C-A/AP/#164
September 2004

Notes on Orbit Equations in the AGS

C.J. Gardner

Collider-Accelerator Department
Brookhaven National Laboratory
Upton, NY 11973

Notes on Orbit Equations in the AGS

C.J. Gardner

August 31, 2004

It has been nearly 20 years since Bleser reviewed and produced two documents [1, 2] on the layout of the AGS magnets. This was based on the original prescription given by Smith [3] nearly 50 years ago. As there has been renewed interest in models of the AGS, I thought it would be useful to revisit these documents. Following are some notes on the orbit equations used. There are no new results here, but there are some details not included in the original documents.

1 Magnet Coordinate System

Particle motion around the AGS is counter-clockwise. The coordinate system in a magnet is illustrated in **Figure 1**. Two survey sockets serve to define the longitudinal axis of each magnet. These are located on top of the magnet three inches from the steel face at each end. The longitudinal axis is the line that lies in the magnetic midplane and passes through the projection of the two sockets onto the midplane. This is the socket line shown in the figure. Unit vector $\mathbf{x}$ lies along the socket line and points in the counter-clockwise direction. Unit vector $\mathbf{y}$ is perpendicular to $\mathbf{x}$, lies in the magnetic midplane, and points in the radially outward direction. x and y are the coordinates along the $\mathbf{x}$ and $\mathbf{y}$ directions respectively.

2 Equation of Motion

The equation of motion for a particle of mass m and charge e in a static magnetic field $\mathbf{B}$ is

$$\frac{d\mathbf{p}}{dt} = \frac{e}{c}\,\mathbf{v} \times \mathbf{B} \tag{1}$$

where

$$\mathbf{p} = m\gamma\mathbf{v}, \quad \gamma = (1 - v^2/c^2)^{-1/2}, \quad v = |\mathbf{v}|. \tag{2}$$

Here $\mathbf{v}$ is the particle velocity which has constant magnitude v. We assume that the magnetic field is everywhere perpendicular to the midplane of the AGS magnets and that the motion is confined to the midplane. We then have

$$\mathbf{v} = \frac{dx}{dt}\mathbf{x} + \frac{dy}{dt}\mathbf{y}, \quad v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \tag{3}$$

and on the midplane

$$\mathbf{B} = B(x, y)\,\mathbf{z} \tag{4}$$

where

$$\mathbf{z} = \mathbf{y} \times \mathbf{x}. \tag{5}$$

Note that $\mathbf{z}$ points up. The $\mathbf{x}$ and $\mathbf{y}$ components of equation (1) are then

$$m\gamma\,\frac{d^2x}{dt^2} = -\frac{eB}{c}\frac{dy}{dt}, \quad m\gamma\,\frac{d^2y}{dt^2} = \frac{eB}{c}\frac{dx}{dt} \tag{6}$$

which we write as

$$\frac{d^2x}{dt^2} = -vK\,\frac{dy}{dt}, \quad \frac{d^2y}{dt^2} = vK\,\frac{dx}{dt} \tag{7}$$

where

$$K = \frac{e}{cp}B(x, y), \quad p = mv\gamma, \quad \gamma = (1 - v^2/c^2)^{-1/2}. \tag{8}$$

3 Equations with s as the Independent Variable

Let s be the distance traveled along the particle trajectory. Then

$$v = \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \tag{9}$$

and

$$\frac{dx}{dt} = v\frac{dx}{ds}, \quad \frac{d^2x}{dt^2} = v^2\,\frac{d^2x}{ds^2}, \quad \frac{dy}{dt} = v\frac{dy}{ds}, \quad \frac{d^2y}{dt^2} = v^2\,\frac{d^2y}{ds^2}. \tag{10}$$

Equations (7) then become

$$\frac{d^2x}{ds^2} = -K\,\frac{dy}{ds}, \quad \frac{d^2y}{ds^2} = K\,\frac{dx}{ds}. \tag{11}$$

We also have

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 = 1. \tag{12}$$

4 First-Order Differential Equations

The set of second-order differential equations (11) can be expressed as a set of first-order equations. Using a prime to denote differentiation with respect to s, and defining

$$Z_1 = x, \quad Z_2 = \frac{dx}{ds} = x', \quad Z_3 = y, \quad Z_4 = \frac{dy}{ds} = y' \tag{13}$$

we have

$$Z_1' = Z_2, \quad Z_2' = -KZ_4, \quad Z_3' = Z_4, \quad Z_4' = KZ_2 \tag{14}$$

where

$$K = \frac{e}{cp} B(Z_1, Z_3). \tag{15}$$

We also have

$$Z_2^2 + Z_4^2 = 1. \tag{16}$$

These equations can be integrated numerically to obtain the particle trajectory in the midplane. This is what is done in the BEAM program [4, 5].

5 The Case of Constant K

For the case in which K is constant we have

$$Z_2'' = -KZ_4' = -K^2 Z_2 \tag{17}$$

which has general solution

$$Z_2(s) = A\cos(Ks + \phi). \tag{18}$$

Thus

$$Z_2' = -AK\sin(Ks + \phi) = -KZ_4 \tag{19}$$

and therefore

$$Z_4(s) = A\sin(Ks + \phi). \tag{20}$$

Since $Z_2^2 + Z_4^2 = 1$ we have $A^2 = 1$. Taking $A = 1$ we have

$$Z_2(s) = \cos(Ks + \phi), \quad Z_4(s) = \sin(Ks + \phi) \tag{21}$$

and

$$Z_2(0) = \cos\phi, \quad Z_4(0) = \sin\phi. \tag{22}$$

Integrating $Z_1' = Z_2$ and $Z_3' = Z_4$ we then obtain

$$Z_1(s) - Z_1(0) = -R\{\sin(Ks+\phi) - \sin\phi\} \tag{23}$$

$$Z_3(s) - Z_3(0) = R\{\cos(Ks+\phi) - \cos\phi\} \tag{24}$$

where

$$R = -1/K. \tag{25}$$

Thus

$$Z_1(s) = Z_1(0) + RZ_4(0) - R\,\sin(Ks+\phi) \tag{26}$$

$$Z_3(s) = Z_3(0) - RZ_2(0) + R\,\cos(Ks+\phi) \tag{27}$$

and using (13) we have

$$x(s) = x_c - R\,\sin(Ks+\phi) \tag{28}$$

$$y(s) = y_c + R\,\cos(Ks+\phi) \tag{29}$$

where

$$x_c = x(0) + R\,y'(0), \quad y_c = y(0) - R\,x'(0) \tag{30}$$

$$\sin\phi = y'(0), \quad \cos\phi = x'(0). \tag{31}$$

The particle trajectory is therefore a circle of radius R centered on the point $(x_c,\, y_c)$.

6 Equations with x as the Independent Variable

Taking x as the independent variable we have

$$\frac{dy}{dx} = \frac{dy}{ds}\frac{ds}{dx} = \frac{y'}{x'} \tag{32}$$

and

$$\frac{d^2y}{dx^2} = \frac{1}{x'}\left\{\frac{y''}{x'} - \frac{y'x''}{x'^2}\right\} = \frac{1}{x'^3}\{x'y'' - y'x''\} \tag{33}$$

where the primes denote differentiation with respect to s. Using

$$x'' = -Ky', \quad y'' = Kx', \quad x'^2 + y'^2 = 1 \tag{34}$$

we have

$$x'y'' - y'x'' = x'(Kx') - y'(-Ky') \tag{35}$$

$$x'y'' - y'x'' = K(x'^2 + y'^2) = K \tag{36}$$

and

$$\frac{d^2y}{dx^2} = \frac{K}{x'^3}. \tag{37}$$

Then since

$$\frac{1}{x'^2} = \frac{x'^2 + y'^2}{x'^2} = 1 + \frac{y'^2}{x'^2} = 1 + \left(\frac{dy}{dx}\right)^2 \tag{38}$$

we have

$$\frac{d^2y}{dx^2} = K\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}. \tag{39}$$

7 AGS Combined Function Magnets

Notation Change: For the remainder of these notes we use a prime to denote differentiation with respect to x. Equation (39) then becomes

$$y'' = K\left\{1 + y'^2\right\}^{3/2} \tag{40}$$

where

$$K = \frac{e}{cp}B(x, y). \tag{41}$$

For the AGS combined function magnets, the field on the midplane is

$$\mathbf{B} = B(x, y)\,\mathbf{z} \tag{42}$$

where

$$B(x, y) = -B_0\left(1 - \frac{ny}{\rho}\right) \tag{43}$$

and

$$\rho = \frac{cp}{eB_0}, \quad \frac{1}{\rho} = \frac{eB_0}{cp}, \quad n = \frac{\rho}{B_0}\frac{dB}{dy}. \tag{44}$$

Thus

$$K = -\frac{1}{\rho}\left(1 - \frac{ny}{\rho}\right). \tag{45}$$

Note that along the socket line we have $y = 0$ and $B(x, 0) = -B_0$. For positive particles going counter-clockwise around the AGS, B_0 is positive. The gradient dB/dy is assumed to be constant. It is negative for focussing magnets and positive for defocussing magnets. Since B_0 and ρ are positive, n has the same sign as dB/dy.

Following Bleser [1, 2] we define

$$\lambda = \frac{\rho}{\sqrt{|n|}}, \qquad \frac{1}{\lambda^2} = \frac{|n|}{\rho^2} = \left| \frac{1}{B_0 \rho} \frac{dB}{dy} \right| . \tag{46}$$

For a focussing magnet we then have

$$K = -\frac{1}{\rho} \left(1 + \frac{\rho}{\lambda^2} \, y \right) \tag{47}$$

and

$$y'' = -\frac{1}{\rho} \left(1 + \frac{\rho}{\lambda^2} \, y \right) \left\{ 1 + y'^2 \right\}^{3/2} . \tag{48}$$

For a defocussing magnet we have

$$K = -\frac{1}{\rho} \left(1 - \frac{\rho}{\lambda^2} \, y \right) \tag{49}$$

and

$$y'' = -\frac{1}{\rho} \left(1 - \frac{\rho}{\lambda^2} \, y \right) \left\{ 1 + y'^2 \right\}^{3/2} . \tag{50}$$

8 Solution for a Pure Dipole

For a pure dipole we have

$$n = 0, \qquad K = -\frac{1}{\rho} \tag{51}$$

and

$$y'' = -\frac{1}{\rho} \left\{ 1 + y'^2 \right\}^{3/2} \tag{52}$$

where ρ is constant. The solution of this differential equation must satisfy

$$(x - x_c)^2 + (y - y_c)^2 = \rho^2 \tag{53}$$

where $(x_c,\, y_c)$ are the coordinates of the center of curvature. Solving for $y - y_c$ we have

$$y - y_c = \left\{ \rho^2 - (x - x_c)^2 \right\}^{1/2} \tag{54}$$

and

$$y' = -\left(\frac{x - x_c}{y - y_c} \right), \qquad 1 + y'^2 = \frac{\rho^2}{(y - y_c)^2} \tag{55}$$

$$y'' = \frac{-1}{y - y_c} + \left\{ \frac{x - x_c}{(y - y_c)^2} \right\} y' = -\frac{\rho^2}{(y - y_c)^3} = -\frac{1}{\rho} \left\{ 1 + {y'}^2 \right\}^{3/2}. \tag{56}$$

This verifies that equation (52) is satisfied.

Consider now the approximation in which the ${y'}^2$ term in (52) is neglected. We then have

$$y'' = -\frac{1}{\rho} \tag{57}$$

where ρ is constant. This equation has general solution

$$y = y_0 + y_0' x - \frac{x^2}{2\rho} \tag{58}$$

and with initial conditions

$$y_0 = \rho, \quad y_0' = 0 \tag{59}$$

we have

$$y = \rho - \frac{x^2}{2\rho}, \quad y^2 = \rho^2 - x^2 + \frac{x^4}{4\rho^2}. \tag{60}$$

Thus

$$x^2 + y^2 = \rho^2 + \frac{x^4}{4\rho^2} = \rho^2 \left\{ 1 + \frac{x^4}{4\rho^4} \right\} \tag{61}$$

and we see that for $x/\rho << 1$ we have $x^2 + y^2 = \rho^2$ to a good approximation. Thus the solution of (57) is a good approximation to the solution of (52).

9 Solution for Focussing Magnet

Following Smith [3] and Bleser [1, 2] we assume that the ${y'}^2$ term in equations (48) and (50) can be neglected. For a focussing magnet in the AGS we then have

$$y'' = -\frac{1}{\rho} \left(1 + \frac{\rho}{\lambda^2} y \right) \tag{62}$$

and

$$y'' + \frac{1}{\lambda^2} y = -\frac{1}{\rho}. \tag{63}$$

This has particular solution

$$y_p = -\frac{\lambda^2}{\rho} \left\{ 1 - \cos \frac{x}{\lambda} \right\} \tag{64}$$

and general solution

$$y = -\frac{\lambda^2}{\rho} + \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\cos\frac{x}{\lambda} + \lambda y_0'\,\sin\frac{x}{\lambda}, \quad y(0) = y_0 \tag{65}$$

$$y' = -\frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\sin\frac{x}{\lambda} + y_0'\cos\frac{x}{\lambda}, \quad y'(0) = y_0'. \tag{66}$$

10 Orbit Conditions at the Upstream and Downstream Ends of the Magnets

The AGS combined function magnets have two different lengths called "long" and "short". For long magnets the length of steel from one end to the other is 90 inches; for short magnets it is 75 inches. We assume that the field on the midplane is independent of x and extends two inches beyond the ends of the magnet as illustrated in **Figure 1**. Outside this region the field is assumed to be zero. This is the hard-edge approximation. The effective length L of the long magnets is then 94 inches; that of the short magnets is 79 inches.

Following Smith and Bleser we require that the design orbit enter and leave the region of nonzero field at the same angle with respect to the socket line as illustrated in **Figure 2**. The derivative y' must then have the same magnitude but opposite signs at the upstream and downstream boundaries of nonzero field. Taking $x = 0$ and $x = L$ at the upstream and downstream boundaries, we have

$$y'(L) = -y'(0) = -y_0'. \tag{67}$$

Equation (66) then gives

$$y'(L) = -\frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\sin\frac{L}{\lambda} + y_0'\cos\frac{L}{\lambda} = -y_0' \tag{68}$$

and

$$y_0' = \frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\left\{\frac{\sin(L/\lambda)}{1 + \cos(L/\lambda)}\right\}. \tag{69}$$

Using

$$\sin\frac{L}{\lambda} = 2\sin\frac{L}{2\lambda}\cos\frac{L}{2\lambda}, \quad 1 + \cos\frac{L}{\lambda} = 2\cos^2\frac{L}{2\lambda} \tag{70}$$

this becomes

$$y_0' = \frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\tan\frac{L}{2\lambda}. \tag{71}$$

Note that as the gradient in the magnet goes to zero, λ goes to infinity and y_0' becomes $L/(2\rho)$ which is independent of y_0. As long as the gradient is nonzero we can solve (71) for y_0. This gives

$$y_0 = \frac{\lambda y_0'}{\tan\{L/(2\lambda)\}} - \frac{\lambda^2}{\rho} = \frac{\lambda^2}{\rho}\left\{\frac{\rho y_0'}{\lambda \tan\{L/(2\lambda)\}} - 1\right\}. \tag{72}$$

Consider now $y(L)$. Equation (65) gives

$$y(L) = -\frac{\lambda^2}{\rho} + \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\cos\frac{L}{\lambda} + \lambda y_0' \sin\frac{L}{\lambda} \tag{73}$$

and using (69) we have

$$\lambda y_0' \sin\frac{L}{\lambda} = \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\left\{1 - \cos\frac{L}{\lambda}\right\}. \tag{74}$$

Thus

$$y(L) = -\frac{\lambda^2}{\rho} + \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\left\{\cos\frac{L}{\lambda} + 1 - \cos\frac{L}{\lambda}\right\} \tag{75}$$

$$y(L) = -\frac{\lambda^2}{\rho} + \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\} = y_0 = y(0). \tag{76}$$

This shows that the condition $y'(L) = -y'(0)$ is equivalent to the condition $y(L) = y(0)$.

11 Orbit Sagitta

Halfway through the magnet we have $x = L/2$ and (66) gives

$$y'(L/2) = -\frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\sin\frac{L}{2\lambda} + y_0'\cos\frac{L}{2\lambda}. \tag{77}$$

Using (71) we have

$$y_0'\cos\frac{L}{2\lambda} = \frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\sin\frac{L}{2\lambda} \tag{78}$$

and therefore

$$y'(L/2) = 0. \tag{79}$$

Thus we see that the condition $y'(L) = -y'(0)$ implies that $y'(L/2) = 0$. Note also that since y'' is less than zero, $y(x)$ reaches its maximum at $x = L/2$.

Now setting $x = L/2$ in (65) we have

$$y(L/2) = -\frac{\lambda^2}{\rho} + \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\cos\frac{L}{2\lambda} + \lambda y_0'\,\sin\frac{L}{2\lambda} \tag{80}$$

and again using (71) we have

$$\lambda y_0'\,\sin\frac{L}{2\lambda} = \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\tan\frac{L}{2\lambda}\,\sin\frac{L}{2\lambda}. \tag{81}$$

Thus

$$y(L/2) = -\frac{\lambda^2}{\rho} + \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\frac{1}{\cos\{L/(2\lambda)\}}. \tag{82}$$

The sagitta of the orbit in the magnet is defined to be

$$\Delta = y(L/2) - y_0 \tag{83}$$

as illustrated in **Figure 2**. Thus (82) gives

$$\Delta = \frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\frac{1 - \cos\{L/(2\lambda)\}}{\cos\{L/(2\lambda)\}}. \tag{84}$$

Using the relation

$$y_0' = \frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\}\tan\frac{L}{2\lambda} \tag{85}$$

we can also express the sagitta in terms of y_0'. We have

$$\frac{\lambda^2}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\,y_0\right\} = \lambda y_0'\cot\frac{L}{2\lambda} \tag{86}$$

and therefore

$$\Delta = \lambda y_0'\left(\frac{1 - \cos\{L/(2\lambda)\}}{\sin\{L/(2\lambda)\}}\right). \tag{87}$$

12 Bend Angles

The condition

$$y'(L) = -y_0' \tag{88}$$

gives the relation

$$y_0 = \frac{\lambda^2}{\rho}\left\{\frac{\rho y_0'}{\lambda\tan\{L/(2\lambda)\}} - 1\right\} \tag{89}$$

but does not fix the values of y_0 and y_0'. Specifying the bend angle θ of each magnet will fix the value of y_0'. Following Smith and Bleser we require that θ be proportional to the effective length L. Thus

$$\theta = L/k \tag{90}$$

where $1/k$ is the constant of proportionality. Since the sum of all the bend angles is 2π for a closed ring, we must have

$$k = \frac{1}{2\pi} \sum L \tag{91}$$

where the sum is taken over all magnets. In the AGS there are 240 magnets. Of these, 144 are long magnets with effective length $L_L = 94$ inches; the remaining 96 are short magnets with effective length $L_S = 79$ inches. This gives

$$\sum L = 144\, L_L + 96\, L_S = 21120 \text{ inches} \tag{92}$$

and

$$k = 3361.35239810 \text{ inches.} \tag{93}$$

The bend angle in the long magnets is then

$$\theta_L = L_L/k = 27.9649346 \text{ mrad} \tag{94}$$

and in the short magnets it is

$$\theta_S = L_S/k = 23.502445 \text{ mrad.} \tag{95}$$

Note that for the numerical results given here and throughout these notes more digits are retained than would be warranted by the approximations we our using. This is done for computational convenience.

Since we have required that the design orbit enter and leave the region of nonzero field at the same angle with respect to the socket line, both angles must equal half the bend angle θ. Thus we have

$$y_0' = \tan\{\theta/2\} = \tan\{L/(2k)\} \tag{96}$$

and (89) becomes

$$y_0 = \frac{\lambda^2}{\rho}\left\{\frac{\rho\tan\{\theta/2\}}{\lambda\tan\{L/(2\lambda)\}} - 1\right\} = \frac{\lambda^2}{\rho}\left\{\frac{\rho\tan\{L/(2k)\}}{\lambda\tan\{L/(2\lambda)\}} - 1\right\}. \tag{97}$$

Using (96) in (87) we also have sagitta

$$\Delta = \lambda \left(\frac{1 - \cos\{L/(2\lambda)\}}{\sin\{L/(2\lambda)\}} \right) \tan\{L/(2k)\} \tag{98}$$

which is independent of ρ.

The values of y_0' for the long and short magnets are

$$\tan\{\theta_L/2\} = 13.983379 \times 10^{-3}, \quad \tan\{\theta_S/2\} = 11.751763 \times 10^{-3}. \tag{99}$$

13 Radius of Curvature ρ

To obtain y_0 from the equation

$$y_0 = \frac{\lambda^2}{\rho} \left\{ \frac{\rho \tan\{L/(2k)\}}{\lambda \tan\{L/(2\lambda)\}} - 1 \right\} \tag{100}$$

we must specify the radius of curvature ρ. Recall that in each magnet ρ is defined to be

$$\rho = \frac{cp}{eB_0} \tag{101}$$

where p is the momentum and $B_0 = -B(x, 0)$. Specifying ρ therefore amounts to specifying the field $B(x, 0)$ on the midplane along the socket line. Since y reaches its maximum at $x = L/2$ and since the sagitta

$$\Delta = y(L/2) - y(0) = \lambda \left(\frac{1 - \cos\{L/(2\lambda)\}}{\sin\{L/(2\lambda)\}} \right) \tan\{L/(2k)\} \tag{102}$$

is independent of ρ, we can minimize the deviation of the design orbit from the socket line by choosing ρ such that

$$y_0 = -\Delta/2. \tag{103}$$

This gives

$$\frac{\lambda^2}{\rho} \left\{ \frac{\rho \tan\{L/(2k)\}}{\lambda \tan\{L/(2\lambda)\}} - 1 \right\} = -\frac{\lambda}{2} \left(\frac{1 - \cos\{L/(2\lambda)\}}{\sin\{L/(2\lambda)\}} \right) \tan\{L/(2k)\} \tag{104}$$

and

$$\rho \tan\{L/(2k)\} = \lambda \left(\frac{2 \sin\{L/(2\lambda)\}}{1 + \cos\{L/(2\lambda)\}} \right). \tag{105}$$

Using the expansions

$$\tan\frac{L}{2k} = \frac{L}{2k} + \frac{1}{3}\left(\frac{L}{2k}\right)^3 + \cdots \tag{106}$$

$$\cos\frac{L}{2\lambda} = 1 - \frac{1}{2}\left(\frac{L}{2\lambda}\right)^2 + \frac{1}{24}\left(\frac{L}{2\lambda}\right)^4 - \cdots \tag{107}$$

$$\sin\frac{L}{2\lambda} = \frac{L}{2\lambda} - \frac{1}{6}\left(\frac{L}{2\lambda}\right)^3 + \frac{1}{120}\left(\frac{L}{2\lambda}\right)^5 + \cdots \tag{108}$$

we then have

$$\frac{\rho L}{2k}\left\{1 + \frac{1}{3}\left(\frac{L}{2k}\right)^2 + \cdots\right\} = \frac{L}{2}\left\{1 + \frac{1}{12}\left(\frac{L}{2\lambda}\right)^2 + \frac{1}{120}\left(\frac{L}{2\lambda}\right)^4 + \cdots\right\} \tag{109}$$

and

$$\rho = k\left\{1 + \frac{1}{12}\left(\frac{L}{2\lambda}\right)^2 + \frac{1}{120}\left(\frac{L}{2\lambda}\right)^4 - \frac{1}{3}\left(\frac{L}{2k}\right)^2 + \cdots\right\}. \tag{110}$$

Here we see that ρ is equal to k plus some correction terms which are small. Following Smith and Bleser we simply choose ρ to be equal to k. Thus

$$\rho = k = 3361.35239810 \text{ inches} = 85.3783509117 \text{ meters} \tag{111}$$

and equation (100) becomes

$$y_0 = \frac{\lambda^2}{\rho}\left\{\frac{\rho\tan\{L/(2\rho)\}}{\lambda\tan\{L/(2\lambda)\}} - 1\right\}. \tag{112}$$

We also have

$$y_0' = \tan\{\theta/2\} = \tan\{L/(2k)\} = \tan\{L/(2\rho)\}. \tag{113}$$

Note that in AGS magnets we have [6]

$$|n| \approx 360 \tag{114}$$

which gives

$$\lambda = \frac{\rho}{\sqrt{|n|}} \approx 177 \text{ inches}, \quad \left(\frac{L_L}{2\lambda}\right)^2 \approx 0.0705 \quad \left(\frac{L_S}{2\lambda}\right)^2 \approx 0.0498. \tag{115}$$

This justifies our use of expansions (107) and (108).

In the next section we show that the choice $\rho = k$ gives to lowest order

$$y_0 = -2\Delta/3 \tag{116}$$

rather than the optimum condition $y_0 = -\Delta/2$ which results from the more complicated choice (105). The practical effects of this on the available beam aperture are negligible.

14 Further Approximations

Using the expansions

$$\tan \frac{L}{2\rho} = \frac{L}{2\rho} + \frac{1}{3}\left(\frac{L}{2\rho}\right)^3 + \cdots \tag{117}$$

$$\tan \frac{L}{2\lambda} = \frac{L}{2\lambda} + \frac{1}{3}\left(\frac{L}{2\lambda}\right)^3 + \frac{2}{15}\left(\frac{L}{2\lambda}\right)^5 + \cdots \tag{118}$$

we find

$$y_0' = \frac{L}{2\rho} + \frac{1}{3}\left(\frac{L}{2\rho}\right)^3 + \cdots \tag{119}$$

$$\rho \tan \frac{L}{2\rho} = \frac{L}{2}\left\{1 + \frac{1}{3}\left(\frac{L}{2\rho}\right)^2 + \cdots\right\} \tag{120}$$

$$\lambda \tan \frac{L}{2\lambda} = \frac{L}{2}\left\{1 + \frac{1}{3}\left(\frac{L}{2\lambda}\right)^2 + \frac{2}{15}\left(\frac{L}{2\lambda}\right)^4 + \cdots\right\} \tag{121}$$

$$\frac{\rho \tan\{L/(2\rho)\}}{\lambda \tan\{L/(2\lambda)\}} = 1 + \frac{1}{3}\left(\frac{L}{2\rho}\right)^2 - \frac{1}{3}\left(\frac{L}{2\lambda}\right)^2 - \frac{1}{45}\left(\frac{L}{2\lambda}\right)^4 + \cdots \tag{122}$$

and

$$y_0 = -\frac{L^2}{12\rho}\left\{1 - \frac{\lambda^2}{\rho^2} + \frac{1}{15}\left(\frac{L}{2\lambda}\right)^2 + \cdots\right\}. \tag{123}$$

Thus to lowest order

$$y_0 = -\frac{L^2}{12\rho} \tag{124}$$

which is independent of λ. For the long and short magnets we have

$$\frac{L_L^2}{12\rho} = 0.219058654, \quad \frac{L_S^2}{12\rho} = 0.154724430 \text{ inches.} \tag{125}$$

Note also that

$$\frac{\lambda^2}{\rho^2} \approx \left(\frac{177}{3361}\right)^2 = 0.00277 \tag{126}$$

and

$$\frac{1}{15}\left(\frac{L_L}{2\lambda}\right)^2 \approx 0.00470, \quad \frac{1}{15}\left(\frac{L_S}{2\lambda}\right)^2 \approx 0.00332. \tag{127}$$

Now using the expansions

$$\cos\frac{L}{2\lambda} = 1 - \frac{1}{2}\left(\frac{L}{2\lambda}\right)^2 + \frac{1}{24}\left(\frac{L}{2\lambda}\right)^4 - \cdots \tag{128}$$

$$\sin\frac{L}{2\lambda} = \frac{L}{2\lambda} - \frac{1}{6}\left(\frac{L}{2\lambda}\right)^3 + \frac{1}{120}\left(\frac{L}{2\lambda}\right)^5 + \cdots \tag{129}$$

we have

$$\frac{1-\cos\{L/(2\lambda)\}}{\sin\{L/(2\lambda)\}} = \frac{1}{2}\left(\frac{L}{2\lambda}\right)\left\{1 + \frac{1}{12}\left(\frac{L}{2\lambda}\right)^2 + \cdots\right\} \tag{130}$$

and the sagitta

$$\Delta = \lambda\left(\frac{1-\cos\{L/(2\lambda)\}}{\sin\{L/(2\lambda)\}}\right)\tan\{L/(2\rho)\} \tag{131}$$

becomes

$$\Delta = \frac{L}{4}\left\{1 + \frac{1}{12}\left(\frac{L}{2\lambda}\right)^2 + \cdots\right\}\left\{\frac{L}{2\rho} + \frac{1}{3}\left(\frac{L}{2\rho}\right)^3 + \cdots\right\} \tag{132}$$

$$\Delta = \frac{L^2}{8\rho}\left\{1 + \frac{1}{12}\left(\frac{L}{2\lambda}\right)^2 + \cdots\right\}\left\{1 + \frac{1}{3}\left(\frac{L}{2\rho}\right)^2 + \cdots\right\} \tag{133}$$

$$\Delta = \frac{L^2}{8\rho}\left\{1 + \frac{1}{12}\left(\frac{L}{2\lambda}\right)^2 + \frac{1}{3}\left(\frac{L}{2\rho}\right)^2 + \cdots\right\}. \tag{134}$$

Thus to lowest order

$$\Delta = \frac{L^2}{8\rho} \tag{135}$$

which is independent of λ. For the long and short magnets we have

$$\frac{L_L^2}{8\rho} = 0.328587982, \quad \frac{L_S^2}{8\rho} = 0.232086645 \text{ inches.} \tag{136}$$

Comparing (135) and (124) we see also that to lowest order $y_0 = -2\Delta/3$.

15 Equations for Defocussing Magnets

The equations for defocussing magnets can be obtained by substituting $i\lambda$ for λ in the equations for focussing magnets. Thus for defocussing magnets we have

$$y'' = -\frac{1}{\rho}\left(1 - \frac{\rho}{\lambda^2}\, y\right), \quad y'' - \frac{1}{\lambda^2}\, y = -\frac{1}{\rho} \tag{137}$$

and using the identities

$$\cos ix = \cosh x, \quad \sin ix = i \sinh x, \quad \tan ix = i \tanh x \tag{138}$$

one finds

$$y = \frac{\lambda^2}{\rho} - \frac{\lambda^2}{\rho}\left\{1 - \frac{\rho}{\lambda^2}\, y_0\right\} \cosh\frac{x}{\lambda} + \lambda y_0' \sinh\frac{x}{\lambda} \tag{139}$$

$$y' = -\frac{\lambda}{\rho}\left\{1 - \frac{\rho}{\lambda^2}\, y_0\right\} \sinh\frac{x}{\lambda} + y_0' \cosh\frac{x}{\lambda}. \tag{140}$$

The condition $y'(L) = -y_0'$ gives

$$y_0' = \frac{\lambda}{\rho}\left\{1 - \frac{\rho}{\lambda^2}\, y_0\right\} \tanh\frac{L}{2\lambda} \tag{141}$$

and

$$\Delta = \frac{\lambda^2}{\rho}\left\{1 - \frac{\rho}{\lambda^2}\, y_0\right\} \frac{\cosh\{L/(2\lambda)\} - 1}{\cosh\{L/(2\lambda)\}}. \tag{142}$$

The requirement

$$y_0' = \tan\frac{L}{2\rho} \tag{143}$$

then gives

$$\tan\frac{L}{2\rho} = \frac{\lambda}{\rho}\left\{1 - \frac{\rho}{\lambda^2}\, y_0\right\} \tanh\frac{L}{2\lambda} \tag{144}$$

and

$$y_0 = \frac{\lambda^2}{\rho}\left\{1 - \frac{\rho \tan\{L/(2\rho)\}}{\lambda \tanh\{L/(2\lambda)\}}\right\}. \tag{145}$$

The corresponding expansions are

$$y_0' = \frac{L}{2\rho} + \frac{1}{3}\left(\frac{L}{2\rho}\right)^3 + \cdots \tag{146}$$

$$y_0 = -\frac{L^2}{12\rho}\left\{1 + \frac{\lambda^2}{\rho^2} - \frac{1}{15}\left(\frac{L}{2\lambda}\right)^2 + \cdots\right\} \tag{147}$$

and

$$\Delta = \frac{L^2}{8\rho}\left\{1 - \frac{1}{12}\left(\frac{L}{2\lambda}\right)^2 + \frac{1}{3}\left(\frac{L}{2\rho}\right)^2 + \cdots\right\}. \tag{148}$$

Note that to lowest order the expressions for y_0 and Δ are the same as those for focussing magnets.

16 Orbit Length in the Magnets

The length of the design orbit in an AGS magnet is

$$S = \int_0^L \left\{1 + y'^2\right\}^{1/2} dx = L + \frac{1}{2}\int_0^L y'^2 dx + \cdots \tag{149}$$

where (for a focussing magnet)

$$y' = -\frac{\lambda}{\rho}\left\{1 + \frac{\rho}{\lambda^2}\, y_0\right\} \sin\frac{x}{\lambda} + y_0' \cos\frac{x}{\lambda}. \tag{150}$$

Using the expansions

$$\cos\frac{x}{\lambda} = 1 - \frac{1}{2}\frac{x^2}{\lambda^2} + \cdots \tag{151}$$

$$\sin\frac{x}{\lambda} = \frac{x}{\lambda} - \frac{1}{6}\frac{x^3}{\lambda^3} + \cdots \tag{152}$$

we have

$$y' = y_0' - \frac{x}{\rho} + \cdots \tag{153}$$

and

$$y'^2 = y_0'^2 - 2y_0'\frac{x}{\rho} + \frac{x^2}{\rho^2} + \cdots \tag{154}$$

which gives

$$S = L + \frac{L}{2}\left\{y_0'^2 - y_0'\frac{L}{\rho} + \frac{1}{3}\frac{L^2}{\rho^2}\right\} + \cdots \tag{155}$$

Using the expansion

$$y_0' = \frac{L}{2\rho} + \frac{1}{3}\left(\frac{L}{2\rho}\right)^3 + \cdots \tag{156}$$

we then have

$$S = L + \frac{L}{2}\left\{\frac{L^2}{4\rho^2} - \frac{L^2}{2\rho^2} + \frac{L^2}{3\rho^2}\right\} + \cdots \tag{157}$$

$$S = L\left\{1 + \frac{1}{24}\frac{L^2}{\rho^2} + \cdots\right\}. \quad (158)$$

Substituting $L_L = 94$ and $L_S = 79$ for L in this equation we obtain the design orbit lengths

$$S_L = 94.00306298047, \quad S_S = 79.00181820120 \text{ inches} \quad (159)$$

in the long and short magnets. Note that to the retained order the lengths are independent of λ.

17 The BEAM Code Axis in Straight Sections

Consider two magnets and the straight section between them as illustrated in **Figures 3, 4, 5** and **6**. In these figures, A is the point where the socket line of the upstream magnet intersects the downstream face of the magnet. Similarly B is the point where the socket line of the downstream magnet intersects the upstream face of that magnet. The line connecting A and B is the BEAM code axis in the straight section.

18 Orbit Length between Long Magnets

Consider two long magnets and the straight section between them as illustrated in **Figure 3**. At the upstream end of the straight we define two coordinate systems, one with straight section coordinates (x_s, y_s) and the other with magnet coordinates (x_m, y_m). The origin of the two coordinate systems is taken to be the point A defined above. The relation between the two sets of coordinates is

$$x_s = Cx_m - Sy_m \quad (160)$$

$$y_s = Sx_m + Cy_m \quad (161)$$

where

$$S = \sin(\theta/2), \quad C = \cos(\theta/2), \quad \theta = L_L/\rho. \quad (162)$$

The point P in the figure is the point where the design orbit leaves the region of nonzero magnetic field in the upstream magnet. Its coordinates in the upstream coordinate systems are

$$x_m = 2 \text{ inches}, \quad y_m = y_0 = -\frac{L_L^2}{12\rho} \quad (163)$$

$$x_s = 2C - y_0 S, \quad y_s = 2S + y_0 C. \tag{164}$$

Similarly, we define two coordinate systems at the downstream end of the straight section with the origin of both taken to be point B. The relation between the two sets of coordinates here is

$$x_s = Cx_m + Sy_m \tag{165}$$

$$y_s = -Sx_m + Cy_m. \tag{166}$$

The point Q in the figure is the point where the design orbit enters the region of nonzero field in the downstream magnet. Its coordinates in the downstream coordinate systems are

$$x_m = -2 \text{ inches}, \quad y_m = y_0 = -\frac{L_L^2}{12\rho} \tag{167}$$

$$x_s = -2C + y_0 S, \quad y_s = 2S + y_0 C. \tag{168}$$

The design orbit in the straight section is the line connecting points P and Q. This is the "OCO" (Optimum Central Orbit) defined by Bleser [2]. Referring to the figure we see that the length of the design orbit in the straight section is

$$T = D - 2|x_s| = D - 2(2C - y_0 S) \tag{169}$$

where

$$C = \cos\left\{L_L/(2\rho)\right\}, \quad S = \sin\left\{L_L/(2\rho)\right\}, \quad y_0 = -\frac{L_L^2}{12\rho}. \tag{170}$$

Putting in numbers for L_L and ρ we obtain

$$T = D - 4.005734749 \text{ inches}. \tag{171}$$

Referring to the figure we also have

$$-b = y_s = 2S + y_0 C = -0.1910732174 \text{ inches}. \tag{172}$$

This is the displacement of the OCO with respect to the Beam code axis in the straight between two long magnets.

The AGS straight sections which occur between two long magnets are the five-foot straights 3, 5, 7, 13, 15 and 17, and the two-foot straights 4, 6, 14 and 16. For the five-foot straights we have

$$D = D_5 = 64 \text{ inches} \tag{173}$$

and for the two-foot straights

$$D = D_2 = 28 \text{ inches.} \tag{174}$$

The orbit length in five-foot straights 3, 5, 7, 13, 15 and 17 is then

$$T_5 = D_5 - 4.005734749 = 59.994265251 \text{ inches.} \tag{175}$$

The orbit length in two-foot straights 4, 6, 14 and 16 is

$$T_2^L = D_2 - 4.005734749 = 23.994265251 \text{ inches.} \tag{176}$$

19 Orbit Length between Short Magnets

Here the length of the orbit in the straight section is

$$T = D - 2|x_s| = D - 2(2C - y_0 S) \tag{177}$$

where

$$C = \cos\{L_S/(2\rho)\}, \quad S = \sin\{L_S/(2\rho)\}, \quad y_0 = -\frac{L_S^2}{12\rho}. \tag{178}$$

Thus

$$T = D - 4.003360139 \text{ inches.} \tag{179}$$

We also have

$$-b = y_s = 2S + y_0 C = -0.131211843 \text{ inches.} \tag{180}$$

This is the displacement of the OCO with respect to the Beam code axis in the straight between two short magnets.

The straight sections which occur between two short magnets are the ten-foot straights 10 and 20, and the two-foot straights 1, 9, 11 and 19. For the ten-foot straights we have

$$D = D_{10} = 123.9907 \text{ inches} \tag{181}$$

and for the two-foot straights

$$D = D_2 = 28 \text{ inches.} \tag{182}$$

The orbit length in ten-foot straights 10 and 20 is then

$$T_{10} = D_{10} - 4.003360139 = 119.987339861 \text{ inches.} \tag{183}$$

The orbit length in two-foot straights 1, 9, 11 and 19 is

$$T_2^S = D_2 - 4.003360139 = 23.996639861 \text{ inches.} \tag{184}$$

20 Orbit Length between Long and Short Magnets

Consider now the case in which a long magnet is followed by a short magnet. **Figures 4** and **5** illustrate the geometry of the magnets and the straight section between them. Figure 5 is an exaggerated view of Figure 4. The relation between the two sets of coordinates at the upstream end of the straight section is

$$X_s = CX_m - SY_m \tag{185}$$

$$Y_s = SX_m + CY_m \tag{186}$$

where

$$S = \sin(\alpha/2), \quad C = \cos(\alpha/2), \quad \alpha = L_L/\rho. \tag{187}$$

The relation between the two sets of coordinates at the downstream end of the straight is

$$x_s = cx_m + sy_m \tag{188}$$

$$y_s = -sx_m + cy_m \tag{189}$$

where

$$s = \sin(\beta/2), \quad c = \cos(\beta/2), \quad \beta = L_S/\rho \tag{190}$$

(Note that we have used upper-case and lower-case letters respectively for the coordinate systems at the upstream and downstream ends of the straight.) The coordinates of the point P are

$$X_m = 2 \text{ inches}, \quad Y_m = Y_0 = -\frac{L_L^2}{12\rho} \tag{191}$$

$$X_s = 2C - Y_0 S, \quad Y_s = 2S + Y_0 C. \tag{192}$$

Those of point Q are

$$x_m = -2 \text{ inches}, \quad y_m = y_0 = -\frac{L_S^2}{12\rho} \tag{193}$$

$$x_s = -2c + y_0 s, \quad y_s = 2s + y_0 c. \tag{194}$$

The length of the orbit in the straight section is then

$$T = D - |X_s| - |x_s| = D - (2C - Y_0 S) - (2c - y_0 s) \tag{195}$$

where

$$C = \cos\left\{L_L/(2\rho)\right\}, \quad S = \cos\left\{L_L/(2\rho)\right\}, \quad Y_0 = -\frac{L_L^2}{12\rho} \tag{196}$$

$$c = \cos\left\{L_S/(2\rho)\right\}, \quad s = \cos\left\{L_S/(2\rho)\right\}, \quad y_0 = -\frac{L_S^2}{12\rho}. \tag{197}$$

Thus

$$2C - Y_0 S = 2.002867374 \text{ inches} \tag{198}$$

$$2c - y_0 s = 2.001680070 \text{ inches} \tag{199}$$

and

$$T = D - 4.004547444 \text{ inches.} \tag{200}$$

We also have

$$d + e = |Y_s|, \quad e = |y_s| \tag{201}$$

and therefore

$$d = |Y_s| - |y_s| = |2S + Y_0 C| - |2s + y_0 c| \tag{202}$$

$$d = 0.1910732174 - 0.131211843 = 0.0598613744 \text{ inches.} \tag{203}$$

This is inward displacement of the short magnet with respect to the long magnet as illustrated in the figures.

The case in which a short magnet is followed by a long magnet is illustrated in **Figure 6**. Here the relation between the two sets of coordinates at the upstream end of the straight section is

$$x_s = c x_m - s y_m \tag{204}$$

$$y_s = s x_m + c y_m \tag{205}$$

where

$$s = \sin(\beta/2), \quad c = \cos(\beta/2), \quad \beta = L_S/\rho. \tag{206}$$

The relation between the two sets of coordinates at the downstream end of the straight is

$$X_s = C X_m + S Y_m \tag{207}$$

$$Y_s = -S X_m + C Y_m \tag{208}$$

where

$$S = \sin(\alpha/2), \quad C = \cos(\alpha/2), \quad \alpha = L_L/\rho. \tag{209}$$

The coordinates of the point P are

$$x_m = 2 \text{ inches}, \quad y_m = y_0 = -\frac{L_S^2}{12\rho} \tag{210}$$

$$x_s = 2c - y_0 s, \quad y_s = 2s + y_0 c. \tag{211}$$

Those of point Q are

$$X_m = -2 \text{ inches}, \quad Y_m = Y_0 = -\frac{L_L^2}{12\rho} \tag{212}$$

$$X_s = -2C + Y_0 S, \quad Y_s = 2S + Y_0 C. \tag{213}$$

The length of the orbit in the straight section is then

$$T = D - |X_s| - |x_s| = D - (2C - Y_0 S) - (2c - y_0 s) \tag{214}$$

which is the same as that for the case in which a long magnet is followed by a short magnet. We also have

$$d + e = |Y_s|, \quad e = |y_s| \tag{215}$$

and therefore

$$d = |Y_s| - |y_s| = |2S + Y_0 C| - |2s + y_0 c| \tag{216}$$

which again is the same as the result obtained for a long magnet followed by a short magnet.

The AGS straight sections which occur between a long and short magnet are the two-foot straights 2, 8, 12 and 18. For these straights we have

$$D = D_2 = 28 \text{ inches} \tag{217}$$

and the orbit length is

$$T_2^{LS} = D_2 - 4.004547444 = 23.995452556 \text{ inches.} \tag{218}$$

21 Orbit Circumference and Radius

Adding together the orbit lengths in all of the magnets and straight sections we obtain the orbit circumference

$$C = 144 S_L + 96 S_S + 72 T_5 + 48 \left\{ T_2^L + T_2^{LS} + T_2^S \right\} + 24 T_{10} \tag{219}$$

where (in inches)

$$S_L = 94.00306298047, \quad S_S = 79.00181820120 \tag{220}$$

$$T_5 = D_5 - 4.005734749 = 59.994265251 \quad (3, 5, 7, 13, 15, 17) \tag{221}$$

$$T_2^L = D_2 - 4.005734749 = 23.994265251 \quad (4, 6, 14, 16) \tag{222}$$

$$T_2^{LS} = D_2 - 4.004547444 = 23.995452556 \quad (2, 8, 12, 18) \tag{223}$$

$$T_2^S = D_2 - 4.003360139 = 23.996639861 \quad (1, 9, 11, 19) \tag{224}$$

$$T_{10} = D_{10} - 4.003360139 = 119.987339861 \quad (10, 20) \tag{225}$$

and

$$D_2 = 28, \quad D_5 = 64, \quad D_{10} = 123.9907. \tag{226}$$

The straight section numbers for the various lengths are listed in parenthesis.

Putting numbers into the expression for C we obtain

$$C = 31775.2440393, \quad R = C/(2\pi) = 5057.18715679 \text{ (inches)} \tag{227}$$

and

$$R = 128.452554 \text{ meters.} \tag{228}$$

These numbers are in agreement with those obtained by Bleser [1].

22 Beam Code Axis between Long and Short Magnets

The dotted green line in **Figures 5** and **6** is the BEAM code axis in the straight section between a long and short magnet. In both figures the angle η is given by

$$\tan\eta = d/D \tag{229}$$

where

$$d = 0.0598613744, \quad D = D_2 = 28 \text{ inches.} \tag{230}$$

Thus

$$\eta = 2.137903 \text{ mrad.} \tag{231}$$

Figure 5 illustrates the case of a long magnet at the upstream end of the straight followed by a short magnet at the downstream end. Here the angle of the BEAM code axis with respect to the x-axis of the long magnet is

$$\alpha/2 + \eta = \theta_L/2 + \eta = 16.120370 \text{ mrad.} \tag{232}$$

The angle of the x-axis of the short magnet with respect to the BEAM code axis is

$$\beta/2 - \eta = \theta_S/2 - \eta = 9.613320 \text{ mrad.} \tag{233}$$

Figure 6 illustrates the case of a short magnet at the upstream end of the straight followed a long magnet at the downstream end. The angle of the BEAM code axis with respect to the x-axis of the short magnet is then

$$\beta/2 - \eta = \theta_S/2 - \eta = 9.613320 \text{ mrad} \tag{234}$$

and the angle of the x-axis of the long magnet with respect to the BEAM code axis is

$$\alpha/2 + \eta = \theta_L/2 + \eta = 16.120370 \text{ mrad.} \tag{235}$$

References

[1] E. J. Bleser, "Where are the AGS Magnets", Accelerator Division Technical Note 215, May 20, 1985.

[2] E. J. Bleser, "The Optimum Central Orbit in the AGS", Accelerator Division Technical Note 217, July 22, 1985.

[3] Lloyd Smith, "Location of Magnets", Accelerator Development Department Internal Report LS-2, May 14, 1956.

[4] E.D. Courant, "Computation of AGS Orbits with 704 Computer", Accelerator Development Department Internal Report EDC-36, January 7, 1960.

[5] G.H. Morgan, "Fortran IV Revision of BEAM, the AGS Orbit Computing Program", Accelerator Department AGS Internal Report, June 1, 1966.

[6] E.D. Courant and H.S. Snyder, "Theory of the Alternating-Gradient Synchrotron", Annals of Physics 3, 1–48 (1958)

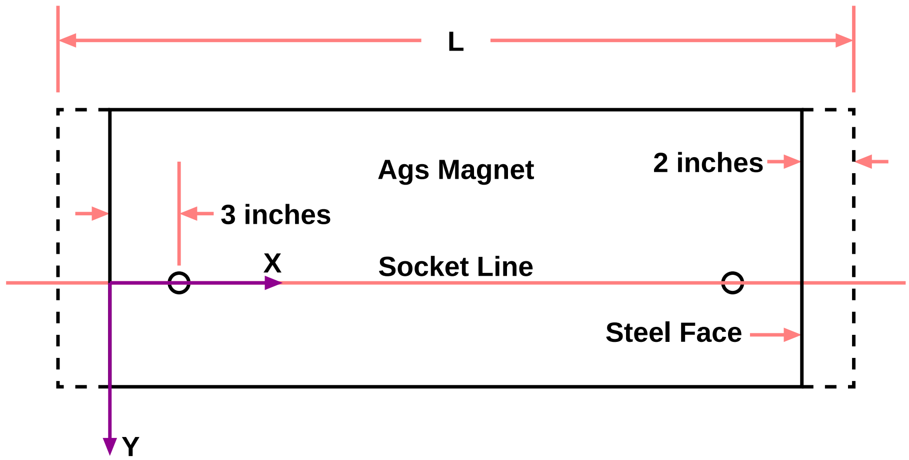

Figure 1: AGS combined function magnet. In the hard-edge approximation, the field extends 2 inches beyond the steel face at each end of the magnet. L is the effective length. This is 94 inches for “long” magnets and 79 inches for “short” magnets.

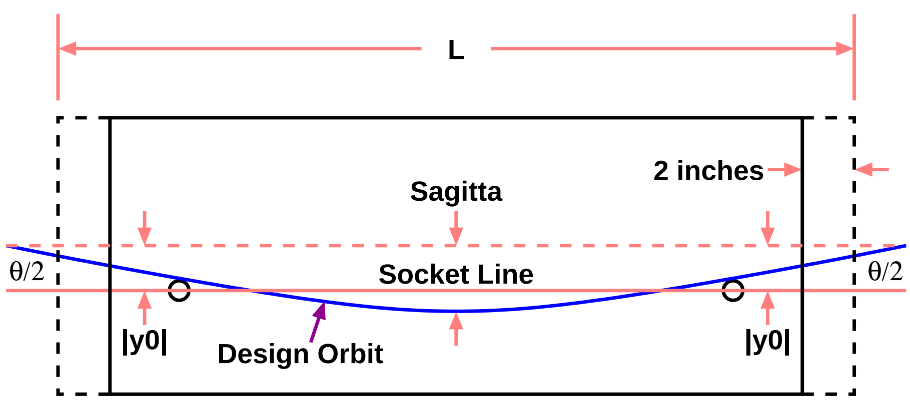

Figure 2: Design orbit in AGS combined function magnet.

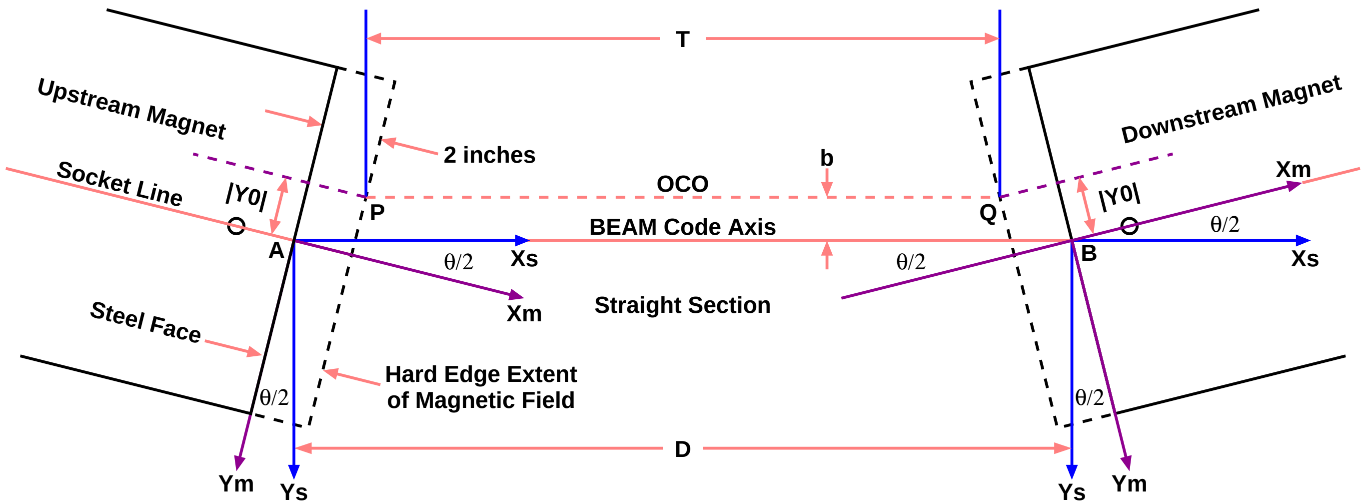

Figure 3: Straight Section between Two Long (or two short) Magnets

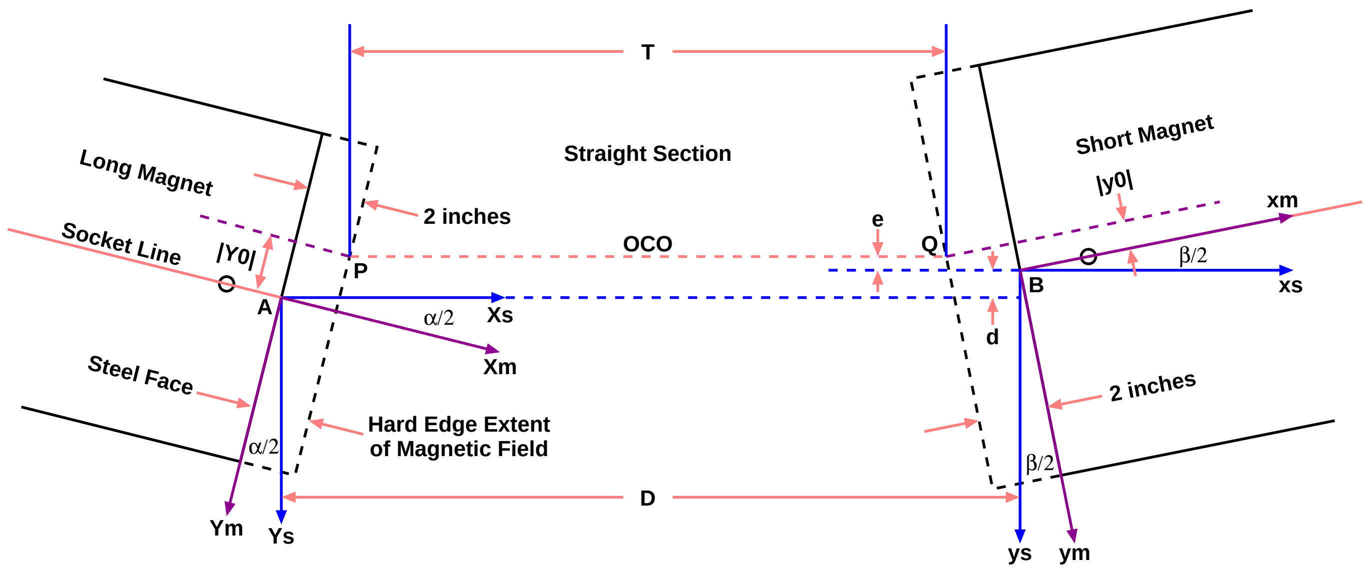

Figure 4: Straight Section between Long and Short Magnets

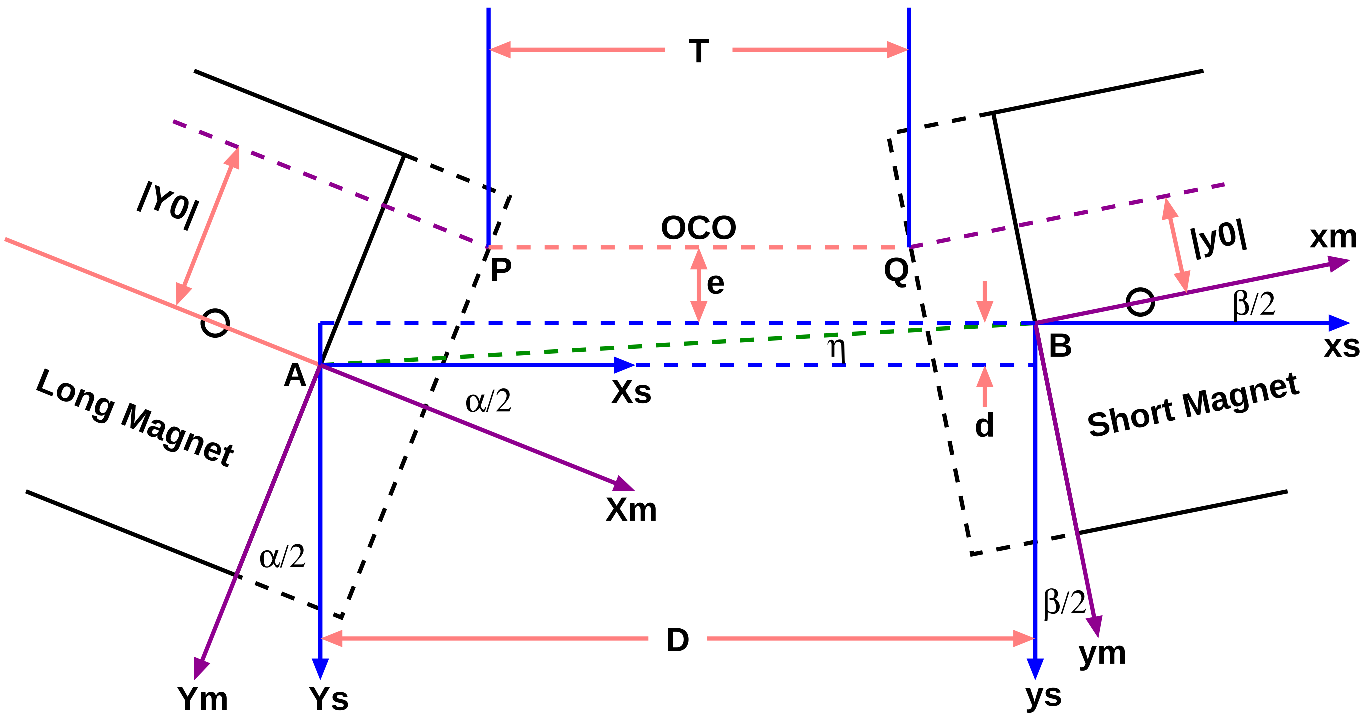

Figure 5: Straight Section between Long and Short Magnets

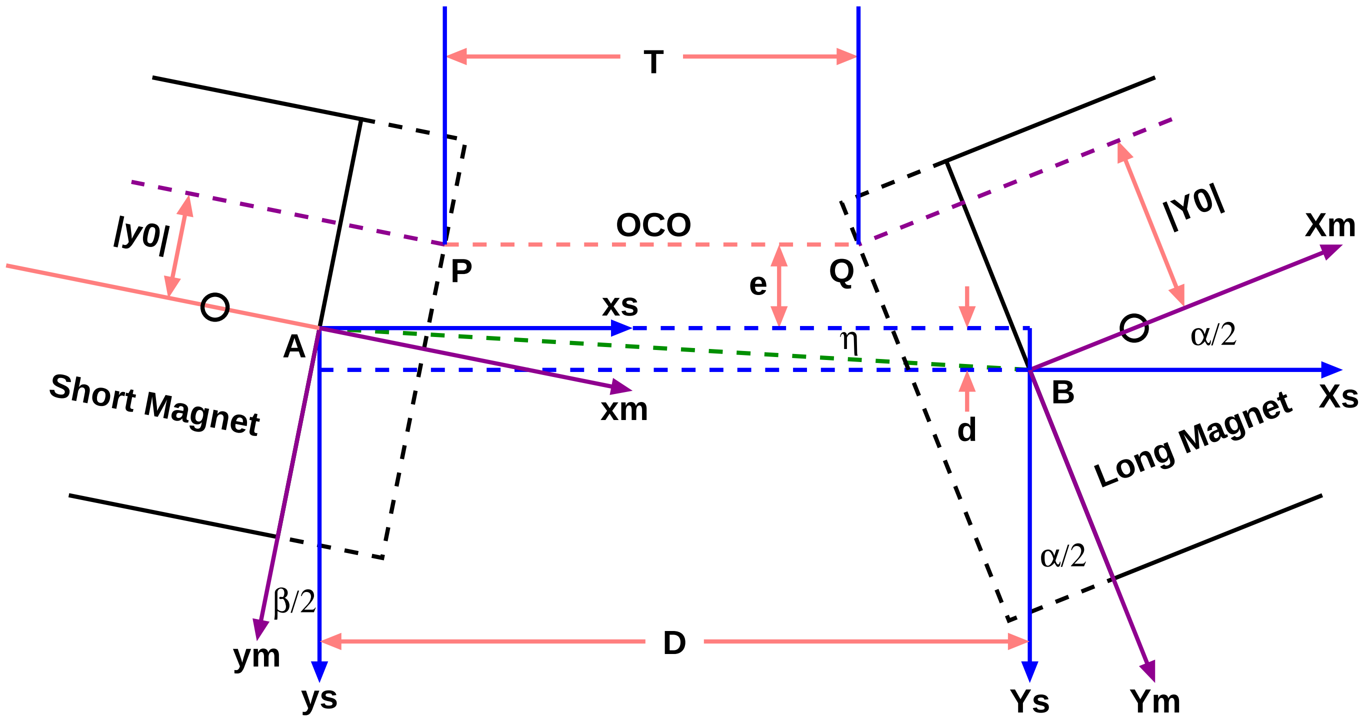

Figure 6: Straight Section between Short and Long Magnets