SANDIA REPORT

SAND2012-6915

Unlimited Release

Printed September 2012

Shaping the Spectrum of Random-Phase Radar Waveforms

Armin W. Doerry, Brandeis Marquette

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Approved for public release; further dissemination unlimited.

Sandia National Laboratories

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

NOTICE: This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government, nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, make any warranty, express or implied, or assume any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represent that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government, any agency thereof, or any of their contractors or subcontractors. The views and opinions expressed herein do not necessarily state or reflect those of the United States Government, any agency thereof, or any of their contractors.

Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from

U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865) 576-8401
Facsimile: (865) 576-5728
E-Mail: reports@adonis.osti.gov
Online ordering: <http://www.osti.gov/bridge>

Available to the public from

U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Rd.
Springfield, VA 22161

Telephone: (800) 553-6847
Facsimile: (703) 605-6900
E-Mail: orders@ntis.fedworld.gov
Online order: <http://www.ntis.gov/help/ordermethods.asp?loc=7-4-0#online>

SAND2012-6915
Unlimited Release
Printed September 2012

Shaping the Spectrum of Random-Phase Radar Waveforms

Armin W. Doerry
ISR Mission Engineering
Sandia National Laboratories
Albuquerque, NM 87185-0519

Brandeis Marquette
Reconnaissance Systems Group
General Atomics Aeronautical Systems, Inc.
San Diego, CA 92127

Abstract

Noise and noise-like waveforms may be generated by random modulation of only the phase of a sequence of samples. Furthermore, the spectral characteristics of resulting waveform may be shaped by suitably constraining the statistics of the random phase modulations. This maximizes the Signal-to-Noise ratio of the output of a matched filter by shaping the autocorrelation of the resulting sequence.

Acknowledgements

The production of this report was funded by General Atomics Aeronautical Systems, Inc. (ASI) Reconnaissance Systems Group (RSG).

GA-ASI, an affiliate of privately-held General Atomics, is a leading manufacturer of unmanned aircraft systems (UAS), tactical reconnaissance radars, and surveillance systems, including the Predator UAS series and Lynx Multi-Mode radar systems.

Contents

Foreword	6
1 Introduction & Background	7
2 Overview & Summary	11
3 Detailed Analysis	13
3.1 FM Chirps	14
3.1.1 LFM Chirp	15
3.1.2 LFM Stepped Chirp	15
3.1.3 NLFM Stepped Chirp	20
3.1.4 NLFM Stepped Chirp – Alternate Technique	21
3.2 Random Phase Increments	26
3.2.1 Random Phase / Random Frequency	27
3.2.2 Limiting Random Phase Increments	27
3.2.3 Random Frequency Chips	30
3.2.4 Random Frequency Chips with Shaped Spectrum	30
3.2.5 Quantized Random Frequency Chips with Shaped Spectrum	31
3.2.6 Randomly Shuffled Quantized Frequency Chips with Shaped Spectrum ..	38
3.2.7 Compound Pulses with Multiple Chip Decks	42
3.2.8 Comments	46
4 Conclusions	49
Appendix A – Matlab Random Number Generator with Specified Density	51
Appendix B – Matlab Random Shuffle Function	53
References	55
Distribution	58

Foreword

General Atomics Aeronautical Systems, Inc., builds the high-performance Lynx SAR/GMTI system.

This report details the results of an academic study. It does not presently describe any modes, methodologies, or techniques employed by any operational system known to the authors.

The specific mathematics and algorithms presented herein do not bear any release restrictions or distribution limitations.

This distribution limitations of this report are in accordance with the classification guidance detailed in the memorandum "Classification Guidance Recommendations for Sandia Radar Testbed Research and Development", DRAFT memorandum from Brett Remund (Deputy Director, RF Remote Sensing Systems, Electronic Systems Center) to Randy Bell (US Department of Energy, NA-22), February 23, 2004. Sandia has adopted this guidance where otherwise none has been given.

This report formalizes preexisting informal notes and other documentation on the subject matter herein.

1 Introduction & Background

A typical pulse-Doppler radar system emits a series of pulses, and collects echo signals. For each pulse, these echo signals are correlated against the transmitted waveform to provide a range sounding, and the range soundings are compared against each other across pulses to discern Doppler information. The correlation function may be implemented as an equivalent matched filter, or as a direct correlation.

Typical radar modes that operate in this fashion include Synthetic Aperture Radar (SAR), Inverse-SAR (ISAR), various Moving Target Indicator (MTI) radars, and coherent search radar systems. This is certainly not an exhaustive list. Herein we concern ourselves with generically range-Doppler radars.

The choice of waveforms to use will depend on the objectives of the radar system with respect to ease of waveform generation, downstream processing issues, and desires for probabilities of detection, interception, spoofing, etc. We typically desire waveforms that offer a large time-bandwidth product to afford both high energy and wide bandwidth for improved range resolution. There are a plethora of waveforms from which to choose. These would include, but are not limited to, the popular Linear-Frequency-Modulated (LFM) chirp, Non-Linear FM (NLFM) chirp, stepped frequency systems, various phase-coded modulation schemes, and even random and pseudo-random noise waveforms. Each has its own set of advantages and disadvantages.

We focus herein on random and pseudo-random noise waveforms; in fact a specific subset of these.

A typical radar system employs a final transmit power amplifier that is normally operated in compression, required to maximize transmitted power output and/or efficiency. That is, the amplifier operates in a non-linear fashion and no longer faithfully reproduces amplitude modulations, severely limiting them. Phase modulations are, however, still easily passed with minimum distortion. This implies that radar systems with such power amplifiers favor phase/frequency modulation rather than amplitude modulations. This is also true for random and pseudo-random noise waveforms.

Accordingly, the principal subject matter of this report is random and pseudo-random phase modulations.

It is well-known that the output of a matched filter, when input with a signal to which it is matched, is the autocorrelation function of the waveform. Furthermore, the autocorrelation of a function is related by the Fourier Transform (FT) to the Energy Spectral Density (ESD) of the waveform. That is, the autocorrelation function and ESD are FT-pairs. We desire matched filters as their principal advantage is to maximize the Signal-to-Noise Ratio (SNR) of energy in the final range-Doppler map. Most radar processing seeks to implement matched filters, or at least nearly so.

One problem with matched filters for many waveforms is undesirably high processing sidelobe levels in the range-Doppler map. These are usually mitigated with additional filtering, often by using data tapering, or window functions, during the processing. Although this somewhat ‘un-matches’ the filter, resulting in some usually slight degradation of the range-Doppler map SNR, this trade is usually deemed worthwhile. While we term this SNR degradation as “slight”, it is typically in the 1-2 dB range.

An alternative is to use waveforms designed to exhibit desirable ESD properties, where the autocorrelation of the waveform exhibits desirable, or at least acceptable processing sidelobe levels directly, that is, without additional filtering and the attendant SNR loss.

Accordingly, we further refine the object of this report to present and discuss random and pseudo-random phase modulations with desirable ESD properties to maximize SNR in a range-Doppler map. Necessarily, we will consider the effects of random modulations as manifest in the entire range-Doppler map, that is, by all pulses in a Coherent Processing Interval (CPI) or synthetic aperture.

Towards this end, we present the following reference material as background information. We note that the literature is rich with publications dealing with noise radar and related topics, with the following representing an incomplete sampling of what is available.

A concise history of noise-radar development, originating in the 1950’s, is given by Lukin & Narayanan.¹ One of the earliest papers, and seminal in the field was written by Horton.²

Much analysis and a number of experimental systems deal with radar waveforms generated by essentially Gaussian Noise sources. These are essentially both amplitude and phase modulated systems. Papers include those by Narayanan, et al.,³ Garmatyuk & Narayanan,⁴ and Bell & Narayanan.⁵

A number of papers also deal with modulating a LFM chirp with noise, both amplitude and phase. These would include papers by Govoni & Li,^{6,7} and Govoni & Moyer.⁸

Stepped-frequency waveforms combined with random signals have been reported by a number of researchers. Lukin, et al.,^{9,10} describe “stepped variation of a single frequency signal over a discrete frequency mesh according to a random law, i.e. frequency hopping with random hops; step-like increase of the central frequency of a narrow band random signal; and frequency hopping of a narrow band random signal according to a random law (stepped frequency), i.e. random frequency hopping of random signals.” Gu, et al.,¹¹ describe what they call “stepped-frequency random noise signal (SFRNS).”

Vela & Lo Monte¹² also describe noise modulated frequency steps and allow that “non-linear steps or spectrally discrete waveforms are also acceptable.” Vela, et

al.,¹³ apply window functions to a “multi-tone” waveform in the time domain to control spectral characteristics.

Hong, et al.,¹⁴ present “a new kind of noise radar called the random binary phase-coded (RBPC) CW radar.”

Zhang & Chen¹⁵ present a waveform “obtained by adding noise frequency modulation to normal LFM signal.”

A particularly nice set of papers is presented by Axelsson^{16,17,18} who analyzes random phase and frequency modulation, including stepped chirps, including shaping the autocorrelation function via adjusting the probability density function of random and shuffled frequencies.

Generating and processing NLFM chirps by shaping the waveform ESD are detailed in a pair of reports written by Doerry.^{19,20} These reports do not however specifically deal with noise waveforms.

A number of papers deal directly with suppressing processing sidelobes when using various random waveforms. These include papers by Jariani, et al.,²¹ Haghshenas & Nayebi,²² and Wu, et al.²³

Chaotic functions applied as phase/frequency modulation has also been reported by a number of researchers. These include papers by Xu & Feng,²⁴ Hall, et al.,²⁵ and Chandra, et al.²⁶

More recently, a tie has been made between noise radar and compressive sensing by Shastry, et al.²⁷

Most papers ignore processing sidelobe suppression entirely. Several treat sidelobe suppression by operating on received data only. These tend to either reduce SNR, or be non-linear in nature with resultant adverse effects to some subsequent exploitation schemes. Some discuss shaping the ESD, but typically limit themselves to specific functions like a Gaussian distribution of frequencies to reduce processing sidelobes in a SAR image. A relatively few mention how a probability distribution in random characteristics of a waveform affects the ESD of the waveform.

What remains missing from these is any thorough discussion of techniques for designing and generating random phase/frequency modulated waveforms (only) with precise ESD characteristics, and hence precise Impulse Response (IPR) shapes particularly in the mainlobe, while retaining maximum SNR in any resulting range-Doppler map.

“Chaos is a friend of mine.”
-- Bob Dylan

2 Overview & Summary

We begin by noting that a random-phase signal will have a white spectrum. Furthermore, a sampled-data random-phase signal will have a white spectrum over the bandwidth equal to the sampling frequency.

A random phase signal will also have random phase differences between two independent random phases. A phase increment across a time increment is in fact a phase-rate, or frequency. A phase-rate change is in fact a frequency-hop.

By controlling the phase-rate, that is the characteristics of the phase increments, we can control the spectrum of the random-phase waveform.

Spectrum precision and sharpness is enhanced by holding a frequency for some ‘chip’ length. For digitally generated phase samples, this means that the chip length needs to be many samples. This is a time-bandwidth issue. The definition of ‘many’ will depend on the sharpness desired, but often several tens’ of samples will be adequate.

To shape the ESD of a random-phase signal, we need to control the average energy at various phase-rates. This can be done with either or a combination of

1. Controlling the likelihood of specific phase increments, and/or
2. Controlling the duration of a specific phase increment chip length.

For range-Doppler images, it is the 2-dimensional IPR that is of principal concern. This will tend to average out the random effects of any single pulse.

“I think there's something strangely musical about noise.”
-- Trent Reznor

3 Detailed Analysis

We provide the following detailed analysis in two parts. We begin by reviewing the relevant characteristics of various FM chirp waveforms. This will provide a baseline reference for the following discussions. Subsequently we develop and present various random-phase waveforms, comparing them to the reference FM chirp waveforms.

We will make heavy use of examples. Unless otherwise noted, we will presume a digitally generated phase with the following pulse-Doppler radar parameters.

$$\begin{aligned} f_s &= 1 \text{ GHz} = \text{sampling frequency,} \\ B_T &= 500 \text{ MHz} = \text{desired waveform bandwidth,} \\ T &= 10.24 \text{ } \mu\text{s} = \text{pulse width,} \\ N &= 1024 = \text{number of pulses.} \end{aligned} \tag{1}$$

The sampling frequency is normally given as a limitation of the waveform generation hardware. The waveform bandwidth is calculated in the customary manner from some required range resolution of the radar. The pulse-width is typically chosen to meet timing requirements for the ranges of interest to prevent occlusion, and the number of pulses is calculated in well-known manners to meet Doppler resolution requirements and/or SNR requirements.

We note that we can define and calculate the fractional bandwidth as

$$b = \frac{B_T}{f_s} = \text{fractional bandwidth.} \tag{2}$$

The sample spacing or period within a pulse is calculated as

$$T_s = \frac{1}{f_s} = \text{sample period within pulse.} \tag{3}$$

The number of samples within a single pulse is calculated as

$$I = \frac{T}{T_s} = \text{number of sample periods within pulse.} \tag{4}$$

We will somewhat arbitrarily presume that we ultimately desire IPR sidelobe mitigation to a level consistent with a -35 dB Taylor weighting ($\bar{n} = 4$). This is a popular IPR shape for SAR, otherwise we could have just as easily chosen a different characteristic. Other modes often do so.

And lastly, we will presume no system noise so that all the uncertainty is purely a function of the waveform, and not of the measurement.

3.1 FM Chirps

We present next some variations of FM chirps. These are typically defined as monotonic frequency slopes from beginning to end of a pulse. More precisely, we are defining them as waveforms with different beginning and ending instantaneous frequencies, with non-reversing frequency slopes (allowing sections of zero slope). The frequency slope may generally be positive or negative (or at times zero), although we will present examples with only non-negative slopes.

FM chirps have deterministic phase functions, which can generally be described as

$$\Phi(i, n) = \phi_n + \sum_{i'=-I/2}^i \Omega(i', n), \quad (5)$$

where

$$\begin{aligned} n &= \text{pulse index, } -N/2 \leq n < N/2 - 1, \\ i &= \text{waveform sample index, } -I/2 \leq i < I/2 - 1, \\ \phi_n &= \text{reference phase for the } n^{\text{th}} \text{ pulse,} \end{aligned} \quad (6)$$

and

$$\Omega(i, n) = \text{an instantaneous frequency function.} \quad (7)$$

The difference between various FM chirps is in the nature of $\Omega(i, n)$.

In general, $\Omega(i, n)$ will also be centered on a reference frequency, and exhibit some chirp rate, where

$$\begin{aligned} \omega_n &= \text{reference frequency for the } n^{\text{th}} \text{ pulse,} \\ \gamma_{i,n} &= \text{chirp rate for the } i^{\text{th}} \text{ sample of the } n^{\text{th}} \text{ pulse.} \end{aligned} \quad (8)$$

The parameters ϕ_n , ω_n , and $\gamma_{i,n}$ may be modulated on a pulse to pulse basis to provide motion compensation.²⁸ In addition, ϕ_n may be additionally modulated to provide other benefits (e.g. ambiguous range mitigation, etc.).

3.1.1 LFM Chirp

The instantaneous frequency of the LFM chirp waveform will have form

$$\Omega(i, n) = \omega_n T_s + \gamma_{0,n} (T_s)^2 i, \quad (9)$$

Figure 1 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 2 shows the autocorrelation function of the waveform of Figure 1. Also shown for comparison is a Taylor window desired compressed pulse response. Figure 3 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 4 shows principal axes cuts of the range-Doppler map in Figure 3.

3.1.2 LFM Stepped Chirp

The instantaneous frequency of the waveform will now have a stepped form

$$\Omega(i, n) = \omega_n T_s + \gamma_n (T_s)^2 f(i), \quad (10)$$

where

$$f(i) = \text{some quantized function of index } i. \quad (11)$$

For the stepped chirp, we set

$$f(i) = \tau_{step} \text{floor}(i / \tau_{step}), \quad (12)$$

where the step length in samples is calculated as

$$\tau_{step} = I / K = \text{step length in samples}, \quad (13)$$

where

$$K = \text{the number of frequency steps in a single pulse}. \quad (14)$$

The number of samples and step-size are chosen to balance the sharpness of the ESD edges with the flatness of the passband.

For our examples $b = 0.5$, and $K = 160$. This allows each step to be a uniform $\tau_{step} = 64$ samples.

Figure 5 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 6 shows the autocorrelation function of the waveform of Figure 5. Also shown is a Taylor window desired compressed pulse response. Figure 7 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 8 shows principal axes cuts of the range-Doppler map in Figure 7.

Figure 1. LFM chirp energy spectrum. Note the nearly rectangular shape.

Figure 2. LFM chirp autocorrelation function. The reference function is a -35 dB Taylor window with $\bar{n} = 4$.

Figure 3. LFM chirp pulses: 2-D IPR of data set of with Taylor windows applied in both fast-time and across pulses. Color units are in dB.

Figure 4. Principal axes cuts of 2-D IPR.

Figure 5. LFM stepped chirp energy spectrum. Note the nearly rectangular shape, albeit with some additional fluctuations compared to Figure 1.

Figure 6. LFM stepped chirp autocorrelation function. The reference function is a -35 dB Taylor window with $\bar{n} = 4$.

Figure 7. LFM stepped chirp pulses: 2-D IPR of data set of with Taylor windows applied in both fast-time and across pulses. Color units are in dB.

Figure 8. Principal axes cuts of 2-D IPR.

3.1.3 NLFM Stepped Chirp

The instantaneous frequency of the NLFM waveform will now still have form

$$\Omega(i, n) = \omega_n T_s + \gamma_n (T_s)^2 f(i), \quad (15)$$

but where the quantization function of index i is now adjusted as a nonlinear function of index i . Here we will adjust the duration of otherwise linearly spaced frequencies. Accordingly, we will quantize the index i as

$$f(i) = \frac{I}{K} k, \quad (16)$$

where k is the minimum integer that satisfies

$$i \leq \sum_{k'=-K/2}^k \tau_{step,k'} = \text{accumulated step length in samples, } -K/2 \leq k < K/2, \quad (17)$$

and where the length of the individual steps or chips themselves are adjusted as a function of the window taper function as

$$\tau_{step,k} = \text{round}\left(\frac{I}{K} w_K(k)\right) = \text{the length of the individual frequency chips}, \quad (18)$$

and

$$w_K(k) = \text{the weighting or window taper function employed}. \quad (19)$$

For our examples $b = 0.5$, $K = 160$, and $w_K(k)$ embodies the Taylor window function. As a practical matter, the rounding operation needs to facilitate the constraint that the sum of all steps equals the total number of waveform samples, that is

$$\sum_{k'=-K/2}^{K/2-1} \tau_{step,k'} = I. \quad (20)$$

Figure 9 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 10 shows the autocorrelation function of the waveform of Figure 9. Also shown is a Taylor window desired compressed pulse response. Note that the autocorrelation function is substantially on top of the reference IPR, although some spikes away from the mainlobe are present. These are due to the quantization of the chip duration steps $\tau_{step,k}$. Figure 11 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 12 shows principal axes cuts of the range-Doppler map in Figure 11.

3.1.4 NLFM Stepped Chirp – Alternate Technique

The instantaneous frequency of the NLFM waveform will now have the more generic form

$$\Omega(i, n) = \omega_n T_s + \Delta\omega_{n,i} T_s, \quad (21)$$

but where

$$\Delta\omega_{n,i} = \text{instantaneous frequency offset for the } i^{\text{th}} \text{ sample in the } n^{\text{th}} \text{ pulse.} \quad (22)$$

Frequency increments will be stepped, uniformly distributed in time, but the steps themselves will be nonlinear, and a function of the desired spectral taper. We begin by determining the instantaneous frequency increments for a non-quantized NLFM chirp. We may use an algorithm as is detailed in the Sandia report by Doerry.¹⁹ This gives us an ideal frequency offset for each sample index i which we denote with

$$\Delta\omega_{n,i,ideal} = \sum_{i'=-I/2}^i \gamma_{n,i,ideal} T_s = \text{ideal frequency offset for the } i^{\text{th}} \text{ sample,} \quad (23)$$

where

$$\gamma_{n,i,ideal} = \text{ideal chirp rate for the } i^{\text{th}} \text{ sample.} \quad (24)$$

We once again calculate the quantized chip index as

$$\begin{aligned} f(i) &= \tau_{step} \text{ floor}(i/\tau_{step}), \\ \tau_{step} &= I/K = \text{step length in samples.} \end{aligned} \quad (25)$$

This allows us now to quantize the frequency offsets as

$$\Delta\omega_{n,i} = \Delta\omega_{n,f(i),ideal} = \text{quantized frequency offset for the } i^{\text{th}} \text{ sample.} \quad (26)$$

For our examples $b = 0.5$, $K = 640$, and $\Delta\omega_{n,i,ideal}$ embodies the Taylor window function.

Figure 13 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 14 shows the autocorrelation function of the waveform of Figure 13. Also shown for comparison is a Taylor window desired compressed pulse response. Note that the autocorrelation function is substantially on top of the reference IPR. Figure 15 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 16 shows principal axes cuts of the range-Doppler map in Figure 15.

Figure 9. NLFM stepped chirp energy spectrum. Note the tapering across the relevant spectrum.

Figure 10. NLFM stepped chirp autocorrelation function. The reference function is a -35 dB Taylor window with $\bar{n} = 4$. They agree quite well except for some spikes away from the mainlobe.

Figure 11. NLFM stepped chirp pulses: 2-D IPR of data set of with Taylor window applied only across pulses. The range sidelobe reduction is due to the spectral shaping of the waveform. Color units are in dB.

Figure 12. Principal axes cuts of 2-D IPR.

Figure 13. Alternate NLFM stepped chirp energy spectrum. Note the tapering across the relevant spectrum.

Figure 14. Alternate NLFM stepped chirp autocorrelation function. The reference function is a -35 dB Taylor window with $\bar{n} = 4$.

Figure 15. Alternate NLFM stepped chirp pulses: 2-D IPR of data set of with Taylor window applied only across pulses. The range sidelobe reduction is due to the spectral shaping of the waveform. Color units are in dB.

Figure 16. Principal axes cuts of 2-D IPR.

3.2 Random Phase Increments

We present next some waveforms where the phase contains some random nature. Specifically, we are interested in waveforms where the frequency is to some degree random. The phase is of course the integral of the frequency. Our ultimate intent is to shape the spectrum of the waveform by manipulating the statistics of the randomness.

Recall that the instantaneous frequency is just the time-rate of change of phase. In a sampled data system, it is the phase increment per sample time. Consequently, we will retain the form of the phase expression in terms of an accumulated instantaneous frequency, that is

$$\Phi(i, n) = \phi_n + \sum_{i'=-I/2}^i \Omega(i', n), \quad (27)$$

where

$$\begin{aligned} n &= \text{pulse index, } -N/2 \leq n < N/2 - 1, \\ i &= \text{waveform sample index, } -I/2 \leq i < I/2 - 1, \\ \phi_n &= \text{reference phase for the } n^{\text{th}} \text{ pulse,} \end{aligned} \quad (28)$$

and

$$\Omega(i, n) = \text{an instantaneous frequency function.} \quad (29)$$

The difference between various waveforms is again in the nature of $\Omega(i, n)$.

In general, $\Omega(i, n)$ will also again be centered on a reference frequency, and exhibit some now random offset, where the instantaneous frequency of the waveform will now have the more generic form

$$\Omega(i, n) = \omega_n T_s + \Delta\omega_{n,i} T_s, \quad (30)$$

where

$$\begin{aligned} \omega_n &= \text{reference frequency for the } n^{\text{th}} \text{ pulse,} \\ \Delta\omega_{n,i} &= \text{frequency offset for the } i^{\text{th}} \text{ sample of the } n^{\text{th}} \text{ pulse.} \end{aligned} \quad (31)$$

As before, the various parameters may be additionally modulated on a pulse to pulse basis to provide motion compensation. In addition, ϕ_n may be additionally modulated to provide other benefits (e.g. ambiguous range mitigation, etc.).

3.2.1 Random Phase / Random Frequency

We begin a staged approach to our investigation by beginning with perhaps the simplest random phase waveform. Accordingly, we set

$$\Omega(i, n) = 2\pi \left(X_u(i, n) - \frac{1}{2} \right), \quad (32)$$

where

$$X_u(i, n) = \text{uniformly distributed random value over the interval } [0,1). \quad (33)$$

To be sure, in this case $X_u(i, n)$ is a different random selection for each index i and n .

Figure 17 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 18 shows the autocorrelation function of the waveform of Figure 17. Also shown for comparison is a Taylor window desired compressed pulse response.

3.2.2 Limiting Random Phase Increments

We next illustrate constraining the frequency increments by setting

$$\Omega(i, n) = 2\pi b \left(X_u(i, n) - \frac{1}{2} \right). \quad (34)$$

We would expect this to alter the ESD and associated autocorrelation function, depending on fractional bandwidth factor b . For our examples $b = 0.5$.

Figure 19 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 20 shows the autocorrelation function of the waveform of Figure 19. Also shown is a Taylor window desired compressed pulse response.

Note that it is difficult to define an edge to the ESD to readily identify the waveform bandwidth. In other words, there is considerable ‘spillage’ outside the band of interest. This is because the frequency is allowed to change over its entire interval for each increment in index i . To sharpen the edges, we need to provide more correlation from sample to sample. One way to do this is with the concept of a ‘chip’, where frequency may randomly jump only at chip boundaries where a chip is some number of individual samples.

The sharpness of the band edges is essentially then a time-bandwidth issue for the chip.

Figure 17. Random phase energy spectrum. Note the ensemble average of the spectra is much flatter than the spectrum of a single pulse.

Figure 18. Random phase autocorrelation function. The reference function is a -35 dB Taylor window with $\bar{n} = 4$. Note how sidelobes diminish with ensemble averages.

Figure 19. Limited random phase energy spectrum. Note the ensemble average of the spectra is much smoother than the spectrum of a single pulse.

Figure 20. Limited random phase autocorrelation function. The reference function is a -35 dB Taylor window with $\bar{n} = 4$. Note the sidelobe reduction and mainlobe broadening.

3.2.3 Random Frequency Chips

We may sharpen the edges of the spectrum by allowing frequency changes only at chip edge boundaries. We model this as

$$\Omega(i, n) = 2\pi b \left(X_u(f(i), n) - \frac{1}{2} \right), \quad (35)$$

where

$$f(i) = \tau_{step} \text{floor}(i / \tau_{step}). \quad (36)$$

This may be viewed as a frequency-hopping spread-spectrum technique. We would expect this to sharpen the edges of the ESD and thereby shape the associated autocorrelation function accordingly, depending on fractional bandwidth factor b . For our examples $b = 0.5$.

Figure 21 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 22 shows the autocorrelation function of the waveform of Figure 21. Also shown is a Taylor window desired compressed pulse response. Figure 23 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 24 shows principal axes cuts of the range-Doppler map in Figure 23.

Immediately obvious is the apparent noise level in the off-axis directions from the mainlobe peak in Figure 23, both as a brighter band at ranges nearer the peak, and a residual lower level more distant from the peak. This is in fact sidelobe energy due to the residual phase noise due to the randomness within the waveform. This off-axis sidelobe energy is at a fairly low level, in our example at approximately -50 dBc in the band and approximately -60 dBc outside the band. These levels will depend on integration gain in general, which in turn depends on number of pulses, number of samples, etc. For many applications these levels will be inconsequential, although some radar modes may be more sensitive than others. A more complete discussion of this noise floor may be found in any of several references listed earlier, and in particular papers by Axelsson.^{16,17}

3.2.4 Random Frequency Chips with Shaped Spectrum

We may further shape the spectrum by altering the statistics of the random values that we select. We now model our instantaneous frequency function as

$$\Omega(i, n) = 2\pi b \left(X_w(f(i), n) - \frac{1}{2} \right), \quad (37)$$

where

$$X_w(i, n) = \text{shaped-distribution random value over the interval } [0,1). \quad (38)$$

For our purposes $X_w(i, n)$ will have a Probability Density Function (PDF) with the same shape as the desired window taper function for the ESD. For our examples, this will be the -35 dB Taylor window with $\bar{n} = 4$. Appendix A details a Matlab function to accomplish this.

Figure 25 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 26 shows the autocorrelation function of the waveform of Figure 25. Also shown for comparison is a Taylor window desired compressed pulse response. Note that they match quite well, especially in the region of the mainlobe. Figure 27 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 28 shows principal axes cuts of the range-Doppler map in Figure 27.

3.2.5 Quantized Random Frequency Chips with Shaped Spectrum

In the previous section, the random frequencies were selected from within the continuum of the passband. Here we now allow the frequencies to be limited to some finite set that are uniformly arrayed within the passband. Although the frequencies themselves are quantized to a linear spacing, the likelihood of selecting a particular frequency is still adjusted to shape the spectrum. We now model our instantaneous frequency function as

$$\Omega(i, n) = 2\pi b \left(g\{X_w(f(i), n)\} - \frac{1}{2} \right), \quad (39)$$

where the quantization function is presumed to allow the same number K steps as there are individual chips, that is

$$g\{x\} = \frac{\text{floor}(x K)}{K}. \quad (40)$$

This isn't absolutely required, but nevertheless convenient.

We also recall the definitions from previous sections

$$\begin{aligned} X_w(i, n) &= \text{shaped distribution random value over the interval } [0, 1), \text{ and} \\ f(i) &= \tau_{step} \text{floor}(i / \tau_{step}). \end{aligned} \quad (41)$$

For our examples, $b = 0.5$, $K = 160$, and the spectral weighting shape will be the -35 dB Taylor window with $\bar{n} = 4$.

Figure 29 shows the energy spectrum of both a single pulse, and the average over all N pulses. Figure 30 shows the autocorrelation function of the waveform of Figure 29. Also shown is a Taylor window desired compressed pulse response. Note again that they match quite well, especially in the region of the mainlobe. Figure 31 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 32 shows principal axes cuts of the range-Doppler map in Figure 31.

Figure 21. Random frequency chip energy spectrum. Note the ensemble average of the spectra is much smoother than the spectrum of a single pulse.

Figure 22. Random frequency chip autocorrelation function. The reference function is a -35 dB Taylor window with $\bar{n} = 4$. Note the sinc nature of the autocorrelation function.

Figure 23. Random frequency chip pulses: 2-D IPR of data set of with Taylor window applied in both dimensions. Color units are in dB.

Figure 24. Principal axes cuts of 2-D IPR.

Figure 25. Shaped-spectrum random frequency chip energy spectrum. Note the ensemble average of the spectra is much smoother than the spectrum of a single pulse.

Figure 26. Shaped-spectrum random frequency chip energy spectrum. Note how well the autocorrelation function matches the reference -35 dB Taylor window with $\bar{n} = 4$.

Figure 27. Shaped-spectrum random frequency chip pulses: 2-D IPR of data set with Taylor window applied only in azimuth dimension. Color units are in dB.

Figure 28. Principal axes cuts of 2-D IPR.

Figure 29. Shaped-spectrum quantized random frequency chip energy spectrum. Note the ensemble average of the spectra is much smoother than the spectrum of a single pulse.

Figure 30. Shaped-spectrum quantized random frequency chip autocorrelation function. Note how well the autocorrelation function matches the reference -35 dB Taylor window with $\bar{n} = 4$.

Figure 31. Shaped-spectrum quantized random frequency chip pulses: 2-D IPR of data set with Taylor window applied only in azimuth dimension. Color units are in dB.

Figure 32. Principal axes cuts of 2-D IPR.

3.2.6 Randomly Shuffled Quantized Frequency Chips with Shaped Spectrum

Heretofore, we have selected random frequencies within the passband without regard for any prior choices. That is, each chip's frequency was chosen independent of any other chip's frequency. Consequently, for any one pulse, a particular part of the desired spectrum might be over-represented, and another part of the spectrum might be under-represented, purely as a matter of chance. Only in the aggregate of all pulses, in a statistical sense, would the desired spectrum be filled in with the appropriate shaping.

We now add another constraint to the randomness of chip frequencies and their energies. Namely, we now insist that each individual pulse contains chips with frequencies that within the pulse cover the entire desired passband. Essentially, we will begin with a NLFM stepped chirp, and just randomly shuffle the chips. Each pulse will be shuffled differently, but each pulse will still contain the same set of chips, identical in both frequency and duration.

The instantaneous frequency of the new waveform will still have form

$$\Omega(i, n) = \omega_n T_s + \gamma_n (T_s)^2 f(i), \quad (42)$$

but where the quantization function of index i is now adjusted in the constrained random fashion. Recall that there will be K total chips, with linearly increasing index $-K/2 \leq k < K/2$.

We now define a function that shuffles the index values, and notate it as

$$\text{Sh}_{k|n}(k) = \text{shuffling function that shuffles index } k \text{ uniquely for pulse } n. \quad (43)$$

Appendix B lists a Matlab function that implements this.

The new sequence of quantized sample indices is then calculates as

$$f(i) = \frac{I}{K} \text{Sh}_{k|n}(k), \quad (44)$$

where k is the minimum integer that satisfies

$$i \leq \sum_{k'=-K/2}^k \tau_{\text{step}, \text{Sh}_{k'|n}(k')} = \text{accumulated step length in samples, } -K/2 \leq k < K/2, \quad (45)$$

where in turn we continue to calculate individual chip durations as

$$\tau_{step,k} = \text{round}\left(\frac{I}{K} w_K(k)\right) = \text{the length of the individual frequency chips}, \quad (46)$$

and the window taper function is given by

$$w_K(k) = \text{the weighting or window taper function employed}. \quad (47)$$

What this says is that a chip's frequency is a draw from the shuffled finite set of allowable frequencies, and the duration of that chip is still corresponding to the actual frequency drawn, specifically its position in the passband.

For our examples $b = 0.5$, $K = 160$, and $w_K(k)$ embodies the Taylor window function. As before, we address a practical matter that the rounding operation needs to facilitate the constraint that the sum of all steps equals the total number of waveform samples, that is

$$\sum_{k'=-K/2}^{K/2-1} \tau_{step,k'} = I. \quad (48)$$

Figure 33 shows the energy spectrum of both a single sample pulse, and the average over all N pulses. Note that this spectrum is somewhat smoother than that in Figure 29.

Figure 34 shows the autocorrelation function of the waveform of Figure 33. Also shown is a Taylor window desired compressed pulse response. Note that the autocorrelation function is substantially on top of the reference IPR. Figure 35 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 36 shows principal axes cuts of the range-Doppler map in Figure 35.

Gratuitous Comments

This specific algorithm just described forces a particular chip frequency to appear once and only once within a pulse. The 'once' criterion gives us a complete spectrum for each pulse, as desired. The 'only once' part can be relaxed without compromising this desire. For example, if each chip frequency appeared exactly twice, albeit with proper duration but equal probability, then we would achieve similar results. Of course, if any chip frequency appeared, say, twice, then for a given pulse width we would need to either reduce the duration of any one chip, or coarsen the frequency quantization. These effects would have to be evaluated with respect to the IPR.

This is akin to choosing a card from two combined and shuffled decks. Each deck is a complete set of chip frequencies. Of course, any number of decks might also be shuffled together to extend this concept.

Furthermore, other variations might also be employed. For example, some integer number of decks for a single pulse might be shuffled together with a random subset of an additional deck. Or a deck of frequency chips might be dealt across multiple pulses. Other combinations are also easily conceived.

Figure 33. Shaped-spectrum randomly shuffled frequency chips energy spectrum. Note the ensemble average of the spectra is much smoother than the spectrum of a single pulse.

Figure 34. Shaped-spectrum randomly shuffled frequency chips autocorrelation function. Note how well the autocorrelation function matches the reference -35 dB Taylor window with $\bar{n} = 4$.

Figure 35. Shaped-spectrum randomly shuffled frequency chips pulses: 2-D IPR of data set with Taylor window applied only in azimuth dimension. Color units are in dB.

Figure 36. Principal axes cuts of 2-D IPR.

3.2.7 Compound Pulses with Multiple Chip Decks

With some malice of forethought, we examine now a particular structure of random-phase waveforms. The structure is designed as follows.

1. Each pulse is divided into two or more contiguous frames.
2. Each frame is an independently shuffled complete deck of frequency chips.
3. Otherwise, each deck has characteristics as defined in the last section.

We illustrate this in Figure 37.

This particular construct forces each frame to have the same spectral characteristics as the entire pulse. This in turn allows a fraction of the entire pulse to offer the same resolution as the entire pulse. We would normally desire that the waveform segments in individual frames to not correlate well with each other, as this would cause enhanced undesirable sidelobes in the overall waveform autocorrelation function. Furthermore, this works better with longer pulses to mitigate quantization effects in both time duration and frequency.

Figure 37. Pulses may be divided into multiple frames, with each frame exhibiting different modulation characteristics.

For our examples the overall compound pulse width is $T = 20.48 \mu\text{s}$ but is made of two equal-length individual frames, with each frame exhibiting $b = 0.5$, $K = 160$, and $w_K(k)$ embodies the Taylor window function.

Figure 38 shows the energy spectrum of both a single compound pulse, and the average over all N pulses. Note that this spectrum is generally equivalent to that in Figure 33, albeit with slightly more energy because of its overall doubled length. Figure 39 shows the autocorrelation function of the waveform of Figure 38. Also shown for comparison is a Taylor window desired compressed pulse response. Note that the autocorrelation function is substantially on top of the reference IPR. Figure 40 shows the 2-D IPR of a range-Doppler map using N pulses. Figure 41 shows principal axes cuts of the range-Doppler map in Figure 40.

Gratuitous Comments

We offer that the basic concept just described may be modified or enhanced in any of a number of ways, including but not limited to the following.

1. Individual frames need not necessarily be the same length or duration.
2. Subsequent pulses need not contain the same number of frames.
3. Individual frames need not have the same frequency chip set, including in either quantization, bandwidth, or spectral regions.

Compound pulses will be addressed more fully in a separate report.

Figure 38. Energy spectrum of compound pulse with two frames, with individual frames shaped-spectrum randomly shuffled frequency chips.

Figure 39. Autocorrelation function of compound pulse with two frames, with individual frames shaped-spectrum randomly shuffled frequency chips. Note how well the autocorrelation function matches the reference -35 dB Taylor window with $\bar{n} = 4$.

Figure 40. Compound pulse with two frames, with individual frames shaped-spectrum randomly shuffled frequency chips: 2-D IPR of data set with Taylor window applied only in azimuth dimension. Color units are in dB.

Figure 41. Principal axes cuts of 2-D IPR.

3.2.8 Comments

We now provide a generalized description of the waveform selection procedure. This is illustrated in Figure 42.

Figure 42. Generalized waveform selection procedure.

We elaborate on the boxes as follows.

Input Parameter Selection: This box represents the specification of waveform parameters, including pulse width, bandwidth, number of frames, number of chips, sampling frequency, etc. We note that generally a chip may be as few as one sample.

Chip Frequency Selection: Based on input parameters, the allowable chip frequencies are specified, and from them chip frequencies are assigned to each chip. Frequencies may be selected based on desired statistics to facilitate ESD shaping.

Chip Duration Selection: Also based on input parameters, allowable chip durations are specified and selected for each chip. Chip durations may be selected based on desired statistics to facilitate ESD shaping.

Chip Randomizer: With individual chip characteristics defined, the chips are then randomized as to their order and concatenated into a larger pulse. In some cases, specific subsets of the chips may first be concatenated into frames, and then the frames concatenated into a larger pulse.

Phase Accumulator: With frequencies chosen for each sample time for each pulse, these are then accumulated within a pulse to provide a phase function of sample index.

Sine ROM: Phase samples are then converted into amplitudes. These may be both Sine and Cosine functions of the phase for Quadrature outputs (single-sideband mixing), or simply one of Sine or Cosine functions of the phase for single-ended output (double-sideband mixing).

Numerous variations and combinations of the aforementioned examples may be conjured.

“So much of life, it seems to me, is determined by pure randomness.”
-- *Sidney Poitier*

4 Conclusions

We have proposed herein the following.

- Random-phase waveforms may still be capable of relatively precise IPR shapes in a range-Doppler map. This is accomplished by controlling the statistical characteristics of the phase ‘increments’, or instantaneous frequencies.
- We may shape the ESD of such a waveform by controlling the instantaneous frequencies themselves, or the durations of the frequency chips, or both. Furthermore, it is possible to shape the ensemble average ESD to match a specified window taper function, thereby precisely providing a desired shape to the IPR, particularly in the mainlobe, in a range-Doppler map.
- The range-Doppler map or image will tend to average out fluctuations in the ESD of any single pulse. Consequently, the CPI of the random signals should be considered as a whole.

“Random numbers should not be generated with a method chosen at random.”
-- Donald Knuth

Appendix A – Matlab Random Number Generator with Specified Density

We present here a Matlab function that creates an array of random values over the interval $[0,1)$ with a specified Probability Density Function (PDF).

```
% randx
%
% create array of random values in interval [0,1) with desired
% distribution
%
% Z = randx(M,N,win)
%
% where
%   M = number of rows
%   N = number of columns
%   win = desired PDF taper (default: taylor(-35,3,1024))
%
% 'win' must have length at least 64
%

% awd 20120529

function Z = randx(M,N,win);

if nargin<=2,
    %%% choose default window function
    Nwin = 1024;
    win = taylor(-35,4,Nwin)';
else
    Nwin = length(win);
    %%% test if negative window values
    if min(win)<0,
        disp('randx: Error: negative PDF not allowed');
        X = NaN;
        return;
    end
    %%% test if inadequate PDF length
    if Nwin<64,
        disp('randx: Error: PDF length < 64');
        X = NaN;
        return;
    end
end

%% normalize taper function to a PDF
win = win/sum(win);

%% calculate CDF of input PDF
cwin = cumsum(win);
```

```

%%% create array of uniform random values
Zin = rand(M,N);

%%% create output array
Z = zeros(M,N);

%%% for each desired output sample
for m=1:M,
    for n=1:N,
        % find index of CDF for corresponding rand value
        z2 = min(find(cwin>=Zin(m,n)));
        z1 = z2-1;
        z2 = min(z2,Nwin);
        z1 = max(z1,1);
        if z1==1,
            z2 = 2;
        elseif z2==Nwin,
            z1 = Nwin-1;
        end
        c2 = cwin(z2);
        c1 = cwin(z1);

        % perform linear interpolation to find non-integer
        % index value
        z = z1 + (z2-z1)*(Zin(m,n)-c1)/(c2-c1);

        % normalize index value to the interval [0,1)
        Z(m,n) = z/Nwin;
    end
end

%%% ensure Z values are in proper interval
Z = max(Z,0);
Z = min(Z,(1-eps));

return;

```

Appendix B – Matlab Random Shuffle Function

We present here a Matlab function that shuffles the column entries in an array of data.

```
% shuffle
%
% shuffle the entries in columns of an input array
%
% [Y,I] = shuffle(X)
%
% X = array of input data
%
% Y = output array with column entries shuffled
% I = mapping of column indices in shuffled array

% awd 20120717

function [Y,I] = shuffle(X)

[M,N] = size(X);

%%% if row vector, reorder into a column vector and set flag
flag = 0;
if M==1,
    flag = 1;
    X = reshape(X,N,M);
    [M,N] = size(X);
end

%%% create random number matrix same size as input matrix
num = rand(size(X));

%%% sort random number matrix -- this in turn causes a random
%%% shuffling of indices in each column of the array
[Y,I] = sort(num);

%%% reorder the elements of the input array X according to the
%%% now random order of the indices in array I
for n=1:N,
    for m=1:M,
        Y(m,n) = X(I(m,n),n);
    end
end

%%% if input was originally a row vector, then order output
%%% into row vector also
if flag,
    Y = reshape(Y,N,M);
end

return;
```

“Those who trust to chance must abide by the results of chance.”
-- Calvin Coolidge

References

- ¹ K.A. Lukin, R.M. Narayanan, "Historical overview and current research on noise radar", Proceedings of the 2011 3rd International Asia-Pacific Conference on Synthetic Aperture Radar (APSAR), 2011.
- ² B. M. Horton, "Noise-modulated distance measuring systems", Proceedings of the IRE, Volume 49, No. 5, pp. 821-828, May 1959.
- ³ R. M. Narayanan, X. Xu, J. A. Henning, "Radar penetration imaging using ultra-wideband (UWB) random noise waveforms", IEE Proceedings – Radar, Sonar, and Navigation, Vol. 151, No. 3, pp. 143-148, June 2004.
- ⁴ Dmitriy S. Garmatyuk, Ram M. Narayanan, "SAR Imaging Using Fully Random Bandlimited Signals", Proceedings of the IEEE Antennas and Propagation Society International Symposium, 2000, Vol. 4, pp. 1948-1951, 2000.
- ⁵ Daryl C. Bell, Ram M. Narayanan, "Inverse synthetic aperture radar imaging using a coherent ultrawideband random noise radar system", SPIE Journal of Optical Engineering, Vol. 40, No. 11, pp. 2612-2623, November 2001.
- ⁶ Mark A. Govoni, Hongbin Li, "Target Detection Using a Pulsed Linear Frequency Modulated Noise Waveform", Proceedings of the SPIE, Radar Sensor Technology XIII Conference, Vol. 7308, 2009.
- ⁷ Mark A. Govoni, Hongbin Li, "Complex Aperiodic Random Signal Modulation on Pulse-LFM Chirp Radar Waveform", Proceedings of the SPIE, Radar Sensor Technology XIV Conference, Vol. 7669, 2010.
- ⁸ Mark A. Govoni, Lee R. Moyer, "Preliminary performance analysis of the advanced pulse compression noise radar waveform", Proceedings of the SPIE, Radar Sensor Technology XVI Conference, Vol. 8361, April 2012.
- ⁹ K. Lukin, A. Mogyla, V. Palamarchuk, V. Cherniy, A. Kravchuk, "Stepped-frequency noise radar with short switching time and high dynamic range", Proceedings of the 11th International Radar Symposium, Vilnius, Lithuania, June 16–18, 2010.
- ¹⁰ K. Lukin, P. Vyplavin, O. Zemlyaniy, V. Palamarchuk, J. P. Kim, C. H. Kim, "L-band stepped frequency noise sar on the basis of arbitrary waveform generator," Proceedings of the 3rd International Asia-Pacific Conference on Synthetic Aperture Radar (SAR), September 26-30 2011.
- ¹¹ X. Gu, Y. Zhang, X. Zhang, "Stepped frequency random noise uwb radar signal", Proceedings of the 3rd International Asia-Pacific Conference on Synthetic Aperture Radar (SAR), September 26-30 2011.
- ¹² Russell Vela, Lorenzo Lo Monte, "Noisy Stepped Frequency (NSF) Waveform in RF Tomography", Proceedings of the SPIE, Radar Sensor Technology XVI Conference, Vol. 8361, April 2012.
- ¹³ Russell Vela, David Erisman, Ram M. Narayanan, "A Technique for the Generation of Customizable Ultra-Wideband Pseudo-Noise Waveforms", Proceedings of the SPIE, Radar Sensor Technology XV Conference, Vol. 8021, April 2011.
- ¹⁴ Gu Hong, Liu Guosui, Zhu Xiaohua, Su Weimin, Li Xi, "A new kind of noise radar-random binary phase coded CW radar", IEEE National Radar Conference, 1997, pp. 202–206, 1997.

-
- ¹⁵ Xinxiang Zhang, Tianqi Chen, "Noise-Linear Frequency Modulation Shared Waveform for Integrated Radar and Jammer System", International Conference on Communications, Circuits and Systems, 2007, ICCAS 2007, pp. 644-648, 2007.
- ¹⁶ S.R.J. Axelsson, "Noise radar using random phase and frequency modulation", IEEE Transactions on Geoscience and Remote Sensing, Vol. 42, No. 11, pp. 2370-2384, November 2004.
- ¹⁷ S.R.J. Axelsson, "Analysis of Random Step Frequency Radar and Comparison With Experiments", IEEE Transactions on Geoscience and Remote Sensing, Vol. 45, No. 4, pp. 890-904, April 2007.
- ¹⁸ S.R.J. Axelsson, "Random Noise Radar/Sodar With Ultrawideband Waveforms", IEEE Transactions on Geoscience and Remote Sensing, Vol. 45, No. 5, Part 1, pp. 1099-1114, May 2007.
- ¹⁹ Armin W. Doerry, "Generating Nonlinear FM Chirp Waveforms for Radar", Sandia Report SAND2006-5856, Unlimited Release, September 2006.
- ²⁰ Armin W. Doerry, "SAR Processing with Non-Linear FM Chirp Waveforms", Sandia Report SAND2006-7729, Unlimited Release, December 2006.
- ²¹ H.H. Jariani, M. Bahadori, E. Tohidi, M.N. Majd, M.M. Nayebi, "Statistical analysis of sidelobes in random phase-modulated radars", Proceedings of the 2011 IEEE CIE International Conference on Radar, Vol. 1, pp. 485-489, 2011.
- ²² H. Haghshenas, M.M. Nayebi, "Suppressing sidelobe levels in random phase modulated radar", Proceedings of the 2011 IEEE Radar Conference, pp. 530-532, 2011.
- ²³ Xin Wu, Chang Liu, Yanfei Wang, "A noise SAR sidelobe suppression algorithm based on stable realization of apodization filtering", IEICE Electronics Express, Vol. 8, No. 21, pp. 1769-1776, November 2011.
- ²⁴ Xiaojian Xu, Xiangzhi Feng, "SAR imagery using chaotic carrier frequency agility pulses", Proceedings of the SPIE, Radar Sensor Technology XV Conference, Vol. 8021, April 2011.
- ²⁵ G. Martin Hall, E. Jeff Holder, Seth D. Cohen, Daniel J. Gauthier, "Low-cost chaotic radar design", Proceedings of the SPIE, Radar Sensor Technology XVI Conference, Vol. 8361, April 2012.
- ²⁶ Pappu S. Chandra, Benjamin C. Flores, Patrick deBroux, "Analysis of the ambiguity function for an FM signal derived from the Lorenz chaotic flow", Proceedings of the SPIE, Radar Sensor Technology XVI Conference, Vol. 8361, April 2012.
- ²⁷ Mahesh C. Shastry, Ram M. Narayanan, Murahlidhar Rangaswamy, "Band-limited Random Waveforms in Compressive Radar Imaging", Proceedings of the SPIE, Compressive Sensing Conference, Vol. 8365, April 2012.
- ²⁸ Armin Doerry, "Bandwidth requirements for fine resolution squinted SAR", SPIE 2000 International Symposium on Aerospace/Defense Sensing, Simulation, and Controls, Radar Sensor Technology V, Vol. 4033, Orlando FL, 27 April 2000.

“Luck can be assisted. It is not all chance with the wise.”
-- Baltasar Gracian

Distribution

Unlimited Release

1	MS 0532	J. J. Hudgens	5240	
1	MS 0519	J. A. Ruffner	5349	
1	MS 0519	A. W. Doerry	5349	
1	MS 0519	L. Klein	5349	
1	MS 0899	Technical Library	9536	(electronic copy)
1		Brandeis Marquette General Atomics ASI, RSG 16868 Via Del Campo Ct San Diego, CA 92127		(electronic copy)
1		John Fanelle General Atomics ASI, RSG 16761 Via Del Campo Ct San Diego, CA 92127		(electronic copy)

;-)