{\infty} R(t - \tau) |A(\tau)|^2 d\tau \right) A(t) \quad (89)$$

$$= N_{nl} \left(|A(t)|^2 (1 - \alpha) + \alpha \int_{-\infty}^{\infty} R(\tau) |A(t - \tau)|^2 d\tau \right) A(t) \quad (90)$$

Where we have used the fact that convolution is commutative. The function $R(t)$ is the Raman response function that represents the vibrations of excited nuclei. This function has the effect of “spreading out” the waveform or pulse. It moves energy from the “front” of the pulse to the “back” of the pulse because it is a “delayed” response (as indicated via the convolution integral description). A common form for the function $R(t)$ is

$$R(t) = \frac{1 + (\omega_r \tau_r)^2}{\omega_r \tau_r^2} \exp(-t/\tau_r) \sin(\omega_r t) \quad (91)$$

where τ_r and ω_r are the characteristic Raman response time and frequency respectively. These will vary between materials.

Normalization

Let's define three physically meaningful normalization lengths

$$\tau_p \quad \text{Characteristic length of the pulse in time.} \quad (92)$$

$$l_D = 2\tau_p^2/\beta_2 \quad \text{Distance proportional to dispersion length.} \quad (93)$$

$$l_{\perp} = \sqrt{l_D/2\beta} \quad \text{Transverse scale factor.} \quad (94)$$

Using these to define the following coordinate transforms

$$\tau = \tau_p \tau' \quad (95)$$

$$\zeta = l_D \zeta' \quad (96)$$

$$rl_{\perp} = r' \quad (97)$$

where r is the transverse distance measure in a cylindrical coordinate system, we have the following differential relationships

$$d\tau = \tau_p d\tau' \quad (98)$$

$$d\zeta = l_D d\zeta' \quad (99)$$

$$dr l_{\perp} = dr' \quad (100)$$

Note that if we set α_m to zero (no loss) and choose to keep only two terms of the $\hat{D}$ operator we have from equation (79)

$$\hat{D} = -\frac{\beta_2}{2} \frac{\partial^2}{\partial \tau^2} - i \frac{\beta_3}{6} \frac{\partial^3}{\partial \tau^3} \quad (101)$$

Multiplication of equation (87) by i and substituting for $\hat{D}$ we obtain with a little algebra

$$i \frac{\partial}{\partial \zeta} A - \frac{\beta_2}{2} \frac{\partial^2}{\partial \tau^2} A - i \frac{\beta_3}{6} \frac{\partial^3}{\partial \tau^3} A + \frac{1}{2\beta_o} \left(1 + \frac{i}{\omega_o} \frac{\partial}{\partial \tau} \right)^{-1} \nabla_{\perp}^2 A + \frac{\beta_o}{2\varepsilon_0 n_o^2} \left(1 + \frac{i}{\omega_o} \frac{\partial}{\partial \tau} \right) B = 0 \quad (102)$$

where we have again neglected loss. We use the temporal coordinate transform to obtain

$$i \frac{\partial}{\partial \zeta} A - \frac{\beta_2}{2\tau_p^2} \frac{\partial^2}{\partial \tau'^2} A - i \frac{\beta_3}{6\tau_p^3} \frac{\partial^3}{\partial \tau'^3} A + \frac{1}{2\beta_o} \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right)^{-1} \nabla_{\perp}^2 A + \frac{\beta_o}{2\varepsilon_0 n_o^2} \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right) B = 0 \quad (103)$$

Noting the coefficient of the second order temporal derivative is really $1/l_D$ we may write

$$i \frac{\partial}{\partial \zeta} A - \frac{1}{l_D} \frac{\partial^2}{\partial \tau'^2} A - i \frac{\beta_3}{6\tau_p^3} \frac{\partial^3}{\partial \tau'^3} A + \frac{1}{2\beta_o} \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right)^{-1} \nabla_{\perp}^2 A + \frac{\beta_o}{2\varepsilon_0 n_o^2} \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right) B = 0 \quad (104)$$

Next we apply the longitudinal transform and multiply by l_D to obtain

$$i \frac{\partial}{\partial \zeta'} A - \frac{\partial^2}{\partial \tau'^2} A - i \frac{l_D \beta_3}{6\tau_p^3} \frac{\partial^3}{\partial \tau'^3} A + \frac{l_D}{2\beta_o} \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right)^{-1} \nabla_{\perp}^2 A + \frac{l_D \beta_o}{2\varepsilon_0 n_o^2} \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right) B = 0 \quad (105)$$

Now we observe the coefficient of the term representing diffraction (transverse Laplacian) as the square of $l_{\perp}$ and apply the final coordinate transform to give

$$i \frac{\partial}{\partial \zeta'} A - \frac{\partial^2}{\partial \tau'^2} A - i \frac{l_D \beta_3}{6\tau_p^3} \frac{\partial^3}{\partial \tau'^3} A + \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right)^{-1} \nabla_{\perp}^2 A + \frac{l_D \beta_o}{2\varepsilon_0 n_o^2} \left(1 + \frac{i}{\omega_o \tau_p} \frac{\partial}{\partial \tau'} \right) B = 0 \quad (106)$$

Finally we define

$$\varepsilon_3 = \frac{l_D \beta_3}{6\tau_p^3} = \frac{2\tau_p^2}{\beta_2} \frac{\beta_3}{6\tau_p^3} = \frac{\beta_3}{3\beta_2 \tau_p} \quad (107)$$

and

$$\varepsilon_{\omega} = \frac{1}{\omega \tau_p} \quad (108)$$

and write

$$i \frac{\partial}{\partial \zeta'} A - \frac{\partial^2}{\partial \tau'^2} A - i \varepsilon_3 \frac{\partial^3}{\partial \tau'^3} A + \left(1 + i \varepsilon_{\omega} \frac{\partial}{\partial \tau'} \right)^{-1} \nabla_{\perp}^2 A + \frac{l_D \beta_o}{2\varepsilon_0 n_o^2} \left(1 + i \varepsilon_{\omega} \frac{\partial}{\partial \tau'} \right) B = 0 \quad (109)$$

This equation compares directly to Zozulya [73] when using equation (88) to describe the proportionality of the non-linear material charge displacement (polarization).

Adding in the ionization terms and using

$$T' \equiv \left(1 + i\varepsilon_\omega \frac{\partial}{\partial \tau'}\right) \quad (110)$$

we have

$$\begin{aligned} i \frac{\partial}{\partial \zeta'} A - \frac{\partial^2}{\partial \tau'^2} A - i\varepsilon_3 \frac{\partial^3}{\partial \tau'^3} A + T'^{-1} \nabla_{\perp}^2 A + \frac{l_D \beta_o}{2\varepsilon_0 n_o^2} T' B \\ - T'^{-1} \frac{l_D}{2\varepsilon_0 \omega_0 n_o c} \left[\frac{e^2}{m} \rho A + i\omega_o \varepsilon_0 c T' \frac{\sum_n \rho_n W_n K_n \hbar \omega_0}{|A|^2} A \right] = 0 \end{aligned} \quad (111)$$

or

$$\begin{aligned} i \frac{\partial}{\partial \zeta'} A - \frac{\partial^2}{\partial \tau'^2} A - i\varepsilon_3 \frac{\partial^3}{\partial \tau'^3} A + T'^{-1} \nabla_{\perp}^2 A + \frac{l_D \beta_o}{2\varepsilon_0 n_o^2} T' B \\ - T'^{-1} \frac{l_D}{2\varepsilon_0 \omega_0 n_o c} \frac{e^2}{m} \rho A - i \frac{l_D}{2n_o} \frac{\sum_n \rho_n W_n K_n \hbar \omega_0}{|A|^2} A = 0 \end{aligned} \quad (112)$$

Let

$$N_{\text{plasma}} = \frac{l_D}{2\varepsilon_0 \omega_0 n_o c} \frac{e^2}{m} \rho \quad (113)$$

and

$$N_{\text{MPA}} = \frac{l_D}{2n_o} \frac{\sum_n \rho_n W_n K_n \hbar \omega_0}{|A|^2} \quad (114)$$

and we have

$$i \frac{\partial}{\partial \zeta'} A - \frac{\partial^2}{\partial \tau'^2} A - i\varepsilon_3 \frac{\partial^3}{\partial \tau'^3} A + T'^{-1} \nabla_{\perp}^2 A + \frac{l_D \beta_o}{2\varepsilon_0 n_o^2} T' B - T'^{-1} N_{\text{plasma}} A - i N_{\text{MPA}} A = 0 \quad (115)$$

For a single species

$$W = \sigma_K I^K = \sigma_K |A|^{2K} \quad (116)$$

with definitions

$$\rho_c \equiv \varepsilon_0 m_e \omega_0^2 / e^2 \quad \beta_K = \rho_{\text{at}} K \hbar \omega_0 \sigma_K \quad \rho = \rho_{\text{at}} - \rho_n \quad (117)$$

we can write

$$N_{\text{plasma}} = l_D \frac{\omega_0}{2\rho_c n_o c} \rho = l_D \frac{k_0}{2\rho_c n_o^2} \rho \quad (118)$$

and

$$N_{\text{MPA}} = \frac{l_D}{2n_o} |A|^{2K-2} K_n \hbar \omega_0 \sigma_K (\rho_{\text{at}} - \rho) = \frac{l_D}{2n_o} |A|^{2K-2} \beta_K \left(1 - \frac{\rho}{\rho_{\text{at}}}\right) \quad (119)$$

Using the Drude model frequency dependence discussed in Couarion [11] we may write the plasma term as

$$N_{\text{plasma}} = -il_D \frac{k_0}{2\rho_c n_o^2} \frac{\omega_0 \tau_c}{1 + \omega_0^2 \tau_c^2} (1 + i\omega_0 \tau_c) \rho \quad (120)$$

Electron Density

Our current model for ionization is

$$\frac{d\rho}{dt} = \frac{\sigma}{n_b^2 U_i} \rho |E_i|^2 + |E_i|^{2K} \frac{\beta_K}{K \hbar \omega_0} \left(1 - \frac{\rho}{\rho_{at}}\right) - \alpha \rho^2 \quad (121)$$

See equation 2 from Berge et. al. [77]. The first and second terms on the RHS represent respectively avalanche and multiphoton ionization respectively. The final term corresponds to radiative recombination of electrons with the atoms or molecules.

Given that

$$F(\rho) = \frac{\sigma}{n_b^2 U_i} \rho |E_i|^2 + |E_i|^{2K} \frac{\beta_K}{K \hbar \omega_0} \left(1 - \frac{\rho}{\rho_{at}}\right) - \alpha \rho^2, \quad (122)$$

with definitions

$$v = \frac{\sigma}{n_b^2 U_i}, \quad K = \left\lceil \frac{U_i}{\hbar \omega_0} \right\rceil, \quad \sigma_k = \frac{\beta_K}{K \hbar \omega_0} \quad (123)$$

we may write more simply

$$F(\rho) = v \rho |E_i|^2 + |E_i|^{2K} \sigma_k \left(1 - \frac{\rho}{\rho_{at}}\right) - \alpha \rho^2. \quad (124)$$

We note that under the Drude model the avalanche ionization rate coefficient can be computed as

$$\sigma = \frac{ke^2 \tau}{\omega m_e \epsilon_0 [1 + (\omega \tau)^2]} \quad (125)$$

However, this relationship will not always hold and so we will not assume it in general.

The Jacobian is then computed as

$$F'(\rho) = v |E_i|^2 - \frac{\sigma_k}{\rho_{at}} |E_i|^{2K} - 2\alpha \rho. \quad (126)$$

where τ is the electron collision time and $k = 2\pi n/\lambda$. Also,

$$m_e = 9.10938188 \times 10^{-31} \text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

The fundamental physics parameters of the ionization model may therefore be chosen as

n_b	Refractive index of medium
ρ_{at}	Number density of neutral atoms at (in the absence of external field)
τ	Electron collision time
σ	Avalanche ionization rate coefficient
U_i	Energy needed to liberate an electron
σ_k	Coefficient of multiphoton ionization rate
α	Recombination coefficient
ρ_b	Initial electron number density (initial condition for ODE)

where the values of τ and K are valid only for a particular angular frequency ω_0 . We could have chosen σ , the cross-section for inverse Bremsstrahlung instead of the electron collision frequency. However, given σ it is possible for physical values of the parameters in equation (125) to obtain two, distinct, real, positive values for the electron collision time. Hence we store the electron collision time and compute the cross-section for inverse Bremsstrahlung.

Model Summary and Fundamental Parameters

The model equations are:

$$\partial_z A = (\mathbf{L} + \mathbf{N})A \quad (127)$$

$$\mathbf{L} = -\frac{i\partial^2}{\partial \tau'^2}A + \epsilon_3 \frac{\partial^3}{\partial \tau'^3}A + iT'^{-1}\nabla_{\perp}^2 A \quad (128)$$

$$\mathbf{N} = N_{\text{Plasma}} + N_{\text{MPA}} + N_{\text{Kerr}} \quad (129)$$

where

$$T' \equiv \left(1 + i\epsilon_{\omega} \frac{\partial}{\partial \tau'}\right) \quad (130)$$

$$N_{\text{MPA}} = -\frac{l_D}{2n_o} |A|^{2K-2} \beta_K \left(1 - \frac{\rho}{\rho_{\text{at}}}\right) \quad (131)$$

$$N_{\text{Plasma}} = -T'^{-1} l_D \frac{k_0}{2\rho_c n_o^2} \frac{\omega_0 \tau_c}{1 + \omega_0^2 \tau_c^2} (1 + i\omega_0 \tau_c) \rho \quad (132)$$

$$N_{\text{Kerr}} = i \frac{2\pi n_2}{\lambda} l_D (|A(t)|^2 (1 - \alpha) + \alpha \int_{-\infty}^{\infty} R(\tau) |A(t - \tau)|^2 d\tau) \quad (133)$$

The first term is the instantaneous Kerr and the second is the delayed Kerr (Raman)

$$R(t) = \frac{1 + (\omega_r \tau_r)^2}{\omega_r \tau_r^2} \exp(-t/\tau_r) \sin(\omega_r t) \quad (134)$$

$$\frac{d\rho}{dt} = \frac{\sigma}{n_b^2 U_i} \rho |E_i|^2 + |E_i|^{2K} \frac{\beta_K}{K\hbar\omega_0} \left(1 - \frac{\rho}{\rho_{\text{at}}}\right) - \alpha \rho^2 \quad (135)$$

The first term is the avalanche ionization rate, the second term is the multiphoton ionization rate and the third term is the recombination rate.

$$\epsilon_3 = \frac{\beta_3}{3\beta_2 \tau_p} \quad (136)$$

$$\epsilon_{\omega} = \frac{1}{\omega \tau_p} \quad (137)$$

$$l_D = 2\tau_p^2 / \beta_2 \quad (138)$$

For these equations β_3 is the third order dispersion and β_2 is the group velocity dispersion.

$$\rho_c \equiv \epsilon_0 m_e \omega_0^2 / e^2 \quad \beta_K = \rho_{\text{at}} K \hbar \omega_0 \sigma_K \quad \rho = \rho_{\text{at}} - \rho_n \quad (139)$$

We note that under the Drude model the avalanche ionization rate coefficient can be computed as

$$\sigma = \frac{ke^2\tau}{\omega m_e \epsilon_0 [1 + (\omega\tau)^2]} \quad (140)$$

The fundamental parameters for the entire propagation model are

n_b	Linear refractive index of medium
GVD	Group velocity dispersion
TOD	Third order dispersion
n_2	Nonlinear refractive index
γ	ratio of instantaneous to delayed Kerr (Raman)
τ_R	Raman time constant
ω_R	Raman frequency
ρ_{at}	Number density of neutral atoms (in the absence of external field)
τ	Electron collision time.
σ	Avalanche ionization rate coefficient
σ_k	Coefficient of multiphoton ionization rate
U_i	Energy needed to liberate an electron
α	Recombination coefficient
ρ_b	Initial electron number density (initial condition for ODE)

5.2 Numerical Methods

The basic equation usually arrived at for non-linear optical propagation is

$$\partial_z A = (\mathbf{L} + \mathbf{N})A \quad (141)$$

where A is the complex field envelope, $\mathbf{L}$ is the linear, dispersive portion and $\mathbf{N}$ the non-linear portion of the propagation operator.

Split-Step

Split-step formulations are based on the idea of “operator-splitting.” That is, the operator is written as the sum of two operators (in this case the linear and non-linear portions of the full operator) and each is applied separately. The basis for the first and second order split-step method is often described physically (as we do here). However, there is an underlying mathematical framework

from ordinary differential equations that depends on the commutativity of the two operators³ and yields higher order split-step methods. The first-order split-step methodology simply solves two separate equations in succession as shown in equation (142)

$$\partial_z A'(z + dz) = \mathbf{L}A(z) \quad (142)$$

$$\partial_z A(z + dz) = \mathbf{N}A'(z + dz) \quad (143)$$

using the output of one as the input to the other. This is equivalent to propagating the pulse a single step distance in the z -direction linearly and then applying the non-linear operator to the result concluding the first complete step. A second-order split step method involves propagating linearly one-half a single z -step distance, applying the non-linear operator for the full-step distance and then completing the full-step by propagating linearly the final half-step distance. A small numerical investigation showed little improvement of the second order method over the first order algorithm and so we use the latter in all of our simulations here. In section 5.2 we describe the Crank-Nicolson discretization and linear system solver used in our simulations. Afterwards in section 5.2 we discuss briefly the non-linear operator.

Linear

The linear operator consists of both temporal and spatial derivatives. We chose in our work to apply the temporal portion of the linear operator in the frequency domain via a common Fourier transform method. This significantly reduces difficulties with temporal numerical dispersion. This means that we advance the linear spatial portion of the operator in the spatical domain once for each frequency in the simulation. If we have N_t samples in time then we must apply the spatial portion of the operator N_t times in order to advance a single step. Here we describe that spatial operator.

Consider a linear partial differential equation of the form

$$\partial_z u = iq_1 u + iq_2 \nabla_\perp^2 u \quad (144)$$

where for our case

$$q_1 \equiv \omega^2 - \varepsilon_3 \omega^3 \quad \text{and} \quad q_2 \equiv 1 + \varepsilon_\omega \omega. \quad (145)$$

Note that we have used a Taylor's series expansion on the inverse shock operator to obtain the value for q_2 . In the absense of shock $q_2 \equiv 1$.

Define the operator $\delta^{-n} d_n^m$ to be an n th order derivative in space (in our case the $\hat{r}$ direction) with support of size m . For example

$$d_1^5 = \frac{1}{2\delta} [0, -1, 0, 1, 0] \quad (146)$$

also note that we have included the grid spacing. In general the operator is inversely proportional to the n th power of the grid spacing for and order n derivative. The actual coefficients are not

³We assume that the operators commutes *sufficiently* over short physical distances in the propagation direction of the pulse.

specified here only their support. We will discuss later how we arrive at the coefficients. Also, we will only concern ourselves with odd support lengths.

We will say by definition the d_0^m is a support centered kronecker delta function

$$d_0^5 = [0, 0, 1, 0, 0] \quad (147)$$

We know that in cylindrical coordinates that

$$\nabla_{\perp}^2 \equiv \partial_r^2 + \frac{1}{r} \partial_r \quad (148)$$

The Crank-Nicholson (CN) approximations are

$$u \rightarrow \frac{1}{2} (u_{n+1,j} + u_{n,j}) \quad (149)$$

$$\partial_z u \rightarrow \frac{1}{\delta_z} (u_{n+1,j} - u_{n,j}) \quad (150)$$

$$\partial_r^2 u \rightarrow \frac{1}{2\delta_r^2} d_2^m \cdot (\mathbf{u}_{n+1,j^m} + \mathbf{u}_{n,j^m}) \quad (151)$$

$$\frac{1}{r} \partial_r u \rightarrow \frac{1}{2j\delta_r^2} d_1^m \cdot (\mathbf{u}_{n+1,j^m} + \mathbf{u}_{n,j^m}) \quad (152)$$

$$(153)$$

where the notation

$$\mathbf{u}_{n,j^m} = [u_{n,j-(m-1)/2}, u_{n,j-(m-1)/2+1}, \dots, u_{n,j}, \dots, u_{n,j+(m-1)/2-1}, u_{n,j-(m-1)/2}]^T \quad (154)$$

describes an m -length vector evaluated at step n in the z -coordinate direction, centered at j in the r -coordinate direction. Substituting into equation (144) gives

$$\frac{2}{\delta_z} d_0^m \cdot (\mathbf{u}_{n+1,j^m} - \mathbf{u}_{n,j^m}) = iq_1 d_0^m \cdot (\mathbf{u}_{n+1,j^m} + \mathbf{u}_{n,j^m}) + i \frac{q_2}{\delta_r^2} d_2^m \cdot (\mathbf{u}_{n+1,j^m} + \mathbf{u}_{n,j^m}) + i \frac{q_2}{j\delta_r^2} d_1^m \cdot (\mathbf{u}_{n+1,j^m} + \mathbf{u}_{n,j^m}) \quad (155)$$

$$\left[\frac{2}{\delta_z} d_0^m - i \left(q_1 d_0^m + \frac{q_2}{\delta_r^2} \left[d_2^m + \frac{1}{j} d_1^m \right] \right) \right] \cdot \mathbf{u}_{n+1,j^m} = \left[\frac{2}{\delta_z} d_0^m + i \left(q_1 d_0^m + \frac{q_2}{\delta_r^2} \left[d_2^m + \frac{1}{j} d_1^m \right] \right) \right] \cdot \mathbf{u}_{n,j^m} \quad (156)$$

Given that q_1 and q_2 are real, if we define a row vector

$$\mathbf{p}_j^m \equiv \frac{2}{\delta_z} d_0^m - i \left(q_1 d_0^m + \frac{q_2}{\delta_r^2} \left[d_2^m + \frac{1}{j} d_1^m \right] \right) \quad (157)$$

then we may write equation (156) as

$$\mathbf{p}_j^m \cdot \mathbf{u}_{n+1,j^m} = \mathbf{p}_j^{m*} \cdot \mathbf{u}_{n,j^m} \quad (158)$$

Note that at $j = 0$ we must use L'Hospital's rule to arrive at

$$\mathbf{p}_0^m \equiv \frac{2}{\delta_z} d_0^m - i \left(q_1 d_0^m + 2 \frac{q_2}{\delta_r^2} d_2^m \right) \quad (159)$$

Equation (158) is an algebraic equation in m unknowns. We know the $\mathbf{u}_{n,j^m}$ (they are the field values at the particular frequency specified by the values of q_1 and q_2 as a function of radial coordinate at z -step index n) and the entries of the row vectors $\mathbf{p}_j^m$. The RHS of equation (158) is a scalar. The m unknowns are $\mathbf{u}_{n+1,j^m}$ (the field values at the particular frequency specified by the values of q_1 and q_2 as a function of radial coordinate at z -step index n). If we add another equation centered at an adjacent value of j (the radial coordinate index) then we have an additional equation and an additional unknown (two equations in six unknowns). Each time we add an adjacent equation we get an additional variable. Let's say we add an adjacent equation until we reach a boundary. The new variable added by each boundary equation is known. At each boundary we add an equation with a known additional variable $(m-1)/2$ times. After reaching both boundaries we have exactly the number of equations we need to solve the system. Solving the system lets us update all the $u_{n,j}$ to $u_{n+1,j}$ for all j at once. This method (CN) is implicit (uses both the current and future time to obtain the future time) and hence requires a solve (in this case a linear solve) at each step. It is unconditionally stable and will tolerate a larger step size than explicit methods (which are only stable if the step size is chosen according to the Courant stability criterion). However, they require a global solve at each step. Also, equation (157) cannot be evaluated at index $j = 0$. This problem results from the definition of the Laplacian operator in equation (148). Here we must use equation (159).

Practical implementation for a grid with J radial samples requires that we use equation (157) and the known current values of $\mathbf{u}_n$ to create a matrix equation

$$\mathbf{L} \mathbf{u}_{n+1} = \mathbf{L}^* \mathbf{u}_n = \mathbf{v}_n \quad (160)$$

where L is an $J \times J$ matrix, $\mathbf{v}_n$ is a known length J vector. The row j of $\mathbf{L}$ is formed from equation (157). The top and bottom $(m-1)/2$ rows are special as they touch the boundaries. If we apply symmetric boundary conditions at the $j = 0$ ($r = 0$ boundary) and zero boundary conditions at the outer boundary and note that

$$\mathbf{p}_j^m = [p_j^{-\ell} \ p_j^{-\ell+1} \ p_j^{-\ell+2} \ \dots \ p_j^0 \ \dots \ p_j^{\ell-2} \ p_j^{\ell-1} \ p_j^\ell] \quad (161)$$

non-linear numerical step. We are unaware of any literature results that contain both these effects simultaneously and so for validation we use the simple exponential method and do not combine these two physics effects. The code has been written so as to facilitate modification to include such an ODE solver in the future.

Application of the instantaneous Kerr effect is accomplished in the common fashion described above. The Raman or delayed Kerr is described via a convolution integral and is implemented as such in the code. Taylor series expansion of the convolution integral creates a simpler expression and is used in some applications [78] but it is less accurate and we were able to observe differences between the two.

Solution of the evolution equation for the electron density requires use of an ODE (Ordinary Differential Equation) solver. The equation itself is *stiff* in the sense that multiple time scales are important (the solution moves very quickly in the presense of the pulse and slowly after the pulse peak has passed). Some investigation was required in order to find solvers the could accurately integrate the equations. We use `ode23` and `ode45` in Matlab depending on the application. In the C++ code we use one of the stiff integrators from the `Sundials` package.

5.3 Validation and Evaluation of Models

The equation for critical power in a Gaussian beam is

$$P_c = \frac{\lambda^2}{2\pi n_o n_2}. \quad (163)$$

The simulation parameters for the figures 27 and 28 are

$$\begin{aligned} \alpha &= 7.0 \times 10^{-13} \text{ m}^{-3}/\text{s} \\ \beta_K &= 3.5 \times 10^{-123} \text{ m}^{13}/\text{W}^7 \\ K &= 8 \\ U_i &= 15.76 \text{ J} \\ \tau_c &= 1.9 \times 10^{-13} \text{ s} \\ \rho_{\text{at}} &= 2.7 \times 10^{25} \text{ m}^{-3} \end{aligned}$$

The material parameters for the figures 29, 30, and 31 are

$$\begin{aligned} n_o &= 1.45 \\ n_2 &= 2.5 \times 10^{-20} \text{ m}^2/\text{W} \\ \text{GVD} &= 3.6 \times 10^{-26} \text{ s}^2/\text{m} \\ \gamma &= 0.15 \\ \tau_R &= 50 \times 10^{-15} \text{ s} \\ \omega_R &= 84 \times 10^{-12} \text{ radians/s} \end{aligned}$$

Figure 25. The plot on the left shows a comparison between the theoretically expected FWHM (full width half max) measurement of a Gaussian beam and the width of the beam as a function of propagation distance obtained in the simulation code. The e^{-1} -amplitude radius is 1.5 mm. The center wavelength is 810 nm. The plot on the right hand side shows a comparison between the theoretical and computed dispersion of a pulse with identical center wavelength and radius having a 30 ns FWHM intensity temporal extent. The stair-step effect is a result of coarse sampling in the simulation. The simulation medium is air.

Figure 26. The left and right plots show beam radius and intensity as a function of propagation distance over approximately 300m in air. The center wavelength is 1024nm. The initial e^{-1} -amplitude radius is 6mm. The input was slightly below critical power for a Gaussian beam as given in equation (163). Here $n_2 = 4 \times 10^{-23} \text{ m}^2/\text{W}$. Clearly the beam is confined nearly indefinitely (we actually used a little less than critical power) due to the balancing of Kerr nonlinearity and diffraction.

Figure 27. The left figure comes from a paper by Mlejnek [74]. Mlejnek's simulation does not include shock or Raman. It does have instantaneous Kerr and ionization, however. Our simulation using the same model is shown at right. The e^{-1} -amplitude radius is 0.2mm. The temporal FWHM intensity extent of the pulse is 200fs. The center wavelength is 586nm. The focus is at 2.5cm. Other relevant simulation parameters are $n_2 = 4.9 \times 10^{-23} \text{ m}^2/\text{W}$ and $\text{GVD} = 2.6 \times 10^{-29} \text{ s}^2/\text{m}$. Excellent agreement is observed.

Figure 28. The left figure comes from a paper by Mlejnek [74]. Mlejnek's simulation does not include shock or Raman. It does have instantaneous Kerr and ionization, however. Our simulation using the same model is shown at right. The e^{-1} -amplitude radius is 0.2 mm. The temporal FWHM intensity extent of the pulse is 200 fs. The center wavelength is 586 nm. The focus is at 2.5 cm. Other relevant simulation parameters are $n_2 = 4.9 \times 10^{-23} \text{ m}^2/\text{W}$ and $\text{GVD} = 2.6 \times 10^{-29} \text{ s}^2/\text{m}$. The propagation distance is 2.3 cm. Excellent agreement is observed.

Figure 29. Calculated on-axis intensity profile (see Zozulya [73] figure 5). The propagation is in fused silica. Each plot considers the following effects (a) Non-linear shock. (b) Linear shock. (c) Both linear and non-linear shock. and (d) Both shock terms and Raman effects. The center wavelength is 800nm. The initial e^{-1} -amplitude radius is $59.45\mu\text{m}$. The temporal FWHM intensity extent of the pulse is 90fs. The input intensity is $85\text{GW}/\text{cm}^2$. Material parameters are described in the text.

Figure 30. The figure on the left is taken from Zozulya [73] figure 3 demonstrating Raman pulse splitting effects. It shows the peak on-axis intensity normalized by the input intensity. The initial pulse parameters are as given in figure 29. Material parameters are described in the text.

Figure 31. Compare Zozulya [73] figure 3. It shows the peak on-axis intensity normalized by the input intensity. No preconditioner was used in the linear solver for the data on the right. On the left a preconditioner was used. After only a few millimeters of propagation differences in the pulse envelope amplitudes are apparent. The initial pulse parameters are as given in figure 29. Parameters are described in the text.

Figure 32. The above simulation used a $(3.5\text{J})/(4\text{ns FWHM})$, 12.5mm $1/e$ amplitude radius pulse at $\lambda = 532\text{nm}$ focused by $f=75\text{cm}$. The approximate $f/\#$ is 30 and hence the (linear) radius at the focal distance should be approximately $(\lambda)(f/\#)$ or about $15\mu\text{m}$. Using approximately 40 samples across the $15\mu\text{m}$ radius at focus requires 100000 radial samples over the extent of the 30mm (to prevent edge effects near the beginning of the simulation). The simulation used 256 temporal samples giving approximately 25 million cells in the simulation at each step. A total of 3000 longitudinal steps were used to propagate 1.5m (twice the focal distance) saving every tenth step. The C++ code used 15 nodes (120 processors) on the Sandia RedSky system and took approximately 15 hours. The structure after focus (not shown here) shows some high frequency modes of the grid that remain undamped by the Crank-Nicolson method. A neutral species number density of $2.75 \times 10^{25}\text{m}^{-3}$ was used. Saturation and second order recombination were turned on in the ODE solver. Observed peak electron number densities were approximately $1 \times 10^{23}\text{m}^{-3}$.

6 Laser-Triggered Switch Modeling

A fully-kinetic simulation of the ZR laser-triggered gas switch (LTGS) has been developed, leveraging expertise in this area from a team at Voss Scientific. This work involves detailed finite-difference time-domain particle-in-cell (PIC) model of the trigger section of the switch and include circuit modeling of other components of the ZR machine from Marx bank through the pulse-forming line (PFL). The simulations presented here build on a recently developed model of electro-negative gas breakdown and streamer propagation [79]. Our motivation for developing a fully electromagnetic (EM) kinetic PIC description of streamer propagation in electronegative gases arises from studies of high-current, laser-triggered gas switches, such as those used on the ZR facility at Sandia National Laboratories [3, 80], where SF_6 is used as the working gas. The goal is to develop 2D and 3D models capable of treating the switch performance through the successive stages of laser-initiation of one or more thin plasma filaments between the electrodes, the subsequent streamer/leader propagation that provides the initial current contact between the electrodes, and finally the formation of a high-current conduction channel. During this final stage, self-magnetic field effects associated with individual and collective conduction channel evolution are expected to be significant and, therefore, warrant a fully EM simulation approach.

The initiation of a plasma channel and subsequent streamer and leader formation in high pressure devices involves coupled kinetic processes on relatively fast time scales. These processes include elastic and inelastic particle collisions in the gas and ambient fields, radiation generation, transport and absorption, and electrode interactions. Accurate modeling of these physical processes is essential for a number of applications, including high current, laser-triggered gas switches [2, 20, 81, 82]. We make use of an implicit particle-in-cell (PIC) simulation model of gas breakdown leading to streamer formation in electronegative gases. Within the framework of the Large-Scale Plasma (LSP) EM PIC code, a Monte Carlo treatment for all particle interactions is used, which includes discrete photon generation, transport, and absorption for ultra-violet (UV) and soft x-ray radiation [79].

The LTGS operation relies on a sequence of conditions in the trigger gap:

1. The electronegative gas prevents breakdown of the switch below an applied voltage consistent with electric field to gas number density $E/N \leq 360 \text{ Td}$ (where the unit Townsend is $1 \text{ Td} = 10^{17} \text{ V} \cdot \text{cm}^2$). On ZR, the trigger section of the LTGS is designed to be operated at a voltage that is 10% of the “pre-fire” or self-breakdown voltage. In terms of E/N , this translates to a reduction of $\sim 50 \text{ Td}$ below the self-triggering value.
2. The laser ionizes a filament of plasma such that the resulting enhancement increasing the E/N to a level that avalanche and UV ionization increase the filament length.
3. The filament spans the anode-cathode (AK) gap entirely and the switch closes.

6.1 Particle-in-Cell Monte Carlo model for kinetic Simulations of laser triggering

The basic MCC model, described in detail for the SF₆ in [79] is an extension of the model described in [83]. This model, including electron elastic, inelastic, and ionization processes with neutrals, was benchmarked for noble gases, such as He and Ar, in which no significant electron sink is present. For an electronegative gas such as SF₆, electron attachment processes play a key role in inhibiting breakdown at low field values. To allow for electronegative gas modeling, an attachment algorithm was added to the MCC model previously developed for noble gases. We have also added an excited state neutral channel. Relatively high energy photons created by relaxation (using 0.3-ns decay time suggested in [79]) of the excited state neutrals can then be propagated through a photon transport model. These photons, propagating at the speed of light, generate electron-ion pairs by neutral photo-absorption. This provides a non-local source of plasma generation in the vicinity of the streamer front. For computational efficiency, a relatively simple chemistry model for SF₆ was developed. The model can easily be extended to include additional reaction channels and particle species beyond those described here. The evolution of six particle species is tracked: electrons, neutral (ground state) SF₆, a single excited neutral state SF₆^{*}, a single positive ion SF₆⁺, a single negative ion SF₆⁻, and photons. We note that there are many dissociative neutral and ionic fragments which can occur in SF₆ reactions [84].

6.2 Triggering with variation of laser parameters

In this section, we discuss idealized 2D simulations of the laser triggering phase of an LTGS leading to current conduction in the trigger section. The simulations are purely electrostatic with axisymmetric geometry shown in Figure 33. We introduce a 3 to 12 ns long, 5 to 20 mJ “laser pulse” into the simulation. The laser model assumes a Gaussian radial laser envelope:

$$w = w_0 \sqrt{1 + \frac{z}{z_R}} \quad (164)$$

$w_0=25\ \mu\text{m}$ is the beam waist, $z_R = \frac{\pi w_0^2}{\lambda}$ is the Rayleigh range (or half the depth of focus), and $\lambda=266\ \text{nm}$ is the laser wavelength. In the simulations, we apply the corresponding electric field intensity of 40 MV/cm over a “sech²(t)” temporal profile for 20 mJ energy and 3 ns pulse length. A multiple-photon ionization probability is calculated via the Ammosov-Delone-Krainov (ADK) model [85], yielding an evolving SF₆⁺-electron plasma within the laser path. This model enables us to skip over frequency and spatial scales associated with the laser beam in the simulation. It does not however allow for plasma feedback on the propagation of the laser pulse. For completeness, we also permit the SF₆⁻ to photoionize via the same model assuming an ionization (or rather detachment) potential of the 1.3-eV binding energy. In both cases, when a photoionization event occurs, the secondary electron is given the energy of the ionization potential with a random direction. This effect is important to the seeding of the filament since it inhibits the negative ion density while the laser is on.

Figure 33. Idealized representation of the LTGS trigger section used for the 2D simulations. The laser envelope is illustrated by the red dashed line. The electron emission site (green) provides the final “electrical” connection for the evolving plasma filament at the start of the conduction phase.

Also shown in Figure 33 is a region on the cathode over which space-charged-limited (SCL) emission is enabled. This region electrically connects the evolving plasma filament to the cathode, enabling the initial conduction of current in the model. This region only begins to emit once the normal electric field at the surface exceeds a preset threshold. We typically set this threshold to a 200 kV/cm value which is generally much larger than the initialized axial field. The electron emission, which typically starts once the filament is within a few millimeters of the cathode, enables a numerically “smooth” connection of the filament to the cathode. In reality, complex surface physics coupled to the UV emission from the approaching filament (and possibly the earlier-time laser pulse) result in some plasma formation along the cathode surface. The SCL electron emission model attempts to crudely mimic this effect.

In the first series of runs, we model the trigger closure of ZR shot 2307 which was run at roughly half voltage (3.12 MV or 460 kV in trigger section). The SF_6 gas pressure in the trigger is 24.3 psi requiring a timestep of 0.067 ps to adequately resolve the collision frequency and show simulation convergence. These parameters give $E/N = 220$ Td, well below that of self-breakdown. The laser parameters we treat as nominal are 20 mJ of injected energy in 3 ns with $w_0 = 25 \mu\text{m}$ which gives the axial extent of the focus $2z_R = 1.5$ cm. The length of the filament relative to the AK gap length is important because it determines the field enhancement. The focal plane was near the anode, mid-gap, and near the cathode ($z = 1.15, 2.3$ and 3.45 cm). The evolution of the plasma filament for these cases is shown in Figure 34. These frames plot the electron density from 4 to roughly 13 ns. The laser reaches peak intensity at 3 ns. In the relatively high E/N , the filament growth is essentially an electron-avalanche rather than a pure streamer. The ADK ionization of the negative ions allows the electron avalanche to grow rapidly with negative ion density only several times that of the electrons while the laser is on. This simulation demonstrates the initial 3 phases

of switching, by establishment of an electrical connection between the anode and cathode after 12 ns and nearly 1 MA/cm^2 current density in the filament. This gives a minimum trigger runtime of 12.5 ns for near ideal conditions, well within the 10-20 ns runtime values for LTGS switches on Z, Z20 and ZR [3].

Figure 34. Electron density contours [$\log_{10}(\text{cm}^{-3})$ contour levels] at various times for Shot 2307 simulations with laser focus at $z = 1.15 \text{ cm}$ (near anode), 2.3 cm (mid gap), and 3.45 cm (near cathode).

The plasma channel seen in Figure 34 increases in radius at the ends and reaches a radius of 0.06 cm by 13 ns. The electron temperature inside and outside the evolving channel is fairly uniform at 4 eV . The ions remain relatively cool, with an average temperature of $\sim 0.035 \text{ eV}$. At the end of the simulation, the plasma density exceeds 10^{18} cm^{-3} or 10% ionized.

We have also simulated ZR shot 2312 which had peak 5.7 MV voltage and 51.5 psi trigger pressure. We kept the same spatial resolution although we reduced the timestep by half due to the 2x larger pressure. The laser energy was 20 mJ and was focused at the gap center or $z = 2.3 \text{ cm}$. We use this shot to illustrate the effect the filament evolution has on the electric field in the gap. In Figure 35, we see the magnitude of the electric field alongside of the electron density from 2 to 9 ns. As the filament grows in density, the electric field within is reduced and the field outside is enhanced. This process occurs once the plasma electrons exceed roughly 10^{13} cm^{-3} density. Beginning with the 186 kV/cm applied field or 200 Td , the peak field increases to that of self-break by 4 ns (before the laser shuts off) and eventually grows to 600 kV/cm or 660 Td by 9 ns. The speed of filament axial growth increases with time as the increasing enhancement would suggest giving an average 0.5 cm/ns . The speed in the cathode and anode directions is roughly equal indicating that the process is dominated by UV photon ionization. As with shot 2307, this process illustrates ideal triggering and gives a trigger runtime of 10 ns.

Figure 35. For shot 2312 with 5.7 MV voltage and 51.5 psi SF_6 pressure, the electric field (left) and electron density (right) in the trigger gap are plotted from 2 to 9 ns.

We have also simulated several less ideal laser trigger parameters and a higher voltage shot (see Table 1). For shot 2307, these include reducing the laser power and energy by a factor of 2 and 4 (denoted 1/2 and 1/4 power). We also include the one simulation of 2312 with nominal laser parameters. We also kept the energy constant by reduced the power by increasing the pulse length to 6 and 12 ns (denoted 6 and 12 ns pulse). In order to compare simulated switch closure or runtimes with measured values, we adopt a technique similar to that used by Schwarz [86] that defines the runtime with the peak rate of increase in the radiation as measured by the laser spark detector (LSD). Because they decay at a given rate and produce a photon, our measure of UV flux is determined by the number of excited or metastable neutrals n_M that when de-excited produce of 15 eV photon. In the LSP simulations, we see a similar abrupt rise or inflection in n_M that directly corresponds to gap closure by the plasma. Note for the simulations with 3 different laser focal planes, the n_M inflects up at $t = 11.5$ ns.

In the center plot of Fig. 36, the results are plotted from the two shots with nominal laser

Laser parameters	Energy (mJ)	Pulsewidth (ns)	Runtime from metastable inflections (ns)
Nominal 2307	20	3	11.5
1/2 power	10	3	13.3
1/4 power	5	3	16.8
6 ns pulse	20	6	14.6
12 ns pulse	20	12	21.7
Nominal 2312	20	3	9.5

Table 1. Summary of laser parameter variation simulations.

parameters which show slightly improved runtime for the higher voltage, higher pressure shot 2312 compared with the mid-gap shot 2307 simulation. Finally we vary the laser parameters as listed in Table 1. Note the increased runtime as defined by the inflection of the radiation flux to nearly 22 ns. Except for the 12 ns laser pulse simulation, these trigger runtimes are all reasonably in agreement with the measured runtimes of 10 to 20 ns as well as current connection times in the simulation. Also, these simulations demonstrate how switch performance can degrade as the laser power diminishes. A 1/10 laser power simulation was not plotted because the plasma filament never sufficiently evolved.

Figure 36. The number of excited SF_6 atoms which relates directly to radiation production. All cases are for shot 2307 except one curve in the center plot for 2312.

6.3 Integrated Simulation of Trigger Section of ZR

We now examine the integrated behavior of the ZR power flow from the Marx bank, through the gas switch, and into the PFL using LSP with detailed circuit modeling of the various components. Shown in Figure 37, the FDTD model of the trigger section is hooked up to the ZR circuit. We terminated the PFL downstream with a $12\ \Omega$ resistive load. This mismatch (the termination is at a $3.3\ \Omega$ impedance line) yields reasonable reflections of 21 ns duration that produce the so-call

squiggle - indicative of the closure of the trigger gap and capacitive couple of the voltage in the cascade section - as well as best modeling the measured voltage in the PFL.

Figure 37. A schematic of the connection of the simulation domain to the ZR circuit upstream to the intermediate store and Marx and downstream through the PFL. The simulation domain includes only the trigger section. The resistances for the trigger and cascade section are shown in upper right corner.

The agreement of the measured data for shot 2307, a purely circuit representation (“circuit”) and the integrated LSP simulation (“integrated”) voltage at the PFL are shown in Figure 38. In this test case for both the circuit and integrated simulation, the trigger section breakdown was modeled with a time varying resistance that decreased from $10^6 \Omega$ to 0.4Ω with 1.8 ns decay time. The circuit and integrated simulation voltages are indistinguishable from each other. The basic rise of the voltage and peak are also similar to the data. Due to the simplicity of the PFL termination, however, the width of the pulse is larger. In Figure 38b, the early capacitive oscillations of the voltage are quite similar to the data indicating we have a similar capacitive coupling after the trigger fires. Furthermore, we have added the signal from the LSD to the figure time-shifted 21 ns to the PFL voltage position. Note the good agreement between the rise of the LSD and the squiggle.

We now replace the resistive medium in the gap with full SF_6 breakdown Monte Carlo model for the two shots. These simulations are fully electromagnetic and require the simulation be run over $1 \mu\text{s}$ before the laser is fired and the gas breakdown begins. These simulations give an excellent agreement with the programmed resistance model and similar squiggle and LSD response. We are estimating switch inductances at trigger time of 75 nH also in agreement with data.

Figure 38. The PFL voltage for shot 2307 is plotted including data, circuit model and LSP model. A close up of the initial rise of the voltage is shown in (b) showing the capacitive ringing of the switch cascade section after the trigger section conduction. In the simulation, the laser fires at $t = 1061$ ns.

6.4 Discussion and Conclusions about the PIC Model of a Z LTGS

Detailed simulations have been presented of the ZR switch with particle-in-cell kinetic modeling of the SF_6 trigger section. We have examined the sensitivity of switch closure or runtime to the trigger laser power, duration and focal plane within the switch. We found that a strong laser pulse of order 5 to 20 mJ and 3 to 6 ns is sufficient to produce runtimes < 20 ns again in agreement with available data. The progression of the laser-ionized filament is roughly of the same speed in the anode and cathode directions indicative of the dominance of the molecular photoionization process. Once the plasma electrons exceed roughly 10^{13} cm^{-3} density, the electric field is damped within the filament and field enhancement occurs at the filament edges. The ionization front accelerates across the gap as the enhancement increases.

We have also examined the correlation between the rapid growth in measured radiation (using the LSD) and trigger gap runtime. In our simulations, the deduced UV photon production rate from de-excitation of the SF_6^* increases rapidly just as significant plasma density from the filament spans the trigger gap and conducts significant current. The time of this conduction and the inflection in the UV production rate correlate to within 1 ns for all simulations largely validating the method used to calculate runtime on ZR.

In the full electromagnetic kinetic simulations of the trigger, we have been able to follow the runtime and conduction phase close to peak power which allows us to calculate the UV photon signal. These calculations must be followed at least 21 ns after conduction to observe the squiggle at the PFL voltage monitor however. Pushing the simulations out to this point is currently being pursued. We have also developed a technique to transition from full kinetic operation to a 2 fluid technique that included radiation via a multi-group diffusion model [87]. We intend to follow the plasma evolution further in time with this technique.

7 Conclusion

Our research was multi-faceted and ultimately did not achieve its goal of triggering longer gaps with a USP laser, largely due to time constraints. Multiple laser plasma diagnostic methods were developed and vetted. The idea of using such techniques to clearly measure laser ionization mechanisms ran afoul of two problems. For nanosecond laser pulses, the issue was that plasma opacity and refraction made interferometry at early times impossible. For USP laser pulses, the issue was that multiple convoluted ionization and nonlinear effects confuse the data interpretation and corrupt the chance of using an equivalent intensity plane measurement. Ultimately, this led to the development of the new in-line wavefront sensor plasma diagnostic method. Using this, we were able to validate that, at our intensity clamped values, the PPT ionization model applied, confirming that it is indeed valid from the low pressure 10^{-7} Torr region up to the higher pressure 10^3 Torr region used in LTGS systems. With this successful, using the same plasma diagnostic approaches, multiple methods of tailoring the laser-generated plasma length, lifetime, and density were developed and/or explored. Length increases largely dealt with the generalized multipulse method wherein each pulse has a different wavefront curvature and thus focuses to a different region, creating a concatenated plasma string. For our work, this involved using telescopes to establish different wavefronts, using chromatic aberration in focal optics to create different wavefronts at different laser harmonics, and using different USP laser energy levels to create different wavefront curvatures from the differences in self-focusing. Simple lens tilt to induce astigmatism was also explored as a method to redistribute laser energy longitudinally. Plasma lifetime was enhanced via multiple USP laser pulses re-ionizing a region of interest within a close timeframe. Plasma lifetime and density were both shown to increase when using a USP laser to seed LIB from a nanosecond laser pulse, but the plasma continuity seems to suffer.

The implementation of the HV switch work was heavily impacted by safety considerations, which wound up reducing the level of HV and stored energy as well as pushing us toward a bulky Faraday enclosure. This apparatus size in turn drove us from using the operational USP laser system (used for the ionization studies) to building up a developmental system where space was available for the HV system. Unfortunately, although the work on the new OPCPA is nearly complete, the laser did not yield USP operation by the completion of the project. As such, nanosecond LTGS testing looking at the impact of the plasma distribution upon performance was conducted. The results were that various aspects of switch performance (runtime, jitter, and HV breakdown threshold), while loosely coupled, can be observed to be adjustable depending on the plasma channel parameters. Multiple laser pulse concatenated plasma channels showed the best performance with respect to jitter and HV breakdown threshold, which (although implemented in the nanosecond laser pulse regime) was at the heart of the USP LTGS concept.

Finally, modeling was broken down to look at laser ionization and switch performance independently. Both efforts benchmark well to reality and offer new tools going forward for future work.

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