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COLLISIONS: IMPLICATIONS ON PION SPECTRA

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FINITE PARTICLE NUMBER EFFECTS IN HIGH-ENERGY
NUCLEAR COLLISIONS: IMPLICATIONS ON PION SPECTRA*

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ABSTRACT

We study pion production from high energy nuclear collisions by means of a simple statistical model. The shapes of the observed spectra exclude that all pions result from freely decaying delta resonances. Rather, they have to participate in kinetic equilibration processes. Finite particle number effects are found to be very important: equilibration does not occur globally but rather in groups of only a few particles. The pion production rates cannot be explained in terms of a chemical equilibrium.

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INTRODUCTION

Reactions between heavy nuclei at incident energies larger than 0.5 GeV/A produce a considerable amount of π -mesons. These pions are of particular interest: while for the nucleons (protons) observed it is difficult to disentangle contributions of different stages of the reaction from distributions that were present in projectile and target initially, the pions are produced during the reaction and should carry some information on the environment in which, and the conditions under which, they have been created [1]. The question is, however, to what degree after their production the pions participate in processes leading to partial or complete equilibrium and by this lose their memory of the initial stages. Theoretical efforts to predict the behavior of pions were undertaken starting from assumptions covering the whole range between extremes: from single scattering models [2,3,4] reducing nucleus-nucleus collisions to elementary events between nucleons that carry Fermi motion over intranuclear cascade calculations [5-9] implying a superposition of independent elementary processes to thermal models [10-14], which assume a complete randomization in all degrees of freedom and by this a complete loss of memory.

Although by now excellent pion data are available for a huge variety of masses and energies ([15-21] may serve as examples), it was not yet possible to make a final decision which of the models comes closest to reality. It is not the purpose of this paper to give a definite answer to this question. Instead, one of the extreme assumptions is taken, that of complete randomization. We show, however, that by simply taking into account the finite number of particles involved in such reactions,

features may result which have been thought of as being indications for non-randomization. This means that finite particle number effects should be considered very carefully when trying to extract information on the reaction dynamics from the data. The following presentation will be restricted to an outline of the basic physical assumptions underlying our study. Technical details will be discussed in the Appendix.

THE MODEL

As one of our objectives is to elucidate the role of finite particle-number effects on the pion spectra, we employ a model suited for that particular purpose: the statistical model [22,23], now extended to include pion production. This model rests on the fairly general assumption that one particle inclusive cross sections can be written as an incoherent sum over contributions arising from different groups of interacting nucleons:

$$\epsilon \frac{d^3\sigma}{dp^3} = \sum_{M,N} \sigma_{AB}(M,N) F_{MN}(\vec{p}) \quad . \quad (1)$$

Here ϵ and $\vec{p}$ denote total energy and momentum of the observed particle

$$\epsilon = \sqrt{m^2 + \vec{p}^2} \quad .$$

The subscripts M and N imply that the contributing subgroups can be labelled by the number of projectile nucleons M and target nucleons N they contain. A and B denote projectile and target, respectively. We factorize the yield of each group into its formation cross section $\sigma_{AB}(M,N)$ and into a properly normalized probability distribution $F_{MN}(\vec{p})$ that one of the particles out of the group (M,N) carries the momentum $\vec{p}$. The particular advantages of this factorization are:

- i) The formation cross sections are related to observables, especially the mean multiplicities of participants [22,23] or the correlation part in two-particle coincidences [24].
- ii) The same formation cross sections apply irrespective of the nature of the particles observed, whether these are protons [5,7,8,22-24] or pions (considered here) or Λ -particles [25].

As mentioned, the above ansatz (1) is fairly general. The generality is lost and one moves on to a particular model as soon as both σ and F are derived from a specific, generally simplifying, physical picture. In the statistical model [22] employed here, the following interpretation is given to these quantities:

Straight line communication limit — $\sigma_{AB}(M,N)$ is the cross section for finding exactly M nucleons out of the projectile A and simultaneously exactly N nucleons out of the target B inside a tube aligned along the beam direction with a section area of the total nucleon-nucleon cross section. This picture becomes valid in the limit of small transverse momentum transfers (cf. Glauber limit). With a nucleon-nucleon cross section of 40 mb appropriate at 800 MeV these cross sections solely reflect the initial nuclear geometry.

The statistical limit — $F_{MN}(\vec{p})$ is the probability of finding one particle with momentum $\vec{p}$ in an ensemble of $M+N$ nucleons. The ensemble has fixed total energy and momentum available and is allowed to occupy all states accessible under the restriction of energy- and momentum-conservation with equal probability.

The latter assumption implies complete randomization: the energy and momentum brought in by the projectile nucleons are shared equally among all nucleons. Such a completely equilibrated ensemble subject to energy- and momentum-conservation is known as a "microcanonical ensemble"; for large particle numbers its features approach those of the "grand canonical ensemble," where temperature, volume, and chemical potentials are the given quantities and energy and momentum are conserved on the average only.

While the basic assumptions (straight line geometry, complete randomization) are essentially the same as those underlying the so-called "firestreak" model [13,14], the latter deviates in one major point: it treats the independently interacting subgroups of nucleons (the "streaks") as grand canonical ensembles. As the average number of particles per subgroup is low (smaller than 10) in most cases, considerable deviations should arise, especially in kinematical regions close to the limits of phase space (e.g., for large energy of the observed particle). A first investigation of this point with respect to proton spectra was undertaken in [23]. The present work extends the analysis to pion spectra.

In the absence of dynamical effects, the Lorentz invariant momentum distribution of K particles (which may all be of different mass) subject to energy- and momentum-conservation is

$$\phi_K(\vec{p}_1 \dots \vec{p}_K) = \delta\left(\sum_{i=1}^K \epsilon_i - E\right) \delta^3\left(\sum_{i=1}^K \vec{p}_i - \vec{P}\right) / I_K(s) \quad . \quad (2)$$

Here E and $\vec{P}$ are total energy and momentum available to the system and ϵ_i is the energy of the i^{th} particle $\epsilon_i = \sqrt{m_i^2 + \vec{p}_i^2}$. The number

$I_K(s)$ is a normalization constant which has the form of a so-called Lorentz invariant phase space integral

$$I_K(s) = \int \frac{d^3 p_1}{\epsilon_1} \dots \frac{d^3 p_K}{\epsilon_K} \delta\left(\sum_{i=1}^K \epsilon_i - \sqrt{s}\right) \delta^3\left(\sum_{i=1}^K \vec{p}_i\right) \quad (3)$$

with the square of the invariant mass of the system

$$s = E^2 - \vec{P}^2 \quad . \quad (4)$$

If one is interested in the momentum distribution of one particle only (say, the first) one has to integrate the above momentum distribution (2) over all unobserved momenta:

$$\phi_K^1(\vec{p}_1) = \int \frac{d^3 p_2}{\epsilon_2} \dots \frac{d^3 p_K}{\epsilon_K} \phi_K(\vec{p}_1 \dots \vec{p}_K) \quad . \quad (5)$$

In the absence of particle production the number of particles per group is conserved during the reaction. One deals with nucleons only and the statistical limit distribution for observing one nucleon out of the (M,N)-group with momentum $\vec{p}_n$ becomes

$$F_{MN}(\vec{p}_n) = (M+N) \phi_{MN}^1(\vec{p}_n) \quad . \quad (6)$$

Here the factor (M+N) accounts for the indistinguishability of the particles.

PIONS AND NUCLEONS IN EQUILIBRIUM

By including pion production, the total number of particles per group is no longer conserved and one has to introduce a probability rate $P_{MNm}(s)$ for producing exactly m π -mesons in the (M,N) -group under consideration. The respective momentum distribution of a single pion out of this group then becomes

$$F_{MN}(\vec{p}_{\pi}) = \sum_m m P_{MNm}(s) \phi_{MNm}^1(\vec{p}_{\pi}) \quad . \quad (6a)$$

Similarly, the one-nucleon distribution changes to

$$F_{MN}(\vec{p}_n) = \sum_m (M+N) P_{MNm}(s) \phi_{MNm}^1(\vec{p}_n) \quad . \quad (6b)$$

The fact that pions as well as nucleons come in different charge states which are well distinguishable leads to a modification of the factors m and $(M+N)$ respectively. Ignoring charge conservation and assuming isospin symmetry leads to an additional factor $1/3$ in the momentum distribution for the pions (6a) if a specific pion charge state is observed and, in the proton case, to a factor considering the charge-to-mass ratio of projectile and target

$$\left(\frac{Z_P}{A_P} \right) M + \left(\frac{Z_T}{A_T} \right) N \quad , \quad (7)$$

replacing the factor $(M+N)$ in the momentum distribution of the protons (6b). We found that taking into account charge conservation exactly had little implication on the production rates in heavy-ion induced reactions. Therefore, we ignore charge conservation in the following (a simplification which might not apply for proton-induced reactions).

The remaining task now is to specify the probability function P . As the main ingredient of our model is "minimum dynamical input" we adopted Fermi's statistical model of pion production in violent hadronic collisions [26] for this purpose. Its basic assumption is that the probability of producing a certain number of pions out of an ensemble with a fixed number of nucleons is proportional to the number of states accessible, i.e., proportional to the available phase space. This model has widely been used to estimate particle multiplicities and spectra from high energy proton-proton collisions (see review articles [27-30] and a series of articles by Hagedorn [31]). Our case as described below in detail is a direct application to high-energy nuclear collisions.

The phase space available to a system with a fixed number of particles K and subject to energy- and momentum conservation is split into a configuration-space part (given by a volume V to which the system is confined) and a momentum-space part (given by total energy and momentum) so that the number of states becomes

$$\Lambda_K = \left(\frac{V}{(2\pi\hbar)^3} \right)^{K-1} J_K(E, \vec{P}) dE \prod_{i=1}^K ((2\sigma_i + 1)(2\tau_i + 1)) \quad (8)$$

Here σ_i and τ_i denote spin and isospin of the i^{th} particle, and J_K the so-called non-invariant phase space integral (which in the non-relativistic case is just the surface of a sphere in momentum space),

$$J_K(E, \vec{P}) = \int d^3p_1 \dots d^3p_K \delta \left(\sum_{i=1}^K \epsilon_i - E \right) \delta^3 \left(\sum_{i=1}^K \vec{p}_i - \vec{P} \right) \quad (9)$$

Here the same notation as for the phase space integral I_K (3) was used. The condition of momentum conservation actually reduces the number of

independent particles from K to $K-1$, as reflected in the powers of the volume V and the quantum volume $(2\pi\hbar)^3$.

The momentum-space volume $J_K dE$ is not a Lorentz invariant quantity, which makes calculations very cumbersome. However, by substituting the individual particle energies ϵ_i in the expression for the Lorentz invariant phase space integral I_K (3) by an appropriate mean value $\bar{\epsilon}_i$, the non-invariant momentum space integral (9) can be related to the invariant phase space integral [32]

$$J_K = I_K \prod_{i=1}^K \bar{\epsilon}_i \quad . \quad (10)$$

(The usefulness of the invariant notation of the phase space integral was first pointed out by Srivastava and Sudarshan [33].) As a result, given the total c.m. energy available to a group, the number of states available is approximately

$$\Lambda_{MNm} = \left(\frac{V}{(2\pi\hbar)^3} \right)^{M+N+m-1} \epsilon_n^{-M+N} \epsilon_\pi^{-m} I_{MNm}(s) dE 2^{2(M+N)} 3^m \quad , \quad (11)$$

if m pions are produced. According to Fermi's statistical theory this directly gives the probability to produce exactly m pions out of the respective ensemble through

$$P_{MNm} = \frac{\Lambda_{MNm}}{\sum_m \Lambda_{MNm}} \quad . \quad (12)$$

This is the way pion production can be treated in a completely statistical picture. The only free parameter entering the model is a configuration space volume V . It is reasonable to assume it to be

proportional to the number of baryons confined in the group under consideration and by this introduce a density parameter ρ_c which, in analogy to other models [11,12,14], may be interpreted as a "freeze-out density." If this interpretation is physically meaningful the value of the density parameter ρ_c should be somewhat below normal nuclear matter density.

The initial Fermi motion of the nucleons in projectile and target gives rise to a finite width for the total energy and momentum available to each group. This is of special importance for pion production near threshold, but also for the yield at high momenta. In all results to be presented this effect was taken into account by an appropriate folding procedure as described in [22]; the saddle point approximation, however, was replaced by an exact numerical integration.

THE ROLE OF THE DELTA RESONANCE

The model presented above only accounts for nucleons and pions in equilibrium. It ignores the production of any other particles besides the pions like composites or nucleon resonances (deltas) which in turn decay into nucleons and pions, and by this contribute to the pion spectra. Neglecting this contribution might be a rather inaccurate treatment. As far as the knock-out component ($M=N=1$) is concerned, we know that pions are produced dominantly via the delta resonance. It turns out, however, that this component is negligibly small in our approach. Still, one might take the viewpoint that the deltas are as important for all other components as they are for the knock-out. Suppose, for instance, a situation which would be the extreme counterpart of the picture outlined

above: nucleons and deltas are in equilibrium at the end of the reaction and all pions result from the final state decay of these deltas. How would pion spectra then look like? After an 800 MeV/A collision, the delta spectrum will occupy relatively moderate rapidities in the c.m. frame of the participant nucleons. By decaying, the deltas produce pions with comparatively large rapidities and the resulting pion spectrum should resemble the delta mass spectrum to a large extent. We calculated such spectra for the reaction $\text{Ne} + \text{NaF}$ assuming that the momentum distribution of the deltas resembles that of the nucleons, except that their mean kinetic energy is about half that of the nucleons [8]. Applying a normal delta mass distribution (Lorentzian shaped with a full width at half maximum of 110 MeV) gave pion spectra which showed a pronounced hole at low c.m. momenta, in strong deviation from the data [15] (see Fig. 1, dashed lines). We therefore reversed the question and asked: how does the delta mass spectrum have to look in order to reproduce the data? The result is shown in Fig. 1 (full lines). It is not an exact fit in the sense of least χ^2 . Rather, it is an optimization guided by the eye, but it indicates that such a delta dominance model as outlined here is not the right mechanism. It seems more probable that the pions in their majority participate in the equilibration process so that it seems appropriate to study the implications of the statistical model described in the previous section.

RESULTS

In the following, a number of calculations with the statistical model of pion production are presented in comparison with data and, in order to elucidate the role of finite particle number effects, are compared with a kind of firestreak simulation. The latter calculations are performed by retaining the geometry coefficients as they are specified above, but replacing the momentum distributions F by the corresponding bulk limit distributions like

$$F_{MN}^{\text{bulk}}(\vec{p}_\pi) = \lim_{\alpha \rightarrow \infty} \sum_m \frac{m}{\alpha} P_{\alpha M \alpha N m} \phi_{\alpha M \alpha N m}^1(\vec{p}_\pi) \quad . \quad (13)$$

In actual calculations the scale parameter α was chosen such that $\alpha(M+N) = 16$.

It turns out that it is very problematic to understand measured pion production rates in the framework of the statistical model. For illustration, Fig. 2 shows the excitation function of the negative pion multiplicity over the participant proton multiplicity together with recently measured data for the system Ar+KCl [16]. The results of the calculation

- i) show an asymptotic behavior similar to the measured one, namely, a linear relation between pion multiplicity and beam energy, but
- ii) are not able to reproduce the slope and magnitude of the measured dependence at the same time.

Moreover, for values of the density parameter ρ_c below or equal to normal nuclear matter density, the statistical model gives much higher pion

multiplicities than observed. This is true for all models based on the equilibrium assumption [11,12,14,34] which may either allow the conclusion that the pion production process cannot be understood in terms of a "chemical equilibrium" and/or absorption processes play a dominant role after the pions have been produced. Recent cascade calculations [35] indicate the possible existence of another explanation: they show that in a state of relatively high density (two times normal nuclear matter density) already the interactions among the particles start to cease so that the following expansion stage resembles an explosion rather than an adiabatic expansion.

As our main interest is not to give a complete understanding of the pion production process (i.e., the probability function P) but rather to study the influence of finite particle numbers on the spectra (i.e., the momentum distributions ϕ), we consequently abandoned any physical interpretation of the density parameter ρ_c . We took it as a free parameter that was fixed once and for all as to reproduce measured pion multiplicities for beam energies around 1 GeV (which we preferably concentrated on) and then kept constant through all calculations to be presented. We chose a value of two times nuclear matter density (cf. Fig. 2).

Figure 3 shows energy spectra of π^- from the collision of Ne at 800 MeV/A with NaF in the laboratory frame at three different angles. The data are from [17], the full curve is the outcome of the statistical model calculation, the dashed curve shows the result of the firestreak simulation. Both models reproduce the data fairly well, although they overestimate the pion yield at high pion energies. Especially the bulk

limit calculations tend to show this effect, whereas the statistical model results come closer to the data. This is a direct consequence of the finite number of particles involved: as the energy of the observed pion rises, the system approaches the limits of the available phase space, and the cross section has to go down.

It is a general trend of the statistical model to give spectra with slopes steeper than given by the bulk limit. This has remarkable consequences for proton spectra viewed at 90° c.m. angle. For the reaction 800 MeV/A Ne+NaF, Fig. 4 shows that the firestreak model gives a proton spectrum (dashed line) which is too flat compared to the data [36] whereas the statistical model (dotted line) copes with the data.

Pion production "cools down" these spectra as kinetic energy goes into pion masses and thus the slopes become steeper. While the firestreak spectrum needs considerable pion cooling to reproduce the measured proton shapes and by this a pion yield far higher than observed the statistical model spectrum is already slightly too steep for the low value of the pion multiplicity observed (full line). (The latter discrepancy may either originate from the omission of transverse momentum transfer between adjacent groups of nucleons in our picture, or from neglecting the production of composites, which is an exothermic process.)

The slope of the associated pion spectrum is very close to the measured one and turns out to be identical to the slope of the calculated proton spectrum (Fig. 4, full lines). The experimentally observed difference [18] cannot be reproduced by the statistical model. However, it can reproduce the mass dependence of the pion spectra (Fig. 5, data from [15]): it has been observed that the slopes are getting steeper

with decreasing mass of the colliding nuclei, a feature not being reproduced by the firestreak limit. Rather, it can be seen as a genuine feature of our statistical approach which retains finite particle number effects. The smaller the incident nuclei are, the lower is the average number of particles interacting. Thus, the limits of phase space are reached earlier and the asymptotic spectra have to become steeper.

Papp et al [19] measured inclusive pion spectra at small forward angles (2.5°) and found that the cross sections, if plotted versus the Feynman scaling variable x_F (which is the pion momentum in the c.m. frame of projectile and target expressed as a fraction of the maximum pion momentum kinematically possible), obviously scales: the cross section is independent of projectile energy [see data points in Fig. 6 for the reaction $p + C \rightarrow \pi^-(2.5^\circ) + X$]. Landau and Gyulassy [37] pointed out that this might simply be a kinematical effect. The fact that the results of our statistical model calculation show scaling features similar to the data (at least for values of x_F close to 1; full lines in Fig. 6) while the firestreak simulation does not (dashed lines) supports this analysis, as the statistical model treats kinematics correctly by definition whereas the firestreak model does not.

In the backward direction inclusive pion spectra obviously do not scale as the data of Schroeder et al [20] in Fig. 7 show for the reaction $p + Cu \rightarrow \pi^-(180^\circ) + X$. Landau and Gyulassy [37] were able to reproduce the energy dependence of the slopes by assuming that the incoming proton scatters on few-nucleon-clusters within the target nucleus. From a fit to the experimentally observed slopes they were able to derive the size

of the clusters mainly responsible for the backward scattering of pions. The main contributions came from clusters with one or two target nucleons. As can be seen in Fig. 7, the statistical model (full lines) as well as the firestreak simulation (dashed lines) are able to roughly reproduce the measured slopes within the experimental uncertainties (at least for 2.1 GeV projectile energy; for 1.05 GeV the statistical model comes closer to the observed value). While in the former the "effective cluster size" (i.e., the average number of nucleons in a subgroup) for a relatively heavy nucleus like copper certainly is larger than two, in the latter no "cluster size" is involved at all. This throws some doubt on the expectation that there exists a unique relation between the slopes of the spectra and the number of interacting nucleons.

Finally, Fig. 8 impressively shows the ability of the statistical model to treat kinematical limits correctly. Recently the production of π^- at 0° in the laboratory frame was investigated for the reaction ${}^3\text{He} + {}^6\text{Li} \rightarrow \pi^- + X$ for an incident energy of 303 MeV/A [21]. The cross section was measured over eight orders of magnitude up to the kinematical limit corresponding to all nine nucleons recoiling coherently (i.e., forming a ${}^9\text{C}$ nucleus in its ground state) and one pion going in the forward direction. Figure 8 shows that as the statistical model by definition treats kinematics correctly, it is able to reproduce the general behavior of the cross section correctly (full line) while the firestreak simulation fails dramatically (dashed line). The normalization, however, is being underestimated considerably; this is due to the fact that no adjustments for the density parameter have been made. As mentioned before its value has been fixed in order to reproduce observed pion

multiplicities for equal mass collisions in the energy range around 1 GeV/A which certainly is a poor guess for this case of nearly sub-threshold pion production (cf. Fig. 2).

CONCLUSIONS

Two questions may be raised in studying pions produced in high energy nuclear collisions:

- i) Which signature do the shapes of the inclusive pion spectra carry? and
- ii) Do we understand the production rates?

In terms of equilibria the first question concerns kinetic, the latter chemical equilibration processes. Both may not be attained on similar scales.

Various pion production models have been studied in the past. Many of them assume the dominance of delta resonances throughout the hot phase of the reaction where the pions are produced in the final stage only via the free decay of the deltas. We have shown that such a picture cannot even cope with the gross structure of the observed spectral shapes unless extremely unphysical assumptions about the delta mass spectrum are made. But far more likely, the pions themselves, whichever way created, to a great extent participate in equilibration processes before reaching the detector. In fact, the spectral shapes seem to carry the signature of equilibrated systems. Models based on the assumption of thermal equilibrium (bulk limit distributions) are well able to reproduce the overall features of inclusive pion spectra, although major deviations remain. It has been demonstrated in this work that these deviations are

not necessarily indications of a non-equilibrium situation but rather may be a consequence of the finiteness of the systems in which equilibration occurs. Cascade calculations [5-9] have already shown that the mean number of collisions one participant nucleon suffers is low (about three). Thus phase space considerations may play an important role.

In this work we have studied the influence of the phase space on the inclusive pion spectra in a simple model [22] and have been able to show its importance even in cases where the role of kinematics is not that obvious. Apparent scaling features at forward angles can be shown to be a consequence of the finite number of particles involved.

Many predictions for new and unusual physical processes critically rely on the temperature that may be reached in (parts of) the system. Usually such temperatures are extracted from the asymptotic slopes of the inclusive spectra. We see these slopes greatly influenced by the number of particles the system contains. Thus finite particle number effects play an especially important role in the determination of apparent temperatures, a feature which should cautiously be considered when trying to extract such information from the data.

As far as the production rates are concerned we found that a chemical equilibration picture is not capable of reproducing the observed yields, at least not within reasonable bounds for the density parameter. In particular, the calculated rates rise more drastically with energy than seen experimentally.

In general it can be said that inclusive pion spectra from high energy nuclear collisions seem to reflect a great deal of features that are typical for equilibrated but finite systems. The pion production

process itself, however, has not been understood yet and remains a field open to further research.

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APPENDIX

The formation cross sections $\sigma_{AB}^{(M,N)}$ are calculated in the same way as in [8] and [22]; a detailed description of the method can be found there.

The Lorentz invariant phase space integral (3) can be calculated recursively [22,31]

$$I_K(s) = \frac{\pi}{s} \int_{b_1}^{b_2} ds' \sqrt{\lambda(s, s', m_K)} I_{K-1}(s') \quad , \quad (A1)$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \quad , \quad (A2)$$

$$b_1 = \left(\sum_{i=1}^{K-1} m_i \right)^2 \quad , \quad b_2 = (\sqrt{s} - m_K)^2 \quad , \quad (A3)$$

$$I_1(s) = 2\delta(s - m_1^2) \quad . \quad (A4)$$

This phase space integral governs both the production probabilities P (12) and the momentum distributions F (6,6a,6b). The latter can easily be shown by evaluating Eq. (5) using Eq. (2), which yields

$$\phi_K^1(\vec{p}_1) = \frac{I_{K-1}(s_1)}{I_K(s)} \quad (A5)$$

with

$$s_1 = (E - \epsilon_1)^2 - (\vec{P} - \vec{p}_1)^2 \quad .$$

In a group of M projectile nucleons, N target nucleons, and m pions the total available energy and momentum E and $\vec{P}$ are affected by the Fermi motion of the nucleons. With $\vec{P}_A$ denoting the sum of the Fermi

motion momenta of the projectile nucleons and $\vec{P}_B$ being the sum of the Fermi motion momenta of the target nucleons the total energy and momentum and thus the variables s and s_1 become functions of $\vec{P}_A$ and $\vec{P}_B$,

$$\begin{aligned} s &= s\left(E(\vec{P}_A, \vec{P}_B), P(\vec{P}_A, \vec{P}_B)\right) = s(\vec{P}_A, \vec{P}_B) \\ s_1 &= s_1(\vec{P}_A, \vec{P}_B) \end{aligned} \quad (A6)$$

As in [22] the explicit form of (A6) considers the nucleons as off-shell particles with $\epsilon_n = m_n$ and $|\vec{p}| \lesssim p_F$ in the respective nuclear rest frame. The momentum distribution of one pion e.g. (6a), results from an appropriate folding over all possible values of $\vec{P}_A$ and $\vec{P}_B$

$$\begin{aligned} F_{MN}(\vec{p}_\pi) &= \sum_m m \int d^3P_A d^3P_B \omega_M(\vec{P}_A) \omega_N(\vec{P}_B) \\ &\times P_{MNm}(s(\vec{P}_A, \vec{P}_B)) I_{MNm-1}(s_1(\vec{P}_A, \vec{P}_B)) / I_{MNm}(s(\vec{P}_A, \vec{P}_B)) . \end{aligned} \quad (A7)$$

Here $\omega_M(\vec{P}_A)$ and $\omega_N(\vec{P}_B)$ denote the distributions of the momenta $\vec{P}_A$ and $\vec{P}_B$ in the respective rest frames of the nuclei. As in [22] we assumed Gaussian distributions with a width given by the parabolic law of independent particle motion

$$\begin{aligned} \sigma_{AM}^2 &= \frac{M(A-M)}{A-1} \frac{1}{5} p_{FA}^2 \\ \sigma_{BN}^2 &= \frac{N(B-N)}{B-1} \frac{1}{5} p_{FB}^2 \end{aligned} \quad (A8)$$

where p_{FA} and p_{FB} are the respective Fermi momenta in projectile and target nucleus.

Starting from expression (A4) with m_1 being the nucleon mass, we calculated a table of values of the phase space integral for nucleon numbers up to 16 and pion numbers up to 10 and a range in the square of the invariant mass s covering equal mass collisions up to 2 GeV/A. Spectra and production probabilities were evaluated using interpolated data from this table. The Fermi motion correction was carried through by a numerical integration instead of using a saddle point approximation as in [22]. This requires more computer time, but has the advantage of being an exact method. Still the time required for calculating one inclusive cross section is comparatively short: three minutes on a VAX 11/780 computer for a collision of Ne with NaF.

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FIGURE CAPTIONS

- Fig. 1. Inclusive pion spectra from the free decay of delta resonances, viewed at 90° in the c.m. system of the colliding nuclei. Data are from [15]. Left: calculated spectra for two different assumptions on the delta mass distribution. Right: the corresponding delta mass distributions. (The dashed line corresponds to the dashed line, the full line to the full line.)
- Fig. 2. The mean number of negative pions over the mean number of participant protons versus projectile energy in a collision Ar+KCl. Data points are from [16]. The curves show the results of a statistical model calculation for three different values of the density parameter ρ_c . ρ_0 is the normal nuclear matter density of 0.16 fm^{-3} .
- Fig. 3. Inclusive pion spectra at three different angles in the laboratory frame. Data are from [17]. Full line: statistical model calculation. Dashed line: firestreak simulation.
- Fig. 4. Proton and pion inclusive spectra as viewed at 90° in the c.m. frame of the colliding nuclei. Data are from [36] and [15]. Dashed line: firestreak simulation for the protons, no pion cooling. Dotted line: statistical model calculation for the protons, no pion cooling. Full lines: statistical model calculation for the protons (including pion cooling effects) and the pions, respectively.

Fig. 5. Inclusive pion spectra at 90° in the c.m. frame of the colliding nuclei for two different projectile (target) masses. Data are from [15]. Full lines: statistical model calculation. Dashed lines: firestreak simulation.

Fig. 6. Inclusive pion spectra at 2.5° for two different projectile energies plotted versus the Feynman scaling variable x_F . Data are from [19]. Full lines: statistical model calculation. Dashed lines: firestreak simulation.

Fig. 7. Inclusive pion spectra at 180° for two different projectile energies plotted versus the Feynman scaling variable x_F . Data are from [20]. Full lines: statistical model calculation. Dashed lines: firestreak simulation.

Fig. 8. Inclusive pion spectrum at 0° in the laboratory frame, measured up to the kinematical limit [21]. Full line: statistical model calculation. Dashed line: firestreak simulation.

Fig. 1

XBL 806-1266

XBL 806-1265

Fig. 2

XBL 806-1271

Fig. 3

XBL 806-1267

Fig. 4

XBL 806-1268

Fig. 5

XBL 806-1269

Fig. 6

XBL 806-1270

Fig. 7

XBL 806-1272

Fig. 8