SANDIA REPORT

SAND2013-1287

Unlimited Release

Printed February 2013

Precomparator and Postcomparator Errors in Monopulse

Douglas L. Bickel

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Approved for public release; further dissemination unlimited.

Sandia National Laboratories

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

NOTICE: This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government, nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, make any warranty, express or implied, or assume any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represent that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government, any agency thereof, or any of their contractors or subcontractors. The views and opinions expressed herein do not necessarily state or reflect those of the United States Government, any agency thereof, or any of their contractors.

Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from
U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865) 576-8401
Facsimile: (865) 576-5728
E-Mail: reports@adonis.osti.gov
Online ordering: <http://www.osti.gov/bridge>

Available to the public from
U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Rd.
Springfield, VA 22161

Telephone: (800) 553-6847
Facsimile: (703) 605-6900
E-Mail: orders@ntis.fedworld.gov
Online order: <http://www.ntis.gov/help/ordermethods.asp?loc=7-4-0#online>

Precomparator and Postcomparator Errors in Monopulse

Douglas L. Bickel
ISR Analysis and Application
Sandia National Laboratories
PO Box 5800
Albuquerque, NM 87185

ABSTRACT

Monopulse radar is a well-established technique for extracting accurate target location information in the presence of target scintillation. It relies on the comparison of at least two patterns being received simultaneously by the antenna. These two patterns are designed to differ in the direction in which we wish to obtain the target angle information. The two patterns are compared to each other through a standard method, typically by forming the ratio of the difference of the patterns to the sum of the patterns. The key to accurate angle information using monopulse is that the mapping function from the target angle to this ratio is well-behaved and well-known. Errors in the amplitude and phase of the signals prior and subsequent to the comparison operation affect the mapping function. The purpose of this report is to provide some intuition into these error effects upon the mapping function.

ACKNOWLEDGEMENTS

The production of this report was funded by General Atomics Aeronautical Systems, Inc. (ASI) Reconnaissance Systems Group (RSG).

GA-ASI, an affiliate of privately-held General Atomics, is a leading manufacturer of unmanned aircraft systems (UAS), tactical reconnaissance radars, and surveillance systems, including the Predator UAS series and Lynx Multi-Mode radar systems.

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

CONTENTS

ABSTRACT	3
1. Executive Summary	7
2. Introduction	9
3. General comments on monopulse ratio and errors	10
4. Effect of precomparator and postcomparator errors	12
5. Other considerations	63
6. Conclusions	67
8. References	68
9. Appendix A: Symbols and terminology	69
10. Appendix B: Equations for precomparator and postcomparator errors and approximations	70
11. Appendix C: Simplified monopulse models	75
12. Distribution	78

This page left intentionally blank.

1. Executive Summary

This document presents the effects of precomparator and postcomparator phase and amplitude errors on monopulse systems. Both amplitude and phase monopulse systems are considered in this document. Equations are given that show the relationship between the complex monopulse ratio with errors as a function of the ideal monopulse ratio without errors. Also, a very simple ideal linear monopulse model is developed and used to present plots and give further insight into the behavior of the various error types. The advantage of this simple model is that deviation from this model is readily apparent.

The monopulse ratio is the key quantity for monopulse radar systems used in location of multiple targets simultaneously and in detection of slower moving targets. Since this is the case, the focus of this document is on the behavior of the complex monopulse ratio in the presence of these errors.

It is shown that in general, there are a lot of similarities between amplitude and phase monopulse. As their names imply, the amplitude and phase monopulse use different methods to provide the input beams, there are some differences. In particular, the roles of precomparator phase and precomparator amplitude errors are approximately reversed between amplitude and phase monopulse systems.

This page left intentionally blank.

2. Introduction

Monopulse radar is a well-established technique for extracting accurate target location information in the presence of target scintillation. It relies on the comparison of at least two differing patterns being received simultaneously by the antenna. These two patterns differ in the direction in which we wish to obtain the target angle information from the monopulse system. The two patterns are compared to each other through a standard method, typically by forming the ratio of the difference of the patterns to the sum of the patterns. The key to accurate angle information from monopulse is that the mapping function from the target angle to this ratio is well-behaved and well-known. Errors in the amplitude and phase of the signals prior and subsequent to the comparison operation affect the mapping function. The purpose of this report is to provide some intuition into these error effects upon the mapping function.

Two different types of monopulse systems will be considered in this document. These are known as amplitude and phase monopulse. In amplitude monopulse, the beams are squinted via amplitude using a common phase center on the antenna. For an ideal amplitude monopulse system the ratio of the difference to sum is purely real. In phase monopulse, the beams are squinted in phase via separate phase centers on the antenna. In an ideal phase monopulse system the ratio of the difference to sum is purely imaginary.

The reader is assumed to have basic familiarity with the monopulse radar concepts (see [1]).

The organization of this report starts with general comments on monopulse and errors; followed by specific discussion of errors in amplitude monopulse, and then in phase monopulse. A brief section of the report that follows includes miscellaneous considerations, such as null-depth. The final section is the conclusions. In addition, Appendix B presents the error equations, and Appendix C presents a simple ideal monopulse model that is used for comparisons throughout this report.

3. General comments on monopulse ratio and errors

The purpose of any monopulse radar is to determine the angle to a target. Since this is the case, the most critical element is the knowledge of how to map the monopulse ratio to target angle. To accomplish this we need not only a well behaved mapping between these quantities, but also a very stable mapping, to avoid errors. Precomparator and postcomparator errors change this mapping function. Often more importantly, if these errors significantly vary (due to temperature, etc.) at a rate faster than the mapping function is estimated, they can cause significant errors in the estimation of the target angle. This memorandum shows how precomparator and postcomparator errors affect the monopulse ratio, and hence its mapping function.

In amplitude and phase monopulse, the comparator generates the sum and difference between the RF signals from two squinted beams. This serves to “compare” the two signals and their respective beam patterns. The RF device used to form the sum and difference is often called a comparator. For example, a common comparator device is the so-called “magic-tee”. The comparator provides a natural demarcation point in a monopulse system. Errors which occur on the signals prior to the comparator are referred to as precomparator errors. Similarly, errors that occur after the comparator are called postcomparator errors.

The monopulse ratio is the voltage ratio response of the difference channel to the sum channel data. In general, this value can be complex, i.e., contain a real and imaginary part. Typically in literature, we focus on the real part of this ratio for amplitude monopulse systems, and the imaginary part of this ratio for phase monopulse systems¹. This is because in the absence of errors and noise, the amplitude monopulse ratio should be real and the phase monopulse ratio should be imaginary. As we will see in this document, in practice the target information is contained in both the real and imaginary parts due to system errors.

Since we wish to find the target, we desire a nice monotonic mapping function between the monopulse ratio and the target angle. In an ideal world this would be a linear relationship given by [1]:

$$r = k_m \left(\frac{\theta}{\theta_{3dB}} \right) \quad (1)$$

where:

k_m - normalized amplitude monopulse

¹ Note that we use the symbol r as the ideal monopulse ratio. In the absence of errors and noise, we will choose r to be a real quantity for both amplitude and phase monopulse; therefore, for a phase monopulse system, the ideal monopulse ratio will be jr where $j = \sqrt{-1}$.

r - ideal monopulse ratio
 θ - the target angle
 θ_{3dB} - the 3 dB beamwidth of the sum pattern

In this document, we have developed an ideal monopulse model that assumes equation (1) is true. This model is derived in Appendix C². The reason for this is that it simplifies the interpretation of the error effects.

² It is worth repeating from Appendix B, that although the plots in this document assume the simple monopulse model in Appendix C, in general the equations in this document do not.

4. Effect of precomparator and postcomparator errors

This section will consider the effects of precomparator and postcomparator errors in monopulse using a simplified model to facilitate insight into the effects on the important monopulse ratio to target location mapping function. We will consider the amplitude and phase monopulse systems separately based upon the equations presented in Appendix B.

It is typical to divide the errors up into the following four categories: 1) precomparator amplitude errors; 2) precomparator phase errors; 3) postcomparator amplitude errors; and 4) postcomparator phase errors. In the following sections we will present the analysis for representative examples of these errors for amplitude, and then phase monopulse systems.

4.1 Amplitude monopulse systems

We start by considering the amplitude monopulse radar system. In the following subsections, we consider the effect of precomparator amplitude, precomparator phase, postcomparator amplitude, and postcomparator phase errors individually and combined, on an amplitude monopulse system.

Before considering the error cases, we want to make some important comments about the ideal monopulse ratio characteristics for an amplitude monopulse system. In order to get a baseline for comparison with the error cases, we start with the simplified model from Appendix C with no errors. Figure 1 illustrates these characteristics. First, the real part of the ideal amplitude monopulse ratio is monotonic (ideally linear) within the angles of regard (typically the beamwidth of the sum pattern). Second, the imaginary part of the ideal amplitude monopulse ratio is zero. Third, the phase of the amplitude monopulse ratio is a very sharp step function going from π radians to zero radians (or vice-versa) at the null location of angle zero. Any deviation from these characteristics will be an indicator of some error that causes the deviation

4.1.1 Effects of individual precomparator and postcomparator errors in amplitude monopulse

Even though all errors exist in some form or another simultaneously, it is informative to consider the effect of each of the four error category types in isolation. The reason is that we can sometimes recognize some of these error components occurring in data from actual monopulse systems. We will extend the analysis to the properties of the combination of these errors. This section considers the error effect of the individual errors on amplitude monopulse.

4.1.1.1 Precomparator amplitude errors in amplitude monopulse

Precomparator amplitude errors were briefly discussed in [2], but we will expand a bit on the effects on the monopulse ratio here using the simplified model. From Appendix B

(also [2]), the monopulse ratio after precomparator amplitude scaling error, g , becomes a function of the ideal monopulse ratio without error, r , given by:

$$\hat{r} = \left[\frac{|g|^2 (1+r)^2 - (1-r)^2}{|g|^2 (1+r)^2 + 2g(1-r^2) + (1-r)^2} \right] \quad (2)$$

Figure 2 shows the sum and difference patterns using the model assuming a 0.5 dB precomparator amplitude error ($g = 1.06$). As pointed out in [2] for the pure precomparator amplitude error case, the peak of the sum and null in the difference patterns shift in opposite directions. The null itself is still infinitely deep. Reference [2] gives a little more intuition into this behavior.

Figure 3 shows the characteristics of the all-important monopulse ratio from equation (2). We will include these plots throughout this document so I will spend a little time explaining them here. In these monopulse ratio plots, we plot: (a) the real part of the monopulse ratio; (b) the imaginary part of the monopulse ratio; and (c) the “angle” or the phase of the monopulse ratio, i.e., the arctangent of the imaginary part over the real part.

Figure 4 shows the difference between the real part of the monopulse ratio with a precomparator error and one without. We see a few things about precomparator amplitude errors from these figures. First, the zero-crossing of the real part of the monopulse ratio is shifted, as is the step function location in the angle of the monopulse ratio. Second, a slight non-linear component is added to the real part of the monopulse ratio³. Third, precomparator amplitude errors by themselves do not add an imaginary part to the resulting monopulse ratio, which is readily observed from equation (2), as well.

We can rewrite $g = 1 + \varepsilon_g$ and then if ε_g is small, we can approximate equation (2) as:

$$\hat{r} \approx r + \frac{\varepsilon_g}{2} \left(1 - \frac{\varepsilon_g}{2} \right) - \left(\frac{\varepsilon_g}{2} \right) r^2 \quad (3)$$

This approximation clearly shows the zero-crossing shift and non-linear component of the monopulse ratio with a precomparator amplitude error. The approximation is shown in Figure 4, as well. Note that equation (3) is an approximation for small values of ε_g and will not be valid everywhere; however, it does provide good insight into effects of small precomparator amplitude errors.

³ This non-linearity is somewhat monopulse model dependent.

Figure 1: (a) real part (b) imaginary part (c) angle of ideal amplitude monopulse ratio

Figure 2: Simulated amplitude monopulse patterns with 0.5 dB precomparator amplitude error (one-way, in dB)

4.1.1.2 Precomparator phase errors in amplitude monopulse

Precomparator phase errors were briefly discussed in [2], and again we will expand on the effects on the monopulse ratio here using the simplified model. From [2] and Appendix B, the monopulse ratio after precomparator phase error, ξ , as a function of the ideal monopulse ratio without errors is:

$$\hat{r} = \left[\frac{2r}{1 + r^2 + (1 - r^2)\cos \xi} \right] + j \left[\frac{(1 - r^2)\sin \xi}{1 + r^2 + (1 - r^2)\cos \xi} \right] \quad (4)$$

Figure 5 shows the sum and difference beams assuming the model with a 10° precomparator phase error. Figure 6 shows the characteristics of the monopulse ratio with the precomparator phase error. Figure 7 shows the deviation of the real part of the monopulse ratio with the precomparator phase error from the ideal model monopulse ratio with no errors.

Figure 3: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio with precomparator amplitude error of 0.5 dB

Figure 4: Change in real part of amplitude monopulse ratio due to 0.5 dB precomparator amplitude error

Figure 5: Simulated amplitude monopulse patterns with 10° precomparator phase error

From Figure 5 we note that the peak of the sum and the null of the difference patterns have not shifted, but that the null depth is no longer infinite. This also follows from discussions in [2] about null depth⁴. Note from this and the other figures, that although the null itself appears to be broader, the slope of the real part of the monopulse ratio has changed only slightly for this small phase error.

Figure 6 highlights the important amplitude monopulse ratio differences. First, from Figures 6 and 7, for a small precomparator phase error there is a relatively small change in the real part of the monopulse ratio, which is the most important factor for target location. Also, the zero-crossing for the real part of the monopulse ratio has not been shifted. Next, Figure 6 shows that there is a new non-linear component added to the imaginary part of the monopulse ratio. This, combined with the loss of the step-function (i.e., smoothing) in the angle of the monopulse ratio (and loss of null depth) are tell-tale signs of a precomparator phase error in amplitude monopulse. Note that the angle of the monopulse is readily calculable from equation (4) as:

$$\angle \hat{r} = \text{atan} \left[\frac{(1-r^2) \sin \xi}{2r} \right] \quad (5)$$

As with the precomparator amplitude error, we now provide a small error approximation formula to equation (4):

$$\hat{r} \approx r \left(1 + \frac{\xi^2}{4} \right) - r^3 \frac{\xi^2}{4} + j \left(\frac{1}{2} \right) (1-r^2) \xi \quad (6)$$

The real part of equation (6) is also shown in Figure 7. From equation (6) we can see the various characteristics discussed above, e.g., zero-crossing, quadratic imaginary part, small cubic and linear changes to the real part of the monopulse ratio, etc.

To a first order the null depth is approximately limited to⁵:

$$\text{amplitude monopulse null depth} \approx 20 \log_{10} |\xi| - 6 \quad \text{in dB} \quad (7)$$

Using equation (7), a 10° precomparator phase error reduces the null depth to 21.2 dB which appears to match Figure 5. Figure 8 shows a comparison of the null depth in amplitude monopulse from the model with the approximation in equation (7) for various precomparator phase errors.

⁴ More discussion about null depth is presented in a section later in this report, as well.

⁵ This is essentially the same equation as can be found in [2].

Figure 6: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio with precomparator phase error of 10°

Figure 7: Change in real part of amplitude monopulse ratio due to 10° precomparator phase error

Figure 8: Comparison of null depth in amplitude monopulse from precomparator phase errors using the model and approximation in equation (7)

4.1.1.3 Postcomparator amplitude errors in amplitude monopulse

Postcomparator amplitude errors are presented in this section. From Appendix B, the monopulse ratio after a postcomparator amplitude scaling error, γ , is:

$$\hat{r} = \gamma r \quad (8)$$

This very simple equation is self-apparent and says that a postcomparator amplitude error simply scales the monopulse ratio. In general this leads to a scaling of the estimated target location. Since this is key in target location, this can be a significant issue. The effects of have been previously discussed in [2].

Obviously, the zero-crossing of the real part of the monopulse ratio does not shift; the imaginary part of the monopulse ratio is zero and the phase is still a nice step function as in Figure 4c.

4.1.1.4 Postcomparator phase errors in amplitude monopulse

Now we examine postcomparator phase errors. From Appendix B, the amplitude monopulse ratio after a postcomparator phase error, ψ , only is:

$$\hat{r} = r e^{j\psi} = r \cos \psi + jr \sin \psi \quad (9)$$

Again, the postcomparator error is a very simple equation, which in this case rotates information from the desired real part of the monopulse ratio to the unwanted imaginary part of the monopulse ratio. Even though the equation is quite simple in this case, we will go ahead and plot the characteristics of the monopulse ratio because it does add a bit of insight. Figure 9 shows the monopulse ratio characteristics in the presence of a 10° postcomparator phase error only. Figure 10 shows the deviation of the real part of the monopulse ratio from the ideal case due to a postcomparator phase error of 10° . From Figure 10, we see that the error in the real part of the monopulse ratio is a linear scaling error just like in the postcomparator amplitude error. The scale factor is now $\cos \psi$ and the scaling error is $\cos \psi - 1$. All the issues for a scaling error in monopulse are the same as with the postcomparator amplitude error. From Figure 9, we have an imaginary part of the monopulse ratio which is also a scaling of the monopulse ratio without an error. If not corrected or accounted for, this can be thought of as a wasted term. Finally, we note in Figure 9c that the angle of the monopulse is still a step function, but that the whole step function is shifted in phase by ψ .

A very interesting case of postcomparator phase error occurs due to differences in timing delays between A/Ds on separate channels. This time delay causes an IF frequency dependent postcomparator phase error, i.e., ψ is changing with range. The result is that the observed monopulse ratio is changing with target range in accordance with the above discussion.

Figure 9: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio with postcomparator phase error of 10°

Figure 10: Change in real part of amplitude monopulse ratio due to 10° postcomparator phase error

4.1.1.5 Summary of individual errors in amplitude monopulse

The above sections have shown the characteristics of isolated precomparator and postcomparator errors with respect to the assumed model. In this section, we summarize the effects of the individual errors on the complex monopulse ratio. Although all such errors tend to be present together, knowing the individual characteristics lends some intuition into the behavior of real system.

The individual errors have the following effects on amplitude monopulse:

- 1) Precomparator amplitude errors shift the relative location of the null of the difference and peak of the sum pattern. They also shift the zero-crossing of the real part of the monopulse ratio, and the step response of the angle of the complex monopulse ratio.
- 2) Precomparator phase errors decrease the null depth in the difference pattern. They do not shift the null location. They change the real part of the monopulse ratio slightly, and add a non-linear term to the imaginary part of the monopulse ratio. They tend to smooth out the step function in the angle of the complex monopulse ratio.

- 3) Postcomparator amplitude errors simply scale the real part of the monopulse ratio.
- 4) Postcomparator phase errors rotate the information from the real part of the monopulse ratio into the imaginary part of the monopulse ratio. This results in scaled values of the original monopulse ratio in both the real and imaginary parts of the resulting monopulse ratio. In doing this, they also shift the angle of the complex monopulse ratio by an amount corresponding to the postcomparator phase error.

4.1.2 Combinations of errors in amplitude monopulse

The situation is complicated in the real world because all of these errors co-exist. One can see from the general equations and the approximations in Appendix B that there are various cross-terms added that make separation more difficult. This section shows some of the complexity that arises due to the errors. To do so, we will first consider the effect of pairwise errors, followed by all errors simultaneously. As before, we use the simple monopulse model from Appendix C to illustrate the issues.

4.1.2.1 Precomparator amplitude and phase errors combined in amplitude monopulse

Figure 11 shows the monopulse ratio plots for a precomparator phase error of 10° and a precomparator amplitude error of 0.5 dB together. We note from Figure 11 that the combined effect of each of the individual errors are observable, namely shifts in the zero-crossing of the real part of the monopulse ratio, shift in the monopulse ratio angle, non-linear effects in the imaginary part of the monopulse ratio, and smoothing of the step function in the phase of the monopulse ratio. Figure 12 shows the combined effects on the sum and difference patterns. These affects are a shift in the location of the difference null relative to the sum peak, and loss in null depth. Figure 13 shows the change in the real part of the monopulse ratio from the ideal due to these errors.

The approximation to the error equation from Appendix B for this case becomes⁶:

$$\begin{aligned} \hat{r} \approx & \frac{\epsilon_g}{2} - \frac{\epsilon_g^2}{4} + r \left(1 - \frac{\epsilon_g^2}{4} + \frac{\xi^2}{4} \right) + r^2 \left(-\frac{\epsilon_g}{2} + \frac{\epsilon_g^2}{4} \right) + r^3 \left(\frac{\epsilon_g^2}{4} - \frac{\xi^2}{4} \right) \\ & + j \left[\frac{\xi}{2} - \left(\frac{\epsilon_g \xi}{2} \right) r - \left(\frac{\xi}{2} \right) r^2 + r^3 \left(\frac{\epsilon_g \xi}{2} \right) \right] \end{aligned} \quad (10)$$

We see from the approximation that for modest errors, the cross-terms in this case are negligible in the real part of the monopulse ratio which explains why in this case we will

⁶ Recall from the Appendix B that these approximations have limitations in the valid range of error parameters so use the approximations presented in this document with that in mind.

tend to observe the superposition of the effects of these individual errors. However, we see that there are cross-terms in the imaginary part of the monopulse ratio that could make separation of the effects difficult in practice.

4.1.2.2 Precomparator and postcomparator phase errors combined in amplitude monopulse

Figure 14 shows the monopulse ratio plots for a combination of a precomparator phase error of 10° and a postcomparator phase error of 10° . Again we see the combined effect of linear and non-linear components in the imaginary part of the monopulse ratio, as well as bias and smoothing of the step function in the phase of the monopulse ratio. Note that the patterns will be the same as in Figure 5 above. Figure 15 shows the change in the real part of the monopulse ratio due to these errors.

The approximation to the error equation for this case becomes:

$$\begin{aligned} \hat{r} \approx & -\frac{\xi\psi}{2} + r\left(1 - \frac{\psi^2}{2} + \frac{\xi^2}{4}\right) + \left(\frac{\xi\psi}{2}\right)r^2 - \left(\frac{\xi^2}{4}\right)r^3 \\ & + j\left[\frac{\xi}{2} + r\psi - \left(\frac{\xi}{2}\right)r^2\right] \end{aligned} \quad (11)$$

It should be pointed out that for this case, we see that the cross-terms in the real part of the monopulse ratio can be important, but not necessarily in the imaginary part.

4.1.2.3 Precomparator amplitude and postcomparator phase errors combined in amplitude monopulse

Figure 16 shows the monopulse ratio plots for a combination of a precomparator amplitude error of 0.5 dB and a postcomparator phase error of 10° . Figure 17 shows the change in the real part of the monopulse ratio from the ideal linear case. Again, we see the combined effects of a shift in zero-crossing and linear and non-linear changes in the real part of the monopulse ratio. We also note that since the postcomparator phase errors do not affect the beam magnitudes, the patterns will follow those in Figure 2 above.

The approximation to the error equation from Appendix B for this case becomes:

$$\begin{aligned} \hat{r} \approx & \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} + r\left(1 - \frac{\varepsilon_g^2}{4} - \frac{\psi^2}{2}\right) + r^2\left(-\frac{\varepsilon_g}{2} + \frac{\varepsilon_g^2}{4}\right) + r^3\left(\frac{\varepsilon_g^2}{4}\right) \\ & + j\left[\frac{\varepsilon_g\psi}{2} + r\psi - \left(\frac{\varepsilon_g\psi}{2}\right)r^2\right] \end{aligned} \quad (12)$$

For modest errors, the cross-terms are negligible in the real part of the monopulse ratio, but may be present in the imaginary part of the monopulse ratio.

Figure 11: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio with a precomparator phase error of 10° and a precomparator amplitude error of 0.5 dB

Figure 12: Simulated amplitude monopulse patterns with 10° precomparator phase and 0.5 dB precomparator amplitude errors

Figure 13: Change in real part of amplitude monopulse ratio due to 10° precomparator phase and 0.5 dB precomparator amplitude errors

Figure 14: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio with a precomparator phase error of 10° and a postcomparator phase error of 10°

Figure 15: Change in real part of amplitude monopulse ratio due to 10° precomparator phase and 10° postcomparator phase errors

Figure 16: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio due to a 0.5 dB precomparator amplitude and 10° postcomparator phase errors

Figure 17: Change in real part of amplitude monopulse ratio due to 0.5 dB precomparator amplitude and 10° postcomparator phase errors

4.1.2.4 Precomparator and postcomparator amplitude errors combined in amplitude monopulse

This section considers the effects of a combination of a precomparator amplitude error of 0.5 dB and a postcomparator amplitude error of 0.5 dB. Figure 18 gives the monopulse ratio plots and Figure 19 presents the change in the real part of the monopulse ratio from the ideal linear case. Since there are no phase errors, the imaginary part and the angle of the monopulse ratio are the same as ideal. The real part of the monopulse ratio is affected as shown in Figure 19.

The approximation to the error equation for this case becomes:

$$\hat{r} \approx \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} + \frac{\varepsilon_\gamma \varepsilon_g}{2} + r \left(1 + \varepsilon_\gamma - \frac{\varepsilon_g^2}{4} \right) + r^2 \left(-\frac{\varepsilon_g}{2} - \frac{\varepsilon_\gamma \varepsilon_g}{2} + \frac{\varepsilon_g^2}{4} \right) + r^3 \left(\frac{\varepsilon_g^2}{4} \right) \quad (13)$$

In equation (13), the cross-terms are modest, but in general we should account for these terms. Also, as was just stated, we observe that there are no imaginary terms in the approximation.

Figure 18: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio due to a 0.5 dB precomparator amplitude and 0.5 dB postcomparator amplitude errors

Figure 19: Change in real part of amplitude monopulse ratio due to 0.5 dB precomparator amplitude and 0.5 dB postcomparator amplitude errors

4.1.2.5 Postcomparator amplitude and phase errors combined in amplitude monopulse

The combination of postcomparator amplitude and postcomparator phase errors is a bit more straightforward to interpret. Figure 20 shows the model under a postcomparator amplitude error of 0.5 dB and a postcomparator phase error of 10° . Figure 21 shows the deviation of the real part of the monopulse ratio from the ideal monopulse ratio. For this combination of errors, the individual effects are readily apparent.

The approximation to the error equation for this case becomes:

$$\hat{r} \approx r \left(1 + \varepsilon_\gamma - \frac{\psi^2}{2} \right) + jr\psi(1 + \varepsilon_\gamma) \quad (14)$$

In general, this combination of errors is the easiest to measure with an internal calibration circuit. Likewise, once we measure these errors, we can remove its effects in the processing. It is important to do so because changes in either of these quantities scale the real part of the monopulse ratio leading to target angle scaling errors [2].

Figure 20: (a) real part (b) imaginary part (c) angle of monopulse ratio with a postcomparator amplitude error of 0.5 dB and a postcomparator phase error of 10°

Figure 21: Change in real part of monopulse ratio due to 0.5 dB postcomparator amplitude and 10° postcomparator phase errors

4.1.2.6 All errors combined in amplitude monopulse

The simultaneous combination of all errors is complicated by cross terms that influence the complex monopulse ratio. We will plot the results from a couple of cases using the model. The first case is for precomparator and postcomparator amplitude errors of 0.5 dB, and precomparator and amplitude phase errors of 10° . The second case is for a precomparator amplitude error of 1 dB, a precomparator phase error of 30° , a postcomparator amplitude error of 0.5 dB, and a postcomparator phase error of -10° . Figures 22 and 23 show the former case. Figures 24 and 25 show the latter case.

Figure 22: (a) real part (b) imaginary part (c) angle of monopulse ratio with pre and postcomparator amplitude errors of 0.5 dB and pre and postcomparator phase errors of 10°

Figure 23: Change in real part of monopulse ratio due to 0.5 dB pre and postcomparator amplitude errors and 10° pre and postcomparator phase errors

Figure 24: (a) real part (b) imaginary part (c) angle of amplitude monopulse ratio with precomparator amplitude error of 1 dB, a precomparator phase error of 30° , a postcomparator amplitude error of -0.5 dB, and a postcomparator phase error of -10°

Figure 25: Change in real part of amplitude monopulse ratio due to precomparator amplitude error of 1 dB, a precomparator phase error of 30° , a postcomparator amplitude error of -0.5 dB, and a postcomparator phase error of -10°

4.2 Phase monopulse systems

This section repeats the previous analysis with the difference being that we examine phase monopulse here, rather than amplitude monopulse. We saw in the previous section that the various errors alter the mapping function between the monopulse ratio and the target angle, including shifting information around between the real and imaginary parts. What we want to understand in this section is what the difference is between the error effects on phase monopulse versus amplitude monopulse.

As with the amplitude monopulse, we illustrate the ideal phase monopulse ratio from Appendix C in Figure 26. Note that the roles of the imaginary and real parts of the phase monopulse ratio are reversed from those in the amplitude monopulse. The other subtle difference is that the angle of the phase monopulse ratio is shifted by $\pi/2$ from the ideal amplitude monopulse ratio shown in Figure 1c.

Following the pattern established above, we will first consider the effects of the individual precomparator and postcomparator errors, and then the combinations of the individual errors.

Figure 26: (a) real part (b) imaginary part (c) angle of phase monopulse ratio for the ideal phase monopulse system

4.2.1 Individual precomparator and postcomparator errors in phase monopulse systems

This section considers the precomparator amplitude and phase, and postcomparator amplitude and phase errors individually to simplify the insight in the effects of each of these errors.

4.2.1.1 Precomparator amplitude errors in phase monopulse systems

Precomparator amplitude errors for phase monopulse model are presented in this section. From Appendix B, the monopulse ratio after precomparator amplitude scaling error, g , becomes a function of the ideal phase monopulse ratio without error, r , given by:

$$\hat{r} = \left\{ \frac{(1+r^2)(g^2-1)}{(1+r^2)(1+g^2)+2g(1-r^2)} + j \frac{4jgr}{(1+r^2)(1+g^2)+2g(1-r^2)} \right\} \quad (15)$$

Figure 27 shows the sum and difference patterns using the model assuming a 0.5 dB. Unlike in the amplitude monopulse case, the null depth for phase monopulse is affected by a precomparator amplitude error. As with an precomparator phase error in amplitude monopulse, the null depth is affected, but not the null location. We will return to this in a while.

Figure 27: Simulated phase monopulse patterns with 0.5 dB precomparator amplitude error

Figure 28 shows the effect on the phase monopulse ratio of a precomparator amplitude error. Note that this is very similar to the effect of a precomparator phase error on the amplitude monopulse ratio shown in Figure 6 above. In this respect, a precomparator amplitude error in phase monopulse is analogous to a precomparator phase error in amplitude monopulse.

The approximation for equation (15) is given by:

$$\hat{r} \approx \left(\frac{\epsilon_g}{2} \right) \left(1 - \frac{\epsilon_g}{2} \right) (1+r^2) + j \left[r - \frac{\epsilon_g^2}{4} r (1+r^2) \right] \quad (16)$$

We can observe the analogy between the precomparator phase error in amplitude monopulse and precomparator amplitude error in monopulse by noting the similarities in the approximations from equations (6) and (16), respectively.

Based upon equation (16), the null depth for phase monopulse is approximately given by:

$$\text{phase monopulse null depth} \approx 20 \log_{10} |\varepsilon_g| - 6 \text{ in dB} \quad (17)$$

For a 0.5 dB precomparator amplitude error, equation (17) says that the null depth is reduced to approximately 30.5 dB which appears to match Figure 27. Also, the approximation says that the zero crossing of the imaginary part of the monopulse ratio is not shifted.

The difference in the imaginary part phase monopulse ratio with a precomparator amplitude error from the ideal phase monopulse ratio is plotted in Figure 29 alongside the approximation from equation (16).

Figure 30 shows a plot of the null depth for phase monopulse using the model and the approximation in equation (17).

4.2.1.2 Precomparator phase errors in phase monopulse systems

In this section the effect of precomparator phase errors on phase monopulse are presented. The equation for the monopulse ratio in this case is given by:

$$\hat{r} = j \frac{\left[(1 - r^2) \sin \xi + 2r \cos \xi \right]}{(1 + r^2) + (1 - r^2) \cos \xi - 2r \sin \xi} \quad (18)$$

Right away, we notice that unlike the amplitude monopulse case, a precomparator phase error does not affect the null depth, but does shift the zero-crossing of the imaginary part of the phase monopulse ratio. Note that in this respect, a precomparator phase error in phase monopulse is analogous to a precomparator amplitude error in amplitude monopulse.

Figure 31 shows the sum and difference patterns based on the models we are using in this document. Interestingly, as opposed precomparator amplitude errors in amplitude monopulse, precomparator phase errors in phase monopulse shift the peak of the sum beam and the null of the difference beam in the same direction.

Figure 32 shows the phase monopulse ratio for a precomparator phase error. We see the shift in the zero-crossing in both the imaginary part of the phase monopulse ratio and the angle of the monopulse ratio. Again we note that this is similar to precomparator amplitude errors in amplitude monopulse.

An approximation to equation (18) is given by:

$$\hat{r} \approx j \left\{ \frac{\xi}{2} + r \left(1 + \frac{\xi^2}{4} \right) + \frac{\xi}{2} r^2 + \frac{\xi^2}{4} r^3 \right\} \quad (19)$$

This approximation permits us to readily interpret some of the previous discussion. Using the approximation in equation (19), we see that there is no real part to the phase monopulse ratio due to a precomparator phase error. This means that as long as the monopulse ratio is reasonably well behaved, there is no effect on the null depth. We also see that the y -intercept (i.e., zero crossing of the imaginary part of the monopulse ratio) is shifted as previously discussed. We also see that additional linear and non-linear components to the imaginary part of the monopulse ratio are added with this error. Figure 33 shows the difference in the imaginary part of the monopulse ratio from the ideal monopulse ratio due to a precomparator phase error. Figure 33 also shows the a comparison with the approximation in equation (19).

Figure 28: (a) real part (b) imaginary part (c) angle of phase monopulse ratio with precomparator amplitude error of 0.5 dB

Figure 29: Change in imaginary part of phase monopulse ratio due to 0.5 dB postcomparator amplitude error

Figure 30: Comparison of null depth in phase monopulse from precomparator amplitude errors using the model and approximation in equation (17)

Figure 31: Simulated phase monopulse patterns with 10° precomparator phase error

Figure 32: (a) real part (b) imaginary part (c) angle of phase monopulse ratio with precomparator phase error of 10°

Figure 33: Change in imaginary part of phase monopulse ratio due to 10° postcomparator phase error

4.2.1.3 Postcomparator amplitude errors in phase monopulse systems

Postcomparator amplitude errors (and as we will soon see, phase errors) are very similar between amplitude and phase monopulse. Other than being in the imaginary part rather than the real part, the phase monopulse ratio after a postcomparator amplitude scaling error, γ , is the same as for amplitude monopulse⁷:

$$\hat{r} = j\gamma r \quad (20)$$

As in the amplitude monopulse case, this error scales the monopulse ratio and has the same effect of a scaling error on the target angle estimation.

The zero-crossing of the imaginary part of the phase monopulse ratio does not shift; the real part of the monopulse ratio is zero; and the phase is still a nice step function as in Figure 26c.

⁷ Recall that for the ideal case of either phase or amplitude monopulse, we have defined r to be real.

4.2.1.4 Postcomparator phase errors in phase monopulse systems

Postcomparator phase errors exhibit the same behavior for phase monopulse as they do for amplitude monopulse. The complex monopulse ratio in this case becomes:

$$\hat{r} = jre^{j\psi} = -r\sin\psi + jr\cos\psi \quad (21)$$

For phase monopulse, the postcomparator phase error rotates information from the desired imaginary part of the monopulse ratio to the unwanted real part of the monopulse ratio.

4.2.1.5 Summary of individual errors in phase monopulse systems

Consistent with amplitude monopulse analysis, we now summarize the effects of the individual errors on the complex monopulse ratio.

The individual errors have the following effects on phase monopulse:

- 1) Precomparator amplitude errors decrease the null depth in the difference pattern. They do not shift the null location. They change the imaginary part of the monopulse ratio slightly, and add a non-linear term to the real part of the monopulse ratio. They smooth out the step function in the angle of the complex monopulse ratio.
- 2) Precomparator phase errors shift the relative location of the null of the difference and peak of the sum pattern. They also shift the zero-crossing of the imaginary part of the monopulse ratio, and the step response of the angle of the complex monopulse ratio.
- 3) Postcomparator amplitude errors scale the imaginary part of the monopulse ratio.
- 4) Postcomparator phase errors rotate the information from the imaginary part of the monopulse ratio into the real part of the monopulse ratio. This scales the original monopulse ratio in both the imaginary and real parts of the resulting monopulse ratio. They also shift the angle of the complex monopulse ratio by an amount corresponding to the postcomparator phase error.

4.2.2 Combinations of errors in phase monopulse systems

As in the amplitude monopulse case, the combination of errors observed in practice complicates the analysis. We repeat the errors examples we used in the amplitude monopulse cases now for phase monopulse.

4.2.2.1 Precomparator amplitude and phase errors combined in phase monopulse

Figure 34 shows the phase monopulse ratio plots for a precomparator phase error of 10° and a precomparator amplitude error of 0.5 dB together. Figure 35 shows the deviation of the imaginary part of the phase monopulse ratio with these errors.

An approximation for the errors in this case from Appendix B is:

$$\begin{aligned} \hat{r} \approx & \frac{\epsilon_g}{2} - \frac{\epsilon_g^2}{4} + r \left(\frac{\epsilon_g \xi}{2} \right) + r^2 \left(\frac{\epsilon_g}{2} - \frac{\epsilon_g^2}{4} \right) + r^3 \left(\frac{\epsilon_g \xi}{2} \right) \\ & + j \left[\frac{\xi}{2} + r \left(1 - \frac{\epsilon_g^2}{4} + \frac{\xi^2}{4} \right) + r^2 \left(\frac{\xi}{2} \right) + r^3 \left(\frac{\xi^2}{4} - \frac{\epsilon_g^2}{4} \right) \right] \end{aligned} \quad (22)$$

Equation (22) implies that modest errors superimpose in the imaginary part of the monopulse ratio and that there are some additional cross-terms that may need to be accounted for in the real part. It is rather interesting to look at the symmetry between equation (22) for the phase monopulse and equation (10) for the amplitude monopulse.

4.2.2.1 Precomparator and postcomparator phase errors combined in phase monopulse

Figure 34 shows the phase monopulse ratio plots for a combination of a precomparator phase error of 10° and a postcomparator phase error of 10° .

The approximation equation for this case becomes:

$$\begin{aligned} \hat{r} \approx & -\frac{\xi \psi}{2} - r \psi - r^2 \left(\frac{\xi \psi}{2} \right) \\ & + j \left[\frac{\xi}{2} + r \left(1 - \frac{\psi^2}{2} + \frac{\xi^2}{4} \right) + r^2 \left(\frac{\xi}{2} \right) + r^3 \left(\frac{\xi^2}{4} \right) \right] \end{aligned} \quad (23)$$

For this case, the error effects combine in the imaginary part of the monopulse ratio, but there are cross-terms than could creep up in the real part.

Figure 34: (a) real part (b) imaginary part (c) angle of phase monopulse ratio with a precomparator phase error of 10° and a precomparator amplitude error of 0.5 dB

Figure 35: Change in imaginary part of amplitude monopulse ratio due to 10° precomparator phase and 0.5 dB precomparator amplitude errors

Figure 34: (a) real part (b) imaginary part (c) angle of phase monopulse ratio with a precomparator phase error of 10° and a postcomparator phase error of 10°

Figure 35: Change in imaginary part of phase monopulse ratio due to 10° precomparator phase and 10° postcomparator phase errors

4.2.2.2 Precomparator amplitude and postcomparator phase errors combined in phase monopulse

Figure 36 shows the phase monopulse ratio plots for a combination of a precomparator amplitude error of 0.5 dB and a postcomparator phase error of 10° . Figure 37 shows the change in the imaginary part of the phase monopulse ratio from the ideal linear case.

The approximation equation in this instance given by is:

$$\begin{aligned} \hat{r} \approx & \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} - r\psi + r^2 \left(\frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} \right) \\ & + j \left[\frac{\varepsilon_g \psi}{2} + r \left(1 - \frac{\varepsilon_g^2}{4} - \frac{\psi^2}{2} \right) + r^2 \left(\frac{\varepsilon_g \psi}{2} \right) - r^3 \left(\frac{\varepsilon_g^2}{4} \right) \right] \end{aligned} \quad (24)$$

The cross-terms are limited to the imaginary part for combined precomparator amplitude and postcomparator phase errors.

Figure 36: (a) real part (b) imaginary part (c) angle of phase monopulse ratio due to a 0.5 dB precomparator amplitude and 10° postcomparator phase errors

Figure 37: Change in imaginary part of phase monopulse ratio due to 0.5 dB precomparator amplitude and 10° postcomparator phase errors

4.2.2.3 Postcomparator amplitude and phase errors combined in phase monopulse

As with amplitude monopulse, the combination of postcomparator amplitude and postcomparator phase errors is straightforward to interpret. Figure 38 shows the model under a postcomparator amplitude error of 0.5 dB and a precomparator phase error of 10° . Figure 39 shows the deviation of the real part of the monopulse ratio from the ideal monopulse ratio. For this combination of errors, the individual effects are readily apparent. The following equation is the approximation for this case:

$$\begin{aligned}
 \hat{r} \approx & \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} - \frac{\xi\psi}{2} + \frac{\varepsilon_\gamma\varepsilon_g}{2} + r \left(-\psi - \varepsilon_\gamma\psi + \frac{\varepsilon_g\xi}{2} \right) \\
 & + r^2 \left(\frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} + \frac{\varepsilon_\gamma\varepsilon_g}{2} - \frac{\xi\psi}{2} \right) + r^3 \left(\frac{\varepsilon_g\xi}{2} \right) \\
 & + j \left[\frac{\xi}{2} + \frac{\xi\varepsilon_\gamma}{2} + \frac{\varepsilon_g\psi}{2} + r \left(1 + \varepsilon_\gamma - \frac{\varepsilon_g^2}{4} - \frac{\psi^2}{2} + \frac{\xi^2}{4} \right) \right. \\
 & \left. + r^2 \left(\frac{\xi}{2} + \frac{\varepsilon_g\psi}{2} + \frac{\xi\varepsilon_\gamma}{2} \right) + r^3 \left(\frac{\xi^2}{4} - \frac{\varepsilon_g^2}{4} \right) \right]
 \end{aligned} \tag{25}$$

Figure 38: (a) real part (b) imaginary part (c) angle of phase monopulse ratio with a postcomparator amplitude error of 0.5 dB and a postcomparator phase error of 10°

Figure 39: Change in imaginary part of phase monopulse ratio due to 0.5 dB postcomparator amplitude and 10° postcomparator phase errors

4.2.2.4 All errors combined in phase monopulse

As with the amplitude monopulse case, the simultaneous combination of all errors is complicated. We will plot the results using the same examples as for the amplitude monopulse case, previously. The first case is for precomparator and postcomparator amplitude errors of 0.5 dB, and precomparator and amplitude phase errors of 10° . The second case is for a precomparator amplitude error of 1 dB, a precomparator phase error of 30° , a postcomparator amplitude error of 0.5 dB, and a postcomparator phase error of -10° . Figures 40 and 41 show the former case. Figures 42 and 43 show the latter case.

Figure 40: (a) real part (b) imaginary part (c) angle of phase monopulse ratio with pre and postcomparator amplitude errors of 0.5 dB and pre and postcomparator phase errors of 10°

Figure 41: Change in imaginary part of phase monopulse ratio due to 0.5 dB pre and postcomparator amplitude errors and 10° pre and postcomparator phase errors

Figure 42: (a) real part (b) imaginary part (c) angle of phase monopulse ratio with precomparator amplitude error of 1 dB, a precomparator phase error of 30° , a postcomparator amplitude error of -0.5 dB, and a postcomparator phase error of -10°

Figure 43: Change in imaginary part of phase monopulse ratio due to precomparator amplitude error of 1 dB, a precomparator phase error of 30° , a postcomparator amplitude error of -0.5 dB, and a postcomparator phase error of -10°

4.3 Sources of precomparator and postcomparator errors

This brief list is by no means exhaustive, nor is it sorted, nor prioritized. Sources of precomparator and postcomparator errors include the radome, antenna, mutual coupling, isolation issues, reflections, timing errors (e.g., A/D timing offsets), various channel balance issues, etc.

Note that we have assumed in this document that the precomparator and postcomparator errors are constant. This is a standard assumption, but does not have to always be the case. For example, radome and mutual coupling errors can be dependent upon angles. In addition, delay differences, etc., can lead to frequency dependent errors.

5. Other considerations

This section discusses implications of precomparator and postcomparator effects, rather than the error analysis. In this section, we discuss null depth as it relates to the types of errors presented above. The null depth is used a common measure of monopulse antenna performance. We also give an incomplete discussion on issues that go into specifying system error requirements, since that would require yet another document, as well as more information on the overall system requirements. This is only meant as a brief introduction to considerations relative to precomparator and postcomparator errors.

5.1 Brief note on null depth and precomparator errors

This section provides a brief note on null depth in monopulse. Since the focus in this document is the precomparator and postcomparator errors, we will only consider their effects on null depth here. Note that there are more contributors to loss of null depth than just precomparator errors, and so a more complete discussion of null depth requires considering all of the causes of loss of null depth [2].

Null depth is traditionally used as an important parameter in specifications for monopulse radar systems. This author believes that we should consider the knowledge of the monopulse ratio for some of the more sophisticated applications as the most important measure. However, null depth will continue to be an important parameter because of its ease of measurement and its historical significance.

From the previous discussion, it is apparent that the null depth can be an indicator of precomparator phase errors for amplitude monopulse, and precomparator amplitude errors for phase monopulse systems. In this context, null depth gives an indication of the fidelity of matching prior to the comparator. It also indicates how much information has been rotated out-of-phase from the desired monopulse ratio phase at the null. A finite null depth indicates that the zero-crossing for the real and imaginary parts of the monopulse ratio do not coincide in angle space.

Figures 44 and 45 show the plots of the difference channel using the model with errors that affect the null depth. Figure 44 is plots of the difference for amplitude monopulse using the model with various precomparator phase errors. Figure 45 shows the difference channel for phase monopulse using the model with various precomparator amplitude errors. We note from these plots that as the null depth decreases, the difference channel curve “flattens” out. Although the difference channel response flattens out and does not appear as “sharp” with decreasing null depth, the width of the null, as it is typically defined, does not appear to increase for the assumed model. A typical null width definition might be the angular separation between the 10 dB points, as an example.

In applications where the radar attempts to track by maintaining a null in power on the target, the sharpness of the null depth and the null depth are important. This appears to be one of the traditional reasons for specifying the null depth.

In more modern applications of monopulse, the target location may be extracted from only the real (for amplitude monopulse) or imaginary (for phase monopulse) parts of the monopulse ratio. In any reasonable monopulse system, these parts of the monopulse ratio will have a zero-crossing, i.e., an infinite “null depth”. Figures 46 and 47 show these parts of the monopulse ratio with the same assumptions and errors as used in Figures 44 and 45, respectively. The slope change of these monopulse ratios does not appear to be dramatic. From this we see that usually, we want as good of a null depth as possible⁸, but a system designer may have some latitude in relaxing the specification. Note that although this is the case, we still need to account for these changes in the function that maps the monopulse ratio to target location. The important point is that we want the precomparator errors to be very stable⁹.

Figure 44: Effect on power in difference channel of amplitude monopulse model for various precomparator phase errors

⁸ It is VERY IMPORTANT to emphasize that we are ONLY considering precomparator errors as sources of the null depth problem in this document. Other sources of null depth loss may not permit this flexibility to the system designer

⁹ If we can estimate the changes in precomparator errors faster than they occur in the system, in theory we can account for these errors. In practice this can be difficult due to several factors, and it is therefore more important to attempt to design the system so that they are stable.

Figure 45: Effect on power in difference channel of phase monopulse model for various precomparator amplitude errors

Figure 46: Real part of amplitude monopulse ratio using the model for various precomparator phase errors

5.2 Issues to consider in error specification

A recurring theme in this document is that for monopulse systems we need to have very accurate knowledge of the function that maps the monopulse ratio to target angle, and that we desire that this function have nearly optimal behavior. It can be common to focus on the latter; however, often the former can be even more critical. In this section, we will briefly try to make this argument.

A typical monopulse ratio has a normalized monopulse ratio slope, k_m , of between 1.6 to 1.8. We will stick with the 1.8 value used in our simple model. Let us assume that one of the errors described in the previous sections causes this value to be reduced to 1.7. The standard angle noise equation given for monopulse from [1] is:

$$\sigma_\theta \approx \frac{\theta_{3dB}}{k_m \sqrt{2snr}} \quad (26)$$

where:

σ_θ - the standard deviation of the angle noise

snr - the sum signal to difference noise ratio

The increase in the angle noise due to this reduction in normalized monopulse ratio is about:

$$\Delta_{\sigma_\theta} \approx \sigma_\theta \left(\frac{\Delta_{k_m}}{k_m} \right) \quad (27)$$

where:

Δ_{σ_θ} - the change in standard deviation of the angle noise

Δ_{k_m} - the change in the normalized monopulse ratio

For our example, this means that the angle noise increases about 6%. For a 20 dB SNR target and a 4° beamwidth, this would change the noise standard deviation from about 0.16° to 0.17°, an increase in error of 0.01° which is a relatively small impact on the angle estimate.

Now let us assume that we do not correct for this same change in monopulse ratio when processing the target location. In this case, we can invert equation (1) and get:

$$\theta \approx \frac{\theta_{3dB}}{k_m} r \quad (28)$$

After manipulation, we can approximate the error in angle estimate is given by:

$$\Delta_{\theta} \approx \theta \left(\frac{\Delta_{k_m}}{k_m} \right) \quad (29)$$

If we do not account for this change in slope in the mapping of the monopulse ratio to target location in this case, the location error at the edge of the scene is 0.1° , a factor of 10 worse than the increase in noise. Therefore, it tends to be more critical in location applications to know the monopulse ratio to angle mapping function very well. It is less critical that the normalized slope be 1.7 or 1.8.

What this means in practice is that we can often live with precomparator and postcomparator errors that may not be ideally zero or some very small value as long as we know what the resulting complex monopulse ratio is very well. This also implies that we do not have to know the exact values of the precomparator and postcomparator amplitude and phase values, as long as we know the resulting monopulse ratio to angle mapping function very well. The system design implication is that, although we would like to keep the precomparator and postcomparator errors low, it is more important that they vary much slower than our ability to estimate the monopulse ratio mapping function.

6. Conclusions

The intent of this report is to give some insight into the effects of precomparator and postcomparator amplitude and phase errors on the complex monopulse ratio mapping function. Both amplitude and phase monopulse are considered. Equations are developed to show the effects of these errors both individually and combined. Many plots are presented using a simple monopulse model to provide insight into the effect of these errors on the mapping function.

Although it is not the main intent of this report, precomparator errors and null depth is briefly presented. Also, an argument is made that knowledge of the exact value of each of the error types is not as critical as precise knowledge of the resulting complex monopulse ratio to target angle mapping function.

8. References

- [1] S. M. Sherman, *Monopulse Principles and Techniques*, 1st Ed., 1984.
- [2] D. L. Bickel, various internal memos on monopulse, 1999-2012.
- [3] D. K. Barton, *Modern Radar System Analysis*, Artech House, 1988.

9. Appendix A: Symbols and terminology

Variable definitions

- Δ - the difference channel
- Δ_θ - the change in estimated target angle given misknowledge of mapping function
- Δ_{σ_θ} - the change in standard deviation of the angle noise
- Δ_{k_m} - the change in the normalized monopulse ratio
- $\varepsilon_g = g - 1$ - deviation of precomparator amplitude gain from ideal (useful in small error approximations)
- $\varepsilon_\gamma = \gamma - 1$ - deviation of postcomparator amplitude gain from ideal (useful in g - precomparator amplitude gain (ideal is 1))
- γ - postcomparator amplitude gain (ideal is 1)
- ψ - postcomparator phase error in radians (ideal is 0)
- k_m - normalized amplitude monopulse slope (used as a simplified mapping from target angle to monopulse ratio) defined in [1]
- θ - the target angle
- θ_{3dB} - the one-way 3 dB beamwidth of the antenna in the angular direction being measured by the monopulse ratio
- $\theta_n = \theta/\theta_{3dB}$ - the normalized target angle
- σ_θ - the standard deviation of the angle noise
- r - ideal monopulse ratio (for amplitude monopulse the ideal is r , for phase monopulse the ideal is jr)
- $\hat{r}$ - complex monopulse ratio in the presence of errors
- snr - the sum signal to difference noise ratio
- Σ - the sum channel
- ξ - precomparator phase error in radians (ideal is 0)

10. Appendix B: Equations for precomparator and postcomparator errors and approximations

It is important to note that the equations in this appendix do not assume any particular model of the monopulse ratio. For example, these equations do not depend upon use of the model in Appendix C.

Also, note that the accuracy of the approximation equations presented depend upon the size of the errors; therefore, the approximations should be used with caution if the errors are anticipated to be large. The exact equations can be used in this case. The approximations often do provide simpler insight.

Amplitude monopulse comparator error equations

If we have an amplitude monopulse system, assume that the true sum and difference patterns are given as functions of the amplitude squinted beams V_A and V_B as:

$$\begin{aligned}\Delta &= V_A - V_B \\ \Sigma &= V_A + V_B\end{aligned}\tag{30}$$

and:

$$r = \text{Re} \left\{ \frac{\Delta}{\Sigma} \right\} = \text{Re} \left\{ \frac{V_A - V_B}{V_A + V_B} \right\}\tag{31}$$

so that:

$$\frac{V_B}{V_A} = \frac{(1-r)}{(1+r)}\tag{32}$$

where r is the true monopulse ratio.

We let the precomparator error be $ge^{j\xi}$ and the postcomparator error be $\gamma e^{j\psi}$ where g is the precomparator amplitude error and ξ is the precomparator phase error; γ is the postcomparator amplitude error and ψ is the postcomparator phase error such that the observed complex monopulse ratio in the presence of errors becomes:

$$\hat{r} = \gamma e^{j\psi} \left(\frac{ge^{j\xi} V_A - V_B}{ge^{j\xi} V_A + V_B} \right)\tag{33}$$

which can be rewritten as:

$$\begin{aligned}\hat{r} = & \gamma e^{j\psi} \left[\frac{g^2 (1+r)^2 - (1-r)^2}{g^2 (1+r)^2 + 2g(1-r^2)\cos\xi + (1-r)^2} \right] \\ & + j\gamma e^{j\psi} \left[\frac{2(1-r^2)g\sin\xi}{g^2 (1+r)^2 + 2g(1-r^2)\cos\xi + (1-r)^2} \right]\end{aligned}\quad (34)$$

Then using some trig and following through with the algebra it can be shown to yield the observed monopulse ratio in the presence of errors, $\hat{r}$:

$$\begin{aligned}\hat{r} = & \left[\frac{\gamma g^2 (1+r)^2 \cos\psi - 2\gamma g(1-r^2)\sin\psi \sin\xi - \gamma(1-r)^2 \cos\psi}{g^2 (1+r)^2 + 2g(1-r^2)\cos\xi + (1-r)^2} \right] \\ & + j \left[\frac{\gamma g^2 (1+r)^2 \sin\psi + 2\gamma g(1-r^2)\sin\xi \cos\psi - \gamma(1-r)^2 \sin\psi}{g^2 (1+r)^2 + 2g(1-r^2)\cos\xi + (1-r)^2} \right]\end{aligned}\quad (35)$$

It is important to notice that this equation does not assume any inherent model of the monopulse ratio (e.g., as in Appendix C).

If we let $g = 1 + \varepsilon_g$ and $\gamma = 1 + \varepsilon_\gamma$ and assume small precomparator and postcomparator errors then we can approximation equation (35) as:

$$\begin{aligned}\hat{r} \approx & \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} - \frac{\xi\psi}{2} + \frac{\varepsilon_\gamma \varepsilon_g}{2} + r \left(1 + \varepsilon_\gamma - \frac{\varepsilon_g^2}{4} - \frac{\psi^2}{2} + \frac{\xi^2}{4} \right) \\ & + r^2 \left(-\frac{\varepsilon_g}{2} - \frac{\varepsilon_\gamma \varepsilon_g}{2} + \frac{\varepsilon_g^2}{4} + \frac{\xi\psi}{2} \right) + r^3 \left(\frac{\varepsilon_g^2}{4} - \frac{\xi^2}{4} \right) \\ & + j \left[\frac{\xi}{2} + \frac{\xi\varepsilon_\gamma}{2} + \frac{\varepsilon_g\psi}{2} + r \left(\psi + \psi\varepsilon_\gamma - \frac{\varepsilon_g\xi}{2} \right) \right. \\ & \left. + r^2 \left(-\frac{\xi}{2} - \frac{\xi\varepsilon_\gamma}{2} - \frac{\varepsilon_g\psi}{2} \right) + r^3 \left(\frac{\varepsilon_g\xi}{2} \right) \right]\end{aligned}\quad (36)$$

With a very simple change, we can rewrite this equation as the deviation of the monopulse ratio with precomparator and postcomparator errors from one without errors:

$$\begin{aligned}
\hat{r} - r \approx & \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} - \frac{\xi\psi}{2} + \frac{\varepsilon_\gamma\varepsilon_g}{2} + r \left(\varepsilon_\gamma - \frac{\varepsilon_g^2}{4} - \frac{\psi^2}{2} + \frac{\xi^2}{4} \right) \\
& + r^2 \left(-\frac{\varepsilon_g}{2} - \frac{\varepsilon_\gamma\varepsilon_g}{2} + \frac{\varepsilon_g^2}{4} + \frac{\xi\psi}{2} \right) + r^3 \left(\frac{\varepsilon_g^2}{4} - \frac{\xi^2}{4} \right) \\
& + j \left[\frac{\xi}{2} + \frac{\xi\varepsilon_\gamma}{2} + \frac{\varepsilon_g\psi}{2} + r \left(\psi + \psi\varepsilon_\gamma - \frac{\varepsilon_g\xi}{2} \right) \right. \\
& \left. + r^2 \left(-\frac{\xi}{2} - \frac{\xi\varepsilon_\gamma}{2} - \frac{\varepsilon_g\psi}{2} \right) + r^3 \left(\frac{\varepsilon_g\xi}{2} \right) \right]
\end{aligned} \tag{37}$$

Phase monopulse comparator error equations

This section presents a discussion of precomparator and postcomparator error equations for phase monopulse. Unlike the idealized amplitude monopulse case, in the idealized phase monopulse the difference channel is 90° out of phase with respect to the sum channel when the precomparator and postcomparator errors and noise are absent. The result is that the true monopulse ratio is a purely imaginary quantity.

Assume that the true sum and difference patterns are given as functions of the phase squinted beams V_A and V_B as:

$$\begin{aligned}
\Delta &= V_A - V_B \\
\Sigma &= V_A + V_B
\end{aligned} \tag{38}$$

and:

$$r = \text{Im} \left\{ \frac{\Delta}{\Sigma} \right\} = \text{Im} \left\{ \frac{V_A - V_B}{V_A + V_B} \right\} \tag{39}$$

so that:

$$\frac{V_B}{V_A} = \frac{(1 - jr)}{(1 + jr)} \tag{40}$$

As in the amplitude monopulse case we have that the observed complex monopulse ratio after precomparator and postcomparator errors is given by:

$$\hat{r} = \gamma e^{j\psi} \left(\frac{g e^{j\xi} V_A - V_B}{g e^{j\xi} V_A + V_B} \right) \tag{41}$$

which can be rewritten as:

$$\hat{r} = \gamma e^{j\psi} \left\{ \frac{(1+r^2)(g^2-1)}{(1+r^2)(1+g^2) + 2g(1-r^2)\cos\xi - 4gr\sin\xi} + j \frac{2g[(1-r^2)\sin\xi + 2r\cos\xi]}{(1+r^2)(1+g^2) + 2g(1-r^2)\cos\xi - 4gr\sin\xi} \right\} \quad (42)$$

After substitution and manipulation we get:

$$\hat{r} = \gamma \left\{ \frac{g^2 \cos\psi(1+r^2) - 2g \sin\psi[(1-r^2)\sin\xi + 2r\cos\xi] - \cos\psi(1+r^2)}{g^2(1+r^2) + 2g[(1-r^2)\cos\xi - 2r\sin\xi] + (1+r^2)} \right\} + j\gamma \left\{ \frac{g^2 \sin\psi(1+r^2) + 2g \cos\psi[(1-r^2)\sin\xi + 2r\cos\xi] - \sin\psi(1+r^2)}{g^2(1+r^2) + 2g[(1-r^2)\cos\xi - 2r\sin\xi] + (1+r^2)} \right\} \quad (43)$$

As a side note, we could show a relationship between the denominator for phase monopulse and amplitude monopulse as:

$$(1+r^2)(1+g^2) + 2g(1-r^2)\cos\xi - 4gr\sin\xi = g^2(1+r)^2 + 2g(1-r^2)\cos\xi + (1-r)^2 - 2r(g^2 + 2g\sin\xi - 1) \quad (44)$$

We note that the former term on the right-hand-side (RHS) of the equation is the same as the denominator of the amplitude monopulse equation. The latter term of the RHS is the new term unique to the denominator of phase monopulse.

We can use the same small error approximation assumptions as we did with the amplitude monopulse system and come up with the equation:

$$\begin{aligned} \hat{r} \approx & \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} - \frac{\xi\psi}{2} + \frac{\varepsilon_\gamma \varepsilon_g}{2} + r \left(-\psi - \varepsilon_\gamma \psi + \frac{\varepsilon_g \xi}{2} \right) \\ & + r^2 \left(\frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} + \frac{\varepsilon_\gamma \varepsilon_g}{2} - \frac{\xi\psi}{2} \right) + r^3 \left(\frac{\varepsilon_g \xi}{2} \right) \\ & + j \left[\frac{\xi}{2} + \frac{\xi \varepsilon_\gamma}{2} + \frac{\varepsilon_g \psi}{2} + r \left(1 + \varepsilon_\gamma - \frac{\varepsilon_g^2}{4} - \frac{\psi^2}{2} + \frac{\xi^2}{4} \right) \right. \\ & \left. + r^2 \left(\frac{\xi}{2} + \frac{\varepsilon_g \psi}{2} + \frac{\xi \varepsilon_\gamma}{2} \right) + r^3 \left(\frac{\xi^2}{4} - \frac{\varepsilon_g^2}{4} \right) \right] \end{aligned} \quad (45)$$

Again, a simple substitution lets us see the difference between the monopulse ratio with errors and one without errors for phase monopulse:

$$\begin{aligned}
\hat{r} - jr &\approx \frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} - \frac{\xi\psi}{2} + \frac{\varepsilon_\gamma\varepsilon_g}{2} + r \left(-\psi - \varepsilon_\gamma\psi + \frac{\varepsilon_g\xi}{2} \right) \\
&+ r^2 \left(\frac{\varepsilon_g}{2} - \frac{\varepsilon_g^2}{4} + \frac{\varepsilon_\gamma\varepsilon_g}{2} - \frac{\xi\psi}{2} \right) + r^3 \left(\frac{\varepsilon_g\xi}{2} \right) \\
&+ j \left[\frac{\xi}{2} + \frac{\xi\varepsilon_\gamma}{2} + \frac{\varepsilon_g\psi}{2} + r \left(\varepsilon_\gamma - \frac{\varepsilon_g^2}{4} - \frac{\psi^2}{2} + \frac{\xi^2}{4} \right) \right. \\
&\left. + r^2 \left(\frac{\xi}{2} + \frac{\varepsilon_g\psi}{2} + \frac{\xi\varepsilon_\gamma}{2} \right) + r^3 \left(\frac{\xi^2}{4} - \frac{\varepsilon_g^2}{4} \right) \right]
\end{aligned} \tag{46}$$

11. Appendix C: Simplified monopulse models

This section presents simplified models for amplitude and phase monopulse used in this document. Although unrealizable, the purpose of this model is to be simple enough to make the effect of system errors readily apparent.

Amplitude monopulse model

This section presents a very simple model for the various voltages in an amplitude monopulse system. The basis of this simple model is twofold: first, the monopulse ratio must be purely linear; second, the sum voltage must be quadratic. The equation of the former is given by¹⁰:

$$r = k_m \theta_n = k_m \left(\frac{\theta}{\theta_{3dB}} \right) \quad (47)$$

where r is the idealized monopulse ratio, k_m is a real constant referred to as the normalized monopulse ratio slope, and θ_n is the target angle θ normalized to the 3 dB angle of the antenna, θ_{3dB} . Likewise, the sum beam voltage is given by:

$$V_\Sigma = 1 - \alpha_0 \theta_n^2 \quad (48)$$

where:

$$\alpha_0 = 2(2 - \sqrt{2})$$

Given that:

$$\begin{aligned} V_\Sigma &= V_A + V_B \\ V_\Delta &= V_A - V_B \end{aligned} \quad (49)$$

then:

$$\begin{aligned} V_A &= \left(\frac{1}{2} \right) (1 + r) V_\Sigma \\ V_B &= \left(\frac{1}{2} \right) (1 - r) V_\Sigma \end{aligned} \quad (50)$$

The following figure shows the different patterns.

¹⁰ Very important to note that the definition of the monopulse ratio in this case is $r = \text{Re} \left\{ \frac{V_\Delta}{V_\Sigma} \right\}$.

Figure 47: Simple simulated patterns from equations for amplitude monopulse

Phase monopulse model

For the phase monopulse model, we will follow the organization for the amplitude monopulse to permit comparisons between phase and amplitude monopulse with errors. We start with defining¹¹:

$$r = k_m \theta_n \quad (51)$$

where, again, k_m is a constant and θ_n is the normalized angle. Similar to the amplitude monopulse model we have:

$$V_\Sigma = 1 - \alpha_0 \theta_n^2 \quad (52)$$

where α_0 was given previously.

¹¹ Very important to note the definition of the monopulse ratio used here is such that $r = \text{Im} \left\{ \frac{V_\Delta}{V_\Sigma} \right\}$.

In this case, the model for the phase monopulse becomes:

$$V_{\Delta} = jrV_{\Sigma} \quad (53)$$

and

$$\begin{aligned} V_A &= \left(\frac{1}{2}\right)(1 + jr)V_{\Sigma} \\ V_B &= \left(\frac{1}{2}\right)(1 - jr)V_{\Sigma} \end{aligned} \quad (54)$$

12. Distribution

Unlimited Release

1	MS 0509	K. D. Meeks	5300	
1	MS 0532	J. J. Hudgens	5340	
1	MS 0532	B. L. Burns	5340	
1	MS 0533	S. Gardner	5342	(electronic copy)
1	MS 0533	T. P. Bielek	5342	(electronic copy)
1	MS 0533	D. Harmony	5342	(electronic copy)
1	MS 0533	J. A. Hollowell	5342	(electronic copy)
1	MS 0533	D. G. Thompson	5342	(electronic copy)
1	MS 0532	W. H. Hensley	5344	
1	MS 0519	A. Martinez	5344	(electronic copy)
1	MS 0519	R. M. Naething	5344	(electronic copy)
1	MS 0519	A. M. Raynal	5344	(electronic copy)
1	MS 0533	K. W. Sorensen	5345	(electronic copy)
1	MS 0533	S. E. Allen	5345	(electronic copy)
1	MS 0533	B. C. Brock	5345	(electronic copy)
1	MS 0533	B. H. Strassner	5345	(electronic copy)
1	MS 0533	D. F. Dubbert	5345	(electronic copy)
1	MS 0519	J. A. Ruffner	5349	
1	MS 0519	A. W. Doerry	5349	
1	MS 0519	L. M. Klein	5349	(electronic copy)
6	MS 0519	D. L. Bickel	5354	
1	MS 0519	R. C. Ormesher	5354	(electronic copy)
1	MS 0899	Technical Library	9536	(electronic copy)