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Modern Modulation Schemes: A Linear Programming Based Tone Injection Algorithm for PAPR Reduction of OFDM and Linearly Precoded Systems

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Modern Modulation Schemes: A Linear Programming Based Tone Injection Algorithm for PAPR Reduction of OFDM and Linearly Precoded Systems

Neil Jacklin, Zhi Ding¹

Abstract

This work investigates the improvement of power amplifier efficiency through the reduction of peak-to-average power ratio (PAPR) of linearly precoded QAM data signals. In particular, we focus on the special cases of linear precoded modulation including the practical OFDM, OFDMA, and SC-FDMA signals that have been widely adopted in W-LAN and W-MAN. We apply the method of tone injection optimization for PAPR reduction. To reduce numerical complexity, we propose a linear programming algorithm which closely approximates the original tone injection optimization problem. Our comprehensive numerical results demonstrate substantial PAPR reduction and superior BER performance using several practical examples.

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Chapter 1. Introduction

Orthogonal Frequency Domain Multiplexing (OFDM) and its variations, OFDMA and SC-FDMA, have been adopted for several wireline and wireless applications, the most well-known of which include the IEEE 802.11a (WiFi) standard, the IEEE 802.16 (WiMAX) standard, LTE cellular, and xDSL. Despite its many superior features such as ease of equalization and capacity approaching bit-loading, OFDM poses a major implementation challenge because of the high peak-to-average power ratio (PAPR) exhibited by its time-domain waveforms. High PAPR results in low power efficiency at the transmitter. In fact, the PAPR reduction is the primary reason that SC-FDMA was chosen for LTE uplink communications [4].

Of course, PAPR reduction of OFDM is not a new subject. Readers might suspect from the number of published works on PAPR reduction that this problem is long solved and little new light can be shed on the subject. However, it is important to note that most works simply illustrate the effect of an algorithm on PAPR reduction and fail to follow through with any meaningful quantization of the resulting power savings. Amazingly, there has been no published work, with the exception of McCune [3] that establishes the true physical impact that PAPR reduction can have on energy efficiency of the RF amplifier and on the practical bit error rate due to RF amplifier's back-off. This present work provides a clear and quantitative illustration of how our PAPR reduction algorithm effectively improves energy efficiency and the resulting BER. Contrary to claims in the literature that PAPR reduction *increases* BER (because of an increase in average transmit power), we show that the BER is in fact decreased when comparing equal power consumed by the RF amplifier in unmodified and PAPR-reduced transmissions. Furthermore, unlike most PAPR reduction algorithms that are tailored only for OFDM signals, we propose a generalized method that is applicable to any linear precoded transmission including as special cases OFDM[A] (used in WiMAX and LTE downlink) and SC-FDMA (single-carrier frequency division multiple access, used in LTE uplink) systems [5].

A number of methods have been proposed to effect PAPR reduction [1, 2, 6–9]. These methods in general modify the OFDM symbol to decrease PAPR in the time-domain without losing the source data information it carries. In particular, several known works have already formulated the PAPR reduction solution into convex optimization algorithms [8, 10, 11]. In general, to reshape the OFDM signal for lower PAPR requires either substantial computation or bandwidth sacrifice for “side information”. For example, the method of tone reservation [1, 7] selects and reserve some of the subcarriers for carrying special PAPR-reducing symbols in each OFDM symbol. Reserved subcarriers represent bandwidth loss since they bear no source information. If the indices of the reserved tones are chosen online, then that side information must also be transmitted to the re-

ceiver for decoding. For more information on PAPR reduction works the reader may refer to more comprehensive review articles in the literature [12, 13].

In order to avoid bandwidth overhead we focus on the tone injection method [1, ch.5]. Tone injection seeks to modify the transmitted subcarrier symbols to achieve PAPR reduction by introducing symbol-by-symbol offsets (or injected tones). As we explain in Chapter 2, this operation can also be viewed as a constellation expansion; see for example [9] and related works in the references of [12] and [13]. Since the transmitter must determine for each OFDM frame how to modify the subcarrier symbols the major implementation challenge in tone injection lies in its computational burden.

In this work we propose a linear programming algorithm as a lower complexity solution to the tone injection problem. In a previous work we proposed a semi-definite programming (SDP) solution through relaxation [10]. Although the performance of the SDP method is very good, its computational complexity can be a detriment to its application to large OFDM frames. In this paper, inspired by ideas from compressive sensing and sparse signal reconstruction, and through the use of multiple linear bounds to approximate the ℓ_2 -norm, we devise a much faster and simpler linear programming formulation that exhibits little loss when compared with the more complex SDP formulation. As another new contribution, we further expand upon the tone injection principle by operating on a subset of subcarrier symbols that naturally applies to multiuser OFDMA wireless uplinks. More generally, our linear programming tone injection algorithm is applicable to any linearly precoded QAM transmissions, including the now popular SC-FDMA, adopted in LTE. We show that SC-FDMA is just another linear precoding scheme which still exhibits high PAPR in some cases of interest. Note that investigations for reducing PAPR in SC-FDMA are relatively new and rare in the literature, one example is the work of [11]. Our methods can also be extended to cover MIMO systems that employ STBC-OFDM transmissions [14].

The organization of this work is as follows. The generic OFDM tone injection problem formulation is presented in Chapter 2. Its extension to multiuser OFDMA is then proposed in Chapter 3. The generalization of tone injection PAPR reduction for general linear precoding and SC-FDMA is discussed in Chapter 4. In each of the different cases, we provide a basic linear programming formulation, analyze the computational complexity, and present numerical results for performance verification. We will compare the performance of our new linear programming approach to our previous SDP work and characterize its effects on power savings and BER performance. Finally, in Chapter 5, we will quantify the power efficiency improvements in terms of BER given equal power input to the PA, or, equivalently, the power savings possible while achieving a given BER.

Chapter 2. OFDM and Tone Injection

Let us formulate the PAPR reduction problem in the context of OFDM. Consider the transmission of a block of N data (subcarrier) symbols $\{X_k | X_k \in T_k \subset \mathbb{C}\}_{k=0}^{N-1}$ from complex QAM constellations T_k . The time-domain OFDM baseband signal over-sampled L -times is

$$x\left[\frac{n}{L}\right] = \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/NL}, \quad n = 0, \dots, NL-1. \quad (2.1)$$

and its PAPR can be defined as

$$\text{PAPR}(x[n/L]) = \frac{\max_n |x[n/L]|^2}{\sum_{k=0}^{N-1} |x[n/L]|^2}. \quad (2.2)$$

With this definition, $\text{PAPR}(x[n/L])$ closely approximates the PAPR of the continuous baseband waveform with $L = 2$ or $L = 4$. The authors wish to stress that evaluating the over-sampled PAPR is important, since it has been shown [1] that the critically-sampled ($L = 1$) PAPR is a very poor approximation to the continuous time PAPR of the transmitted time-domain waveform that is of interest as a continuous, analog waveform ultimately transmitted by the RF amplifier. In turn, the baseband PAPR bounds the passband PAPR and so for mathematical tractability we consider the baseband PAPR throughout this work; c.f. [1, 12].

The goal of tone injection is to send offset/shifted symbols

$$\{Y_k = X_k + C_k\}_{k=0}^{N-1} \quad (2.3)$$

in place of the original symbols $\{X_k\}$ which yields the modified time-domain waveform

$$y\left[\frac{n}{L}\right] = \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} (X_k + C_k) e^{j2\pi kn/NL}. \quad (2.4)$$

The terms C_k are called the injected tones (since each C_k modulates only one subcarrier) and must be chosen to reduce the PAPR of (2.4). An arbitrary choice of $\{C_k\}$ would make it hard to recover

the underlying symbols $\{X_k\}$. Therefore, consider as in [1]

$$C_k = D_k p_k + j D_k q_k, \quad (2.5)$$

$$D_k = \rho \Delta_k \sqrt{M_k} \quad (2.6)$$

$$p_k, q_k \in \{-1, 0, +1\} \quad (2.7)$$

where M_k is the size of the QAM constellation T_k (e.g. $M_k = 16$ for 16-QAM), Δ_k is the minimum inter-symbol distance, and $\rho \geq 1$ is a real scaling factor. By restricting $\{C_k\}$ in this way, the receiver can unambiguously decode a received symbol by mapping it onto to the original constellation using a simple modulo step. For an intuitive explanation of this procedure, refer to Fig. 2.1. Although new errors can arise from smaller decision regions for the “outer” constellation points, the BER is dominated by the minimum intersymbol distance, which remains unchanged.

In practice, tone injection algorithms seek the best choice of $\{C_k\}$ to decrease the peak power of the modified waveform $|y[n/L]|^2$ instead of the actual PAPR. This does not hurt performance because nonzero C_k ’s tend to increase the average power of transmission, thereby reducing the PAPR. Of course, large increase in average power is not desirable. By leaving the average power out of the objective function, we gain mathematical tractability and do not “reward” PAPR reductions through higher average power. As will be shown in Chapter 5, the efficiency gained at the PA from reducing the PAPR requirement more than offsets the small increase in average power experienced by our tone injected waveforms.

In short, the tone injection problem is

$$\min_{\{p_k, q_k\}} \max_n |y[n/L]|^2 \quad (2.8)$$

subject to the constraint (2.7). In this form the problem is an NP-hard combinatorial optimization problem [1, p. 98]. All practical attempts at implementing tone injection solve some related approximation to this problem, trading optimal performance for manageable computational complexity. First, we will provide a vector/matrix framework for this problem to simplify notation and to cast the problem in a more convenient optimization context. We will then propose a novel sparse signal processing-inspired approach that will transform tone injection into a linear programming problem.

2.1 Compact Formulation

Let us recast the tone injection problem using the more compact vector and matrix notations. We will follow the convention of reserving boldface font for vector and matrix quantities, and an over-

arrow, e.g. $\vec{v}$ or $\vec{V}$, will always denote a column vector. We also will use the notations

$$\vec{V}_{Re} = \text{Real}\{\vec{V}\}, \quad \vec{V}_{Im} = \text{Imag}\{\vec{V}\}$$

so that $\vec{V} = \vec{V}_{Re} + j\vec{V}_{Im}$ since it will be convenient to write the problem entirely in terms of real quantities. Superscripts on vector or matrix quantities denote different, related versions, not exponent powers. The k -th element of $\vec{v}$ will be referred to as v_k when needed (with matrices usually indexed by (k, l)).

Equation (2.4) can be rewritten as

$$\vec{y} = \mathbf{W}^I \vec{Y} = \mathbf{W}^I (\vec{X} + \mathbf{D} \vec{p} + j \mathbf{D} \vec{q}) \quad (2.9)$$

where $\vec{y}$ is a $(NL \times 1)$ complex vector, and the (n, k) element of matrix $\mathbf{W}^I$ is denoted by

$$w_{n,k}^I = \sqrt{\frac{1}{NL}} e^{j2\pi nk/NL} \quad (2.10)$$

for $n = 0, \dots, (N \times L) - 1$, $k = 0, \dots, N - 1$, and $\mathbf{D}$ is a diagonal matrix with D_k as the k th diagonal entry. Equation (2.9) shows that each element of $\vec{y}$ is a function of given data vector $\vec{X}$ and the tone injection decision variables, $\{\vec{p}, \vec{q}\}$. To make this relationship explicit we can group the decision variables into a vector $\vec{r} = [\vec{p}; \vec{q}]$ and write

$$\begin{aligned} \vec{y} &= \mathbf{W}^I (\vec{X} + \mathbf{D} \vec{p} + j \mathbf{D} \vec{q}) \\ &= \mathbf{W}^I \vec{X} + \begin{bmatrix} \mathbf{W}^I \mathbf{D} & j \mathbf{W}^I \mathbf{D} \end{bmatrix} \begin{bmatrix} \vec{p} \\ \vec{q} \end{bmatrix} \\ &= \vec{V}^C + \mathbf{V}^D \vec{r} \end{aligned} \quad (2.11)$$

in which $\vec{V}^C = \mathbf{W}^I \vec{X}$ and $\mathbf{V}^D = [\mathbf{W}^I \mathbf{D} \quad j \mathbf{W}^I \mathbf{D}]$.

2.2 Linearization of Constraints

In the place of

$$\min_{\{p_k, q_k\}} \max_n |y[k]|^2 \quad (2.12)$$

consider

$$\begin{aligned} &\min \rho \\ &(\text{Real}\{y_k\})^2 + (\text{Imag}\{y_k\})^2 \leq \rho = \tau^2. \end{aligned} \quad (2.13)$$

The above constraints each y_k to within a circle in the complex plane of radius $\tau = \sqrt{\rho}$, as shown in Fig. 2.2. Of course, this is a quadratic constraint on y_k and it is a quadratic constraint in terms of the discrete decision variables $\{p_k, q_k\}$. A semidefinite relaxation approach to approximately solving the non-convex tone injection problem has already been presented in [10].

Alternatively we can apply linear constraints to approximate the quadratic one. As shown in Fig. 2.2, we can closely approximate the ℓ_2 constraint with a series of joint box constraints. Specifically, we will take the intersection of the constraints

$$|\text{Real}\{y_k e^{j\theta}\}| \leq \tau, \quad |\text{Imag}\{y_k e^{j\theta}\}| \leq \tau \quad (2.14)$$

for $\theta = r/R \times (\pi/2)$, $r = 0, \dots, R-1$, where R denotes the total number of rotations of the constraints in (2.14). For example, if $\theta = \pi/4$, the box constraint becomes the ℓ_1 constraint on y_k . Each box constraint yields 4 linear constraints. Even for R as small as 2 or 3, the resulting intersection well approximates the ℓ_2 constraint.

2.3 Integer Constraint Relaxation

Despite the use of linear constraint (2.14), the discrete (integer) constraint on elements of $\vec{r}$ in (2.7) is highly non-convex yet crucial for decoding the received OFDM symbols. Let r_k be the k -th element of $\vec{r}$. A common relaxation of $-1 \leq r_k \leq +1$ by itself does not lead to consistent PAPR reductions.

Here we take an approach inspired by observations made in our previous work on convex optimization solutions to the PAPR problem [10]. A key observation from that work is that the typical tone injection solution only has a 2-3% increase in average power which implies that few subcarrier symbol shifts are needed to achieve good PAPR reduction. In terms of the decision vector $\vec{r}$, most of its entries are zeros. In compressive sensing terminology, the resulting solution is a “sparse” vector (few nonzero elements). The most celebrated result in compressive sensing is that in many cases of interest a sparse solution can be obtained as the result of minimizing the ℓ_1 norm of the vector of interest [15]. Once the decision vector $\vec{r}$ is obtained, we then use the magnitude $|r_k|$ as the probability of injecting on the k -th tone, whereas $\text{sgn}(r_k) = \pm 1$ denotes the direction of the tone injection.

We therefore propose an algorithm for the tone injection problem as follows. **First**, solve the

Figure 2.1: Constellation expansion [1, 2] interpretation of (2.5)-(2.6) under 16-QAM and $\rho = 1$. The original constellation points are shown inside the dotted box, and a representative symbol is shown as an empty dot and labeled X_k . Other empty dots are potential alternative constellation points to which X_k could be moved.

Figure 2.2: Approximation of ℓ_2 norm with rotations of ℓ_1 norm.

linear program (LP)

$$\begin{aligned}
& \min_{\{r_k\}} \quad \|\vec{\mathbf{r}}\|_1 \\
& \text{s.t.} \quad \left| \text{Real} \left\{ \left(\vec{\mathbf{V}}^C + \mathbf{V}^D \vec{\mathbf{r}} \right) e^{j(m/R)\pi/2} \right\} \right| \leq \tau_N, \\
& \quad \left| \text{Imag} \left\{ \left(\vec{\mathbf{V}}^C + \mathbf{V}^D \vec{\mathbf{r}} \right) e^{j(m/R)\pi/2} \right\} \right| \leq \tau_N, \\
& \quad m = 0, \dots, R-1. \\
& \quad -1 \leq \vec{\mathbf{r}} \leq +1
\end{aligned} \tag{2.15}$$

where $\vec{\mathbf{V}}^C$ represents constants and $\mathbf{V}^D$ represents coefficients on the decision variables in $\vec{\mathbf{r}}$. The inequalities in (2.15) are taken in an element-by-element sense. The parameter τ_N is picked offline as a function of N and has a simple relationship to the maximum PAPR target. This maximum PAPR corresponds directly to the PAPR back-off for which the PA is designed and operated. The solution to the LP problem will be a vector $\vec{\mathbf{r}}$ with continuous entries between -1 and $+1$.

Second, to obtain integer-constrained solutions the sign of r_k is interpreted as the shift direction and random candidate solutions are generated with each shift occurring with probability $|r_k|$. Thus, multiple random candidate solutions are generated. Finally, the candidate that results in the lowest PAPR is then selected to perform the final tone injection. We name our proposed approach Linear Programming with Randomized Tone Injection (LP-RTI).

2.4 Computational Complexity of (2.15)

The linear programming problem of (2.15) can be efficiently solved by applying simplex or interior point methods. The complexity of its solution is of the order $\mathcal{O}(v^2c)$ where v denotes the number of variables in the LP and c denotes the number of constraints [16]. There are $2N$ real variables in (2.15) and $4RN + 4N$ constraints, for an overall complexity of $\mathcal{O}(N^3RL)$.

2.5 Simulation Results of PAPR Reduction for OFDM

Figure 2.3 shows the PAPR reduction achieved under OFDM for $N = 64$ and $N = 128$ subcarriers at $L = 2$ over-sampling. All data symbols are drawn uniformly from a 16-QAM constellation. Following conventions in the literature [1], the results are shown in terms of the complementary cumulative distribution function (CCDF) of the PAPR of the modified and unmodified waveforms. At a clipping rate of 10^{-3} , the PAPR of the unmodified OFDM waveforms is 8.75 dB for $N = 64$ and 10.5 dB for $N = 128$. After tone injection using our LP-based algorithm, the PAPR is reduced to 6.75 dB and 7.5 dB, respectively. We compare the performance of LP-RTI to our SDP algorithm

from [10]. That algorithm reduces the PAPR by an additional 1.1 dB with $N = 64$ subcarriers, but solves a much more complex second-order cone program and is not tractable even for $N = 128$ subcarriers. In order to compare fairly with existing works [1, 2, 7], we did not account for the effects of pulse-shaping filters before the PA since pulse-shaping filters are usually designed not to enhance PAPR [1].

2.6 Further Reduction of Linear Constraints

The LP (2.15) grows in size with the number of subcarriers N by increasing the size of $\vec{r}$ and the number of constraints (2.14). Some systems, such as DVB or WiMax, can use up to 1024 or 2048 subcarriers [17]. Fortunately, most power sensitive applications use only a fraction of the subcarriers as in OFDMA. We shall consider the case of multiuser OFDMA transmissions in the next chapter.

Another simplification can be made by reducing the number of constraints (2.14) to enforce. In fact, many such constraints in (2.15) are easily satisfied because the peaks of the signal are infrequent (thus causing large PAPR). The time samples at which the peaks in the power envelope occur correspond to constraints that are active in the LP. Thus in the first iteration of the LP we could try to enforce only those active constraints. By deleting unenforced constraints we expect a large reduction in the iteration time of the LP solver. In practice, for large OFDM frames, we can order the time indices by the magnitude of the power envelope of the signal at that time and enforce $N_s \leq NL$ of the corresponding box constraints. Although this procedure is only optimal when all peaks larger than τ_N are captured and only in the first iteration of the LP, it provides reasonable results for the overall problem. For example, for $N = 64$ subcarriers, LP-RTI when only enforcing the $N_s = 64$ constraints corresponding to the largest power envelope magnitude is 14x faster (an observed 93% reduction in run time) with a loss of less than 0.5 dB in PAPR reduction.

Figure 2.3: PAPR reduction in OFDM system with $N = 64$ and $N = 128$ subcarriers (at over-sampling of $L = 2$).

Chapter 3. Multiuser OFDMA PAPR Reduction

3.1 LP-RTI for PAPR Reduction in OFDMA

When multiple users are sharing the same channel bandwidth a fraction of the subcarriers may be assigned for each user in orthogonal frequency division multiple access, or OFDMA. This is the primary multiple user access scheme adopted in WiMAX and LTE/LTE-Advanced for downlink transmissions and for uplink and downlink transmissions in WiMAX. The PAPR reduction in uplink is particularly important to improve the mobile users' power efficiency.

Application of our LP-RTI algorithm for PAPR reduction of OFDMA is straightforward. For downlink, the problem is exactly the same at the transmitter. The user allocation of subcarriers has no effect on how tone injection should proceed. For uplink, a simple modification can be used that saves computational complexity by reducing the number of decision variables. Consider an OFDMA user that is allocated $M \leq N$ subcarriers on which to transmit. The user sets the remaining $N - M$ data symbols in the OFDM frame to 0 and then transmits as before. In the context of tone injection we now only have to consider tone injection of these M symbols. Thus we define $\vec{\mathbf{r}}$ as a $2M$ -vector instead of a $2N$ vector and delete the columns of $\mathbf{W}^{\mathbf{I}}$ that do not correspond to the M user's symbols. Call the new $(NL \times M)$ matrix obtained in this way $\mathbf{W}^{\mathbf{IM}}$. The resulting LP has the same structure as in (2.15) by changing $\vec{\mathbf{V}}^C = \mathbf{W}^{\mathbf{IM}}\vec{\mathbf{X}}$ and $\mathbf{V}^D = [\mathbf{W}^{\mathbf{IM}}\mathbf{D} \ j\mathbf{W}^{\mathbf{IM}}\mathbf{D}]$. In practice this improves run-time of the algorithm significantly. The big-O complexity of the LP in the OFDMA case decreases to $\mathcal{O}(M^2RNL)$.

3.2 Numerical Results in OFDMA

The PAPR reduction for OFDMA with $N = 128$ subcarriers and a user assignment of $M = 32$ subcarriers was conducted with interleaved and contiguous block subcarrier assignment. Little difference was noted between the two subcarrier assignment schemes. Our proposed LP-RTI algorithm reduces the PAPR by approximately 0.75-1.0 dB in this case, from 8.0 dB to 7.0 dB at a clipping rate of 10^{-3} . For space considerations no figure is presented for this experiment, but some OFDMA results can be seen for other parameter settings in Fig. 4.2a.

The proposed LP-RTI algorithm can also process large OFDMA frames when coupled with our constraint reduction technique. For example, when tested on an OFDMA frame with $N = 1024$ subcarriers and a user assignment of $M = 64$, the PAPR of the OFDMA frame can be decreased from 7.5 dB to 6.7 dB at a clipping rate of 10^{-3} .

Chapter 4. SC-FDMA and General Linear Precoding

As mentioned previously, the LTE and LTE-Advanced standards have adopted single-carrier frequency division multiple access (SC-FDMA) as the multiuser access scheme for cellular uplink communications. The most commonly cited reason for this choice is reduction of the PAPR requirement for the power amplifiers at the mobile equipments [4]. A schematic diagram of the SC-FDMA signal processing chain is shown in Fig. 4.1. Also called DFT-precoded OFDM[A] in the literature [4], the major innovation in SC-FDMA is the addition of an M -point FFT before the OFDMA procedure of subcarrier mapping and inverse Fourier transform (IFFT). The additional FFT spreads each incoming symbol over multiple subcarriers and removes the multi-carrier behavior of the transmitted waveform which aids, but does not eliminate the need for, PAPR reduction.

SC-FDMA falls within our linear precoding framework. Let $\vec{\mathbf{Y}}$ be the M -vector of modified data-bearing symbols that a user wishes to transmit, where again $\mathbf{Y}$ is defined analogously to (2.3). Under SC-FDMA, the data are first transformed by an M -point DFT. Let that DFT operation be denoted by multiplication with a matrix $\mathbf{W}^M$. The output of that DFT is then mapped onto the N inputs of the IFFT block. We can represent any mapping (e.g. interleaved or contiguous, including arbitrary re-ordering or scrambling) by an $(N \times M)$ selection matrix, $\mathbf{P}$, that has a single “1” in each column and M “1”s among the N rows. Finally the IFFT and oversampling operation can be accomplished with $\mathbf{W}^I$ as defined in Chapter 2.1. Thus the baseband signal is

$$\vec{\mathbf{y}} = (\mathbf{W}^I \mathbf{P} \mathbf{W}^M) \vec{\mathbf{Y}} = \mathbf{V} \vec{\mathbf{Y}} \quad (4.1)$$

where $\mathbf{V}$ denotes the effective precoding matrix.

4.1 Simulation Examples for SC-FDMA

We will compare the PAPR of SC-FDMA with and without LP-RTI and against that of OFDMA for comparison. We will use the same parameters, $N = 128$, $M = 32$, $L = 2$, and interleaved subcarrier mapping as in the previous example. Fig. 4.2a shows that, in this special case, SC-FDMA has virtually no advantage over OFDMA, and that the PAPR of both OFDMA and SC-FDMA can be reduced by 1.0 dB using LP-RTI.

For the case of $N = 1024$ and $M = 64$, we employ our constraint reduction method and utilize only 25% of the constraints that correspond to the highest power envelope magnitude. SC-FDMA by itself reduces the PAPR relative to OFDMA by 1.5 dB, and with the application of LP-RTI the PAPR can be reduced an additional 0.5 dB (see Fig. 4.2b). Whether the additional reduction in PAPR is worth the extra computational cost of LP-RTI depends on their relative importance in the trade-off.

Figure 4.1: Transmitter block diagram comparing OFDM, OFDMA, and SC-FDMA. Digital datapath widths are in subcarrier symbols. In OFDMA and SC-FDMA, the subcarrier (SC) mapping block inserts $N - M$ zeros into the data stream in a pre-defined pattern (interleaved or block).

(a) $N = 128$ subcarriers and $M = 32$ user assignment. Note that in this case SC-FDMA does not reduce the PAPR much compared to OFDMA.

(b) $N = 1024$ subcarriers and $M = 64$ user assignment.

Figure 4.2: PAPR reduction for multiple access.

Chapter 5. Power Efficiency and BER Performance

Reduction of PAPR can lead to substantial power savings at the transmitter since its power amplifier's (PA) efficiency is a function of its peak power/PAPR back-off [2,3]. Fig. 5.1 shows a typical energy efficiency curve of a prototypical Class A RF power amplifier. The input-output operating characteristic of an ideal linear amplifier is represented by the dotted line and an almost-ideal saturating linear amplifier is represented by the light gray line. The thick black line represents the efficiency of a typical PA. Near-saturation efficiency of modern PAs approach 60%, but degrades quickly with peak power back-off. For unprecoded QAM transmissions, the ordinary back-off (OBO) is typically set to 2 dB, which results in efficiency of approximately 50%. In general precoding increases PAPR of transmitted waveforms for amplification¹ and for the IDFT precoding in OFDM, this can result in PAPR of 10-15 dB. When operated at 11 dB back-off, for example, PA efficiency is extraordinarily low such that as little as 5% of the energy input to the PA is converted to useful signal transmission energy. Reducing the PAPR by several dB, as is possible with methods such as the ones proposed in this work, can double or even quadruple power amplifier (PA) efficiency by lowering the necessary PAPR back-off. PAPR reduction also reduces “clipping” at the HPA (hard-limiting of the peak amplitude of the transmitted waveform when the signal exceeds the designed PAPR back-off). Clipping injects in-band noise that decreases SNR at the receiver [1] and can generate out-of-band interference to other transmissions.

We report the BER in terms of PA input power to noise ratio (PNR) as defined [10]. We use the PA efficiency information provided in [3]. The relationship between PNR and SNR is based on the model

$$P_s = \eta P_{in} \quad (5.1)$$

and

$$\begin{aligned} \text{PNR} &= 10\log_{10} \left(\frac{P_{in}}{N_0} \right) \\ &= 10\log_{10} \left(\frac{P_s}{\eta N_0} \right) = \text{SNR} - 10\log_{10} \eta \end{aligned}$$

where P_s represents the signal power, P_{in} is the input power to the PA, and η is the efficiency which

¹The samples of the time-domain transmit waveform for a linearly precoded system with i.i.d. QAM symbols have the distribution of a sum of random variables. By the central limit theorem, the distribution of the sum approaches Gaussian. Hence, there is a non-trivial probability that some samples will be large in magnitude compared to the mean.

ranges from about 0.05 to 0.20 based on the PAPR back-off operating point. For each comparison the PAPR back-off is taken as the PAPR at a clipping rate of 10^{-3} . The clipping rate is roughly a floor on the OFDM frame error rate. We should stress that, at a higher PA efficiency, the SNR of the transmitted waveform is higher given the same PNR. For this reason our simulations will establish that the PAPR reduced waveforms tend to have lower BER at the same PNR as the unmodified ones.

To provide some concrete estimates of the power savings and BER rates that are achievable, we consider an OFDMA system with $N = 128$ subcarriers in which $M = 64$ are assigned to a particular user. From simulations we find that the PAPR of the unmodified waveforms is 10.0 dB at a clipping rate of 10^{-3} while the tone injected waveforms have PAPR of 6.75 dB. From the PA efficiency data from [3], that gives a PA efficiency of 6.7% at 10.0 dB PAPR and 13.7% at 6.75 dB PAPR. Simulation results show that the PNR savings is approximately 1.3 dB at constant BER (in the high PNR/SNR regime), which translates to a power savings of 25.9% at the transmitter. Additional simulation results are shown in Table 5.1 for OFDMA and SC-FDMA systems with varying numbers of subcarriers and user assignments. The PAPR at a clipping rate of 10^{-3} is reported along with the power amplifier (PA) efficiency at that operating point. The power saved by using the LP-RTI algorithm at constant BER of 10^{-4} , is reported in the last column.

This performance study shows that while LP-RTI can be useful for some sets of SC-FDMA system parameters, the gains of LP-RTI under SC-FDMA are less than under OFDMA. For large numbers of subcarriers, we find that the computational complexity is too high to handle all constraints, thereby lowering the PAPR reduction. In such cases, the improvement in PA efficiency is small, and may be outweighed by average power increase.

Table 5.1: PAPR Reduction and BER performance summary

N	M	Unmodified		LP-RTI		Power Saved
		PAPR	PA Eff.	PAPR	PA Eff.	
OFDMA						
128	16	8.75 dB	8.8%	6.50 dB	14.4%	16.8%
128	32	9.50 dB	7.4%	6.50 dB	14.4%	42.5%
128	64	10.00 dB	6.7%	6.75 dB	13.7%	25.9%
256	64	10.00 dB	6.7%	7.00 dB	12.9%	18.7%
256	128	10.25 dB	6.3%	7.00 dB	12.9%	29.2%
1024	64	9.50 dB	7.4%	7.75 dB	11.0%	20.6%
SC-FDMA						
128	16	8.75 dB	8.8%	6.50 dB	14.4%	20.6%
128	32	8.00 dB	10.4%	7.00 dB	12.9%	10.9%
128	64	7.75 dB	11.0%	7.00 dB	12.9%	<5.6%

Figure 5.1: Efficiency of a typical OFDM system's power amplifier as a function of PAPR back-off [3, Fig. 2.30]. Unmodified OFDM waveforms can have PAPR as high as 11 dB for reasonable numbers of subcarriers.

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Chapter 6. Conclusion

In this work we proposed a linear programming-based algorithm that approximately solves the tone injection optimization for the PAPR reduction problem efficiently. Our LP-RTI algorithm utilizes the observation of a typically sparse tone injection solution to formulate a linear programming solution that determines the soft information of the tone injection decision variables. Our proposed algorithm is applicable to all linearly precoded systems for PAPR reduction. In particular, we demonstrate the utility of LP-RTI in well known practical systems including traditional OFDM, multi-user OFDMA and SC-FDMA systems. Our results show that the proposed approach is effective at improving the transmitter power efficiency which result in significant power savings for OFDM-based transmission systems. Furthermore, the ability of LP-RTI to effectively reduce the probability of clipping by the power amplifier can also improve the BER at the receiver. For future works, we should investigate additional complexity reductions and faster implementations of algorithm improvement in real-time hardware.

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References

- [1] J. Tellado, *Multicarrier Modulation with Low PAR Applications to DSL and Wireless*. Kluwer Academic Publishers, 2000.
- [2] B. Krongold and D. Jones, “PAR reduction in OFDM via active constellation extension,” *IEEE Trans. Broadcast.*, vol. 49, pp. 258 – 268, Sept. 2003.
- [3] E. McCune, *Practical Digital Wireless Signals*. Cambridge University Press, 2010.
- [4] F. Khan, *LTE for 4G Mobile Broadband*. Cambridge University Press, 2009.
- [5] “Ts 36.211 physical channels and modulation.” Technical Specification, E-UTRA, Revision 10.4.0, Dec. 2011.
- [6] Y.-S. Park and S. Miller, “Peak-to-average power ratio suppression schemes in DFT based OFDM,” in *Proc. IEEE 52nd VTS-Fall VTC 2000*, vol. 1, pp. 292–297, 2000.
- [7] S. Hu, G. Wu, Y. L. Guan, C. L. Law, and S. Li, “Analysis of tone reservation method for WiMAX system,” in *Proc. Intl. Symp. Commun. and Inform. Tech. (ISCIT '06)*, pp. 498–502, Sept. 2006.
- [8] A. Aggarwal and T. Meng, “A convex interior-point method for optimal OFDM PAR reduction,” in *Proc. ICC 2005*, vol. 3, pp. 1985–1990, May 2005.
- [9] Y. Kou, W.-S. Lu, and A. Antoniou, “A new peak-to-average power-ratio reduction algorithm for OFDM systems via constellation extension,” *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1823 –1832, May 2007.
- [10] N. Jacklin and Z. Ding, “A convex optimization approach to reducing peak-to-average-power ratio in OFDM,” in *Proc. IEEE Intl. Sym. on Circuits and Syst. (ISCAS 2011)*, pp. 973 –976, May 2011.
- [11] G. Chen, S. Song, and K. Letaief, “A low-complexity precoding scheme for PAPR reduction in SC-FDMA systems,” in *Proc. IEEE Wireless Commun. and Networking Conf. (WCNC 2011)*, pp. 1358 –1362, Mar. 2011.
- [12] T. Jiang and Y. Wu, “An overview: Peak-to-average power ratio reduction techniques for OFDM signals,” *IEEE Trans. Wireless Commun.*, vol. 54, pp. 257–268, June 2008.
- [13] S. H. Han and J. H. Lee, “An overview of peak-to-average power ratio reduction techniques for multi-carrier transmissions,” *IEEE Trans. Wireless Commun.*, vol. 12, pp. 56–65, Apr. 2005.
- [14] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. Cambridge University Press, 2003.
- [15] D. Donoho, “Compressed sensing,” *IEEE Trans. Inf. Theory*, vol. 52, pp. 1289 –1306, Apr. 2006.

- [16] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [17] “802.16-2005 part 16: Air interface for fixed and mobile broadband wireless access systems,” December 2005.

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