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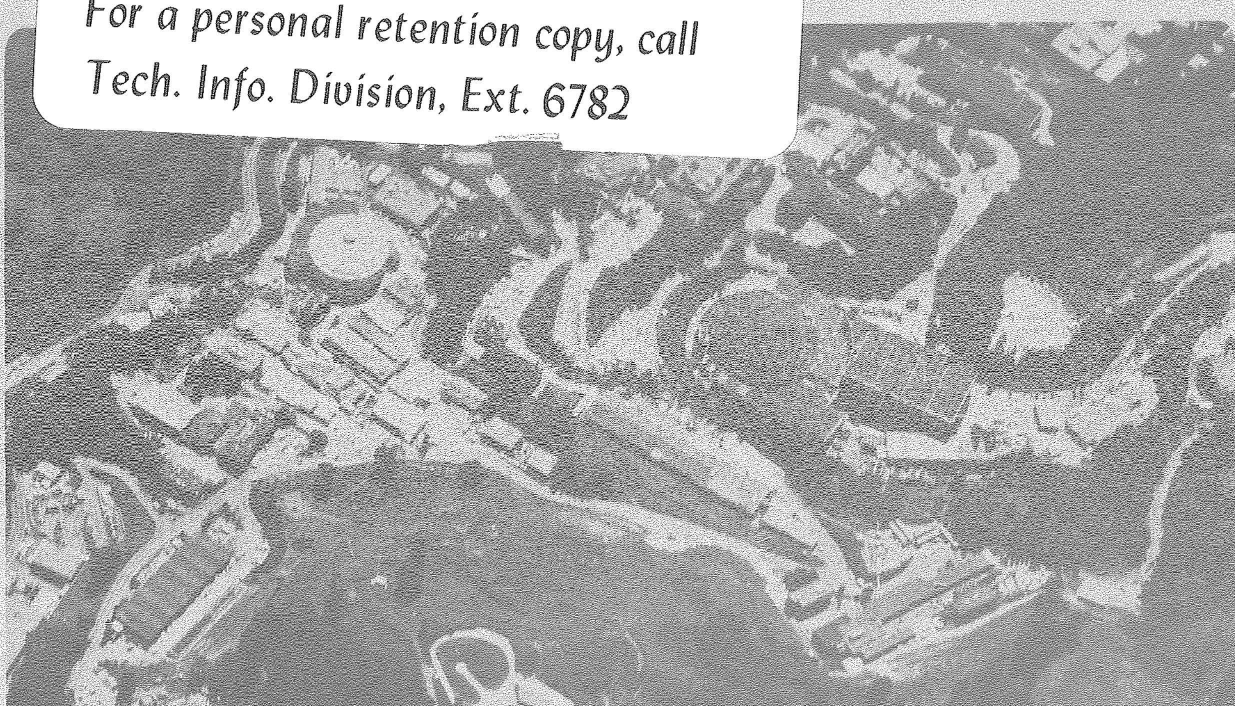

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TOPOLOGICAL ELECTROMAGNETISM FOR QUARKS AND LEPTONS

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Abstract

As outgrowth of a topological bootstrap theory of strong interactions and precursor to a corresponding theory of weak interactions, we propose a representation of electromagnetic interactions for "elementary" hadrons and leptons through combinatorial topology. The representation supports the prediction of four lepton doublets.

A hopefully complete topological bootstrap theory of hadrons has been developed.[1] In this theory quarks appear as oriented triangles that build hadron discs, which in turn cover closed 2-dimensional orientable "quantum surfaces" corresponding to strong-interaction amplitudes. A single triangle cannot be a hadron disc--quarks are confined--but certain properties of topological quarks are so suggestive of physical leptons that we have sought to extend our theory to achieve a topological representation for the electromagnetic and weak interactions of both leptons and "elementary" hadrons. This paper proposes for the electromagnetic current-current interaction a structure where individual leptons have the same topolgoical form as individual quarks.

In our proposal the difference between a quark and a lepton arises in the relation of the corresponding triangular patches to neighbors, a relation connected to the distinction between strong and electromagnetic interactions. Certain triangular patches on any quantum surface are cut by an oriented one-dimensional "belt" which enters at a vertex and departs through the opposite edge. The belt is the intersection between the quantum surface (carrying "internal" quantum numbers through its orientations) and a "classical" surface (carrying direct observables through embedded Landau and Harari-Rosner graphs)--the belt constituting the classical-surface boundary. In a strong-interaction surface pair, belt orientation is continuous throughout each particle disc--reversing if at all only at trivial vertices which appear on hadron-disc perimeters. "Transition arcs," defining the boundaries of oriented patches on the classical surface, terminate at belt-reversal points. We propose that an electromagnetic interaction involves at least one "forbidden" transition arc incident on an edge opposite to a trivial vertex (Fig 1a).

A closed portion of quantum surface, completely covered by only two triangles with opposing belt orientations, as in the example of Fig. 1(b), corresponds to a lepton-antilepton pair. Leptons occur only in such surface-exhausting pairs which make contact with forbidden transition arcs. If more than two triangles cover a closed quantum surface-portion or if there are no forbidden classical transition arcs, the triangles correspond to quarks rather than to leptons. Our proposal for the quantum surface belonging to an electromagnetic current is a belted sphere with one forbidden belt reversal. If the sphere is covered by two triangles we have a leptonic current; if by more than two triangles (4, 6, 8 ...) the current is hadronic. We expect a corresponding structure for weak currents but with a quantum surface more complex than a sphere.

The motivation for our proposal stems from the (strong-interaction) representation of a bare-pomeron mediated interaction--a pair of belted spheres (no forbidden reversals) in contact at north and south poles, the east-west belt of each being its equator.(1) Each sphere carries zero total flavor and zero baryon number, energy-momentum being transferred by the two-boundary-component classical surface (cylinder) that intersects both. Thus in the reaction of Fig. 2(a) the particles ABC cover one sphere and the particles DE cover the other. Each particle cluster constitutes an effective "scalar current." It is natural to guess that an electromagnetic interaction such as depicted in Fig. 2(b) corresponds to a similar topological structure; we postulate two differences: (1) that on each classical boundary component there is now one forbidden belt reversal and (2) that the quantum spheres are not in direct contact. Their contact is only through arcs on the classical surface.

The disconnected nature of the quantum electromagnetic surface represents the physical fact that electromagnetic interactions may occur when the currents are macroscopically separated. Each of the two currents in Fig. 2(b) then corresponds to a separate "quantum event," in contrast to Fig. 2(a) which represents a single event.*

Fig. 3(a) shows one of our proposed classical surfaces corresponding to Fig. 2(b) when all four particles a,b,c,d are leptons. The classical surface is a sphere with two discs (heavily shaded in Fig. 3a) removed, the surface being divided into two oppositely-oriented patches separated by transition arcs (dashed lines in Fig. 3a) that terminate on the boundary at points where the belt reverses. There is one allowed reversal and one forbidden reversal on each boundary component. The forbidden reversal points are connected by the same (forbidden) transition arc. The dotted curves in Fig. 3(a) are Harari-Rosner (HR) arcs which carry spin.(1)

Fig. 3(b) shows the (closed) quantum sphere containing leptons a and b, the triangle corresponding to "a" covering the "back side" of the sphere. The belt reversal points marked 1 and 2 correlate Figs. 3a and 3b, the forbidden reversal occurring at point 2. Edge orientations on the quantum sphere, which differentiate lepton types (electron, muon, etc.) are not shown.

There are four different surface pairs corresponding to the reaction of Fig. 2b; these carry different belt orientations and

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*The simplest classical surface belonging to the reaction of Fig. 2(a) has a single Landau vertex, while that for Fig. 2(b) has two Landau vertices, as explained below.

put the leptons into different orders. The corresponding four amplitude components are not separately C,P,T invariant but their sum is invariant. The leptons in each are either "ortho" or "para" in character depending on the relative orientation of belt and HR arc.(1) In the surface pair of Fig. 3 (a,b), for example, lepton b is ortho, lepton a is para, lepton c is ortho and lepton d is para. Spin "propagators" $\frac{1}{2}(1+\gamma_5)$ or $\frac{1}{2}(1-\gamma_5)$ are attached to ortho or para quarks or leptons.(2) The electromagnetic coupling of two currents always changes ortho to para or vice versa, so the photon vertex (see below) effectively is like γ_μ or $\gamma_5\gamma_\mu$. In the sum of the four topological components belonging to an electromagnetic current-current interaction, all γ_5's cancel out so the physical electromagnetic coupling is pure vector.

In contrast to strong-interaction classical surfaces, the classical surface belonging to a current-current interaction requires no separate Landau graph, momentum-energy flow being representable through transition arcs and HR arcs. The photon Landau arc may be identified with the transition-arc portion that connects two HR arcs*--each belonging to a separate boundary component (a separate current). In the example of Fig. 3(a,b) the "photon" couples two lepton HR arcs, delivering energy-momentum and "flipping spin." The photon appears nowhere on the quantum surface, in sharp contrast to leptons and

*Transition arcs connecting "allowed" belt-reversal points also occur on strong-interaction surfaces, where they may be associated with "vector gluons." Gluon arcs, however, transmit only spin, not momentum-energy. Photon or gluon helicity corresponds to the sense of the classical transition arc, which is tied to the sense of the belt components and to the classical patch orientation.

hadrons; the photon resides entirely within the classical surface, supporting the notion that electromagnetism is in origin a classical phenomenon.

Fig. 3(a') shows a representative classical surface when particles a and b are mesons,(1) while c and d are leptons and Fig. 3(b') shows the corresponding quantum sphere for the hadronic current. The forbidden belt reversal occurs at point #2, indicated in both Figs. 3(a') and 3(b'). We generally demand that the hadronic electromagnetic current be "local" in the sense that, regardless of how many triangles occur, both transition arcs attach to the same quantum triangle. Even so, note that there is another surface pair, in addition to that of Fig. 3(a',b'), where the "photon" couples to the same HR arc but where the forbidden reversal occurs at point #3. This latter topological doubling, which is in addition to the fourfold multiplication already discussed, always occurs for hadronic currents. This is the ortho-para doubling of "spectator" quarks that do not directly interact with the photon.

The distinguishability in our electromagnetic topologies of quark and lepton triangles that individually are indistinguishable supports the proposal that the four flavor doublets corresponding to quark-edge orientations(1) will be matched--one to one--by four physical lepton doublets. Although the edge-orientation topology of flavor and lepton type has played no role in this paper, beyond guaranteeing that electromagnetic currents carry zero internal quantum numbers, a critical role for edge orientations arises when the quantum surface becomes more complex. The topology of weak currents will be the subject of a separate paper.

Electromagnetic surface pairs are not generated by unitarity from zero-entropy(1) surface pairs, so our proposal does not give a bootstrap explanation of the photon's existence. Correspondingly we are unable at the present stage to understand the value of the elementary electric charge,* although it is tempting to suppose the small value of $e^2/\hbar c$ to be somehow related to the topological complexity associated with forbidden belt reversal. In Ref. (1) we speculate that the classical meaning of electromagnetism, together with that of the space-time continuum, lies in high complexity. If so, the proposal presented here, even if it survives, constitutes only one piece of a full topological picture of electromagnetism.

Acknowledgment

Discussions with J. Finkelstein and F. Capra have been influential in the proposal presented here.

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*We also do not yet understand why lepton charges are $\mp$ 1, 0 while quark charges are $\pm$ 2/3, $\mp$ 1/3, although combinatorial topology plausibly has the capacity to explain such ratios.

References

1. G. F. Chew and V. Poénaru, "Topological Bootstrap Theory," Lawrence Berkeley Lab. preprint LBL-9768, Sept. 1979, submitted to Nuclear Physics B; "Topological Bootstrap Prediction of 3-colored 8-flavored Quarks," Lawrence Berkeley Lab. preprint LBL-10665, March 1980, submitted to Physical Review Letters.

2. The ortho-para spin dependence of topological amplitudes will be described in a separate paper. This question has been independently studied by H. P. Stapp, "Spins and Baryons in the Topological Expansion," Lawrence Berkeley Laboratory preprint LBL-10774, April 1980, to be submitted to Nucl. Phys. B. Stapp's advice to us in this connection has been invaluable.

Figure Captions

1. (a) A forbidden belt reversal. On the transverse classical surface a forbidden transition arc is incident on the reversal point. (See Fig. 3.)

   (b) A lepton antilepton pair covering a sphere.

2. (a) Schematic representation of pomeron exchange.

   (b) Schematic representation of virtual photon exchange.

3. (a) Representative classical surface for electromagnetic interaction of two leptonic currents.

   (b) Quantum surface corresponding to 3(a).

   (a') Representative classical surface for electromagnetic interaction between a mesonic current and a leptonic current.

   (b') Quantum surface belonging to 3(a').

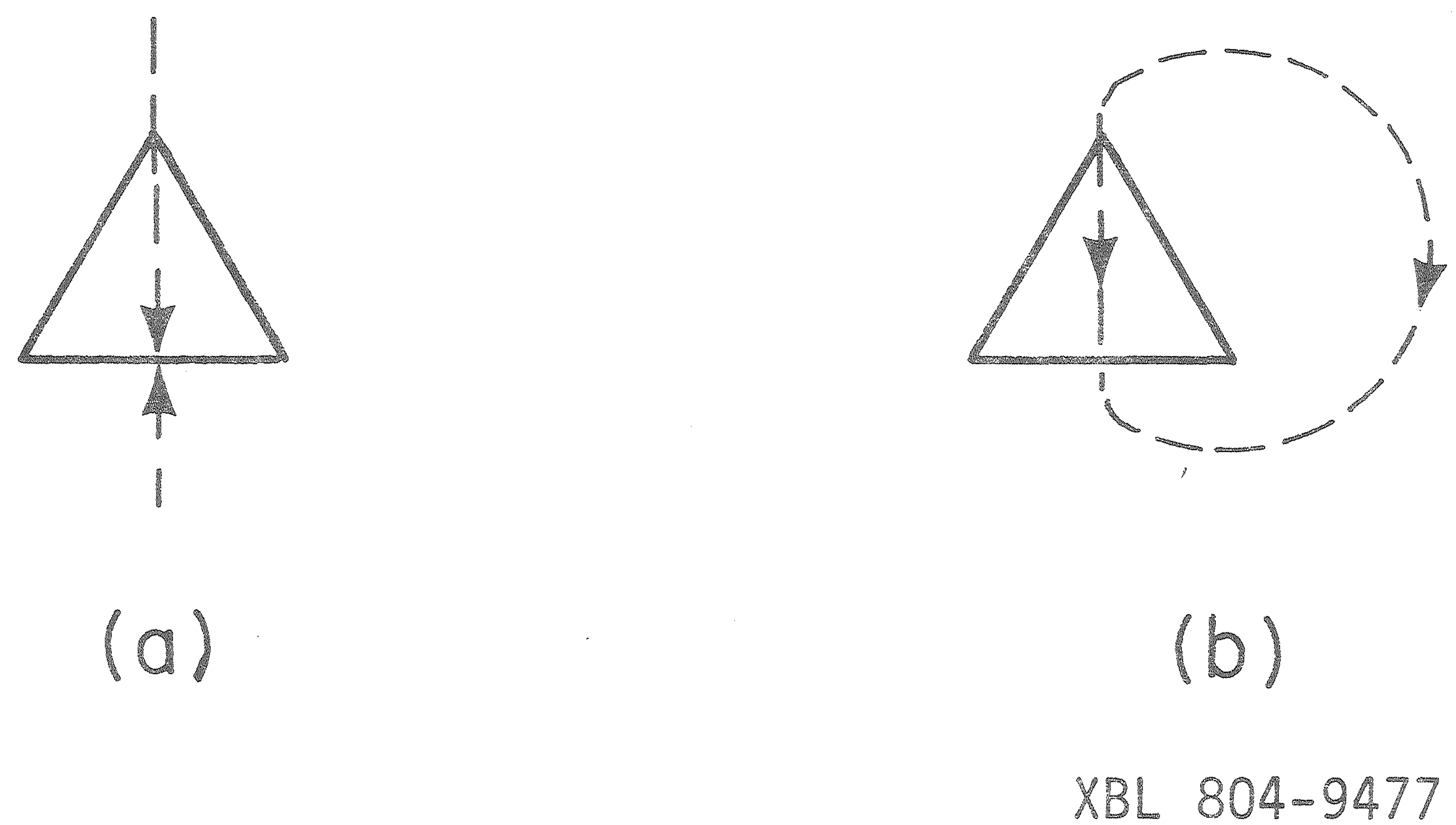

FIG. 1

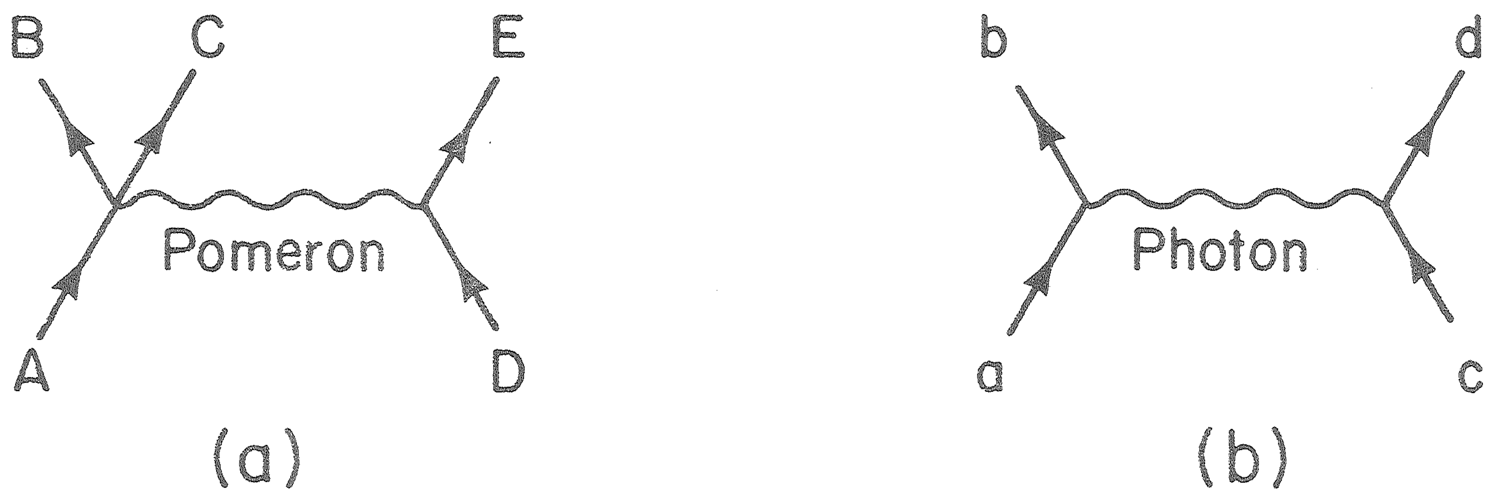

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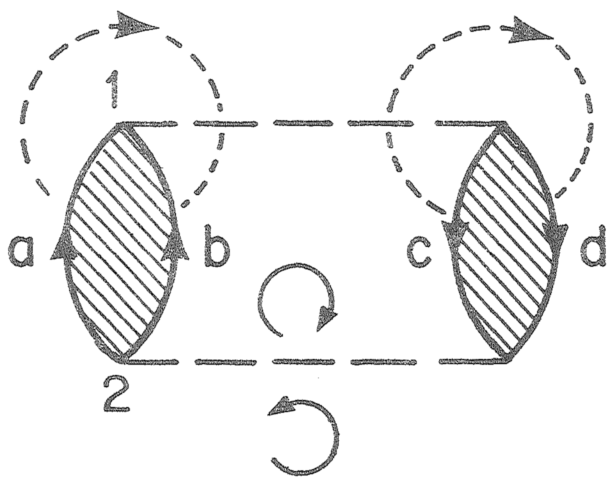
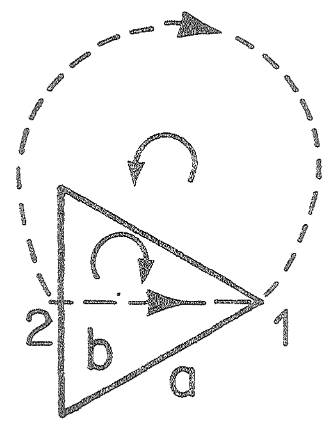
FIG. 2



(a)



(b)

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FIG. 3

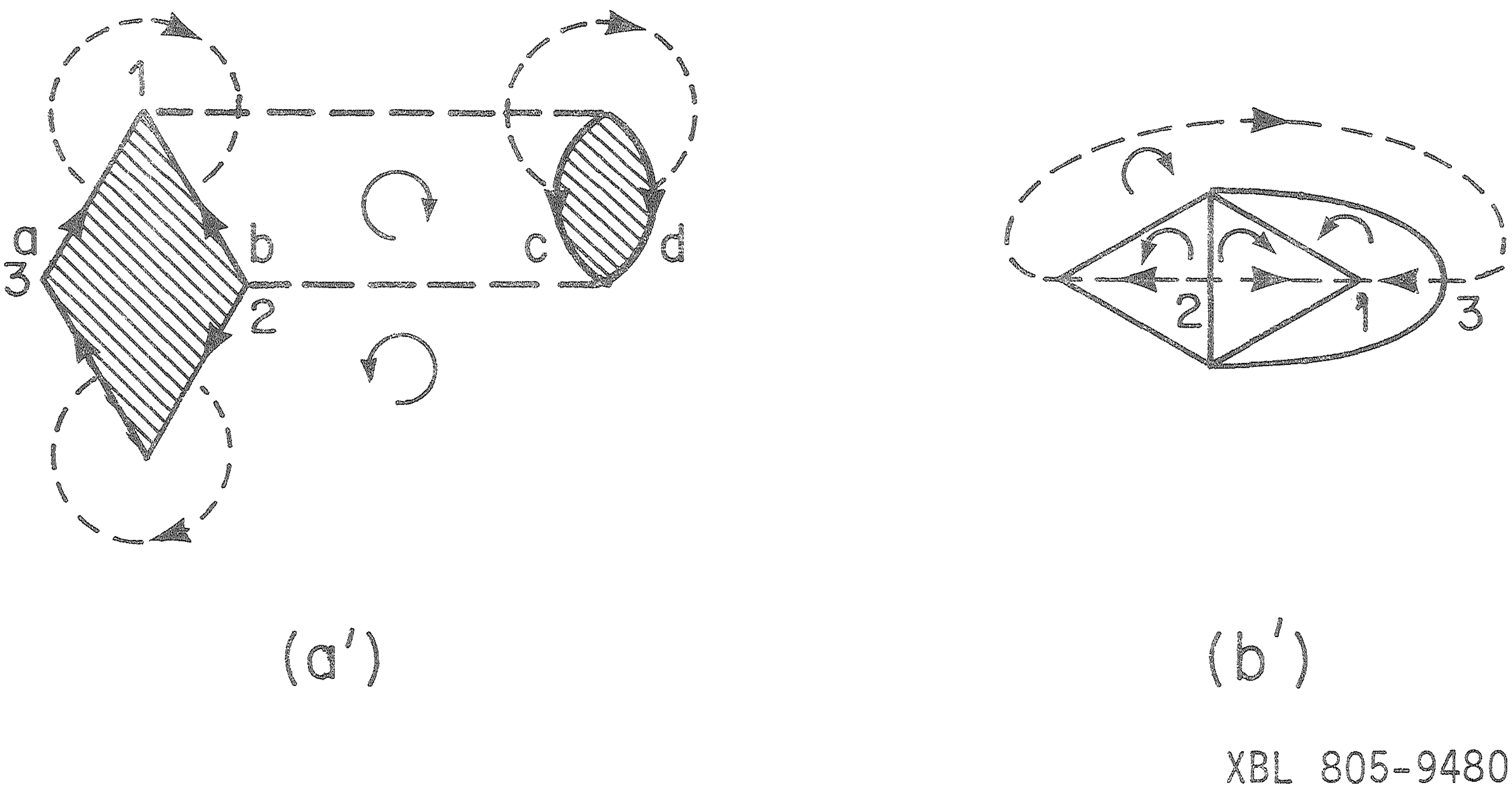

FIG. 3