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ANGULAR DISTRIBUTIONS AND THE PHYSICS OF CHARMED MESON PRODUCTION AT THE 4.028 GeV RESONANCE

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ABSTRACT

A detailed study of angular distributions arising from $D\bar{D}$, $D^*\bar{D}$, $\bar{D}^*D$, and $D^*\bar{D}^*$ production at $\sqrt{s} = 4.028$ GeV is made, including the subsequent decays $D^* \to D\pi$ and $D^* \to D\gamma$. The production amplitudes are unique except for the $D^*\bar{D}^*$ case, where there are two p-wave amplitudes $(S = 0, 2)$ and one small f-wave amplitude $(S = 2)$. It is shown that observations of the angular distributions and correlations of the π^0's and γ's from the $D^* \to D\pi^0$ and $D^* \to D\gamma$ decays provide an effective way of measuring the p-wave amplitudes. These amplitudes are a reflection of the underlying hadronic interactions among the charmed and uncharmed quarks.

I. INTRODUCTION

Just above the threshold for charmed particle creation in electron-positron collisions are several resonance peaks which serve as "factories" for production of the charmed mesons D and D^*. Angular distribution measurements made at the 3.772 GeV and 4.028 GeV peaks have already yielded evidence on the D and D^* spins. New measurements promise to reveal the details of the resonant production process itself. A confrontation between theories of charmed particle production in e^+e^- collisions and experiment will then become possible.

In this paper we focus on the resonance at 4.028 GeV, where both D and D^* are produced. We obtain expressions for the angular distributions and correlations which may be measured, in terms of the production spin amplitudes. In Section II, we give the production angular distributions and D^* polarization distributions for the three final states which dominate[1] at 4.028 GeV: $D^*\bar{D}^*$, $D^*\bar{D}$ (or $\bar{D}^*D$), and $D\bar{D}$. In Section III, the single pion angular distributions resulting from $D^*\bar{D}^*$ or $D^*\bar{D}$ production followed by D^* decay to $D\pi$ are found. So too are the analogous single photon distributions corresponding to the alternative decay $D^* \to D\gamma$. Section IV presents the correlated angular distributions for two pions, two photons, or a pion and a photon, resulting from a production and decay sequence such as $e^+e^- \to D^*\bar{D}^* \to (D\pi)(\bar{D}\gamma)$. The single- and two-particle distributions arising from $D^*\bar{D}^*$ decays are collected in Appendix A.

As we shall see, the two particle correlations and single particle distributions depend quite differently on the unknown production spin amplitudes. Thus, measuring the correlations and distributions could prove to be a very effective way to determine the amplitudes accurately. The various distributions and their usefulness are discussed in Section V.

The 4.028 GeV resonance is only barely above the threshold for production of a $D^*\bar{D}^*$ pair. In fact, at 4.028 GeV the D^* in such a pair will have velocity $\beta < 0.1$, and to a good approximation $D^*\bar{D}^*$ production and decay may be treated non-relativistically. It is not so obvious that a non-relativistic analysis is also valid for $D^*\bar{D}$ production and decay, or for $D\bar{D}$ production, at this same energy, since $M_D < M_{D^*}$, and $\beta = 0.27$ for a D^* accompanied by a $\bar{D}$, and $\beta = 0.38$ for the members of a $D\bar{D}$ pair. Consequently, in Appendix B we give a fully relativistic treatment of the processes involving $D^*\bar{D}$ or $D\bar{D}$ pairs, and show explicitly that the relativistic effects are negligible. Armed with this result, we present in the main body of the paper an analysis which is non-relativistic throughout.

Past experiments carried out at the "factories" at 4.028 GeV and 3.772 GeV support the conventional spin-parity assignments of 0^- for D and 1^- for D^*[2,3]. We shall assume that these assignments are correct. However, in Section VI the evidence for them is reviewed and a further test which could be carried out at 3.772 GeV is suggested.

II. PRODUCTION AMPLITUDES

The production amplitudes for $e^+e^- \rightarrow D\bar{D}, D^*\bar{D}, \bar{D}^*D$, and $D^*\bar{D}^*$ are easily described in non-relativistic language. We assume the conventional assignments: $J^P(D) = 0^-$, $J^P(D^*) = 1^-$. The $D\bar{D}$ production is purely p-wave with an amplitude

$$\mathcal{M}_{D\bar{D}} \propto \underset{\sim}{\eta}\cdot\underset{\sim}{p} \,, \qquad (1)$$

where $\underset{\sim}{\eta}$ is the γ^* polarization vector and $\underset{\sim}{p}$ is the three momentum of the D in the center of mass frame. If the e^+ and e^- are unpolarized, the coupling to the virtual photon gives transverse polarization:[4]

$$\sum_{pol} \eta_i\eta_j = \delta_{ij} - \hat{n}_i\hat{n}_j \,, \qquad (2)$$

where $\hat{n}$ is the e^+ or e^- beam direction. From Eqs. (1) and (2), we see that the angular distribution of D's from $D\bar{D}$ production is

$$dN_{D\bar{D}} \propto 1 - (\hat{n}\cdot\hat{p})^2. \qquad (3)$$

For $D^*\bar{D}$ or $\bar{D}^*D$ production, we have $S = 1$. To get the correct parity we need L odd, and since $J = 1$, $L \leqslant 2$. Thus $L = 1$. The amplitude is of the form

$$\mathcal{M}_{D^*\bar{D}} \propto \underset{\sim}{\eta}\cdot\underset{\sim}{p} \times \underset{\sim}{\varepsilon} \qquad (4)$$

where $\underset{\sim}{\eta}$ and $\underset{\sim}{p}$ are defined as before, and $\underset{\sim}{\varepsilon}$ is the polarization vector of the D^*, obeying

$$\sum_{pol} \varepsilon_i\varepsilon_j = \delta_{ij} \qquad (5)$$

Doing the sum over the photon polarizations,

$$dN_{D^*\bar{D}} \propto 1 - (\hat{p}\cdot\varepsilon)^2 - [\underset{\sim}{\varepsilon}\cdot(\hat{n}\times\hat{p})]^2. \qquad (6)$$

In particular, if the polarization of the D^* is not observed, the angular distribution is

$$dN_{D^*\bar{D}} \propto 1 + (\hat{n}\cdot\hat{p})^2 \,. \qquad (7)$$

If, on the other hand, the direction of the D is not observed, we have a correlation between the beam direction and the D^* spin:

$$dN_{D^*\bar{D}} \propto 1 + (\hat{n}\cdot\boldsymbol{\varepsilon})^2. \qquad (8)$$

For $D^*\bar{D}^*$ production, S = 0,1, and 2 are possible. However, since $P = (-1)^L$ and $C = (-1)^{L+S}$, L is odd and S is even. If S = 0, then L = 1. If S = 2, then L = 1 or 3. Since the $\psi(4028)$ is so near the $D^*\bar{D}^*$ threshold, we ignore the f-wave production. Then the production amplitude may be written as

$$\mathcal{M}_{D^*\bar{D}^*} = A_o\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}}\ \mathbf{p}\cdot\mathbf{n} + A_2\left(\tfrac{1}{2}\,\boldsymbol{\varepsilon}\cdot\mathbf{p}\ \bar{\boldsymbol{\varepsilon}}\cdot\mathbf{n} + \tfrac{1}{2}\ \boldsymbol{\varepsilon}\cdot\mathbf{n}\,\bar{\boldsymbol{\varepsilon}}\cdot\mathbf{p} - \tfrac{1}{3}\,\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}}\,\mathbf{p}\cdot\mathbf{n}\right), \qquad (9)$$

where A_o is the S = 0 amplitude and A_2 is the S = 2 amplitude with L = 1. The $\boldsymbol{\varepsilon}(\bar{\boldsymbol{\varepsilon}})$ is the $D^*(\bar{D}^*)$ polarization vector and $\mathbf{p}$ is the D^* three-momentum in the center of mass frame.

The most general expression for the observables is obtained by summing over the photon polarizations to obtain

$$\begin{aligned} dN_{D^*\bar{D}^*} \propto\ & |A_o|^2(\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}})^2\,[1 - (\hat{n}\cdot\hat{p})^2] \\ & + 2\mathrm{Re}A_oA_2^*\,\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}}\ [\boldsymbol{\varepsilon}\cdot\hat{p}\ \bar{\boldsymbol{\varepsilon}}\cdot\hat{p} - \tfrac{1}{3}\,\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}} \\ & + \hat{n}\cdot\hat{p}(\tfrac{1}{3}\,\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}}\ \hat{n}\cdot\hat{p} - \tfrac{1}{2}\,\boldsymbol{\varepsilon}\cdot\hat{n}\ \bar{\boldsymbol{\varepsilon}}\cdot\hat{p} - \tfrac{1}{2}\,\boldsymbol{\varepsilon}\cdot\hat{p}\ \bar{\boldsymbol{\varepsilon}}\cdot\hat{n})] \\ & + |A_2|^2\Big[\tfrac{1}{4}(\boldsymbol{\varepsilon}\cdot\hat{p})^2 + \tfrac{1}{4}\,(\bar{\boldsymbol{\varepsilon}}\cdot\hat{p})^2 + \tfrac{1}{9}(\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}})^2 - \tfrac{1}{4}\,(\boldsymbol{\varepsilon}\cdot\hat{p})^2(\bar{\boldsymbol{\varepsilon}}\cdot\hat{n})^2 \\ & - \tfrac{1}{4}\,(\bar{\boldsymbol{\varepsilon}}\cdot\hat{p})^2(\boldsymbol{\varepsilon}\cdot\hat{n})^2 - \tfrac{1}{9}\,(\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}})^2\,(\hat{n}\cdot\hat{p})^2 - \tfrac{1}{6}\,\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}}\ \ \boldsymbol{\varepsilon}\cdot\hat{p}\ \bar{\boldsymbol{\varepsilon}}\cdot\hat{p} \\ & + \tfrac{1}{3}\,\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}}\ \hat{n}\cdot\hat{p}\ \boldsymbol{\varepsilon}\cdot\hat{p}\ \bar{\boldsymbol{\varepsilon}}\cdot\hat{n} + \tfrac{1}{3}\,\boldsymbol{\varepsilon}\cdot\bar{\boldsymbol{\varepsilon}}\ \hat{n}\cdot\hat{p}\ \boldsymbol{\varepsilon}\cdot\hat{n}\ \bar{\boldsymbol{\varepsilon}}\cdot\hat{p} \\ & - \tfrac{1}{2}\,\boldsymbol{\varepsilon}\cdot\hat{p}\ \ \bar{\boldsymbol{\varepsilon}}\cdot\hat{p}\ \ \boldsymbol{\varepsilon}\cdot\hat{n}\ \ \bar{\boldsymbol{\varepsilon}}\cdot\hat{n}\Big]\ . \end{aligned} \qquad (10)$$

This cumbersome expression yields useful results upon further reduction. Suppose for example, that the D^* and $\bar{D}^*$ polarizations are not observed. Then using Eq. (5),

$$\begin{aligned} dN_{D^*\bar{D}^*} \propto\ & 3|A_o|^2\,[1 - (\hat{n}\cdot\hat{p})^2] \\ & + \tfrac{1}{6}\,|A_2|^2\,[7 - (\hat{n}\cdot\hat{p})^2]\ . \end{aligned} \qquad (11)$$

The relative normalization of A_o and A_2 is clarified by averaging over $\hat{p}$ or $\hat{n}$, using $\hat{p}_i\hat{p}_j \to \frac{1}{3}\delta_{ij}$ or $\hat{n}_i\hat{n}_j \to \frac{1}{3}\delta_{ij}$, to obtain the full rate,

$$dN_{D^*\bar{D}^*} \propto |A_o|^2 + \tfrac{5}{9}\,|A_2|^2\ . \qquad (12)$$

The ratio $|A_2/A_o|$ can be obtained by measuring the angular distribution of D^*'s in $e^+e^- \to D^*\bar{D}^*$. Actually, after the decay $D^* \to D\pi$, the D has nearly the same direction as its parent D^*, so it is possible to use identified D's in the place of D^*'s. From Eq. (11) we see that

$$dN_{D^*\bar{D}^*} \propto 1 + \alpha\,(\hat{n}\cdot\hat{p})^2, \qquad (13)$$

where

$$\alpha = -\,\frac{18 + |A_2/A_o|^2}{18 + 7|A_2/A_o|^2}\ . \qquad (14)$$

Thus α is constrained by $-1 \leqslant \alpha \leqslant -1/7$ in our approximation (that is, ignoring f-wave production)[5]. Reference 2 gives $\alpha = -0.30 \pm 0.33$ as deduced from the angular distribution of D's in $e^+e^- \to D^*\bar{D}^* \to DX$. If this result is interpreted as showing $\alpha > -0.63$, the conclusion is $|A_2/A_o|^2 > 1.95$.

An alternative for measuring A_2/A_0 (including its phase) is to use the information carried by the spins of the D^* and $\overline{D}^*$, even without measuring their momenta. This can be done by observing the π's and γ's from the decays $D^* \to D\pi$ and $D^* \to D\gamma$. If the direction $\hat{p}$ is averaged over, Eq. (10) becomes

$$dN_{D^*\overline{D}^*} \propto |A_0|^2 \, \frac{2}{3} (\boldsymbol{\varepsilon} \cdot \overline{\boldsymbol{\varepsilon}})^2 + 2\mathrm{Re} A_0 A_2^* \left(\frac{1}{9} \boldsymbol{\varepsilon} \cdot \overline{\boldsymbol{\varepsilon}} - \frac{1}{3} \overline{\boldsymbol{\varepsilon}} \cdot \hat{n} \, \boldsymbol{\varepsilon} \cdot \hat{n} \right) \boldsymbol{\varepsilon} \cdot \overline{\boldsymbol{\varepsilon}}$$

$$+ |A_2|^2 \left[\frac{1}{6} + \frac{1}{54} (\boldsymbol{\varepsilon} \cdot \overline{\boldsymbol{\varepsilon}})^2 - \frac{1}{12} (\boldsymbol{\varepsilon} \cdot \hat{n})^2 - \frac{1}{12} (\overline{\boldsymbol{\varepsilon}} \cdot \hat{n})^2 + \frac{1}{18} \boldsymbol{\varepsilon} \cdot \overline{\boldsymbol{\varepsilon}} \, \boldsymbol{\varepsilon} \cdot \hat{n} \, \overline{\boldsymbol{\varepsilon}} \cdot \hat{n} \right]. \quad (15)$$

As we shall see in subsequent Sections, the $\boldsymbol{\varepsilon}$ and $\overline{\boldsymbol{\varepsilon}}$ will be replaced by observable momenta when the decays $D^* \to D\pi$ or $D^* \to D\gamma$ occur. If one decay (say that of the $\overline{D}^*$) is unobserved, Eq. (15) is further simplified using $\varepsilon_i \varepsilon_j \to \delta_{ij}$. Then we obtain

$$dN_{D^*\overline{D}^*} \propto \frac{2}{3} |A_0|^2 + 2\mathrm{Re} A_0 A_2^* \left[\frac{1}{9} - \frac{1}{3} (\hat{n} \cdot \boldsymbol{\varepsilon})^2 \right]$$

$$+ |A_2|^2 \left[\frac{47 - 21 (\hat{n} \cdot \boldsymbol{\varepsilon})^2}{108} \right] . \quad (16)$$

III. SINGLE PARTICLE DISTRIBUTIONS

The D^* decays into $D\pi$ or $D\gamma$. Each decay has only a single amplitude. For $D^* \to D\pi$ we write

$$\mathcal{M}_{D^* \to D\pi} \propto \boldsymbol{\varepsilon} \cdot \mathbf{q} \quad (17)$$

where $\boldsymbol{\varepsilon}$ is the D^* polarization vector and $\mathbf{q}$ is the pion three momentum. For the photonic decay we have the M1 amplitude

$$\mathcal{M}_{D^* \to D\gamma} \propto \left(\boldsymbol{\varepsilon} \cdot \mathbf{k} \times \hat{E} \right) \propto \left(\boldsymbol{\varepsilon} \cdot \hat{B} \right) \quad (18)$$

where $\hat{k}$ is the γ direction, and $\hat{E}$ and $\hat{B}$ represent the photon's electric and magnetic polarization vectors. If the photon's polarization is not measured, we have the summation relations

$$\sum_{pol} \hat{E}_i \hat{E}_j = \sum_{pol} \hat{B}_i \hat{B}_j = \delta_{ij} - \hat{k}_i \hat{k}_j \, . \quad (19)$$

Because of the simple forms of Eqs. (17) and (18), the distributions of Section II may be easily translated into distributions for final state π's or γ's. Summing over the polarization of the decaying D^* is equivalent to replacing $\boldsymbol{\varepsilon}$ by $\hat{q}$ for $D^* \to D\pi$ and to replacing $\boldsymbol{\varepsilon}$ by $\hat{B}$ for $D^* \to D\gamma$. For example, using Eq. (8) we determine the distribution of pions relative to the beam axis in $e^+e^- \to D^*\overline{D} \to (D\pi)\,\overline{D}$:

$$dN_{D^*\overline{D}} \propto 1 + (\hat{n} \cdot \hat{q})^2 . \quad (20)$$

For $e^+e^- \to D^*\overline{D}^* \to (D\pi)\overline{D}^*$, using Eq. (16), we have

$$dN_{D^*\overline{D}^*} \propto \frac{2}{3} |A_0|^2 + 2\mathrm{Re} A_0 A_2^* \, \frac{1}{9} [1 - 3 (\hat{n} \cdot \hat{q})^2]$$

$$+ |A_2|^2 \cdot \frac{1}{108} [47 - 21 (\hat{n} \cdot \hat{q})^2] . \quad (21)$$

This distribution is again of the form $1 + \alpha(\hat{n}\cdot\hat{q})^2$, where α is a function of the complex number A_o/A_2. Since our concern is with the resonance at 4.028 GeV, we expect A_o and A_2 to have approximately the same (resonant) phase, assuming both the $S = 0$ and $S = 2$ channels couple strongly to the resonance. If one channel is effectively decoupled, its amplitude will be small and the relative phase between the amplitudes will be unimportant. We thus expect A_o/A_2 to be nearly real, but its sign must be determined by measurements.

The angular distributions of photons relative to the beam axis may be determined in a similar fashion. For the process $e^+e^- \to D^*\overline{D} \to (D\gamma)\overline{D}$ we have by analogy with Eq. (20)

$$dN_{D^*\overline{D}} \propto \frac{1}{3}[1 + (\hat{B}\cdot\hat{n})^2] . \qquad (22)$$

Using Eq. (19), this becomes

$$dN_{D^*\overline{D}} \propto 1 - \frac{1}{3}(\hat{k}\cdot\hat{n})^2 . \qquad (23)$$

Similarly, for $e^+e^- \to D^*\overline{D}^* \to (D\gamma)\overline{D}^*$, the single photon distribution is

$$dN_{D^*\overline{D}^*} \propto \frac{4}{3}|A_o|^2 + 2\mathrm{Re}A_oA_2^* \ \frac{1}{9}[-1 + 3(\hat{n}\cdot\hat{k})^2]$$

$$+ \ |A_2|^2 \cdot \frac{1}{108}[73 + 21(\hat{n}\cdot\hat{k})^2] . \qquad (24)$$

IV. TWO PARTICLE CORRELATED ANGULAR DISTRIBUTIONS

Processes like $e^+e^- \to D^*\overline{D}^* \to (D\pi_1)(\overline{D}\pi_2)$ or $(D\pi)(\overline{D}\gamma)$ are very powerful analyzers of the production amplitudes. The angular

distributions are obtained from the formulae of Section II by the replacements $\underset{\sim}{\varepsilon} \to \hat{q}_1$, $\underset{\sim}{\overline{\varepsilon}} \to \hat{q}_2$, or $\underset{\sim}{\varepsilon} \to \hat{B}$, etc. Thus for example, using Eq. (15), we have for the final state $(D\pi_1)$ $(\overline{D}\pi_2)$

$$dN_{D^*\overline{D}^*} \propto \frac{2}{3}(\hat{q}_1\cdot\hat{q}_2)^2 \ |A_o|^2 \ + 2\mathrm{Re}A_oA_2^*\hat{q}_1 \cdot \hat{q}_2 \cdot \frac{1}{9}(\hat{q}_1\cdot\hat{q}_2 - 3\hat{q}_1\cdot\hat{n} \ q_2\cdot\hat{n})$$

$$+ |A_2|^2\left[\frac{1}{6} + \frac{1}{54}(\hat{q}_i\cdot\hat{q}_2)^2 - \frac{1}{12}(\hat{q}_1\cdot\hat{n})^2 - \frac{1}{12}(\hat{q}_2\cdot\hat{n})^2 + \frac{1}{18}\hat{q}_1\cdot\hat{q}_2\hat{q}_1\cdot\hat{n} \ \hat{q}_2\cdot\hat{n}\right]. \qquad (25)$$

If the beam direction is ignored, the correlation is

$$dN_{D^*\overline{D}^*} \propto \frac{2}{3}(\hat{q}_1\cdot\hat{q}_2)^2 \ |A_o|^2 + \frac{1}{27}|A_2|^2\left[3 + (\hat{q}_1\cdot\hat{q}_2)^2\right] . \qquad (26)$$

The analogous form for $e^+e^- \to D^*\overline{D}^* \to (D\pi)(\overline{D}\gamma)$ is

$$dN_{D^*\overline{D}^*} \propto \frac{2}{3}|A_o|^2[1 - (\hat{q}\cdot\hat{k})^2] + 2\mathrm{Re}A_oA_2^*\left[\frac{1}{9} - \frac{1}{9}(\hat{q}\cdot\hat{k})^2 - \frac{1}{3}(\hat{q}\cdot\hat{n})^2 + \frac{1}{3}\hat{k}\cdot\hat{q}\hat{q}\cdot\hat{n}\hat{n}\cdot\hat{k}\right]$$

$$+ |A_2|^2\left[\frac{29}{108} - \frac{1}{54}(\hat{q}\cdot\hat{k})^2 - \frac{1}{9}(\hat{q}\cdot\hat{n})^2 + \frac{1}{12}(\hat{k}\cdot\hat{n})^2 - \frac{1}{18}\hat{q}\cdot\hat{n}\hat{n}\cdot\hat{k} \ \hat{q}\cdot\hat{k}\right] , \qquad (27)$$

where $\hat{q}$ and $\hat{k}$ are the directions of the pion and photon. Similarly, for the final state $(D\gamma_1)$ $(\overline{D}\gamma_2)$ we have

$$dN_{D^*\overline{D}^*} \propto \frac{2}{3}|A_o|^2[1 + (\hat{k}_1\cdot\hat{k}_2)^2] + 2\mathrm{Re}A_oA_2^*\left[-\frac{2}{9} + \frac{1}{9}(\hat{k}_1\cdot\hat{k}_2)^2 + \frac{1}{3}(\hat{k}_1\cdot\hat{n})^2\right.$$

$$\left. + \frac{1}{3}(\hat{k}_2\cdot\hat{n})^2 - \frac{1}{3}\hat{k}_1\cdot\hat{k}_2\hat{k}_1\cdot\hat{n} \ \hat{k}_2\cdot\hat{n}\right] \qquad (28)$$

$$+ |A_2|^2\left[\frac{22}{54} + \frac{1}{54}(\hat{k}_1\cdot\hat{k}_2)^2 + \frac{1}{9}(\hat{k}_1\cdot\hat{n})^2 + \frac{1}{9}(\hat{k}_2\cdot\hat{n})^2 + \frac{1}{18}\hat{k}_1\cdot\hat{k}_2\hat{k}_1\cdot\hat{n} \ \hat{k}_2\cdot\hat{n}\right].$$

The angular distributions and correlations of pions and photons arising from $D^*\overline{D}^*$ production and decay are summarized in Appenidix A.

V. DISCUSSION

Among the dominant charmed meson production processes at 4.028GeV, only the reaction $e^+e^- \to D^*\bar{D}^*$ has more than one invariant amplitude. Dropping the f-wave, this freedom is characterized by the parameter $z \equiv A_0/A_2$. From the angular distribution for this reaction, as inferred from that for $e^+e^- \to D^*\bar{D}^* \to DX$, there is some weak evidence that $|z| \leq 0.7$ (see paragraph following Eq. (12) and Ref. 2). More definitive information may come from the pions and photons resulting from the D^* and $\bar{D}^*$ decays.

Being easier to study, the inclusive single π and γ distributions will probably be obtained before the two-particle correlations. From Eq. (21), the angular distribution of pions from $e^+e^- \to D^*\bar{D}^* \to (D\pi)\bar{D}^*$ will be $1 + \alpha_\pi(z)(\hat{q}\cdot\hat{n})^2$, where

$$\alpha_\pi(z) = \frac{-21 - 72\mathrm{Re}\,z}{47 + 24\mathrm{Re}\,z + 72|z|^2} . \qquad (29)$$

Similarly, from Eq. (24), the angular distribution of photons from $e^+e^- \to D^*\bar{D}^* \to (D\gamma)\bar{D}^*$ will be $1 + \alpha_\gamma(z)(\hat{k}\cdot\hat{n})^2$, with

$$\alpha_\gamma(z) = \frac{21 + 72\mathrm{Re}\,z}{73 - 24\mathrm{Re}\,z + 144|z|^2} . \qquad (30)$$

The functions $\alpha_\pi(z)$ and $\alpha_\gamma(z)$ are plotted for real z in Figures 1 and 2. From the Figures, we see that if z is real as expected, α_π and α_γ must lie in the intervals $-0.74 \leq \alpha_\pi(z) \leq 0.54$ and $-0.21 \leq \alpha_\gamma(z) \leq 0.59$. Furthermore, a given value of either α corresponds in general to two values of z. This ambiguity cannot be resolved by measuring both α_π and α_γ. Indeed, $\alpha_\gamma = -\alpha_\pi/(2 + \alpha_\pi)$, so the two z- values which correspond to a given value of one α also

correspond to a common value of the other. The ambiguity can be resolved by studying the two-particle correlations. The $\pi\pi$ correlation, $1 + (\frac{1}{3} + 6|z|^2)(\hat{q}_1\cdot\hat{q}_2)^2$ (see Eq. (A2)), is particularly sensitive to $|z|^2$. From this expression and Fig. 1, we see that a measurement of $\alpha_\pi(z)$ and of the $\pi\pi$ correlation would make possible a rather precise determination of z.

To measure $\alpha_\pi(z)$, one must be able to separate single pions due to $e^+e^- \to D^*\bar{D}^* \to (D\pi)\bar{D}^*$ from those due to $e^+e^- \to D^*\bar{D} \to (D\pi)\bar{D}$. This can, to a large extent, be done. At 4.028 GeV, neutral pions from $D^*\bar{D}^*$ production must have 138 MeV $< E <$ 147 MeV, while those from $D^*\bar{D}$ production can have 135 MeV $< E <$ 160 MeV, where E is the pion's laboratory energy. Since the total rates for $D^*\bar{D}^*$ and $D^*\bar{D}$ production are comparable[1], the slow pions ($E <$ 147 MeV) will be mostly from $D^*\bar{D}^*$[6,7,8].

What does one expect theoretically for the spin structure of $e^+e^- \to D^*\bar{D}^*$ at a resonance peak? One popular approach to this problem[9] views the resonance at 4.028 GeV as a $^3S_1$ $c\bar{c}$ state. Its decay to charmed mesons entails the creation of a light quark-antiquark pair from the vacuum, and it is assumed that there is no correlation between the light quark spins and those of the c and $\bar{c}$. This picture predicts that the production amplitudes for all the allowed $D^*\bar{D}^*$ helicity states have the same magnitude[10]. Using Eq. (9), one easily finds that this implies that $z = -1/6$[11,12]. From Eqs. (29), (30), (A2), (A6) and (A9), we see that this picture then predicts that the one-pion distribution will be

$$dN_{D^*\bar{D}^*} \propto 1 - \frac{1}{5}(\hat{q}\cdot\hat{n})^2, \qquad (31)$$

the one-photon distribution

$$dN_{D^*\bar{D}^*} \propto 1 + \frac{1}{9}(\hat{k}\cdot\hat{n})^2 , \qquad (32)$$

the pion-pion correlation

$$dN_{D^*\bar{D}^*} \propto 1 + \frac{1}{2}(\hat{q}_1\cdot\hat{q}_2)^2, \qquad (33)$$

the pion-photon correlation

$$dN_{D^*\bar{D}^*} \propto 1 - \frac{1}{5}(\hat{q}\cdot\hat{k})^2, \qquad (34)$$

and the photon-photon correlation

$$dN_{D^*\bar{D}^*} \propto 1 + \frac{1}{9}(\hat{k}_1\cdot\hat{k}_2)^2. \qquad (35)$$

However, we note from Figures 1 and 2 that α_π and α_γ vary extremely rapidly with z around $z = -1/6$. Thus, if there is a small error in the theoretically predicted z, the true single-pion and -photon angular distributions could look very different from the predicted ones.

It has been argued[13] that the apparent large rise in $D^*\bar{D}^*$ production at 4.028GeV, so close to $D^*\bar{D}^*$ threshold, may mean that an s-wave final state is involved. This being impossible for $e^+e^- \rightarrow \gamma^* \rightarrow D^*\bar{D}^*$, it was suggested[13] that this process is in reality dominated by $e^+e^- \rightarrow D^{**}\bar{D}^*$, where D^{**} is a 0^+ meson which is degenerate in mass with D^*, and decays exclusively to $D\pi$. To accommodate the relative one-pion and one-photon rates seen at 4.028GeV,[14] the D^* must then decay mostly to $D\gamma$. Using the methods of the previous Sections, we can easily show that in

this heretical scheme, the slow pions resulting from $D^{**}\bar{D}^*$ production will have the angular distribution $1 - \varepsilon^2(\hat{q}\cdot\hat{n})^2$. Here ε is a small number, the distribution being dominated by an isotropic part from $e^+e^- \rightarrow D^{**}\bar{D}^* \rightarrow (D\pi)(D\gamma)$. Clearly this scheme would be ruled out by an observed pion distribution which grows with $(\hat{q}\cdot\hat{n})^2$. However, an observed distribution consistent with the prediction of the canonical view, Eq.(31), would probably also be consistent with this unorthodox picture. One could then try to eliminate the latter by studying the two-pion correlation. In the unorthodox scheme, one would expect less correlation than when $D^*\bar{D}^*$ is the two-pion source, because now $D^{**}\bar{D}^*$ and $D^{**}\bar{D}^{**}$ are the major sources, and neither mode leads to any correlation.[15]

VI. SPIN-PARITY OF D AND D^*

We have been taking for granted that $J^P(D) = 0^-$ and $J^P(D^*) = 1^-$. Here we briefly review the evidence for this assignment, and propose a further test based on two-body correlations.

What is known is that <u>if</u> $(J(D) + J(D^*)) < 2$, <u>then</u> the D(1865) is spinless, the $D^*(2007)$ has spin one, and the $D - D^*$ relative parity is even. The possibility that both D and D^* have spin one, or that some higher spins are involved, has not really been considered.

The D and D^* mesons cannot both be spinless because of any one of the following obervations: (a) $D^* \rightarrow D\gamma$, (b) $e^+e^- \rightarrow D^*\bar{D} \rightarrow (D\pi)\bar{D}$, (c) the angular distribution for $e^+e^- \rightarrow D^*\bar{D}$ is inconsistent with $1 - (\hat{p}\cdot\hat{n})^2$.[2] If one then assumes that one of these mesons is spinless and the other has spin one, it follows from the occurrence of $D^* \rightarrow D\pi$ that they have the same parity, conventionally taken to be negative. Given the relative parity, Jackson has predicted the joint production-decay distribution for $e^+e^- \to D^*\bar{D}$ followed by the weak decay $\bar{D} \to K\pi$.[16] The result,[2] expressed in terms of the D^* production angle $\Theta = \cos^{-1}(\hat{p}\cdot\hat{n})$ and the spherical angles θ and ϕ of the K in the $\bar{D}$ helicity frame[17] is

$$\frac{d^3\sigma}{d\cos\Theta\, d\cos\theta\, d\phi} \propto \begin{cases} 1 + \cos^2\Theta & \text{if } \begin{array}{l} J(D(1865)) = 0, \\ J(D^*(2007)) = 1. \end{array} \quad (36a) \\ \\ \sin^2\theta(\cos^2\phi + \cos^2\Theta\sin^2\phi) & \text{if } \begin{array}{l} J(D(1865)) = 1, \\ J(D^*(2007)) = 0. \end{array} \quad (36b) \end{cases}$$

For the conventional spin assignment, this result is just our Eq. (7) for $D^*\bar{D}$ production. There can be no dependence on the kaon angles in this case because the parent $\bar{D}$ is spinless. For $J(D) = 1$, $J(D^*) = 0$, the result may be obtained by multiplying the amplitude of Eq. (4) for $D^*\bar{D}$ production by that of Eq. (17) for decay of a spin-one particle (here $\bar{D}$) to two spinless ones (here K and π). Summing over the $\bar{D}$ polarizations gives the amplitude $\underset{\sim}{n} \cdot \underset{\sim}{p} \times \underset{\sim}{q}$, where $\underset{\sim}{q}$ is now the momentum of the K. Squaring and summing over the γ^* polarizations then yields the distribution

$$|\hat{p}\times\hat{q}|^2 - |\hat{p}\times\hat{q}\cdot\hat{n}|^2 = \sin^2\theta\,(\cos^2\phi + \cos^2\Theta\sin^2\phi),$$

as in Eq. (36b).

A number of experimental tests performed near 4.03GeV have found the distribution of Eq.(36a) to be consistent with the data, and that of Eq. (36b) to be inconsistent.[2] Furthermore, as stated after Eq. (14), the $D^*\bar{D}^*$ production angular distribution, $1 + \alpha(\hat{n}\cdot\hat{p})^2$, is found to have $\alpha = -0.30 \pm 0.33$.[2] This value of α is two standard deviations away from -1, which from Eq. (3) is the value required for production of a pair of spinless mesons. The assignment $J(D) = 1$, $J(D^*) = 0$ has been ruled out.

Now could one rule out the possibility that both D and D^* have spin one, as would be the case if, for example , they were different radial excitations of a $^3S_1$ quark-antiquark bound state? If $J(D) = J(D^*) = 1$ and the $D - D^*$ relative parity is even, then near threshold the angular distribution for $e^+e^- \to D^*\bar{D}$ would be what we calculated for $e^+e^- \to D^*\bar{D}^*$: $1 + \alpha(\hat{n}\cdot\hat{p})^2$, with $-1 \leqslant \alpha \leqslant -\frac{1}{7}$ (see Eqs. (13) and (14)). If the relative parity is odd, then the $D^*\bar{D}$ final state would be predominantly s-wave, and the angular distribution isotropic. The data taken near 4.03GeV^2 are somewhat inconsistent with both of these possible distributions, but not conclusively so.[18]

Evidence concerning the spin of the D has come from the study of $e^+e^- \to D\bar{D}$ at the 3.772 GeV resonance. It is found that the angular distribution for this process is $1 - (\hat{n}\cdot\hat{p})^2$.[3] If the D is spinless, such a production angular distribution is <u>required</u> (see Eq. (3)), as is an isotropic distribution for any particle created in the subsequent decay of the D or $\bar{D}$. However, this same production angular distribution is also possible if the D has spin one. Indeed, from Eqs. (13) and (14), we see that a pair of spin-one mesons will have this distribution if the amplitude A_o for $S = 0$ production happens to dominate. Furthermore, if A_o dominates, then kaons and pions resulting from $e^+e^- \to D\bar{D} \to (\bar{K}\pi)\bar{D}$ will be isotropically distributed, just as if D were spinless. This may be seen from Eq. (21), if we read D^* as D and $D\pi$ as $\bar{K}\pi$.

To distinguish unambiguously between a spin-one and a spin-zero D, one should measure the $K - \bar{K}$ correlation in $e^+e^- \to D\bar{D} \to (\bar{K}\pi_1)(K\pi_2)$. If the D is truly spinless, there will be no correlation. Suppose, however, that the D is spin-one, but has a production angular distribution $\propto 1 - (\hat{n}\cdot\hat{p})^2$. Then A_0 must dominate, and from Eq. (26) (with D^* read as D) we see that the $K - \bar{K}$ correlation will be $\propto (\hat{q}_K \cdot \hat{q}_{\bar{K}})^2$, rather than isotropic. We observe that in the $K - \bar{K}$ (or $\pi_1 - \pi_2$) correlation, there is a big difference between a spinless D and its spin-one imposter. When the D has spin one and is produced with $S(D\bar{D}) = 0$, the D and $\bar{D}$ spins are correlated (antiparallel), even though each spin may point with equal probability in any direction. Since the distribution of decay kaons from a D or $\bar{D}$ will reflect the spin of the parent (see Eq. (17)), the kaons from, say, the D will be isotropic, but will be correlated with those from the $\bar{D}$.[19]

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APPENDIX A: COLLECTED ANGULAR DISTRIBUTIONS

We list here the angular distributions for pions and photons produced in the processes $e^+e^- \to D^*\bar{D}^* \to (D\pi_1)(\bar{D}\pi_2)$, $(D\pi)(\bar{D}\gamma)$, and $(D\gamma_1)(\bar{D}\gamma_2)$. Both two-particle and inclusive one-particle distributions are given. The beam direction is $\hat{n}$. Pion momenta are denoted by $\hat{q}$, and photon momenta by $\hat{k}$. The ratio of amplitudes $A_0/A_2 = z$ is expected to be nearly real.

$$\frac{dN}{d^3q_1 d^3q_2} \propto 1 + \frac{1}{9}(\hat{q}_1\cdot\hat{q}_2)^2 - \frac{1}{2}(\hat{q}_1\cdot\hat{n})^2 - \frac{1}{2}(\hat{q}_2\cdot\hat{n})^2 + \frac{1}{3}\hat{q}_1\cdot\hat{q}_2\,\hat{q}_2\cdot\hat{n}\,\hat{q}_1\cdot\hat{n}$$
$$+ (2\mathrm{Re}z)\,\frac{2}{3}\,\hat{q}_1\cdot\hat{q}_2(\hat{q}_1\cdot\hat{q}_2 - 3\hat{q}_1\cdot\hat{n}\;\hat{q}_2\cdot\hat{n})$$
$$+ |z|^2\, 4(\hat{q}_1\cdot\hat{q}_2)^2, \qquad (A1)$$

$$\frac{dN}{d(\hat{q}_1\cdot\hat{q}_2)} \propto 1 + \frac{1}{3}(\hat{q}_1\cdot\hat{q}_2)^2 + |z|^2\, 6(\hat{q}_1\cdot\hat{q}_2)^2, \qquad (A2)$$

$$\frac{dN}{d(\hat{q}\cdot\hat{n})} \propto 1 - \frac{21}{47}(\hat{q}\cdot\hat{n})^2 + (2\mathrm{Re}z)\,\frac{12}{47}[1 - 3(\hat{n}\cdot\hat{q})^2]$$
$$+ |z|^2\,\frac{72}{47}, \qquad (A3)$$

$$N \propto 1 + \frac{9}{5}|z|^2, \qquad (A4)$$

$$\frac{dN}{d^3q\,d^3k} \propto 1 - \frac{2}{29}(\hat{q}\cdot\hat{k})^2 - \frac{12}{29}(\hat{q}\cdot\hat{n})^2 + \frac{9}{29}(\hat{k}\cdot\hat{n})^2 - \frac{6}{29}\hat{q}\cdot\hat{n}\;\hat{n}\cdot\hat{k}\;\hat{k}\cdot\hat{q}$$

$$+ (2\mathrm{Re}z)\left[\frac{12}{29} - \frac{12}{29}(\hat{q}\cdot\hat{k})^2 - \frac{36}{29}(\hat{q}\cdot\hat{n})^2 + \frac{36}{29}\hat{q}\cdot\hat{n}\,\hat{n}\cdot\hat{k}\;\hat{k}\cdot\hat{q}\right]$$

$$+ |z|^2\,\frac{72}{29}\left[1 - (\hat{q}\cdot\hat{k})^2\right], \qquad \text{(A5)}$$

$$\frac{dN}{d(\hat{q}\cdot\hat{k})} \propto 1 - \frac{1}{7}(\hat{q}\cdot\hat{k})^2 + |z|^2\,\frac{18}{7}\left[1 - (\hat{q}\cdot\hat{k})^2\right], \qquad \text{(A6)}$$

$$\frac{dN}{d(\hat{k}\cdot\hat{n})} \propto 1 + \frac{21}{73}(\hat{k}\cdot\hat{n})^2 + |z|^2\,\frac{144}{73} + (2\mathrm{Re}z)\left[-\frac{12}{73} + \frac{36}{73}(\hat{k}\cdot\hat{n})^2\right], \qquad \text{(A7)}$$

$$\frac{dN}{d^3k_1\,d^3k_2} \propto 1 + \frac{1}{22}(\hat{k}_1\cdot\hat{k}_2)^2 + \frac{3}{11}(\hat{k}_1\cdot\hat{n})^2 + \frac{3}{11}(\hat{k}_2\cdot\hat{n})^2 + \frac{3}{22}\hat{k}_1\cdot\hat{k}_2\hat{k}_1\cdot\hat{n}\hat{k}_2\cdot\hat{n}$$

$$+ (2\mathrm{Re}z)\left[-\frac{6}{11} + \frac{3}{11}(\hat{k}_1\cdot\hat{k}_2)^2 + \frac{9}{11}(\hat{k}_1\cdot\hat{n})^2 + \frac{9}{11}(\hat{k}_2\cdot\hat{n})^2 - \frac{9}{11}\hat{k}_1\cdot\hat{k}_2\;\hat{k}_1\cdot\hat{n}\;\hat{k}_2\cdot\hat{n}\right]$$

$$+ |z|^2\,\frac{18}{11}\left[1 + (\hat{k}_1\cdot\hat{k}_2)^2\right], \qquad \text{(A8)}$$

$$\frac{dN}{d(\hat{k}_1\cdot\hat{k}_2)} \propto 1 + \frac{1}{13}(\hat{k}_1\cdot\hat{k}_2)^2 + |z|^2\,\frac{18}{13}\left[1 + (\hat{k}_1\cdot\hat{k}_2)^2\right]. \qquad \text{(A9)}$$

APPENDIX B: RELATIVISTIC CALCULATIONS

$e^+e^- \to D\bar{D}$

For the Lorentz-invariant amplitude we have

$$\mathcal{M}_{D\bar{D}} \propto \eta_\mu (p_D - p_{\bar{D}})^\mu, \qquad \text{(B 1)}$$

where η is the γ^* polarization vector, and p_D and $p_{\bar{D}}$ are the D and $\bar{D}$ momenta. In the γ^* rest frame, $\mathcal{M}_{D\bar{D}}$ reduces to $2\,\underset{\sim}{\eta}\cdot\underset{\sim}{p}$, which is just the non-relativistic result, and gives the same angular distribution.

$e^+e^- \to D^*\bar{D}$

We first consider just the production process. The Lorentz-invariant amplitude is of the form

$$\mathcal{M}_{D^*\bar{D}} \propto \varepsilon^{\alpha\beta\mu\nu} Q_\mu \eta_\nu p_\beta \varepsilon_\alpha, \qquad \text{(B 2)}$$

where Q is the γ^* momentum, p the D^* momentum, and ε the

D^* polarization. In the e^+e^- center-of-mass frame this is

$$\mathcal{M}_{D^*\bar{D}} \propto \sqrt{s}\, \underset{\sim}{n} \cdot \underset{\sim}{p} \times \underset{\sim}{\varepsilon} \ , \qquad \text{(B 3)}$$

where $\sqrt{s}$ is the center-of-mass energy. This agrees with the non-relativistic amplitude of Eq.(4), except that now $\underset{\sim}{\varepsilon}$ must be understood to be properly boosted from the D^* rest frame to the e^+e^- c.m. frame. However, Eq. (B 3) states that the production amplitude vanishes unless $\underset{\sim}{\varepsilon}$ is transverse to $\underset{\sim}{p}$ (that is, unless the D^* helicity is ± 1). A vector transverse to $\underset{\sim}{p}$ will not be affected by a boost along $\underset{\sim}{p}$. Thus, the relativistic production amplitude is identical to the non-relativistic one, and gives the same angular distribution.

$\underline{e^+e^- \to D^*\bar{D} \to (D\pi)\bar{D}}$

For sequential processes such as this, the non-relativistic methods used in the main text give the approximate angular distribution, but not the energy distribution, for the outgoing pion or photon. Indeed, those methods are accurate and useful when the speed β of the D^* in the e^+e^- c.m. is sufficiently small, relative to that of the π or γ in the D^* rest frame, so that the π or γ energy in the e^+e^- c.m. is sharply determined, and there is no energy distribution to be predicted. By contrast, the relativistic calculation we report here gives the joint distribution in angle and energy for the outgoing particle. To obtain this distribution, it is convenient to represent $e^+e^- \to D^*\bar{D} \to (D\pi)\bar{D}$ by a Feynman diagram, with the D^* as a virtual particle.

For $D^* \to D\pi$, the vertex is

$$\varepsilon^\rho \, (q - r)_\rho \ , \qquad \text{(B 4)}$$

where q and r are, respectively, the π and D momenta. From (B 2) and (B 4), we find that, apart from an overall constant, the invariant amplitude for $e^+e^- \to D^*\bar{D} \to (D\pi)\bar{D}$ is

$$\mathcal{M} = \frac{\underset{\sim}{n} \cdot \underset{\sim}{p} \times \underset{\sim}{q}}{p^2 - m_{D^*}^2 + i\Gamma_{D^*}M_{D^*}} \ . \qquad \text{(B 5)}$$

Here the three vectors are to be evaluated in the e^+e^- c.m., and M_{D^*} and Γ_{D^*} are the D^* mass and width. Except for the D^* propagator, this amplitude is just what one would get non-relativistically by replacing $\underset{\sim}{\varepsilon}$ with $\underset{\sim}{q}$ in Eq. (4). The integration of $|\mathcal{M}|^2$ from (B 5) over final state phase space is tedious but straightforward. In performing it, we use the fact that the D^* propagator is very sharply peaked. For the arbitrarily normalized pion distribution in $\hat{q}\cdot\hat{n}$ and energy E, we find

$$dN \propto [\, d(\hat{q}\cdot\hat{n}) \ (1 + (\hat{q}\cdot\hat{n})^2)] \ [\, dE \ \vec{q}^{\,2}(1 - (\hat{q}\cdot\hat{p})^2) \,] \ . \qquad \text{(B 6)}$$

In this expression, the cosine of the angle between the π and its parent D^* is related to the pion energy by

$$\hat{q}\cdot\hat{p} = \frac{2EE_{D^*} - M_{D^*}^2 + M_D^2 - m_\pi^2}{2|\vec{q}|\,|\vec{p}|} \ , \qquad \text{(B 7)}$$

where M_D and m_π are the D and π masses, and E_{D^*} is the (fixed) D^* energy in the e^+e^- c.m. frame.

We see that the energy and angular distributions factorize, and that the relativistic angular distribution agrees precisely with the non-relativistic one. The energy distribution is new information, but it can easily be understood in non-relativistic terms, at least in part. Namely, the factor $(1 - (\hat{q}\cdot\hat{p})^2)$ simply reflects the fact that the D^* is produced with helicity ± 1, so that its decay to $D\pi$ is forbidden when $\hat{q}$ lies along $\hat{p}$.

$e^+e^- \to D^*\bar{D} \to (D\gamma)\bar{D}$

This process involves the $D^*D\gamma$ coupling described by (B 2), twice. Apart from an overall constant, the invariant amplitude for the process is found to be

$$\mathcal{M} = E_{D^*}\omega \frac{(\underset{\sim}{p} \times \underset{\sim}{n}) \cdot (\hat{k} \times \hat{E} - \beta\hat{p} \times \hat{E})}{p^2 - M_{D^*}^2 + i\Gamma_{D^*}M_{D^*}} \quad . \tag{B 8}$$

Here ω, $\hat{k}$, and $\hat{E}$ are, respectively, the energy, direction, and electric polarization vector of the outgoing photon, and β is the speed of the D^* in the e^+e^- c.m. The first term in the numerator of Eq. (B 8) is the non-relativistic amplitude obtained by substituting $\hat{B}(\propto \hat{k} \times \hat{E})$ for $\underset{\sim}{\varepsilon}$ in Eq. (4). However, we see that there is an additional term, of order β relative to the first, so that the outgoing photon distribution in angle and energy will involve order β and order β^2 corrections to its non-relativistic limit. The summation of $|\mathcal{M}|^2$ from Eq. (B 8) over the polarizations $\hat{E}$ and $\hat{n}$, and its integration over final state phase space, is an extremely involved calculation. However, it is straightforward and we shall quote only the result. For the arbitrarily normalized outgoing photon distribution in direction cosine $\hat{k}\cdot\hat{n}$ and energy ω, we find

$$\begin{aligned} dN \propto d(\hat{k}\cdot\hat{n})\omega^2 d\omega \Big\{ &\left[(1 - (\hat{k}\cdot\hat{n})^2) + 2(\hat{k}\cdot\hat{n})^2(\hat{k}\cdot\hat{p})^2\right] \\ &- \beta\, \hat{k}\cdot\hat{p} \left[(3 - (\hat{k}\cdot\hat{n})^2) - (1 - 3(\hat{k}\cdot\hat{n})^2)(\hat{k}\cdot\hat{p})^2\right] \\ &+ \frac{\beta^2}{2}\left[(1 + (\hat{k}\cdot\hat{n})^2) + 2(1 - (\hat{k}\cdot\hat{n})^2)(\hat{k}\cdot\hat{p})^2 - (1 - 3(\hat{k}\cdot\hat{n})^2)(\hat{k}\cdot\hat{p})^4\right]\Big\} . \end{aligned} \tag{B 9}$$

Here $\hat{k}\cdot\hat{p}$ is given in terms of ω by

$$\hat{k}\cdot\hat{p} = \frac{2\omega E_{D^*} - M_{D^*}^2 + M_D^2}{2\omega|\vec{p}|} . \tag{B 10}$$

The maximum and minimum possible values of ω may be found from Eq. (B 10) by setting $\hat{k}\cdot\hat{p} = \pm 1$, corresponding to photons emerging parallel or antiparallel to their parent D^* mesons. At 4.028 GeV, the allowed photon energy range is $103\ \text{MeV} < \omega < 182\ \text{MeV}$. In an experiment where this is a large interval compared to the photon energy resolution, and where there are high statistics, one may compare Eq. (B 9) to the data directly. Alternatively, one may wish to study only the average angular distribution obtained by integrating over ω. To integrate Eq. (B 9) over ω we require the integrals

$$J_n \equiv \int_{\omega_{min}}^{\omega_{max}} (\hat{k}\cdot\hat{p})^n \omega^2 d\omega ;\ n = 0,\ldots,4. \tag{B 11}$$

These can be done analytically. In terms of β, $\gamma \equiv (1 - \beta^2)^{-\frac{1}{2}}$, and $a \equiv (M_{D^*}^2 - M_D^2)/2|\vec{p}|$ they are

$$J_o = \frac{2}{3} a^3 \gamma^6 \beta^4 (3 + \beta^2)$$

$$J_1 = \frac{8}{3} a^3 \gamma^6 \beta^5$$

$$J_2 = \frac{2}{3} a^3 \gamma^6 \beta^4 (1 + 3\beta^2) \qquad \text{(B 12)}$$

$$J_3 = \frac{1}{3} a^3 \gamma^6 \left[2\beta(3 - 8\beta^2 + 9\beta^4) - \frac{3}{\gamma^6} \ln \frac{1+\beta}{1-\beta} \right]$$

$$J_4 = \frac{2}{3} a^3 \gamma^6 \left[12 - 32\beta^2 + 27\beta^4 - 3\beta^6 - \frac{6}{\gamma^6} \frac{1}{\beta} \ln \frac{1+\beta}{1-\beta} \right] .$$

At 4.028 GeV, $\beta = 0.27$, so that the $j_n \equiv J_n / (\frac{2}{3} a^3 \gamma^6 \beta^4)$ are:

n	j_n
0	3.07
1	1.08
2	1.22
3	0.67
4	0.78

Using these values, we find from Eq. (B.9) that the energy-averaged angular distribution is

$$dN \propto d(\hat{k} \cdot \hat{n}) \left[1 - 0.31 (\hat{k} \cdot \hat{n})^2 \right]. \qquad \text{(B 13)}$$

This is amazingly close to the non-relativistic result, $(1 - \frac{1}{3}(\hat{k} \cdot \hat{n})^2)$, of Eq. (23). In practice they are indistinguishable.

To understand why the agreement is so close, note first that the leading term in Eq. (B 9), $[(1 - (\hat{k} \cdot \hat{n})^2) + 2(\hat{k} \cdot \hat{n})^2 (\hat{k} \cdot \hat{p})^2]$,

does agree with the non-relativistic result for $dN / d(\hat{k} \cdot \hat{n}) d(\hat{k} \cdot \hat{p})$ which one would obtain from Eqs. (4) and (18). Secondly, despite appearances, for small $\beta, j_{0,2,4} \propto 1$ and $j_{1,3} \propto \beta$. Finally, for small β, j_2/j_o differs from the non-relativistic average of $(\hat{k} \cdot \hat{p})^2$ over all directions of $\hat{p}$, namely $\frac{1}{3}$, by a term of order β^2. Thus, from Eq. (B 9) we see that all of the many corrections to the leading energy-averaged angular distribution are actually of order $\beta^2 \simeq 0.07$. We then indeed expect the $\sim 7\%$ change in the distribution.

REFERENCES

1. G. Goldhaber et al., Phys. Lett. 69B, 503 (1977).
2. H. K. Nguyen et al., Phys. Rev. Lett. 39, 262 (1977).
3. I. Peruzzi et al., Phys. Rev. Lett. 39, 1301 (1977).
4. For our purposes it will suffice to consider all polarization vectors to be real. This will be assumed throughout.
5. Equations (13) and (14) have been obtained previously by J. D. Jackson (unpublished) and R. Bhandari, Phys. Rev. D17, 2965 (1978).
6. We thank D. Coyne for this observation.
7. The distribution of the background pions from $D^*\bar{D}$ is given by Eq. (B 6), which includes the energy-dependence over the relatively large energy range involved. In determining the normalization of this background relative to the signal from $D^*\bar{D}^*$ recall that "$D^*\bar{D}^*$" can consist of $D^{*+}D^{*-}$ or $D^{*0}\bar{D}^{*0}$, and "$D^*\bar{D}$" of $D^{*+}D^-$ or $D^{*0}\bar{D}^0$. When integrated over pion energy and angle, the pion contribution from each of these four states must yield the production cross section for that state, times the branching ratio for the D^* involved to decay to a pion with the charge of interest.
8. The analogous separation of photons due to $e^+e^- \to D^*\bar{D}^* \to (D\gamma)\bar{D}^*$ from those due to $e^+e^- \to D^*\bar{D} \to (D\gamma)\bar{D}$ may not be too easy. At 4.028 GeV, photons from these two processes have energies ω in the ranges $125 \text{ MeV} < \omega < 151 \text{ MeV}$ and $103 \text{ MeV} < \omega < 182 \text{ MeV}$, respectively.
9. See, for example, J. L. Rosner "Heavy Quarks and New Particles", Invited talk given at the Annual Meeting of the Division of

Particles and Fields of the APS, Montreal, Canada, Oct. 25-27, 1979, and references therein. See also A. DeRujula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 37, 398 (1976) and R. N. Cahn, Y. Eylon, and S. Nussinov. Phys. Rev. D21, 82 (1980).

10. J. Rosner, private communication
11. A direct argument that $|z| = 1/6$ was given by Cahn et al., Ref. 9.
12. We thank C. Carlson for pointing out that the "Cornell model" [see, for example E. Eichten et al., Phys. Rev. D21, 203 (1980)] also predicts $z = -1/6$.
13. R. Bhandari, Ref. 5.
14. G. Goldhaber et al., Ref. 1.
15. Other ways of distinguishing between the two views are given by R. Bhandari, Ref. 5.
16. J. D. Jackson, LBL Internal Note JDJ/76-1, unpublished.
17. This frame has the z-axis along the $\bar{D}$ momentum in the overall c.m. and the y-axis along the normal to the production plane. See Ref. 2.
18. The Dalitz plot reported for $D \to \bar{K}\pi\pi$ by J. Wiss et al., Phys. Rev. Lett. 37, 1531 (1976) is compatible with $J(D) = 1$.
19. J. Rosner has pointed out to us that if the decay $D^0 \to \pi^0\pi^0$ were seen it would prove that the D^0 spin is not one.

FIGURE CAPTIONS

Fig. 1. The function $\alpha_\pi(z)$ versus $z(=A_0/A_2)$. The angular distribution of π's from $e^+e^- \rightarrow D^*\bar{D}^* \rightarrow (D\pi)\,\bar{D}^*$ is proportional to $1 + \alpha_\pi(z)(\hat{q}\cdot\hat{n})^2$, where $\hat{q}$ is the pion direction and $\hat{n}$ is the beam direction. See Eq. (29).

Fig. 2. The function $\alpha_\gamma(z)$ versus $z(=A_0/A_2)$. The angular distribution of γ's from $e^+e^- \rightarrow D^*\bar{D}^* \rightarrow (D\gamma)\,\bar{D}^*$ is proportional to $1 + \alpha_\gamma(z)(\hat{k}\cdot\hat{n})^2$, where $\hat{k}$ is the photon direction and $\hat{n}$ is the beam direction. See Eq. (30).

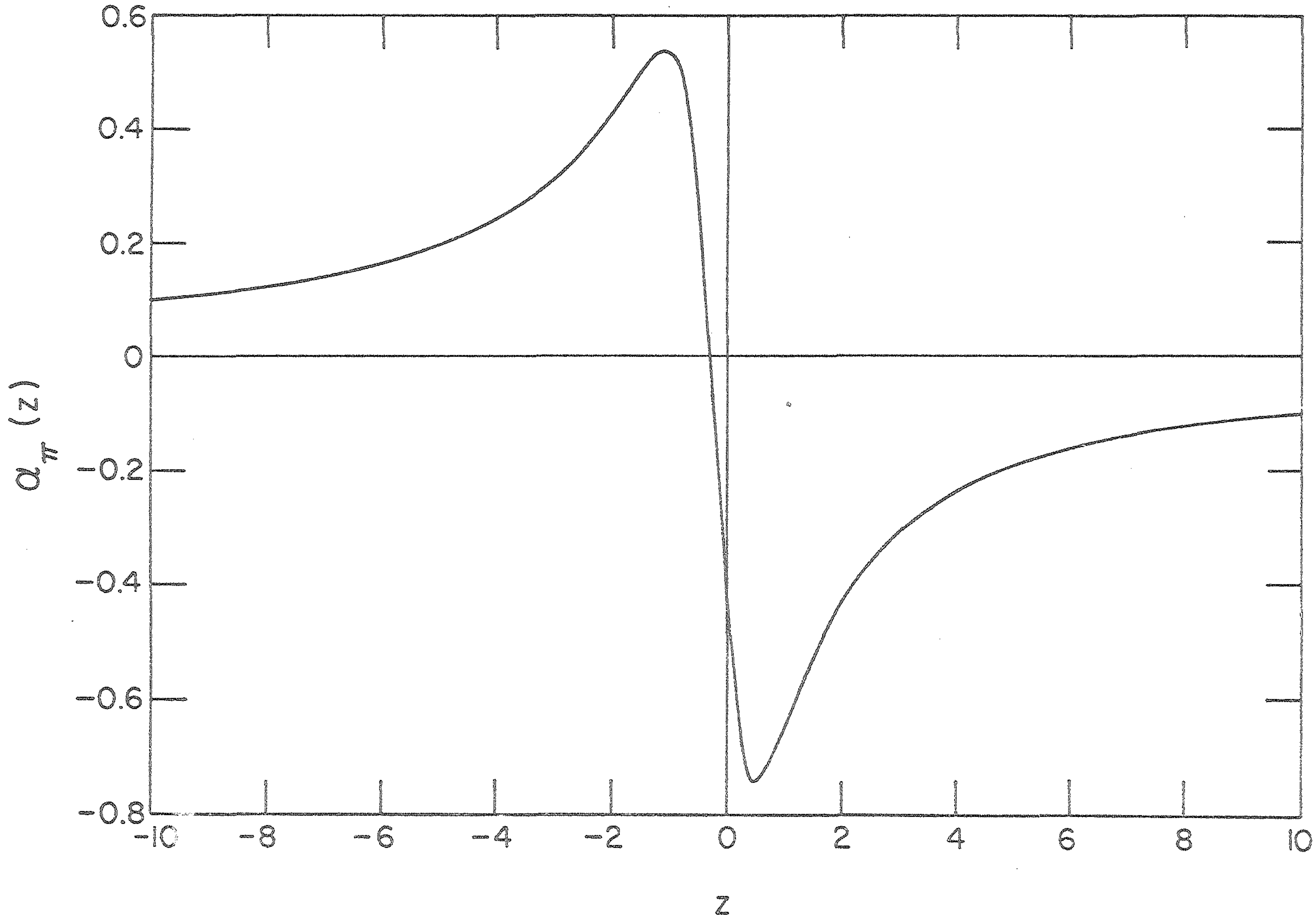

Figure 1

XBL 804-763

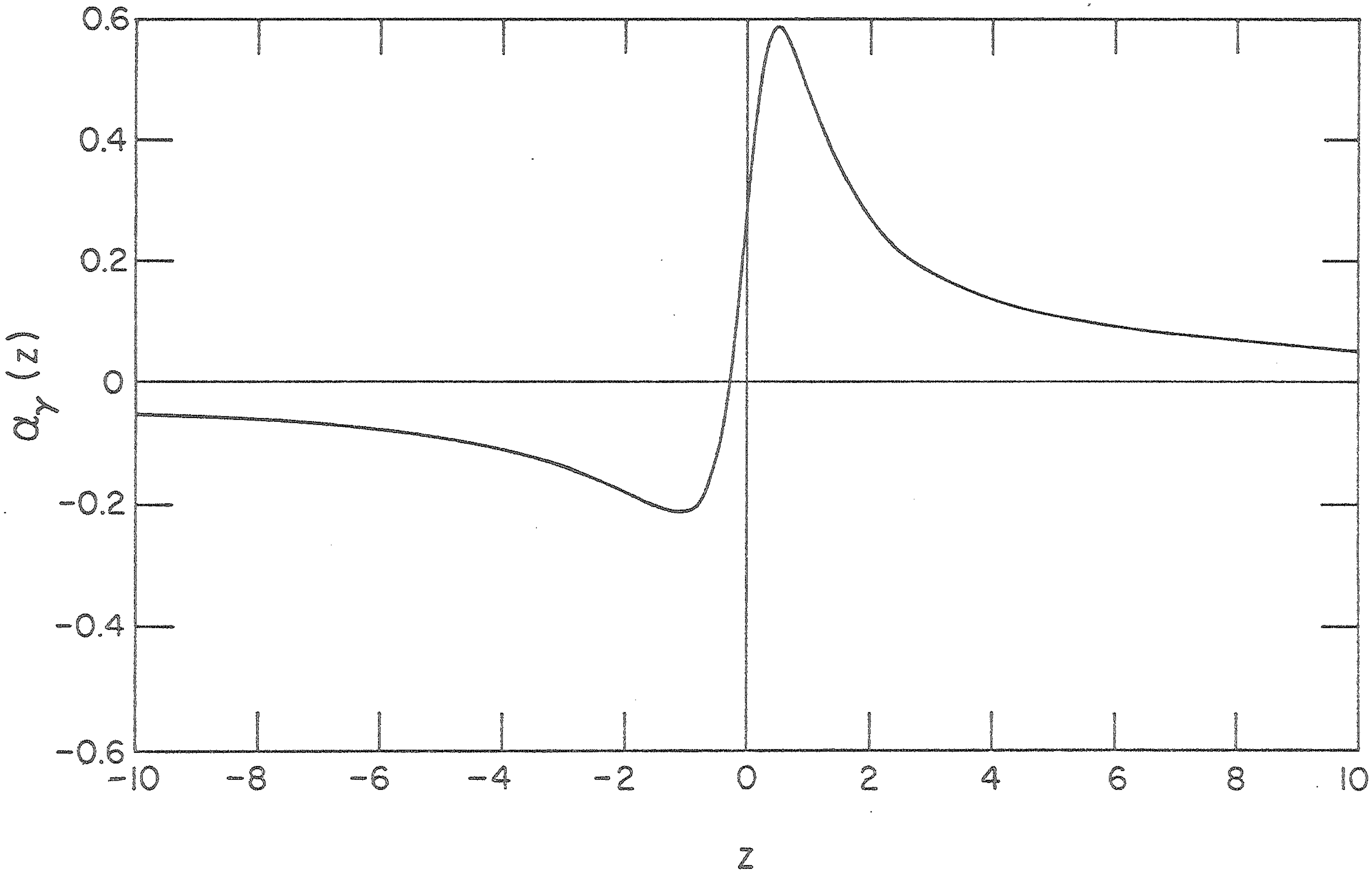

Figure 2

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