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Kinetic Energy Principle And Neoclassical Toroidal Torque in Tokamaks

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Abstract

It is shown that when tokamaks are perturbed the kinetic energy principle is closely related to the neoclassical toroidal torque by the action invariance of particles. Especially when tokamaks are perturbed from scalar pressure equilibria, the imaginary part of the potential energy in the kinetic energy principle is equivalent to the toroidal torque by the Neoclassical Toroidal Viscosity (NTV). A unified description therefore should be made for both physics. It is also shown in this case that the potential energy operator can be self-adjoint and thus the stability calculation can be simplified by minimizing the potential energy.

A tokamak maintains hot plasmas in a toroidal vessel and thus the stability of the plasma confinement is always an important issue. The essence of the stability can be obtained with the linear method, by investigating plasma responses to small perturbations. Especially the energy principle based on the ideal MagnetoHydroDynamic (MHD), which determines the stability using the ideal potential energy associated with perturbations, has been extensively studied and applied to various stability problems in tokamaks. The ideal MHD however only gives the lower bound of the stability and thus the kinetic modification to the energy principle has also been studied to find the stability bound more precisely [1–6].

The kinetic energy principle takes into account the perturbed particle orbits, which in a guiding center plasma drive a pressure anisotropy $\delta p_{\|} - \delta p_{\perp}$ and change the energy associated with perturbations. The perturbed energy is mostly positive definite, so it is known as the kinetic stabilization increasing the stability bound [2]. On the other hand, a similar approach has been used to describe the toroidal torque arising due to perturbations. This is often called the Neoclassical Toroidal Viscosity (NTV) torque [7–9] in tokamaks, and is the momentum part of the non-ambipolar transport in general [10–13]. Although the kinetic stability theory is based on a randomly exerted perturbation and the NTV theory is based on an externally applied perturbation, their physics origins are equivalent; The *kinetic potential energy* δW_k, in the kinetic energy principle, and the *neoclassical toroidal torque* T_φ are both required to perturb the tokamak plasma, in order to conserve the action of particles.

The action invariance in a guiding center plasma gives

$$T_\varphi = 2in\delta W_k, \tag{1}$$

where n is the toroidal harmonic number of perturbations, especially if the unperturbed plasma equilibrium obeys $\vec{\nabla} p = \vec{j} \times \vec{B}$. The complex number i means that the part of the distortion involves the displacement orthogonal to the direction of the perturbed force and becomes the toroidal torque. This close relation between δW_k and T_φ will be proved and discussed in this paper.

The kinetic potential energy and the neoclassical toroidal torque in a guiding center plasma are in fact analogous to the additional physics of a spinning ball on a hill, Figure 1, compared to a non-spinning ball on a hill, which is frequently used to describe ideal MHD stability. The illuminating feature can be obtained using three Eulerian angles $(\psi, \vartheta, \alpha)$ where α is the body axis normal to the surface of the hill. Assuming a no slip condition,

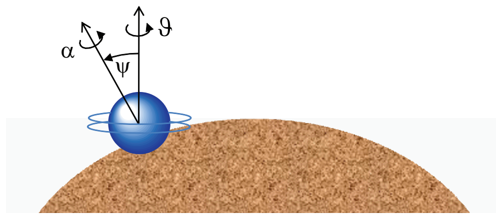

FIG. 1. The stability in a rotating tokamak plasma is similar to a spinning ball on a hill. The action conservation in a guiding center plasma plays a similar role to the momentum conservation of the ball. The three angles are Eulerian angles $(\psi, \vartheta, \alpha)$.

the Lagrangian of the system is $L = (1/2)I_T(\dot{\vartheta}^2 \sin^2\psi + \dot{\psi}^2) + (1/2)I(\dot{\vartheta}\cos\psi + \dot{\alpha})^2 - U(\psi)$. Here I is the moment of inertia and $I_T = I + MR^2$ with the mass M and the radius R of the ball, and $U(\psi)$ is the gravitational energy that can be determined by the shape of the hill. The Lagrangian is cyclic in the α-coordinate, so a constant of the motion exists as $p_\alpha = I(\dot{\vartheta}\cos\psi + \dot{\alpha})$. Therefore, the second term of the Lagrangian is invariant and the effective potential of the system is $V(\psi, \vartheta) = (1/2)I_T(\dot{\vartheta}^2 \sin^2\psi) + U(\psi)$. The additional term is always stabilizing against a perturbation and thus the ball does not fall if the spin is fast enough. The perturbation also produces a torque on the body, $\dot{\alpha}$ of $p_\alpha = I(\dot{\vartheta}\cos\psi + \dot{\alpha})$, which is self-conserved and thus not participating in the energy stabilization. A guiding center plasma exhibits similar physical phenomena, but by a different constant of the motion - the action of particles $J = \oint M v_\parallel d\ell$.

The action invariance of particles results in the anisotropic tensor pressure in a perturbation $\overleftrightarrow{\Pi} = \delta p_\perp \overleftrightarrow{I} + (\delta p_\parallel - \delta p_\perp)\hat{b}\hat{b}$. It will be shown that the kinetic potential energy is

$$\delta W_k = \frac{1}{2}\int dx^3 \left[(\delta p_\parallel - \delta p_\perp)\frac{\delta B_L}{B} + \delta p_\parallel(\vec{\nabla}\cdot\vec{\xi})\right], \tag{2}$$

and the neoclassical toroidal torque is

$$T_\varphi = -\int dx^3 \left[(\delta p_\parallel - \delta p_\perp)\frac{1}{B}\frac{\partial \delta B_L}{\partial \varphi} + \delta p_\parallel \frac{\partial}{\partial \varphi}(\vec{\nabla}\cdot\vec{\xi})\right], \tag{3}$$

when a tokamak is perturbed with a plasma displacement $\vec{\xi}(\psi, \vartheta, \varphi) \sim e^{i(m\vartheta - n\varphi)}$. Equation (1) is obvious by these two equations. The variation in the field strength δB_L is the Lagrangian quantity, that is, the variation in the field strength measured on the perturbed field lines. It is different from the variation in the field strength at a fixed spatial point

$\delta B_E = |\vec{\nabla} \times (\vec{\xi} \times \vec{B})|$ as $\delta B_L = \delta B_E + \vec{\xi} \cdot \vec{\nabla} \vec{B}$. A convenient form is given by

$$\frac{\delta B_L}{B} = \hat{b} \cdot (\vec{\nabla} \vec{\xi}) \cdot \hat{b} - (\vec{\nabla} \cdot \vec{\xi}), \tag{4}$$

where $\hat{b}$ is the unit vector of the magnetic field $\vec{B}$.

The simplest derivation to Equation (2) is to use $\delta W_k = -(1/2) \int dx^3 \vec{\xi} \cdot (\vec{\nabla} \cdot \overleftrightarrow{\Pi})$. Using the several integrations by parts and vector relations, it is straightforward to show that the non-zero integrand is $-(\delta p_\parallel - \delta p_\perp)(\hat{b} \cdot (\vec{\nabla} \vec{\xi}) \cdot \hat{b}) - \delta p_\perp (\vec{\nabla} \cdot \vec{\xi})$. The details are skipped here since this is in fact identical to the form derived by Rosenbluth and Rostoker [2], and Taylor and Hastie [3] or Antonsen [4] when the unperturbed equilibrium is assumed as $\vec{\nabla} p = \vec{j} \times \vec{B}$. Using Equation (4), one can show Equation (2).

Equation (3) can be obtained using the general relation by Boozer [14]

$$T_\varphi = \int dx^3 \left[\frac{\partial \vec{x}}{\partial \varphi} \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi} \right] = -\frac{1}{2} \sum_{ij} \int dx^3 \frac{\partial g_{ij}}{\partial \varphi} \Pi^{ij}, \tag{5}$$

where the metric tensor $g_{ij} = (\partial \vec{x} / \partial x^i) \cdot (\partial \vec{x} / \partial x^j)$, since $\overleftrightarrow{\Pi} = 0$ on the surface. Using the expression for the magnetic field $2\pi \mathcal{J} \vec{B} = (\partial \vec{x} / \partial \varphi) + \iota (\partial \vec{x} / \partial \vartheta)$ with the Jacobian $\mathcal{J}$ and the rational transform ι, the metric tensors have the relations $g_{\varphi\varphi} + \iota g_{\vartheta\varphi} = 2\pi \mathcal{J} \vec{B} (\partial \vec{x} / \partial \varphi)$ and $g_{\varphi\vartheta} + \iota g_{\vartheta\vartheta} = 2\pi \mathcal{J} \vec{B} (\partial \vec{x} / \partial \vartheta)$. If the particle pressure tensor is expressed by $\overleftrightarrow{\Pi} = \overleftrightarrow{\Pi}_\parallel + \delta p_\perp \overleftrightarrow{I}$, then $\Pi_\parallel^{\vartheta\vartheta} = \iota \Pi_\parallel^{\varphi\vartheta} = \iota^2 \Pi_\parallel^{\varphi\varphi}$. Using these relations, the identity $I^{ij} = g^{ij}$, and $(\partial g_{ij} / \partial \varphi) g^{ij} = (2/\mathcal{J})(\partial \mathcal{J} / \partial \varphi)$, one can obtain

$$T_\varphi = -\frac{1}{2} \int dx^3 \left[\frac{\partial}{\partial \varphi} (2\pi \mathcal{J} B)^2 \Pi_\parallel^{\varphi\varphi} + \delta p_\perp \frac{2}{\mathcal{J}} \frac{\partial \mathcal{J}}{\partial \varphi} \right], \tag{6}$$

where the tensor component is

$$\Pi_\parallel^{\varphi\varphi} = (\delta p_\parallel - \delta p_\perp)(\vec{\nabla} \varphi \cdot \hat{b})^2 = \frac{(\delta p_\parallel - \delta p_\perp)}{(2\pi \mathcal{J} B)^2}. \tag{7}$$

To the first order, one should consider the arc-length change of the integration due to the perturbed flux surfaces as $\mathcal{J} \rightarrow \mathcal{J}(1 + \vec{\nabla} \cdot \vec{\xi})$. Combining results, one can show Equation (3). Note in this case the frame is naturally along the perturbed field lines $\hat{b}$.

Equations (2) and (3) can be rewritten into a more insightful form if one uses the definitions $\delta p_\parallel \equiv \int dv^3 M v_\parallel^2 \delta f$ and $\delta p_\perp \equiv \int dv^3 (1/2) M v_\perp^2 \delta f$ for a species and uses the variation in the action $\delta J = 2\pi \oint d\vartheta (M / \psi_p') \delta(\mathcal{J} B v_\parallel)$ on magnetic coordinates $(\psi, \vartheta, \varphi)$ that make $\vec{B} = 2\pi \psi_p' \vec{\nabla} \psi \times \vec{\nabla} \alpha$, where $\alpha \equiv q\vartheta - \varphi$, ψ_p is the poloidal flux, and $\psi_p' = d\psi_p / d\psi$. Using $\vec{v}(E, \mu)$ with the energy E and the magnetic moment μ of particles, and assuming δf

averaged over ϑ, one can show that the kinetic potential energy is

$$\delta W_k = \frac{1}{2M^2} \int d\psi_p d\varphi dE d\mu \left(\delta J \delta f\right), \tag{8}$$

and the neoclassical toroidal torque is

$$T_\varphi = \frac{1}{M^2} \int d\psi_p d\varphi dE d\mu \left(\frac{\partial \delta J}{\partial \alpha}\right) \delta f. \tag{9}$$

Equations (8) and (9), which also prove Equation (1), imply that the kinetic potential energy and the neoclassical toroidal torque both depend on the variation in the action, but different part by the phase. Consequently, they are dominated by different particles in δf as will be shown later.

The presented relations imply that studies of the kinetic energy stabilization and the NTV should be consistent. Recent numerical and empirical studies for the kinetic energy stabilization for Resistive Wall Modes (RWMs) revealed a complex dependence of stability on rotation [15], and also found that the fast ion contributions are perhaps important [16]. On the other hand, the NTV magnetic braking experiments have shown the toroidal momentum dissipation [17], the neoclassical offset rotation [18], and the resonance between the electric and magnetic precessions [19], as predicted by the analytic theory. Analytic and numerical NTV studies have also been rapidly evolved to achieve more precise prediction, by calculating the perturbed distribution function with a more complex collisional operator [20, 21]. Progress in any of these efforts should be reflected on both studies.

Note that a subtlety arises in the calculations of the variation in the action δJ given a perturbation $\vec{\xi}$. The variation in the action is given by

$$\delta J = \frac{2\pi M}{\psi_p'} \oint \mathcal{J} d\vartheta \left[\frac{d(Bv_\parallel)}{dB}\delta B + Bv_\parallel \frac{\delta \mathcal{J}}{\mathcal{J}}\right]. \tag{10}$$

The Lagrangian variation in the field strength $\delta B = \delta B_L$ can be written as $\delta B_L = \vec{\xi}_\parallel \cdot \vec{\nabla} B_0 - B_0(\vec{\nabla} \cdot \vec{\xi}_\perp + \vec{\xi}_\perp \cdot \vec{\kappa}_0)$ [22], with the curvature $\vec{\kappa}_0 = \hat{b}_0 \cdot \vec{\nabla}\hat{b}_0$. One can see that the variation in the field strength δB_L depends on the parallel displacement $\vec{\xi}_\parallel$. However, the variation in the action δJ is independent of $\vec{\xi}_\parallel$ due to the variation in the Jacobian, as expected since the choice of $\vec{\xi}_\parallel$ defines the choice of the coordinate used for the integration variable. Therefore, one can use $\delta B_L = \delta B_L(\vec{\xi}_\perp)$ for δJ. The proof is as follows: Using $\delta\mathcal{J}/\mathcal{J} = \vec{\nabla} \cdot \vec{\xi} = \vec{\nabla} \cdot \vec{\xi}_\parallel + \vec{\nabla} \cdot \vec{\xi}_\perp$, and denoting $F(B) = Bv_\parallel(B)$, the part of Equation (10) that depends on the parallel displacement is

$$\oint \mathcal{J} d\vartheta \left[\frac{dF}{dB} \vec{\xi}_{\|} \cdot \vec{\nabla} B + F \vec{\nabla} \cdot \vec{\xi}_{\|} \right] = \oint \mathcal{J} d\vartheta \vec{\nabla} \cdot (F \vec{\xi}_{\|}). \tag{11}$$

Now $\vec{\xi}_{\|} \cdot \vec{\nabla}\alpha = (\xi_{\|}/B)\vec{B} \cdot \vec{\nabla}\alpha = 0$, and $\vec{\xi}_{\|} \cdot \vec{\nabla}\psi = 0$, so $\mathcal{J}\vec{\nabla} \cdot (F\vec{\xi}_{\|}) = \partial(\mathcal{J}F\vec{\xi}_{\|} \cdot \vec{\nabla}\vartheta)/\partial\vartheta$. At the end points of the integral F vanishes, so $\oint \mathcal{J} d\vartheta \vec{\nabla} \cdot (F\vec{\xi}_{\|}) = 0$.

The derived Equations (8) and (9) require the calculation of δf. Here an analytic example will be used to illustrate basic features in δW_k and T_φ, but any advanced calculation of δf [21, 23] can be applied consistently for both. Analytically δf is solvable with the pitch-angle collision operator if the regimes are separately treated. However, recently it has been noticed that the regimes are significantly overlapping and thus combining the regimes is more important even if sacrificing the accuracy of the collision operator [9]. Using a Krook collision operator $\nu_K(E)$ with an effective correction, it can be shown that the bounce-average perturbed distribution function $\delta f = \langle \delta f \rangle_b$ in a slowly varying perturbation is given by

$$\delta f_\ell = \frac{(\omega_b/e)}{i\ell\omega_b - in(\omega_E + \omega_B) - \nu_K} \left(\frac{\partial J_\ell}{\partial \alpha} \right) \frac{\partial f_0}{\partial \psi_p}, \tag{12}$$

for the ℓ-class of particles, with the bounce frequency $\omega_b(E,\mu)$, the electric precession frequency ω_E, and the magnetic precession frequency $\omega_B(E,\mu)$. The f_0 is the zeroth-order distribution in the equilibrium state. The bounce-average action for the ℓ-class of particles is given by

$$\delta J = \frac{2\pi}{\omega_b} \left\langle (2E - 3\mu B) \frac{\delta(B_L \mathcal{P}^\ell)}{B} + (2E - 2\mu B)\, \vec{\nabla} \cdot \vec{\xi}_\perp \right\rangle_b, \tag{13}$$

with an appropriate phase factor $\mathcal{P}^\ell$ [9].

The δf is proportional to the action variation. By simplifying the notation $\delta f = \mathcal{R}(\partial \delta J/\partial \alpha)(\partial f_0/\partial \psi_p)$ and omitting ℓ, one obtains the kinetic potential energy

$$\delta W_k = \frac{1}{2M^2} \int d\psi_p d\varphi dE d\mu \mathcal{R} \left(\delta J \frac{\partial \delta J}{\partial \alpha} \right) \frac{\partial f_0}{\partial \psi_p}, \tag{14}$$

and the neoclassical toroidal torque

$$T_\varphi = \frac{1}{M^2} \int d\psi_p d\varphi dE d\mu \mathcal{R} \left(\frac{\partial \delta J}{\partial \alpha} \frac{\partial \delta J}{\partial \alpha} \right) \frac{\partial f_0}{\partial \psi_p}. \tag{15}$$

Note that Equations (14) and (15) are independent of $\vec{\xi}_{\|}$, as often implicitly claimed in the literature. Also, one can see in this case that the energy operator $\delta W_k(\vec{\xi}, \vec{\eta})$ for any arbitrary displacements $\vec{\xi}$ and $\vec{\eta}$ is symmetric, and thus the operator is self-adjoint. This means that one can use the energy principle together with the ideal MHD energy operator δW_p. That is, the minimized potential $\delta W_p + \delta W_k$ will determine the stability and also give the perturbed force balance.

The practical forms for each can also be obtained if one performs the further ordering by $\epsilon = r/R_0$ for the field, $B = B_0(1 - \epsilon\cos\vartheta) + B_0\sum_{nm}\delta_{nm}e^{i(m-nq)\vartheta+in\alpha}$, and assumes the Maxwellian $f_0 = f_M$ and the contributions only by trapped particles. Using the normalized variables $x \equiv E/T$ and $\kappa^2 \equiv (E - \mu B_0(1-\epsilon))/2\mu B_0\epsilon$, where T is the temperature, one can show the term $2E - 2\mu B$ becomes the next order and thus $\delta J \propto \delta B_L$, $\delta W_k \propto (\delta B_L)^2$, and $T_\varphi \propto (\delta B_L)^2$. Combining Equation (12) with Equations (14) and (15), with several approximations, one can show the kinetic potential energy

$$\delta W_k = \sum_{\ell nmm'} \int dV \int_0^1 d\kappa^2 \int_0^\infty dx \frac{\epsilon^{1/2} p}{2\sqrt{2}\pi^{3/2}} \delta_{w\ell}^2 \mathcal{R}_{e\ell}\omega_{e\ell}^\varphi, \tag{16}$$

and the neoclassical toroidal torque

$$T_\varphi = \sum_{\ell nmm'} \int dV \int_0^1 d\kappa^2 \int_0^\infty dx \frac{\epsilon^{1/2} p}{\sqrt{2}\pi^{3/2}} \delta_{w\ell}^2 \mathcal{R}_{t\ell}\omega_{t\ell}^\varphi, . \tag{17}$$

Here the weighted variation in the field strength is

$$\delta_{w\ell}^2 = \delta_{nmm'}^2 \frac{F_{nm\ell}^{-1/2} F_{nm'\ell}^{-1/2}}{4K(\kappa)}, \tag{18}$$

with $\delta_{nmm'}^2 \equiv Re(\delta_{nm})Re(\delta_{nm'}) + Im(\delta_{nm})Im(\delta_{nm'})$, and the special function $F_{nm\ell}^y(\kappa) = \int_{-\vartheta_t}^{\vartheta_t} d\vartheta(\kappa^2 - \sin^2(\vartheta/2))^y \cos[\Theta_{nm\ell}(\vartheta)]$, $\Theta_{nm\ell}(\vartheta) \approx (m - nq - \sigma\ell)\vartheta$ with the sign function σ that $\sigma = +1$ for co-rotation relative to the plasma current, and the complete elliptic integral of the first kind K. The resonant terms are

$$\mathcal{R}_{e\ell} = \frac{1}{2}\frac{n[n(\omega_E + \omega_B) - \ell\omega_b](x^{5/2}e^{-x})}{[\ell\omega_b - n(\omega_E + \omega_B)]^2 + \nu_{D\ell}^2 x^{-3}}, \tag{19}$$

$$\mathcal{R}_{t\ell} = \frac{1}{2}\frac{n^2\nu_{D\ell}(xe^{-x})}{[\ell\omega_b - n(\omega_E + \omega_B)]^2 + \nu_{D\ell}^2 x^{-3}}, \tag{20}$$

where the effective collision frequency $\nu_{K\ell} = \nu_{D\ell}x^{-3/2}$ and $\nu_{D\ell} = (\nu/2\epsilon)[1 + (\ell/2)^2]$. The $\omega_{e\ell}^\varphi$ and $\omega_{t\ell}^\varphi$ are toroidal rotation frequencies with the neoclassical offset for each, and defined as

$$\omega_{(e,t)\ell}^\varphi = \omega^\varphi + \sigma\frac{\int_0^\infty (x - 3/2)\mathcal{R}_{(e,t)\ell}dx}{\int_0^\infty \mathcal{R}_{(e,t)\ell}dx}\left|\frac{2\pi}{e}\frac{dT}{d\psi_p}\right|. \tag{21}$$

Equations (16) and (17) are intrinsically similar, but the different resonant terms $\mathcal{R}_{(e,t)\ell}$ can give different consequences in the parametric space of the rotation or the collisionality.

The largest contribution to the energy is produced by particles with $\ell\omega_b - n(\omega_E + \omega_B) \sim \nu_{K\ell}$, while the torque is dominated by $\ell\omega_b - n(\omega_E + \omega_B) \sim 0$, that is, the resonant particles. The neoclassical offsets are also different over the parameter space of rotation and collisionality. Each offset is $(2 \sim 5)|(2\pi/e)(dT/d\psi_p)|$ for the energy, and $(0.5 \sim 3.5)|(2\pi/e)(dT/d\psi_p)|$ for the torque, considering that the smallest offset is given when $\nu \to 0$ (ν-regime) [24] and the largest offset is when $\nu \to \infty$ ($1/\nu$-regime) [8]. When a perturbation gives a continuous torque and consequently the rotation becomes $\omega^{\varphi} \sim -\omega^{\varphi}_{t\ell}$, the torque becomes zero for any perturbation, but the kinetic stabilizing effect can still exist.

The kinetic potential energy and the neoclassical toroidal torque arise for any perturbation, and can change any of the ideal MHD modes including kink, peeling, and ballooning modes. It will also change the plasma response to external perturbations and thus RWMs. The general relation between the plasma and the external perturbation can be derived by integrating the Maxwell stress tensor, similarly to the particle pressure tensor, and the characteristic plasma response is given by $1/(s + i\alpha)$ [25, 26]. Here s is the negative of the normalized energy and α is the normalized toroidal torque. The prediction of the plasma response using only the ideal MHD energy s_p has been well validated when $s_p < 0$, that is, below the no-wall stability limit [26, 27]. When $s_p \to 0$, the plasma response will be dominated by the kinetic potential energy and the neoclassical toroidal torque $1/(s_k + i\alpha)$, and the toroidal phase shift will be $45° \sim 90°$. This means that the no-wall limit is actually greater than the ideal no-wall limit. Eventually the perturbation in the plasma will modify itself from the ideal state towards $s_p + s_k \to 0$. The torque can remain, and then the plasma response will exhibit a phase shift up to 90°. This is the true no-wall limit. However, the torque will also be decreased when approaching the true no-wall limit since minimizing potential requires minimizing δB_L as well as other quadratic terms in ideal MHD.

This paper demonstrates that the kinetic potential energy and the neoclassical toroidal torque in tokamaks are closely related to each other, and should be studied consistently. Both arise due to the action invariance in perturbations and depend quadratically on the variation in the action. They however have different dependencies on the phase in the action variation, and different particles are dominant for each. The resulting energy operator can be self-adjoint, and thus more accurate stability boundary can be studied in the same way as ideal MHD. The plasma response to external perturbations is also modified by both, and can be studied beyond the ideal stability limit.

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