ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60440

NUCLEAR SPECTROSCOPY WITH DIRECT REACTIONS
II. PROCEEDINGS

Edited by
F. E. Throw

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Organizing Committee.
J. B. French
D. Kurath
L. L. Lee, Jr.
M. H. Macfarlane
J. P. Schiffer
R. Sherr
F. E. Throw

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FOREWORD

The Symposium on Nuclear Spectroscopy with Direct Reactions, sponsored and organized by Argonne National Laboratory under the auspices of the U. S. Atomic Energy Commission, was held on 9—11 March 1964 at the Center for Continuing Education, University of Chicago. In order to leave ample time for discussion, the contributed papers were collected in advance and issued to registrants in the form of an Argonne report: Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, ANL-6848. The present volume contains the invited papers along with abstracts or summaries of the few short papers selected for their special relevance to the topics of the invited lecturers. Edited versions of the discussions are also included. Since this Symposium was accepted as a topical conference of the American Physical Society, the abstracts of the contributed papers will be printed in the Bull. Am. Phys. Soc. 9, 674-680 (October 1964).

The names and addresses of the 338 registrants are listed at the end of this volume. Of these, 268 were from outside the Chicago area and 29 were from outside the United States.

In preparing this volume of Proceedings for publication, the edited manuscripts and edited discussions were returned to the speakers and discussers for their comments and corrections. In many cases, the discussers quite thoroughly revised their statements in accordance with the announced desire that the Proceedings should "say what [each author] wished he had said at the meeting." The editor has made a determined effort to follow the authors' instructions exactly—but a few minor changes had to be made to preserve the connections between independently revised statements, and in this very large volume of material some mistakes and oversights are inevitable. Since it was not feasible to send final page proof to the authors, the responsibility for

such errors must be accepted by the editor. To any who have been misquoted or may be misled thereby, he offers his sincere apologies.

The scientific secretaries and editor are most appreciative of the courtesy, patience, and cooperation of the speakers who contributed manuscripts and of the participants in the discussions. An astonishing amount of correspondence was involved, especially in the editing of the discussions. The editor is especially grateful for the promptness and good nature of those who were pestered during or just before vacations or trips abroad.

Finally, it is a pleasure to acknowledge the contributions of Mrs. Dorothy Carlson, Mr. Leon Markheim, and Mrs. Judy Miller of Argonne and of Mr. John F. Dezauche of the Center for Continuing Education for arranging the local transportation, housing, meals, and facilities for the meeting. Transcripts of the discussions were supplied by Mr. W. G. Terry and Mr. George Kozuch of Leon M. Golding and Associates, court reporters. The experience and skills of the Argonne Graphic Arts Department, particularly of Mr. Frank M. Gentile, Mr. James W. Henning, and Mr. Joseph G. Fierce, have been invaluable at several stages of preparing this volume. And most special thanks are due to Mrs. Kay M. Pemble for efficiently managing the editorial office during the Symposium, for handling the voluminous editorial correspondence, and for her patience and precision in typing the multilith plates.

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SESSION A

MONDAY MORNING

9 MARCH 1964

CHAIRMAN: D. R. INGLIS

SCIENTIFIC SECRETARIES

R. E. SEGEL

A. BODMER

DIRECT REACTIONS AND NUCLEAR STRUCTURE

N. Austern

University of Pittsburgh, Pittsburgh, Pennsylvania

If a system of many interacting particles is confined in a box, and if at some time a quantity of energy is added to this system, then we may be sure that the system will evolve in a statistical fashion and that in due time the energy will be shared statistically among all the various degrees of freedom of the system of particles. In general, systems reach statistical equilibrium if they are confined.

Nuclei are not confined systems. Of course nuclear bound states are localized in space, and some conclusions may be based upon this fact. However, the subject of nuclear reactions concerns continuum states, and these are neither bound nor localized. For these reasons, although it often is interesting to employ statistical ideas in the subject of nuclear reactions, the use of such ideas must require special justification for each application. Strong nonstatistical effects are to be expected. Under suitable circumstances such effects may become dominant.

In a reaction, energy enters the nuclear system in an extremely nonstatistical fashion. Two particles which have definite internal states are made to collide. The particles approach each other in a definite "collision alternative."¹ I shall prefer to be loose in my terminology and say that they approach in a definite "channel." The pattern of motion in such a channel is very simple, and is in general only one of a vast number of different kinds of motions, at or near the energy in question, in which the system of colliding nucleons may engage.

¹A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).

The wave function which describes the entrance channel is a product function, namely

$$\xi_{aA}(\text{rel.})\phi_{aA}(\text{int.}) . \quad (1)$$

It is the product of the wave function that describes the internal states of the colliding particles times a wave function that describes their relative motion. If other motions of the system are to become excited, so that the energy brought in in the entrance channel shall be distributed among many motions, then the wave function (1) must be made to couple to the other energetically-possible wave functions of the system.

Most of the other possible wave functions are more complicated than (1). They probably involve excitation of one or more particle-hole pairs. Matrix elements that couple Eq. (1) to such other wave functions are nonvanishing, but are not likely to be very large. Nevertheless strong excitation of such wave functions could take place if the system were confined, so that the small overlap integrals would be compensated with long time intervals. Most of the wave functions that would be excited in such "compound nucleus formation" would not be wave functions of open channels.

However, there probably are other open channels besides the entrance channel, aA ; these also are described by simple product wave functions, and sometimes these product wave functions will have very good overlap with Eq. (1). We can say there is a strong "parentage" relationship between two such channels. A close parentage connection between two channels that both have low excitation energies is quite likely. Under the circumstance that matrix elements that couple Eq. (1) to other open channels might be large, whereas matrix elements that couple (1) to closed channels probably are small, we see that it is very likely that the energy that arrives in the entrance channel will depart through one or more open exit channels, long before the complicated compound-nucleus (CN)

motions have been excited. Such simple coupling between entrance and exit channels is spoken of as a "direct reaction" (DI).

Figure 1 is a diagram of the picture I have just been describing. We see, first of all, why DI is not likely at very low energy,

Fig. 1. Diagram showing the distribution among the possible reaction modes.

namely at energies at which penetration coefficients are small. Under such circumstances the amplitudes that exit-channel wave functions have in the region of reaction are very small, and therefore transitions to the already-localized CN intermediate states are more likely than are the DI transitions. So this is one reason for avoiding low energy when studying DI transitions. We see an additional fact which helps to isolate the DI transitions as the bombarding energy is raised, namely the rapid increase in the number of open exit channels. Most of this increase is in channels that can be reached only through complicated CN intermediate states. Since the CN process is expected to populate all exit channels with equal likelihood, it is seen that the CN contribution to any one exit channel must decrease as the energy is increased. In particular, therefore, increasing the energy must decrease the CN contribution to the few exit channels that do have good overlap with the entrance channel, and that are the exit channels of the DI process.

Evidently DI tends to be favored over CN (a) for certain exit channels, (b) at sufficiently high bombarding energy E , (c) at sufficiently high mass number A . The systematic investigation of the competition between these two processes as a function of A and E is only just now getting under way.

Evidently at very high bombarding energies most of the energy leaves the reaction site via DI exit channels. The tendency toward confinement is very slight when we are dealing with large amounts of energy. It is easier to make such a statement with regard to energy than with regard to probability flux. Most of the incident energy leaves the reaction through DI channels by the emission of just one or two particles. However, a part of the energy always tends to remain at the reaction site in the form of CN excitations, and the eventual boil-off of this small part of the energy may lead to the emission of many more particles than came out in the first stage. The two stages may be studied separately.

At all energies an analysis into partial waves tends to separate DI and CN transitions (Fig. 2). Flux that enters a reaction in a high partial

Fig. 2. The influence of angular momentum in the DI-CN competition.

wave will tend to leave in a low partial wave if it makes a transition through the CN. In contrast, if a DI transition is made, then the flux will tend to leave the reaction through a partial wave which is a near neighbor of the one in which it enters. Thus the higher partial waves in the exit channels are more likely to be populated through the DI process than through the CN process. It is interesting that these partial waves also have high statistical weight.

I have discussed the circumstances under which DI transitions are isolated for study. Obviously the study of such transitions yields useful spectroscopic information, as this conference recognizes. We are dealing with simple matrix elements between product wave functions, and these matrix elements can be studied for information about the wave functions. Intermediate states seldom are involved. Parentage relations between the entrance and exit channels play a central role.

Let us look at the reaction amplitude for the transition

$$\frac{d\sigma_{ab}}{d\Omega_b} = \frac{M_{aA} M_{bB}}{(2\pi\hbar)^2} \cdot \left(\frac{k_b}{k_a} \right) \cdot \sum_{Av} |T_{ab}|^2 . \quad (3)$$

For brevity the channel $(a + A)$ is just denoted $\underline{a}$, and the channel $(b + B)$ is denoted $\underline{b}$. However, M_{aA} and M_{bB} are the reduced masses in the two channels. The reaction amplitude is T_{ab} .

Let us study how to calculate T_{ab} by examining a simple version of the wave function for the reaction

$$\Psi = \xi_a \phi_{aA} + \xi_b \phi_{bB} + \Phi_{CN} . \quad (4)$$

I am supposing that channel b is the only one which is strongly coupled to channel a, so that the only DI transition is the one between these two channels (in the distorted-wave Born-DWB-approximation, this assumption will not matter), and that Φ_{CN} corresponds to CN excitations and is all the rest of the wave function. Equation (4) sets up a "unified theory." It should be noted that Eq. (4) is not antisymmetrized. Reliable investigations of the consequences of using antisymmetric wave functions are just now starting to be made.

Normally we assume that Φ_{CN} does not contribute to the channel b, and we only to some extent take into account the influence of Φ_{CN} on ξ_a and ξ_b by treating these functions as subject to some absorption. I think it would be very interesting to treat Φ_{CN} as a single, sharp, localized, quasi-bound state and to use Eq. (4) to study DI-CN interference for that model. That model, in which Eq. (4) would be regarded as a variational wave function, would be a slight generalization of Bethe's 1937 derivation of the Breit-Wigner formula.² This derivation is reviewed in the second edition of the book by Mott and Massey.³ One would merely carry Φ_{CN} in the coupled equations that otherwise are studied.

However, let us follow the usual procedure, and ignore any explicit effects due to Φ_{CN} . Then

$$(H - E) [\xi_a \phi_{aA} + \xi_b \phi_{bB}] = 0 \quad (5)$$

so in schematic notation

$$\begin{cases} \langle \phi_{aA} | (H - E) | \phi_{aA} \rangle \xi_a + \langle \phi_{aA} | H | \phi_{bB} \rangle \xi_b = 0, & (6a) \\ \langle \phi_{bB} | (H - E) | \phi_{bB} \rangle \xi_b + \langle \phi_{bB} | H | \phi_{aA} \rangle \xi_a = 0. & (6b) \end{cases}$$

²H. A. Bethe, Rev. Mod. Phys. 9, 101 (1937).

³N. F. Mott and H. S. W. Massey, Theory of Atomic Collisions (Oxford University Press, New York, 1949), 2nd ed., pp. 158-162.

Equations (6a,b) are the equations of the "coupled-channels method," which currently is receiving much investigation. These simultaneous differential equations may be solved numerically. By the time all necessary conserved quantum numbers have been taken into account, the job of finding the solution is quite difficult. It is to be noted that the exact simultaneous solution of Eqs. (6a,b) treats the interchannel coupling of a,b to infinite order—far more exactly than the coupling to any other channel is treated.

The asymptotic amplitude of ξ_b is the transition amplitude T_{ab} . It is easy enough to solve Eq. (6b) and to display T_{ab} , assuming that ξ_a is known. The result is

$$T_{ab}^{\text{coupled}} = \langle \chi_b^{(-)} \phi_{bB} | H | \xi_a \phi_{aA} \rangle. \quad (7)$$

Here $\chi_b^{(-)}$ is an eigensolution of the first term of Eq. (6b). It is a typical "distorted wave." It is interesting to contrast Eq. (7) with the result

$$T_{ab}^{\text{DWB}} = \langle \chi_b^{(-)} \phi_{bB} | H | \chi_a^{(+)} \phi_{aA} \rangle \quad (8)$$

obtained in the distorted-wave Born (DWB) approximation. Here $\chi_a^{(+)}$ is the wave function for the relative motion of the particles a,A in the entrance channel. It describes the elastic scattering. Of course ξ_a also describes the relative motion in the entrance channel and the elastic scattering. However, the influence of the reaction channel upon the elastic channel is taken into account exactly in the calculation of ξ_a , so that Eq. (7) must be regarded as more accurate than Eq. (8). Having said this, it should then also be noted that the quantitative difference between Eq. (7) and Eq. (8) is much less than one at first supposes. The function $\chi_a^{(+)}$ should not be regarded as merely an eigenfunction of the first term of Eq. (6a), with the coupling disregarded. Instead, in all practical calculations the function $\chi_a^{(+)}$ is obtained from experiment by a phenomenological procedure of fitting

to the observed elastic scattering. By basing $\chi_a^{(+)}$ upon the measured elastic scattering, the coupling to reaction channels is partly taken into account. For this reason Eq. (8) normally is a very good approximation to Eq. (7). This is fortunate, because the calculation of $\chi_a^{(+)}$ in Eq. (8) does not require the solution of coupled equations.

A quantitative comparison between Eqs. (7) and (8) was recently published by Perey and Satchler⁴ and Table I is taken from their article. It is seen that the use in Eq. (8) of a $\chi_a^{(+)}$ which disregards the coupling gives a serious overestimate of the cross section. However, when a $\chi_a^{(+)}$ that fits the elastic scattering is used in Eq. (8), the result now is a slight underestimate of the cross section. Coupled-equations calculations are better than DWB calculations, and must be made for some cases. Nevertheless DWB calculations will be accurate enough for most applications.

It is well known, since the work of Chase, Wilets, and Edmonds,⁵ that the coupling must be treated especially carefully when the bombarding energy is very low. Shape resonances are important. This is not a problem at the energies which interest this conference.

Let us therefore regard DWB as the standard method for studying direct reactions, and spend our remaining time in discussing this method. As a personal note, I first became convinced of the basic importance of DWB back when we were trying to formulate an attitude about (α, α') reactions. There was the Blair theory, and this seemed very different from DWB, perhaps even in contradiction to DWB. However, it was then found that the Blair theory was, on the whole, just an elegant approximation to DWB. By now it is known that for collective (α, α') excitations DWB provides an almost exact theory, provided careful calculations are performed.

⁴F. G. Perey and G. R. Satchler, Phys. Letters 5, 212 (1963).

⁵D. M. Chase, L. Wilets, and A. R. Edmonds, Phys. Rev. 110, 1080 (1958).

TABLE I. $\text{Fe}^{56}(\text{p}, \text{p}')$, exciting the 2^+ vibrational state at 0.85 MeV. The effective deformations were obtained by DWBA calculations. The "true" deformation β is that used in the coupled-equations calculations; β_1 is required to give the same peak cross section in DWBA using the same optical parameters as with coupled equations, and β_2 is required in the self-consistent DWBA using the adjusted values of V' and W_D' in the last two columns which give the same elastic scattering as the coupled equations.

E	β	β_1	β_2	V'	W_D'
10	0.1	0.099	0.102	49.4	12.0
	0.2	0.188	0.213	50.2	13.4
	0.3	0.260	0.321	51.9	15.7
	0.4	0.313	0.413	54.1	18.5
12	0.1	0.099	0.101	48.8	11.9
	0.2	0.192	0.208	49.4	13.0
	0.3	0.272	0.315	50.8	14.6
	0.4	0.337	0.402	52.5	16.0
14	0.1	0.099	0.101	48.2	11.9
	0.2	0.193	0.209	48.7	13.0
	0.3	0.277	0.318	49.9	14.6
	0.4	0.346	0.419	51.5	16.2
17	0.1	0.100	0.101	47.3	11.9
	0.2	0.194	0.208	47.8	13.0
	0.3	0.281	0.320	48.9	14.7
	0.4	0.354	0.413	50.7	16.1
22	0.1	0.099	0.102	46.7	11.7
	0.2	0.195	0.201	47.1	12.4
	0.3	0.283	0.302	47.5	13.2
	0.4	0.361	0.400	47.8	14.2

DWB is a theory of nuclear structure, on a par with other theories of nuclear structure. It is employed in the continuum, and therefore concerns reactions. This theory works badly if it is evaluated carelessly, if bad wave functions are used. Not only must good scattering wave functions $\chi_a^{(+)}$ and $\chi_b^{(-)}$ be used in DWB calculations, but good bound-state wave functions ϕ_{aA} and ϕ_{bB} must be used. By virtue of the work of many people, flexible DWB computer codes have been constructed, so that bad approximations to $\chi_a^{(+)}$ and $\chi_b^{(-)}$ need no longer be made. In particular, there is no reason for any further use of plane-wave approximations to these scattering wave functions. Formulas derived with the use of plane waves may resemble results of experiment; however, they are not expressions of any valid physical theory. Plane waves are passé.

The interesting open questions nowadays concern: (1) the consequences of careful construction of the bound-state functions, ϕ_{aA} and ϕ_{bB} ; (2) improved integration schemes for the DWB matrix element, e. g., taking into account the finite range of the nuclear force; (3) deeper understanding of the radial shape of the wave functions, e. g., by the study of nonlocal optical potentials; and (4) DWB studies of antisymmetrization and recoil effects.

Polology alternatives to the DWB theory also are interesting. However, these theories do not differ in spirit from the one we are discussing and need not be pursued here.

(1) Let us discuss these questions one at a time. The bound-state functions enter into the DWB calculations through the quantity called the "form factor." Mostly, simple models of this form factor are used in calculation. It is very desirable that accurate microscopic theories of ϕ_{aA} and ϕ_{bB} be used to generate form factors, and that these be used in as many calculations as possible. Accurate angular distributions and spectroscopic factors are not to be expected unless form factors are obtained from accurate bound-state wave functions.

The excitation of collective states by inelastic scattering has been treated with great success with the use of an elementary deformed optical potential as a means of generating the form factor. Microscopic studies of the collective states have not yet demonstrated the extent to which this procedure may be expected to be reasonable.

Single-nucleon stripping theories give the form factor a special name. For (d,p), say, it is called "the wave function of the captured neutron," and it is computed as an eigenfunction for a single particle in a potential well. It is not clear that this picture is adequate if great accuracy in the extraction of spectroscopic factors is desired. Rost and others have recently been working on this question.

(2) Finite-range evaluations of the DWB matrix element are starting to be performed^{6,7} On the whole, there is no great alteration of deuteron stripping results, but important effects are found in other reactions.

Very recently it has been found possible to perform quite good approximate finite-range calculations, which reduce the six-dimensional finite-range integration to a three-dimensional zero-range integration.^{8,9}

(3) All derivations of the average potential experienced by a particle in the nucleus show that it is nonlocal. For convenience of calculation we nearly always ignore this fact, and use instead a local potential whose shape is roughly that of the distribution of nuclear matter. If all we desire is a good representation of elastic-scattering data, say, then there is no doubt that either sort of potential can give an equally good representation. The two potentials are equivalent also for other applications.

⁶A. Agodi, R. Giordano, and G. Schiffrer, Phys. Letters 4, 253 (1963).

⁷N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. 133, B3-B16 (1964).

⁸P. J. A. Buttle and L. J. B. Goldfarb (to be published).

⁹Gy. Bencze and J. Zimanyi (to be published).

However, even if the wave functions associated with a local potential and with a nonlocal potential are made identical outside the nucleus, so that they give the same scattering, the two wave functions will differ inside the nucleus. This is not surprising, since the two wave functions refer to different theories of the particle-nucleus interaction, which only are rigged to give the same scattering. Study of a representative sort of nonlocal potential led Perey¹⁰ to the very interesting discovery that the two wave functions differ in a systematic manner. If we fit a local potential to the scattering predicted by a nonlocal potential, we always alter the theory so as to exaggerate the magnitude of the wave function in the nuclear interior. For nucleons, this exaggeration seems to be by about 20%. For other particles, the effect may be larger. There is no doubt that Perey's effect raises important questions. The exaggeration of the wave function in the nuclear interior is seen as a defect of the use of local potentials. This may at last be the explanation for the "cutoff radius" which is used in stripping theories.

(4) "Recoil and antisymmetrization" raises the question, among others, of understanding "heavy-particle stripping." People seem to divide into two camps, with regard to this effect: those who take it for granted in very elementary terms, and those who believe it does not happen. Of course neither view is correct. Heavy-particle stripping does happen, but it is difficult to construct an adequate theory for its evaluation. Simply to decompose a target nucleus into the structure (core + orbital particle) and to ask how the core recoils when it is struck does not give an adequate theory.¹¹ At the very least, such a picture disregards the internal coordinates of the core.

¹⁰ F. G. Perey, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3—8, 1962 (Gordon and Breach, Science Publishers, New York, 1963), p. 125.

¹¹ T. B. Day, L. S. Rodberg, G. A. Snow, and J. Sucher, Phys. Rev. **123**, 1051-1053 (1961).

* * *

A. K. KERMAN, Massachusetts Institute of Technology: I noticed that the effect of mocking up the coupling by using the distorted wave for the elastic-scattering exit channel produced something like a 50% effect. For $\chi^{(-)}$ on the left side of the matrix element in Eq. (8), did that calculation use a solution of the uncoupled equation or a function which mocked up the elastic scattering in channel A? That is what is usually done in fitting data. My feeling is that if there was a 50% effect in changing the wave functions $\chi^{(+)}$ of channel A there also would have been an effect from changing the wave function $\chi^{(-)}$ in channel B. That would imply that the coupled-channel calculation is absolutely necessary if you want to get the accuracy that you are talking about.

AUSTERN: That is so. Perhaps Dr. Satchler would like to comment.

G. R. SATCHLER, Oak Ridge National Laboratory: The calculation which Perey and I have performed was of the conventional distorted-wave type and the "fitted" elastic-scattering wave function was used in both channels—in just the same way that we do when fitting experiment.

AUSTERN: I didn't understand that. Do you mean that the distorted-wave equivalent of the coupled calculation used potentials which fit the elastic scattering in both the A and B channels?

SATCHLER: Yes.

KERMAN: That answers my question.

L. R. B. ELTON, Battersea College of Technology, London: What, in fact, is the Hamiltonian in the equations that you have written down? In the usual calculations, as we have now learned, one gets the wave functions from an optical-model analysis; but in your coupled equations, the Hamiltonian is obviously not obtained from the optical-model analysis. What is the Hamiltonian?

AUSTERN: Well, it is whatever your insight into the problem makes it be in any given problem that you are dealing with. That is an intentionally vague statement. What the Hamiltonian often will consist of, at least in the calculations that are most widely advertized right now, is the interaction of protons with a deformed rotator. The interaction is pictured as being in fact an optical interaction. However, it is an optical interaction with a deformed crystal ball—a deformed potential. What one then has is that the distorting potential in either channel will be the spherical part of this interaction. The nonspherical part is treated as the coupling between states of the rotator.

ELTON: If I understand this correctly, which I am not quite sure I do, it is that this Hamiltonian is what must be used in calculating $\chi^{(-)}$ but the Hamiltonian obtained from experimental optical analyses is the one used for $\chi^{(+)}$. However, this is not what Perey and Satchler did.

AUSTERN: I think not. I think a correct statement of the position is that if one uses the solutions of the coupled problem, then the function $\chi^{(-)}$ used in channel B is an eigenfunction governed by the spherical potential only; and the solution function used in channel A is the very accurate one. What these people did was to fit (both in channel B and in channel A) equivalent spherical potentials which describe the elastic scattering as it would occur accurately in channel B alone and in channel A alone.

R. SHERR, Princeton University: I wanted to ask whether you could elaborate on the last remark of your paper about antisymmetrization and recoil. Are these expected to be large effects?

AUSTERN: The antisymmetrization and recoil business has been a subject of grief for many years because it obviously ought to be treated. I think you may have noticed in the course of the talk that the equations which I wrote down were not antisymmetrical. We all know this is wrong. One obviously ought to take account of this difficulty. Now, the principal excuse for using wave functions that are not antisymmetrical is supposed to be that

exchange terms involve a great deal of rearrangement. The wave functions that describe the exchange are the ones that involve displacing several particles to other orbits, and as a consequence overlap integrals are supposed to be very small. This is a good excuse, except for the fact there has been very little investigation of the truth of it. It also may not be very true.

The principal reason why the exchange contributions have not been treated with much care is that the integrals are difficult. The finite-range distorted-wave calculation has been needed in order to approach the exchange terms. Now that we have finite-range distorted-wave calculations, it is possible to tackle the exchange terms intelligently.

There is this tradition which has existed with us for many years, which is called "heavy-particle stripping." I think there is obviously a lot of sense in the idea, but I am afraid that the company of physicists seems to be divided into two more or less religious sects on the subject. There are those who, without benefit of calculation, say it cannot be true and there are those who say it has to be true in the most elementary conceivable fashion. Those in the latter sect have not tried to improve the theory, even though they know improvements are necessary. Both of these positions are highly objectionable. It is only quite recently that people have attempted to look seriously for the effects of recoil. The paper of Day, Rodberg, Snow, and Sucher¹¹ is unfortunately not an adequate investigation of the recoil question. It is a correct calculation, but it is an oversimplified picture of what a nucleus looks like. The idea that the recoil effects are very small is, I think, not adequately assessed. At the same time, the antisymmetrization effects are not adequately assessed, because the usual calculations of these have been done with very bad wave functions. One is looking for small overlap integrals here, and small overlaps cannot be calculated with bad wave functions.

F. S. LEVIN, Brookhaven National Laboratory: I have first a comment and then a question. It seems to me that one basic point has been overlooked by every one who has done these exchange-type calculations, including myself. It is that if you approximate the exact wave function for the system by your optical-model wave function, then you must realize that this optical wave function already has exchange effects in it. So far as I know, all the calculations done fail to include these. You take the amplitude, break it in two parts, and approximate the exact wave function in each part (the exchange part and the nonexchange part) by an optical-potential wave function. This is the wrong approach; one can do better.

The question I have to ask is the following. Suppose you have a particle-transfer reaction, and you approximate your exact wave function ξ by your optical-model wave function. This is the beginning of the distorted-wave approximation. Is there a number (a coupling constant which can be exhibited) such that if we expand in powers of this thing then the corrections that result from higher powers are of small order? It seems to me that physically what you are saying is that as long as the elastic cross section is the largest part of the total cross section, it is a good approximation to use the optical model; but I do not think you can transfer this from cross sections back to the amplitude. I am wondering if there is a sort of coupling constant that you can use to measure the reliability of the optical-model approximation.

AUSTERN: Let me answer the two parts of your question separately.

The first question concerns whether people have considered whether exchange makes contributions to the elastic scattering. Of course people have. In fact, just yesterday I was reading a 1962 paper by ROBERTSON in which he makes quite a point of this. I don't know that I feel that any of these papers have answered all of the questions that we want to see answered; but I think that they are aware of the point you mentioned.

The other question you ask is whether there is a parameter in which one can expand. I think what you are inquiring about is whether what I have called the "compound nucleus piece" of the wave function is to be regarded as a higher term in a power-series expansion in which the channel-B and channel-A parts are treated as the first term. I think it would not be possible to make such a simple description of this complicated situation. Power series have much to recommend them, but I do not think that we normally are privileged to write down one particular series and look at the coupling constant in it. What happens, you see, is that the separation between the two processes is not that one is big and one is small. Rather one is found in some circumstances and the other is found in other circumstances, and if you rig the experiment right you can look for one effect rather than the other; but they are both present in the wave function. Is that an answer to the question that you asked?

LEVIN: In part, but normally one sees some exact wave function which you cannot handle, and if you do a distorted-wave calculation, you approximate that wave function by (let's say) the optical part. How good an approximation is that? This is the kind of question I am asking. It seems to me that in order to answer that question you have to know what happens to the part you are leaving out. I was characterizing the part that you are leaving out. Is it then possible to say anything about that in general?

AUSTERN: I do not think you can characterize it in terms of a coupling constant. The optical part is in the first place supposed to be related to some energy-averaging considerations. The compound part is present, and is large; but by tricks, you can cause yourself not to look at it.

E. P. WIGNER, Princeton University: I have a question that has bothered me for a long time. I ask you this because I think other people are puzzled about it. In your Eq. (6b) there is a matrix which is not a

number because if it were a number it would imply that a function of the radial separation R_a is a function of the radial separation R_b . What is meant by that matrix?

AUSTERN: As you are aware, in the description of this term we normally regard different variables as being internal and external in the different channels. The one case in which the variables are the same in both channels is that of inelastic scattering. Let me answer your question for this one simple case. Then we have a projectile a moving with respect to a target system. The relative coordinate between a and A is the variable in both ξ_a and ξ_b . The internal variables of the target nucleus are the ones that we integrate over.

WIGNER: Then the matrix element is still a function of what may be called R_a ?

AUSTERN: Yes. If we have the situation which I think you wanted me to talk about, i. e., if we have a rearrangement, then the argument of ξ_a is not the same as the argument of ξ_b . The only way that we can establish good correspondence between the arguments of these two terms is by setting up a fairly complete model for the reaction that is going on, and deciding quite clearly in this model what linear combinations of the coordinates we are using in ξ_a , what linear combinations we are using in ξ_b , and what linear combinations we are integrating over. That is where you get into the finite-range code which is now running.

WIGNER: Thank you very much for your answer. In the case of inelastic scattering this is clear and I understand. But I still do not understand it in the case of exchange, or what you call rearrangement. I know how to solve and how to obtain the final results; and most of us can calculate it for an approximation also. But what do those matrix elements depend upon?

AUSTERN: In order to describe all the motions, I think I would like very much to regard this wave function as an expression of a dynamical

model of the reaction, in which we are looking at a very small selection of the possible motions of the system. We are looking at some half a dozen degrees of freedom, perhaps, and we make suitable algebraic descriptions of all of these degrees of freedom, of these coordinates which are introduced, and various linear combinations of these coordinates appear in the wave functions. One linear combination appears in ξ_a , another appears in ξ_b ; and we hope to find that we are integrating over whichever linear combinations are not currently being used in these two functions.

WIGNER: But over what coordinates did you not integrate?

F. COESTER, Argonne National Laboratory: Perhaps it would be helpful to notice that the second so-called matrix element is an operator that operates on functions of R_b and produces functions of R_a .

KERMAN: To talk about a particular case (such as stripping) in which one of the particles has two coordinates and the other has one, I think the only thing you can do easily is to eliminate all coordinates but the two. Then it is a function of these coordinates, and is a three-body problem. Stripping is the simplest example, and you get functions of the two variables everywhere.

AUSTERN: That is what I meant by putting it on a picture.

WIGNER: The question has been answered with the statement that ξ_a and ξ_b are both two-body functions. In that case, I understand, you do not integrate over either R_a or R_b . However, I never saw a calculation being carried out by solving a set of differential equations with two bodies. It is a very theoretical answer but a logically consistent one.

CHAIRMAN: We are grateful for both the questions and the answers. Along with discussions of the validity of the derivations and these very interesting techniques, I think it is valuable to try to keep our feet on the ground by looking at simplified pictures wherein the simplest notions of scattering and the availability of momentum coordinates from the wave

functions of the participating systems appear in a very simple fashion. I have asked Harry Lipkin to give a 5-min summary of some interesting recent work along these lines in Israel.

H. J. LIPKIN, Weizmann Institute of Science, Rehovoth: I would like to mention some exciting work done by Arnon Dar^{1 2} at the Weizmann Institute. He has taken a black-sphere diffraction model similar to that used by Blair and has applied it to a wide variety of reactions, including elementary-particle reactions as well as nuclear reactions, and has had remarkable success in fitting the data. He also fits cases in which you would not naturally expect an approach based on black absorption to be good. For example, he fits the elastic scattering of 14-MeV neutrons for all nuclei from beryllium to uranium with a one-parameter model and gets agreement which is comparable to that obtained with the optical model with many more parameters. It is not obvious why the nucleus should be black for 14-MeV neutrons.

In treating some stripping and pickup reactions, he gets results which are certainly much better than the Butler theory. Although no one is claiming this as a substitute for distorted-wave calculations, it may be useful as an interpolation formula for the experimentalist who wants to have a quick look at his data with a one-parameter model.

In the fitting of the 14-MeV neutron data, he found a remarkable fact, namely that the elastic-scattering cross section for 14-MeV neutrons in the backward direction is 10 mb for all nuclei from beryllium to uranium. I don't know why; but I think it is an interesting point. He then puts this into his model as a phenomenological reflection, i. e., his model is a black sphere with a weakly reflecting mirror surface. This

^{1 2}A. Dar, Phys. Letters 7, 339 (1963); Nucl. Phys. (to be published); A. Dar and W. Tobocman, Phys. Rev. Letters 12, 511 (4 May 1964). Further remarks on Dar's work can be found in the invited paper by H. Lipkin, starting on p. 481 of these Proceedings.

adds one parameter for fitting all nuclei. Therefore, in effect there is still only one free parameter for each nucleus. This improves the fit to the data and fills in the minima.

His work with elementary particles is the sort of thing many people here would think was obvious if they thought about elementary particles. Many of these elementary-particle reactions are cases in which you expect strong absorption. He finds that he gets good agreement with experimental data without using Regge poles and other things that other people have tried to use. Out of a set of about 20 figures I have picked two, more or less at random, to serve as typical examples of Dar's results.

The first (Fig. 3) compares the diffraction curve with the Butler curve for the $F^{19}(d,t)F^{18}$ reaction at 14 MeV. He has one parameter, a radius which comes out to be similar to the Butler radius. The fit is considerably better, particularly for the larger angles.

Figure 4 is a typical example of neutron scattering at 14 MeV. I happen to have picked carbon. The radius parameter is the one parameter to fit the data. Comparing it with the better optical-model

Fig. 3. Comparison of diffraction and Butler curves for $F^{19}(d,t)F^{18}$ at 14.8 MeV.

Fig. 4. Comparison of diffraction and optical-model curves for $C^{12}(n,n)C^{12}$ at 14 MeV.

calculation, you see that although this is a one-parameter fit, it is not much worse than these others. You also notice the characteristic leveling off of the cross section at 10 mb in the backward direction.

DISTORTED-WAVE CALCULATIONS AND SPECTROSCOPIC FACTORS

G. R. Satchler

Oak Ridge National Laboratory,
Oak Ridge, Tennessee

I. Introduction

The inelastic scattering of various particles has become a very powerful tool of nuclear spectroscopy, and one which emphasizes the collective aspects. (The corresponding spectroscopic factors are usually expressed as deformations or deformabilities.) It has also provided some of the best examples of the usefulness of the distorted-wave method. This field, however, will be covered by other talks, and I will restrict myself to rearrangement collisions of the pickup and stripping type. Most of the emphasis will be on the transfer of a single nucleon. In contrast to inelastic scattering, these reactions emphasize the "single particle" aspects of the nuclear states involved.

We should address ourselves to two questions. How well does the theory predict (i) the shape, and (ii) the magnitude, of the differential cross section? These two questions are partially decoupled; the shape depends upon the right selection of angular momenta (the right combination of partial waves) while the magnitude depends upon interaction strengths and various nuclear overlaps. This is fortunate, because often the greatest uncertainty is over these latter quantities. (Of course, we should not be satisfied with a theory which gave the right shape if the magnitude it predicted was quite unrealistic.)

The early plane-wave stripping theories were remarkably successful as a tool for identifying ℓ values from measured angular

distributions from light nuclei, and even gave good fits to the shape of the main peak. However, it was quickly noted that the predicted absolute cross sections for (d,p) reactions were frequently too large by an order of magnitude or more, so that absolute values of spectroscopic factors could not be obtained, and semi-empirical methods based on relative cross sections had to be devised.¹ But the failure to predict absolute cross sections also cast doubt on the ability of the plane-wave theory to give a good account of relative cross sections and their dependence on energy and Q value. Experimental data for heavier nuclei made it clear that distortion by the Coulomb field was important, and we now know that distortion by nuclear scattering and absorption is always important. Under no circumstances met with experimentally are distortion effects negligible, so that a plane-wave theory is never a good approximation.

The distorted-wave method is intended to give a more accurate description. It takes account of the scattering and absorption of the particle before and after stripping (or pickup) by using distorted waves instead of plane waves. These are generated by optical-model potentials which are chosen to reproduce the observed elastic scattering from the same nucleus at the same energy. It is not appropriate here to discuss the various ways of deriving a distorted-wave Born approximation,² but I want to present one schematic derivation just to illustrate possible limitations the theory may have. To be specific let us consider the A(d,p)B deuteron-stripping reaction, where $d = p + n$ and $B = A + n$. One exact expression for the transition amplitude is

$$T_{d,p} = \int \chi_p(\vec{k}_p, \vec{r}_p) \langle B | V_{pB} - U_{pB} | A \rangle \Psi_d, \quad (\text{EXACT})$$

¹ M. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960).

² N. Austern, in Selected Topics in Nuclear Theory, edited by F. Janouch (International Atomic Energy Agency, Vienna, 1963), and references therein.

where Ψ_d is the (exact) total wave function, with deuterons incident on nucleus A and outgoing waves in all other channels. V_{pB} is the actual interaction between the proton and nucleus B, while U_{pB} is the proton optical potential which generates χ_p , the proton distorted wave. If we assume elastic scattering is the dominant process, we may approximate Ψ_d by χ_d , a distorted wave describing the deuteron elastic scattering. This gives the distorted-wave amplitude

$$T_{d,p} \approx \int \chi_p(\vec{k}_p, \vec{r}_p) \langle B | V_{pB} - U_{pB} | A \rangle \chi_d(\vec{k}_d, \vec{r}_d) . \quad (\text{DWBA})$$

Next it is usual to write the interaction as

$$V_{pB} = V_{pn} + V_{pA},$$

and neglect the contributions from $(V_{pA} - U_{pB})$. The justification for this last step is not completely clear, although there is obviously considerable cancellation between the two terms. I prefer, at this stage, to regard the resulting amplitude

$$T_{d,p} \approx \int \chi_p \langle B | V_{pn} | A \rangle \chi_d$$

as a model to be tested against experiment. Now V_{pn} does not depend upon the internal coordinates ξ of nucleus A, so we may integrate over these. The resulting overlap defines a spectroscopic factor

$$\int \psi_B^*(\mathbf{r}_n, \xi) \psi_A(\xi) d\xi = \sum_{\ell} \beta_{\ell} \psi_{\ell}(\mathbf{r}_n).$$

If ψ_{ℓ} is a shell-model orbital, then

$$S_{\ell} = n |\beta_{\ell}|^2$$

is the spectroscopic factor for that orbit. (Here n is the number of equivalent nucleons in B , and all angular-momentum factors are discarded for simplicity.¹⁾ The resulting cross section then takes the form

$$\frac{d\sigma}{d\omega} = \frac{2I_B + 1}{2I_A + 1} S_\ell \sigma_\ell(\theta) .$$

The reduced cross section $\sigma(\theta)$ only depends on the orbital ψ_ℓ and the distorted waves; all the nuclear-structure information (except for the form of ψ_ℓ) is contained in the factor S .

In practice we usually make an additional approximation, that of "zero range." The product of V_{np} and the internal deuteron wave function is replaced by a delta function

$$V_{np}(r_{np}) \psi_d(r_{np}) \approx D_0 \delta(\vec{r}_n - \vec{r}_p) .$$

This tremendously simplifies the calculations. When we do not make this approximation, we say we are doing a "finite-range" calculation. Of course, the strength D_0^2 enters as a factor in σ , the reduced cross section. It is known quite accurately for deuteron stripping, but not so well for the stripping of other projectiles, such as in (He^3, d) . In these cases, D_0 includes the overlap of the internal states of the two free particles, which may be regarded as another kind of spectroscopic factor. In (He^3, d) for example, it is an indication of the extent to which He^3 consists of a free deuteron and proton.

II. Some Typical Results

Since most of this discussion is devoted to difficulties and uncertainties in the application of the theory, we should compensate by

Fig. 1. Comparison of plane-wave and zero-range distorted-wave predictions, using $S = 1$ and the same $1f$ neutron wave function for both. The plane-wave cutoff was chosen to give the same peak angle as the experimental data.

first briefly surveying some typical cases to demonstrate its success. In each case (except the last), the optical potentials were obtained from fits to elastic scattering from the same or nearby nuclei, and not adjusted in any way to fit the reaction cross sections.

Figure 1 is intended mainly to emphasize how the plane-wave version gives quite wrong magnitudes in (d,p) reactions; the same neutron bound state and S factor were used in both calculations. Figure 2 compares measurements³ and distorted-wave predictions for deuteron stripping on Cd, for $l = 0$ to 5. I might add the calculations and measurements were made independently! I understand also that the absolute spectroscopic factors extracted are in good agreement with pairing-model calculations. The following curves (Fig. 3) show the predictions of a finite-range calculation which used optical potentials including spin-orbit coupling, obtained by fitting to elastic-scattering data. The dashed curves are similar except that contributions from the nuclear interior are excluded; the agreement with experiment⁴ is slightly improved. This

³ R. J. Silva (private communication).

⁴ L. L. Lee, Jr., J. P. Schiffer, B. Zeidman, R. H. Bassel, R. M. Drisko, and G. R. Satchler (to be published).

Fig. 2. Comparison of theoretical predictions and measurements for several groups from $\text{Cd}^{114}(d, p)$ at 12 MeV.

Fig. 3. Comparison of experiment and distorted-wave predictions for $\text{Ca}(d,p)$. Finite-range and spin-orbit coupling were included, and the optical potentials used were those which reproduce the observed scattering.

situation is not unusual, although often the price of a slight improvement in shape is a poorer agreement in magnitude. In some cases a cutoff qualitatively reproduces the effects of finite range.⁵ Another reason for damping interior contributions is that we should probably be using nonlocal potentials to generate our distorted waves and bound-state orbital, and it is known that the effect of nonlocality is to reduce the magnitude of these wave functions inside the nucleus.⁶ In addition, it is perhaps questionable whether a scattering wave function, which only depends upon the position of the center of mass of a complex particle such as a deuteron, is adequate inside the nucleus. However, I don't intend by this to advocate the general use of a cutoff, but rather to indicate that our treatment of the nuclear interior is not as good as it might be. Figure 2 does indicate over-all agreement with experiment over a range of two orders of magnitude, so there is nothing radically wrong with the theory! It also demonstrates the effect at wide angles for $\ell = 1$, pointed out by Schiffer and Lee⁷; the $p_{1/2}$ shows a minimum around 100° while the $p_{3/2}$ does not. In the calculation, an effect of the right magnitude appears when one uses spin-orbit coupling.

Next, Fig. 4 indicates the range of uncertainties in applying the theory to an $\ell = 0$ capture.⁸ These $\ell = 0$ transitions, at least for heavier nuclei, almost invariably require a radial cutoff in order to reproduce the ratio of first and second peaks, and the correct amount of oscillation at wide angles. Even so, the position of the second peak is not quite right and

⁵N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. 133, B3 (1964).

⁶F. G. Perey, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3-8, 1962 (Gordon and Breach, Science Publishers, New York, 1963), p. 125.

⁷J. P. Schiffer and L. L. Lee, Phys. Letters 12, 108 (1964).

⁸The experimental data were kindly made available by D. W. Miller.

Fig. 4. Range of uncertainties in applying DWBA theory to an $\ell = 0$ capture. The shaded areas represent the range of cross sections and polarizations predicted by potentials which fit the elastic scattering of deuterons from Sr^{88} . $S = 1$ is used in all cases.

this is also characteristic. I take this again as an indication that we are not treating the nuclear interior correctly. The shaded bands in the figures encompass all the polarizations, and differential cross sections predicted for a spectroscopic factor $S = 1$, by using 6 different deuteron potentials (all giving good fits to the elastic-scattering data), deuteron spin-orbit strengths from zero to 10 MeV, and cutoffs between 5 and 6 F, and by employing both zero and finite range. The agreement with the differential cross section is satisfactory, but the polarization is very poorly fitted.

We turn now to other single-nucleon-transfer reactions. Figure 5 shows some preliminary results for (He^3, d) . The theoretical

Fig. 5. Comparison of experiment and theory using zero-range approximation and without spin-orbit coupling. Potentials A and B give equally good fits to He^3 scattering from Ca^{40} .

normalization is less well known here, but the fits to experiment⁹ shown here are consistent (to within 50%) with analyses of (d, He^3) pick-up at lower energies.¹⁰ Figure 6 shows results for (He^3, α) obtained some time

⁹ H. E. Wegner and W. S. Hall, Phys. Rev. 119, 1654 (1960).

¹⁰ B. Cujec, Phys. Rev. 128, 2303 (1962); J. L. Yntema and G. R. Satchler (to be published).

Fig. 6. Comparison of experiment and distorted-wave predictions for $\text{Fe}^{56}(\text{He}^3, \alpha)$ at 14 MeV, after Blair and Wegner, Ref. 11.

ago by Blair and Wegner,¹¹ again using potentials which fit elastic-scattering data. The normalization is even less well known in this case. The crudest zero-range estimate predicts cross sections about 1/10 of those observed. A more realistic estimate would reduce this discrepancy, but whether it would remove it all remains to be seen.

Multinucleon transfer presents a field of considerable interest, and one in which nucleon correlations or clustering could be explored. This subject will be discussed in a later talk, but a quick look may be taken at Figs. 7 and 8. The (p, α) calculation assumes the pickup of a proton from the $2p_{1/2}$ orbit and two neutrons from $1g_{9/2}$; harmonic-oscillator wave functions were used, and the $2p_{1/2}$ proton orbit was assumed to be 70% full in the Zr^{90} ground state. A Gaussian was used for the internal state of the alpha. The agreement in shape with the experimental data^{1,2} is remarkably good, while the number N is the "unhappiness factor"; that is, the ratio of observed to predicted magnitude. It seems likely that the use of harmonic-oscillator wave functions, with their short tails, is responsible for part of this factor. Incidentally,

¹¹A. G. Blair and H. E. Wegner, Phys. Rev. 127, 1233 (1962).

¹²C. B. Fulmer (private communication).

Fig. 7. Comparison of experiment and distorted-wave predictions for "triton" pickup from Zr^{90} . Here β is the parameter for the single-particle oscillator wave functions, and N is the factor by which the theoretical predictions have been multiplied.

Fig. 8. Preliminary calculations for "alpha" pickup from F^{19} . The alpha cluster is assumed to be picked up from a 4p orbit.

it is interesting to note here that the use of finite range has little effect on the angular distribution, but reduces the cross section uniformly by a factor of 2. The (d, Li^6) alpha pickup shown in Fig. 8 is much more preliminary.¹³ Elastic-scattering data for Li were not available, so the potentials were chosen to be "reasonable." The results are very encouraging, and seem to be giving the right order of magnitude for the cross sections.

¹³ L. J. Denes, M.S. Thesis, University of Pittsburgh, 1963. See also R. M. Drisko, G. R. Satchler, and R. H. Bassel, Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, 14-18 April 1963, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), p. 85.

III. Detailed Study of the $\text{Ca}^{40}(\text{d},\text{p})$ Reaction

After this rapid survey of some typical applications of the distorted-wave method, let us now turn to a more detailed analysis of one reaction,¹⁴ namely deuteron stripping on Ca^{40} . This is a good target to study because we believe it is a doubly magic nucleus and the neutron should be captured into single-particle levels with $S = 1$. (Of course, the existence of two low-lying $p_{3/2}$ levels is already a warning that we may be over-optimistic.) It is also sufficiently heavy not to be plagued with the difficulties we always seem to run into with very light nuclei. Measurements of the elastic scattering and stripping were made at Argonne⁴ at 7, 8, 9, 10, 11, and 12 MeV, so that a fairly complete set of data was available for analysis. After presenting the results of that analysis, I think it is instructive to examine some of the attendant uncertainties and ambiguities, and also the consequences of simplifying assumptions such as zero range and neglect of spin-orbit coupling.

First, Table I is a summary of the spectroscopic factors obtained from this analysis. The "best" deuteron potential means that the parameters used at each energy are those that give the best fit to the deuteron scattering at that energy, while "Av" means an average of these which gives a good over-all fit at all energies. A spin-orbit strength of 5 MeV was assumed for the deuterons. The largest part of the "errors" quoted arises from uncertainties and fluctuations in the data from energy to energy; the larger errors for S values obtained with the "best" potentials

¹⁴ The results of a less extensive study on $\text{Zr}^{90}(\text{d},\text{p})$ have been submitted as a contribution to this conference by W. R. Smith (Vol. I, Contributed Papers. Argonne National Laboratory Report ANL-6848, March 1964, p. 30). New data on this reaction recently obtained at Oak Ridge are being subjected to similar analysis.

TABLE I. Spectroscopic factors for $\text{Ca}^{40}(\text{d}, \text{p})\text{Ca}^{41}$, $E_{\text{d}} = 7$ to 12 MeV. These were calculated by use of Z-type potentials, finite range, and spin-orbit coupling, with $r_{0n} = 1.2$, $a_n = 0.65$ for the neutron.

Q (MeV)	Orbit	"Av." D potential	"Best" D potential
6.14	$1f_{7/2}$	0.84 ± 0.03	0.86 ± 0.07
4.19	$2p_{3/2}$	0.67 ± 0.06	0.72 ± 0.12
3.67	$2p_{3/2}$	0.30 ± 0.03	0.32 ± 0.04
4.19 + 3.67	$2p_{3/2}$	0.97 ± 0.07	1.03 ± 0.15
2.19	$2p_{1/2}$	0.66 ± 0.08	0.78 ± 0.04

reflect similar variations in the elastic analyses. This emphasizes the possible dangers of placing too much weight on a measurement taken at a single energy, particularly for the lighter nuclei and at moderately low energies. It also emphasizes that the optical potential that gives the very best fit to scattering data at one energy is not necessarily the most physically correct. Forcing the optical model in this way may bias the parameters because of errors or idiosyncracies in the data, and an average fit (for several close energies, or neighboring elements) may be more physically significant.

This is also the time to emphasize that the definition of the spectroscopic factors is not completely independent of the choice of parameters for the rest of the calculation. In this sense, in (d,p) stripping, for example, we assume that the neutron is captured into a shell-model orbit. This orbit we take to be an eigenstate in a potential well, so the spectroscopic factor obtained is somewhat dependent upon the choice of parameters for this well, particularly the radius and spin-orbit coupling. In principle these are properties of the nucleus under study, but in practice we do not

know their values too closely. Table I was obtained by use of a Saxon well with radius $1.2 A^{\frac{1}{3}}$ and spin-orbit coupling of 25 times the Thomas value. This radius is probably a lower limit, so the spectroscopic factors obtained are upper limits in this respect. An increase of this radius to $1.25 A^{\frac{1}{3}}$ would reduce the S factors of Table I by about 15% for $\ell = 3$ and 10% for $\ell = 1$.

The values given in Table I are close to the "single particle" value $S = 1$; in particular, the two $p_{3/2}$ levels together just about exhaust the $p_{3/2}$ strength. The $f_{7/2}$ and $p_{1/2}$ numbers are, perhaps, significantly less than unity, but whether this is a consequence of our neglecting effects like the nonlocality of the potentials, or whether it really reflects a property of Ca^{41} , remains to be seen. We can certainly say that the model being used for the reaction is giving the expected results to within 20% or so.

Now to examine some of the work behind these results. It is not appropriate to go into this in great detail here, but a more detailed account will be given elsewhere.⁴ The first problem which arises is due to ambiguities in the deuteron optical potentials. As is well known, a whole series of these can be found (only differing, crudely, by the number of half-wavelengths in the well); for Ca we have a series, with real well depths from 30 MeV to 450 MeV in steps of about 40 MeV, which give almost identical scattering. They do not predict the same stripping, however, for reasons which are evident from Fig. 9. This shows a cross section of the distorted waves in the nuclear interior for three of these potentials. The potential Z of depth about 110 MeV is most satisfactory for stripping; shallower potentials give unacceptable angular distributions (unless a radial cutoff is used), Fig. 10, while deeper potentials give nearly the same results as using Z with a cutoff, and require a spectroscopic factor greater than unity for the ground-state transition. If the potential has

Fig. 9. Magnitude of deuteron distorted wave along the incident direction passing through the center of the nucleus. Positive radii correspond to the shadow side, negative radii to the illuminated side. Potentials X, Y, and Z have depths of approximately 30, 70, and 110 MeV, respectively.

any physical significance at all, it would be difficult to understand it being much deeper than 100 MeV, so the Z potential was adopted as standard. Another potential ambiguity arises from the choice of surface or volume absorption for the deuterons. Potentials with either can be found to give almost identical scattering; fortunately this ambiguity has rather little effect on the stripping predictions.

Two other features we should examine are not so much physical as calculational; namely the consequences of using the zero-range approximation and of neglecting spin-orbit coupling. The results of Table I take both these effects into account, but many groups have access to computer codes which do not. Figure 11 shows the effects of finite range; these curves are not the results of the final analysis but represent some early work. The zero-range predictions using a radial cutoff at 4 F are also shown for comparison. The effect of finite range in this case is to more or less uniformly reduce the cross section, more so for the "shallow" (Y) than the "deep" (Z) deuteron potential. Its effect is also considerably smaller for the $\ell=1$ transition, for which the interior contributes less. (At least part of the reason for this is the node in the 2p bound wave function and the somewhat smaller Q value.) These results are typical for deuteron stripping in this energy range, so the chief effect of neglecting finite range would be to lead to an underestimate of the spectroscopic factor. (The situation is more complex at higher energies and for other reactions.)

Fig. 10. Results of using several potentials which give the same deuteron scattering from Ca^{40} . Zero range was used and spin-orbit coupling was neglected. Potential G is approximately 150 MeV deep, J is about 230 MeV deep.

Fig. 11. Finite-range effects in Ca(d,p). The "deep" potential is of type Z, the "shallow" potential is of type Y. LCO refers to a radial lower cutoff. (The experimental data shown should be increased in magnitude by about 60%.)

The inclusion of spin-orbit coupling has almost negligible effect on the angular distribution shapes, unless one is concerned with small differences at wide angles (such as the $p_{1/2}$ - $p_{3/2}$ effect⁷). This is illustrated in Fig. 12, where various proton and deuteron spin-orbit strengths are simply added in a zero-range calculation using the Z potential. It also compares the curves calculated by use of the spin-independent Z, obtained by fitting to elastic data without spin-orbit, and those calculated with a similar potential Z3S which includes about 5 MeV of spin-orbit and gives the same elastic scattering as Z. (At this energy, they differ mainly in Z3S having a weaker absorptive potential than Z.) Most of these shape differences would be invisible in a linear plot; however, the spin-orbit coupling does have important consequences for the peak magnitudes, and hence for the spectroscopic factors extracted. This is emphasized in Fig. 13, where linear plots for the $\ell=3$ peak are presented, all using $S = 1$. Simply including spin-orbit coupling for the neutron bound state ($Z + N$) has the effect of "expanding" the bound-state wave function when $j = \ell + \frac{1}{2}$, and "contracting" it when $j = \ell - \frac{1}{2}$; for $f_{7/2}$ this leads to a 24% increase in peak cross section over the case (Z) in which this is neglected. Then adding 8 MeV of proton spin-orbit coupling further increases the cross section, while the addition of 5 MeV of deuteron spin-orbit brings it down again. Because the deuteron spin-orbit strength is relatively unknown at this time, the comparison of cases ($Z + NP$) and ($Z3S + NP$) is of interest, representing the best we can do when we use best fits to the elastic deuteron scattering without and with spin-orbit coupling, respectively. We see that there is roughly a 20% difference, which arises because including spin-orbit coupling in the optical-model analysis necessitates readjusting the other parameters slightly in order to regain a good fit for the elastic data. A similar analysis at 12 MeV leads to a similar effect, but of opposite sign! This partly reflects variations in the elastic data, and partly the difficulty of determining the spin-orbit strength from analysis of differential cross sections alone. It

Fig. 12. Spin-orbit effects on Ca(d,p). Here V_{sp} and V_{sd} are the proton and deuteron spin-orbit strengths, respectively, in a zero-range calculation using the Z potential. A neutron spin-orbit strength of 8 MeV is used in all cases.

Fig. 13. Various spin-orbit effects on the magnitude of the $\ell=3$ peak cross section. N, P, and D refer to the particles on which spin-orbit coupling is acting. Zero range was used. Ratios of peak cross sections: $(Z + N)/Z = 1.24$, $(Z + NP)/Z = 1.30$, $(Z + NPD)/Z = 1.21$, $(Z + NP)/(Z3S + NP) = 1.18$.

also re-emphasizes the dangers of using absolute best-fit parameters in a reaction calculation. The summary of these effects given in Table II is based upon calculations using the average Z potential. The numbers are intended only to be order-of-magnitude values; their precise values depend upon all the other parameters. Also included is a rough indication of the effect of varying the proton and deuteron absorptive potentials W , since these parameters are usually determined with least accuracy in an optical-model analysis. The stronger dependence of $\ell=3$ capture on the deuteron absorption W_d presumably reflects the larger contributions from the nuclear interior for this transition.

It is relevant at this point, to enquire what would be the consequences of using some average set of deuteron optical-potential parameters deduced from scattering on nuclei in another region of the periodic table, or based upon qualitative analysis with a simplified potential. As an example of the first, we may use the parameters suggested for heavier nuclei by the Pereys,¹⁵ while for the second we may take

¹⁵ C. M. Perey and F. G. Perey, Phys. Rev. 132, 755 (1963).

TABLE II. $\Delta\sigma(\text{peak})\%$ for $\text{Ca}^{40}(\text{d},\text{p})$ at $E_{\text{d}} \approx 10$ MeV. The calculations were based on zero range with a 110-MeV Z-type potential.

Addition	$Q = 6.14, 1f_{7/2}$	$Q \approx 4, 2p_{3/2}$	$Q = 2.19, 2p_{1/2}$
Finite-range	- 15	+ 1	+ 3
Neutron radius $1.2 \rightarrow 1.25$	+ 15	+ 10	+ 10
Neutron spin-orbit, 8 MeV	+ 25	+ 4	- 5
Proton spin-orbit, 8 MeV	+ 5	+ 3	+ 1
Deuteron spin-orbit, 5 MeV	- 7	- 2	+ 4
Deuteron absorption	$\sigma \approx 1/W_{\text{d}}$	$\sigma \approx 1/\sqrt{W_{\text{d}}}$ $\sigma \approx 1/\sqrt{W_{\text{p}}}$	
Proton absorption	$\sigma \approx 1/\sqrt{W_{\text{p}}}$		

Over-all uncertainty in D potential: $\Delta\sigma \lesssim \pm 20\%$

the Saxon potential recommended by Hodgson.¹⁶ The elastic scattering predicted by these gives a poor fit to the measured values (Fig. 14); it is known^{4,15} that the reason for this is that the data require the imaginary potential to extend to considerably larger radii than the real potential. Figure 15 shows the result of using these potentials in a stripping calculation, in comparison with the use of the best-fit Z. The over-all fit to the data is not bad, although the $\ell=3$ peak is shifted a few degrees with Hodgson's potential. Other calculations, not shown, indicate they also reproduce reasonably well (to 10 or 20%) the variation of peak cross section with energy and Q value. However, the absolute magnitudes (and hence the spectroscopic factors) are in serious error. In particular, the ratio of

¹⁶ P. E. Hodgson, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3-8, 1962 (Gordon and Breach, Science Publishers, New York, 1963), p. 103.

Fig. 14. Elastic scattering predicted by the type-B potential of Perey (PB) and that recommended by Hodgson (H), in comparison with the data and the "best-fit" Z potential.

Fig. 15. Stripping predictions for the $\text{Ca}(d,p)$ reaction. Zero range and the potentials of Fig. 14 were used.

$\ell=1$ to $\ell=3$ intensity is in error by 50% with potential PB and by a factor of nearly 2 with potential H. We conclude that reasonable guesses at the potentials will probably give a fit to the main peak of the angular distribution and predict energy dependence and Q dependence with fair accuracy, but may be in serious error for both absolute magnitudes and relative magnitudes for different ℓ captures.

IV. Other Effects

Before concluding, I should briefly mention some other effects which can influence the extraction of spectroscopic factors. One

which is perhaps least well understood is what I like to call the use of an "effective binding energy." A three-body model for the deuteron-stripping reaction $A(d,p)B$ may be expressed by

$$d = p + n, \quad B = A + n.$$

The overlap of initial and final nuclear states defines the wave function ϕ_ℓ of the captured neutron. (We assume one ℓ value for simplicity.) This wave function is

$$\int \psi_B^*(r_n, \xi) \psi_A(\xi) d\xi = \phi_\ell(r_n) = \sum_N \beta_N \psi_{N\ell}(r_n).$$

In general, this can be expanded in terms of shell-model single-particle states $\psi_{N\ell}$, that is, in eigenstates of a single-particle potential well. The sum is over principal quantum numbers and, of course, continuum states if the well is finite. We would like to think that only one such term was important. The question then arises, what binding energy should be associated with this term? If the target A were a closed-shell nucleus and the residual state B was well described as a single neutron outside this closed shell, then its binding energy ϵ is equal to its separation energy S (if there is negligible rearrangement of the closed-shell core). This, we hope, is the situation in $\text{Ca}^{40}(d,p)$. Figure 16 shows schematically a different situation, as might occur, for example, in $\text{Ca}^{41}(d,p)$.

Fig. 16. Schematic representation of a situation in which an effective binding energy may be required.

Because of the residual interaction v_{12} between the two extra neutrons, the states of various total angular momentum are split, and the separation energy of a neutron is no longer equal to ϵ . However, we still like to think of that neutron as moving in the shell-model orbit with binding energy ϵ . Of course, there have to be small corrections, because the tail of the overlap function ϕ must fall off exponentially in a way consistent with the actual separation energy. In this simple case there are two extreme possibilities; we may use for ϕ the shell-model orbital with a binding B equal to the actual separation energy S (the usual procedure), or one with a binding B equal to the zero-order model energy ϵ (an "effective" binding energy). Most situations are much more complicated than this, especially if configuration mixing is involved.

Why is it important to worry about this? Well, the magnitudes of the calculated stripping and pickup cross sections depend quite sensitively upon the choice of effective binding energy B . An increase in B contracts the wave function and reduces the cross section, and vice versa. This effect has been invoked, for example, to obtain a reasonable internal consistency in the interpretation of the (d,t) pickup reaction on various Ti isotopes, and also to reconcile these results with spectroscopic factors for (d,p) transitions connecting the same states.¹⁷ These experiments seemed to require the neutron to be bound by approximately the same energy in all the isotopes, while the separation energy varies by several MeV.

Somewhat clearer evidence was obtained recently from (p,d) pickup reactions on the Fe isotopes.¹⁸ Here one excites the $T \pm \frac{1}{2}$ states of the same shell-model configuration which, however, differ in energy by

¹⁷J. L. Yntema, Phys. Rev. 131, 811 (1963). Similar effects were seen with the (d,t) reaction on the Fe isotopes.

¹⁸E. Rost, B. Bayman, and R. Sherr, Bull. Am. Phys. Soc. 9, 458 (1964).

Fig. 17. Change in shape of an $\ell=3$ angular distribution with two different effective binding energies B (after Rost, Ref. 19).

several MeV. In order to fit these results, it was necessary to assume the same effective binding in both these transitions. Naively this is very reasonable, since the neutron is picked from the same orbit in each case.

An additional effect on the shape of the angular distributions was recently pointed out by Rost.¹⁹ If two states have roughly the same Q values and separation energies, even the peak shapes of the angular distributions may differ if sufficiently different effective binding energies are used. Figure 17 shows an example given by Rost for two $\ell=3$ pickup transitions in which one is $f_{7/2}$ and the other $f_{5/2}$. The binding energies are adjusted so that both orbits are eigenstates of the same shell-model potential well. This could be a tool for determining nuclear spins.

One final effect I should mention, which is currently being explored, concerns the possible nonlocality of the optical potentials and shell-model potential being used. It is known that elastic scattering can

¹⁹R. Sherr, E. Rost, and M. E. Rickey, Phys. Letters 12, 420 (1964).

Fig. 18. Ratio of wave functions for local and nonlocal potentials that give the same scattering (after Perey, Ref. 6).

Fig. 19. Effect of damping the contributions from the nuclear interior, such as might arise from nonlocality of the potentials.

be fitted equally well with a local or nonlocal potential; however, the corresponding wave functions differ inside the nucleus.⁶ The nonlocal wave function is reduced in comparison with the local one which has the same asymptotic form, as shown in Fig. 18. The same feature is shown for a bound-state wave function, although to conserve normalization the magnitude of the tail has to be increased also. Since a product of three of these functions enters the distorted-wave matrix element, a considerable damping of the interior contributions could result if nonlocal potentials were used. Figure 19 indicates the sort of effect this could have; it is assumed that the interior is smoothly damped by 50%. The over-all reduction in cross section would be at least partly compensated for by the increase in magnitude of the bound-state tail, which was not included in the calculation shown. A local approximation to this nonlocal damping is now being studied numerically.²⁰

²⁰ F. Perey (private communication).

V. Conclusions

If I may risk sticking my neck out, let me conclude by saying it looks as though we can predict the absolute magnitudes of deuteron stripping cross sections to within something like 20% if we take care to include the various effects I have been discussing. In view of my introduction to this talk, this is probably better than we had any right to expect! I don't pretend to understand why this is so. The situation is less clear for other reactions; it looks as though (d, He^3) and (d, t) reactions are coming out right to within a factor of 2, while (He^3, α) and (p, α) perhaps leave a factor between 2 and 10 to be accounted for. In fairness though, one should say that less work has been done on these latter reactions than on deuteron stripping.

Acknowledgment

None of the work reported here would have been possible without the cooperation of my colleagues, R. H. Bassel and R. M. Drisko.

* * *

M. H. MACFARLANE, Argonne National Laboratory: You mentioned a factor of 2—10 bonus on the uncertainty in reactions such as (α, t) and (He^3, d) . How much of this comes from the uncertainties about the internal wave functions of the complex projectiles?

SATCHLER: You mean the overlap of the He^3 and the deuteron wave functions? I don't know. This is something that we have been meaning to do for a long time, but have never gotten around to. In (He^3, d) the uncertainty is not worse than a factor of 2, and possibly is better than that.

N. K. GLENDENNING, Lawrence Radiation Laboratory: What kind of potential do you use in the (p, α) reaction?

SATCHLER: If you are using zero range, then, of course, all you have is a strength; and the strength one gets could be deduced crudely from the binding energy of the alpha particle, i. e., the binding of the triton to the proton. In this particular case, Drisko did use wave functions for the alpha particle to calculate the overlap more carefully, and this number was used. In this case, I think it is a realistic number.

GLENDENNING: What did you use for a range? Something like 1.5 F?

SATCHLER: Yes.

A. G. BLAIR, Los Alamos Scientific Laboratory: I have two questions. My first one is that you mentioned that for the $\ell=3$ transition there is about a 22% $\vec{\ell} \cdot \vec{s}$ effect for the captured neutron in (d,p) stripping. How does the magnitude of this vary with ℓ for (d,p)?

SATCHLER: It increases fairly linearly with ℓ simply because the spin-orbit splitting increases with ℓ . The spin-orbit interaction expands the $\ell + \frac{1}{2}$ wave function and contracts the $\ell - \frac{1}{2}$. For the p state, it is about 10%.

BLAIR: My second question is this: What does one do about distorted-wave calculations for unbound states? Are there calculations that have been tried for this?

SATCHLER: Huby at Liverpool had a theory for this and was proposing to do calculations. Whether they have been done, I do not know.

H. E. GOVE, University of Rochester: Does the effect of ignoring spin-orbit interaction and finite range change the cross section in the same direction?

SATCHLER: The finite range tends to reduce the cross section. The spin-orbit coupling on the bound state tends to increase it for $\ell + \frac{1}{2}$. It decreases it for $\ell - \frac{1}{2}$. The spin-orbit effects in the scattering also depend on the j value, as shown in Table II of my talk.

D. R. INGLIS, Argonne National Laboratory: I presume that the finite range decreases the cross section largely because it means that the interaction reaches out across a node and takes in some wave function of the opposite sign on the other side, so as to have more chance to cancel positive and negative contributions.

SATCHLER: Yes.

F. S. LEVIN, Brookhaven National Laboratory: It might be theoretically advantageous to consider the final-state distorted wave functions as being generated by a real potential rather than an optical potential. Have you tried any calculations in which the imaginary part is zero? If so, what did they look like?

SATCHLER: No. I never felt sympathetic towards the arguments in favor of that.

K. R. GREIDER, Yale University: In connection with the non-uniqueness of the optical potential, have you any criterion for choosing between the many sets of optical-model parameters?

SATCHLER: Only the criteria that I gave, namely, the shallow potentials do not give the right angular distributions. The deeper ones tend not to give the right angular distributions and tend to give too small cross sections. If the deuteron optical potential has any physical significance, I find it hard to believe it would be much more than about 100 MeV deep. Presumably it would be about 100 MeV deep.

GREIDER: In the last analysis, does it not depend on the fit to the particular rearrangement scattering process?

SATCHLER: 90% on that, 10% on prejudice.

F. G. PEREY, Oak Ridge National Laboratory: Concerning this ambiguity of the deuteron optical potential, I would like to say that the data at 27 MeV tend to show that 30- and 60-MeV potentials do not fit the data as well as the 100-MeV one and the one above. In other words, the data at 27 MeV now seem to indicate that the lower two potentials (30 and 60 MeV deep) do not fit the experiment.

E. V. IVASH, University of Texas: I wonder if you would comment about the importance of exchange effects—knock-out processes and similar sorts of effects? These have been neglected in your calculation.

SATCHLER: No, I am not prepared to comment on it for the reasons Norman Austern stated. We really do not understand how to do it yet.

INGLIS: In that connection, since we have now come a little closer to the comparison with experiment than we were in the preceding discussion, may I ask a question? In connection with the exchange, Norman Austern mentioned that this comes in most importantly when we talk about effects toward the rear (the heavy-particle stripping and the like). Is there any indication that it is more needed there than elsewhere? In particular, in your distorted-wave calculations there must be important effects which McCarthy has described in terms of this reflection from the far surface being focused backwards again. It is something like the "cat's eye" optics in the highway-sign reflectors. This seems to take care of some of the needs of backward intensity. Is it enough or does one still need something else from the exchange?

SATCHLER: I think the answer is that we do not really know. It can produce effects which are large enough; but whether it can do so when the correct parameters for any particular case are used, I do not know.

It does call to mind one feature I should have pointed out when comparing theory with the experiments. At least in the deuteron stripping cases, we were seeing agreement between theory and experiment over a range of magnitudes of some two orders; this suggests that there is nothing radically wrong with the form of the theory that is being used.

H. C. NEWNS, University of Liverpool: In your finite-range calculations, did you include the projection operator which means that only terms which are nondiagonal in the states of the final nucleus contribute to the matrix element? What I have in mind is Mittleman's

projection operator. If you assume that the proton wave function obtained from the optical model reproduces exactly the elastic scattering part of the exact proton/final-nucleus wave function then the normal DWBA matrix element is obtained except that the interaction is multiplied by a projection operator which means only nondiagonal elements in the states of the final nucleus contribute. This comes out quite naturally from the theory and should be included in the finite-range calculation.

SATCHLER: No, that was not included.

R. R. CARLSON, State University of Iowa, Iowa City: In the calculation for the $F^{19}(d, Li^6)$ reaction, did you use a Gaussian dependence for the α -d separation in your model for the Li^6 ? If so, how dependent is the angular distribution on the particular form you use for the Li^6 wave function?

SATCHLER: The particular curves I showed were calculated with zero range, which corresponds to a delta function for the product of the interaction times the Li^6 wave function. We have made one finite-range calculation with a Gaussian which, surprisingly enough, had rather small effect on the cross section. We have not explored the sensitivity to the finite-range parameters at all.

J. Y. PARK, North Carolina State University, Raleigh: I would like to ask one question. You mentioned in the beginning of the talk that you were going to talk mostly on deuteron-stripping reactions. May I assume that the general conclusions you have made on the effect of binding energy (nonlocality of optical potential and so forth) are equally applicable to other types of reactions, such as the $C^{13}(\alpha, n)O^{16}$ reaction? Have you investigated numerically some reactions other than stripping?

SATCHLER: We have not looked at that particular reaction, but quite clearly the effects will come in because I am talking about properties of radial wave functions which enter the calculation. Certainly the effects on the distorted waves will be present in any calculation which uses distorted waves.

PARK: I would like to comment that our finite-range distorted-wave analysis of the $C^{13}(\alpha, n)O^{16}$ reaction in the knockout model²¹ indicates that some of your conclusions are also applicable to reactions other than stripping. For example, we find that the finite-range effect reduces the magnitude of cross sections at all energies we have considered, namely, 12.8, 14.1, and 22.5 MeV. We get quite good agreements in absolute magnitudes. We find also that the spin-orbit effect on the angular distributions is very small.

E. GUTH, Oak Ridge National Laboratory: I would like to comment on the question of the nonuniqueness of the potential. Norman Francis, David Goldman, and I have used DWBA for (γ, n) reactions. This case is simpler than stripping, etc., because at least we know the interaction: it must be electromagnetic. We have for photoeffects also the nonuniqueness of the potential, but it can (at least partially) be resolved by looking at the polarization. For stripping, etc., there are no data on polarization as far as I know. (I have checked this with Francis Perey just this morning.) It seems to me, that until such measurements have been done, we have to reserve judgment about the physical significance of DWBA calculations. Probably, one can always represent the data—at least in a limited range of the variables involved—by some DWBA calculation; but whether the parameters to be used are "realistic" or not, remains to be seen.

N. M. HINTZ, University of Minnesota: I would like to ask about the other third of what you covered in your talk. This is on the question of how you treat the binding energy of a single-particle state in a nucleon-transfer reaction. I have heard at least three prescriptions for going through a series of isotopes where you want to extract meaningful spectroscopic factors. One is to leave the binding energy alone. Another is

²¹J. Y. Park, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848 (March 1964), p. 226.

to take the experimental separation energy which usually decrease as you increase A , let's say. The final thing you can do is to keep the well depth fixed and let the radius go as $A^{1/3}$. This causes the binding energy to increase with A . Which of these three do you think is the most meaningful procedure?

SATCHLER: That is a good question!

HINTZ: Is there any experimental evidence?

SATCHLER: Well, I don't think the experimental evidence distinguishes yet between the first and last of your alternatives. If you really think of this involving one shell-model orbital, one would perhaps think of having a constant well depth for all isotopes rather than a fixed binding energy. The evidence that is available now seems to indicate that either the first or the last is what is required, at least for that particular set of data.

R. SHERR, Princeton University: Dr. Henley asked me to report on the Princeton work on this question tomorrow. We have examined the (p,d) reaction on some 9 elements, and with Q values over a range some 10 MeV wide. There it turns out that using a shell-model separation energy, one accounts sensibly for the observations.

HINTZ: By that you mean you let the potential go as $A^{1/3}$ and let the binding energy go as for the single-particle wave functions in that well?

SHERR: We use a constant well depth and a radius which increases as $A^{1/3}$, independent of the actual Q value for the state in question. Maybe one had better wait until the experimental evidence is presented tomorrow.

J. G. CRAMER, JR., Indiana University: I would like to comment on Dr. Guth's question of a moment ago as to whether polarization measurements can be used to eliminate the nonuniqueness of the optical potential. My comments concern alpha-particle scattering, not (d,p) reactions, but I believe the problem is the same. We have evidence, based on DWBA analysis of our cross-section and angular-correlation measurements of the

elastic and inelastic scattering of 22.5-MeV alpha particles from Mg^{24} and Si^{28} , that the polarization of the excited nucleus following inelastic scattering, as reflected in the angular correlations between scattered particles and de-excitation gamma rays, is extremely sensitive to which of the ambiguous potentials is used. In the case of Si^{28} , we found that real optical potentials of 23, 50, and 85 MeV could all predict with reasonable accuracy the experimental elastic-scattering differential cross section. We found that the best fit to the experimental inelastic-scattering cross section was obtained from the lowest potential, while the worst fit came from the highest potential. Even more striking, however, were the corresponding predictions for the angular correlations. The correlation predictions of the two highest potentials were completely unsatisfactory, and looked nothing whatever like the data, while the prediction of the 23-MeV potential agreed very well with the data up to scattering angles in the neighborhood of 90° . We have obtained similar results in analyzing our data from Mg^{24} and Fe^{56} ; in all cases, the lowest of the ambiguous potentials seems to do the best job on the inelastic cross sections and angular correlations. Some of this is discussed in more detail in our contributed paper.²² I find it interesting that we needed potentials on the order of 20 MeV for alpha scattering, since you indicated that you needed 100-MeV potentials for your deuteron calculations.

SATCHLER: I think I should make two comments here. If the different potentials do give different inelastic scattering, one would expect polarization effects such as you are describing to also show differences. Experience (for example in alpha scattering at 40 MeV, where all of the equivalent potentials gave identical inelastic scattering) has been that the equivalent potentials also gave the same gamma-ray correlations.

²²J. G. Cramer, Jr., W. W. Eidson, and D. E. Blatchley, Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, p. 153.

If I may make the other point, your experimental data at 22.5 MeV, those you are referring to, show considerable fluctuations in that energy range. We have had a hard time trying to make some meaningful average of data like these on magnesium, for example, at Oak Ridge. The preference for the shallow potential may arise because of this.

CRAMER: I am familiar with some cross-section fluctuation measurements on Mg^{24} which were done by Kim at Oak Ridge, but these were at a somewhat lower energy than ours—18 MeV as I recall. However, I agree that there are going to be fluctuations in our energy region also, and it may well be that our beam spread and target thickness are not averaging them adequately. On the other hand, there is some indirect evidence that this may not be responsible for the effect that I mentioned. In examining the results of our DWBA calculations, we have found that there is an angular-momentum peaking in the partial-wave expansion, so that only a few partial waves (localized about what corresponds to the nuclear surface) are responsible for most of the inelastic scattering. This is just what one would expect for a direct reaction with strongly absorbed particles. Now my point is that the shallowest of the ambiguous potentials gives the purest example of this ℓ -value peaking, in that there are virtually no contributions from the lower partial waves. On the other hand, the deeper potentials show progressively larger contributions from the low partial waves and progressively poorer fits to the inelastic data. Now it seems to me that if compound-nucleus contributions to the reaction process were prejudicing one's choice of potentials, as you suggest, these contributions would introduce larger contributions from low-order partial waves into the experimental data and favor the choice of the higher, not the lower, potentials.

P. GOULD, University of Minnesota: This is a simple-minded question and you probably mentioned it in your talk; but I did not notice it.

Do you expect any lack of uniqueness in the elastic-scattering analysis of tritons?

SATCHLER: Our belief would be any complex strongly-absorbed particle would show this effect. It certainly shows up for He^3 , deuterons, and alpha particles; and tritons are presumably like He^3 .

E. ROST, Princeton University: I would like to comment on Dr. Hintz's question. We have some evidence that when one is treating (p,d) reactions in the same nucleus, he should use the same neutron wave function. Whether one obtains this from using a constant binding energy or by using a constant well depth is not clear. In one case there seems to be evidence that when you are comparing isotopic-spin analogues over a wide range in A, it is better to use a constant well depth than constant binding energy, but this is by no means conclusive.

SESSION B

MONDAY AFTERNOON

9 MARCH 1964

CHAIRMAN: H. E. GOVE

SCIENTIFIC SECRETARIES

R. E. HOLLAND

H. E. JACKSON

J. L. YNTEMA

STRIPPING REACTIONS WITH DEUTERONS

L. L. Green

University of Liverpool, Liverpool, England1. Distorted-Wave Born Approximation for the
 $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ Reaction

I think some of the more interesting things which are happening in (d,p) reactions at the moment are being covered in talks in other sessions—for instance, the spin effects which John Schiffer will be talking about later.

It is clear from the abstracts submitted and recent published work that with solid-state counter systems and with several multigap magnetic spectrometers in operation, a large quantity of high-resolution data is being obtained from (d,p) reactions. For instance, in addition to the $\text{Ca}^{40}(\text{d}, \text{p})$ work which Ray Satchler discussed this morning, work on the nuclei above Ca^{40} is appearing from M.I.T., Argonne, Michigan, Liverpool, and probably other places. DWBA analysis of these data with programs using zero-range approximation is necessary even for l -value extraction and is usual. Ray Satchler discussed the important effects of spin-orbit terms and finite-range effects on the Ca^{40} data. Since most people have access to zero-range programs without spin-orbit terms, I want to discuss the analysis of these results with one of these latter programs. The experiment I want to use as an example is the $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ reaction which we have studied at Liverpool over the deuteron energy range from 7 to 11 MeV and where the deuteron and proton parameters can be measured. In fact, I want to look at what happens when you do the best you can without finite-range spin-orbit terms. In the Ca^{40} case, we are adding a nucleon to a doubly closed shell; there one may expect spectroscopic

factors approaching unity for some states. In Cr^{52} , although we are dealing with a closed neutron shell there is evidence that the single-particle strengths are distributed, so it is more difficult to compare with any theoretical spectroscopic factors; but one can obtain the optical parameters from both the deuteron and proton scattering experiments and can investigate the dependence of angular distributions on these parameters. One can also investigate the dependence of the spectroscopic factors on energy and on the parameterization of the scattering.

The results on $\text{Cr}^{52}(\text{d},\text{p})\text{Cr}^{53}$, $\text{Cr}^{52}(\text{d},\text{d})$, and $\text{Cr}^{53}(\text{p},\text{p})$ were all obtained with surface-barrier detectors: a thin detector which measured the rate of loss of energy $\Delta E/\Delta x$, and a thick total-energy counter. The ΔE pulse was used for particle identification and the summed pulse for energy measurement. The energy resolution was good, so groups due to light- and heavy-element impurities could be accurately subtracted. Before embarking on optical-model and DWBA analysis of the results, we need to know that the excitation function does not contain large fluctuations. The excitation functions for the (d,p) reaction and deuteron elastic scattering are shown in Fig. 1.

Fig. 1. Excitation functions for $\text{Cr}^{52}(\text{d},\text{p})$ and $\text{Cr}^{52}(\text{d},\text{d})$.

Fig. 2. Deuteron elastic-scattering results.

At the time this experimental work was started, the optical model for deuteron scattering was not well understood. The deuteron parameters were thus most in need of determination so we measured the elastic deuteron scattering on all separated chromium isotopes at intervals from 7 to 11 MeV. At the energy at which the (d,p) reaction was studied, the lack of structure in the elastic-scattering angular distribution makes it fairly insensitive to the deuteron parameters. Typical results are shown in Fig. 2. To obtain the deuteron parameters from these results, an optical-model search program written by R. N. Maddison was used. This varies the optical-model parameters to find a minimum in χ^2 . For fitting the deuteron results, no spin-orbit term was used. The usual form factors for the optical model were used. For the real potential a Woods-Saxon form factor was used; for the imaginary potential either a Woods-Saxon form factor or a surface Gaussian was used. The same quality of fit to the experimental results could be

Fig. 3. (a) Satisfactory fits shown in the $U r_u$ plane. (b) Fits to deuteron elastic scattering with different potentials.

obtained with either of the absorptive potentials. There is of course always the $U r_u^n$ ambiguity (where U is the real potential and r_u is the radius of the real potential) so that regions of good fit correspond to a long narrow valley in the $U r_u$ plane. In addition there is a real ambiguity in the potentials giving a satisfactory fit to the data which Satchler mentioned at Padua¹ and which has also since been investigated by the Oak Ridge group. The regions of good fit in the $U r_u$ plane of the parameter space are shown in Fig. 3(a). The phase shifts for these different solutions are all very similar. Parameters lying between these valleys give angular distributions which are out of phase with the measured angular distributions. The Oak Ridge group have discussed the differences between these solutions in terms of the WKB approximation in which, for

¹G. R. Satchler, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3—8, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 80.

TABLE I. Distinct sets of parameters giving good fits to the 11-MeV deuteron elastic-scattering data on Cr^{52} .

	U (MeV)	W (MeV)	r_u (fm)	r_w (fm)	a_u (fm)	a_w (fm)	σ_R (mb)	Δ
6(G)C11	21.19	5.56	1.275	1.659	0.973	1.746	1505	448
6(G)A11	45.92	10.90	1.275	1.365	0.835	1.900	1383	210
6(G)B11	78.93	15.22	1.275	1.203	0.726	1.955	1325	217
6(G)D11	119.38	23.45	1.275	1.102	0.646	1.829	1248	329

the relevant ℓ values, a factor π is added to the phase shift. This can be seen in terms of the partial wave functions for each of these solutions for the chromium results. Only the first six ℓ waves have reflection coefficients η_ℓ appreciably different from 1 and the potential including an extra $\frac{1}{2}\lambda$ for the s wave does so quite well for all the relevant ℓ waves.

Figure 3(b) shows the fits produced by a number of the potentials which are specified in Table I. The parameters are plotted in the $U r_u$ plane of Fig. 3(a) and Figs. 4(a, b) show the $\ell = 0$ and $\ell = 4$ partial waves for these solutions. The potentials are labeled C, A, B, and D and correspond to real well depths of 21, 46, 79, and 119 MeV, respectively, with $r_0 = 1.275$ fm and correspond to 2, 3, 4, and 5 half waves in the nuclear well. [The notation used to describe the potentials is that 5(SW)B8 means a 5-parameter fit with a Woods-Saxon imaginary potential to the data at 8 MeV. A surface Gaussian form factor is denoted by G.] The quality of the fit with the C potential is worse than that obtained with the deeper potentials. This is probably because of the size of U compared with the centrifugal potential. There are of course many deeper potentials which will give equally good fits. These potentials all give the same asymptotic wave function, and elastic scattering cannot decide between them. It is questionable whether the wave function in the interior is very meaningful because of the polarization of the deuteron which must occur in

Fig. 4. The $\ell = 0$ and $\ell = 4$ partial wave functions for different potentials.

and close to the nucleus, but the different interior wave functions will give different DWBA matrix elements.

For the proton potentials we need elastic-scattering measurements at 14 MeV. We have measured the proton scattering up to 11 MeV to check these parameters. The elastic-scattering data were fitted with the same search program both with and without a spin-orbit potential since the DWBA calculations did not include spin-orbit terms in the distorting potentials. The same type of ambiguity as that encountered in the deuteron scattering is present at this energy. The usual solution for protons close to 50 MeV provides one fit and a potential with a 107-MeV real part includes an extra half wave in the nuclear well and gives an equally good fit to the data. Here, of course, one has no difficulty in choosing the 56-MeV potential. These results are shown in Fig. 5 with the two solutions. Table II gives the potentials obtained from the elastic-scattering experiments, which are used in the DWBA calculations with no further adjustment.

Fig. 5. Proton elastic-scattering results and optical-model fits.

TABLE II. Typical optical-model parameters from elastic-scattering measurements.

Scattering	No. of parameters (well shape) ^a classification	Well shape					
		Real well			Imaginary well		
		U	r _u	a _u	W	r _w	a _w
Cr ⁵² (d,d)	6(G)A	45.916	1.275	0.8346	10.090	1.365	1.900
E _d = 11.0 MeV	6(G)B	80.092	1.260	0.7304	15.116	1.198	1.962
	6(G)C	20.821	1.290	0.9552	5.575	1.650	1.746
Cr ⁵³ (p,p)	4(SW) U = 56	56.463	1.200	0.5704	9.686	r _u	a _u
E _p = 10.029 MeV	4(SW) U = 107	106.970	1.196	0.6558	13.230	r _u	a _u

^aFor the well shape, G denotes Gaussian, SW denotes Saxon-Woods.

We have measured the angular distributions for a number of proton groups for the $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ reaction for deuteron energies from 7 to 11 MeV and used the measured parameters at the appropriate energy to calculate the DWBA angular distributions. In this reaction there are $\ell = 1, 2, 3$, and 4 transitions. The DWBA program used was originally written by Buck and Hodgson² and extended to $\ell = 3$ and 4 by Scott.³ It uses the zero-range approximation for the product $V_{\text{np}} \cdot \psi_{\text{D}}$ and has no spin-orbit terms. The aim here was to compare the DWBA calculations with the different deuteron potentials with the experimental results and to see how the spectroscopic factors depended on the potentials and the bombarding energy. The groups for which the comparisons are made are all relatively strong. There are a number of weak groups (a factor of 10 weaker in intensity) whose angular distributions would not be at all well fitted by the DWBA curves.

Figure 6 shows the $\ell=1$ ground-state group with the first three potentials (C, A, and B) without radial cutoff in the integration. The B potential gives consistently better fits to the $\ell=1$ angular distributions independent of the parameterization of the elastic scattering—but the normalization is different for the different parameterizations of the scattering. This apparent choice of the B potential is stronger for higher ℓ values. Figure 7 shows an $\ell=3$ group with different potentials. Of course if we take a cutoff in the integration at a large enough radius, the different deuteron potentials must give the same DWBA angular distributions. If a cutoff is used in the integration at the Gamow-Critchfield radius $r = 6.25$ fm (well outside the nuclear surface for chromium), then the C, A, and B potentials all give the same angular distribution. Perhaps surprisingly, these give a

² B. Buck and P. E. Hodgson, *Phil. Mag.* 6, 1371 (1961).

³ H. D. Scott, Ph.D. thesis, University of Liverpool, 1961.

Fig. 6. DWBA for the ground-state group with C, A, and B potentials.
 $E_D = 8$ MeV.

Fig. 7. DWBA for $l=3$ group with C, A, and B deuteron potentials.
 $E_D = 8$ MeV.

Fig. 8. DWBA for $\ell = 1$ with radial cutoff at $r = 6.25$ fm. $E_D = 8$ MeV.

good fit (Fig. 8) to the $\ell = 1$ —quite comparable with that obtained with the B potential.

The (d,p) measurements were made at a number of energies. At 11 MeV the fitting is less sensitive because more of the cross section has moved into the main peak. The B potential still gives a slightly better fit without radial cutoff and again all the potentials give equally good fits with a radial cutoff at the Gamow-Critchfield radius. Figures 9 and 10 show the results and DWBA calculations at 11 MeV. The different shapes of the differential cross sections arise from the variation of the overlap integral with ℓ in the different potentials. This arises from the behavior of the deuteron partial waves in the different potentials. It is possibly fortuitous cancellations that cause the B potential to give a result similar to that obtained with a cutoff and agreeing with experimental results in this

Fig. 9. DWBA curves for $\ell = 1$ at $E_D = 11$ MeV without cutoff and with cutoff at $r = 6.25$ fm.

Fig. 10. DWBA curves for $\ell = 3$ at $E_D = 11$ MeV without cutoff and with cutoff at $r = 6.25$ fm.

Fig. 11. DWBA calculations with and without cutoff for $\text{Zr}^{90}(\text{d}, \text{p})\text{Zr}^{91}$.

mass region. At a higher mass, e.g., in Zr where there is a larger number of deuteron half waves in the well, the cancellations will be different. Here in fact (Fig. 11) the DWBA with cutoff at the Gamow-Critchfield radius does not appear to give quite such a good fit to the data as a B potential without cutoff.

One question affecting these results is that of the effect of the zero-range-interaction approximation. Buttle and Goldfarb⁴ report that if $V_D^{\text{opt}} \approx V_n^{\text{opt}} + V_p^{\text{opt}}$, then the finite-range angular distribution is very similar to the angular distribution with finite-range interaction. This is approximately so for the B deuteron potential. Their calculations on the effect of the finite-range approximation on the $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ calculation is shown in Figs. 12 and 13. These are not full finite-range calculations but rather are a method of simulating effects of the finite range while maintaining the form of the zero-range calculation. They contain some approximations which are difficult to estimate.

Although the shape of the angular distribution is correctly predicted either with the B potentials with no radial cutoff or with all the

⁴P. J. A. Buttle and L. J. B. Goldfarb, Proc. Phys. Soc. (London) 83, 701 (1964).

Fig. 12. The $\ell=1$ effect of zero-range approximation. After Buttler and Goldfarb, Ref. 4.

Fig. 13. The $\ell=3$ effect of zero-range approximation. After Buttler and Goldfarb, Ref. 4.

potentials with the cutoff far enough out, the energy variation of the peak cross section from 7 to 11 MeV is not correctly predicted. The experimental peak cross section for the ground state stays almost constant from 7 to 11 MeV whereas the DWBA peak cross section for the best fits increases by nearly a factor of 2 from 7 to 11 MeV. This effect shows in the change in the absolute width extracted from the data. These are shown in Table III where the absolute widths extracted from the data are shown for various levels at $E_d = 7, 8$, and 11 MeV. The relative reduced widths for the $\ell=1$ states, on the other hand, are fairly independent of

TABLE III. Absolute reduced widths for the $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ reaction at $E_d = 7, 8, \text{ and } 11 \text{ MeV}$. The analysis used the 6(G)B parameters (no cutoff).

State	ℓ_n	Spin and parity	Absolute $\theta_{j\ell}$		
			$E_d = 7 \text{ MeV}$	$E_d = 8 \text{ MeV}$	$E_d = 11 \text{ MeV}$
Ground	1	$3/2^-$	0.69	0.58	0.45
0.57	1	$1/2^-$	0.62	0.46	0.36
1.00	3	$5/2^-$	0.47	0.35	0.27
2.23	1	$3/2^-$	0.48	0.31	0.24
3.61	1	$3/2^-$		0.27	
3.70	4	$(9/2^+)$		1.40	
4.12	2	$(5/2^+)$		0.12	
4.21	2	$(3/2^+)$		0.12	

energy though there is a slight decrease in the values for the $\ell = 3$ with respect to those for the $\ell = 1$ transitions (Table IV).

The sensitivity of the relative and absolute reduced widths can be seen from Tables V and VI. The $\ell = 3$ in particular is quite sensitive to the A, B, or C deuteron potential used but is not particularly sensitive to how the deuteron optical model is parameterized. The predicted peak cross section (see last line of Table V) does vary with the quality of the fit to elastic scattering and with the different deuteron potentials. The relative and absolute widths predicted with a cutoff at 6.25 fm with all the deuteron potentials which fit the elastic scattering are quite close to the results obtained with the B potential without cutoff.

Another paper on this reaction has been submitted by Legg, Scott, and Mehta⁵ to this conference. They have studied this reaction at

⁵J. C. Legg, H. D. Scott, and M. K. Mehta, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 63.

TABLE IV. Relative reduced widths for the $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ reaction at $E_d = 7, 8$, and 11 MeV. The analysis used the 6(G)B parameters (no cutoff).

State	ℓ_n	Spin and parity	Relative $(2J + 1) \theta_{j\ell}^2$		
			$E_d = 7 \text{ MeV}$	$E_d = 8 \text{ MeV}$	$E_d = 11 \text{ MeV}$
Ground	1	$3/2^-$	100	100	100
0.57	1	$1/2^-$	44.5	39	38 ± 1.5
1.00	3	$5/2^-$	101	87	76 ± 5
2.23	1	$3/2^-$	68	53	53

energies between 3 and 6 MeV and report that reasonable fits to their angular distributions can only be obtained if a radial cutoff at 7.6 fm is used. They also suggest that the variation of peak cross section with energy is well reproduced, but the elastic scattering at these energies is very insensitive to parameter variations.

2. Stripping to Rotational States

In making spin assignments from (d, p) reactions on heavy nuclei, DWBA calculations are very necessary. Recently a number of studies on the two groups of heavy rotational nuclei have been made. The theory of stripping cross sections for reactions leading to rotational levels was discussed by Satchler⁶ some years ago. Studies in this region are useful for obtaining further information on these rotational nuclei, and where the Nilsson band ordering has been studied by other techniques for

⁶G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

TABLE V. DWBA predicted reduced widths for the $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ reaction at $E_{\text{d}} = 8.0$ MeV. The values were obtained with the indicated sets of parameters for the deuteron potentials.

State	ℓ_n	Spin and parity	Exptl. max.	Relative $(2J + 1) \theta_j^2$							Absolute θ 5(SW)B8 $\pm 15\%$	
				4(SW)B8	5(SW)B8	6(G)B8	4(SW)C8	6(G)A8	4(SW)B8(1)	4(SW)B7		5(SW)B8(2)
gnd.	1	$3/2^-$	22 080	100	100	100	100	100	100	100	100	0.50
0.57	1	$1/2^-$	9 467	39.0	39.5	40.0		39.0		39.0		0.40
1.00	3	$5/2^-$	1 583	91	89	87	39	62	105			0.30
1.29	?(1)		1 504		4.0							
2.32	1	$3/2^-$	16 791	53	53	54						0.27
2.66	(1)	$(1/2^-)$	4 012		12.0							
3.61	1	$(3/2^-)$	17 867	46	46	47						0.23
3.70	4	$(9/2^+)$	3 352	570	595	600	275					1.21
4.12	2	$(5/2^+)$	7 090	33	32	30	30					0.10
4.21	2	$(3/2^+)$	4 340		19.5							0.10
$\sigma_R(\text{mb})$				942	1152	1260	695	1340	1211		1192	
Δ				385	117.3	158.8	802	119.0	247		130.2	
$d\sigma/d\Omega$ at peak of gnd state (mb/sr)				23.59	18.06	15.56	27.37	15.18	16.40	27.08	17.89	

TABLE VI. Relative reduced widths for the $\text{Cr}^{52}(\text{d}, \text{p})\text{Cr}^{53}$ reaction at $E_d = 7.0$ MeV. The values were obtained with the indicated sets of parameters for the deuteron potentials.

State	ℓ_n	Spin and parity	Exptl. max.	Relative $(2J + 1) \theta_j^2$			
				4(SW)B7	5(SW)B7	4(SW)C7	5(SW)B8
Ground	1	$3/2^-$	10 500	100	100	100	100
0.57	1	$1/2^-$	5 104	44.5	44.5	43.5	44.5
1.00	3	$5/2^-$	1 074	111	101	54	119
2.32	1	$3/2^-$	10 510	69	68		
σ_R				771	1025	586	1018
Δ				169	89.5	243	143.1
$d\sigma/d\Omega$ at peak of ground state (mb/sr)				20.94	15.01	23.16	14.47

checking the predictions of stripping theories. With an even-even target (for instance, with $I_1 = K_1 = \Omega_1 = 0$) the nucleon is captured with j, ℓ to form the even-odd final nucleus in the various members of a rotational band with $I_2 = \Omega_2, \Omega_2 + 1, \dots$. To conserve angular momentum, the captured nucleon must have $j = I_2$. But the nucleon enters the same orbit (with Ω_2) for all levels of the band. So capture into successive levels explores one at a time the $j = \Omega_2, \Omega_2 + 1, \dots$ parts of the wave function for that orbit. There are also several sum rules and selection rules which apply.

In a paper contributed to this symposium, Macefield and Middleton⁷ discuss a study of the reaction $\text{U}^{238}(\text{d}, \text{p})\text{U}^{239}$ by use of a

⁷B. E. F. Macefield and R. Middleton, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 52.

multigap broad-range spectrograph. The angular distributions are somewhat insensitive to ℓ value but the ratio of the peak cross section to the 180° cross section can be used to identify the ℓ value. In this way a number of levels are identified with appropriate Nilsson bands and there is reasonable agreement between theoretical and measured cross sections.

Erskine⁸ has also used the (d,p) reaction to study the rotational levels in the tungsten isotopes. W^{183} is a well studied nucleus and was discussed in Kerman's paper⁹ on band mixing. Good agreement is reported between calculations using Kerman's parameters for W^{183} and the observed cross sections. This gives confidence in applying similar methods to the W^{185} and W^{187} isotopes where the levels up to 0.5 MeV could be interpreted as arising from three intrinsic states and good fits to the cross sections were obtained. Similar studies have been made on Ta^{182} by the $Ta^{181}(d,p)Ta^{182}$ reaction.

We can clearly expect application of stripping reactions to these nuclei. In conclusion I would like to show some curves due to Macefield, on the best energy at which to study these nuclei. These curves (Fig. 14) show calculations with typical optical-model parameters for $U^{238}(d,p)U^{239}$ for $\ell = 0, 2$, and 4 transitions over the energy range from 7 to 40 MeV.

⁸J. R. Erskine, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 86.

⁹A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 30 (15) (1956).

Fig. 14. DWBA calculations for $\ell = 0, 2$, and 4 transitions in $U^{238}(d,p)U^{239}$.

* * *

D. A. BROMLEY, Yale University: This morning we heard about cases in which a deuteron potential somewhere between 100 and 140 MeV in depth has been adopted; the alpha potential was reported to be about 20 MeV in some cases. You appear to come up with 70 to 80 MeV. You also mentioned that Goldfarb shows that the finite range would not be expected to change significantly, if I understand you correctly.

GREEN: For one particular potential, where $V_d^{\text{opt}} \approx V_n^{\text{opt}} + V_p^{\text{opt}}$.

BROMLEY: I have two questions. First, would you attribute the shift from perhaps 70 to something over 100 MeV to some spin-orbit effect which might be included later?

GREEN: No, this shift is quite trivial; the difference between 78 MeV and the range above 100 MeV comes merely from using a different point within the ambiguity in $U \cdot r^n$. There is no essential difference between the two potentials.

Satchler uses a small radius. In fact, if one uses that and looks at the calculations done at the point on the U - r valley with the

smaller radius, the fits are slightly better and there is a slight preference for the potentials with the smaller radius.

BROMLEY: My second question is this: To what extent does the apparent fluctuation in this deuteron potential represent sweeping all the other uncertainties in the problem under the rug by reflecting them instead in an effective deuteron potential? Could you, for example, hold the deuteron potential constant and vary something else?

In other words: To what extent is your work actually sensitive to the deuteron potential—as opposed to being generally sensitive to various parameters throughout the problem, the net effects being lumped in the deuteron potential for calculational purposes?

GREEN: This is an ambiguity of just solving a potential-well problem in which the number of l values contributing to the scattering is small.

A. G. BLAIR, Los Alamos Scientific Laboratory: Even though these angular distributions are considerably peaked near 0° at higher energy, is it not true that they are quite distinct?

B. E. F. MACEFIELD, Oxford University: The curves you saw were for one particular set of parameters. Those were really applicable only to a deuteron energy of about 12 MeV. If you used the scattering parameters for 30 or 40 MeV, you would find that all the reaction strength was within the first $10-15^\circ$. It is almost impossible to determine the l values unambiguously. I think that 20—25 MeV would be the optimum energy to study these reactions.

E. P. WIGNER, Princeton University: Is there any possibility of experimentally finding maxima and minima in the deuteron cross section similar to the Barschall maxima or the Peterson maxima in neutron cross sections? It is surprising that although you use the same model, I, at least, never saw such maxima. Are there any?

GREEN: I know of no experimental data on deuteron scattering which show these maxima through the periodic table.

WIGNER: If you made the calculation, would there be maxima?

GREEN: Yes, I guess there would. In fact, this is not a new problem. It did crop up in the early days. I think Adair was aware of it in the neutron strength functions. The original neutron-strength optical models were worked out about 19 MeV for the neutron-well data and it was on the grounds of the strength function that it was changed to the deeper potential which gives exactly equivalent results.

WIGNER: Yes, the neutron situation is clear; but one is surprised that the same model applies in this case, and yet nobody has demonstrated the maxima.

J. P. SCHIFFER, Argonne National Laboratory: I would just like to comment on the same point. I think the imaginary part of the deuteron potential is usually found to be much larger than for the neutron or proton potentials so that the widths of giant resonances would be very large compared with the spacings. Therefore, they would not show up in the same sense for deuterons as they do for neutrons. The existing experimental data indicate no evidence for such resonances, but since large imaginary parts of the potential are needed to fit the elastic scattering, there is no need to expect giant resonances with deuterons.

D. R. INGLIS, Argonne National Laboratory: I wonder if Professor Wigner's question, which seems to cast some doubt on the physical validity of this deuteron optical-model potential, does not deserve at least some further emphasis. As Schiffer said, there is so much absorption that it does not really act as an optical-model potential capable of showing resonances. Is it not related to the question of the cutoff that seems to be very prevalent in Dr. Green's work?

In view of the fact that one finds so much agreement by artificially cutting off the contributions from the interior of the nucleus, one

cannot help but wonder if it is not a mistake to start the calculation with the fundamental assumption that the whole "valley floor" of the optical potential contributes to the perturbing potential (or the common equivalent alternative assumption in the backward or pickup calculation). In the perturbation theory, should not the most important contribution come at the edge of the nucleus where the deuteron is experiencing a force from the steep derivative of the potential at the edge? Perhaps there should be a more specific contribution of the edge rather than the middle. One wonders if the success of the cutoff procedure is not a reflection of this aspect of the fundamental assumptions.

GREEN: Could I make a brief remark on that? The cutoff, as I see it, has nothing to do with the ambiguity in the deuteron potential that is present in the optical-model analysis of the deuteron elastic-scattering results. I would just like to emphasize the point I was trying to make about cutoffs. Good agreement with the results is obtained with the B potential of 78 MeV which is probably the potential one would pick on physical grounds. If one takes a cutoff far enough out, the different potentials must give the same DWBA curves. Rather surprisingly, at the cutoff radius where this happens the DWBA results are very similar to those obtained with the B potential without cutoff.

N. AUSTERN, University of Pittsburgh: I guess the only last thing I wanted to comment about, after hearing all of this interesting discussion, concerns the nature of this ambiguity in the deuteron potentials. Frankly, I think there has been too much made of it today. First of all, it is not remarkable that there should be such an ambiguity. One can say quite accurately that the elastic-scattering experiment determines the wave function everywhere outside the nucleus, with absolutely no ambiguity at all. In order to extend the wave function inside the nucleus, one must have more than the elastic-scattering experiment. One must have some understanding as to what is going on. The experimental values can be fitted only

for certain discrete values of the potential depth; there is no continuous ambiguity except for the simple one that is associated with the nuclear radius. Let us discuss the discrete ones. These represent different points of view about the nature of the interaction of the particle with the nucleus. We are not completely at sea regarding the nature of that interaction. We actually have some rather strong opinions as to what that interaction ought to be, and as pointed out this morning, it ought to be roughly the sum of the neutron and proton optical potentials; and these are very well known to us from many sources of information. There is every reason to think that the 100-MeV depth is proper, and that the others are extremely interesting curiosities.

GREEN: I agree with those remarks. The 100-MeV one corresponds to the 79-MeV one in our results. There is nothing particularly special about this ambiguity. It occurs in the neutron elastic scattering and in the proton elastic scattering.

B. L. COHEN, University of Pittsburgh: In reply to Prof. Wigner's question, it seems to me that you do not expect to get giant resonances in the deuteron cross section because there are waves corresponding to many values of ℓ . The thing that gives them in the Barschall case is that only $\ell = 0$ contributes. When you start getting to higher energies, you get many higher values of ℓ and the waves get out of phase with one another. Even for neutrons there are no giant resonances above 1—2 MeV. The effect for neutrons up to 20 MeV was investigated in some very beautiful experiments by Peterson¹⁰ at Livermore a few years ago. It turns out that you get things that look like optical-model resonances except that their shift with A is in the opposite direction. They are explained as a nuclear Ramsauer effect.

¹⁰ J. M. Peterson, Phys. Rev. 125, 955 (1962).

A STUDY OF THE REACTION $U^{238}(d,p)U^{239}$ †

B. E. F. Macefield

Nuclear Physics Laboratory, University of Oxford, England

and

R. Middleton*

Atomic Weapons Research Establishment, Aldermaston, EnglandAbstract of Report in "Vol. I. Contributed Papers"¹

Spectra and angular distributions for the $U^{238}(d,p)U^{239}$ reaction have been obtained and compared with the predictions of the Nilsson model and DWBA calculations. Predicted and observed intensities agree within the experimental uncertainties.

This reaction study arose from a program originally instigated to obtain accurate relative masses for the uranium isotopes from a series of (d,p) and (t,p) reactions together with their inverses. I shall discuss this afternoon some aspects of the (d,p) work that has been done and also add a few points in amplification of the contributed paper.

This work was carried out by use of the Aldermaston tandem generator and multichannel spectrograph. In a typical exposure, 5 mC of charge were deposited on the target over a period of approximately 12 hr.

† Contributed paper. Presented by B. E. F. Macefield.

* Present address: Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania.

¹ B. E. F. Macefield and R. Middleton, in Argonne National Laboratory Report ANL-6848, March 1964, p. 52. The part that follows the abstract is in amplification of this earlier report.

Fig. 1. DWBA angular distributions for the reaction $U^{238}(d,p)U^{239}$; $E_d = 12.0$ MeV, $Q = 2.59$ MeV. A reduced width of unity was assumed for each transition.

I shall take the contributed paper as read and just add a comment on Fig. 1 (a repetition of Fig. 3 on page 58 of the contributed papers¹). It is interesting to compare this series of curves with those that would be obtained if the calculations were performed for a lighter target element, say magnesium. In Fig. 1 we see an enhancement of the high- l transitions. In the magnesium case the ratio of the peak cross section for $l = 0$ to that for $l = 8$ would be approximately 10^5 , in contrast to the approximately 10^1 in the uranium case. This can be understood physically in that the radius of the uranium nucleus is approaching the classical capture radius for these high angular momentum exchanges.

I should now like to leave the contributed paper and indicate some results we have obtained for the $U^{235}(d,p)$ reactions at a deuteron energy of 12 MeV. Figure 2 shows an example of the spectra we obtained for this reaction. In this figure are indicated the positions where we expected to find the ground-state group together with the first group that

Fig. 2. Spectrum of the reaction $U^{235}(d,p)U^{236}$ at a deuteron energy $E_d = 12.09$ MeV. The data were taken at a laboratory angle of 80° .

we can reliably attribute to the $U^{235}(d,p)$ reaction. This discrepancy could obviously be due to an error in the masses. It is thought, however, that this is unlikely, since the Q value of the $U^{238}(d,p)$ reaction agreed with that calculated from the mass data, albeit within the latter's rather large uncertainty.

Table I shows the calculated transition strength to the ground-state rotational band of U^{236} on the assumption that the ground state of U^{235} is the Nilsson orbit [743]. This table indicates a further selection rule on the total spin of the captured nucleon, imposed by the angular-momentum coupling scheme.² The peak cross sections indicated in Table I are below the sensitivity of the one exposure yet obtained for the $U^{235}(d,p)$ reaction.

In conclusion, therefore, it seems that a combination of DWBA calculations and the Nilsson model enable one to make estimates of absolute cross sections which agree with observation to within the error of experiment.

² G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

TABLE I. Predicted intensities for transitions to the ground-state rotational band of U^{236} .

J_{π}^a	L_c^b	J_c^c	$\sigma(\theta)^d$ (μb)	$\sigma(\theta)^e$ (μb)
0^+	3	$7/2$	0.1	0.3
2^+	1	$3/2$	J forbidden ^f	
4^+	1	$1/2$		
6^+	3	$5/2$		
8^+	5	$9/2$	0.03	0.1
10^+	7	$13/2$	0.03	0.08
12^+	9	$17/2$	ℓ forbidden ^g	

^aThe spin of a member of the ground-state band of U^{236} .

^bThe ℓ value of the captured neutron.

^cThe J value of the captured neutron.

^dThe predicted peak differential cross section for $\eta = 4$.^g

^eThe predicted peak differential cross section for $\eta = 6$.^g

^fReference 2.

^gS. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, 16 (1955).

* * *

M. H. MACFARLANE, Argonne National Laboratory: One of the interesting things that emerged from a series of studies of (d,p) reactions on deformed nuclei by Sheline³ was that the excitation spectrum of (d,p) reactions to even-even final nuclei usually show a step shape. In other words, strongly excited states do not show up until the energy reaches about 1.5—2 MeV, about the energy expected for the onset of the new intrinsic configurations. In the U reaction the formation of low-lying states is further inhibited by selection rules so that the effect in question is even clearer.

³For example, W. N. Shelton and R. K. Sheline, Phys. Rev. 133, B624 (1964).

NUCLEAR STRUCTURE STUDIES WITH (d,p) AND (d,t) REACTIONS

Bernard L. Cohen

University of Pittsburgh, Pittsburgh, Pennsylvania

The deuteron stripping reactions, especially (d,p) and (d,t), have been very versatile tools in studies of nuclear structure. Some of the most important applications of them have been: (A) location of single-particle (S.P.) states, (B) location of single quasi-particle states, (C) degree of filling of S.P. states, (D) distribution of components of S.P. states, (E) spectroscopy with odd target nuclei, (F) neutron binding energies, and (G) miscellaneous. The items on the above list form the sections of this paper.

A. Location of Single-Particle States

A knowledge of the location of the single-particle states is a very basic ingredient in almost any nuclear-structure calculation. These may be determined from (d,p) or (d,t) reactions on closed-shell nuclei, or at least on nuclei in which the shell in question is completely full or completely empty. For example, the S.P. states in the $N = 50-82$ shell were determined¹ from (d,p) reactions on Zr^{90} . The states of Zr^{91} were thus located, and from the proton angular distributions, each is assigned to a S.P. state. In principle, one cannot distinguish between $d_{5/2}$ and $d_{3/2}$, but in practice this is rarely a problem because these are well separated in energy. Measurements of (d,t) reactions on Zr^{92} can also be used to

¹ B. L. Cohen, Phys. Rev. 125, 1358 (1962); B. L. Cohen and O. V. Chubinsky, Phys. Rev. 131, 2184 (1963).

Fig. 1. Locations of single-particle states in various nuclei.

resolve this problem—the ratio of excitation cross sections in (d,p) reactions to those in (d,t) reactions is much larger for $d_{3/2}$ than for $d_{5/2}$. The sum rule on S may be used to check whether all components of each S.P. state have been found. The "location" of a S.P. state is then just the center of gravity of its components.

A large amount of work of this type has been carried out at the University of Pittsburgh, the University of Michigan, and other laboratories, so that the locations of individual S.P. states can be traced through extensive mass regions (Fig. 1). For example, the $3s_{1/2}$ and $2d_{5/2}$ states have been traced from $A = 58$ to 140. An analysis of these locations² has given us a great deal of information about the shell-model potential from which S.P. levels are derived. This includes:

1. Symmetry energy. Locations of S.P. states in different isotopes of the same element indicate that the shell-model potential well for neutrons is shallower in a neutron-rich than in a neutron-poor isotope.

2. Self-binding. A single-particle state is more strongly bound when it is full than when it is empty. This is directly determined in the Ca and Zr isotopes, and is very strongly evidenced by the increased spin-orbit splittings where one member of a doublet is full and the other is empty; and by the lowering of states with large l relative to those with small l when a group of levels fills.

² B. L. Cohen, Phys. Rev. 130, 227 (1963).

3. Special neutron-proton interactions. A neutron state is lowered when a proton state of the same ℓ fills. This is observed for $g_{7/2}$ neutrons as $g_{9/2}$ protons fill, for $h_{9/2}$ neutrons as $h_{11/2}$ protons fill, and for $f_{5/2}$ neutrons as $f_{7/2}$ protons fill.

4. Effective mass. The shell-model potential is velocity dependent such that the effective mass for neutrons bound by a few MeV is about 1.5 times the actual mass. This is evidenced by the rate of change of binding energy of S.P. states with A , and by the energy spacings between major shells.

5. Spin-orbit force. In more than 20 cases, spin-orbit splittings of S.P. states are determined. The data are shown in Table I. When corrected for self binding, they exhibit roughly the $(2\ell + 1)$ dependence expected from an $\vec{\ell} \cdot \vec{s}$ force. The A dependence indicates that this is a surface force rather than a volume force.

6. Shape of shell-model potential. There is no appreciable lowering of high- ℓ relative to low- ℓ S.P. states when they are empty. This indicates that the shell-model potential is much like an oscillator potential—not midway between this and a square well as often stated.

B. Location of Single Quasi-Particle States

By methods similar to the above (except for the sum rule), the location of single quasi-particle states may be found in nuclei in which S.P. states are partly full. An example of this³ is shown in Fig. 2. These are of interest for comparisons with pairing theory or more sophisticated calculations. The agreement is usually very good.

³B. L. Cohen and R. E. Price, Phys. Rev. 121, 1441 (1961); E. J. Schneid, A. Prakash, and B. L. Cohen (to be published).

TABLE I. Spin-orbit splittings. All energies are in MeV.

Mass	ℓ	ΔE (MeV)	$\frac{\Delta E}{2\ell + 1}$	$\frac{\Delta E}{2\ell + 1} \cdot A^{1/3}$	Mass	ℓ	ΔE (MeV)	$\frac{\Delta E}{2\ell + 1}$	$\frac{\Delta E}{2\ell + 1} \cdot A^{1/3}$
208	1	0.90	0.30	1.77	96	4	6.74	0.75*	3.4
	2	0.96	0.19	1.14					
	3	1.76	0.26	1.51	90	2	2.70	0.54	2.4
	4	2.47	0.28	1.62		4	6.74	0.75*	3.4
	6	5.71	0.44*	2.6	48	1	2.03	0.68	2.5
140	1	1.37	0.46	2.4		3	8.17	1.17*	4.3
	2	2.2	0.44	2.3	40	1	1.8	0.60	2.0
	3	1.88	0.27	1.40		2	7.0	1.4	4.6
	5	6.05	0.55*	2.9		3	6.5	0.93	3.2
					16	1	6.16	2.05	5.2
138	2	2.4	0.48	2.5		2	5.08	1.02	2.6

Fig. 2. Locations of single quasi-particle states in the tin isotopes.

C. Degree of Filling of Single-Particle States

The sum of the S values gives a direct determination of how full a S.P. state is. Unfortunately, this depends on DWB calculations which are not always sufficiently accurate. In cases in which data on (d,p) and (d,t) reactions are available on several isotopes, these may be used directly. An example of such determinations³ is shown in Fig. 3. Other methods for reducing the dependence on DWB calculations are also available.⁴ These data may again be compared with pairing theory or

Fig. 3. Degree of filling of various single-particle states in the tin isotopes.

⁴B. L. Cohen, S. Mayo, and R. E. Price, Nucl. Phys. 20, 360 (1960); 20, 370 (1960); P. Mukherjee and B. L. Cohen, Phys. Rev. 127, 1284 (1962).

more sophisticated calculations, and they may be correlated with data from Sec. B above. Agreement usually is reasonably good, but many anomalies have been found.

In cases in which there are only two particles (or holes) outside a closed shell, determinations of the fullness are equivalent to a determination of the complete wave function. For example, for the ground state of Pb^{206} we found⁴

$$\begin{aligned}\psi(\text{Pb}^{206}) = & \sqrt{0.54} \, p_{1/2}^{-2} + \sqrt{0.20} \, f_{5/2}^{-2} + \sqrt{0.12} \, p_{3/2}^{-2} \\ & + \sqrt{0.12} \, i_{13/2}^{-2} + \sqrt{0.03} \, f_{7/2}^{-2}\end{aligned}$$

as compared with the theoretical expression

$$\begin{aligned}\psi(\text{Pb}^{206}) = & \sqrt{0.72} \, p_{1/2}^{-2} + \sqrt{0.13} \, f_{5/2}^{-2} + \sqrt{0.13} \, p_{3/2}^{-2} \\ & + \sqrt{0.01} \, i_{13/2}^{-2} + \sqrt{0.01} \, f_{7/2}^{-2},\end{aligned}$$

which had been expected to be quite accurate in this nucleus. The agreement is not very good. However, the experimental result has been confirmed by several independent methods.

Work of this sort has been done in Ce^{142} , Ba^{136} , Ni^{58} , etc., but in none of these cases is it as complete as in Pb^{206} .

D. Distribution of Components of S. P. States

The first step in any study of stripping reactions is to assign the nuclear levels to S. P. states; this amounts to a determination of the distribution of the components of S. P. states. One use of these data is to

compare with detailed nuclear-structure calculations where they are available. Another use is to compare with the prediction of giant-resonance theory⁵ that the distribution of components of a given S. P. state has a half width of about W , the depth of the imaginary optical-model potential. The depth W is well known as a function of excitation energy E^* from elastic-scattering and neutron giant-resonance studies. It is well approximated⁶ by

$$W \approx \frac{1}{3} E^* . \quad (1)$$

One might hope that an extrapolation of Eq. (1) would be valid in the low-energy region where neutrons are bound. This is tested in Fig. 4 where the application of Eq. (1) gives the horizontal bars. It is seen that these give a reasonably good representation of the widths of the experimental distributions. Once it is accepted, Eq. (1) is a very useful tool. For example, one knows in what energy region to expect levels of a given spin and parity. Another application of (1) is to predict whether a S. P. state is mixed into more than one nuclear level. For example, the $g_{7/2}$ S. P. state in Pb^{209} at 2.5 MeV has the configuration

$$[\text{Pb}^{208} (0^+) + g_{7/2}] 7/2^+; \quad E = 0 + 2.5 = 2.5 \text{ MeV}.$$

The lowest state with which it could mix is

$$[\text{Pb}^{208} (?^+) + g_{9/2}] 7/2^+ ,$$

⁵A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).

⁶B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 126, 698 (1962).

Fig. 4. Distribution of components of some single-particle states.

Heights of vertical lines are S values. Open circles are "centers of gravity," and horizontal bars indicate $\pm W$, where W is from Eq. (1).

a combination of the lowest positive-parity excited state in Pb^{208} (which is above 4.0 MeV excitation) and a $g_{9/2}$ neutron (the lowest S.P. state in the shell). From Eq. (1), there should be little mixing of this state if it is above $2.5 + (\frac{1}{3} \times 2.5) = 3.3$ MeV; since it is above 4.0 MeV, little mixing is expected. Experimentally none is found.

There are many other examples of this kind, including all the particle and hole states near Pb^{208} , the $g_{9/2}$ state in the $A \approx 60$ region, etc. In other cases one can understand the splitting into two levels.

Formula (1) is clearly not the whole story on "range of mixing." An interesting situation occurs in the comparison⁷ of isotones

⁷S. Hjorth and B. L. Cohen, "Nuclear Structure Studies in the Molybdenum Isotopes with (d,p) and (d,t) Reactions," Phys. Rev. (to be published).

Fig. 5. Levels excited in the 50-neutron and 56-neutron isotopes of Zr and Mo. Numerals are l values; 2^* is $d_{5/2}$ and 2 is $d_{3/2}$. Note that although the single-particle levels are nearly identical, as evidenced from the 51-neutron isotope, the degree of mixing is very much greater in the 57-neutron isotope of Mo than in that of Zr.

of Zr and Mo (Fig. 5). The great similarity between the states of Zr^{91} and Mo^{93} indicates that the S.P. states are nearly the same. However, the structures of Zr^{97} and Mo^{99} are entirely different; in the former, the $d_{5/2}$ is filled and all others are empty, but in the latter, many S.P. states (including $d_{5/2}$) are partly full. This indicates that the range of mixing is much larger in Mo, probably because the proton subshell is no longer closed. The quadrupole interaction predicts such an effect.⁸

E. Spectroscopy with Odd-A Target Nuclei

It is too well known to need discussion that when a 0^+ target nucleus is bombarded, the spins and parities of the states in the final (odd-A) nucleus are directly and simply determined by stripping-reaction studies. I will therefore concentrate on the more complex problem of spectroscopy with odd-A target nuclei.

A (d, p) [or (d, t)] reaction on an odd-proton target nucleus leads to the various states that can be formed by coupling the odd proton with the inserted neutron. For example, (d, p) reactions on Y^{89} (whose ground state contains an odd $p_{1/2}$ proton) lead to the configurations $(p_{1/2} d_{5/2})$, $(p_{1/2} s_{1/2})$, $(p_{1/2} g_{7/2})$, etc.; the identity of the configuration

⁸R. A. Sorenson (private communication).

Fig. 6. Levels found from Zr^{90} and $\text{Y}^{89}(\text{d}, \text{p})$ reactions by E. W. Hamburger. Spin assignments are obtained by assuming that the cross sections for exciting the various members of a given configuration are proportional to $(2I + 1)$.

is easily determined from the angular distribution. The experimental situation, as found by Hamburger, is shown in Fig. 6. Each configuration leads to several nuclear states with different spins; for example the $(\text{p}_{1/2} \text{d}_{5/2})$ gives 2^- and 3^- states. The cross sections for exciting these levels are proportional to $(2I + 1)$, so that the spins are easily identified if there is not too much mixing. Work of this type has been done in Sao Paulo, Pittsburgh, Florida State, M.I.T., Argonne, etc.

By studying (d, p) or (d, t) reactions on an odd-neutron target nucleus, one can often determine spins and parities of levels in the final even-even nucleus. The simplest example of this type occurs when the target nucleus is $s_{1/2}$ and the angular distribution indicates that the inserted (or withdrawn) neutron has $\ell = 0$; the state is then $(s_{1/2})^2$ which must be 0^+ . Many 0^+ states have been assigned in this way,⁹ including very interesting states in the two-phonon regions of Cd^{112} and Cd^{114} .

⁹ B. L. Cohen and R. E. Price, Phys. Rev. 118, 1582 (1960).

Any level excited in stripping reactions on an odd-neutron target nucleus is immediately assigned a parity and a range of spins. The cross sections for exciting states of a given spin in this range are subject to simple sum rules. In some cases, only one combination of spin assignments can satisfy these sum rules, so that these assignments can then be made.¹ Some examples of this are shown in Table II.

Nuclear-structure calculations usually determine wave functions for states; a knowledge of the wave functions immediately gives a prediction of the stripping cross section,¹⁰ which can then be tested experimentally. This has been especially useful in studies of collective levels, the understanding of which have been very important. Many interesting features can be understood without complex theory. For example, a (d, p) reaction on Pd^{105} (whose ground state is $d_{5/2}$) leading to the lowest 2^+ state of Pd^{106} goes principally by $\ell=0$ neutron transfer. This indicates that the 2^+ state is predominantly $(d_{5/2} s_{1/2})_2$ rather than $(d_{5/2})_2^2$.

F. Neutron Binding Energies

Odd-even mass differences play a very important part in nuclear-structure calculations; for example, the parameter Δ which plays a central role in pairing theory¹¹ is directly determined from them. They, in turn, are determined experimentally from neutron binding energies. More sophisticated nuclear-structure calculations now predict neutron binding energies with uncertainties less than 100 keV.¹²

¹⁰ S. Yoshida, Nucl. Phys. 25, 674 (1961).

¹¹ L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat-fys. Medd. 32, 9 (1960).

¹² I. Talmi, Rev. Mod. Phys. 34, 704 (1962); R. Avieu, E. Baranger, M. Veneroni, M. Baranger, and V. Gillet, Phys. Letters 4, 119 (1963); E. Baranger (private communication).

TABLE II. Data from $\text{Zr}^{91}(\text{d}, \text{p})\text{Zr}^{92}$ reactions. The levels in the first group have predominantly the configuration $(\text{d}_{5/2})^2$; the expected sums of cross sections for exciting states of $I = 0, 2$, and 4 are proportional to $(2I + 1)$. These sum rules are best fulfilled if the unknown level at 2.40 MeV is assigned 2^+ . The states excited by $\ell=0$ stripping have the configuration $(\text{d}_{5/2} \text{ s}_{1/2})$ which can couple to 2^+ or 3^+ ; the ratio is governed by the $(2I + 1)$ rule. The 2.07-MeV level is known to be 2^+ . If the 2.91-MeV level were also 2^+ , the sum of 2^+ would be greater than that of 3^+ , so the 2.91-MeV level must be 3^+ . Granting this, the sum of 3^+ would be excessive if either the 3.06- or 3.03-MeV states were 3^+ . They must therefore be 2^+ .

Excitation energy	ℓ	I_f	S'	$\sum S'$		Excitation energy	ℓ	I_f	S'	$\sum S'$	
				Exp.	Theor.					Exp.	Theor.
0	2	0	0.24	~ 0.34	0.33	3.49	2	1—4	0.46		
1.38	2	0	~ 0.10			3.69	2	1—4	0.33		
0.94	2	2	1.11			3.81	2	1—4	0.44		
1.88	2	2	0.36	1.70	1.67	4.03	2	2—4	0.69	4.26	4.00
2.40	2	(2)	0.23			4.14	2	3—4	0.93		
						4.50	2	1—4	0.44		
1.50	2	4	2.9	2.9	3.00	4.65	2	1—4	0.23		
2.07	0	2	0.30			4.97	2	1—4	0.35		
2.66	0	2,3	0.15			5.30	2	1—4	0.39		
2.91	0	3	1.24	2.79	2.00	4.80	4	(6)	2.4	2.4	8.0
3.06	0	2	0.50			5.10	1	1—4	0.21	0.46	4.0
3.30	0	2	0.60			5.50	1	1—4	0.25		

It is widely believed that neutron binding energies are well known from mass data. This definitely is not the case. Firstly, mass data are not known with sufficient accuracy in the heavy-element region; and secondly, pairs of nuclei differing by one neutron are rarely both stable, so that the two masses cannot be directly determined. One of the best ways of determining neutron binding energies is from the Q values of (d, p) and (d, t) reactions. Examples of this type of data are shown¹³ in Fig. 7. Note the breaks in the smooth curves at 56 neutrons in Zr (but to a much lesser extent in Mo) and at 64 neutrons in Sn (but not in Cd). These are indications of closed subshells. The Zr data have been fitted in accurate detail by Talmi,¹² and there has been considerable progress in fitting the Sn data.¹²

G. Miscellaneous

Many specialized nuclear-structure problems receive important illumination from stripping reactions. We mention a few of them here.

The purity of the so-called single-particle states in closed-shell-plus-one nuclei is an interesting question. For example, the ground state of Zr^{91} is widely assumed to be purely

$$\text{Zr}^{90} (\text{Ground}) + d_{5/2}.$$

However, (d, t) reactions on Zr^{91} go to excited states to some extent; for example, they go to the 2.2-MeV 2^+ state with 5% probability.¹ This indicates that 22% of the Zr^{91} wave function is

¹³ B. L. Cohen, R. Patell, A. Prakash, and E. J. Schneid, "Neutron Binding Energies in Heavy Nuclei," Phys. Rev. (in press).

Fig. 7. Neutron binding energies as determined from the Q values of (d,p) and (d,t) reactions.

$$[\text{Zr}^{90}(2^+) + d_{5/2}]_{5/2^+}.$$

It is often assumed that a level in an odd nucleus can be either "collective" or "single-particle," but not both. However, this has turned out not to be the case. Stripping-reaction studies¹³ showed the principal collective level in Sn^{117} to be the principal $d_{5/2}$ single-particle level. In Sn^{119} , on the other hand, this is not the case.

Since a (d,p) reaction is essentially a technique for inserting a neutron into a nucleus, it seems reasonable that it should be connected with neutron cross sections. Indeed, there should be a relationship between the neutron reduced width θ_0 and the S function from stripping. This relationship can be predicted theoretically but with a large uncertainty. We have recently succeeded in establishing an experimental connection¹⁴ by observing two levels in Ni which are known from neutron cross-section work. It agrees with the theoretical prediction within the expected error. We have used this to locate the $s_{1/2}$ single-particle state in Ni, and thus to tie down the location of the neutron giant resonance.

We thus see that stripping-reaction studies are a very valuable tool in a wide range of nuclear-structure investigations.

* * *

N. M. HINTZ, University of Minnesota: You made a comment that you can determine the wave function completely by determining the occupation amplitudes. That is true only if you assume pairing wave functions of the kind you wrote down.

I think you need something else to determine the signs of the terms in the wave function. For instance, two-nucleon-transfer reactions.

COHEN: How else can you get a 0^+ state unless everything is paired?

HINTZ: You are undoubtedly correct for 0^+ states. I am talking about other states.

COHEN: Every case I have talked about is a 0^+ state. A 2^+ state is much more complicated.

B. BAYMAN, Princeton University: I think even for 0^+ states, Norton Hintz's point is well taken. You have studied the Pb^{206} wave function

¹⁴R. H. Fulmer, A. L. McCarthy, B. L. Cohen, and R. Middleton, Phys. Rev. 133, B995 (1964).

by the Pb^{207} (p,d) reaction. I think we all agree that you know the absolute magnitudes of the (j^2 I=0) components. Your experiment does not give you the relative phases of these components, and these are really an essential part of the wave function. Before you can say you know the wave function, you have to know these relative signs. This might be related to what Norton said.

COHEN: I agree. That is right.

BAYMAN: Since you have found a 2^+ proton component in the Zr^{91} state, does that not cause you to worry a little bit about your use of the $(2I + 1)$ rule? Isn't it right that the $(2I + 1)$ rule depends on the model and rests on the assumption that your target state is a 0^+ proton part plus just a single neutron?

COHEN: Yes. However, it does rest on the square of the amplitude. So it is really as though it is 5% non-single-particle for that purpose.

F. S. LEVIN, Brookhaven National Laboratory: Going back to that discrepancy between the theoretical calculations and your experimental number in Pb^{207} , isn't it true that all the experimental methods that you used rely on a particular physical model for the analysis, and it could be that the model that you consider is just too simple? After all, there is nothing sacred, yet, about the present form of the distorted-wave theory and small corrections could change your numbers. I don't think it is heresy to suggest that the model is not yet completely understood nor in final form.

COHEN: I would say that that number is at least as sure as anything that will be said at this entire conference.

N. AUSTERN, University of Pittsburgh: Is the use of the $(2I + 1)$ rule cast in doubt by the effect discussed this morning, namely the manner in which the radial shape of the wave function affects the spectroscopic factor? In particular, the Q changes as the j changes; and therefore the

wave function pulls in and out. This may affect the strength of the cross sections rather appreciably. I wonder if you have observed any systematics on this effect.

COHEN: I do not see what this has to do with it. The $(2I + 1)$ rule applies when you have a definite configuration which is at a definite energy. The question is: What value of I do they couple to? This is where we use the $(2I + 1)$ rule.

J. P. SCHIFFER, Argonne National Laboratory: I think your $(2I + 1)$ rule is correct only if the shell-model state into which the neutron goes is empty. I do not think it has anything to do with the binding energy. It can be completely wrong if you use it to assign spins in cases in which the single-particle state which you are populating is partially occupied. There is no reason to expect that the total summed strengths for the particular final states will be proportional to $(2I + 1)$ in general.

COHEN: Actually, in every case I talked about, they were empty.

SCHIFFER: I would like to comment on one other thing, namely your rule of W being one-third of the excitation energy. I think there are quite a few cases now which indicate that there are rather large additional terms which would have to be added onto such an approximation to W . In particular, if you want to talk about W at low excitation energies, it is really quite large. Even at the ground state it can be 2—3 MeV in many nuclei.

COHEN: Figure 4 had many cases that said that this was not true. Of course, you are used to light nuclei, while I am used to heavy nuclei. In heavy nuclei at least, it is not so.

It is true, of course, that this $W = \frac{1}{3} E^*$ is far from the whole story. In fact, Fig. 4 shows some deviation from this simple rule—but not too drastic, I think.

R. G. WINTER, Pennsylvania State University, University Park: I have heckled you before with the argument that a spin-orbit splitting

proportional to $(2\ell + 1)$ requires that the $\vec{\ell} \cdot \vec{s}$ interaction is a volume effect. If it were a surface effect, then the splitting would be expected to increase a little more rapidly with ℓ .

My question is this: Have you considered your experimental spin-orbit splitting to be something that might tell you about how the spin-orbit interaction is distributed through the nucleus?

COHEN: You catch me unprepared on this matter of the splitting not being proportional to $(2\ell + 1)$ for a surface force. I did check to see if there were any systematic deviations from the $(2\ell + 1)$ rule, i. e., to see if the splittings were too large or too small for the higher values of ℓ . I found no simple trend.

K. K. SETH, Northwestern University: How large an effect of symmetry energy did you find? What was the depth of the part of the potential that is proportional to $(N - Z)/A$?

COHEN: The real potential changed by a couple of MeV over 6—8 mass units. Actually, it is very similar to the symmetry in the optical potential.

SETH: How well does the symmetry energy agree with such estimates as Lane's?

COHEN: The one I found to be in agreement with it was the one that Satchler was using.

DIRECT REACTION STUDY OF THE Ca^{41} AND Ca^{49}
SHELL-MODEL LEVELS*†

T. A. Belote, E. Kashy,† A. Sperduto, H. A. Enge,
and W. W. Buechner

Department of Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology
Cambridge, Massachusetts

Abstract of Report in "Vol. I. Contributed Papers"¹

The shell-model level structures of Ca^{41} and Ca^{49} have been studied by the stripping reactions $\text{Ca}^{40}(\text{d},\text{p})\text{Ca}^{41}$ and $\text{Ca}^{48}(\text{d},\text{p})\text{Ca}^{49}$. An analysis using DWBA has permitted the identification of single-particle level structure in the two nuclei, and a comparison between the positions and spectroscopic factors of the levels is given. The possible role of shell-model hole states is also discussed.

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† Contributed paper. Presented by T. A. Belote.

† Now at Princeton University, Princeton, New Jersey.

¹ T. A. Belote, E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, in Argonne National Laboratory Report ANL-6848, March 1964, p. 172. The text that follows the abstract is in amplification of this earlier report.

The deuteron-stripping $\text{Ca}^{40}(\text{d}, \text{p})\text{Ca}^{41}$ reaction² and the $\text{Ca}^{48}(\text{d}, \text{p})\text{Ca}^{49}$ reaction³ have been used for an investigation of the shell-model levels of Ca^{41} and Ca^{49} . These reactions should serve as a sensitive tool for determining shell-model levels of these nuclei, in particular, those states corresponding to the single-particle levels. In these experiments, the DWBA direct-reaction code JULIE (originated by Satchler et al. at Oak Ridge National Laboratory) was used to extract captured-neutron angular momenta and spectroscopic factors for the states showing forward stripping angular distributions. The results of this work are summarized in Fig. 1 which shows the presumed components of the single-particle levels of Ca^{41} and Ca^{49} , together with the sums of their measured spectroscopic factors. The $\text{Ca}^{49}(2\text{p}_{3/2})$ ground state is shown displaced to the center of gravity of the $(2\text{p}_{3/2})$ level in Ca^{41} . Only the single-particle levels are shown in the figure. In the region of excitation up to 6.825 MeV, a total of 120 states was observed in Ca^{41} ; whereas only eleven states were seen in Ca^{49} below 6.1 MeV. It is interesting, in addition, to note the degree of fragmentation of the $(2\text{p}_{3/2})$ and $(2\text{p}_{1/2})$ single-particle levels in Ca^{41} compared with the one component corresponding to these orbitals in Ca^{49} .

In addition to the stripping reactions that leave the neutron in a higher shell-model orbital, it is also possible that the stripping takes place by lower shell-model orbitals—for example, by $(1\text{d}_{3/2})$. This would be the case where the ground-state wave functions are not described as simply the lowest possible shell-model orbitals. Such stripping appears to take place for the 2.017-MeV $\ell_n = 2$ level in Ca^{41} .

²T. A. Belote, J. Rapaport, and W. W. Buechner, Bull. Am. Phys. Soc. 9, 79 (1964).

³E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, Phys. Rev. (to be published).

Fig. 1. The presumed components of single-particle levels in Ca^{41} and Ca^{49} , together with the sums of their measured spectroscopic factors. The Ca^{49} ground state is displaced to the center of gravity of the $2p_{3/2}$ level in Ca^{41} .

* * *

A. K. KERMAN, Massachusetts Institute of Technology: You remarked that you observed some 100 levels all together in Ca^{41} and that you were able to identify a few of these as having single-particle strength. What was the density of levels in the neighborhood of those single-particle levels you identified?

BELOTE: There are 25 levels up to about 4.2 MeV excitation. Between 4.2 and 6.8 MeV, we see about 95 more levels. The average spacing above 4.2 MeV is about 25 keV. We are working near the limit of resolution of the spectrograph, and we see lots of very close doublets and partially resolved levels.

KERMAN: I would like to draw attention to the fact that you do not see many levels, each with only a little bit of the particle strength; you only see 3 or 4 in a region of high level density and these have the entire single-particle strength. I think this deserves comment.

N. S. WALL, Massachusetts Institute of Technology: Kerman has just pointed out that you see a few strong single-particle states and that their strength is not shared among the many nearby levels. Is this not a result of the technique used to extract the information? How do you know that the S values that you get are correct? Suppose they are off by a factor of 2, or by 50% or 20%—which Satchler said would not be unreasonable—and the strength is spread over these 20 levels. Would such a difference show up in terms of the stripping pattern?

BELOTE: Yes, it depends on the technique used to extract the spectroscopic factors; the important point here is the comparison between the spectroscopic factors in Ca^{41} and Ca^{49} .

P. AXEL, University of Illinois: Just to clarify Kerman's point, is there any evidence that any of the levels in that region have the same spin and parity as the ones that have the single-particle strength?

BELOTE: No, we don't see any evidence in this experiment, but some of the weak levels may have the same spin and parity and also some of the strength. However, the strength of these weak levels is expected to be only a negligible part of the total single-particle strength.

WALL: I would like to know if anyone has analyzed the level density by putting in the known angular-momentum states and deducing a nuclear

temperature. If so, how does that temperature compare to what one gets from, let us say, slow-neutron experiments?

BELOTE: A level-density plot seems to indicate that we may be missing some levels in the region of high excitation. They may be too weak or too close together to be observed.

B. E. F. MACEFIELD, Oxford University: Is the difficulty you encounter in examining the nature of the weak levels simply that of finding acceptable distorted-wave fits or is it that the angular distributions do not show stripping characteristics.

BELOTE: The weak levels are fairly isotropic. They do not seem to have stripping characteristics.

E. P. WIGNER, Princeton University: I am puzzled by the fact that even in Ca⁴¹ there are levels that we do not understand. For instance, there are two $p_{3/2}$ states close together at the bottom where one would expect only one such level to exist. Does the second level have a shell-model assignment? The lowest configuration that I can think of that would give a $p_{3/2}$ level is $(d_{3/2})^2 (f_{7/2})^2 p_{3/2}$. It would seem to be a high level. Is there an interaction that can push it down so much? (There is, of course, the interaction with the regular $p_{3/2}$ level, but that would push it up.)

KERMAN: I cannot answer Prof. Wigner's question specifically, but I believe that this is evidence that Ca⁴⁰ is not as good a closed shell as is ordinarily assumed. This is also shown by some of the properties of the ground state itself. For example, its magnetic moment cannot be explained by any simple configuration mixing.

K. K. SETH, Northwestern University: In partial answer to Dr. Wall's question, some people indeed have tried to compare level densities as determined by "measurements" of nuclear temperatures with those determined from counting neutron levels. However, it is a rather discouraging comparison since both processes are selective in the nature of the levels they populate. For example, at low neutron energies you see

only s-wave levels; resonances of higher angular momentum, which are certainly there, do not show up at all. On the other hand, (d,p) reactions easily transfer neutrons with high angular momenta. Because of this, it is very difficult to compare the densities obtained by the two ways of counting levels.

The problem is further complicated by the big gap in excitation energy over which we know nothing about level spacings; there is usually quite a large gap between the energy at which (d,p) work leaves off and 8 MeV where the counting of neutron levels begins. Thus the relationship between the two counts is difficult to establish unambiguously.

N. K. GLENDENNING, Lawrence Radiation Laboratory, Berkeley: Another possibility in connection with the point that Prof. Wigner raised is one that we would all like to forget about. This is the possibility that the outgoing proton interacts with the core of the nucleus and excites it; i. e., the nondiagonal part of the proton-target interaction cannot be subtracted by the optical potential but nevertheless is always neglected. This, of course, allows a stripping reaction to place a particle into a given shell-model orbit; but at the same time it excites a state other than the fundamental one. Possibly this is the mode of excitation of the weaker levels.

B. ZEIDMAN, Argonne National Laboratory: Recent work at Argonne on pick-up reactions indicates that the energy necessary to break the Ca closed shell is in fact rather small. Thus the difficulty with the two $p_{3/2}$ levels pointed out by Prof. Wigner may not be so serious.

J. L. YNTEMA, Argonne National Laboratory: You observe an $\ell_n=2$ level at 2 MeV, which is approximately the position in which the hole state occurs in the reaction from Ca^{42} to Ca^{41} . What is the strength of the transition?

BELOTE: The $\ell_n=2$ level at 2.017 MeV has a spectroscopic factor equal to 0.07 on the assumption that $J=\frac{3}{2}^+$.

COMPLEX SINGLE-NUCLEON STRIPPING AND PICKUP
REACTIONS $[(\alpha, t), (d, \text{He}^3), \text{ETC.}]^*$

Allen G. Blair

Los Alamos Scientific Laboratory, University of California,
Los Alamos, New Mexico

The subject of my talk is the study of nuclear spectroscopy with the less common single-nucleon stripping reactions (He^3, d) , (α, t) , and (α, He^3) , and the inverse pickup reactions. These are the analogs of the (d, n) and (d, p) reactions and their inverses, but there are special features which sometimes make them more attractive than the simpler reactions. For example, the (He^3, d) and (α, t) reactions transfer a proton to the target nucleus, as does the (d, n) reaction; but the former involve an outgoing charged particle, with the associated ease and efficiency of detection and higher energy resolution. As another example, the study of the (He^3, α) reaction may in some cases be preferable to that of the (p, d) or (d, t) reactions because of its much higher Q (18.3 MeV and 14.3 MeV higher, respectively). A similar argument holds for the (t, α) reaction vs the (n, d) reaction.

Let me illustrate a very few of the more recent applications of some of these reactions to studies of level schemes. Fisher and Whaling,¹ at the California Institute of Technology, have investigated the level structure of B^9 by observing the (He^3, α) reaction on B^{10} . Using a 10-MeV He^3 beam, they were able to search from zero excitation to 17 MeV.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

¹T. R. Fisher and W. Whaling, Bull. Am. Phys. Soc. 8, 598 (1963).

They located one new B^9 state at 11.62 MeV. Clayton,² also at the California Institute of Technology, investigated the level scheme of N^{14} in the 4—8 MeV range of excitation by means of the (He^3, α) reaction on N^{15} , taking advantage of the 9.7-MeV Q of this reaction. In this case the energy of the He^3 beam was about 3 MeV. Of particular interest was the possible existence of a level near 7.6 MeV of excitation, since such a state could serve as a resonance for the $C^{13}(p, \gamma)N^{14}$ reaction for low-energy protons. Such a resonance is of interest to cosmologists, since the rate of this (p, γ) reaction is instrumental in determining the amount of C^{13} that can persist in stellar interiors. Since no such level was found from the (He^3, α) reaction, the conclusion could be drawn that the so-called "CNO" cycle adequately explains the observed C^{12}/C^{13} abundance ratio in stars.

Taylor,³ at Manchester, has compared the spectra of Ar^{38} and K^{38} , obtaining the level positions by observing the $K^{39}(t, \alpha)Ar^{38}$ and $Ca^{40}(d, \alpha)K^{38}$ reactions. He made $T=1$ assignments to states of K^{38} on the basis of low cross sections from the (d, α) reaction and the positions of these states relative to the low-lying ($T = 1$) states in Ar^{38} .

Hinds, Marchant, and Middleton⁴ at Aldermaston have investigated the energy level schemes of several of the light-to-medium-mass nuclei by means of one or more of the reactions under discussion. An example of the kind of results they have achieved is shown in Table I. Note that the high Q of the (t, α) reaction allows an additional 2 MeV of excitation to be studied. Above this the level density becomes too great for suitable

²D. D. Clayton, Phys. Rev. 128, 2254 (1962).

³I. J. Taylor, Nucl. Phys. 41, 227 (1963).

⁴See, for example, S. Hinds, H. Marchant, and R. Middleton, Nucl. Phys. 38, 81 (1962); Proc. Phys. Soc. (London) 78, 473 (1961).

TABLE I. The energy levels of O^{18} . All energies are in MeV.

Group number	From $O^{16}(t,p)O^{18}$	From $F^{19}(t,\alpha)O^{18}$	Ajzenberg-Selove and Lauritsen
1	1.980 ± 0.005	1.97	1.980
2	3.549 ± 0.005	3.55	3.552
3	3.627 ± 0.005	3.63	3.634
4	3.915 ± 0.005	3.92	3.916
5	4.449 ± 0.005	4.45	4.448
6	5.090 ± 0.005	5.10	5.067
7	5.247 ± 0.007	5.25	5.170
8	5.329 ± 0.007	5.33	5.311
9	5.368 ± 0.010	5.37	5.456
10	5.521 ± 0.010	5.51	
11	6.189 ± 0.010	6.18	6.190
12	6.341 ± 0.010	6.34	6.328
13	6.391 ± 0.010	6.39	
14		6.86 ± 0.02	
15		7.10 ± 0.02	7.115
16		7.60 ± 0.03	7.622
17		7.75 ± 0.02	
18		7.84 ± 0.02	
19		7.96 ± 0.02	
20		8.02 ± 0.02	8.041
21		8.11 ± 0.02	
22		8.19 ± 0.02	8.213
23		8.26 ± 0.02	8.286
24		8.39 ± 0.02	(8.403)
25		8.48 ± 0.02	
26		8.64 ± 0.02	

analysis. In another experiment, Hinds and Middleton⁵ have located a previously undetected state at 6.345 MeV in C^{11} , by means of the (He^3, d) reaction on B^{10} . This state had been the subject of several unsuccessful searches via the (d, n) reaction. (It had been obscured by a strong transition to the 6.476-MeV level.)

When one examines the mechanism by which these reactions proceed, the situation for low-mass targets is somewhat confused. It seems likely that (at least in some cases) clustering effects in the nuclei are important, and the reactions may proceed by knockon and heavy-particle stripping.⁶ However, for heavier nuclei (say, above Ne) and at higher energies (above, say, 5 or 6 MeV) the dominant mechanism is probably one of simple stripping or pickup. It is mainly to a discussion of these cases that I shall confine the remainder of my talk.

Although distorted-wave (DW) calculations are being used in increasing frequency in the analysis of stripping and pickup reactions, there are many laboratory groups who still analyze their data by means of the plane-wave (Butler) calculation, most often because they do not have ready access to a large computer or to a friend who does. In addition, there is frequently insufficient information on the optical-model parameters used in the DW calculation. This gap, however, is rapidly being filled. There is also the criticism that an optical-model calculation lacks validity for the lighter nuclei. In principle, it should be possible to analyze the reactions under discussion by means of a plane-wave calculation and to extract information of the same quality as has been obtained in the past for

⁵ S. Hinds and R. Middleton, Proc. Phys. Soc. (London) 78, 81 (1961).

⁶ H. D. Holmgren and E. A. Wolicki, Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961, edited by J. B. Birks (Heywood and Co., Ltd., London, 1961), p. 541.

(d,p) and (d,t) reactions. However, even for (d,p) and (d,t) reactions the results are often conflicting, and in the more complicated reactions there are additional difficulties. I shall discuss some of these difficulties, but I would first like to say that I would advise trying to get either a computer or a friend.

I shall first consider the possible application of a plane-wave analysis to the (d, He^3) pickup reaction; the extension to the other reactions will be obvious. One would like to obtain a normalization factor between the (d, He^3) reaction and the corresponding (n, d) reaction in order to extract spectroscopic factors from the angular distributions. This is done for the (d, t) and (p, d) reactions and their inverses; an average normalization value of $\Lambda \approx 200$, in the notation of Macfarlane and French,⁷ is obtained. There is, however, a great amount of scatter, individual cases often vary by a factor of 2 or more from this value.^{7,8} One should not expect to use the value $\Lambda \approx 200$ for (d, He^3) reactions, largely because of the difference in the Coulomb field. Indeed, in many cases the normalization factor for this reaction is only a quarter or a fifth of that for the (d, t) reaction; there is at present insufficient information to allow the establishment of a preferred value.

Also, because the classical Coulomb barrier is twice as high for the outgoing He^3 particle as for an outgoing singly-charged particle of the same energy, the cross section for a (d, He^3) reaction will begin to drop off at a higher energy than that for a (d, t) reaction on the same nucleus. A few MeV below the Coulomb barrier, the cross section may be quite small. For example, Cujec⁹ found that it was not possible to

⁷M. H. Macfarlane and J. B. French, *Rev. Mod. Phys.* 32, 567 (1960).

⁸R. N. Glover, Vol. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 104.

⁹B. Cujec, *Phys. Rev.* 128, 2303 (1962).

Fig. 1. Comparison of the $\text{Zn}^{68}(\text{d}, \text{He}^3)\text{Cu}^{67}$ excitation function with theoretical predictions: (1) experimental; (2) statistical model $\times 4100$; (3) pickup mechanism with transitions to all excited states up to 2 MeV allowed; (4) single-particle pickup model. All curves are normalized to the experimental value at 15.4 MeV.

observe the $\text{Zr}^{90}(\text{d}, \text{He}^3)\text{Y}^{89}$ reaction at a deuteron energy of 14 MeV. From the inverse of this reaction, to be discussed later, the ground-state transition is known to be strong, but in Cujec's experiment the ground-state He^3 particles emerged with an energy approximately 5 MeV below the Coulomb barrier. The study of Williams and Irvine¹⁰ of the excitation functions for the (d, He^3) reaction on Ar^{40} and Zn^{68} provides another example. At the high-energy end of the excitation functions (Fig. 1), the maximum energy available for the He^3 particle was about equal to the classical Coulomb barrier height. As the incoming deuteron energy was lowered, the cross section fell, and had dropped two orders of magnitude in 4 MeV. In all these cases, the deuteron energy was well above the barrier.

The importance of the Coulomb barrier has sometimes been ignored. Table II shows the results of some studies of the (n, d) and (d, He^3) reactions on Al^{27} . Note that to all three states of Mg^{26} the orbital angular momentum transfer is $\ell = 2$. In the last three columns of the table are shown ratios of spectroscopic factors for the excited-state transitions relative to the ground-state transition. The results in the fourth column are from a plane-wave analysis of a 14-MeV (n, d) study,

¹⁰ D. C. Williams and J. W. Irvine, Jr., Phys. Rev. 130, 259 (1963).

TABLE II. Transition strengths from the $\text{Al}^{27}(\text{n}, \text{d})\text{Mg}^{26}$ and $\text{Al}^{27}(\text{d}, \text{He}^3)\text{Mg}^{26}$ reactions.

Final state (MeV)	J^π	l	S/S_g		
			GW ^a	PW ^b	C ^c
0	0^+	2	1	1	1
1.83	2^+	2	2.2	1.53	1.87
2.97	2^+	2	. . .	0.32	0.64

^aR. N. Glover and E. Weigold, Nucl. Phys. 24, 630 (1961).

^bF. Pellegrini and S. Wiktor, Nucl. Phys. 40, 412 (1963).

^cB. Cujec, Phys. Rev. 128, 2303 (1962).

those in the fifth column are from a plane-wave analysis of a 12.8-MeV (d, He^3) study, and those in the final column are from a DW analysis of a 15-MeV (d, He^3) study. For the (d, He^3) reaction to the 2.97-MeV state, the value from the plane-wave analysis is only half that obtained from the DW analysis. It seems reasonable to attribute the difference mainly to the fact that the Coulomb barrier for the outgoing He^3 particle is about 8 MeV, which is about 1 MeV higher than the actual energy of the He^3 particle from this level in the 12.8-MeV experiment.

A further difficulty with plane-wave analyses appears in studies of the (He^3, α) and (t, α) reactions or their inverses. It has been shown that for these reactions which involve an incoming or outgoing alpha particle, the cross sections for transitions in which the orbital angular momentum is changed by three or four units are enhanced over those for

Fig. 2. Angular distributions for the $\text{Fe}^{56}(\text{He}^3, \alpha)\text{Fe}^{55}$ reaction to several groups of states of Fe^{55} . The laboratory energy of the He^3 beam was 14.40 ± 0.15 MeV. The curves on this and the following angular-distribution figures result from a DW calculation, and are normalized to the data in the forward-angle region. The error bars indicate typical standard deviations associated with the datum points.

which $\ell = 0$ or 1 ,¹¹⁻¹³ in contrast to the behavior of the simple reactions such as (d,p) and (d,t). Figure 2 shows an example of this behavior. The spectroscopic factor for the $\ell=1$ transition to the ground state and first excited state is approximately equal to that for the $\ell=3$ transition to the 3-MeV group, but the cross section is lower by a factor of two or three in the first 40° of angular distribution. This behavior is adequately predicted by a DW calculation, but if one analyzes such data by means of a plane-wave calculation, the values used for the single-particle reduced width $\theta_0^2(\ell)$ will have to be much different from those used for (p,d) or (d,t) reactions.

This rather unique behavior of the (He^3, α) and (t, α) reactions and their inverses has not been greatly exploited. Experimentally, by proper choice of emulsions and/or development techniques, the alpha particles from these two pickup reactions can be detected by means of the

¹¹ J. Saudinos, R. Beurtey, P. Catillon, R. Chaminade, N. Crut, H. Faraggi, A. Papineau, J. Thirion, *Compt. Rend.* **252**, 3776 (1961).

¹² J. L. Yntema, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Birks (Heywood and Co., Ltd., London, 1961), p. 513.

¹³ A. G. Blair and H. E. Wegner, *Phys. Rev.* **127**, 1233 (1962).

magnetic spectrograph/nuclear emulsion technique nearly as easily as can protons from a (d,p) reaction. A paper on the experimental results and DW analysis of the $O^{16}(He^3, \alpha)O^{15}$ ground-state reaction, by Alford, Blau, and Cline,¹⁴ appears in the Contributed Papers for this conference.

Let me return to the (d, He^3) and (He^3, d) reactions. Although the (d, He^3) reaction should provide information about ground-state wave functions and proton hole states in the same way that (d,t) reactions provide information about neutron configurations, it has been studied far less than the (d,t) reaction, partly because of experimental difficulties. Included among the rather few recent studies is the work of Cujec,⁹ at the University of Pittsburgh, who has obtained angular distributions of low-lying states from the (d, He^3) reaction on a variety of nuclei from O^{16} to Cu^{63} . She has extracted spectroscopic factors with the aid of DW calculations, and from these has deduced information on proton configurations for the target ground states. At the Argonne Laboratory, the (d, He^3) reaction has been studied by Zeidman, Yntema, and co-workers.^{15,16} Target nuclei have included F^{19} , Al^{27} , and nuclei at or near $Z = 28$. Among the results of the studies has been the indication that the 28-proton shell in the Ni nuclei is not well closed. This is to be contrasted to neutron-pickup results for the $N = 28$ nucleus Fe^{54} , which indicate a well-closed $N = 28$ shell.^{13,17,18}

¹⁴W. P. Alford, L. M. Blau, and D. Cline, Vol. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 8.

¹⁵B. Zeidman and T. H. Braid, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3-8, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, New York, 1963), p. 566.

¹⁶J. L. Yntema, T. H. Braid, B. Zeidman, and H. W. Broek, Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961, edited by J. B. Birks (Heywood and Co., Ltd., London, 1961), p. 521.

¹⁷M. H. Macfarlane, B. J. Raz, J. L. Yntema, and B. Zeidman, *Phys. Rev.* 127, 204 (1962).

¹⁸C. D. Goodman, J. B. Ball, and C. B. Fulmer, *Phys. Rev.* 127, 574 (1962).

Fig. 3. The $\text{Ni}^{62}(\text{He}^3, \text{d})\text{Cu}^{63}$ spectrum at $\theta_{\text{lab}} = 27^\circ$. The laboratory energy of the He^3 beam was 22.05 ± 0.20 MeV. The curves through the datum points on this and following figures of spectra result from a calculation with a digital computer program, as described in the text.

There have been a few (He^3, d) experiments performed lately, but I shall restrict myself to some recent work at Los Alamos. We are studying the (He^3, d) reaction on the Mg isotopes, four of the $N=28$ nuclei, the four even- A Ni isotopes, Y^{89} , and a few others.

A deuteron spectrum at 27° laboratory angle from the $\text{Ni}^{62}(\text{He}^3, \text{d})\text{Cu}^{63}$ reaction is shown in Fig. 3. The energy of the He^3 beam is 22 MeV. The curves through the datum points are the results of a digital-computer-program fit to the spectrum. This program fits peaks with skewed Gaussians plus exponential tails, and computes the area under each peak.¹⁹ Note especially the relatively large cross section of the first and second excited states, at 0.67 and 0.96 MeV.

Figure 4 shows the angular distributions of deuterons from the ground state and first few excited states of Cu^{63} . The curves through the datum points represent the results of a distorted-wave calculation for the reaction, normalized in magnitude to the experimental distributions at their peaks in the region of 20° to 30° . The orbital angular-momentum transfers are indicated adjacent to the curves. The calculation comes from a computer program based on the zero-range Gibbs-Tobocman

¹⁹ P. T. McWilliams, W. S. Hall, and H. E. Wegner, Rev. Sci. Instr. 33, 70 (1962); W. S. Hall (private communication).

Fig. 4. Angular distributions for the $\text{Ni}^{62}(\text{He}^3, d)$ reaction to the ground state and first few excited states of Cu^{63} . The error bars on this and the following angular-distribution figures show the standard deviations associated with the respective datum points; in the present figure nearly all are smaller than the diameters of the points. The scale of absolute cross section has been determined to an accuracy of $\pm 15\%$.

program,²⁰ modified at Los Alamos.²¹ The integration of the wave functions is cut off near the surface of the nucleus.^{22,23} The optical-model parameters used in the calculation are those obtained from fits to elastic-scattering data.

It can be seen that the DW calculation yields curves which are in good agreement with the angular distributions. In the spectra, the third and fourth excited states are not well resolved, and the analysis of the spectra yields some scatter of points in their angular distributions. However, the observed energies of the peaks are within 10 keV of the energies quoted in the literature and indicated on Fig. 3. At nearly all angles, the analysis indicates that the higher state at 1.41 MeV is excited

²⁰ W. Tobocman, Phys. Rev. **115**, 98 (1959); W. R. Gibbs, Ph.D. thesis, Rice University (1961).

²¹ W. R. Gibbs and W. S. Hall (private communication).

²² N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. **133**, B3 (1964).

²³ G. R. Satchler (private communication).

Fig. 5. Angular distributions for the $\text{Ni}^{62}(\text{He}^3, d)\text{Cu}^{63}$ reaction to states or groups of states in Cu^{63} at 2.06, 2.36, and 2.51 MeV. The excitation energies have been determined to an accuracy of ± 0.03 MeV.

three to four times as strongly as the lower state at 1.33 MeV. The distribution labeled 1.4 MeV in the figure is the sum distribution of the two states. It is probably entirely $\ell = 3$ in character. The spin and parity of the stronger state at 1.41 MeV is thus restricted to $\frac{5}{2}^-$ or $\frac{7}{2}^-$, as is (with only a little less certainty) that of the weaker 1.33-MeV state. Thus, these results are consistent with the recent assignment²⁴ of $\frac{7}{2}^-$ for the 1.33-MeV state, but in disagreement with the often-proposed assignment of $\frac{3}{2}^-$ for the 1.41-MeV state.^{25,26}

Figure 5 shows the next several excited-state distributions and the DW fits. A distribution peaking at about 25° and corresponding to an $\ell=4$ DW calculation is found at 2.51 MeV. Two more distributions and the DW fits are shown in Figure 6. The distribution for the 3.48-MeV

²⁴H. E. Gove, Phys. Letters 4, 249 (1963).

²⁵F. Perey, R. J. Silva, and G. R. Satchler, Phys. Letters 4, 25 (1963).

²⁶R. D. Lawson and J. L. Uretsky, Phys. Rev. 108, 1300 (1957).

Fig. 6. Angular distributions for the $\text{Ni}^{62}(\text{He}^3, d)\text{Cu}^{63}$ reaction to states or groups of states in Cu^{63} at 2.79 and 3.48 MeV. The excitation energies have been determined to an accuracy of ± 0.04 MeV.

group peaks further out in angle than any observed $\ell = 1, 3$, or 4 distribution, and is tentatively assigned as an $\ell=2$ transition. It is possible that the expected minimum of the $\ell=2$ distribution at approximately 27° is filled in by a weaker $\ell=1$ or $\ell=4$ transition to a group at approximately the same energy.

These figures show the DW fits to the strongest deuteron groups up to approximately 4 MeV of excitation. Most of the weaker groups have been similarly analyzed.

To obtain quantitative results for spectroscopic factors from the present experiment is somewhat more difficult. The DW calculation does not yet predict absolute cross sections for the (He^3, d) reaction, and the normalization factor is not known very accurately. We have chosen to assume that all observed $\ell=1$ transitions except that to the ground state are $p_{1/2}$. We also assume that we have actually identified all of the important $\ell=1$ transitions. The assumptions are probably correct in the main; from the analysis of the (He^3, d) reaction on the $N = 28$ nuclei, currently being studied by Armstrong,²⁷ it seems likely that the transition strengths obtained from the present reaction are accurate to within 30%.

²⁷ D. D. Armstrong (private communication).

TABLE III. Transition strengths for the $\text{Ni}^{62}(\text{He}^3, d)\text{Cu}^{63}$ reaction.

Assumed s. p. state	E_x (MeV)	C^2S (Exp.)	$\sum(C^2S)$	
			(Exp.)	(Pred.)
$p_{3/2}$	g. s.	0.53	0.53	0.96 (0.60)
$p_{1/2}$	$\left\{ \begin{array}{l} 0.668 \\ 1.547 (?) \\ 2.06 \\ 2.79 \\ 3.06 \end{array} \right\}$	$\left\{ \begin{array}{l} 0.58 \\ 0.01 \\ 0.18 \\ 0.12 \\ 0.03 \end{array} \right\}$	0.92	0.92
$f_{5/2}$	$\left\{ \begin{array}{l} 0.961 \\ 1.412 \\ 2.36 \end{array} \right\}$	$\left\{ \begin{array}{l} 0.40 \\ 0.40 \\ 0.10 \end{array} \right\}$	0.90 (0.80)	0.93
$f_{7/2}$	1.327	0.07		...
$g_{9/2}$	2.51	0.64		0.86
$d_{5/2}$	3.48	0.21		0.86

Table III shows the results of the data analysis on the basis just outlined. The first column lists the single-particle states, and the second column shows the observed states and their single-particle assignment. The quantity actually extracted from the normalization of the DW curves to the experimental distributions is NC^2S , where S is the spectroscopic factor, C is the isobaric-spin Clebsch-Gordan coefficient, and N is the factor multiplying the calculated DW curves such that the calculated cross section equals the experimental cross section for a full single-particle transition, i. e., for a transition having $C^2S = 1$. We have also assumed a cross-section dependence on $\vec{l} \cdot \vec{s}$ for the captured proton of about $\pm 15\%$, as $j = l \pm \frac{1}{2}$, respectively, as indicated by recent work of Satchler.²³

The sum of the $p_{1/2}$ transition strengths is actually expected to be slightly less than unity, as shown in the final column. The reasons for this are that, first, the value of the isobaric-spin coupling factor C^2 is $6/7$, and second, a correction must be made for the partial occupation of the $p_{1/2}$ neutron states in the Ni^{62} nucleus. The other 8% of the full $p_{1/2}$ strength will lie in $T=T_z+1$ states, many MeV higher in excitation.

Thus, we adjust the value of the normalizing constant N such that the sum of the $p_{1/2}$ transitions, when compared to the DW calculation, yields a total strength of 0.92. This value of N will then be used for all the other transitions, and for the other Ni isotopes. The experimental values for the transition strengths for each state or group are given in column 3 and their sums in column 4.

The experimental strength of 0.53 obtained for the $p_{3/2}$ ground state is to be compared with the expected value of 0.96 if the full strength were available to this state. This discrepancy can be partially accounted for in several ways, but the two which are probably the most important are (1) some of the higher $\ell=1$ transitions may actually be $p_{3/2}$ rather than $p_{1/2}$, and (2) there is configuration mixing of $p_{3/2}$ protons in the ground-state wave function of Ni^{62} . Previous evidence for this latter observation has already been mentioned. If one assumes that the low value of the ground-state transition strength is due completely to this latter factor, one can calculate the occupation number of $p_{3/2}$ protons in the Ni^{62} ground-state wave function. The result is that on this basis the $p_{3/2}$ proton orbital is approximately 40% full. This value can be considered as an approximate upper limit to the amount of $p_{3/2}$ admixture.

If the $p_{3/2}$ orbital were 40% full, it follows that the $f_{7/2}$ orbital would be 20% empty (the $p_{3/2}$ orbital can hold four protons; the $f_{7/2}$ can hold eight). Thus, a fraction of the $\ell=3$ transitions must be $f_{7/2}$ rather than $f_{5/2}$. I have already mentioned that the third excited state at 1.33 MeV

has recently been assigned a spin and parity of $\frac{7}{2}^-$. In our work, although this state is not well resolved from the stronger transition to the 1.41-MeV state, we can still deduce an approximate value for its strength. The number so obtained is about 7% of the full $f_{7/2}$ single-particle strength, as indicated in Table III. Thus, we conclude that the $f_{7/2}$ orbital is at least 7% empty, and probably not more than 20% empty; i. e., the $p_{3/2}$ orbital is between 14% and 40% filled in the Ni^{62} ground state. If we take an average between these numbers, we obtain the predicted $p_{3/2}$ strength shown in parentheses. (To be consistent, we must say that a small additional fraction of the $\ell=3$ transfers labeled as $f_{5/2}$ must be $f_{7/2}$, and this accounts for the number in parentheses in the $f_{5/2}$ row.) The agreement for the $p_{3/2}$ and $f_{5/2}$ states is satisfactory.

The present analysis also yields 0.64 for the $g_{9/2}$ transition strength to the 2.51-MeV level, approximately three-quarters of the predicted full strength of 0.86. The $d_{5/2}$ transition at 3.48 MeV includes only about one-quarter of the total predicted $d_{5/2}$ strength. These results are reminiscent of the work on the location of single-particle neutron states by Fulmer et al.²⁸ In their study of (d,p) reactions in this region of the periodic table, they found that the $g_{9/2}$ neutron strength was nearly entirely concentrated in one (or, at the most, two) nuclear levels, while the $d_{5/2}$ neutron strength was split over many nuclear levels.

The results from the reaction on Ni^{58} , Ni^{60} , and Ni^{64} are quite similar. For all four Cu isotopes, large fractions of the available $p_{1/2}$ and $f_{5/2}$ single-particle strengths are found in the first two excited states. On the other hand, the results of proton inelastic-scattering experiments by Perey et al.,²⁵ alpha-particle inelastic-scattering

²⁸R. H. Fulmer, A. L. McCarthy, B. L. Cohen, and R. Middleton, Phys. Rev. **133**, B955 (1964) R. H. Fulmer and A. L. McCarthy, Phys. Rev. **131**, 2133 (1963).

Fig. 7. The $\text{Ni}^{58}(\text{He}^3, d)\text{Cu}^{59}$ spectrum at $\theta_{\text{lab}} = 27^\circ$. The laboratory energy of the He^3 beam was 22.00 ± 0.20 MeV.

experiments by Bruge *et al.*²⁹ and Harvey *et al.*,³⁰ and Coulomb-excitation and resonance-fluorescence experiments,²⁴ are consistent with a nuclear model that in essence couples a $p_{3/2}$ proton to an excited core, with very little, if any, coupling of $p_{1/2}$ and $f_{5/2}$ protons. These results have not yet been reconciled with those of the present (He^3, d) study.

The $\text{Ni}^{58}(\text{He}^3, d)\text{Cu}^{59}$ reaction is interesting in a further respect. Figure 7 shows a spectrum of this reaction. In the 4-MeV excitation region, in the neighborhood of channels 130–140 in the figure, strong peaks appear which are missing in the spectra of the three other Cu isotopes. At least a part of this difference can probably be attributed to the excitation of $T = T_z + 1$ isobaric states. In Cu^{59} , where there is only one more neutron than proton, the isobaric analog to the Ni^{59} ground state lies quite low in energy, namely at an excitation of about 3.9 MeV. It is expected to be fairly strongly excited in the (He^3, d) reaction. By definition, the state has the $\frac{3}{2}^-$ spin and parity of the Ni^{59} ground state. Indeed, there is a transition at 3.9 MeV, and its angular distribution,

²⁹G. Bruge, J. C. Faivre, M. Barloutaud, H. Faraggi, and J. Saudinos, *Phys. Letters* 7, 203 (1963).

³⁰B. G. Harvey, J. R. Meriwether, and D. J. Horen, *Bull. Am. Phys. Soc.* 8, 612 (1963).

Fig. 8. Angular distribution for the $Ni^{58}(He^3, d)Cu^{59}$ reaction to the state or group of states in Cu^{59} at 3.87 ± 0.04 MeV.

shown in Fig. 8, most strongly resembles an $l=1$ transition. The state is unbound, so that no DW calculations are available, but the bumps fall in the correct place, and, because of its low Q , its structure should be washed out considerably from that of the $Ni^{62}(He^3, d)Cu^{63}$ distributions. It has approximately the expected cross section. Other transitions a few hundred keV higher in excitation may be roughly identified as analogs of the lowest excited states in Ni^{59} .

In the case of $Mg^{26}(He^3, d)Al^{27}$, a transition has similarly been seen which can be identified as proceeding to the analog of the Mg^{27} ground state.

Another interesting application of the (He^3, d) reaction has been in the study of the low-lying states of Zr^{90} . One of the successes of the shell-model theory has been its ability to account in a quantitative manner for the positions of these states. Since the neutron number (50) for this nucleus is a magic number representing the filling of a closed shell, one would not expect the low excited states to be described by neutron excitations. Ford³¹ was the first to point out that the low states should be describable as two protons in almost pure configurations of the form $(p_{1/2})^2$, $(p_{1/2} g_{9/2})$, and $(g_{9/2})^2$. Subsequent theoretical work of

³¹K. W. Ford, Phys. Rev. 98, 1516 (1955).

Fig. 9. The $Y^{89}(\text{He}^3, d)\text{Zr}^{90}$ spectrum at $\theta_{\text{lab}} = 27^\circ$. The laboratory energy of the He^3 beam was 22.00 ± 0.20 MeV.

a more detailed nature showed that the agreement between theory and experiment was quite good.

The experimental work up until recently was able to identify all of the states arising from the configurations just listed except for the 4^- state of the $(p_{1/2} g_{9/2})$ configuration. On the basis of somewhat indirect arguments, Wagner *et al.*³² concluded that the missing level was probably located at an excitation energy of 2.75 MeV. We have recently observed the $Y^{89}(\text{He}^3, d)\text{Zr}^{90}$ reaction, and have definitely identified the 4^- level and confirmed this assignment.

The nucleus Y^{89} has a $p_{1/2}$ proton ground state, and can be considered the parent of the $(p_{1/2})^2$ and $(p_{1/2} g_{9/2})$ states in Zr^{90} , which will be excited in the reaction. A pulse-height spectrum is shown in Fig. 9. All four of the strong deuteron groups seen have the same width and shape; thus they appear to represent the excitation of single, well-separated states. Figure 10 shows the angular distributions of these states along with the results of a DW calculation. Both the ground state and first excited state have the same angular distribution, in agreement with the DW calculation for an $\ell=1$ transition. Although in the zero-order shell-model theory the ground state would be pure $(p_{1/2})^2$ and the first excited state pure $(g_{9/2})^2$, they are known to be strongly mixed. Our

³²R. T. Wagner, E. R. Shunk, and R. B. Day, Phys. Rev. 130, 1926 (1963).

Fig. 10. Angular distributions for the $Y^{89}(\text{He}^3, d)$ reaction to several states of Zr^{90} . The upper two curves result from $\ell=1$ DW calculations, the lower two curves from $\ell=4$ DW calculations.

results give a measure of this mixing. The angular distributions to the next two levels are also similar to each other, and agree well with an $\ell=4$ DW calculation. Since the first of these corresponds to the well-known 5^- state of the $(p_{1/2} g_{9/2})$ configuration at 2.32 MeV, the second (at 2.74 MeV) is clearly the missing 4^- state of this configuration. The spectroscopic factors for these two transitions are equal to each other to within 3%.

It is to be expected that over the next few years, advances both in experimental techniques and in theoretical treatment of the data will make studies of the reactions I have been discussing more popular among physicists.

* * *

B. E. F. MACEFIELD, Oxford University: Your experimental data seem to be restricted to angles between 10° and 35° . How sure of the ℓ value can you be with this in mind?

BLAIR: Pretty sure. In some experiments we have gone on to about 55° , and nothing unexpected happens at these larger angles. An $\ell=1$ transition, for example, looks like an $\ell=1$ transition all of the way out to 55° . The forward-angle data appear to be extremely representative in determination of ℓ values. (Since the Symposium, however, we have obtained preliminary results which indicate that for $\theta \gtrsim 65^\circ$, $p_{3/2}$ and $p_{1/2}$ transitions are out of phase with each other.)

D. R. INGLIS, Argonne National Laboratory: As one who is trying to understand the rapidly developing state of the art, I would like to ask when you see a row of points which decrease monotonically toward the right in a logarithmic graph of the angular distribution for a Ni^{58} target (Fig. 8), how can you so glibly say $\ell=1$? I could not see a peak or anything.

BLAIR: I meant that the angular distribution I showed looks much more like $\ell=1$ than any other ℓ value. If you compare this distribution either with an experimental $\ell=1$ distribution or with a calculated $\ell=1$ distorted-wave cross section, you find close agreement in the positions of the maxima and minima. Distributions with other ℓ values have different shapes with the bumps displaced by $7-8^\circ$.

INGLIS: Was this assuming that the left-hand point is a maximum?

BLAIR: No, this is assuming that there is a bump at approximately 26° . I do not pay much attention to regions of maximum slope. What I look at myself is the first maximum (if you want to call it one) that I can see. It is most representative.

M. H. MACFARLANE, Argonne National Laboratory: Perhaps we should remember that an experimenter does not assign ℓ values on a desert island. He does not have to rely entirely on the comparison of measured angular distributions and calculated differential cross sections. For each ℓ value, there are numerous transitions involving states of known spin for which the ℓ assignment is fixed by selection rules. These transitions can

then be used to provide typical shapes for each ℓ value, and ℓ assignments can be made by finding the standard curve (if any) whose shape is closest to the measured one.

BLAIR: For example, the spins and parities of the first four states of Cu^{63} are known.

D. A. BROMLEY, Yale University: In the early part of your talk, you commented that transitions involving higher orbital angular-momentum transfer were enhanced over those with lower. I think some related experiments which Sachs, Chasman, and I have carried out at Yale³³ fall in the class you listed as "miscellaneous." In particular, we have examined ($\text{B}^{10}, \text{Be}^9$) and ($\text{B}^{11}, \text{Be}^{10}$) reactions involving single-nucleon transfer. We have compared our results with your corresponding (He^3, d) reactions and find that where the same states are populated they are populated with almost precisely the same yield, although your reactions are somewhat more selective. If we look at the Be^{10} or Be^9 product nucleus coming out from the reaction, the fact that it has survived the interaction at all means that we are dealing with a rather pure surface interaction.

If we make that assumption, then we argue that the only partial waves that are strongly involved are those corresponding to a grazing impact parameter in both entrance and exit channels. If we couple these angular momenta to give that transferred to the core during the reaction and if the transferred nucleons are in orbits corresponding to a different angular-momentum transfer, then this momentum mismatch is reflected in enhancement of the orbital transfers that minimize the mismatch. This quite frequently results in inhibition of the lower orbital values and effective enhancement of the higher ones. I think probably this same mechanism shows in your He^3 data.

³³ M. W. Sachs, C. Chasman, and D. A. Bromley, Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, California, 14—18 April 1963, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), p. 90; D. A. Bromley, paper on p. 353 of these Proceedings.

BLAIR: I agree. This interpretation is discussed briefly in Ref. 13.

A. K. KERMAN, Massachusetts Institute of Technology: I would like to make some comments about the Cu^{63} data, which have some interest for nuclear models. It has been suggested that the first four excited states can be understood by assuming that a $p_{3/2}$ particle is coupled to the collective 2^+ vibration. In this particular case there would be four states ($\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^+$) and the (α, α') data should then show E2 strengths to those states, as it indeed does. There has been a question as to why the supposed $\frac{3}{2}^+$ state is not seen in the (α, α') reactions when the simple model indicates it should be. Of course, the states are more complicated than that. If we can trust the distorted-wave fitting, this experiment indicates that these levels have large single-particle admixtures in them, and that the simple description above is to be taken with a grain of salt.

H. E. GOVE, University of Rochester: In connection with Kerman's remarks, I would like to comment that the values of $B(E2)_{\downarrow}$ for the only states in Cu^{63} that are observed by Coulomb excitation (namely the $\frac{1}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^+$ states) are equal to within a few percent; but, even more remarkable, they are also equal to the $B(E2)_{\downarrow}$ for the $2^+ \rightarrow 0^+$ transition in Ni^{62} , again to within a few percent. To me, that seems like more than chance.

B. ZEIDMAN, Argonne National Laboratory: In connection with this same issue, a few years ago some very preliminary (d, He^3) work on Cu^{63} and Cu^{65} showed pickup to the ground state and the 2^+ state, both with $\ell = 1$. You can take the ratio of these cross sections to give a sort of admixture of the 2^+ state in the Cu ground state—a back-of-the-envelope type of calculation. The admixture is 25%. On this basis, the wave function constructed for the ground state of Cu^{63} is

$$\frac{1}{2} \sqrt{3} (0 + p_{3/2}) + \frac{1}{2} (2 + p_{1/2})$$

and that for the $\frac{1}{2}^+$ state is

$$\frac{1}{2} \sqrt{3} (0 + p_{1/2}) + \frac{1}{2} (2 + p_{3/2});$$

and if the transition strengths relative to the $0^+ \rightarrow 2^+$ transition strength for Ni^{62} is calculated with the $B(E2)$ for $p_{3/2} \rightarrow p_{1/2}$ taken as about a fourth of $B(E2) 2 \rightarrow 0$, then the $B(E2)$ for the $\text{Cu}^{63} \frac{1}{2} \rightarrow \frac{3}{2}$ is found to be exactly the same as for the $\text{Ni}^{62} 2 \rightarrow 0$. This value is very dependent upon the ratio of the cross sections for pickup to the ground state and 2^+ state of Ni^{62} . If you change the admixture ratio even slightly, then you do not get good agreement at all. It should be emphasized that this is a very simple-minded approach and should not be relied upon.

BLAIR: This kind of calculation has been done by Bouten and van Leuven.³⁴ They coupled a $p_{3/2}$ proton to the zero-, one-, and two-phonon quadrupole states of the core and got a bit of mixing like this. Our results showed $1\frac{1}{2}$ — 2 times as much single-particle admixture as they predicted.

³⁴ M. Bouten and P. van Leuven, Nucl. Phys. 32, 499 (1962).

STUDIES OF THE $\text{O}^{16}(\text{He}^3, \alpha)\text{O}^{15}$ AND
 $\text{O}^{16}(\text{He}^3, \text{He}^3)\text{O}^{16}$ REACTIONS^{*†}

W. P. Alford, L. M. Blau, and D. Cline

University of Rochester, Rochester, New YorkAbstract of Report in "Vol. I. Contributed Papers"¹

Angular distributions and excitation functions have been measured for the reaction $\text{O}^{16}(\text{He}^3, \alpha)\text{O}^{15}$ g.s. and for the elastic scattering of He^3 on O^{16} over the range of bombarding energies from 8.00 MeV to 10.00 MeV and the angular range from 2.5° to 174° . The excitation functions for the elastic scattering showed only slight variation with energy, while the angular distributions exhibited a strong backward peak over the entire energy range. At all energies the angular distributions of the alphas showed the forward peaking expected for an $L=1$ pickup reaction. The magnitude of the integrated cross section varied by a factor of two, peaking at 8.5 MeV. An analysis of the reaction was attempted by use of the distorted-wave programs of the Oak Ridge National Laboratory group. The optical-model parameters for the input channel of the distorted-wave program JULIE were extracted from the elastic-scattering data by use of the search program HUNTER. Reasonable agreement between the data and the calculations was obtained at all energies.

We recently completed some precisely analogous measurements on Ca^{40} in which things seem to be much pleasanter. The angular distributions do not seem to show such an energy dependence as they do for O^{16} , but unfortunately we have not yet had time to make the calculations for

^{*}Work performed under the auspices of the U. S. Atomic Energy Commission.

[†]Contributed paper. Presented by W. P. Alford.

¹W. P. Alford, L. M. Blau, and D. Cline, in Argonne National Laboratory Report ANL-6848, March 1964, p. 8. The paragraph that follows it is in amplification of this earlier report.

these. Hence we do not have a number for the spectroscopic factors in these reactions at the moment, but all these results will be submitted for publication in the near future.

* * *

J. P. SCHIFFER, Argonne National Laboratory: I am a little confused. You showed a sharp energy variation in the excitation function; it seems to be a factor of three at some angles and almost a factor of 10 at others. You also say that you can extract spectroscopic factors to 30% from such data. How are these two statements compatible?

ALFORD: The excitation functions that I showed were at angles of 45° and larger, and at large angles the cross sections may show large variations with energy. The spectroscopic factors that were quoted were obtained by fitting the calculations to the forward peak.

SCHIFFER: The forward peak does not show resonances?

ALFORD: As can be seen from Fig. 1, the peak cross section varies between about 10 and 20 mb/sr. At all energies except 8.50 MeV, the shape of the peak was fitted quite well by the calculations. We concluded that the best way to extract spectroscopic factors was to calculate the cross section by use of a fixed set of optical-model parameters for input and output channels. When these calculations were compared with the data, the spectroscopic factors that were obtained varied by $\pm 30\%$ about their mean value, corresponding to the variation in the cross section by a factor of two.

SCHIFFER: But your curve between 9 and 9.5 MeV showed sharp changes also. Before that they do not?

ALFORD: The shape of the angular distribution changes most markedly around 8.5 MeV. Between 9 and 9.5 MeV, the shape of the cross section changes very little, while the magnitude of the forward peak increases from about 17 to 23 mb/sr.

Fig. 1. Angular distributions of the $\text{O}^{16}(\text{He}^3, \alpha)\text{O}^{15}$ ground-state reaction between 8 and 10 MeV. The indicated errors are statistical, and absolute cross sections are accurate to 20%.

G. RAWITSCHER, Yale University: Since the overlap between the He^3 and the alpha particle enters into the determination of the spectroscopic factor, it might be interesting in a future calculation to use a quarter of the total mass defect instead of the energy to separate an alpha into a neutron and He^3 in the hope of obtaining closer agreement with experiment.

ALFORD: We do not feel that the present calculation of the He^3 -alpha overlap is very satisfactory, and this might well account for part of the discrepancy found here. For the moment, our feeling is that even if this overlap is not calculated very accurately, it will introduce an error which is independent of the nuclear states involved, or of the angular momentum of the transferred particle. Thus if the ratio of calculated to measured cross sections could be normalized by measurements of the (He^3, α) reaction on simple nuclei such as O^{16} or Ca^{40} , it could be used as an empirical correction in spectroscopic measurements on other nuclei. As for the possibility of a better calculation of the He^3 -alpha overlap, I must refer you to Dr. Satchler and Dr. Bassel of Oak Ridge, who set up the calculations. We simply used their DWBA program under their direction.

J. Y. PARK, North Carolina State University, Raleigh: Have you looked at the reaction to study the reaction mechanism, e. g., the possibility of a knockout process?

ALFORD: We have not looked at such things as knockout processes because it does not appear to be necessary. We can get reasonable fits to angular distributions just by using simple-minded distorted-wave calculations.

SESSION C

TUESDAY MORNING

10 MARCH 1964

CHAIRMAN: E. M. HENLEY

SCIENTIFIC SECRETARIES

R. D. LAWSON

L. MEYER-SCHÜTZMEISTER

INELASTIC SCATTERING

J. S. Blair

University of Washington, Seattle, WashingtonIntroduction

Experiments involving inelastic excitation of discrete nuclear levels can reveal much information about these levels. Analyses of the scattering cross sections not only may determine such quantities as spins, parities, and deformation parameters, but also may lead to rather detailed descriptions of the nuclear wave functions.

In this talk I cannot hope to give a comprehensive review of the spectroscopic information which emerges from these studies. Rather I will limit myself to some specific experiments which illustrate the variety of problems which have been attacked by use of this technique. In so doing I will, of course, neglect many important contributions by many individuals. To these people I tender my apologies in advance.

A. Theory of the Reaction Mechanism

Although we shall emphasize the spectroscopic utility of inelastic scattering, we must begin with some comments on the theory of the reaction mechanism itself; since there is such a proliferation of theoretical models which enjoy at least partial success in explaining the experimental data, a brief classification of these models may prove helpful.

1. The extended optical model. This is generally recognized as the natural and most successful description for direct inelastic

processes.¹ In this model the normal spherical optical potential is extended by including terms that explicitly contain dynamical nuclear coordinates and thus couple different states of the target nucleus. The Schrödinger equation and the boundary conditions of the projectile-target system yield a set of coupled differential equations in the projectile coordinates, infinite in number since the nuclear coordinates presumably comprise a complete set.

Most applications have dealt with excitations of the collective surface modes for which it is assumed that the potential is a function of the coordinates describing the displacement of the nuclear surface. A standard but by no means mandatory assumption is that the potential has the form $V(R + \alpha(\hat{r}), r)$, where R is the radius parameter of a typical spherical optical potential and $\alpha(\hat{r})$ is the displacement of the nuclear surface in the direction $\hat{r}$. This displacement may be related to the canonical collective coordinates $\zeta_{\ell, m}$ through the multipole expansion

$$\alpha(\hat{r}) = \sum_{\ell, m} \zeta_{\ell, m} Y_{\ell}^{m*}(\hat{r}). \quad (\text{A. 1})$$

Frequently the potential is then expanded in powers of $\alpha(\hat{r})$, i. e. ,

$$\begin{aligned} V(R + \alpha(\hat{r}), r) &= V(R, r) + \Delta V(R, r, \alpha(\hat{r})) \\ &= V(R, r) + \alpha(\hat{r}) \frac{\partial V(R, r)}{\partial R} + \frac{1}{2} \alpha^2(\hat{r}) \frac{\partial^2 V(R, r)}{\partial R^2} . \end{aligned} \quad (\text{A. 2})$$

(a) What precisely we mean by the distorted-wave Born approximation (DWBA) depends to some extent on the nuclear model. I believe it is generally accepted that for the case of collective excitations the DWBA solves the coupled differential equations to first order in $\alpha(\hat{r})$ but

¹ See for example the review article by N. Austern, in Selected Topics in Nuclear Theory (International Atomic Energy Agency, Vienna, 1963).

not in ΔV ; thus the DWBA transition amplitude T_{ba} between two nuclear collective states a and b has the familiar form²

$$T_{ba} = \int d\vec{r} \chi^{(-)*}(\vec{k}_b, \vec{r}) \langle b | a(\hat{r}) | a \rangle \frac{\partial V}{\partial R} \chi^{(+)}(\vec{k}_a, \vec{r}). \quad (\text{A. 3})$$

Because it is proportional to the matrix element of the first power of $a(\hat{r})$, this amplitude will be referred to as a single-excitation amplitude. Of course the DWBA calculations are not limited to collective excitations but have also been carried out for models in which there is interaction with the individual nucleons.

The DWBA amplitude is often evaluated with the aid of further approximations. In the WKB approximation, the exact spherical solutions $\chi^{(+)}$ and $\chi^{(-)*}$ are replaced by their WKB counterparts; this approximation is particularly appropriate for the scattering of nucleons with energy in excess of 100 MeV. The plane-wave Born approximation (PWBA) has semiquantitative success for such nucleons although it supplies only qualitative orientation with nucleons of lower energy and is downright deceiving when the projectiles are more complex.

(b) The Tamm-Dancoff approximation³ has been the principal method by which contributions to the transition amplitudes of higher order in $a(\hat{r})$ have been investigated. In this procedure there is a restriction in the number of nuclear levels which are coupled; but within this restricted set the coupled equations are solved exactly. Buck,⁴ Wills,⁵ Stamp,⁶ and

²R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. 128, 2693 (1962).

³D. M. Chase, L. Wilets, and A. R. Edmonds, Phys. Rev. 110, 1080 (1958).

⁴B. Buck, Phys. Rev. 127, 940 (1962).

⁵J. Wills, Ph.D. Thesis, University of Washington, 1963 (unpublished).

⁶A. P. Stamp, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3—8, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, New York, 1963).

Tamura⁷ have all constructed elaborate numerical codes so that the Tamm-Dancoff procedure may be applied to the scattering of medium-energy projectiles.

For cases in which the matrix element of $\alpha(\hat{r})$ between the ground state and excited states is nonvanishing but at the same time not too large, there is little difference between the Tamm-Dancoff calculation and the DWBA single-excitation amplitudes. When, however, the matrix element of $\alpha(\hat{r})$ vanishes while that of $[\alpha(\hat{r})]^2$ does not, the Tamm-Dancoff procedure leads to results qualitatively different from those found for single excitation of the same multipolarity.

2. The adiabatic method.⁸ This has been the basis for several approximate treatments and is appropriate to situations in which the excited levels may be regarded as degenerate in energy with the ground-state level.

(a) The sharp-edge Fraunhofer model is the crudest of the adiabatic theories.⁹⁻¹¹ At this time we discuss neither the assumptions underlying the derivation nor the resulting formulas, but content ourselves with the following observations: (i) The method is most appropriate for strongly absorbed nuclear projectiles. (ii) At large orders in the diffraction pattern, the single-excitation angular distributions exhibit the familiar phase rule; i. e., for even l the oscillations are out of phase with those of

⁷ T. Tamura, in Nuclear Spectroscopy with Direct Interactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 24.

⁸ D. M. Chase, Phys. Rev. 104, 838 (1956).

⁹ S. I. Drozdov, J. Exptl. Theoret. Phys. (U.S.S.R.) 28, 734 and 736 (1955); [translation: Soviet Phys. —JETP 1, 588 and 591 (1955)].

¹⁰ E. V. Inopin, J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 901 (1956); [translation: Soviet Phys. —JETP 4, 784 (1957)].

¹¹ J. S. Blair, Phys. Rev. 115, 928 (1959).

the elastic scattering while the reverse is true for odd l . (iii) At smaller angles, the angular distributions are uniquely related to the multipole order.

(b) Double-excitation cross sections may also be calculated in the sharp-edge Fraunhofer model.^{12,13} The most important result is that the phase relations for double excitation, in this case of strong absorption, are reversed from those for single excitation of the same multipolarity.

(c) A smooth-edge Fraunhofer model proposed by Inopin¹⁴ remedies a major deficiency of the sharp-edge theory, namely its failure to give cross sections which decrease fast enough at higher orders in the diffraction pattern.

(d) Alternative smooth cutoff theories^{13,15,16} have been constructed which completely avoid the Fraunhofer approximation and instead are couched in terms of partial-wave expansions.

That there is a close connection between the extended optical model and adiabatic calculations has been appreciated for some time. The single-excitation phase relations of the Fraunhofer theory are similarly

¹²S. I. Drozdov, J. Exptl. Theoret. Phys. (U.S.S.R.) 38, 499 (1960); [translation: Soviet Phys. — JETP 11, 362 (1960)].

¹³J. S. Blair, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3—8, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, New York, 1963), p. 669.

¹⁴E. V. Inopin (to be published).

¹⁵J. S. Blair, D. Sharp, and L. Wilets, Phys. Rev. 125, 1625 (1962).

¹⁶N. Austern and J. S. Blair (to be published).

found in DWBA calculations involving strongly absorbed projectiles.^{17,2} Further the reversal of the phase relations in double-excitation angular distributions, which was predicted by the Fraunhofer calculations,^{12,13,16} is also obtained from the Tamm-Dancoff calculations with strongly absorbed projectiles.^{4,5} That these connections have more than qualitative significance has been demonstrated through comparison of smoothed cutoff and DWBA calculations^{15,18} and also through explicit derivation of the Fraunhofer formulas from the DWBA expressions.¹⁶

In what follows we shall regard the extended optical model as a generally satisfactory theory for describing direct inelastic scattering but will also use the fact that this theory is frequently well approximated by simpler formulations.

B. Single Excitation of Strong Collective Levels

1. Even-mass nuclei. The observed strong excitation of the first 2^+ levels and low-lying 3^- levels and the success of DWBA calculations in describing these results² are familiar enough so that the following remarks should suffice.

(a) Compilations of nuclear matrix elements, usually expressed in terms of β_l (the deformation parameter of the strong-coupling model), are given in two contributions to this conference. Fricke¹⁹ has

¹⁷E. Rost and N. Austern, Phys. Rev. 120, 1375 (1960).

¹⁸E. Rost, Phys. Rev. 128, 2708 (1962).

¹⁹M. P. Fricke, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 130.

made DWBA analyses of the proton-scattering data of Hintz and Stovall²⁰ on iron and nickel isotopes. Faraggi and Saudinos²¹ have used adiabatic theories to analyze alpha-particle scattering from iron, nickel, and zinc isotopes. On the whole, the values of β_ℓ and $\beta_{\ell R}$ obtained with different projectiles are very similar; it is not clear to me at this stage whether the differences that are found reflect real physical effects or varying methods of analysis. Many other determinations of β_ℓ are contained in published^{2, 18, 22} and unpublished work of the Oak Ridge group.

(b) It is interesting to note that the strong octupole and $\ell = 4$ transitions seen in intermediate-mass nuclei also appear to be present in the middle of the s-d shell. In Fig. 1 are shown some angular distributions, obtained by Naqib and Farwell,²³ of 42-MeV alpha particles scattered from Mg^{26} ; the locations of the oscillations agree well with the predictions of the Fraunhofer theory for $\ell = 4$ transitions. The arrows indicate the predicted locations of maxima and minima according to this theory. Similarly Fig. 2 contains some angular distributions for Mg^{26} which correspond well to the predictions for octupole excitations. In most intermediate-mass nuclei only a single strong octupole transition is observed; however, Ne^{20} , Mg^{24} , and Mg^{26} (even-mass nuclei which are presumably permanently deformed), all show more than one strong octupole

²⁰ T. Stovall and W. Hintz, University of Minnesota Linear Accelerator Laboratory Annual Progress Report, November 1962 (unpublished), p. 37.

²¹ H. Faraggi and J. Saudinos, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 130.

²² G. R. Satchler, R. H. Bassel, and R. M. Drisko, *Phys. Letters* 5, 256 (1963).

²³ I. Naqib, Ph.D. Thesis, University of Washington, 1962 (unpublished); I. M. Naqib and G. W. Farwell, *Bull. Am. Phys. Soc.* 8, 318 (1963) and private communication.

Fig. 1. Differential cross sections for inelastic excitation of $\ell=4$ transitions in Mg^{26} by 42-MeV alpha particles (Ref. 23).

Fig. 2. Differential cross sections for inelastic excitation of octupole transitions in Mg^{26} by 42-MeV alpha particles (Ref. 23).

transition; the octupole excitation energies for these nuclei, as well as for Al^{27} , lie in the range from 5.5 to 8.5 MeV.

One reason I have for presenting these results is to emphasize that our information concerning the occurrence of higher order multipolarities throughout the periodic table is still quite fragmentary.

(c) A question which naturally occurs is: "Are some projectiles and bombarding energies better than others for making spin and parity assignments?" My own prejudiced feeling is that complex projectiles are usually better for this purpose. Because of their short wavelengths and strong absorption, these particles show considerable structure in their angular distributions. By comparison, nucleon angular distributions tend to be rather washed out and characterless. At high

Fig. 3. Differential cross sections for inelastic excitation of levels in Al^{27} and Si^{28} by 150-MeV protons (Ref. 24).

energies, the most prominent feature of the nucleon angular distributions is a single forward peak and the angular-momentum assignment depends sensitively on the location of this maximum. One instance, for example, in which proton angular distributions appear to be misleading occurs in a recent report from the synchrocyclotron group at Orsay.²⁴ Analysis of the scattering of 150-MeV protons suggests that a level at 4.5 MeV in Al^{27} undergoes octupole excitation (Fig. 3) while analysis²³ of the highly structured alpha-particle angular distributions to this level points rather clearly to an $\ell=4$ assignment.

This comparative lack of structure persists for protons of lower energy. Figure 4 shows the angular distributions²⁵ of 17.45-MeV protons which excite 2^+ levels in Fe^{54} ; the solid curve is the DWBA prediction for $\ell=2$. These $\ell=2$ assignments agree with those made on the basis of alpha-particle scattering.²⁶ Figure 5 shows excitations which

²⁴J. C. Jacmart, M. Liu, R. A. Ricci, M. Riou, and C. Ruhla, Phys. Letters **8**, 273 (1964).

²⁵H. O. Funsten, N. R. Roberson, and E. Rost, Phys. Rev. **134**, B117 (1964).

²⁶M. Barloutaud, R. Chaminade, H. Faraggi, D. Garreta, and B. G. Harvey, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3—8, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, New York, 1963).

Fig. 4. Differential cross sections for inelastic excitation of $\ell=2$ transitions in Fe^{54} by 17.45-MeV protons (Ref. 25). The solid curves result from DWBA calculations for an $\ell=2$ collective surface excitation.

are presumably $\ell = 4$ in nature; however, later examination of the data showed that there was contamination in the 4.76-MeV group at the two smallest angles and thus the cross sections at these angles should be reduced. The new angular distribution was now in good agreement with the DWBA curve for $\ell = 3$. Perhaps there is a moral to be drawn, if the alteration of two points can make such a clear change in assignment.

2. Odd-A nuclei; weak particle-core coupling. A thought

long prevalent in the field of inelastic scattering has been that the collective excitations observed in even-mass nuclei should also appear in their odd-mass neighbors; it usually is presumed that the total strength

Fig. 5. Differential cross sections for inelastic excitation of $\ell=4$ transitions in Fe^{54} by 17.45-MeV protons (Ref. 25). The indicated cross section to the 4.76-MeV level is known now to be too large at the two smallest angles and the corrected angular distribution is in accord with the DWBA predictions for an $\ell=3$ transition. The solid curves result from DWBA calculations for an $\ell=4$ collective surface excitation.

of such excitations is little influenced by the addition of an odd particle or hole but that the distribution of strength reflects the mode of particle-core coupling.

When this coupling is assumed to be weak (an assumption of wider validity than the extreme vibrational model), the inelastic cross section in the odd-mass nucleus is simply related to the cross section that would be found were this nucleus of even mass.²⁷ The relation is

$$\frac{d\sigma_{\ell}(I=j \rightarrow I')}{d\Omega} = \frac{(2I' + 1)}{(2\ell + 1)(2I + 1)} \frac{d\sigma(0 \rightarrow \ell)}{d\Omega}, \quad (\text{B. 1})$$

where j is the angular momentum of the odd particle or hole, I and I' are the angular momenta of the ground state and excited state, respectively, and ℓ is the multipolarity of the transition. In the weak-coupling picture, $I = j$.

The scattering cross sections from many odd-mass nuclei have been cited as providing evidence for this weak-coupling picture. The copper isotopes, Cu^{63} and Cu^{65} , have been perhaps the most popular targets for such studies. The first three excited levels of these isotopes, shown in Fig. 6, have spin $\frac{1}{2}^-$, $\frac{5}{2}^-$, and, presumably, $\frac{7}{2}^-$. Proton

Fig. 6. Levels in the even-A nuclei, Ni^{62} and Ni^{63} , and in the neighboring odd-A nuclei, Cu^{63} and Cu^{65} , which are preferentially excited by alpha particles (Ref. 29).

²⁷ A. de Shalit, Phys. Rev. 122, 1530 (1961).

experiments at Oak Ridge²⁸ and alpha-particle experiments at Saclay²⁹ have indicated that these levels are populated in accord with the $(2I' + 1)$ intensity rule, although it does appear that the transition to the $\frac{7}{2}^-$ level is only slightly more intense than that to the $\frac{5}{2}^-$ level. Contrary to previous thinking, it is most likely that the 1.41-MeV level is not a member of the multiplet.²⁹ In both the proton and alpha-particle experiments it is found that the shapes of the angular distributions and the summed intensities are nearly equivalent to those found in the neighboring Ni⁶² nucleus. There have been some recent indications, however, that these nuclei do not provide the best examples of weakly-coupled particle-core motion. (a) The (He^3, d) investigations of Allen Blair³⁰ indicate a large admixture of single-particle configurations in the presumed particle-core states. (b) The Saclay group²⁹ has found that the octupole strength in Cu⁶⁵ now lies in at least five levels (Fig. 6). This is one more than is compatible with the extreme weak-coupling model. Further, this same superfluity of octupole levels has been observed in Cu⁶³ by Harvey, Horan, Meriwether, Bussiere, and Mahoney³¹ at Berkeley working with 50-MeV alpha particles from the 88-in. cyclotron; the angular distributions to these five levels are shown in Fig. 7. This work makes one particularly appreciate the value of good energy resolution in spectroscopic studies.

Naive considerations suggest that the weak-coupling model should be most successful for nuclei which are one particle or one hole removed from a major shell closing. In some of the first investigations of proton scattering, Cohen and Mosko³² found that the energy spectra

²⁸F. Perey, R. J. Silva, and G. R. Satchler, Phys. Letters 4, 25 (1963).

²⁹G. Bruge, J. C. Faivre, M. Barloutaud, H. Faraggi, and J. Saudinos, Phys. Letters 7, 203 (1963).

³⁰A. G. Blair, Phys. Letters 9, 37 (1964).

³¹B. Harvey, D. Horan, J. Meriwether, Bussiere, and Mahoney (to be published).

³²B. L. Cohen and S. W. Mosko, Phys. Rev. 106, 995 (1957).

from the three lead isotopes, $\text{Pb}^{206, 207, 208}$, and Bi^{209} were almost identical; the octupole excitation at 2.6 MeV in Pb^{208} was particularly intense and occurred with equal intensity and with no apparent shift of energy in the other three nuclei. Similarly, experiments with high-energy electrons^{33, 34} have also indicated that the core excitations of doubly-magic Pb^{208} are present in the neighboring odd-mass nuclei with almost the same strength and position.

Taking advantage of the improved energy resolution now obtainable through use of solid-state detectors, Jonas Alster³⁵ at the University of Washington has recently performed similar experiments in which he scattered 42-MeV alpha particles from Pb^{208} , Pb^{207} , and Bi^{209} . Alster observes strong excitation of the 3^- state in Pb^{208} , equally strong excitation of a group at 2.6 MeV in Pb^{207} , and equally strong excitation of a group occurring near this energy in Bi^{209} . The levels in Pb^{207} , presumably only two in number, are split by less than 100 keV. Some splitting is observed in Bi^{209} , however; the levels appear to be clustered into two peaks, at 2.50 and 2.67 MeV, respectively.

In Fig. 8 are shown the ratio of the elastic cross section for Pb^{207} to the Rutherford cross section, and the three inelastic cross sections for the groups near 2.6 MeV. In support of

³³H. W. Kendall and J. Oeser, Phys. Rev. 130, 245 (1963).

³⁴H. Crannell, R. Helm, H. W. Kendall, J. Oeser, and M. Yearian, Phys. Rev. 123, 923 (1961).

³⁵J. Alster (to be published).

Fig. 7. Preliminary cross sections (Ref. 31) of alpha particles (50-MeV incident energy) which excite octupole transitions in Cu^{63} . Each curve has been displaced by a decade. Also shown is the elastic angular distribution.

Fig. 8. Differential cross sections for excitation of groups near 2.6 MeV in Pb^{207} , Pb^{208} , and Bi^{209} by 42-MeV alpha particles (Ref. 35). Also shown is the ratio of the elastic-scattering cross section from Pb^{207} to the Rutherford cross section.

the weak-coupling interpretation, we observe that the three inelastic cross sections are practically equivalent and (in contrast to the usual nuclei to which the weak-coupling model is applied) that the relative splitting and the shift of the core excitation energy are very small.

3. Odd-mass nuclei with strong particle-core coupling.

If there is strong coupling between the odd nucleons and a permanently deformed core, and if the adiabatic approximation is valid, the inelastic cross section within the ground-state band is related to the cross section that would be found were this nucleus of even mass by the expression

$$\frac{d\sigma_{\ell}(I, K \rightarrow I', K)}{d\Omega} = (I, \ell, K, 0 | I', K)^2 \frac{d\sigma(0 \rightarrow \ell)}{d\Omega}, \quad (\text{B.2})$$

where K is the projection of the total angular momentum on the symmetry axis.

The nucleus Mg^{25} provides an excellent example of this situation. The energy levels of this nucleus are shown in Fig. 9. The $\frac{7}{2}^{+}$ level at 1.61 MeV and the $\left(\frac{9}{2}^{+}\right)$ level at 3.4 MeV are presumably members of the ground-state band and as such should be strongly excited in inelastic-scattering experiments. Blair and Hamburger³⁶ found that

³⁶A. G. Blair and E. W. Hamburger, Phys. Rev. 122, 566 (1961).

Fig. 9. The energy levels, spins, and parity assignments for Mg^{25} .

these two levels were those most strongly excited by 15-MeV deuterons. Similarly, Schrank, Warburton, and Daehnck³⁷ observed this to be true when Mg^{25} was bombarded with 17-MeV protons. In both the deuteron and proton experiments, the ratio of the cross section for exciting the 3.4-MeV level to that for exciting the 1.61-MeV level was found to be about 0.41, in good agreement with the value 0.35 predicted by Eq. (B.2).

Additional experiments utilizing alpha particles bolster this conclusion. Figure 10 shows the angular distributions of 42-MeV alpha particles exciting these two levels, as measured by Naqib and Farwell.²³ Not only are the shapes of these angular distributions nearly indistinguishable but also they are very close to those obtained for the first 2^+ levels in the neighboring Mg^{24} and Mg^{26} nuclei. The ratios of

Fig. 10. Differential cross sections for inelastic excitation of quadrupole transitions within the ground-state band of Mg^{25} produced by 42-MeV alpha particles (Ref. 23). Also shown is the elastic-scattering cross section.

³⁷G. Schrank, E. K. Warburton, and W. W. Daehnck, Phys. Rev. 127, 2159 (1962).

the 3.4- and 1.61-MeV cross sections at the first four maxima are found to be 0.33, 0.32, 0.35, and 0.33. The resulting nuclear deformation is $\beta_2 R_0 = 1.39$ fm, somewhat less than the value 1.66 fm found for Mg^{24} .

The effects of strong coupling should also appear in the elastic cross section. Both simple models¹¹ and more general theoretical considerations³⁸ lead to the prediction that the elastic cross section from an odd-mass nucleus should be

$$\frac{d\sigma}{d\Omega}(I \rightarrow I) = \frac{d\sigma}{d\Omega}(0 \rightarrow 0) + \sum_{\text{even } \ell} (IK\ell 0 | IK)^2 \frac{d\sigma}{d\Omega}(0 \rightarrow \ell), \quad (\text{B.3})$$

where $\frac{d\sigma}{d\Omega}(0 \rightarrow 0)$ and $\frac{d\sigma}{d\Omega}(0 \rightarrow \ell)$ are the elastic and inelastic cross sections, respectively, this nucleus would have if it were of even mass. Besides the assumption of strong coupling, the derivations of both this result and Eq. (B.2) require only that the scattering amplitudes may be computed in the adiabatic approximation; they hold to all orders of the deformation parameter and are not restricted to first- and second-order contributions.³⁸ Figure 11 contains universal plots of the measured cross sections²³ for 42-MeV alpha particles elastically scattered from Mg^{24} and Mg^{25} . Also

Fig. 11. "Universal" plot of elastic-scattering cross sections (Ref. 23) of 42-MeV alpha particles from Mg^{24} and Mg^{25} . This figure illustrates the "inelastic" contribution to the elastic-scattering cross section for an odd-mass nucleus.

³⁸ G. R. Satchler, Nucl. Phys. 45, 197 (1963).

shown is the "even-mass" elastic-scattering cross section obtained by subtracting 0.75 times the measured inelastic cross section to the 1.61-MeV level from the observed Mg^{25} elastic curve. This new curve agrees well with the Mg^{24} cross section; on the other hand, we see that only a rough estimate of β_2 could be made were the Mg^{24} curve subtracted from the observed Mg^{25} cross section in order to obtain the "inelastic" contribution to the Mg^{25} cross section.

Let us also consider scattering from nearby Al^{27} and Si^{28} . The level structure of Al^{27} is given in Fig. 12. The ordering of the positive-parity levels of Al^{27} with energy less than 3 MeV is very similar to that found in Al^{25} and Mg^{25} . Since the strong-coupling model has been applied with considerable success to these latter nuclei (as well as to many other representatives from the s-d shell), it was expected that the inelastic cross sections would be close to those found for corresponding levels in Mg^{25} .

Experiments, again carried out by Naqib and Farwell with 42-MeV alpha particles, show that this is not the case. The inelastic

Fig. 12. The energy levels, spins, and parity assignments for Al^{27} .

Fig. 13. Differential cross sections for inelastic excitation of $\ell=2$ transitions in Al^{27} produced by 42-MeV alpha particles (Ref. 23). Also shown is the elastic-scattering cross section.

cross sections to the levels with excitation energy less than 3 MeV are shown in Fig. 13.

Concerning these cross sections we make the following comments: (a) All angular distributions display the familiar $\ell=2$ shape. (b) In contrast to the situation in Mg^{25} , the cross sections to the first $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states now have modest size; their values at the maxima are at least 3–5 times as large as the upper limits to the corresponding cross sections in Mg^{25} .

(c) The ratio of the cross sections for exciting the 2.99-MeV doublet (presumably containing the $\frac{9}{2}^+$ level) and the 2.21-MeV level (which is most probably a $\frac{7}{2}^+$ state) is close to unity, in contrast to the value 0.35 predicted by the extreme strong-coupling model.

(d) Except for the transition to the 2.73-MeV level, which has the same spin and parity as the ground state, the cross sections correspond roughly to the $(2I' + 1)$ intensity rule. All of

these features then suggest that an appropriate crude model for Al^{27} is that of the $d_{5/2}$ hole weakly coupled to a Si^{28} core. Certainly this model enjoys no less success for Al^{27} than it does for the copper isotopes.

Experiments with 12.8-MeV deuterons carried on at Cracow by Niewodniczanski, Nurzynski, Strzalkowski, and Wilczynski^{39,40} bear on this same question. The inelastic-scattering cross sections to these same levels in Al^{27} and to the first 2^+ state of Si^{28} are displayed in Fig. 14. Several comments are appropriate. (a) The relative strengths of the Al cross sections are nearly identical to what was found in the alpha-scattering experiment. (b) Although the angular distributions are not too well described by the adiabatic strong-absorption formulas, the deformation parameters obtained by matching these predictions to the first maxima are very close to those obtained in the alpha experiments; the corresponding deformation parameters for the three strongest transitions differ by no more than 20% in these two experiments.

Fig. 14. Differential cross sections for inelastic excitation of $\ell=2$ transitions in Al^{27} and Si^{28} produced by 12.8-MeV deuterons (Refs. 39, 40).

³⁹ H. Niewodniczanski, J. Nurzynski, and J. Wilczynski, in Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, Italy, September 3–8, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, New York, 1963).

⁴⁰ H. Niewodniczanski, J. Nurzynski, and A. Strzalkowski, J. de Physique 24, 944 (1964).

Fig. 15. Differential cross sections for elastic scattering and for inelastic excitation within the ground-state rotational band of Ho^{165} by 17.5-MeV protons (Ref. 41). The solid curves are the predictions of Tamura's coupled-channel calculations (Ref. 7).

(c) The summed Al^{27} intensity is only 60% of that to the 2^+ level in Si^{28} . This reduced intensity is consistent with the fact that the center of gravity of the Al^{27} levels lies at 2.32 MeV while the 2^+ state in Si^{28} occurs at 1.78

MeV, a fact which implies a smaller deformation for Al^{27} .

The strong-coupling model is most firmly established among the rare-earth and actinide nuclei. Until recently, however, detectors have been unable to resolve the elastic and inelastic levels within the ground-state bands of these nuclei. Using cooled solid-state detectors, Lieber and Whitten⁴¹ have effected such a separation; Fig. 15 shows the cross sections for 17-MeV protons scattered from Ho^{165} . A very significant observation is that the ratio of the $\frac{9}{2}^-$ to $\frac{11}{2}^-$ cross sections is about 2 although the adiabatic prediction for this ratio, Eq. (B.2), is nearly 4. Also shown are the results of coupled-channel calculations by Tamura⁷; because a single calculation took seven hours on the computer, no variation in parameters was attempted to improve the fit. Perhaps the most important result from this calculation is that the computed ratio of cross

⁴¹ A. Lieber and C. A. Whitten, Phys. Rev. 132, 2582 (1963).

Fig. 16. Differential cross section for inelastic excitation of an $I=4^+$ level in Ni^{58} produced by 43-MeV alpha particles (Ref. 42). The solid curve is the prediction of Buck's coupled-channel calculation (Ref. 4).

sections is now close to the experimentally observed value. This is a significant justification for performing the coupled-channel computations. I do find it surprising though that experiments and calculations show such a large departure from the adiabatic predictions for Ho^{165} while the Mg^{25} results agree so well with Eq. (B.2).

C. Multiple Excitation of Collective Transitions

One of the most interesting phenomena in the field of inelastic scattering has been the observation of double excitation, a process whose signature is the reversal of the usual asymptotic phase relations of complex projectiles. Double excitation processes are of particular spectroscopic importance since they place strong limitations on the form of the wave function of the excited state.

To indicate how well these reactions are described by coupled-channel calculations, let us first examine the cross sections (Fig. 16) for excitation of the 4^+ level at 2.47 MeV in Ni^{58} as measured by Yntema and Broek⁴² of the Argonne National Laboratory. The heavy line represents coupled-channel computations performed by the Oak Ridge group using Buck's code.⁴ It is seen that the computed curve follows the data out to rather large angles, although it should be pointed out that

⁴²J. L. Yntema and H. W. Broek (to be published).

Fig. 17. Differential cross sections for inelastic excitation of $I=2^+$ and 4^+ levels in Fe and Ni isotopes by 44-MeV alpha particles (Ref. 43). The top curves are representative of undiluted single excitation of $\ell=2$ transitions while the bottom curves show the presence and absence of double excitation.

structure in the experimental pattern is lost beyond 80° . The coupled-channel fits to the elastic and first 2^+ inelastic cross sections, not shown here, are excellent.

The absence or dilution of double excitation can be just as interesting as its presence. Figure 17 summarizes some researches by the Saclay group⁴³ for Fe and Ni isotopes, together with sharp-edge Fraunhofer predictions for double and single excitation. The 4^+ level in Fe^{56} at 2.04 MeV remains the prototype for pure double excitation. But the unresolved 2^+ and 4^+ levels in Ni^{62} at 2.30 MeV and the level in Ni^{64}

⁴³M. Barloutaud-Crut, G. Bruge, J. C. Faivre, H. Faraggi, and J. Saudinos, Phys. Letters 6, 222 (1963).

at 2.62 MeV conform much better to the single-excitation $\ell=4$ curves. The measured angular distributions at the top of the figure all correspond to $\ell=2$ single excitation. The dilution of double excitation in Ni^{62} is corroborated by recent work at Berkeley.³¹ Indeed, in the Berkeley data it is difficult to find any contenders for double excitation among the levels up through 5 MeV. It had been proposed that Ni^{62} was a classic illustration of the extreme vibrational model with the 0^+ level at 1.84 MeV and the doublet at 2.30 MeV forming a 2-phonon triplet; in support of this, coupled-channel calculations assuming this nuclear model were found to be in accord with 12-MeV proton experiments.⁴⁴ The alpha-particle results, however, may force a revision of the vibrational description of this nucleus or, on the other hand, it may develop, as Dr. Perey has suggested privately, that double excitation is enhanced relative to single excitation when protons are the bombarding particles. Unfortunately, it does seem difficult to discriminate between single- and double-excitation mechanisms on the basis of proton angular distributions, since these lack the characteristic signature of more strongly absorbed projectiles.

D. Single-Particle Transitions

One of the major dogmas concerning inelastic scattering is the thought, first emphasized by Cohen⁴⁵ in his pioneering studies, that collective transitions are greatly enhanced over single-particle transitions. This idea seems to be borne out in countless experiments; transitions for which we expect only a single particle to be involved are either not observed

⁴⁴J. K. Dickens, F. G. Perey, R. J. Silva, and T. Tamura, Phys. Letters 6, 53 (1963).

⁴⁵B. L. Cohen and A. G. Rubin, Phys. Rev. 111, 1568 (1958).

or are down in the mud of experimental background. Indeed, although inelastic scattering has been an active research field for several years, I do not know of a single good angular distribution for an unequivocal single-particle transition. The most promising examples would be transitions near closed shells between single-particle or hole states involving a change of parity since then no quadrupole enhancement could be present. Certainly²⁹ single-particle transitions in Pb^{207} or Bi^{209} are weak, as is also the $p_{1/2} \rightarrow g_{9/2}$ transition in Y^{89} , but I am not aware that meaningful angular distributions for these cases have been measured. The single-particle transitions in Pb^{207} and Bi^{209} appear to be inhibited by factors of 10 or 20.

Other good candidates for single-particle transitions are interband transitions in odd-mass nuclei which are well described by the strong-coupling model. Indeed, Naqib and Farwell²³ find that the cross sections to the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ levels at 0.84 and 1.01 MeV, respectively, in Mg^{25} are less than 1/30 of the cross section for the strong collective transition at 1.61 MeV. However, the $\frac{5}{2}^+$ member of this $K=\frac{1}{2}^+$ band, occurring at 1.94 MeV, is not similarly inhibited; its rather moderate cross section suggests that there is band mixing between this level and the ground state.

Definitive measurements of single-particle cross sections would be invaluable to the theoreticians. Such determinations would enable them to calibrate other transitions against single-particle transitions and are essential in the construction of any microscopic model of inelastic scattering.

There have been several attempts recently to discuss inelastic-scattering cross sections in terms of interactions between the

projectile and a target nucleon. Wall^{46,47} has contributed such a microscopic interpretation of collective excitations in Ca^{40} to this conference. I also know of similar unpublished work on microscopic descriptions by V. Madson at Oregon State University and D. Jackson at the University of Washington.

A joint experimental-theoretical study by Funsten, Roberson, and Rost,²⁵ who analyze proton scattering from Ti^{50} , Cr^{52} , Fe^{54} , and V^{51} , comes very close to explaining the magnitudes of cross sections in terms of a fundamental nucleon-nucleon interaction. The nuclei Ti^{50} and Fe^{54} are described as having two particles and two holes, respectively, in the $f_{7/2}$ shell. In the shell model, the lowest 2^+ , 4^+ , and 6^+ levels are excited through recoupling of these $f_{7/2}$ orbitals. When the two-nucleon interaction is taken to be a pure Wigner force, the strength of the interaction potential necessary to fit the magnitude of 2^+ inelastic cross sections is only 30% higher than the free two-body strength. Now perhaps we should not be too impressed by this near agreement; potentials with exchange mixtures will require larger strengths. Further, these effective potentials should be regarded as parameterizations of the 2-body t matrix rather than the 2-body potential.

More important are two further results of this paper.

(i) The effective strength necessary to fit the 4^+ angular distributions is only larger by another factor of 5/4. (ii) The scattering cross sections for the odd nucleus V^{51} did not fit the weak-coupling model in regard to either shapes of angular distributions or relative intensities to the different levels. However, using a two-body interaction with the same strength

⁴⁶E. A. Sanderson and N. S. Wall, *Phys. Letters* 2, 173 (1962).

⁴⁷N. S. Wall, in *Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers*, Argonne National Laboratory Report ANL-6848, March 1964, p. 108.

Fig. 18. Differential cross sections for inelastic excitation of low-lying levels in V^{51} by 17.45-MeV protons (Ref. 25). The curves at the right are the result of DWBA calculations in which the shell model is used to describe the nuclear levels.

as that found in the even-mass experiment, these authors did obtain good agreement between the predictions of shell-model theory and experiment. This comparison is shown in Fig. 18.

E. Concluding Remarks

Two additional topics which will undoubtedly receive much attention before long are: (a) excitation of unnatural-parity levels and (b) scattering to the first excited 0^+ levels.

For spin-zero projectiles, transitions to unnatural-parity levels are forbidden in a single excitation involving a local interaction.⁴⁸ Small cross sections have been observed, though, and at this time it is not clear what is the reaction mechanism.

The variation in energy of the lowest excited 0^+ levels suggest that they all may be of similar character, and electromagnetic transitions involving these levels indicate considerable collective enhancements. However, inelastic-scattering studies show a rather varied behavior; some measured cross sections provide perfect examples of single excitation, some are suggestive of double excitation, and others are unobservably small. It is clear that inelastic-scattering studies will be of great assistance in dispelling the mystery surrounding these states.

⁴⁸ W. W. Eidson and J. G. Cramer, Jr., Phys. Rev. Letters 9, 497 (1962).

* * *

N. M. HINTZ, University of Minnesota: I would like to defend nucleons. It is true that if you don't want much information, (α, α') reactions are the way to get it. They give spin, parity, and maybe the strength of the coupling constant without much trouble. If you want to know something about the wave function, you need something which penetrates inside the nucleus like electrons or protons. Electrons are very sensitive to the detailed configuration assumed in the wave functions. This is not true for the strongly absorbed particles in general. Your Fig. 18 is a good example of this.

F. G. PEREY, Oak Ridge National Laboratory: Did you say that the values of β_l obtained in this experiment were equal or only very similar?

BLAIR: When I said "similar" I meant equality within $\pm 30\%$.

G. T. GARVEY, Princeton University: It would seem that the similarity of angular distributions (say, of alphas from lead) could not be too meaningful in terms of the similarity of l value because the scattering will be dominated by the Coulomb terms.

BLAIR: It is definitely true that in the cases in which the Coulomb field is large, the structure is washed out to a large extent; but it is interesting to see that rather strong oscillations do appear here in the inelastic cross sections where the elastic angular distributions do not show much structure.

M. A. NAGARAJAN, Case Institute of Technology: I would like to ask about the (α, α') experiment on the copper isotopes at Berkeley. I noticed that five levels had been excited, whereas only four would be expected. Does this represent a breakdown of the weak coupling?

BLAIR: Yes, my point was that the weak coupling does seem to break down and that I have no ready explanation for the presence of the extra level.

S. MESHKOV, U. S. National Bureau of Standards: Your Fig. 11 showed the elastic scattering by Mg^{24} and Mg^{25} . Do you have some corresponding data for Mg^{26} so that one could see whether the little dip that you ascribe to the Clebsch-Gordan coefficient occurs for these also?

BLAIR: If the Mg^{26} were placed on the same universal plot, the difference between it and Mg^{24} would not be very large. I could have shown the effect just as well by comparing Mg^{25} with Mg^{26} instead of with Mg^{24} .

H. E. GOVE, University of Rochester: Isn't it true that Mg^{26} and Mg^{24} are vastly different nuclei? I do not think one expects Mg^{26} to have the same distortion as Mg^{24} .

BLAIR: The Mg^{26} certainly has a smaller deformation. The deformation $\beta_2 R_0$ may be extracted from an adiabatic smooth-cutoff calculation. The value is 1.66 fm for Mg^{24} , 1.39 fm for Mg^{25} , and for Mg^{26} it is 1.34 fm. It is definitely less.

However, I think there is a very confused situation concerning the proper model for Mg^{26} . Actually, it may be a little closer to Mg^{24} than we have thought previously. It certainly is not vibrational.

GARVEY: On a spin-zero nucleus, the deformation would have to come in second order in a calculation of the elastic scattering. Have you looked at any strongly deformed nuclei with spin zero? If so, have you been able to see the second-order effects of the deformation?

BLAIR: The Mg^{24} may be considered as a case in point. One gets a good fit with a spherical calculation; but on the other hand one also gets good fits (with different parameters) with a coupled-channel calculation. Now, what does that mean? I don't know.

What would be nice would be to compare a deformed nucleus with a spherical one that has very nearly the same A. In the Contributed Papers for this symposium, Perey⁴⁹ has considered just this point in his

⁴⁹F. G. Perey, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 114.

analysis of Shrank's data on the elastic scattering of 17-MeV protons on nuclei ranging from spherical ones to the rare earth nuclei. This analysis is not concerned with the quadrupole contribution to scattering from odd-mass nuclei but bears on just the point you make. He finds that with the same optical parameters he can understand elastic angular distributions that are qualitatively rather different. The only difference in the analysis is that the rare earths have a deformation while lead does not.

B. G. HARVEY, Lawrence Radiation Laboratory, Berkeley: I would like to comment again on the weak-coupling model in Cu^{63} . In the inelastic spectra we see five (or possibly six) levels of octupole type in Cu^{63} and Cu^{65} . If you invoke the presence of more than one 3^- state in Ni^{62} and if you take weak coupling seriously, then, of course, you can build a quartet of levels on each one of these 3^- states. We know that there are at least two 3^- levels in Ni^{62} , one of them much more strongly excited than the other, so in the weak-coupling limit, you would expect one strong quartet of octupole states in Cu^{63} and one weak quartet. This is exactly what was in fact shown by the calculations of Lemmer and Beres that were discussed by Kerman yesterday.

What we see in our experiment, however, is not quite this because the five or six levels that are present naturally do not fall into two quartets. Of course, some of them may be unresolved doublets. However, we do not see one group that is weak and then another group that is much stronger. Thus one would have to suppose that the E3 strength was somehow redistributed between these levels in such a way as to even it out a little bit. But the situation is clearly much more complicated than just a quartet based on a single 3^- state in the core.

A. BERNSTEIN, Massachusetts Institute of Technology: I would just like to comment on the excitation of 0^+ states. A group of us at MIT have scattered alpha particles from Ca^{40} . Much to our surprise, we do not see any excitation of the 0^+ state at all.

BLAIR: I only comment that angular distributions or cross sections for these 0^+ states sometimes show beautiful single-excitation patterns, sometimes they clearly show double excitation, and sometimes (as you mentioned for Ca^{40}) no patterns are seen at all. There is a great deal of variety in these 0^+ cross sections.

C. D. ZAFIRATOS, Los Alamos Scientific Laboratory: Some question was raised earlier about Coulomb effects on the alpha-particle inelastic-scattering angular distributions for the excitation of 3^- collective states in the lead region, in particular on the similarity of the observed angular distributions. We have seen the same sort of effect with 8-MeV neutrons. We see one strong 3^- level in Pb^{206} , Pb^{208} , and Bi^{209} . The transition strengths are virtually identical and the angular distributions are similar.

R. D. LAWSON, Argonne National Laboratory: Seven of the levels below 2 MeV in Cu^{63} can be attributed to the core-coupling model plus single-particle states. The fact that these all show up in inelastic scattering implies only that the states are not as simply described as one would like. For example, since the $f_{5/2}$ single-particle level is not far from the $p_{3/2}$ single-particle level, it follows that the lowest two $5/2$ levels in Cu^{63} will be described by a linear combination of wave functions in which an $f_{5/2}$ proton couples to the $J=0$ Ni^{62} core, a $p_{3/2}$ proton couples to the $J=2$ Ni^{62} core, and additional possibilities which probably play a less important role in the description of the lowest two spin- $\frac{5}{2}$ states. Thus both these states will show up strongly in the scattering. Hence one sees that, in total, more than four states should show up at excitation energies of the order of the excitation energy of the 2^+ state in Ni^{62} . A similar situation is bound to persist for the states arising from the coupling to the 3^- level in Ni^{62} .

BLAIR: I do not think it is at all surprising that this situation occurs in Cu^{63} . The only point I was trying to make here was that this nucleus is not a good candidate for the extreme weak-coupling model, but that there are some other good candidates found in nature.

HARVEY: You very briefly mentioned one point about the single-particle transitions. I would like to further comment that there is one thing that we do know about these. Nobody doubts that one can excite a single-particle transition by inelastic alpha scattering. It is interesting to note that the wave functions of the 6.23- and 6.44-MeV levels in N^{14} are fairly well established, and that both of these states would have to be made from the ground state of N^{14} by exciting two particles simultaneously. These levels are very weakly excited in inelastic alpha scattering. Their cross sections are very roughly one tenth of those for neighboring levels that represent single-particle excitation. Just by looking for levels that are unusually weakly excited, one can pick out those that may be more complicated than single-particle excitation.

INELASTIC SCATTERING ON Ti^{48} [†]

Larry Zamick

Princeton University, Princeton, New Jersey

I am going to speak about inelastic scattering on Ti^{48} and certain selection rules that can be explained by the shell model. If we represent Ti^{48} by a model in which nucleons are filling the $f_{7/2}$ shell, then the nucleus is described as having two protons and six neutrons in this shell. If we assume that the nuclear forces are charge independent, then we can say (1) that under the interchange of neutrons and proton holes and of protons and neutron holes, Ti^{48} goes into itself and (2) the Hamiltonian would not change under this interchange. What, then, are the wave functions of Ti^{48} ?

In the paper of McCullen, Bayman, and myself,¹ we first of all considered separately the neutrons and the protons. There are two protons, and they can couple to angular momentum 0, 2, 4, or 6. There are two neutron holes, and they can also couple to 0, 2, 4, or 6. Consider the ground state of Ti^{48} , which is spin zero. What are the ways to form spin zero? One way is to take the protons and couple them to zero, and the neutron holes and couple them to zero. Another is to take the protons and couple them to 2, and the neutron holes and couple them to 2. We also form the combinations 4,4 and 6,6. The true wave function will be some linear combination of these four basic states. By simply looking at this wave function, we can see that regardless of what the linear combination is, the wave function will remain unchanged under the interchange of neutrons and proton holes and of protons and neutron holes.

[†]Contributed paper (not in Vol. I. Contributed Papers).

¹J. D. McCullen, B. F. Bayman, and Larry Zamick, Phys. Rev. 134, 515 (1964).

The spin-2 state can be formed by having the two protons coupled to 0 and the two neutron holes coupled to 2. Alternatively, the two protons could be coupled to 2 and the two neutron holes to 0. (The full spin-2 wave function will, of course, also have components 22, 24, 42, 44, 46, 64, and 66.) Now because of the symmetry under the interchange of neutrons and proton holes, neither of these wave functions by itself is a state of Ti^{48} , but rather the state will be

$$[\psi_p^0 \psi_{nh}^2] \pm [\psi_p^2 \psi_{nh}^0],$$

where the subscripts p and nh designate proton and neutron-hole wave functions, respectively. That is, one state is the symmetric combination and the other is the antisymmetric combination. In this calculation we have found that all the states of Ti^{48} in this $f_{7/2}$ -shell picture either change sign under this interchange or they do not change sign.

Now it is interesting to notice that the spin-0 state has even parity under this interchange, and so also does the first spin-2 state. In inelastic scattering, we find that if the operator that changes the ground state of Ti^{48} from a spin-0 state into a spin-2 is assumed to come from a simple Wigner force, then this operator would be simply $\sum_i Y_2(\Theta_i)$ times some radial factor, the sum being over all the neutrons and the protons in the $f_{7/2}$ shell. It is easy to show that this operator, which will connect from the spin-0 state to the spin-2 state, will not connect states of the same parity under the interchange of protons and neutron holes. Hence there should be no connection from the ground state to the symmetric spin-2 state. On the other hand, there will be a strong connection to the antisymmetric state which has opposite parity under this interchange.

The operation of the selection rule is seen in Fig. 1, which shows some unpublished Ti^{48} inelastic-scattering data taken by G. T. Garvey, G. M. Crawley, H. O. Funsten, and N. R. Roberson at Princeton.

Fig. 1. Inelastic scattering of protons on Ti^{48} . The spins, parities, and excitation energies of the states of interest have been included. These data were taken at the Princeton cyclotron by G. T. Garvey and G. M. Crawley using 17.5-MeV protons.

The first excited state in Ti^{48} is at about 1 MeV and has spin 2^+ .

According to our model, this state is the antisymmetric combination of proton-particle and neutron-hole wave functions. According to the simple theory just outlined, it should therefore be excited in inelastic scattering. Experimentally, it is strongly excited. This agrees with our prediction, although we would not be able to explain the magnitude of the cross section, which is much larger than a pure $f_{7/2}$ model of the state would predict.

The next two peaks are O^{16} and C^{12} impurities. The states at 2.31 and 2.43 MeV can be attributed to Ti^{48} . According to our model, these states are symmetric under interchange of protons and neutron holes and therefore should not be excited in the inelastic-scattering process. This is precisely what is observed experimentally.

The average cross section for excitation of the 4^+ state is ≤ 0.2 mb. The corresponding state in Ti^{46} , to which this selection rule does not apply, has a cross section of 0.8 mb. Thus this transition is inhibited by about a factor of 4 in Ti^{48} .

The state at 3.24 MeV is a 4^+ level which is predicted to be antisymmetric. This level should therefore be appreciably excited by inelastic scattering; and this is indeed what is observed experimentally. Finally, the peak at 3.36 MeV is a 3^- collective oscillation. Although this state is extremely interesting, its interpretation necessarily takes us outside the $f_{7/2}$ shell.

For Coulomb excitation, we would have roughly the same operator, but would be summing only over the protons. Therefore such a selection rule would not be expected in Coulomb excitation.

* * *

C. K. BOCKELMAN, Yale University: I would like to mention that in a contributed paper² on the $\text{Ti}^{47}(\text{d},\text{p})\text{Ti}^{48}$ reaction, it is clear that $\ell_n=1$ angular distributions are observed in the cases of the second 2^+ and the second 4^+ states. It is true that this reaction is particularly sensitive to the $\ell_n=1$ state and the $\ell_n=3$ may also be there in significant proportion. But it does suggest that the wave functions for these states are not quite as simple as you put them.

ZAMICK: Yes. I believe both of these states are basically $\ell_n=3$ states, although they may have small $\ell_n=1$ admixtures.

BOCKELMAN: We have not yet tried to separate the angular distribution to see what the proportion is. It is possible that this is true.

²P. D. Barnes, C. K. Bockelman, O. Hansen, and A. Sperduto, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 79.

J. S. BLAIR, University of Washington, Seattle: I recall that H. W. Broek did some inelastic alpha scattering from Ti^{48} . I wonder if Dr. Yntema would care to make a comment.

J. L. YNTEMA, Argonne National Laboratory: In our inelastic-scattering experiment, the excitation of the states near 2.3 MeV is about as intense as the excitation of the 4^+ level near 2.0 MeV in Ti^{46} . However, the angular distribution of the Ti^{48} states is very different from the one observed for the Ti^{46} level. In the case of Ti^{46} , one apparently has the mixing of the one-step and two-step excitations predicted by the vibrational model. In the case of Ti^{48} , the angular distribution indicates that one of the two processes is much stronger than the other.

ZAMICK: An alternative explanation of the reduced cross section is that this is a 2-quantum vibration and cannot be reached by direct interaction in the first order.

E. P. WIGNER, Princeton University: If you take exchange or antisymmetry into account, would your rule still hold?

ZAMICK: I think not.

WIGNER: Can one estimate what fraction of the cross section is due to exchange or antisymmetry and what fraction is due to admixtures of higher configurations?

ZAMICK: Calculations in progress are taking account of the admixture of $p_{3/2}$; but I haven't the faintest idea what the relative amount due to the antisymmetry would be. I have not done this at all. I hope the $p_{3/2}$ calculations will be finished soon; but at the moment I still have no good feeling for what the relative amounts would be.

WIGNER: Can your concept of the quasi-isotopic spin be generalized, or is it strictly confined to the cases you considered?

ZAMICK: You can also do it if you have the same number of particles (neutrons and protons), e. g., in the unstable nucleus Ti^{44} . In this case the selection rule is less interesting because states of different

signature also have different isotopic spin. I think the only difference there would be that the inelastic-scattering operator $\sum_i Y_2(\Theta_i)F(r_i)$ would connect states of the same symmetry rather than states of different symmetry because you deal with a particle-particle configuration rather than the particle-hole case. So the only places you can apply this reasoning is where you have the same number of neutrons and protons, or the same number of holes and particles.

WIGNER: There must be cases in which you have states of different isotopic spin. These can be excited only by antisymmetry or exchange. Do we have any information on how strongly these are excited?

ZAMICK: In our calculations (in which, of course, we use the central force), we predict zero excitation for $\Delta T > 0$ and the states seem to lie too high. There are no experimental analyses of these states at all. There are some interesting structures in these experiments at higher energies, but the natures of these states have not been ascertained.

E. KASHY, Princeton University: With regard to configuration mixing, the calculations based on the $(f_{7/2})^n$ configuration give very poor predictions of the spectroscopic factor for the $Ti^{49}(p,d)Ti^{48}$ reaction going to the 2.43-MeV level. The predicted value of the $\ell_n=3$ spectroscopic factor is considerably larger than the experimental value. Therefore, the configuration of this level may be quite different from the one assumed.

ISOBARIC-SPIN EFFECTS IN NUCLEAR REACTIONS^{*}

J. B. French

University of Rochester, Rochester, New York

I. Introduction

I propose to discuss in an elementary way, in terms of the shell model, certain aspects of the relationships between neutrons and protons, particularly in nuclei which have a sizeable neutron excess. I shall not say much specifically about (p,n) reactions or elastic proton scattering since Dr. Anderson and Dr. Fox will, to judge by their titles, discuss these matters in detail. I shall also not discuss the complicated question of the goodness of the isobaric-spin quantum number since it seems fairly clear that for the present purposes this question is largely irrelevant.[†]

One of the special features about a heavier nucleus is that, because of the operation of Coulomb effects, many of the nucleons in it (namely the excess neutrons) have their isobaric spins aligned, the resultant ground-state T value then being equal to its z projection, i. e., $T = T_z$. As a result of this, when we couple a further nucleon, or nucleon-hole, to this group we encounter a large interaction energy, and the sign of this energy will depend on the alignment of the extra isobaric spin with the original group, being positive for parallel alignment and negative for antiparallel. This gives rise then to an isobaric-spin splitting of single-particle and single-hole states. This phenomenon seems first to

^{*}Supported in part by the Atomic Energy Commission.

[†]Some comments are given in an Appendix.

have been discussed in papers by Macfarlane and the present speaker,¹⁻³ but the notion is such an obvious one that quite probably anybody who considered the situation in heavier nuclei must have thought of it. It is only during the past two years, however, that measurements of the splitting (a very rough value for which is 1 MeV per excess neutron) have become available.

Measurements of the splitting give us values of certain average interactions and because of this they are important. If one considers the details of the spectra which result from coupling to a $J \neq 0$ group, or if one examines certain fine structure which should show up especially in the smaller-T member of the doublet, one can learn more detailed things about the interaction. These questions have been recently considered by Pinkston⁴ and by Bansal.⁵

Besides the splittings, one can measure also the reaction strengths which pertain if the particle or hole is added by means of a stripping reaction (or a resonant reaction in the case of a particle). The application of sum-rule techniques here can measure the separate proton and neutron occupancy of the orbit concerned, in the target, as well as give information about the mode of coupling. It is important also that because of the splitting one will encounter transitions with the same ℓ value (and in many cases with predictable strengths) but far separated in energy; this circumstance has been shown recently⁶ to be very helpful in

¹ M. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960).

² J. B. French, Ned. Tijdschr. Natuurk. 27, 185 (1961).

³ J. B. French and M. H. Macfarlane, Nucl. Phys. 26, 168 (1961).

⁴ W. T. Pinkston, Nucl. Phys. (in press).

⁵ R. K. Bansal (private communication).

⁶ R. Sherr, the next paper presented in these Proceedings. See also E. Rost, B. Bayman, and R. Sherr, Bull. Am. Phys. Soc. 9, 458 (1964).

Fig. 1. Possible neutron and proton excitations that maintain $T = T_z$. Proton excitations to higher orbits and neutron excitations from lower orbits will not ordinarily give states of good T . The cross hatching denotes closed shells.

examining certain difficult questions about the proper analysis of direct-reaction results.

I have said that in a heavier nucleus the ground state has $T = T_z$. But the same is true for the low-lying excited states as well. The situation is as shown in Fig. 1. Any excitations that still leave all the protons in orbits that are filled for neutrons produce states in which we still have $T = T_z$; this follows easily from the fact that the nucleons in any one orbit then form a group of identical nucleons or identical holes, each group having $T^{(i)} = T_z^{(i)}$. Of course, instead of considering excitations as in the figure, we can consider the addition or removal of a nucleon. If we add a proton in a sufficiently high orbit, or take away a neutron from a sufficiently low one, we can produce states with $T = T_{\text{target}} \pm \frac{1}{2}$ because in both of these cases, we have the possibility of n-p exchanges producing a state linearly independent of the first. An example of this is given in Fig. 2; if we have a $p_{3/2}$ proton circulating around a nucleus (Ca^{48}) which has a set of fully closed shells (closed for both protons and neutrons) and an $f_{7/2}$ shell closed for neutrons, then

$$\begin{array}{c}
{}^{48}_{20}\text{Co}_{28} + \text{proton } (p_{3/2}) \\
\hline
\begin{array}{c}
p_{3/2} \\
f_{7/2}
\end{array}
= \sqrt{\frac{1}{9}} \left\{ \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right\} \xrightarrow{J_0=0, T_0=4} (T=\frac{3}{2}) \\
+ \sqrt{\frac{8}{9}} \left\{ \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right\} \xrightarrow{J_0=0, T_0=4} (T=\frac{5}{2}) \\
\hline
\psi = \sqrt{\frac{1}{9}} \begin{array}{c} \triangle \\ T_0=4 \\ T=\frac{3}{2} \end{array} + \sqrt{\frac{8}{9}} \begin{array}{c} \triangle \\ T_0=4 \\ T=\frac{5}{2} \end{array}
\end{array}$$

Fig. 2. The expansion (in both the n-p and T formalisms) of the $[Ca^{48} + \text{proton } (p_{3/2})]$ state in terms of states with good T.

according to the Pauli principle plus charge independence, we must imagine that it will be continually interchanging with an $f_{7/2}$ neutron. This exchange produces a state in which a $p_{3/2}$ neutron circulates around a Sc^{48} nucleus containing an $f_{7/2}$ hole-particle pair coupled to $J_0 = 0$ (and hence $T_0 = 4$). The two eigenstates of the system, with $T = T_0 \pm \frac{1}{2}$, are as indicated in Fig. 2 which gives the expansion of the original function, which does not have a good T, in terms of these eigenstates. The numbers which appear in the expressions for these eigenstates are the obvious isobaric-spin C.G. coefficients; we might remark parenthetically, that the usual C.G. coupling applies in the n-p formalism,* which we are now using, as long as the two groups being coupled have no orbit in common (though ordinarily not otherwise⁷).

The important thing to remember is that when we deal with configurations that do not automatically preserve T, it is usually misleading to speak of single-proton or neutron-hole states, because such states do not have good isobaric spin. We can however speak of single-nucleon states if we use an isobaric-spin formalism as also shown in Fig. 2. Each formalism has its own advantages, and quite often things that are simple in one are relatively complicated in the other. This point should be borne in mind when, for some reason, we wish to describe a

*In this formalism neutrons and protons are separately numbered, the basis states being then antisymmetrized separately in the two kinds of particles.

⁷J. B. French, Nucl. Phys. 26, 161 (1961).

Fig. 3. The states of the same nucleon configuration that can admix with the single-nucleon $T = \frac{7}{2}$, $p_{3/2}$ state.

phenomenon in terms of the formalism not better adapted to it. As a specific example of this consider, via Fig. 2, the $T = \frac{9}{2}$ Sc^{49} state and its isobaric analogue in Ca^{49} . The relationship between them is simple and obvious in the T formalism, but not obvious at all in the n-p formalism.

The n-p hole-particle pair needed in the above example is zero-coupled because the operation of n-p interchange does not affect the angular momenta. There is, however, another set of states in which the hole-particle pair is coupled to $J_0 \neq 0$ (and hence $T_0 = 3$). These states are as listed in Fig. 3; three of them (with $J_0 = 1, 2, 3$) have $J = \frac{3}{2}$, $T = \frac{7}{2}$. If the $\frac{3}{2}$ states are being formed by the process $\text{Ca}^{48} + \text{proton}$, these "core-polarization" states obviously carry none of the strength. But they may admix with the $T = \frac{7}{2}$ single-nucleon state, thereby fragmenting the $(T_0 - \frac{1}{2})$ strength and producing a fine structure in this member of the doublet. This is, of course, merely an example of the Lane-Thomas-Wigner giant-resonance phenomenon⁸ but it is of some special interest because, for one reason, the excitations involved are all within the same configuration, and, for another, we can hope to study the effective interaction by means of this fine structure.

II. The Isobaric-Spin Doublet Splitting

Figure 4 shows some relatively early results for $p_{1/2}$ hole states in Zr^{89} , the two members of the doublet having been located by two

⁸A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).

(p,n) Anderson, Wong & McClure

(p,d) Goodman

Fig. 4. An example of isobaric-spin splitting. The (p,n) reaction specifically selects the $T_{>}$ state; the (p,d) reaction favors the $T_{<}$ state but in any case does not extend to high enough excitation to show the upper member of the doublet.

different experiments^{9,10} as indicated.* The elementary view of these states is that they are describable as a $p_{1/2}$ hole weakly coupled to a Zr^{90} -ground-state core ($T_0 = 5$). The upper state then has $T = \frac{11}{2}$ and is in fact the isobaric-spin analogue of the ground state of $_{39}Y^{89}$; the lower state has $T = \frac{9}{2}$. The isobaric-spin doublet splitting is then about 7 MeV.

In order to calculate this splitting and thus to decide what information about the effective nucleon-nucleon interaction is given by the splitting, consider the energies for two particles in different orbits j_1, j_2 as shown in Fig. 5. Let $\overline{E^{(2)}}$ be the center of gravity of the states [with a $(2J+1)(2T+1)$ weighting], and $\Delta E^{(2)}$ the difference between the separate $T = 1$ and $T = 0$ centers of gravity. If now we consider only the monopole parts of the interaction of a j_2 particle with a $(j_1^{n_1})_{J_0 T_0}$ group, then for the interaction energy we have

⁹ J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 126, 2170 (1962) and later papers by the same group.

¹⁰ C. D. Goodman (private communication).

* I have been informed by Dr. C. D. Goodman that some fine structure has been observed for the $T = \frac{9}{2}$ member and that he has been making a theoretical analysis of the more detailed results.

Fig. 5. Schematic representation of the energies of a two-nucleon system, one nucleon in each orbit, and the definitions of the average parameters.

$$H_{\text{Int}}^{(\text{Monopole})} \equiv n \overline{E^{(2)}} + \overline{\Delta E^{(2)}} (\vec{T}_0 \cdot \vec{t}),$$

$$E^{(n+1)} = n \overline{E^{(2)}} + \frac{\overline{\Delta E^{(2)}}}{2} \left\{ T(T+1) - T_0(T_0+1) - \frac{3}{4} \right\},$$

$$\overline{E^{(n+1)}} = n \overline{E^{(2)}}, \quad (1)$$

$$E_{T>}^{(n+1)} - E_{T<}^{n+1} = \frac{(2T_0+1)}{2} \overline{\Delta E^{(2)}}.$$

The isobaric-spin splitting, of course, is given by the last equation. The same equations, but with $\overline{E^{(2)}} \rightarrow -\overline{E^{(2)}}$, apply to the interaction of a hole with a group.

We can identify the parameters $\overline{E^{(2)}}$ and $\overline{\Delta E^{(2)}}$ for a given interaction H_{ij} as follows. Let*

$$H_{ij} (1 - P_{ij}) = H_{ij}^{(0)} + H_{ij}^{(1)} \vec{t}_i \cdot \vec{t}_j, \quad (2)$$

where $H^{(0)}$ and $H^{(1)}$ contain no space-exchange or $\vec{t}_i \cdot \vec{t}_j$ operators. Then the two parameters are respectively the integrals, over a two-particle

* P_{ij} is the operator which exchanges all the coordinates of i, j . The exchange terms which it contributes to the energy are usually small (say 10% of the total). With this way of writing the interaction (appropriate only for the interaction between groups having no orbits in common), the antisymmetrization between the groups is to be ignored.

state, of the space-spin monopole parts of $H^{(0)}$ and $H^{(1)}$. Any term that is linear in the spin operators of each particle (e. g., a $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ or tensor-force term) gives no contribution to the monopole parts. Thus, for example, if we are dealing with a central interaction defined, along with its exchange part, as the sum of V_0 , V_σ , V_τ , $V_{\sigma\tau}$ interactions, then $\overline{E^{(2)}}$ depends only on the V_0 part and $\overline{\Delta E^{(2)}}$ only on the V_τ part.

The monopole requirement in Eq. (1) is no restriction if the $j_1^{n_1}$ group has $J_0 = 0$ and the coupling is weak, because then only the monopole term can contribute, but if $J_0 \neq 0$ (and $T_0 \neq 0$) the coupling of the particle will produce two sets of states, each spanning the range of J , i. e., the range $|j_1 - j_2|, \dots, (j_1 + j_2)$. It is pretty obvious that the results of Eq. (1) apply then if we consider the centers of gravity of these sets. Nonweak coupling implies that the matrix elements of the interaction which are off-diagonal in the states of the core are significant, and thus once again we will see a proliferation of states. It can be shown that the monopole result applies here also as long as (i) the core state has no particles in the j_2 orbit (or holes if we consider the coupling of a hole) and (ii) we consider the centers of gravity of the states produced, the a th state being weighted with $(2J_a + 1)\theta_a^2$, where θ_a^2 is the reduced width in the reaction $j_1^{n_1} + j_2 \rightarrow j_1^{n_1} j_2$.

To the extent that the parameters are independent of the j_1 orbit, the same results apply to the interaction of j_2 with n particles in various orbits (though strictly speaking we should consider separately the case $j_2 = j_1$). If we were to consider the scattering of the $(n + 1)$ th particle by the nucleus, we could represent the $\overline{E^{(2)}}$ and $\overline{\Delta E^{(2)}}$ interactions by optical wells and in so doing would be led to the so-called Lane term¹¹ in the optical-model potential. (For an earlier discussion, see the paper of

¹¹ A. M. Lane, Phys. Rev. Letters 8, 171 (1962).

Kerman, McManus, and Thaler.¹²) For the orbit pairs for which values are known, $\overline{\Delta E^{(2)}}$ turns out to be 1.5—3.0 MeV and, assuming that this range of values applies to other orbits as well, we can deduce reasonable values for the strength of the $(\vec{T}_0 \cdot \vec{t})$ term for any given radial dependence. Incidentally $\overline{E^{(2)}}$ is much smaller and negative; $|\overline{E^{(2)}}| \leq 0.25$ MeV for the known cases. It is in principle a more complicated parameter than $\overline{\Delta E^{(2)}}$ since it must be regarded as including single-particle kinetic-energy effects.

An application of Eq. (1) to the interesting case of the low-lying even-parity hole states in Sc and Ti isotopes has been recently made by R. K. Bansal and the present author.¹³ The hole states in question are those found in (d, He^3) and (p, d) reactions on Ti isotopes (see the papers of Yntema and Satchler¹⁴ and of Kashy and Conlon¹⁵); they thus refer to the coupling of a hole ($d_{3/2}$ or $s_{1/2}$ as identified by the l values) to the ground states of various Ti isotopes. The low-lying states found via (d, He^3) in Sc isotopes are those with $T = T_{>} \equiv (T_0 + \frac{1}{2})$, while those found via (p, d) have $T = T_{<} \equiv (T_0 - \frac{1}{2})$. The (p, d) experiments do not reach high enough in excitation to see the $T_{>}$ hole states but by a Coulomb correction to the Sc hole states (which are isobaric analogues of the $T_{>}$ states in Ti) we can safely locate them in the Ti isotopes.

The main point now is that the energy of a $d_{3/2}$ hole state (interpreted where necessary as the center of gravity of a pickup giant

¹² A. K. Kerman, H. McManus, and R. M. Thaler, *Ann. Phys. (N.Y.)* **8**, 551 (1959).

¹³ R. K. Bansal and J. B. French (to be submitted to *Phys. Letters*).

¹⁴ J. L. Yntema and G. R. Satchler, *Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers*, Argonne National Laboratory Report ANL-6848, March 1964, p. 177, *Phys. Rev.* **134**, B976 (1964).

¹⁵ E. Kashy and T. W. Conlon (to be published).

resonance) can be calculated in terms of known binding energies and the two monopole parameters of the $d_{3/2}$ - $f_{7/2}$ interaction. For the excitation energy E^* of the hole state in the nucleus $(f_{7/2})^n \times \text{Ca}^{40}$, we have

$$\begin{aligned}
 E^* &= E \left\{ j^{-1} \times (f_{7/2}^{n+1} \times \text{Ca}^{40})_{x_0 J_0 T_0} \right\}_{JT} - E \left\{ f_{7/2}^n \times \text{Ca}^{40} \right\}_{g.s.} \\
 &= E \left\{ f_{7/2}^{n+1} \times \text{Ca}^{40} \right\}_{x_0 J_0 T_0} - E \left\{ f_{7/2}^n \times \text{Ca}^{40} \right\}_{g.s.} + E \left\{ j^{-1} \times \text{Ca}^{40} \right\} \\
 &\quad - E \left\{ \text{Ca}^{40} \right\} + E_{\text{Int.}} \left\{ j^{-1} \times (f_{7/2}^{n+1})_{x_0 J_0 T_0} \right\}_T + \epsilon_c. \quad (3)
 \end{aligned}$$

The hole states are defined here by the quantum numbers $(x_0 J_0 T_0)$ of the core, the l and j values of the hole orbit, and the resultant values J, T .

The next to the last term above is the interaction term which we take from Eq. (1). The last term is a Coulomb correction for the j^{-1} - $f_{7/2}$ interaction, which we exhibit explicitly since we want the multipole parameters to refer to the specifically nuclear interaction. For ϵ_c we take -400 keV for the interaction of each $f_{7/2}$ proton with the proton hole. As an application of Eq. (3), we calculate the $d_{3/2}$ hole state produced by the (d, He^3) reaction on Ti^{46} . We have

$$\begin{aligned}
 E^* &= E(\text{Ti}^{46}) - E(\text{Sc}^{45}) + E(\text{K}^{39}) - E(\text{Ca}^{40}) \\
 &\quad - 6 \overline{E^{(2)}} + \frac{1}{2} \overline{\Delta E^{(2)}} - 0.8 \text{ MeV} \\
 &= -6 \overline{E^{(2)}} + \frac{1}{2} \overline{\Delta E^{(2)}} - 2.8 \text{ MeV}, \quad (4)
 \end{aligned}$$

where the numerical values for the ground-state energies shown are taken from the binding-energy tables.¹⁶

¹⁶ L. A. Konig, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. 31, 18 (1962).

Fig. 6. The calculated and observed $d_{3/2}$ -hole states in Sc isotopes. The T doubling shown for $A = 41$ and 42 comes about because the corresponding Ti-core nuclei have a proton excess instead of a neutron excess. Note that the prediction is for the center of gravity of the $d_{3/2}$ hole strength.

It should be remembered that in those cases for which $T_z = T_0 - \frac{1}{2}$, the hole (either for $T_>$ or $T_<$) is not purely a neutron hole but is rather partially a proton and partially a neutron hole with, of course, a concomitant variation in the charge of the core. The weighting coefficients are squares of the obvious isobaric-spin Clebsch-Gordan coefficients as indicated by the coupling in Eq. (1). Alternatively, one can agree first to locate the $T_>$ state in the nucleus with $T_z = T_0 + \frac{1}{2}$ (in the usual case that implies a proton pickup); this state can then be transferred to other nuclei by Coulomb correction and the $T_<$ states located by subtracting $\frac{1}{2}(2T_0 + 1)\overline{\Delta E^{(2)}}$.

Figure 6, calculated with $\overline{E^{(2)}} = -0.25$ MeV and $\overline{\Delta E^{(2)}} = 2.9$ MeV, shows the expectation for the $d_{3/2}^{-1}$ holes in the scandium isotopes. The agreement with experiment is quite satisfactory. (Note that for the even Sc isotopes the core, being odd, is easily excited. The strength is thereby distributed, and it is not certain at all that it has been all observed.)

Similar results for the neutron-pickup cases are shown in Fig. 7. By Coulomb corrections on the Sc results, we also locate the $T_>$ states and explicitly show the isobaric-spin splitting increasing regularly with the neutron excess. The large odd-even fluctuation in the excitation energy of the $T_<$ states is due to the $f_{7/2}^n$ identical-particle pairing effect; the promoted particle is strongly attracted by an odd-n group but is repelled or only weakly attracted by an even-n group. For the $T_>$

Fig. 7. Calculated and observed $d_{3/2}$ hole states in Ti isotopes. The observed $T_{<}$ states (crosses) are compared with calculation (dots). Also shown are the predicted positions ($—$) of the $T_{>}$ levels which are the isobaric analogues of the Sc ground states, and the $T_{>}$ hole states ($\otimes$) as well. The heavy vertical lines show the isobaric-spin splitting.

states, this does not show up because in each case the promoted proton sees only a single like particle.

Some $s_{1/2}$ holes are also located in these experiments. Since the $s_{1/2}$ -hole state in $A = 40$ occurs 2.6 MeV above the $d_{3/2}$ -hole state, the separation in other $f_{7/2}$ nuclei will be the same except for an obvious modification due to the fact that the $d_{3/2}$ - $f_{7/2}$ interaction may be different from the $s_{1/2}$ - $f_{7/2}$. In fact, the results suggest that for $s_{1/2}$ - $f_{7/2}$ we have $\overline{E(2)} \approx 0$, $\overline{\Delta E(2)} \approx 2.5$ MeV.

Various other aspects of these hole states are discussed in the paper of Bansal and the author.¹³ The same procedures can be used in other cases, for example, when we add inequivalent particles to $f_{7/2}^n$ nuclei, or in the regions around $A = 90$ and $A = 208$. By putting together parameter values for varying orbit pairs, we might hope to learn something about the radial forms and the exchange nature of the shell-model interaction.

Fig. 8. The coupling of a $p_{3/2}$ proton to a Ca^{48} core; the $T_0=4$ core state is shown as well as the $T_0=3$ states which can be admixed. Observe the negative isoscalar monopole interaction which is the same for all states, and the splitting of the $T_0=4$ state by means of the isovector monopole interaction. The strength in the $(\text{Ca}^{48} + \text{proton})$ reaction is shown schematically as a doublet (appropriate if there were no core polarization) and also as a doublet fragmented by the polarization.

III. Effects of Fine Structure and Core Polarization

When the coupling is not weak, the core state changes when we add a particle or hole to it, and at the same time the strength in the corresponding reaction is distributed. The single-particle states which, for a $J_0=0$ core, would form a sharp doublet in the weak-coupling case, now become "giant" resonances. This is of course the usual phenomenon of Lane, Thomas, and Wigner. There are a few particular points involving isobaric spin which are of some interest. Consider again, for example, the coupling of a $p_{3/2}$ proton to a Ca^{48} core, considering only those core "polarizations" that still involve $f_{7/2}^8$ nuclei. Taking into account at first only the internal "core" energies, we note that in the $T = \frac{7}{2}$ system there is a strong tendency for the core to "relax down" from its $J_0=0$, $T_0=4$ state to a lower $J_0 \neq 0$, $T_0=3$ state, since the internal energy of the latter states is lower by 3—6 MeV. In other words, it is economical of internal energy to open up a neutron-proton hole-particle pair in the $f_{7/2}$ shell. The interaction energy with the $p_{3/2}$ proton, however, opposes this tendency since the coupling energy is positive for $T_0=3$ and negative for $T_0=4$, as indicated schematically in Fig. 8. It is then a relatively delicate matter which of the two effects will dominate;

if, for example, the external interaction energy is dominant, the lowest ($T_0 - \frac{1}{2}$) state will carry the major strength as shown in Fig. 8. From the experimental results of Erskine, Schiffer, and Marinov,¹⁷ it appears in fact that the two interactions are comparable and one seems to see a considerable fragmentation of the $T_{<}$ strength. This same fragmentation shows up in β decay as discussed by Fujita, Fujii, and Ikeda.¹⁸ The detailed fine structure obviously carries information about the effective interaction, specifically about the higher terms in the multipole expansion. To extract this information, one can compare experiment with calculation for assumed interactions, as has been done to some extent by Pinkston⁴ and by Bansal.⁵ An interesting alternative procedure involves the use of energy-weighted sum rules; in this case one can ask what information about the target state and about the effective interaction is implied by a given set of energy levels and corresponding reduced widths. An account of this procedure will be given elsewhere.

Figure 5 showed a "complete" two-nucleon spectrum for a pair of orbits j_1 and j_2 . It is clear that if the complete spectrum (together with the equivalent particle spectra) were known, we could explicitly solve for the interaction of any number of particles in the two orbits. Unfortunately there is not a single case in which such a complete inequivalent-particle spectrum is known. Because of the great amount of information about $f_{7/2}^n$ nuclei that is now available and the considerable interest in the coupling of other holes and particles to these nuclei, a serious attempt to determine the $f_{7/2}$ - j spectra for $j = s_{1/2}, d_{3/2}, p_{3/2}, p_{1/2}$, and $f_{5/2}$ would be very well worth the trouble. The $d_{3/2}$ - $f_{7/2}$ spectrum is accessible via an examination of the $\ell=3$ levels in the $S^{33}(\text{He}^3, d)$ experiment, and one

¹⁷J. R. Erskine, J. P. Schiffer, and A. Marinov, Bull. Am. Phys. Soc. 9, 80 (1964).

¹⁸J. Fujita, S. Fujii, and K. Ikeda, Phys. Rev. 133, B549 (1964).

Fig. 9. The predicted $d_{3/2} - f_{7/2}$ levels in Cl^{34} . The separate $T=0$ and $T=1$ spectra are not predictable (although their centers of gravity are) but the average spectrum [without a $(2T + 1)$ weighting] should be identical with the Cl^{38} spectrum shown.

might be able to make a fair assignment of the spins by the usual statistical-weight argument. One expects to see, as shown in Fig. 9, a $T=0$ spectrum centered at approximately 3.3 MeV and a $T=1$ spectrum about 2.9 MeV higher.* The $T=1$ spectrum could, of course, be just as well determined by $S^{33}(d, p)$.

Figure 9 also shows the well known ground-state $d_{3/2} - f_{7/2}$ spectrum of Cl^{38} . The Cl^{38} case differs from the Cl^{34} case in that, since the $d_{3/2}$ neutron shell is filled in Cl^{38} , we are dealing with the spectrum $d_{3/2}(\text{proton}) \times f_{7/2}(\text{neutron})$ (an example of what we call an n-p two-particle spectrum) rather than with the general two-nucleon spectrum. It is easy to see that, to within a J-independent constant, we have for the n-p spectrum

$$E_J^{(np)} = \frac{1}{2} \{ E_J^{T=1} + E_J^{T=0} \} \quad (5)$$

and this relationship is schematically shown in Fig. 9. A determination of the separate $T=0, 1$ spectra would enable a test of Eq. (5). But alternatively we could use the Cl^{38} spectrum and (for example) the $\ell=3$ $S^{33}(d, p)$ spectrum, if somebody would only determine it, to supply

*These values come immediately from the $A = 32, 33, 34$ binding energies and the $d_{3/2} - f_{7/2}$ monopole parameters. Since the latter parameters come from nuclei with $A \approx 45$, we cannot rely too heavily on the predicted excitations and must be prepared to give or take several hundred keV; the separation between the two spectra should be a better prediction than the position with respect to the ground state.

ourselves with all the parameters of the $d_{3/2}$ - $f_{7/2}$ interaction. This kind of thing was discussed by the author several years ago^{2,7}; it seems more interesting now than it did then.

The monopole average of the n-p interaction (as opposed to the T=0 or T=1 interactions) is given by the parameter $\overline{E}_{np}$ which from Eqs. (1,5) is given in terms of the isoscalar and isovector monopole strengths by

$$\overline{E}_{np} = \overline{E}^{(2)} - \frac{1}{4} \Delta \overline{E}^{(2)}. \quad (6)$$

A value for $\overline{E}_{np}$ can be found from the binding energies of S^{36} , S^{37} , Cl^{37} , and Cl^{38} and the known spectrum of Cl^{38} . This value turns out to be -0.95 MeV, in pleasing agreement with the value -0.97 MeV obtained by inserting the values of the nucleon-nucleon parameters into Eq. (6).

Once again we remark that as more data become available the arguments used above can be applied for other regions of A. And of course some of the levels and some of the reaction strengths can be determined by elastic nucleon scattering¹⁹ instead of by stripping reactions.

IV. Neutron-Proton Counting

By this time the use of sum-rule procedures to determine the average nucleon occupancy of a given orbit has become commonplace; the sum involved is the sum of all the reduced widths corresponding to a

¹⁹D. Robson, J. D. Fox, J. A. Becker, C. F. Moore, P. Richard, D. Long, S. I. Hayakawa, G. Vourvopoulos, and C. E. Watson, Technical Report #6, Tandem Laboratory, Florida State University, January 1964.

given orbit. A division of this sum into two separate sums according as $T = T_{<}$ or $T_{>}$ informs us about the separate neutron and proton occupancy of the orbit. The results here,³ for the usual case in which $T = T_Z$ for the target, are^{*}

$$\begin{aligned} G_n &= \langle \text{neutron holes} \rangle_j, \\ G_p &= \langle \text{proton holes} \rangle_j, \\ G_p(T_{>}) &= \frac{1}{(N - Z + 1)} \langle \text{neutron holes} \rangle_j, \\ G_p(T_{<}) &= \langle \text{proton holes} \rangle_j - \frac{1}{(N - Z + 1)} \langle \text{neutron holes} \rangle_j, \end{aligned} \tag{7}$$

where the strength G is defined³ as the sum of the "spectroscopic factors" times the appropriate J and T statistical factors, $\langle \text{neutron holes} \rangle_j$ is the average number of j holes in the target, and N and Z refer to the target. G_p is for the addition of a proton and G_n for a neutron. Equation (7) applies for stripping or for resonant scattering; a similar set of equations is given for pickup processes.

The two independent sum rules of Eq. (7) may be classified as the non-energy-weighted (n.e.w.) isoscalar and isovector space-spin-multipole sum rules. It is perhaps obvious that for reactions involving a $J_0=0$ target these are the only n.e.w. sum rules; for $J_0 \neq 0$ there is, however, a set of such rules corresponding to the various possible space-spin multipoles. The set of such sum rules can be derived very easily once it is recognized that a "spectroscopic amplitude" $\mathcal{S}^{1/2}$ (the square root of the so-called "spectroscopic factor" $\mathcal{S}$ but with the appropriate

^{*} $G_p(T_{>})$ and $G_p(T_{<})$ are interchanged in Eq. (7) of Ref. 3.

phase) is intimately related, in the language of second quantization, to a simple vacuum expectation value (v. e. v.). In fact we have

$$\mathcal{G}^{1/2}(n, y, \Gamma_0 + \rho \rightarrow n+1, x, \Gamma) = [\Gamma]^{-\frac{1}{2}} \left\langle \left(\bar{\psi}_{y\Gamma_0}^n \times B^\rho \times \psi_{x\Gamma}^{n+1} \right)^0 \right\rangle_{\text{v. e. v.}}, \quad (8)$$

where B^ρ is a destruction operator for a particle in the ρ th orbit, Γ and Γ_0 define the angular-momentum-like quantum numbers J and T , x, y define the non-angular-momentum ones, the superscript zero on the right-hand side stands for coupling to tensorial rank zero (in both the j and t spaces), and $[\Gamma] \equiv (2J+1)(2T+1)$. Then if P and Q are arbitrary operators, the Δ 's give their tensorial ranks in the jt space, and U is a product of two normalized Racah coefficients; and if we combine Eq. (8) with the general v. e. v. theorem

$$\begin{aligned} & \left\langle \left(\bar{\psi}_{x'\Gamma'}^{n'} \times (P^{\Delta_1} \times Q^{\Delta_2})^\Delta \times \psi_{x\Gamma}^n \right)^0 \right\rangle_{\text{v. e. v.}} \\ &= \sum_{y\Gamma_0 m} [\Gamma_0]^{-\frac{1}{2}} U(\Gamma' \Delta_1 \Gamma \Delta_2 : \Gamma_0 \Delta) \left\langle \left(\bar{\psi}_{x'\Gamma'}^{n'} \times P^{\Delta_1} \times \psi_{y\Gamma_0}^m \right)^0 \right\rangle_{\text{v. e. v.}} \\ & \quad \times \left\langle \left(\bar{\psi}_{y\Gamma_0}^m \times Q^{\Delta_2} \times \psi_{x\Gamma}^n \right)^0 \right\rangle_{\text{v. e. v.}} \end{aligned} \quad (9)$$

and its angular-momentum inverse, it is easy to produce the set of n. e. w. sum rules. It seems quite probable that these rules will before too long be of considerable value.

Until very recently there have been no good tests of the sum-rule equations beyond the very simplest one which gives the total nuclear occupancy of an orbit. But in a few minutes Rubby Sherr will discuss an ingenious and fruitful application of the space-spin-monomer rules, which he and his colleagues have recently made.

It should be stressed, since the contrary has recently been stated, that the relationship between $G(T_>)$ and $G(T_<)$ in Eq. (7) is not simply an isobaric-spin Clebsch-Gordan relationship. It does have that simple form when the target has no particles (holes) equivalent to the particle (hole) which is transferred; but in other cases the equations give a kind of "blocking effect" which destroys the Clebsch-Gordan ratio. As an extreme example of this, consider the $f_{7/2}$ strength in the $\text{Ca}^{48} + p$ reaction. All the strength goes into a $T=T_0-\frac{1}{2}$ state, there being in fact no $(T_0+\frac{1}{2})$ states allowed by the Pauli principle.

One general feature of the results of Eq. (7) is of practical significance, namely that when the target has a sizeable neutron excess, most of the strength goes into the $T_<$ component (the ratio being roughly equal to the neutron excess). Since, for purposes of the kind we have been discussing, the $T_<$ component is both more interesting than the $T_>$ and at the same time more easily fragmented by core admixing, the fact that it carries most of the strength is quite fortunate. The $T_>$ component (often the only one that can be reached by elastic proton scattering) can be studied most easily by means of the (d,p) reaction to the isobaric-analogue states.

V. Final Remarks

Let me now turn briefly to a number of other topics. As I indicated at the beginning, I say very little about the beautiful (p,n) experiments done at Livermore⁹ and the elastic-scattering experiments at Tallahassee,¹⁹ and therefore little also about the theoretical work of Lane and Soper²⁰ concerning the (p,n) results.

²⁰A. M. Lane and J. M. Soper, Nucl. Phys. 37, 506 (1962), and other papers.

Most of the discussion above has been concerned specifically with the interaction of a single nucleon or hole with a single shell. The extension to the case of a nucleon interacting with several shells is immediate, as long as we ignore the orbit dependence of the interaction. (See, for example, the discussion above of Lane's term in the optical potential.) If we allow such a dependence we can consider a new question, namely the way in which the isobaric spins of equivalent groups of nucleons couple in a nucleus. To start with a trivial case, if the nucleus in question consists, as in Fig. 1, of fully closed shells (closed for neutrons and protons) plus a number of neutron shells, then the coupling is simply a "straight-line" coupling to maximum T . Let us now add a proton in a high enough orbit, or convert a neutron into a proton. Then, even if we ignore the core excitations that involve higher orbits, and also the excitations that produce neutron-proton hole-particle pairs with $T=1$ coupling, the chain of isobaric spins can "buckle" at every joining to produce new sets of states; in these states the T 's for each equivalent group maintain their values, but the internal couplings between groups (one at a time) have a value reduced by unity. It is at present unknown whether this simple type of "isobaric-spin core polarization" is of any consequence, i. e., whether there are states which have predominantly this structure. If the interaction were well enough known in any given case, one could make shell-model calculations to see.

An alternative procedure²¹ is to make a more detailed study of the monopole (p,n) reaction, namely to study $0^+ \rightarrow 0^+$ transitions to final states that are not isobaric analogues of the targets. The monopole (p,n) reaction operator is of course a close analogue of the operator for the allowed Fermi β decay, the essential difference being that the interaction strength may depend on the orbit. Whereas in the β -decay case a measurement of such a T -forbidden transition informs us mainly about the goodness

²¹ J. B. French and M. H. Macfarlane, Phys. Letters 2, 255 (1962).

of T , in the (p,n) case it will tell us about the orbit dependence of the interaction operator. To the extent that one sees only the isobaric-analogue excitation, the operator is independent of orbit and simply a multiple of $\vec{T} \cdot \vec{t}_0$; in this case, we would learn nothing about the T coupling in the nucleus. If, however, we should see other monopole excitations, we could conclude that the interaction depends on the orbit (and there is no reason why it should not be), the ratio of the summed strengths involving the nonanalogue levels to the analogue strength would tell us about the magnitude of the orbit dependence, and the degree of fragmentation would tell us about the T coupling in the nucleus. At present the main difficulty in such experiments is in finding 0^+ states in appropriate odd-odd nuclei.* Along the same line, the direct-reaction monopole (n,p) or (t, He^3) reactions, which should be very highly forbidden, are also of interest. I refer you also to the paper of Garvey and Bayman^{2,2} for a further discussion of the excitation of $T=T_z+2$ states.

It is clear, of course, that two-nucleon-transfer reactions $[(\text{He}^3, p), (t, p), (d, \alpha), \text{ and so on}]$ will also inform us about neutron-proton relationships in nuclei, provided that they can be well described by means of the usual DWBA procedure. There is an obvious set of isobaric-spin Clebsch-Gordan intensity-ratio and selection rules⁵ by means of which one can, for example, separate the $T=0$ and the $T=1$ channels in such reactions as (He^3, p) . The $T=0$ channel can, of course, also be examined via the (d, α) reaction. In the same way, we can hope eventually to separate the isovector

* If the DWBA analysis is eventually able to distinguish monopole transitions by their angular distributions, one could perhaps look for a monopole giant resonance away from the region of the analogue state.

^{2,2} G. T. Garvey and B. F. Bayman, Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 125.

excitations in inelastic scattering from the isoscalar ones by comparing (p, p') and (α, α') .

The (d, α) reaction has long been used to study the goodness of T, but the results are still to a considerable extent mystifying. A careful experiment in which the cross section for a T-forbidden reaction is measured over a wide range of deuteron energy (say up to 25 MeV or higher) would be very valuable. In this way, one might see whether the T-forbidden reaction becomes less important as the reaction becomes more and more nearly a pure direct-reaction process.

From the theoretical standpoint, there are a number of questions of interest concerning the relationship between neutrons and protons in nuclei. These range from rather obvious formal shell-model questions to much more difficult ones of the interplay between neutrons and protons in collective excitations, and other things of that sort. One of the simpler questions of some interest is the relationship between the n-p and isobaric-spin formalisms. On looking into this, one finds a number of selection rules which arise when we deal with states that can be regarded as consisting of two identical systems (neutron and proton), or two closely related systems, coupled together. The paper by Garvey et al.²³ relates to these rules.

The question of pairing in n-p systems is also of interest. In its simplest form, in which all the particles are in the same orbit, the question here is that of symplectic symmetry which is the natural generalization of the zero-coupled pairing encountered for identical particles in the same orbit. It is easy to show that independent pairing of neutrons and protons is incompatible with charge independence. If we add a zero-coupled

²³G. T. Garvey, G. M. Crawley, H. O. Funsten, N. R. Robertson, and L. Zamick, Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 168.

pair of protons to a seniority-zero or seniority-one neutron group, the resultant wave function is given by

$$\begin{aligned}
 (j^n)_{s=0} \times (j^2)_0 &= \left[\frac{n(2j+2)}{(n+1)(2j+1)} \right]^{1/2} \Psi_{T=\frac{1}{2}n-1} + \left[\frac{2j+1-n}{(n+1)(2j+1)} \right]^{1/2} \Psi_{T=\frac{1}{2}n+1}, \\
 (j^n)_{s=1} \times (j^2)_0 &= \left[\frac{(n-1)(2j+2)}{n(2j+1)} \right]^{1/2} \Psi_{T=\frac{1}{2}n-1} + \left[\frac{4(j+1)}{n(n+2)(2j+1)} \right]^{1/2} \Psi_{T=\frac{1}{2}n} \\
 &\quad + \left[\frac{(2j-n)}{(n+2)(2j+1)} \right]^{1/2} \Psi_{T=\frac{1}{2}n+1}, \tag{10}
 \end{aligned}$$

which illustrate the point.* This comes about because a zero-coupled pair behaves like a vector in the isobaric-spin space; the coupling of such vectors is complicated, and there is no reason why the coupling implied by the independent pairing of neutrons and protons should lead to a fixed resultant T . From the recent work on $f_{7/2}^n$ nuclei by Bayman et al.²⁴ and by Ginocchio,²⁵ we can learn a great deal about the n - p coupling in this shell, while recent formal work, also by Ginocchio,²⁶ is useful in making clear the group-theoretical structure of systems containing neutrons and protons in the same orbit. For n - p pairing and collective effects involving several orbits, there has been a little theoretical work (e. g., Ref. 27) but the subject is largely unexplored.

*The vanishing of the coefficient for the $T=n/2$ wave function in the first equation of (10) is an example of the special selection rules referred to above.

²⁴J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964).

²⁵J. N. Ginocchio, Ph. D. Thesis, University of Rochester (1964).

²⁶J. N. Ginocchio (private communication).

²⁷S. Yoshida, Nucl. Phys. 33, 685 (1962).

Acknowledgment

I am indebted to Mr. R. K. Bansal for much help in preparing this account.

Appendix: The Goodness of Isobaric Spin

Although most of the things we have discussed are highly insensitive to even large isobaric-spin admixtures, the question of the goodness of T is of considerable theoretical interest. The experimental results that cast light on this subject have recently been reviewed by Bloom.²⁸ T admixtures of the type that can contribute to the T -forbidden pure Fermi β transitions are found to be very small (intensity $\approx 10^{-4}$). This is not, however, a measure of the total admixture and, moreover, refers to low-lying states. On the other hand, calculations in which the Coulomb interaction is treated as a perturbation also give small admixtures. These calculations include the conventional perturbation calculations (see MacDonald²⁹) as well as the unconventional one of Iwao³⁰ in which, to take account of a major part of the Coulomb interaction, different radial functions are taken for the proton and neutron functions. And for the particular case of heavy nuclei, there is the sum rule of Lane and Soper³¹ who show that a perturbation calculation of the total admixture also gives a very small result.

²⁸ S. D. Bloom, *Nuovo Cimento* 32, 2417 (1964).

²⁹ W. M. MacDonald, *Phys. Rev.* 100, 51 (1955); 101, 271 (1956).

³⁰ S. Iwao, Ph.D. Thesis, University of Rochester (1960).

³¹ A. M. Lane and J. M. Soper, *Phys. Letters* 1, 28 (1962).

If then a perturbation procedure is adequate, one does expect small admixtures. There seems, however, to be no clear proof that it is adequate; and one sometimes has an uneasy feeling that such small admixtures are hard to reconcile with the very large Coulomb energies. (The single-particle Coulomb energy at $A = 40$ is 7 MeV.) The following points however should be remembered. The Coulomb interaction consists of isobaric scalar, vector, and tensor parts, the third part of which is small and the first part of which does not contribute to any admixing (nor of course to the single-particle Coulomb energy). The most significant interaction is then the isobaric vector. It seems probable that a large part of this may be written as a multiple of the T_z operator and thus contributes to the energy but produces no admixing at all.

As far as the high-lying isobaric-analogue states found in elastic scattering are concerned, it should be remembered that an off-diagonal Coulomb matrix element can scarcely be larger than 100 keV and is often smaller than that by an order of magnitude or more. Also, while it is true that the isobaric-analogue states with $T = T_z + 1$ are located in a sea of states with $T = T_z$, one has an a priori expectation that individual T admixings will be small because the Coulomb interaction admixes, relatively weakly, states involving different orbits. (Remember that the $T_{<}$ single-particle orbit lies many MeV below the $T_{>}$.)

We propose soon to look quantitatively into some of these matters.

* * *

B. J. RAZ, State University of New York, Stony Brook: Could you say something about $d_{3/2}$ hole states in the calcium isotopes on the basis of this sort of picture?

FRENCH: One can, of course, just put the numbers in. However, we came across both varieties of data only a week or two ago; and it takes

a few days to realize that only arithmetic is needed for the analysis. This will be done by Bansal.

J. L. YNTEMA, Argonne National Laboratory: Would it be possible to make the predictions before you know the results?

FRENCH: In answer to that, the predictions in the last figure on Cl^{34} (Fig. 9) are from a paper written in 1960. The strength equations [Eq. (7)] were given in a paper²¹ written by Macfarlane and myself in 1962. So it is possible to make predictions. As far as the hole states are concerned, the answer is that the whole thing is rather trivial and, of course, one can.

EXCITATION OF ANALOGUE STATES IN (p,d) REACTIONS*

R. Sherr

Princeton University, Princeton, New Jersey

This work is a joint venture between Boulder and Princeton. The experimental work was carried out with the 28-MeV proton beam of the new and wonderful cyclotron at the University of Colorado at Boulder. The people involved in the experiment and its interpretation are C. Hoot and M. E. Rickey (University of Colorado), and B. Bayman, E. Rost, and myself (Princeton University).

Dr. French just pointed out that the neutron pickup in a (p,d) reaction can lead not only to the $T_0 - \frac{1}{2}$ states, where T_0 is the isotopic spin of the target nucleus, but also to a $T_0 + \frac{1}{2}$ state that is the analog of one in the nucleus with one less proton than the target nucleus. This is certainly not a new reaction by any means. As a matter of fact, three years ago Bennett¹ observed that $T=0$ and $T=1$ states are excited in a number of isotopes—for example, in the pickup reactions on Li^7 , N^{15} , F^{19} , and P^{31} with 18.5-MeV protons. So there is nothing novel in reaching both of these states in the (p,d) reaction. However, when one tries to reach these states in heavier elements, the Coulomb energies get so high that one needs a higher proton energy. With the 18.5-MeV proton beam of the Princeton cyclotron we have never been able to see them in the medium-heavy nuclei; but at 28 MeV, we will see that they are quite easily excited.

Now, there are a number of things that one would like to know about these analog states. First of all, there is a question of the

* Contributed paper (not in Vol. I. Contributed Papers).

¹ E. F. Bennett, Phys. Rev. 122, 595 (1961).

excitation energy E_x at which they appear in the residual nucleus. E_x is given by

$$E_x = \Delta E_c + S_p - S_n ,$$

where S_n and S_p are the neutron and proton separation energies for the target nucleus and ΔE_c is the difference in Coulomb energies of the $(A - 1)$ isobars. The masses and Coulomb energies are well known in the mass region of interest here, so that E_x can be calculated, usually to within 100 keV.

We not only want to know the excitation energy, but we would also like to know the angular distribution for these particular states. In the cases that I will be talking about, the protons are in the $f_{7/2}$ shell, while the neutrons can either fill this shell and go into the next shell, or the $f_{7/2}$ shell can be only partially filled. In all cases N is, of course, greater than Z . Thus the angular distributions for the analogue states will be characterized by an $\ell=3$ transfer.

A third item of interest will be the strengths of the transitions for the pickup of an $f_{7/2}$ neutron. For the even-even nuclei with $N \leq 8$, one will excite the isobaric analogue state ($J = \frac{7}{2}^-$ and $T = T_0 + \frac{1}{2}$) and in addition several other possible states with spin $\frac{7}{2}^-$ and $T = T_0 - \frac{1}{2}$. French and Macfarlane² have shown that the shell-model spectroscopic factors are

$$S_{>} \equiv S(T_0 + \frac{1}{2}) = \frac{\pi}{N - Z + 1}$$

and

$$S_{<} \equiv S(T_0 - \frac{1}{2}) = \nu - S_{>} ,$$

where π (ν) is the number of protons (neutrons) in the $f_{7/2}$ shell for the target nucleus (N, Z) , and $S_{<}$ is the sum of the strengths for the $T_0 - \frac{1}{2}$

²J. B. French and M. H. Macfarlane, Nucl. Phys. 26, 168 (1961).

Fig. 1. Deuteron spectrum at 25° from the $\text{Fe}^{58}(\text{p,d})\text{Fe}^{57}$ reaction at 28 MeV. The excitation energies of the various peaks are given in MeV. The lower portion of the figure shows the angular distribution. The energy-level diagram on the right shows the strengths and ℓ values reported for the $\text{Fe}^{56}(\text{d,p})\text{Fe}^{57}$ reaction.

states. For nuclei with $N > 8$, $\nu = 8$ in the above formula. Thus in examining the experimental results we shall be interested in the excitation energy, the angular distributions, and the strengths for the transitions to the analogue states.

The 25° deuteron spectrum from Fe^{58} is shown in Fig. 1. The peaks are labeled with the excitation energy in MeV. The angular distributions are shown at the bottom; the corresponding ℓ values are given. (The curves are drawn through the data points and do not represent calculations.) On the right are given the energies, relative strengths, and ℓ values for the states of Fe^{57} which have been reported for the $\text{Fe}^{56}(\text{d,p})\text{Fe}^{57}$ reaction. Note that there is no clearly identifiable peak in the spectrum above 5.27 MeV until we reach 10.41 MeV. This peak caught our attention and we soon realized that it had the correct energy to be the analogue of the ground state of Mn^{57} .

Fig. 2. Deuteron spectrum at 35° from the $\text{Fe}^{54}(\text{p},\text{d})\text{Fe}^{53}$ reaction at 28 MeV.

is much greater than that of the 10.41-MeV state in Fig. 1 and consequently a much better angular distribution was obtained. Its similarity to the ground-state transition is excellent.

Figure 3 shows the angular distributions for a strong low-lying $\ell=3$ state and for the analogue state for various nuclei. The distributions for the analogue states are shown as dashed curves and have been multiplied by 10. The energy ranges from 4.24 up to 10.43 MeV. The excitation energy varies from one target to another; however, these states are easy to spot because they show up very strongly even at very high energies where nothing else seems to be excited. The curves are theoretical DWBA calculations for $\ell=3$ transitions and show a reasonable fit to the data for both sets of states.

The 0.14-MeV state is the only state with $\ell = 3$ strongly excited in both the (p,d) and (d,p) reactions. It has spin $\frac{5}{2}^-$ and its ℓ is designated as 3_- . The remaining $\ell=3$ states seen in the (p,d) spectrum are assumed to be $\frac{7}{2}^-$, with ℓ designated by 3_+ . In summary, we have the analogue state at 10.41 MeV and configuration states at (0.71), 2.21, 3.19, and (4.97) MeV.

Figure 2 shows the results for a target of Fe^{54} . In this nucleus all particles are in the $f_{7/2}$ shell and all angular distributions, except for the state at 2.95 MeV, are $\ell=3_+$ transitions to $\frac{7}{2}^-$ states. The analogue state is at 4.24 MeV. Its strength is

Fig. 3. Experimental and DWBA angular distributions for the analogue state and for a $T = \frac{1}{2}$ configuration state for various residual nuclei in the (p,d) reaction at 28 MeV. The dashed curves and crosses designate the analogue states.

Fig. 4. Coulomb energies as a function of Z for $N = Z$. The experimental energies for a set of isotopes have been multiplied by $(A/2Z)^{1/3}$ to eliminate the $A^{1/3}$ dependence.

From the energies for these states, and using the known separation energies, we can calculate the Coulomb energies. These are shown on Fig. 4. The "reduced Coulomb energy difference" is the actual energy difference multiplied by $(A/2Z)^{1/3}$ in order to remove the $A^{1/3}$ dependence for a set of isotopes. The three nickel isotopes give Coulomb energies which agree within about 50 keV. The energies for the three iron nuclei agree within about 50 keV as do the two values for titanium. Also shown on this plot are the values for Coulomb energy obtained from mirror nuclei, and the zero-zero beta transitions measured by Miller and Sutton³ many years ago for Sc^{42} , V^{46} , Co^{50} , and Mn^{54} . In the boxes are shown the values from the (p,n) work of Anderson, Wong, and McClure.⁴ The over-all agreement is very good.

³ J. H. Miller and D. C. Sutton, Bull. Am. Phys. Soc. 2, 206 (1958).

⁴ J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 126, 2170 (1962).

Fig. 5. Ratio of experimental to theoretical spectroscopic factors for the $T+\frac{1}{2}$ and for the $T-\frac{1}{2}$ states in various nuclei. The upper set of points is based on the use of neutron wave functions determined by the separation energy; for the lower set, a neutron wave function based on a constant well depth was used.

I think there is little question that these states we are concerned with are the analogue states. The third thing we would like to understand is the degree of applicability of the previous formula for the spectroscopic factors. The results are shown in Fig. 5. This reveals the sort of ambiguities that arise in the distorted-wave calculations that Dr. Satchler talked about yesterday. S_{exp} is obtained by dividing the experimental cross section by the distorted-wave calculation, while S_{th} is computed from the theoretical formula given above. Plotted are the ratios $S_{\text{exp}}/S_{\text{th}}$; the dots are for $T_0 - \frac{1}{2}$, summed over the $T_0 - \frac{1}{2}$ states, and the triangles for the single $T_0 + \frac{1}{2}$ state. If one uses the normal prescription for doing distorted-wave calculations, where the neutron wave function is determined by the separation energy, one sees there are huge discrepancies between theory and experiment. The strength of the analogue state is simply much too great and the strength of the $T-\frac{1}{2}$ states decreases with Z . Thus the prescription gives too high a Q dependence for the cross section and appears to give an incorrect Z dependence. On the other hand, if one assumes that the well depth is constant for all of these nuclei, and that the width increases as $A^{1/3}$, and computes the position and wave function of the $f_{7/2}$ neutrons in the well, then one obtains the lower diagram in Fig. 5. One sees that things level out in a beautiful way; the absolute value here is a little fortuitous perhaps. We have used a cutoff at $r = a$. If we had used another cutoff, the whole line of points

could be shifted up or down, but the consistency of theory and experiment over the whole range of targets is maintained.

You will notice that where there is a difference (such as for Ni^{62}) the $T-\frac{1}{2}$ strength is a little too small perhaps. However, it is easy to miss some weak $7/2$ states in the spectra.

In conclusion, if one accepts the validity of the shell model in describing the many states involved in the present experiment, the analysis just given seems to indicate that the shell-model wave function for the picked-up neutron is more appropriate than the one obtained from the separation energy. The latter depends on the excitation energy of the particular final state, while the former is independent of excitation energy.

(p,n) REACTIONS AND ISOBARIC ANALOGUE STATES

J. D. Anderson

Lawrence Radiation Laboratory, University of California
Livermore, California

Previously reported¹⁻⁴ neutron spectra all showed one strong neutron group whose excitation energy in the final nucleus corresponds to the isobaric counterpart (analogue state) of the ground state of the target nucleus, i. e., the Q value for the reaction is the usual Coulomb displacement energy.

Many of the properties of these isobaric-analogue states have been the subject of recent theoretical investigations.⁵⁻⁸ Of particular interest is the suggestion of Lane⁹ that this direct (p,n) reaction may be considered as a quasi-elastic process and that the angular distributions may be computed with an optical potential.^{10,11}

¹J. D. Anderson and C. Wong, Phys. Rev. Letters 7, 250 (1961).

²J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 126, 2170 (1962).

³J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 129, 2718 (1963).

⁴C. J. Batty, G. H. Stafford, and R. S. Gilmore, Phys. Letters 6, 292 (1963).

⁵A. M. Lane and J. M. Soper, Phys. Rev. Letters 7, 420 (1961); Phys. Letters 1, 28 and 33 (1962).

⁶K. Ikeda, S. Fujii, and J. I. Fujita, Phys. Letters 2, 169 (1962); 3, 271 (1963).

⁷J. B. French and M. H. Macfarlane, Phys. Letters 2, 255 (1962).

⁸A. M. Lane and J. M. Soper, Nucl. Phys. 37, 506 (1962).

⁹A. M. Lane, Phys. Rev. Letters 8, 171 (1962); Nucl. Phys. 35, 676 (1962).

¹⁰R. M. Drisko, R. H. Bassel, and G. R. Satchler, Phys. Letters 2, 318 (1962).

¹¹P. E. Hodgson and J. R. Rook, Nucl. Phys. 37, 632 (1962).

Using time-of-flight techniques and the higher energy protons available as a result of the recent modifications of the 90-in. cyclotron, we have measured the angular distributions of the neutrons from the isobaric (p,n) reaction for 18.5-MeV protons on 19 targets between Be⁹ and Nb⁹³.

In order to facilitate the measurement of the angular distribution from the isobaric (p,n) reaction, considerable modification of the Livermore time-of-flight instrumentation has been necessary.¹² In particular, these modifications include the construction of collimated 10-m flight paths at 3°, 30°, 60°, 90°, 120°, and 135° and the installation of an additional bending magnet and auxiliary equipment to double bend the proton beam so that its angle of incidence on the target is 18° to the normal beam line, thus doubling the number of angles for which measurements can be made.

Angular-distribution data (Figs. 1—4) were taken in two steps: 3°, 30°, 60°, 90°, 120°, and 135° were obtained simultaneously and then the proton beam was doubly bent such that it struck the target at 18° to the normal beam line and the 18°, 48°, 78°, 108°, 138°, and 158° data were obtained.

The angular distributions of neutrons from the mirror nuclei (Fig. 1) are quite similar with the exception of N¹⁵. The forward peaking of the cross sections seen here has previously been noted at lower bombarding energies.¹³ Since the integrated N¹⁵ cross section (12 mb) is actually larger than the Be cross section (11 mb), it is suggested that the inversion in shape of the N¹⁵ cross section does not imply a different

¹²B. D. Walker, J. D. Anderson, J. W. McClure, and C. Wong, University of California Radiation Laboratory Report UCRL-7612, to be published in Nucl. Instr. Methods.

¹³C. Wong, J. D. Anderson, S. D. Bloom, J. W. McClure, and B. D. Walker, Phys. Rev. 123, 598 (1961).

Fig. 1. Angular distributions of the neutrons from the (p,n) reaction on light nuclei for an incident-proton energy of 18.5 MeV.

reaction mechanism but is rather an interference phenomenon. The A dependence of the 0^0 cross sections of the true mirror nuclei are then characterized by large fluctuations (a factor of 10), while the integrated cross sections vary less than a factor of 2.

Fig. 2. Angular distributions of the neutrons from the isobaric-state (p,n) reaction on the medium-A nuclei between Sc^{45} and Co^{59} for an incident-proton energy of 18.5 MeV.

Figure 2 presents the data on the medium-A nuclei ($A = 40-60$). Here one notes a characteristic minimum between 30 and 45° , and a second pronounced minimum at about 140° . There is also some evidence for an additional minimum at 100° (e.g., in Cr, Fig. 2). In addition to the pronounced forward peaking of the cross section, there is evidence for backward peaking ($\theta > 150^\circ$). Again we note large fluctuations in the differential cross sections, e.g., the variation for Sc is a

Fig. 3. Angular distributions of the neutrons from the isobaric-state (p,n) reaction on nuclei between Ni and Nb⁹³ for an incident-proton energy of 18.5 MeV.

factor of 5, while for cobalt the variation is almost two orders of magnitude. However, even though there are large fluctuations in the shapes of the distributions, the integrated cross sections vary by only a factor of 2 (e. g., for Sc, $\sigma = 5$ mb and for Co, $\sigma = 4$ mb). One should also note that there are no systematic differences between the odd-even and even-even angular distributions.

As one proceeds to heavier nuclei (Fig. 3) the first minimum near 40° , which was so prominent for Ti and V, gradually disappears. For Zr and Nb the second prominent minimum, which was at 140° for V, has now moved in to 120° . Again one sees that the general features of the differential cross sections change more radically as a function of A than do the integrated cross sections. One also notes relatively little difference in shape between the angular distributions for the predominantly even-even nucleus Zr and the odd-even nucleus Nb.

Figure 4 presents the data for 17-MeV incident protons. One notes that although the 3° cross section for B¹¹ has changed almost a factor of 2, the other angular cross sections are the same and thus the integrated cross section is virtually unchanged. The 17-MeV data are in general unchanged as compared to the 18.5-MeV data. In addition to the angular distribution for the ground state, those for the first excited state were also obtained for B¹¹ and Fe⁵⁶. An interesting feature of these measurements is that the ratio of ground state to first excited state is approximately the same for the two nuclei, i. e., for B¹¹ the ratio is about 4 while for Fe it is roughly 2.

Lane⁹ noted that an isotopic-spin-dependent potential arises straightforwardly in a potential calculation taken as a sum of two-body forces with Heisenberg components and averaged over a Fermi gas. He proposed that the optical potential should be of the form

$$V = V_0 + (\vec{t} \cdot \vec{T}) \frac{V_1}{A}, \quad (1)$$

where V_0 is the ordinary optical potential, t is the isotopic spin of the incident particle, and T is the isotopic spin of the target nucleus of mass number A . In addition to reproducing the proton potential anomaly,^{14,15} the isospin term $(\vec{t} \cdot \vec{T})$ acting on an incident proton can convert it into a neutron, and turn the target into the corresponding isobaric state, thus resulting in a quasi-elastic (p,n) reaction.

If the incident energy is high enough for Coulomb effects to be small (the Coulomb effects include the distortion of the proton wave and the fact that the neutron energy is less than the proton energy by the Coulomb displacement energy), then the coupled equations resulting from the inclusion of the isospin potential $(\vec{t} \cdot \vec{T}) V_1$ can be solved to yield

$$\sigma_{PN}(\theta) = \frac{2T_0}{(2T_0 + 1)^2} \left| f_{T_0 + \frac{1}{2}}(\theta) - f_{T_0 - \frac{1}{2}}(\theta) \right|^2,$$

Fig. 4. Angular distributions of the neutrons from the isobaric-state (p,n) reaction on selected nuclei for an incident-proton energy of 17 MeV. The angular distributions of the neutrons from the first excited isobaric states resulting from the (p,n) reaction on B^{11} and Fe^{56} are also shown.

¹⁴ A. E. S. Green and P. C. Sood, Phys. Rev. 111, 1147 (1958).

¹⁵ P. C. Sood, Nucl. Phys. 37, 624 (1962).

Fig. 5. Comparison between the experimental data and optical-model calculations neglecting Coulomb effects for the true mirror nuclei. The optical-model parameters used are those of H. F. Lutz, J. B. Mason, and M. D. Karvelis, Nucl. Phys. 47, 521 (1963).

where the $f(\theta)$'s are the scattering amplitudes corresponding to the two values of the total isotopic spin. Assuming that the form factor of the isospin term is the same as that of the real part of the optical potential, Hodgson and Rook¹¹ noted the sensitivity of the (p,n) cross section to this potential. However, with the limited data² available (a partial angular distribution for 12-MeV protons on vanadium) no accurate estimate of the magnitude of V_1 nor its form factor was possible.

The optical-model code LOKI,¹⁶ including spin orbit, has been modified to allow calculations similar to those of Hodgson and Rook.¹¹ The results of these calculations for the true mirror nuclei are shown in Fig. 5. It is interesting to note that although the calculations do not yield a particularly good fit to the data, the magnitude of the cross section is approximately correct for a value $V_1 = 90$ MeV except for Be where a value of 60 MeV gives an appreciably better fit to the data. It is

also clear that the optical calculations are capable of reproducing the prominent features of the angular distributions although those employed here do not reproduce all the maxima and minima found in the data. It is concluded that the data on true mirror nuclei can be adequately described with the optical model although the agreement is not overly impressive.

¹⁶ We are indebted to Dr. E. Schwarcz for modifying the optical-model code to include the isospin potential.

For the nonmirror nuclei the basic assumption in these calculations (i. e., that the Coulomb effects are small) is no longer valid. In trying to account at least partially for the Coulomb effects, calculations using the average nucleon energy were made; i. e., the incident energy used in the optical model is assumed to vary as $E_p - \frac{1}{2}\Delta E_c$, where ΔE_c is the Coulomb displacement energy.

Figure 6 displays the results of optical-model calculations using both volume and surface absorption. Although there is good agreement with the Cr data and fair agreement with the Nb, the other data are not well fitted. Again, as in the light nuclei, the optical model is reasonably able to predict the magnitude of the cross sections but in general does not fit the detailed shape of the angular distribution. The over-all agreement of the volume absorption term¹⁷ is slightly better than that for the surface-centered absorption.¹⁸

With volume absorption and the assumption that the isospin potential has the same form factor as the real potential, one concludes that V_1 varies from 60 to 90 MeV. Where the over-all fit to the data is good (as it is for Cr), the value obtained for V_1 is 90 ± 10 MeV.

¹⁷M. Gursky and C. E. Porter, in Proceedings of the International Conference on the Nuclear Optical Model, Tallahassee, Florida, 1960, p. 85.

¹⁸F. G. Perey, Phys. Rev. 131, 745 (1963).

Fig. 6. Comparison between the experimental data and optical-model calculations neglecting Coulomb effects for medium-A nuclei. The optical-model parameters for surface absorption are those of Perey (Ref. 18) while those for volume absorption are similar to those of Gursky and Porter (Ref. 17).

In general the optical model including the isospin potential but neglecting Coulomb effects is capable of predicting the order of magnitude of the (p,n) cross section, the diffraction character of the angular distribution, and the rapid variation of angular shape as a function of the atomic weight of the bombarded nucleus. These calculations do not, however, give detailed agreement with the data. In the light nuclei this disagreement may be due to the rapid fluctuation of optical parameters with energy; indeed the shell-model formalism of Bloom *et al.*¹⁹ may be a more valid description. In the medium-A nuclei, the assumption that the Coulomb effects are negligible is of course no longer valid since the incident energy is not large compared with the Coulomb displacement energy.

Drisko *et al.*¹⁰ have studied the effects of the isospin term in the optical potential in distorted-wave Born approximation (DWBA). Using the DWBA they were able to account for the Coulomb distortion of the incident proton and for the fact that the neutron energy may be significantly less than the incident proton energy (which differs from the incident energy by the Coulomb displacement energy).

In Fig. 7 the optical-model calculations neglecting Coulomb effects with a volume isospin potential and surface absorption are compared with the corresponding DWBA calculations. As expected, in the light elements there is little difference between the two calculations. Even for nuclei in the range $A = 40-60$, the agreement between calculations is sufficiently good that values of V_1 inferred from the optical model (neglecting Coulomb effects) are probably reliable. It is clear, however, that for $A > 60$ the neglect of Coulomb effects renders the optical calculations unreliable and one must employ the DWBA in order to obtain a reliable estimate of the isospin potential.

¹⁹S. D. Bloom, N. K. Glendenning, and S. A. Moszkowski, Phys. Rev. Letters 3, 98 (1959).

Fig. 7. Comparison between the quasi-elastic (p,n) optical-model calculations neglecting Coulomb effects and the results obtained using the DWBA which includes the Coulomb effects.

Fig. 8. Comparison between the experimental data and the DWBA calculations of Satchler for both a volume and a surface-centered isospin potential.

In addition to producing reliable estimates of the strength of the isospin potential, the most recent DWBA calculations of Satchler *et al.*,²⁰ using a surface-centered isospin potential, yields a significantly better fit to the medium-A nuclei than the volume isospin term. The calculations for several nuclei are shown in Fig. 8.

Conclusions

It is clear that the inclusion of the isospin term in the optical potential⁹ is capable of explaining the diffractive nature of the (p,n)

²⁰G. R. Satchler, R. M. Drisko, and R. H. Bassel (to be published).

angular distribution and the absolute magnitude of the cross section. The results of these calculations are summarized as follows.

(1) The optical model neglecting Coulomb effects and the DWBA yields comparable results when the Coulomb displacement energy is small compared with the incident proton energy, i. e., for $\Delta E_c < \frac{1}{2} E_p$.

(2) Assuming volume absorption and that the isospin potential has the same form factor as the real potential, and neglecting Coulomb effects, one obtains values for V_1 which vary as a function of A from 70 to 90 MeV.

(3) The value for V_1 (70—90 MeV) inferred from the (p,n) cross sections is in good agreement with the determination of the proton potential anomaly (~ 100 MeV).¹⁵ It is seen then that the isospin term in the optical potential proposed by Lane⁹ accounts both for the proton potential anomaly and for the isobaric (p,n) process.

(4) A surface-centered isospin potential is able to produce a significantly better fit to the data for medium- A nuclei than can the volume term.²⁰

* * *

M. BÖHNING, University of Heidelberg: Have you evidence that there is not a large amount of compound processes in the reactions you have shown? The angular distributions of your cross sections do not show in all cases the characteristic forward peaking normally observed in direct interactions. (In some of your cases the ratio of the peak values in forward angles and at 90° seem to be only of the order of two.) Wouldn't this fact indicate that the mechanism which leads to an excitation of the isobaric state is to a considerable degree compound-nucleus rather than direct reaction?

Fig. 9. The neutron spectra transformed to the center-of-mass system and divided by the energy of the emitted neutron, plotted as a function of the excitation of the residual nucleus Cr^{51} . The arrow denotes the position of the analogue state. The neutron spectra, with the exception of the analogue-state neutrons, are in agreement with the predictions of the statistical model of the compound nucleus.

ANDERSON: The variation in most cases is two orders of magnitude. The smallest variation, a factor of 5, occurs in the case of scandium. Another line of evidence is based on the population of the adjacent states. That is, on a compound-nucleus picture this state would not be preferred and the neutron spectrum would be as shown in Fig. 9. This indicates the compound-nucleus contribution in that region. Insofar as we have to use a line shape when the cross section becomes small, there may be some contribution from compound-nucleus decay; but it is an experimentalist's difficulty.

E. P. WIGNER, Princeton University: I am confused on this question of the compound-nucleus formation. It seems to me that the principal question concerns the density of levels of lower isobaric spin at the place where you have the level with the higher isobaric spin. It is quite possible that the density is low in the region of light nuclei and is much greater in the heavier region. There seems to be a transition in your graph.

ANDERSON: In many of these cases, the compound-nucleus neutrons are from (p,2n) reactions. In some of the heavier elements, I believe they would come from (p,p'n) reactions. The actual density of the first neutrons decaying from a compound system is exceedingly small,

i. e. , just taking the neutrons and distributing them by using a statistical model of the compound nucleus.

WIGNER: I do not think that the "final level" is one single state; possibly the situation is more complex. You may be dealing with the transition region between formation of a single resonant state and formation of a giant resonance. The regular resonance corresponds to a single final state, whereas in the giant resonance the final state is dissolved into many levels.

ANDERSON: I agree. We cannot tell whether there is a single state or whether there is a narrow band of states.

WIGNER: It may have a considerable effect on the angular distribution and also on the point which Dr. Austern mentioned, namely that perhaps in some cases you should calculate with a definite state and a definite overlap rather than with a group of states.

J. P. SCHIFFER, Argonne National Laboratory: I have one comment about light nuclei. You said that you were sure that there is no compound-nucleus contribution. However, a recent article from your laboratory²¹ presented angular distributions from the $N^{15}(p,n)$ reaction at a series of energies. The energy dependence seems to indicate many resonances. It would seem that in light nuclei there could be very large contributions from something that resonates with energy; perhaps this should be called a compound-nucleus process.

ANDERSON: As a matter of fact, we are above the N^{15} resonances.

SCHIFFER: A second point is that if an analog state occurs in a compound nucleus (as in the cases to be discussed by Fox), the evidence is quite good that the analog states are very sharp, possibly even single resonances. The limits set on the width can be much lower than those from your work.

²¹ L. F. Hansen and M. L. Stelts, Phys. Rev. 132, 1123 (1963).

P. AXEL, University of Illinois: I should like to comment on whether the state and reaction you are investigating can be interpreted in terms of the compound nucleus. First, I do not think that the level density in the composite system can provide an answer about your reaction mechanism. If there were isolated levels evident in the composite system, one could infer that the compound nucleus was playing a role. But one would expect to find overlapping levels in the composite system formed when 18-MeV protons interact with nuclei; this would be true even in light nuclei because even if the level spacing is large, the widths could be expected to be larger.

However, your experimental data do give a clear indication that the analogue state is excited by a direct reaction. Your neutron spectra (Fig. 9) clearly show a spike superimposed on a background; the background is the relatively smooth decrease in the number of neutrons with increasing neutron energy, while the spike is an obvious concentration of neutrons at a particular energy. Most of the neutrons you saw, which form the background over which the analogue state appears, might originate from the decay of the compound nucleus. A decaying compound nucleus would give the smooth rapid decrease in neutron number with increasing neutron energy. Since I do not remember the angle at which you were measuring, I am not sure of whether one would be justified in suspecting that there are also some contributions to these background neutrons from direct interactions. But any ambiguity about the reaction mechanism responsible for the background should not obscure the fact that the excitation of the analogue state is clearly due to a direct reaction.

It is not at all clear to me that the positions or characteristics of the final levels in your reaction are relevant to the question of whether your reaction mechanism is compound nuclear. The situation is quite different when one considers reactions such as Fox is going to talk about in which the analogue state is reached in the composite rather than

the final system. In that case the interpretation of the reaction depends on the level density near the analogue state. It would seem to me that this level density would not influence the interpretation of your reactions.

There is a separate problem of explaining why the analogue-state strength is concentrated in a single level or in a very narrow energy region despite neighboring states of the correct spin and parity. Fox will talk about this concentration of strength. But this problem is not related to your reaction mechanism. Your data indicate that some of the reactions select a very special energy region in the final nucleus. Whether there is just one level or several levels in that energy region is not relevant. The facts are that this energy region is distinctive because it has some special strength, and that this type of strength could govern a direct reaction but would not influence what is usually thought of as the decay of the compound nucleus.

S. MESHKOV, U. S. National Bureau of Standards: Did not French and Vogt once calculate the $C^{14}(p,n)N^{14}$ reaction in which N^{15} is the compound nucleus? Would Prof. French indicate how their predicted angular distributions compared with experiments?

J. B. FRENCH, University of Rochester: Vogt, Iwao, and I²² did indeed use a three-level Wigner-Eisenbud formalism to analyze the $C^{14}(p,n)$ and $C^{14}(p,\gamma)$ reactions going through resonances in O^{15} . One of these resonances is the analog state of the C^{15} ground state. The proton energies were quite low, below 2 MeV, and no angular distributions were measured. The three-level formalism gave a quite remarkably good fit.

Of course, the calculations included no direct reaction at all. The fit to the data was excellent, although the energy variations of the total cross section for both the (p,n) and the (p,γ) reactions were quite

²² J. B. French, S. Iwao, and E. Vogt, Phys. Rev. 122, 1248 (1961).

complex. I believe the (p,n) cross section varied by a factor of 2000. Thus I would conclude that there was virtually no direct reaction.

ANDERSON: I would like to add two side comments. We have almost completed a series of measurements on the titanium isotopes from Ti^{46} to Ti^{50} to obtain more information about the spectroscopic factor in terms of (N - Z). In addition, the higher proton energies obtainable from our machine have enabled us to extend the measurements of the Coulomb energy up to Ho^{165} . Shell effects have begun to show up; their absence in the earlier measurements was due, at least in part, to the sparsity of data.

ISOBARIC-SPIN ANALOGUE STATES

J. D. Fox

Florida State University, Tallahassee, Florida

It has been nearly two years since we discovered for ourselves that certain proton reaction cross sections were not exactly smooth—or even fluctuating. It was already known that neutron "evaporation" spectra were capable of showing interesting structure (as was just described by Dr. Anderson) but the connection between the two phenomena was only recently established. We decided just last fall that we were looking at isobaric-analogue states that were being excited as compound-nucleus resonances in proton reactions. This is certainly not a new phenomenon—this sort of thing is well known for light nuclei of course—but there was a certain tendency on the part of experimenters to regard nuclear cross sections in heavy nuclei at high excitations as smooth, or at most, fluctuating.

Since the theoretical aspects of this work have been discussed in New York recently,¹ I will devote most of the time today to a discussion of the experiments as they now stand, and discuss the possible future of this type of work.

Our experiments are really quite simple. We study both (p,n) and (p,p) reactions with proton beams accelerated by a 12-MeV tandem Van de Graaff. We use thin targets evaporated onto thin carbon backings. The target thicknesses are of the order of 20 to 100 $\mu\text{g}/\text{cm}^2$. Proton beams from the accelerator are collimated onto a spot about 2 mm in diameter at the center of a scattering chamber for the elastic-scattering experiments. We usually get beams of the order of 1 μA with

¹D. Robson, Bull. Am. Phys. Soc. 9, 89 (1964).

this arrangement, and this is sufficient. Some of the targets do not tolerate even this amount of beam. We are at present using three surface-barrier counters, each with a collimating aperture 0.067 in. in diameter placed about 6 cm from the target spot. The counters are positioned at 90° , 125° , and 165° to the incident beam. Counting is done with multichannel analyzers with the elastic and inelastic scattering peaks of oxygen and carbon in the spectrum biased out. In this way, counting losses due to dead time are negligible. We usually run for something of the order of 100 μC per point and this takes of the order of 2—3 min per point. The beam energy can be changed while the analyzers are being printed out. We try to accumulate about 10 000 counts at each of the angles; this determines our running time.

When looking at (p,n) reactions, the beam is uncollimated but is focused by a quadrupole lens to a beam spot of 2-mm diameter. The beam then proceeds to a Ta beam stop located 3 m away. Neutrons are detected by a Hanson-McKibben long counter, the nose of which is about 4 in. away from the target. We have not used collimation here because by about 7 MeV, the background due to collimation near the target and counter becomes too large. A more elaborate experimental setup would allow the use of collimation and make running much easier. As it is now, the beam wanders a bit over the target and this causes trouble when we are using isotopically enriched targets whose area of uniform thickness is not too large. We are able to certify the existence of a resonance with the (p,n) reaction but, because of the crudity of the setup and because we do not have the Rutherford cross section to normalize to, we cannot determine absolute cross sections. If we confined ourselves to lower beam energies, we could measure the (p,n) cross section at the same time as the (p,p) and this would be more useful. Lee, Marinov, and Schiffer,² in their study of proton reactions through isobaric analogue

²L. L. Lee, Jr., A. Marinov, and J. P. Schiffer, Phys. Letters 8, 352 (1964).

Fig. 1. Excitation function for $\text{Sn}^{120}(\text{p}, \text{p})$ at 165° near the analogue of the ground state of Sn^{121} .

states with targets of Ni^{64} , used this technique and confined their energy of bombardment to below 6 MeV.

We have seen resonances corresponding to isobaric-analogue states excited in the compound nucleus in 21 nuclides, averaging about 5 levels per nuclide. Striking results, such as we recently published for Y^{89} and Sr^{88} ,³ are not particularly unusual. Figure 1 shows a large s-wave resonance in $\text{Sn}^{120} + \text{p}$. This is the lowest energy structure we see in this isotope, although it is apparently lying right on top of the d-wave ground-state analogue. Note that the magnitude of the elastic-scattering wiggle is about 75% of the smooth Rutherford cross section in this region.

The actual size and shape of each resonance is determined by a set of parameters such as E_{res} , ℓ_p , Γ_p , Γ_n , J^π ; the reaction depends very sensitively on them. All the resonances seem to have a reduced width which is an appropriate fraction of the single-particle reduced width. We are able to detect proton elastic-scattering resonances down to about 4.5 MeV in nuclei such as the Zr isotopes and (p,n) reactions appear to be still useful at much lower energies. We do not learn as much from the (p,n) reaction because of the limitations just described, but it is useful in finding the resonances.

We are still very much behind in our resonance analysis. Some simple cases with spin-zero targets have been analyzed and it is from this analysis that we know that the reduced proton widths can be

³J. D. Fox, C. F. Moore, and D. Robson, Phys. Rev. Letters 12, 198 (1964).

Fig. 2. Excitation function for $\text{Mo}^{94}(\text{p},\text{p})$ at 125° and 165° .

compared to the reduced neutron widths found in (d,p) work. Analysis to determine ℓ_p is not really necessary since the characteristic shape of the resonance at three angles is usually enough to determine this. One trouble, of course, is the problem of overlapping resonances which occurs when the neutron width broadens the level out at high energy. We are at present working on programs to facilitate the unfolding of resonances in such cases.

I would like to describe some of our most recent work.

Our current experiments in the $\text{Zr}-\text{Mo}$ region have yielded striking agreement with the spectroscopic results on the (d,p) reaction. Figure 2 shows the data on the $\text{Mo}^{94}(\text{p},\text{p})$ reaction between the incident-proton energies of 4.5 and 8 MeV. Only two angles, 90° and 165° , are available here. We see much structure in the excitation function. The resonance near 4.9 MeV is presumably the ground-state analogue with excited states at higher energy. The effect of the proton Coulomb barrier is apparent here because the ground state has a large reduced width but the resonance is practically undetectable because of the low energy. One interesting feature is that although the resonance structure gets more complicated at higher excitation energies, presumably because of a higher density of states, the analogue resonances are still quite sharp.

Fig. 3. Energy level diagrams of the states of Mo⁹⁵ and the observed analogues in Tc⁹⁵. The date (3-64 for this figure) is given on this and several other figures because of the preliminary nature of the data.

Fig. 4. Energy level diagrams of the states of Zr⁹³ and their analogues observed in Nb⁹³.

Figure 3 shows the level scheme of Mo⁹⁵ and the states in Tc⁹⁵ as given by the data in Fig. 2. We are able to compare our results with the (d,p) work by subtracting the center-of-mass proton energy needed to reach the ground-state analogue from the resonance energies. The data on Mo⁹⁴(d,p) was supplied by Prof. Cohen⁴ before publication and we wish to acknowledge his cooperation. The (d,p) work shows that the levels we have missed are fairly weak. It is possible that we are resolving the 0.806-MeV d+s level into levels at 0.76 and 0.86 MeV. Both the (p,p) and (d,p) reactions miss the known first excited state at 0.204 MeV and this state is probably fairly complicated.

Figure 4 shows the energy levels of Zr⁹³ and their analogues in Nb⁹³ excited by proton elastic scattering. Between the ground state and

⁴S. Hjorth and B. L. Cohen (to be published).

Fig. 5. Excitation functions for $\text{Zr}^{91}(\text{p}, \text{p})$ at 90° , 125° , and 165° .

3 MeV excitation, the only states that are seen in the (d,p) work but missed in the proton elastic scattering work are g-wave. Above 3 MeV, we are apparently missing some weak d-wave levels. The first excited state is not seen in the (p,p) work and, like the $(d_{5/2})^3$ first excited state of Mo^{95} , is probably a complicated state.

Some of the most striking agreement that we get with the (d,p) reaction data is for the case of Zr^{91} as a target. Figure 5 shows the elastic-scattering excitation functions at the three angles over a range of more than 4 MeV. These data were taken in about 36 hr. The lowest state seen in elastic scattering is the second excited state near 4.7 MeV—the first excited state and probably the ground state have been identified by use of the (p,n) reaction. Figure 6 compares the energy levels of Zr^{92} and its analogue levels in Nb^{92} . Only very weak levels have been missed up to 4.12 MeV excitation and one can see that there is excellent agreement with both the energies and particle orbital angular momenta.

One of the numbers that can be extracted from the data is the Coulomb energy involved when a neutron is replaced by a proton in the nucleus. Since we measure the proton energy required to produce a given proton analogue, and the neutron binding energy belonging to the neutron analogue is measured in (d,p) reactions, we can determine the energy required to produce a hypothetical (p,n) reaction between the analogues by merely adding the center-of-mass proton energy to the neutron binding energy of the neutron analogue. This may be seen as follows. If the proton analogue is formed when a proton of mass M_p and energy E_p is incident on a target nucleus of mass M_t , the mass of the proton analogue P is

$$M_P = M_t + M_p + E_p.$$

Similarly, if the neutron analogue is formed when the same target nucleus captures a slow neutron of mass M_n , the mass of the neutron analogue N is

$$M_N = M_t + M_n - B_n,$$

where B_n is the binding energy of the neutron. Then in the (p,n) reaction

between the analogues N and P, the Q value Q_{pn} is just

$$Q_{pn} = -(B_n + E_p).$$

Fig. 6. Energy level diagrams of the states of Zr^{91} and the observed analogues in Nb^{92} .

Fig. 7. Plot of the measured Coulomb constant as a function of A.

The mass values recently published by the Minnesota group⁵ for our region of masses has been especially useful. There is no need to lecture here on the importance of good data on masses and Q values; anyone who feels that his measurements of the Q value of a nuclear reaction have an error only of the order of 20 keV will be troubled by the previously available mass data. In our experiment, it is probably possible to make absolute measurements to ± 20 keV and, with reasonable errors in the mass data, to get values for the Coulomb energy with errors of the order of ± 50 keV out of about 12 MeV—less than $\frac{1}{2}\%$. As I said, the new mass data have helped pull things together a great deal. Looking now at Fig. 7, we see a definite difference in the Coulomb constant between the Zr—Mo region and the Sn region. Otherwise, the Coulomb-energy data for each

⁵R. R. Ries, R. A. Damerow, and W. H. Johnson, Jr., Phys. Rev. 132, 1662 (1963).

set of nuclides are in reasonable agreement. The crosses on the figure are from the data of Anderson, Wong, and McClure⁶ based on the (p,n) reaction measuring the energy of the outgoing neutron to the analogue state. The apparent variation in the constant C when going from Zr to Sn will have to be explored further and we may examine the nuclei in between.

I would like to comment now on the future of such measurements and their possible usefulness. This reaction (proton elastic scattering through analogue resonances) is in essence equivalent to the (d,p) reaction on the same target for purposes of nuclear spectroscopy. It differs, of course, in the reaction mechanism and this can serve as an advantage for nuclear spectroscopy. While the (d,p) reaction method makes use of distorted-wave Born approximation for its interpretation, our assignment of the value of ℓ_p involved in the resonances is quite certain; we come away from a night of data taking feeling much better about our angular distributions than distorted-wave Born approximators usually do. It is also true that our data are often obtained more rapidly than good (d,p) reaction spectra.

There are certainly some disadvantages to this method as applied to nuclear spectroscopy. The effect of neutron emission, even though forbidden for isotopic-spin reasons, nonetheless serves to broaden the levels, making them unresolvable at high excitation energies. This may cause us considerable trouble in the higher masses for which excitation energies are high and (p,n) thresholds are low. More careful computer analysis can help a great deal in the unfolding of overlapping levels, but this is still ahead of us. We are certainly missing small levels at present; this may be due to a variety of causes.

⁶J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 126, 2170 (1962).

Our measurements of Coulomb energy lead to interesting things. They are certainly more accurate than the recent data of Anderson, Wong, and McClure and will provide the best measure of this quantity in the foreseeable future. It is a little early to say whether or not there will be interesting shell or deformation effects in this Coulomb constant. We intend to proceed with this work rather systematically.

The technique of measurement of analogue states in heavy nuclei as compound-nucleus resonances is only 6 months old. We have had a lot of fun with it and it may prove an interesting and useful tool for nuclear-structure studies. The existence of sharp structure such as we have found (structure that is anything but random in nature at excitations above 12 MeV in heavy nuclei) will prove a considerable hazard to experimenters trying to perform fluctuation measurements.

It is not at all inconceivable that the power of this technique will be improved by at least an order of magnitude within a reasonable time. At any rate, this work should serve to warn the (d,p) reaction spectroscopists that their technique is not unique. Figures 8 through 12 present additional data and energy level diagrams for several Zr isotopes.

* * *

R. E. SEGEL, Argonne National Laboratory: When your data are compared with (d,p) reaction data, do your spacings between levels for heavy nuclei come out as well or better than they do in light nuclei?

FOX: We have observed that our spacings are closely related to the (d,p) level spacings, i.e., the mean errors are ± 20 keV of excitation and the spacings are of the order of 1 MeV. In light nuclei, differences of 100 keV are not unexpected.

Fig. 8. Excitation functions for $Zr^{92}(p,p)$. Fig. 9. Excitation functions for $Zr^{96}(p,p)$.

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Fig. 10. Energy levels in Zr^{97} and their observed analogues in Nb^{97} .

Fig. 11. Excitation functions of $\text{Zr}^{94}(p,p)$.

M. K. BRUSSEL, University of Illinois: Why do you get such large cross sections only for these analog states in this region of excitation? There are many other levels in this region of excitation. What are the comparative proton and total widths of the analog states and these others?

FOX: Of course, we do see these and other levels in the (p,n) reaction. The point is not that they are excluded; rather the configurations of these other levels, being at very high excitation, are quite complicated. The only strong states that we see are analogues of what we think of as very simple states in nuclei. Probably the configurations alone are enough to account for the big cross sections observed—

resonances with an appreciable fraction of the single-particle width. The unresolved states with lower T appear as a smooth background.

P. AXEL, University of Illinois: Some of your compound nuclei definitely have many overlapping levels with the lower T. For such nuclei, your data appear analogous to sharp levels superimposed on potential scattering; similar experimental curves with well-defined resonances are seen often when lighter nuclei are studied.

My main question is this. I noticed that Professor French was unwilling to say whether T is or is not a good quantum number. But

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$$E_p^{cm} - 5.13$$

Fig. 12. Energy levels of Zr^{95} and their analogues observed in Nb^{95} .

why do these analogue states of higher T not mix with the numerous levels of lower T ? Lane and Soper⁷ have discussed this, and I realize that formally no mixing would be expected if T were a good quantum number. But, the Coulomb interaction does mix different T values and I wonder how high in excitation energy one must go for the mixing to be significant?

FOX: I cannot comment on the theory, but there is some evidence that T is a fairly good quantum number and prevents mixing. It is known experimentally that the levels are not broad and do not disappear into this wealth of levels that are underneath.

A. K. KERMAN, Massachusetts Institute of Technology: I would like to ask a related question. How sure are we that there is very little mixing? Does the bump come up to the unitary limit when the background is subtracted? Is it a real resonance or is it some kind of a strength function? Can you tell that from your experiment?

FOX: Experimentally the total width is very narrow—in some cases of the order of a few keV. It is 10–20 keV in the figures presented. The implication is that the neutron reduced width is therefore very narrow indeed, of the order of 0.01–0.001 of a neutron single-particle reduced width. If there were considerable mixing, you would expect that the width would be larger than the value observed.

E. P. WIGNER, Princeton University: The theory may not be right, but I think that its predictions are unequivocal. If many levels are in the region of the "analogue state," one should have a giant resonance. Admittedly, it is very surprising that the experiments show such narrow lines, but the widths of the usual giant resonances are also surprisingly low.

The usual giant resonance has a width of the order of magnitude of a few MeV. The Coulomb interaction is only a fortieth of

⁷A. M. Lane and J. M. Soper, Nucl. Phys. 37, 663 (1962).

the nuclear interaction, and the width of the giant resonance caused by it should be proportional to the strength of the interaction. In addition, and perhaps somewhat paradoxically, the longer range of the Coulomb interaction renders it less effective in causing an admixture of one state to another. Evidently, a constant potential, no matter what its magnitude, does not cause any mixing of states. Hence, the giant resonance, caused by the electrostatic interaction between the "analogue state" and the many nearby states with one-lower isotopic spin, can be expected to be very narrow.

I do not see very good evidence that there is only one level at which the cross sections are high, i. e. , that the analogue state remains simple. It could well have a structure within the observed width, as every giant resonance does. This should be the situation, at least, if the density of the levels with the isotopic spin of the normal state (i. e. , an isotopic spin one below that of the "analogue state") is so high that several of these levels fall within the expected width of the giant resonance. This situation should prevail in all not-too-light nuclei. Only in the case of light nuclei can one expect a sufficiently low density of the levels with the isotopic spin of the normal state for the analogue state to be left alone, i. e. , to remain a single level rather than to form a giant resonance. It would be interesting to investigate the transition region between light and not-too-light nuclei, for which the spacing of the levels with the lower isotopic spin is of the order of magnitude of the giant resonance that could be expected.

C. D. GOODMAN, Oak Ridge National Laboratory: I would like to present a little picture (Fig. 13) that may show the relationships of these states to other states, in the hope that it may clarify some of these questions. I take your example of Sr^{88} as the target nucleus. In this nuclide, the $p_{3/2}$ shell is filled and there are neutrons filling the $p_{1/2}$

Fig. 13. Diagrammatic representations of the wave functions for Sr^{88} (ground state), $\text{Sr}^{89} \frac{5}{2}^+$ (ground state), $\text{Y}^{89} \frac{5}{2}^+$ (isobaric analog of Sr^{89} ground state), and Sr^{88} plus a $d_{5/2}$ proton. For an explanation of the notation, see C. D. Goodman, Nucl. Phys. 55, 449 (1964).

and $g_{9/2}$ levels. The $\text{Sr}^{88}(\text{d}, \text{p})\text{Sr}^{89}$ reaction might put in a $d_{5/2}$ neutron to form a $\frac{5}{2}^+$ state of Sr^{89} . The isobaric analogue of this state in Sr^{89} would be a state in Y^{89} , which could be obtained by operating on the $\frac{5}{2}^+$ Sr^{89} wave function with the T_- operator. In the notation of Fig. 13, the wave function would consist of thirteen terms, each term being obtained by moving one of the excess neutrons to a proton state. This $\frac{5}{2}^+$ analogue state in Y^{89} is what I think is seen in the kind of resonance that Fox was talking about. In that kind of reaction one is adding a $d_{5/2}$ proton to the target Sr^{88} . The product wave function has an overlap with one of the thirteen terms in the wave function of the analogue states. I think that is why one should expect to excite the analogue state.

D. ROBSON, Florida State University: I would like to refer to Prof. Wigner's comment on whether these states are in fact giant resonances or just single levels. In the region in which Fox et al. are working, I think that it is almost impossible to tell experimentally. However, for the levels we have analyzed, the sum of the partial widths is very close to the total width of the level. I do not think this is true for a giant resonance. In fact, the theory of Lane, Thomas, and Wigner⁸ has an extra width which appears in the denominator. We apparently do not observe a term of this type in the cases we have analyzed; if it is present, it is only of the order of a few keV.

⁸A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).

SESSION D

TUESDAY AFTERNOON

10 MARCH 1964

CHAIRMAN: W. C. PARKINSON

SCIENTIFIC SECRETARIES

J. R. ERSKINE

S. P. PANDYA

SINGLE-NUCLEON SPECTROSCOPIC FACTORS AND NUCLEAR MODELS*

M. H. Macfarlane

Argonne National Laboratory, Argonne, Illinois1. Introduction

1960 was the year 10 from the first realization¹ that deuteron stripping reactions could be of great value in nuclear spectroscopic studies. The necessary foundations had been securely laid. It had been shown that the angular distributions of reaction products in deuteron stripping and similar processes provide an ideal way of measuring transfer of angular momentum and parity. The relative magnitude of the cross sections for different transitions had been extensively analyzed on the basis of various nuclear models.² The analysis had been purely phenomenological and only the lightest nuclei ($A < 40$) had received detailed attention. Yet a lot of useful information had been obtained about nuclear levels and, most important of all, this information was in excellent over-all agreement with that gleaned from other sources. Finally, the distorted-wave Born approximation (DWBA) for (d,p) reactions had been examined in some detail³ and seemed to provide a basically correct description of the

*Work performed under the auspices of the U. S. Atomic Energy Commission.

¹S. T. Butler, Proc. Roy. Soc. (London) A208, 559 (1951).

²References to work published during the 1950's will be found in M. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960).

³A detailed and critical review has been given by N. Austern, "Direct Reaction Theories," Sec. V.D of Fast Neutron Physics, Part II, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1963).

reaction mechanism. It remained to combine the underlying principles of the analysis of relative cross sections with the DWBA theory of the reaction process.

Since 1960, this consolidation has taken place. The use of a reasonably correct reaction theory eliminates the most obvious shortcomings of the old phenomenology of cross sections. We think here of the inability of the latter to say anything useful about absolute cross sections, of its inadequate treatment of more complex stripping and pickup reactions such as (He^3, d) or (t, d) , and of its very restricted region of applicability, namely to deuteron-nucleon reactions in which both projectiles are of 8—16 MeV caliber and in which the transferred nucleon is neither too weakly nor too strongly bound. Of course the smooth energy dependence of the peak cross sections, which was the underlying cause of the rather astonishing success of the phenomenology of cross sections, still works in our favor. It enables us to obtain reliable relative spectroscopic factors in situations in which the appropriate optical-model parameters are poorly known and perhaps even when subtler effects are making their presence felt in the reaction process.

Thus the field of nuclear spectroscopy with single-nucleon stripping and pickup reactions has changed and developed radically in the past three or four years. Improvements in experimental technique have more than kept pace with increased theoretical insight. As one result, the old restriction of attention to the lightest nuclei has vanished without a trace. Among the contributions to this conference, we find high-precision studies of the structure of nuclei with mass 12 and mass 238.

Some of the more interesting recent developments have been or will be covered by other survey speakers or in research contributions. Thus my talk will in a certain respect resemble a Swiss cheese. Before presenting the part of this Swiss cheese complementary to the holes, it will be necessary to give a few basic definitions.

2. Single-Particle States, Single-Hole States, and Spectroscopic Factors

A one-nucleon stripping reaction reveals the structure of a nucleus by forcing it to accept a nucleon in a state of specified orbital angular momentum and spin. This act of acceptance is described formally by the creation operator $a_{lj}^\dagger$ for the appropriate orbit, which operates on the target ground state $\Psi_{J_0}(A)$ to produce the single-particle state

$$a_{lj}^\dagger \Psi_{J_0}(A) .$$

The single-particle state so defined is to be interpreted after the fashion of Lane, Thomas, and Wigner; it is not necessarily an eigenstate of the residual nucleus but may be distributed over numerous physical states. The spectroscopic factor for the transition between the target ground state and one of these final levels is simply a measure of the amount of single-particle excitation which the final state in question contains.

Before giving the formal definition of spectroscopic factor, two remarks should be made. Firstly, the entire discussion of single-particle states carries through for single-hole states; the creation operator $a_{lj}^\dagger$ is replaced by the corresponding annihilation operator. Secondly, it should be noted that the single-particle and single-hole states written down above are properly normalized only for the special cases of capture into an empty orbit and pickup from a filled orbit. Attention to the proper normalization gives rise to sum rules which express the obvious physical statements that the total single-particle strength measures the number of holes in the relevant orbit in the target ground state, while the single-hole strength gives the number of particles.

We consider, then, single-nucleon transfer reactions of the form

$$A + a \rightleftharpoons (A + 1) + (a - 1),$$

where the forward sense of the arrows refers to stripping, backward to pickup. Clearly not one, but two spectroscopic factors enter here. However, when a and $(a - 1)$ refer to very light nuclei (nucleons to α particles) it is customary to insert their wave functions explicitly into the calculation and to avoid the more symmetric procedure. If a and $(a - 1)$ are, for example, N nuclei or F nuclei the two spectroscopic factors must come forward into the light of day. The extraction of spectroscopic factors from heavy-ion transfer reactions has recently reached the level of quantitative investigation—the possibility has of course long been realized. A research contribution by McIntyre⁴ deals with this subject. Here, however, we confine attention to light projectiles and write down the differential cross section in DWBA as

$$\frac{d\sigma}{d\omega} = R \sum_{j\ell} \mathcal{S}_{j\ell} \sigma_{j\ell}(\theta),$$

where R is a statistical factor, $\sigma_{j\ell}(\theta)$ is the distorted-wave angular distribution, and $\mathcal{S}_{j\ell}$ is our main quarry, the spectroscopic factor. With one exception to be mentioned later, different single-particle orbits contribute incoherently to the cross section.

The spectroscopic factor for a given pair of states and for the single-particle orbit ℓj of the transferred nucleon is defined by

$$\begin{aligned} \mathcal{S}_{\ell j}(A, J_0 \rightleftharpoons (A + 1), J_1) \\ = (A + 1) \left| \langle \Psi(J_1) | \{ \phi_{\ell j} [\#(A + 1)] \times \Psi(J_0) \}_{J_1} \rangle \right|^2 \\ = \left| \langle \Psi(J_1) | a_{\ell j}^\dagger | \Psi(J_0) \rangle \right|^2, \end{aligned}$$

⁴J. A. McIntyre, in I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 160.

where $\times$ denotes vector coupling, $\langle \parallel \parallel \rangle$ is a reduced matrix element in the sense of the Wigner-Eckart theorem,⁵ and magnetic quantum numbers (whose role is inessential) have been eliminated. The first equality gives the conventional definition in terms of overlap integrals²; the second, which is equivalent to the first, expresses the physical interpretation of the spectroscopic factor as the squared amplitude of a fragment of single-particle state.

Thus, as has often been emphasized, one-nucleon capture and pickup reactions study nuclei by producing single-particle and single-hole excitations. Cohen⁶ has already discussed many of the gross properties of these excitations—centroid positions, widths, and total strengths. Most of what I have to say concerns the fine structure, whose analysis clearly requires the use of a detailed nuclear model. Comparison of measured and predicted spectroscopic factors then provides an exacting test of one aspect—the single-particle aspect—of the model wave functions.

It should be stressed that a spectroscopic factor is uniquely determined by the structural relationship between two nuclear states. The same factor will enter whenever these two states are connected by a nucleon-transfer reaction—stripping, pickup, or resonant.

3. Two Unrelated Matters Concerning Light Nuclei

Direct-reaction spectroscopy—at least at moderate energies—has tended of late to pay less attention to light nuclei than it

⁵ For the purposes of this paper, the Wigner-Eckart theorem is written in the form

$$\langle jm | T_q^k | j'm' \rangle = C_{m'qm}^{j'kj} \langle j || T^k || j' \rangle ,$$

where T_q^k is the component of a spherical tensor of rank k , C is a Wigner coefficient, and $\langle j || T^k || j' \rangle$ is the reduced matrix element.

⁶ B. L. Cohen, these Proceedings, p. 91 .

Fig. 1. Levels of F^{17} and O^{17} .

did a few years ago. There are, however, two unrelated topics concerning light nuclei which I would like to broach. The first involves j -forbidden transitions which refuse to be forbidden. The second is a very brief comment on the one really new indignity suffered by light nuclei in recent years, their assault by energetic nucleons of 150 MeV or more.

3.1. j -Forbidden

Transitions in Light Nuclei. The main qualitative conclusion from stripping and pickup at moderate

energies on light nuclei ($A < 30$) is that simple models describe the salient experimental facts with surprising fidelity.² The intermediate-coupling shell model gives an adequate account of strong $\ell=1$ transitions in $1p$ -shell nuclei and of $\ell=0$ and $\ell=2$ transitions to low-lying states of the isotopes of O and F. As a rule, j -forbidden transitions are strongly inhibited (e. g., the $0^+ \rightarrow \frac{3}{2}^+$ ground-state transition⁷ in $Ne^{20}(d,p)Ne^{21}$). There are, however, some puzzling exceptions to this general rule; we now discuss two of them, suggesting (faintly but distinctly) that they may be related.

We turn our attention first to the negative-parity states at about 3.85 MeV in the mirror nuclei O^{17} and F^{17} (Fig. 1). Until about a year ago, the spin of this state was firmly believed⁸ to be $\frac{7}{2}^-$. It appeared quite strongly in the reaction $O^{16}(d,p)O^{17}$, the indicated

⁷H. B. Burrows, T. S. Green, S. Hinds, and R. Middleton, Proc. Phys. Soc. (London) A69, 310 (1956).

⁸See for example, F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 1 (1959).

spectroscopic factor being somewhere in the range 0.2—1. This had been confirmed in independent experiments^{9,10} at deuteron energies of 8 and 15 MeV. As a result the level in question was usually regarded as a fairly good single-particle $f_{7/2}$ level.

It has now been established^{11,12} beyond reasonable doubt that the spin of the 3.85-MeV level in O^{17} and of its analogue in F^{17} is $\frac{5}{2}^-$. The lowest $\frac{7}{2}^-$ level¹³ in F^{17} seems to be at around 5.7 MeV; its resonant width is some 50 times that of the f state at 3.86 MeV. This latter item of information establishes that the lower state cannot have an $\mathcal{S}$ value much greater than 0.01, in a manner that is almost independent of uncertainties in the values of the single-particle resonant widths.^{14,15} An $\ell=3$ transition with $\mathcal{S} \approx 0.01$ is too weak by a factor of at least 5 to show up clearly in an $O^{16}(d,p)O^{17}$ experiment at $E_d \approx 10$ MeV. Thus the transition in question is probably a j -forbidden transition with an unexpectedly large cross section of something very like the "normal" stripping form.

Let us now recall similar difficulties that were encountered in dealing with the j -forbidden transition

⁹ T. S. Green and R. Middleton, Proc. Phys. Soc. (London) A69, 28 (1956).

¹⁰ E. L. Keller, Phys. Rev. 121, 820 (1961).

¹¹ C. Broude, T. K. Alexander, and A. E. Litherland, Bull. Am. Phys. Soc. 8, 26 (1963).

¹² R. E. Segel, P. Singh, R. G. Allas, and S. S. Hanna, Phys. Rev. Letters 10, 345 (1963).

¹³ S. R. Salisbury and H. T. Richards, Phys. Rev. 126, 2147 (1963).

¹⁴ E. W. Vogt, Phys. Letters 1, 84 (1962).

¹⁵ J. P. Schiffer, Nucl. Phys. 46, 246 (1963).

This problem was much discussed¹⁶ in the late 1950s but never really settled. The difficulty was not so much in proposing mechanisms that would permit the transition to proceed but rather that of explaining why the proposed mechanisms never seemed to appear in other j-forbidden transitions. Perhaps the $O^{16}(d,p)$ transition to the 3.85-MeV state provides a companion example.

As a footnote it should be remarked that the reaction $O^{18}(d,t)O^{17}$ also excites¹⁷ the 3.85-MeV $\frac{5}{2}^-$ state; the angular distribution again is roughly that of an $\ell=3$ transition. It is probably advisable in this situation to tackle one problem at a time! However, until we are really sure what is going on here it will be impossible to be entirely confident about quantitative spectroscopic studies of light nuclei by means of moderate-energy stripping and pickup reactions.

3.2. Single-Nucleon Pickup at High Energy; Levels of B^{11} and C^{11} . Since 1960, conventional deuteron stripping experiments have added relatively little to our understanding of the spectroscopy of light nuclei. In compensation, experimental techniques with electrons and with nucleons of 100-MeV caliber have developed to a point where it is possible to resolve individual levels in light nuclei. Thus a lively new byway of nuclear spectroscopy has come into being. Detailed discussion

¹⁶See, for example, the discussion on p. 731 of M. K. Banerjee, "The Theory of Stripping and Pickup Reactions," Chap. V.B.2 of Nuclear Spectroscopy, edited by F. Ajzenberg (Academic Press, Inc., New York, 1960).

¹⁷J. C. Armstrong and K. S. Quisenberry, Phys. Rev. 122, 150 (1961).

of the objectives and achievements of these higher energy experiments lies outside the scope of this talk; nevertheless I would like to describe one particular episode partly for its intrinsic interest and partly to emphasize that the higher energy pickup reactions measure precisely the same spectroscopic factors as do the more familiar experiments in the 10-MeV region.

Consider next the lower reaches of the spectra of the mirror nuclei B^{11} and C^{11} (Fig. 2). The spectroscopic factors shown to the left of the level diagrams are those calculated from intermediate-coupling shell-

model wave functions¹⁸ with $|a/K| \approx 6$; the relative values given are normalized to unity for the ground-state transition. Qualitatively, the predictions are clear. The ground-state transition should predominate, with three weaker transitions to $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states below 8 MeV. Transitions to the levels at 4.46 ($\frac{5}{2}^-$), 6.76 ($\frac{7}{2}^-$), and 6.81 MeV ($\frac{3}{2}^+$) in B^{11} or to their analogues in C^{11} are forbidden by j-selection rules.

Five different experiments have been performed which bear directly on this problem:

$$\begin{aligned} C^{12}(p,d)C^{11}, & \quad E_p = 57 \text{ MeV} \quad (\text{Ref. 19}); \\ C^{12}(p,d)C^{11}, & \quad E_p = 155 \text{ MeV} \quad (\text{Ref. 20}); \end{aligned}$$

Fig. 2. Levels of B^{11} and C^{11} .

¹⁸ D. Kurath, Phys. Rev. 101, 216 (1956).

TABLE I. Cross-section ratios for pickup to various levels in C^{11} and B^{11} . (References given in text.)

J^π	Excitation in B^{11} (MeV)	(p,d) $E_p = 57 \text{ MeV}$	(p,d) 155 MeV	(p,pn)	$(p,2p)$	(n,np)	δ_{theor} (Relative)
$3/2^-$	0	1	1	1	1	1	1
$1/2^-$	2.14	0.13	0.25	0.08	0.08	0.08	0.08
$5/2^-$	4.46	...	...	0.09	0.04	0.04	...
$3/2^-$	5.03	0.15	0.12	0.04	0.04	0.04	0.12
$7/2^-$	6.76	...	...	0.04	0.03	0.03	...
$(3/2^+)$	6.81						
$3/2^-$	7.30	weak	0.06	<0.01	<0.02	<0.02	

$$C^{12}(p,pn)C^{11}, \quad E_p = 153 \text{ MeV} \quad (\text{Ref. 21});$$

$$C^{12}(p,2p)B^{11}, \quad E_p = 153 \text{ MeV} \quad (\text{Ref. 21});$$

$$C^{12}(n,np)B^{11}, \quad E_n = 120 \text{ MeV} \quad (\text{Ref. 21}).$$

The results of these experiments are summarized in Table I, which shows the cross-section ratios for transitions to the various states in B^{11} and C^{11} . These cross-section ratios should be very roughly proportional to the ratios of the appropriate spectroscopic factors. Certainly the small differences in the energies of the projectiles emerging from the different

¹⁹K. Matsuda, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 499.

²⁰D. Bachelier, M. Bernas, C. Détraz, J. Génin, J. Haag, and P. Radvanyi, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 1141.

²¹S. M. Austin, G. L. Salmon, A. B. Clegg, K. J. Foley, and D. Newton, Proc. Phys. Soc. (London) **80**, 383 (1962).

final states can be ignored with impunity; the same is not so likely to be true of the effects of differences in binding energy of the picked-up neutrons corresponding to the various final states. At any rate, no serious flaw in the qualitative arguments to be presented should result from identifying ratios of cross sections with ratios of spectroscopic factors for transitions with the same orbital angular-momentum transfer.

Perhaps the most obvious feature of Table I is the way in which the five sets of experimental results fall into two internally compatible groupings. The cross-section ratios from the two (p,d) experiments agree rather closely, as do those measured in the experiments in which two nucleons emerge separately. It will be interesting to see whether this superficial impression survives a proper extraction of the various spectroscopic factors. It very probably will.

The over-all picture for $\ell=1$ transitions is quite well described by the intermediate-coupling shell model; in fact the quality of agreement is much the same as that obtained for $\ell=1$ transitions to the same states in B^{11} and C^{11} in low-energy (d,p) and (d,n) experiments. It is, however, noteworthy that the four p-wave angular distributions in the 155-MeV (p,d) experiment are not identical. Instead the transitions to the two lowest 1p states in B^{11} have an angular distribution which is distinctly different from that of the two higher 1p states. The implication of this is probably that the high-energy (p,d) angular distribution is quite sensitive to details of the neutron bound-state wave function, in particular to the pulling-in of wave-function tails consequent on increased binding. Similar effects are well established for low-energy (p,d) and (d,t) reactions; there, however, the main manifestation is a sharp reduction in absolute cross section. The angular distributions seem to be less critically influenced.

The (p,2p), (p,pn), and (n,np) reactions populate the j-forbidden $\frac{5}{2}^-$ and $\frac{7}{2}^-$ (and/or $\frac{3}{2}^+$) states with about the same intensity as

the higher $1p$ states. The fact that these model-unfavored transitions are not detected in the (p,d) experiments counsels wariness in making quantitative interpretations involving $1f$ and $1d$ admixtures in the C^{12} , C^{11} , and B^{11} wave functions.

It is interesting that just as for the lowly 10-MeV deuteron so also is it for the 150-MeV nucleon; j -forbidden transitions sometimes decline to be forbidden.

4. Studies of $f_{7/2}$ -Shell Nuclei

Discussion of the fine structure of a single-particle or single-hole resonance requires a detailed nuclear model. The comparison of what the model predicts for the fine structure with what is in fact observed then yields information of various sorts about the nuclear wave functions. Recent work on $1f_{7/2}$ -shell nuclei clearly illustrates the character of such studies.

The basic assumption involved in talking about " $1f_{7/2}$ -shell nuclei" is that nuclei beyond Ca^{40} can be described in terms of an inert Ca^{40} core with the additional nucleons in the $f_{7/2}$ orbit. At least, it is hoped that some properties of low-lying states can be adequately described by such a model. Since the necessary two-body interaction energies can be determined directly from the observed spectra of Sc^{42} and Ca^{42} , the $f_{7/2}$ shell-model calculations are of particular interest. Once the basic restriction to $f_{7/2}^n$ configurations has been made, the calculation is free of adjustable parameters. Calculations of this sort have recently been carried out by McCullen, Bayman, and Zamick and by French and Ginocchio.²²

²²J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964); J. N. Ginocchio and J. B. French, Phys. Letters 7, 137 (1963).

The shell-model predictions of McCullen, Bayman, and Zamick for the fine structure of the $1f_{7/2}$ single-hole resonance in various f-shell nuclei are compared with experiment in Figs. 3 and 4. The experimental data involved are those of Kashy²³ for the reaction $Sc^{45}(p,d)Sc^{44}$ and of Kashy and Conlon²⁴ for the (p,d) reactions on the stable isotopes of Ti. Spectroscopic factors were extracted from the measured differential cross section by a DWBA analysis in which the pertinent optical-model parameters were taken, without further adjustment, from studies of elastic scattering. Thus the experimental spectroscopic factors are free of parameter adjustment in the same sense as are their theoretical counterparts. The quality of agreement between theory and experiment for the structure of the $f_{7/2}$ single-hole resonance exhibited in Figs. 3 and 4 is therefore very encouraging. Even the absolute values of the spectroscopic factors are given with fair accuracy by the shell-model wave functions.

Finally, let us briefly mention a very interesting qualitative result which was deduced by Lawson and Zeidman²⁵ in a direct analysis of (d,t) spectroscopic factors. They demonstrated that the (d,t) data clearly imply that symplectic symmetry (the generalization of seniority to mixed neutron-proton systems) is very strongly mixed in the wave functions of $f_{7/2}$ -shell nuclei. This result was derived with the aid of wave functions obtained by projecting angular-momentum eigenstates from determinants of deformed-oscillator eigenfunctions. Its validity has since been confirmed by the results of detailed shell-model calculations.²²

²³ E. Kashy, Phys. Rev. 134, B378 (1964).

²⁴ E. Kashy and T. W. Conlon, Phys. Rev. 135, B389 (1964).

²⁵ R. D. Lawson and B. Zeidman, Phys. Rev. 128, 821 (1962).

Fig. 3. The $1f_{7/2}$ single-hole resonance in $Sc^{45}(p,d)Sc^{44}$ (Ref. 23).

Fig. 4. The $1f_{7/2}$ single-hole resonance in the Ti isotopes (Ref. 24).

Fig. 5. Single-particle levels pertinent in the vicinity of mass 90.

5. Levels of Nb⁹², Nb⁹³

The work to be described next is a minor classic in the extraction of a maximum of nuclear-structure information from a minimum of experimental data. It concerns the interpretation by Sweet, Bhatt, and Ball²⁶ of a (p,d) experiment on Nb⁹³ by Sweet, Ball, and Goodman²⁷ and of the analogous (d,t) experiment of Sheline, Watson, and Hamburger.²⁸

First let us recall the qualitative features of the pertinent single-particle shell-model states (Fig. 5). The nearest closed-shell nucleus is Zr⁹⁰ with its proton shells filled up to the end of the fp shell, and with its neutrons filling also the g_{9/2} level. Thus for the Nb isotopes (Z = 41) the simplest model would place one proton in g_{9/2} and n neutrons in 2d_{5/2}. The pickup reaction Nb⁹³(p,d) [or(d,t)] should then proceed according to the scheme

$$\begin{array}{ccc}
 \text{Nb}^{93} & & \text{Nb}^{92} \\
 J = \frac{9}{2}^+ & \longrightarrow & J_0 = 2, 3, 4, 5, 6, 7 \\
 \sum_{J=0,2,4} c_J \left[(g_{9/2}^\pi) \times (d_{5/2}^2)_J^\nu \right]_{9/2} & & \left[g_{9/2}^\pi \times d_{5/2}^\nu \right]_{J_0}
 \end{array}$$

$$\mathcal{S}_{J_0} = \left| \sum_c c_J U\left(\frac{5}{2} \frac{5}{2} \frac{9}{2} \frac{9}{2} : J J_0\right) \right|^2.$$

In the Nb^{93} ground-state wave function it is not assumed that the two neutrons are coupled to spin zero; as we shall see, the ratios of spectroscopic factors are acutely sensitive to $J \neq 0$ admixtures.

The simple model just described thus predicts six $\ell=2$ transitions to low-lying states of Nb^{92} with relative spectroscopic factors which depend on the mixing coefficients c_J . Six transitions are indeed observed (Table II) to levels in Nb^{92} below 500 keV; the next level strongly

TABLE II. Relative cross sections and spectroscopic factors for $\text{Nb}^{93} \rightarrow \text{Nb}^{92}$.

Excitation energy (MeV)	Relative $d\sigma/d\omega$		Measured relative $\mathcal{S}$
	(p, d)	(d, t)	
0.0	0.36	0.43	0.40
0.135	0.21	0.20	0.21
0.286	0.12	0.10	0.11
0.357	0.13	0.11	0.12
0.479	0.18	0.08	0.08
0.500		0.08	0.08

excited in the pickup reactions is at about 3 MeV. The relative measured intensities in the (p, d) and (d, t) studies are in very close agreement. Since the transitions in question span an energy region of only 500 keV, it is reasonable to equate ratios of cross sections to ratios of spectroscopic factors. The ratios in Table II have been arbitrarily normalized to $\sum \mathcal{S} = 1$.

²⁶R. F. Sweet, K. H. Bhatt, and J. B. Ball, Phys. Letters 8, 131 (1964).

²⁷J. B. Sweet, J. B. Ball, and C. D. Goodman (to be published).

²⁸R. K. Sheline, C. Watson, and E. W. Hamburger, Phys. Letters 8, 121 (1964).

It remains now to determine the mixing coefficients c_J by a simple shell-model calculation for the Nb^{93} ground state and to deduce the relative spectroscopic factors. For this shell-model calculation we need to know the nn interaction matrix elements

$$\langle (d_{5/2}^2)_J^\nu | V | (d_{5/2}^2)_J^\nu \rangle \quad (J = 0, 2, 4),$$

which we obtain directly from the observed²⁹ spectrum of Zr^{92} and the np matrix elements

$$\langle [d_{5/2}^\nu \times g_{9/2}^\pi]_{J_0} | V | [d_{5/2}^\nu \times g_{9/2}^\pi]_{J_0} \rangle \quad (J_0 = 2-7).$$

The np matrix elements could be read directly from the observed Nb^{92} spectrum if only the spin sequence were known. One procedure is to try all the possible spin orderings (all but a few can be eliminated on rather general theoretical grounds) until we find the one that fits best. Another possibility, adopted by Sweet, Bhatt, and Ball, is to assume that the spin sequence predicted by Kim³⁰ in a recent shell-model study of Nb^{92} is correct. The excellent agreement so obtained for the intensity ratios then confirms the assumed spin assignments.

All the necessary two-body-interaction matrix elements can now be taken directly from experiment and the 3×3 Nb^{93} matrix diagonalized. The mixing coefficients and ratios of spectroscopic factors so obtained are shown in Table III. Agreement with the measured spectroscopic factors is in essence perfect, certainly within the uncertainties in the "experimental"

²⁹M. E. Bunker, B. J. Dropesky, J. D. Knight, and J. W. Starnes, Phys. Rev. 127, 844 (1962).

³⁰Y. E. Kim, Phys. Rev. 131, 1712 (1963).

TABLE III. Mixing coefficients for Nb^{93} and $\text{Nb}^{93} \rightarrow \text{Nb}^{92}$ spectroscopic factors. The Nb^{93} ground-state parameters used in the calculations were $c_0 = 0.949$, $c_2 = 0.297$, and $c_4 = 0.106$.

State	J_0	$\mathcal{S}$	
		Exp.	Theor.
0.000	7	0.40	0.40
0.135	2	0.21	0.22
0.286	5	0.11	0.11
0.357	3	0.12	0.11
0.479	4	0.08	0.09
0.500	6	0.08	0.08

quantities in every case. The success of this simple model seems all the more surprising when it is recalled that shell-model studies³¹ of the low-lying states of Nb^{91} indicate that the proton configuration $(p_{1/2})^2_0 g_{9/2}$ mixes quite strongly with $(g_{9/2})^3_{9/2}$. Perhaps the point here is that the above arguments demand no more of the proton state than that it be the same for the pertinent states of Nb^{92} as in the Nb^{92} ground state and that it have spin $\frac{9}{2}^+$. At any rate, from the intensity ratios of six nucleon pickup transitions Sweet, Bhatt, and Ball were able to extract six spin assignments, six two-body energies, and the mixing coefficients in the Nb^{93} ground-state wave function.

As an aesthetic bonus, we obtain here a striking illustration of the extent to which a small admixture of $(d_{5/2})^2_{J \neq 0}$ can influence the ratios of spectroscopic factors. If the Nb^{93} ground-state function were pure $(d_{5/2})^2_0 \nu \times g_{9/2}^\pi$, then we would have

³¹S. Cohen, R. D. Lawson, M. H. Macfarlane, and M. Soga, Phys. Letters 10, 195 (1964).

TABLE IV. Influence of a 10% admixture of $[(d_{5/2}^2)_J^\nu \times g_{9/2}^\pi]_{9/2}$ with $J \neq 0$ on spectroscopic factors.

J_0	Relative $\mathcal{S}_{J_0}$ with $c_0 = 1$	Relative $\mathcal{S}_{J_0}$ with best-fit c_J
2	0.08	0.22
3	0.12	0.11
4	0.15	0.09
5	0.18	0.11
6	0.22	0.08
7	0.25	0.40

$$\mathcal{S}_{J_0} \propto (2J_0 + 1) .$$

In Table IV we see how greatly the 10% admixture of $J_0 \neq 0$ has distorted the $(2J_0 + 1)$ dependence.

6. Stripping Reactions and Rotational Nuclei

Until about a year ago, nuclear direct-reaction studies had scarcely touched the rare-earth and heavy deformed nuclei. This situation is changing very rapidly. Several interesting reports³²⁻³⁹ of

³²A. Isoya, Phys. Rev. 130, 234 (1963).

³³G. L. Struble, W. N. Shelton, and R. K. Sheline, Phys. Rev. Letters 10, 58 (1963).

³⁴R. K. Sheline, W. N. Shelton, and R. A. Kenefick, Phys. Letters 5, 129 (1963).

inelastic-scattering and (d,p) stripping experiments on rotational nuclei have already appeared. It is clear that this will be one of the most active fields of study with the new Emperor tandems; the whole subject is still in its infancy.

The theoretical treatment of single-nucleon stripping or pickup on a deformed nucleus is in principle very simple. The single-particle states which are appropriate here are the Nilsson states,⁴⁰ eigenfunctions of a deformed-oscillator Hamiltonian with spin-orbit coupling. These Nilsson functions are linear superpositions of spherical single-particle functions. The single-nucleon spectroscopic factors directly measure the amplitudes of the various spherical constituents of the deformed single-particle states.

Consider then the transition

$$\left\{ \mathcal{D}_{M_0 K_0}^{I_0} \chi_{K_0} \right\} \xrightarrow{\alpha \Omega} \left\{ \mathcal{D}_{M_1 K_1}^{I_1} \chi_{K_1} \right\}$$

in which the target intrinsic state χ_{K_0} acquires a single nucleon in the Nilsson orbital α with projection Ω . We talk in terms of a stripping reaction, but as usual pickup is simply the same thing backwards. The

³⁵ R. A. Kenefick and R. K. Sheline, Phys. Rev. 133, B25 (1964).

³⁶ J. R. Erskine and W. W. Buechner, Phys. Rev. 133, B370 (1964).

³⁷ W. N. Shelton and R. K. Sheline, Phys. Rev. 133, B624 (1964).

³⁸ J. R. Erskine, in I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 86.

³⁹ M. N. Vergnes and R. K. Sheline, Phys. Rev. 132, 1736 (1963).

⁴⁰ S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, No. 16 (1955).

spectroscopic factor for transfer of given orbital angular momentum ℓ and spin j is then

$$\mathcal{S}_{j\ell} = \rho^2 \left(\frac{2J_0 + 1}{2J_1 + 1} \right) \left(C_{K_0 \Omega K_1}^{J_0 j J_1} a_{n\ell j}(\Omega) \right)^2 \mathcal{S}_{\text{intr}}(\Omega) \delta(\Omega, K_1 - K_0) .$$

Here

$$\rho^2 = 1 + \delta(K_0 0) + \delta(K_1 0) ,$$

the intrinsic spectroscopic factor is

$$\mathcal{S}_{\text{intr}}(\Omega) = \langle \chi_K | a_{\Omega}^{\dagger} | \chi_{K_0} \rangle^2$$

and $a_{n\ell j}$ is the appropriate Nilsson coefficient. Recall that different values of ℓ_j contribute incoherently to the cross sections; band mixing is taken into account in the spectroscopic factors in exactly the same way as configuration mixing is treated in the spherical shell model.

Theoretical analysis of stripping on deformed nuclei thus falls into two stages. The first concerns those predictions that follow directly from the rotational form of the wave function and do not depend on the details of the intrinsic states; here we think of branching ratios and sum rules for states of a rotational band.⁴¹ These general considerations are unlikely to be as fruitful in the analysis of stripping widths as they have been, for example, in the study of E2 rates. The purity of the rotational predictions is usually marred either by the possible contribution of more than one ℓ value to a given transition or, when this is prevented from happening by angular momentum selection rules, by the fact that the various states of a band are populated by transitions with different ℓ values. Clearly interest focusses on the second stage of the analysis—that which concerns the particle structure of the participating states. This of course is always

⁴¹ G. R. Satchler, Ann. Phys. (N.Y.) 3, 257 (1958).

the main point of a stripping study and the reason why such studies of rotational nuclei should be of great value.

What information has already been gained from (d,p) experiments on deformed nuclei? Firstly, of course, swarms of new energy levels have been found; these can often be parceled into rotational bands purely on the basis of the energy systematics. This latter practice is a dangerous one if unaccompanied by detailed scrutiny of the transition strengths in the (d,p) reaction. Here I would like to concentrate on the spectroscopic factors; the type of analysis possible at present depends on whether the residual nucleus is odd-mass, even-even, or odd-odd.

The situation is simplest when the target nucleus is even-even. Later in this session Erskine^{3 8} will describe his analysis of the (d,p) reactions on W^{182} , W^{184} , and W^{186} . Many features of the observed level structure and spectroscopic factors can be described quite adequately by an extreme single-particle model for the intrinsic structure, provided that band mixing in the residual nucleus is correctly taken into account. The details of the intrinsic structure of the core seem to manifest themselves primarily in simple multiplicative factors modulating all the stripping widths for a given target in similar fashion. We can of course expect this simple situation to break down at excitation energies above about 1.5 MeV where core breakup should be important.

Significantly, an attempt^{3 6} to apply similarly simple ideas to the reaction $Ta^{181}(d,p)Ta^{182}$ was much less successful. Odd-odd nuclei evidently demand more sophisticated treatment of their intrinsic structure. Part of the trouble may stem from the fact that proton single-particle excitations are just as important as neutron excitations, and about these the (d,p) reaction has nothing useful to say.

The work of Sheline and his associates at Florida State University has revealed intriguing systematic features of stripping and inelastic scattering to final states in even-even nuclei^{3 4} and of the

spectroscopy of the transition region between vibrational and deformed nuclei.³⁵ It is noticed in particular that no strong stripping transitions are observed to levels below about 2 MeV in even-even deformed nuclei—about the energy at which we expect the lowest two-quasiparticle excited intrinsic states.

The study of deformed nuclei by stripping reactions is clearly very promising. Most of the work remains to be done. Experimentally, proton stripping studies, for example by (He^3, d) , are badly needed; pickup reactions should be rewarding when high-resolution experiments with sufficiently energetic projectiles are possible. Theoretically, it is clear that the extensively-studied superfluid model for the intrinsic structure of deformed nuclei will be important in interpreting spectroscopic factors. Very simple expressions are thus obtained for $\mathcal{S}_{\text{intr}}$, namely

$$\mathcal{S}_{\text{intr}}(\Omega) = V_{\Omega}^2(1)$$

when the lighter nucleus (nucleus 0) is odd-mass; and

$$\mathcal{S}_{\text{intr}}(\Omega) = U_{\Omega}^2(0)$$

when the mass of the lighter nucleus is even. Preliminary discussions along these general lines have already been reported.^{32,38,39}

In studies of rotational nuclei the artificial separation of reaction-mechanism and nuclear-model analysis of spectroscopic factors has disappeared. The angular distributions are no longer adequate "l meters" so that the nuclear model is needed to identify the contributing angular-momentum transfers. Conversely, crude semiempirical treatments of the cross sections are valueless so that quantitative discussion of the spectroscopic factors is impossible without adequate treatment of the reaction mechanism.

7. The Pairing Model and Ground-State Occupation Numbers

The various shell-model approximations⁴² for identical-nucleon systems based on the BCS theory of superconductivity have often been used to discuss the results of stripping and pickup experiments. Formal expressions for the spectroscopic factors have been given by Yoshida.⁴³ The underlying notions are of course the same as in any shell-model analysis of spectroscopic factors. Most practical applications of the formalism have not been so much concerned with the fine structure as with the total strengths of the single-particle and single-hole resonances. Extensive work of this kind has been done by Cohen and Price⁴⁴ who use (d,p) and (d,t) reactions to measure the average occupation numbers of the single-particle orbits in the ground states of various isotopes of Sn. In order to use the approximations under consideration to discuss the fine structure of single-particle and single-hole resonances with reasonable accuracy, it will probably be necessary to include 3- and 4-quasiparticle contributions to their excited-state wave functions.

8. Coherence and Conclusion

We have already remarked that single-particle orbits which differ in either of their angular-momentum quantum numbers ℓ and j contribute incoherently to the differential cross section of single-nucleon

⁴² M. Baranger, Phys. Rev. 120, 657 (1960).

⁴³ S. Yoshida, Phys. Rev. 123, 2122 (1961).

⁴⁴ B. L. Cohen and R. E. Price, Phys. Rev. 121, 1441 (1961).

stripping and pickup reactions. On the other hand, coherent contributions are obtained from orbits with the same (ℓ, j) but different principal quantum numbers n .

Such an effect may have something to do with the unexpected strength⁴⁵ of the $\ell=0$ pickup in the reaction $O^{16}(He^3, \alpha)O^{15}$ to the $\frac{1}{2}^+$ state at an excitation energy of 5.17 MeV in O^{15} . This s-wave transition must involve either an admixture of the 1s-hole configuration $(1s_{1/2})^{-1}$ in the pertinent O^{15} wave function or of the core-excited configuration $(1p_{1/2})^{-2}_0(2s_{1/2})^2_0$ in the O^{16} ground state. Order-of-magnitude estimates indicate that neither admixture can plausibly be made large enough to account by itself for the full strength of the transition. The most probable explanation is of course that we are dealing with a coherent superposition of roughly comparable 1s and 2s spectroscopic amplitudes.

Quantitative analysis of this and similar transitions in which the orbits (n, ℓ) and $(n+1, \ell)$ contribute coherently is at present impossible. For one thing, such an analysis would require a more accurate treatment of the process of single-nucleon capture than current DWBA calculations provide.⁴⁶ It is clear that the coherent effect in question makes it very hard to estimate weak core excitation near major closed shells.

Spectroscopic analysis of reactions in which coherent effects can arise necessarily makes severe demands of the underlying reaction theory. It is in this respect that single-nucleon stripping and pickup reactions differ most radically from reactions in which two, three, or

⁴⁵S. Hinds and R. Middleton, Proc. Phys. Soc. (London) 74, 762 (1959).

⁴⁶Such matters are discussed by N. Austern in a recent preprint entitled "Configuration Mixing Effects in Stripping" (September 1964).

more nucleons are transferred.⁴⁷ The range of phenomena revealed by a single-nucleon reaction is comparatively restricted. On the other hand, within its narrow area of usefulness, the single-nucleon reaction is a very penetrating means of studying nuclear spectroscopy. It yields information about the single-particle aspect of nuclear structure and in so doing relies very little on the details of the underlying reaction theory.

Helpful conversations with B. F. Bayman, D. Kurath, and R. D. Lawson are gratefully acknowledged.

* * *

J. CERNY, Lawrence Radiation Laboratory, Berkeley: In our $C^{12}(p,d)$ work with 40-MeV protons, we see both of the states that were not observed at the higher excitation energies. I would guess that they are populated at least a third as much as the other excited states. The population agrees with the $(p,2p)$ data in your table.

S. MESHKOV, U. S. National Bureau of Standards: There is a place where you can really examine rotational structure from many points of view, namely, in the well-known deformed region in the $2s1d$ shell in which there are many nice experiments. There you can use SU_3 to describe the rotational level schemes and can also compare with the results obtained by using Nilsson functions. With SU_3 , the puzzle about why the $d_{3/2}$ comes into mass 21 is taken care of. If you calculate the level structure of mass 21 in lowest order of SU_3 , you can reproduce the level scheme because you have incorporated the sd mixing into your basis functions. This region is a fertile one because you have many cross checks. In the region in which you use the Nilsson functions, you have no such backlog.

⁴⁷B. F. Bayman, these Proceedings, p. 335.

MACFARLANE: I did not mention the rotational deformed nuclei in the center of the $d_{5/2}$ shell because in certain respects they obviously are not going to be at all characteristic of what would happen in heavy deformed nuclei. Furthermore, in terms of band mixing, it will be clear that the effects are greater. Also, I think most of the data refer to Mg^{25} , which is a special case because there is a K difference of 2 or 3. Don't you agree?

MESHKOV: No. The point is that even in Mg^{25} or Al^{25} , by using the $(\lambda, \mu) = (6, 6)$ representation together with the $(9, 3)$ you can work out the structure properly with what is in essence a shell-model calculation.

F. S. LEVIN, Brookhaven National Laboratory: One may be able to understand the whole $B^{10}(d, p)B^{11}$ reaction in terms of a model in which the target nucleus is excited by the outgoing proton. So far, this is somewhat qualitative; but calculations have been carried out to test this idea. I think one can have some qualitative understanding of the reaction, including the energy dependence.

M. A. NAGARAJAN, Case Institute of Technology: I have one question about the pickup reaction on C^{12} . In the last column of your table, you mentioned the ratios of cross sections. In that column I noted three numbers: one corresponding to the ground state of B^{11} , one to the $\frac{1}{2}^-$ state, and the third one to the $\frac{3}{2}^-$ level. How do you theoretically predict the $\frac{3}{2}^-$ level?

MACFARLANE: An intermediate-coupling shell-model calculation simply predicts something like seven or eight $\frac{3}{2}^-$ states. In pure j - j coupling, the ground state will have all the pickup strength from C^{12} ; but reasonable values of intermediate-coupling parameters will give some of the strength to higher states. In fact, the relative strength for the latter is also in fair agreement with Kurath's calculations.

A HIGH-RESOLUTION STUDY OF (d,p) REACTIONS ON TARGETS OF W^{182} , W^{184} , AND W^{186} ^{†*}

John R. Erskine

Argonne National Laboratory, Argonne, Illinois

Abstract of Report in "Vol. I. Contributed Papers"[†]

The $W^{182}(d,p)W^{183}$, $W^{184}(d,p)W^{185}$, and $W^{186}(d,p)W^{187}$ reactions have been studied with a magnetic spectrograph at a bombarding energy of 12 MeV. The ground-state Q values for the three reactions were found to be 3.967 ± 0.005 , 3.524 ± 0.005 , and 3.236 ± 0.005 MeV, respectively. The angular-distribution data distinguished between $\ell=1$ and $\ell=3$ transitions for most of the lower excited states. Computer calculations of the theoretical differential cross sections and excitation energies were made with a code which uses the single-particle rotational model including band mixing. All levels observed in W^{183} , W^{185} , and W^{187} up to an excitation energy of 0.5 MeV could be interpreted as arising from the $[510 \frac{1}{2}^-]$, $[512 \frac{3}{2}^-]$, and $[503 \frac{7}{2}^-]$ intrinsic states. Good agreement between the observed and calculated cross sections is obtained for W^{183} by the use of Kerman's parameters. Similar good fits are obtained for W^{185} and W^{187} .

The final report, which is nearly ready for publication, will cover the material presented here and in addition will compare the energies of the intrinsic states with certain theoretical predictions. Nilsson's theory for single-particle states in deformed potentials is used in the

[†]Work performed under the auspices of the U. S. Atomic Energy Commission.

*Contributed paper.

[†]J. R. Erskine, in Argonne National Laboratory Report ANL-6848, March 1964, p. 86. The text that follows the abstract is in amplification of this earlier report.

theoretical calculations of intrinsic energies, and refinements to take account of pairing correlations are included.

* * *

M. H. MACFARLANE, Argonne National Laboratory: Can you study pickup reactions with your present techniques?

ERSKINE: Yes, I actually have some data on pickup reactions. Such work is certainly more difficult, but it can be done.

SPIN MEASUREMENTS AND STRIPPING REACTIONS^{*}

John P. Schiffer

Argonne National Laboratory, Argonne, Illinois

and

Palmer Physical Laboratory, Princeton University
Princeton, New JerseyI. Introduction

The general success of the Butler theory of deuteron stripping and pickup reactions has provided much information on the energy levels of nuclei throughout the periodic table. The significance of the l values extracted from (d,p) and other similar reactions has been considerable. Such measurements provide much of the experimental foundation on which the theory of nuclear structure rests today. While the measurement of l values was the purpose of most of the early (d,p) experiments, it was realized from the beginning that additional information could also be extracted from the data; in particular, the absolute cross section was proportional to a reduced width or, as it has become more fashionable to call it, a "spectroscopic factor."

Since stripping or pickup reactions have in the past only yielded information on the orbital angular momentum of the captured particle, the identification of levels could still involve ambiguities because of lack of knowledge of their total angular momenta. For spin-zero targets and the single-nucleon transfer reaction, to which I shall largely confine myself for the sake of simplicity, two spin states can appear with a given

* Work performed under the auspices of the U. S. Atomic Energy Commission.

orbital angular momentum of $J = \ell \pm \frac{1}{2}$. It is the purpose of my talk to discuss various experimental methods whereby this ambiguity can be removed.

II. Measurements of Gamma-Ray Angular Correlations

The most general technique is to study the gamma rays emitted in the de-excitation of a state. The angular distribution of such gamma rays will in general be given by

$$W(\theta) = \sum_{L=0}^{L_{\max}} a_{2L} P_{2L}(\cos \theta),$$

where $L_{\max} = J - \frac{1}{2}$ for J half integral. It is clear that if a given value of ℓ is established from a direct reaction, the presence of a Legendre polynomial of order 2ℓ will immediately prove that $J = \ell + \frac{1}{2}$. However, the absence of such a term does not conclusively establish that $J = \ell - \frac{1}{2}$. In the latter case, additional information is needed on the mixture of multipolarities in the emitted gamma rays. This can sometimes be derived from other measurements such as the internal-conversion coefficients or, for the case of ground-state transitions, from the combination of the half-life of the state and its Coulomb-excitation cross section. (This latter information is useful only for sorting out E2-M1 admixtures.)

This use of the maximum complexity in the angular correlation of gamma rays is, of course, perfectly general and independent of the details of the reaction mechanism. In fact, it is irrelevant whether the state is prepared by means of a direct reaction or some other process such as beta decay. The method becomes increasingly difficult to apply as the spin of the state becomes higher, because it is experimentally very

difficult to determine the presence of high Legendre polynomials. An additional difficulty with higher spins is that the maximum complexity of the correlation may be limited by the multipolarity of the transition rather than by the spin of the decaying state ($L_{\text{max}} = K$, where K is the multipolarity). In such cases, the knowledge of multipole admixtures may help to remove the ambiguity. I do not want to go into any further details on this since I would have to cover the whole field of beta-gamma and gamma-gamma correlations.

I would like to go on to mention two types of experiments that have helped to clear up some of the ambiguities for final states in stripping reactions which show $\ell = 1$ orbital-angular-momentum transfer and where $J = \frac{3}{2}$ or $\frac{1}{2}$. This is the simplest case for angular correlations because $J = \frac{1}{2}$ requires isotropy and $J = \frac{3}{2}$ does not. The (d, py) reaction has been observed on Ca^{40} and Fe^{54} in an attempt to identify the J of three states in Ca^{41} and five in Fe^{55} , all of which were identified as giving rise to $\ell_n = 1$ transitions in the stripping reaction.¹ The results of these measurements are shown in Table I. I want to note that the two states showing isotropic correlations are not necessarily $J = \frac{1}{2}$; on the other hand, the chances of such accidental isotropy for $J = \frac{3}{2}$ states are rather small. One might have been suspicious had too many $J = \frac{1}{2}$ states turned up, but this was not the case here. The accumulation of such data is a slow and painful process; many days of measuring are required to obtain even this much information reliably. If one were to try to establish the presence or absence of higher order Legendre polynomials, considerably more information would be needed. It may be possible to do experiments of this type to determine the coefficient of the polynomial

¹ L. L. Lee, Jr., J. P. Schiffer, and D. S. Gemmell, Phys. Rev. Letters 10, 496 (1963); D. S. Gemmell, L. L. Lee, Jr., A. Marinov, and J. P. Schiffer, Bull. Am. Phys. Soc. 8, 523 (1963).

TABLE I. Results of (d, p γ) correlation measurements.

Nuclide	Excitation (MeV)	Anisotropy	Spin
Ca ⁴¹	1.95	0.17 $\pm$ 0.04 ^a	3/2
Ca ⁴¹	2.47	0.75 $\pm$ 0.10	3/2
Ca ⁴¹	3.95	0.06 $\pm$ 0.07	(1/2)
Fe ⁵⁵	0.41	0.017 $\pm$ 0.02	(1/2)
Fe ⁵⁵	2.49	0.29 $\pm$ 0.09	3/2
Fe ⁵⁵	3.56	0.35 $\pm$ 0.06	3/2
Fe ⁵⁵	3.80	0.31 $\pm$ 0.04	(3/2)

^aR. T. Taylor, Phys. Rev. 113, 1293 (1959).

of next higher order so that $J=\frac{3}{2}$ states could be distinguished from $J=\frac{5}{2}$ in $\ell_n=2$ transitions. It seems doubtful, however, that such measurements will be profitable for still higher ℓ values, although the recent developments in high-resolution low-background gamma-ray detectors may change this situation drastically in the future.

The other type of experiment, which has made use of gamma-ray correlations to measure the spins of states seen in (d, p) reactions, is the study of the correlations of gamma rays following thermal-neutron capture. It has been known for many years that the compound states formed in thermal-neutron capture decay preferentially to those states that are strong in $\ell_n=1$ (d, p) reactions. This has been explained in terms of the single-particle E1 transition operator selecting those transitions that relate the s-wave component of the captured neutron in the compound state to the p-wave component of the neutron in the state to which it decays. By studying the correlation between this initial gamma ray and the subsequent

ones, the spin of the intermediate state can be determined if it is known from the (d,p) reaction that $\ell_n = 1$ and therefore $J = \frac{3}{2}$ or $\frac{1}{2}$. Such measurements have been carried out for perhaps a dozen nuclei around $A = 50$ by Bartholomew et al. at Chalk River² and by Coté et al. at Argonne.³ Spins of a number of states in this region have been identified. This method is unfortunately also limited to $\ell_n = 1$ states and furthermore to nuclei that are otherwise favorable for studying neutron capture.

To summarize, gamma-ray correlation measurements are easiest to apply in deciding ambiguities between relatively low spins; this is, however, the technique that requires the fewest assumptions and in which one can have the most confidence.

III. Polarization Measurements

A number of simple qualitative statements have been made about the dependence of the polarization in a (d,p) experiment on the J of the final state. Perhaps the most general of these is that if all other contributing factors in the reaction are the same, two states with the same ℓ_n and opposite J will give rise to protons with polarizations that are equal in magnitude but opposite in sign. A number of experiments have been performed to measure the polarization of protons from (d,p) reactions. I think that those who have done them would agree with the statement that these are difficult and time-consuming measurements. They can usually

²G. A. Bartholomew and G. A. Andrews, Bull. Am. Phys. Soc. 8, 367 (1963), and private communication.

³R. E. Coté, H. E. Jackson, Jr., L. L. Lee, Jr., and J. P. Schiffer, Bull. Am. Phys. Soc. 7, 551 (1962), and to be published.

Fig. 1. Polarization measurements from Ref. 4 for (d,p) reactions initiated by 15-MeV deuterons and populating final states with the same ℓ but different J .

be performed only for states that give rise to a high yield of protons, and the time involved in the measurements is very great. I have picked two cases in which the polarization for two different J states with the same ℓ_n was measured in experiments at Pittsburgh.⁴ It is clear from Fig. 1 that the polarization does not depend on J in the simple way expected.

Many other measurements qualitatively similar to these also indicate that polarization cannot yet be used to measure spins of unknown states, although if two states in the same nucleus show similar polarizations at all angles one might regard this as a strong hint that their spins are the same, and vice versa. It is possible that the nuclei studied

⁴A. Isoya, S. Micheletti, and L. H. Reber, Phys. Rev. **128**, 806 (1962); L. H. Reber and J. X. Saladin (private communication).

so far are too light, and that the polarization will show more consistency for heavier nuclei. This remains to be seen. It would seem, however, that a lot of difficult experiments need to be done before the situation is clarified and before one can use polarization as a spin-measuring tool. Even then, one would shudder at the amount of time it would take to measure the spins of any appreciable number of levels by this technique. Of course, the situation may become simplified overnight should someone succeed in getting a reasonably healthy beam of polarized deuterons. With such a beam, these measurements could be performed quickly and reliably and this technique could become useful. At the moment though, I think it is fair to say that polarization in (d, p) reactions is not well enough understood experimentally or theoretically to be of much use in measuring spins.

IV. Use of Spectroscopic Factors for Spin Measurements

A simple technique has been used by several investigators to determine the spins of states in nuclei that are accessible by both a neutron-stripping and a neutron-pickup reaction. Such will be the case for a nucleus of atomic weight A for which the neighboring nuclei of weights $(A + n)$ and $(A - n)$ are both stable. If the stripping and pickup reactions leading to the same final states have both been measured, then one can compare the relative cross sections of those states that (according to the shell model) are partially filled. If, for instance, the nucleus is in a region where the $2p_{3/2}$ state is filled with four neutrons but the $2p_{1/2}$ state is not, then the (d, p) reaction will populate a $\frac{3}{2}^-$ state in the final nucleus only very weakly, or not at all, while a $\frac{1}{2}^-$ state will be populated strongly. In the pickup reaction, on the other hand, the $\frac{3}{2}$ states will show up strongly,

because all the neutrons are there to be picked up, and $\frac{1}{2}$ states will not appear at all because there are no $p_{1/2}$ neutrons. Therefore, a comparison of relative intensities for states with the same l_n in the two reactions can yield assignments of J.

The only difficulty with this method is that it relies heavily on the assumption, derived from the simplest shell model, that nucleons fill the single-particle states in their proper order and that there is no mixing between these states. In practice, some assignments based on such arguments have turned out to be wrong; many others have not yet been tested. Perhaps this method will be more reliable for states with higher angular momentum, for which the spin-orbit splitting is large enough that sequential filling is likely to be a good approximation. However, in a real nucleus there are many other factors which may change the relative populations of particular states in the two reactions. In the absence of other information, these considerations may allow one to make a reasonable guess; but for a definitive assignment, they cannot be relied on.

V. The J Dependence of Angular Distributions

Finally, I would like to present some recent experimental results which lead one to believe that the spin of a final state can be measured by a much simpler method than the complicated techniques I have described so far.

After making the rather messy (d, p γ) measurements I described earlier, we noticed that the two states with $J = \frac{1}{2}$ in Ca⁴¹ and Fe⁵⁵ both had sharp minima in the backward angular distributions. This is shown in Figs. 2 and 3. L. L. Lee and I had actually measured these angular distributions more than a year ago; but not knowing the J values

associated with the states, we did not assign any significance to this variation in the backward structure—especially since DWBA calculations by Satchler and coworkers for these measurements indicated that the backward angles were very sensitive to the choice of the distorting parameters. With the J values measured, we decided that this looked interesting enough to explore further. We accordingly picked a number of nuclei whose spins were known from gamma-ray correlation experiments, mostly from the $(n, \gamma\gamma)$ experiments of Refs. 2 and 3. The results of this survey are shown in Figs. 4 and 5. It is clear from these results that all the $J=\frac{1}{2}$ states exhibit a sharp minimum at an angle greater than 90° and that none of the $J=\frac{3}{2}$ states show this. The data on Cr^{53} are those of Andrews *et al.*,⁵ who noted independently that this may be a J -dependent effect. The spins of the two states in Ca^{49} are not known definitely but are inferred from the simple single-particle-like structure of the low-lying states in this nucleus.

The minimum in the $J=\frac{1}{2}$ states does not occur at the same angle in each case but, if one discounts the two Ca isotopes, it remains between 130° and 140° from Ti through Ni. In all these cases, then, we observed a consistent pattern; there is no exception to the rule. I should

Fig. 2. Angular distributions for the various $\ell_n=1$ states in the $\text{Ca}^{40}(\text{d}, \text{p})\text{Ca}^{41}$ reaction at 12 MeV.

⁵P. T. Andrews, R. W. Clift, L. L. Green, and J. F. Sharpey-Schafer (to be published).

Fig. 3. Angular distributions for the various $l_n=1$ states in the $Fe^{54}(d,p)Fe^{55}$ reaction at 10 MeV.

Fig. 4. Angular distributions in the backward quadrant for (d,p) reactions populating final states with $\ell_n = 1$ and known J in Ca, Ti, Cr, and Fe nuclei. The J for the Ca^{49} states is not known but was inferred from the simple structure of that nucleus.

Fig. 5. Angular distributions in the backward quadrant for $\text{Ni}(d,p)$ reactions populating final states with $\ell_n = 1$ and known J .

Fig. 6. Angular distributions in the backward quadrant for the (d,p) reaction on Ca^{42} at 10 MeV. The anomalous angular distribution for the known $J=\frac{3}{2}^-$ state is discussed in the text.

point out, though, that since this first measurement was done we have found one exception in Ca^{43} which is shown in Fig. 6. Here the first excited state is known to have spin $\frac{3}{2}$ and its angular distribution shows a minimum at 120° . However, one should note that this state appears very weakly in stripping; its cross-section scale is plotted a factor of twenty lower than the strong $l_n=1$ state at 2.05 MeV. The reason for the low cross section is that the state seems to consist of the $(f_{7/2})^3_{3/2}$ configuration. Since the stripping cross section is so low, the compound-nucleus part of the reaction could dominate at back angles and it is not too disturbing that this state violates our rule.

Further evidence that this effect is not due to chance is that it persists as a function of energy. This is shown in Fig. 7 for Ca^{40} and in Fig. 8 for Fe^{54} . It can be seen that while the effect diminishes at the lower energies it is qualitatively still unmistakable.

We have therefore pretty well established the reality of this effect in stripping for $l_n=1$ orbital-angular-momentum transfer in the region of $40 < A < 65$, and for spin-zero targets. In this region of the periodic table, we also measured the backward angular distributions for a number of states that had previously been shown to be $l_n=1$ but for which J was not known. Except for a few states, all of which are weak in stripping, the backward angular distributions fall into one category or the other. The results are given in Table II. One can regard these assignments with a fair degree of confidence.

Fig. 7. Angular distributions for the $\text{Ca}^{40}(\text{d},\text{p})\text{Ca}^{41}$ reaction at various energies. These curves show the persistence of the J-dependent difference in the angular distribution at backward angles.

Fig. 8. Angular distributions for the $\text{Fe}^{54}(\text{d},\text{p})\text{Fe}^{55}$ reaction at various energies. The curves show the persistence of the J-dependent difference in the angular distribution at backward angles.

TABLE II. Spin assignments based on (d,p) angular distributions measured at backward angles. The transitions were known to be $l_n = 1$ but the J values of the final states were unknown.

Nuclide	Excitation energy (MeV)	Spin	Nuclide	Excitation energy (MeV)	Spin
Ca^{41}	1.95	$3/2^a$	Fe^{55}	0	$3/2$
	2.47	$3/2^a$		0.41	$1/2^a$
	3.95	$1/2^a$		2.49	$3/2^a$
Ca^{43}	2.05	$3/2$		3.56	$3/2^a$
			3.80	$3/2^a$	
Ca^{45}	1.43	$1/2$	Ni^{59}	0	$3/2^a$
	1.90	$3/2$		0.47	$1/2^a$
Ca^{49}	0	$3/2$		0.89	$3/2^a$
	2.03	$1/2$	1.32	$1/2^a$	
Ti^{47}	1.56 1.80 2.58 3.31 3.95	$3/2$ $1/2$ $1/2$ $3/2$ $3/2$	Ni^{61}	0	$3/2$
				0.29	$1/2^a$
			Ni^{63}	0	$1/2$
				0.16	$3/2^a$
				0.53	$3/2^a$
Ti^{49}	1.38 1.72 3.17 3.26	$3/2^a$ $1/2^a$ $1/2$ $3/2$	Ni^{65}	1.01	$1/2^a$
				0.063	$1/2$
				0.325	$3/2$
				0.703	$3/2$
Ti^{51}	0 1.16 2.13	$3/2$ $1/2$ $3/2$	1.42	(1/2)	

^a Confirms assignment based on earlier angular-correlation work.

The next question that occurs is: How general is this J dependence? Will it show up for other ℓ values, will it show up for targets with nonzero spin, and will it persist through other regions of the periodic table?

Very recently we have obtained some partial and preliminary experimental answers to all of these questions. Figure 9 shows some measurements of the (p,d) angular distributions measured in collaboration with Kashy at the Princeton cyclotron. The two final states are the same as in some of the earlier examples; but since this is a (p,d) reaction, the deuteron is interacting with the nucleus with spin and the proton is interacting with the spin-zero one. Because of detailed balancing, this demonstrates that a (d,p) reaction on an odd-spin target would show the same qualitative features as we have seen for spin-zero targets.

Figure 10 shows some angular distributions for $\ell_n=3$ (d,p) transitions which L. L. Lee measured on the Argonne tandem Van de Graaff. For these transitions, J is either known or its value is guessed with considerable confidence. Unfortunately, the (d,p) reaction does not go to two states with J known to be $\frac{5}{2}$ and $\frac{7}{2}$ in the same nucleus. Therefore this comparison is not as strong as the one for $\ell_n=1$. There does seem to be a qualitative difference in these angular distributions, although perhaps it is not as pronounced as for the $\ell_n=1$ states. The (p,d) reaction again can help out here because there are cases in which it does proceed

Fig. 9. Angular distributions in the backward quadrant for the $\text{Fe}^{56}(p,d)$ and $\text{Ni}^{60}(p,d)$ reactions populating final states with $\ell_n = 1$ and known J .

Fig. 10. Angular distributions in the backward quadrant for the (d,p) reaction on various isotopes. The final states have $\ell_n = 3$ and J either known or guessed with high probability.

Fig. 11. Angular distributions in the backward direction for the (p,d) reaction on Fe^{56} and Ni^{60} . The final states have $\ell_n = 3$ and J either known or guessed with high probability.

with appreciable cross section to both $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states in the same nucleus. The results of these measurements from Princeton, shown in Fig. 11, indicate that there is indeed a qualitative difference between the two types of states. There is a general downward slope between 100° and 150° for $\frac{7}{2}^-$ states but not for $\frac{5}{2}^-$ states. More experimental evidence is needed to further substantiate this effect.

Finally, L. L. Lee measured the angular distributions for the (d,p) reaction at 12-MeV bombarding energy for two Zr isotopes. In each case there are two states that are strongly suspected to have $J = \frac{5}{2}$ and $\frac{3}{2}$ and show $\ell_n = 2$ stripping patterns in the forward direction. In the

backward distributions shown in Fig. 12, there again seems to be a clear qualitative difference; but one would again like to see more evidence.

In light nuclei, the angular distributions at reasonably high energies have not been measured systematically, but what evidence there is indicates that this simple effect does not exist. This is not surprising because in light nuclei compound-nucleus formation is relatively more important, and distortions can vary from nucleus to nucleus. It is likewise not clear whether J-dependent effects would appear in nuclei that are strongly deformed, as they are in the s-d shell, but it would be extremely interesting to make such a systematic study. Unfortunately, not much experimental information is available. Most stripping experiments were done at angles forward of 60° (or, at best, 90°) because it was well known that the backward angles do not contain "interesting" information. One relevant experiment is shown in Fig. 13, which shows the angular distributions for the only two strong $\ell_n=1$ states seen in the reaction $\text{Mg}^{24}(\text{d},\text{p})\text{Mg}^{25}$. The upper of the two states is seen to exhibit a sharp minimum in the backward direction. From simple arguments one would have predicted this state to have $J = \frac{1}{2}$.

All the material I have presented here is in essence empirical. One may well ask whether there is any reasonable theoretical explanation of this effect. What we observe is that there seems to be a

Fig. 12. Angular distributions in the backward direction for (d, p) reactions on Zr isotopes. The final states have $\ell_n = 2$ and the value of J is guessed with high probability.

Fig. 13. Angular distributions of the $Mg^{24}(d,p)Mg^{25}$ reaction to the only two strong $\ell_n=1$ states seen. The source of these data is given in the figure.

J-dependent effect in the stripping and pickup reactions, but can the origin of this be understood? It is clear that in the DWBA formalism it is possible to get three spin-independent effects: one due to some sort of spin-dependent potential for the incident deuteron (possibly a spin-orbit force), second, a spin-orbit effect on the captured neutron, and finally, a spin-orbit term in the distorting potential for the protons. Whatever the source of the observed effect, it must explain the persistence of at least the $\frac{1}{2} - \frac{3}{2}$ difference over a relatively large range of nuclei. I should mention that both Satchler and Rost have been able to fit these J-dependent effects qualitatively with reasonable assumptions

about the spin-orbit term in the distorting proton potential. A calculation by Rost for the $\ell_n=1$ states in the $Fe^{56}(p,d)$ reaction is shown in Fig. 14. The experimental angular distributions are a composite of the data in Fig. 9 and measurements of C. A. Whitten, Jr.⁶ The parameters used in the calculation were the "Perey" parameters for the deuteron and proton potentials. These include a spin-orbit term for protons but not for deuterons. The qualitative features of the angular distributions are clearly reproduced. Similar calculations for (d,p) reactions on other nuclei indicate that J dependences could also be expected in heavier nuclei and for higher ℓ values. Whether this will really turn out to be the case

⁶C. A. Whitten, Jr. (private communication).

or not can only be determined by experiments. We seem to have reached a stage at which these rules have to be established experimentally because there are too many poorly-understood parameters in the theory.

I would like to mention at this point that very recently Sherr *et al.*,⁷ using the 28-MeV proton beam of the University of Colorado cyclotron, have found what appears to be a different type of J dependence at forward angles for $\ell_n = 3$ transitions.

It is clear then that in the very near future we may have several tools which will allow the assignment of total angular momenta to states by means of simple stripping and pickup

reactions. I have talked here only of (d,p) and (p,d) reactions, but the same considerations should apply to (d,n) reactions also. It was not so clear whether they would apply to reactions such as (d,t) or (He^3 ,d) because the distortions would no longer be those for single nucleons. However, Fulmer and Daehnck have very recently observed a backward spin dependence in the $\ell_n = 1$ (d,t) transition on Ni isotopes. Their effect is qualitatively similar to the one seen in (d,p) and (p,d) reactions. One may perhaps say very naively that spin-orbit effects on a triton would be

Fig. 14. Angular distributions for the $\text{Fe}^{56}(\text{p},\text{d})\text{Fe}^{55}$ reactions to two final states with $J = \frac{3}{2}$ and $\frac{1}{2}$, and a theoretical curve calculated with "reasonable" optical parameters.

⁷R. Sherr, E. Rost, and M. Rickey (private communication).

similar to those on a proton because the paired neutrons will not show such an effect. All this needs experimental investigation; it is quite clear that the theories are on shaky ground in making predictions.

VI. One Application

It is, of course, obvious that once one knows the angular momenta of the states the restrictions on theoretical models and speculations become much tighter. As an illustration, I show the $\ell_n = 1$ states in Fe^{55} which have perhaps received the greatest concentration of experimental attention of any medium-weight nucleus in recent years. The distribution of $\ell_n = 1$ states is shown in Fig. 15 along with what one would see in a poor-resolution experiment. A few years ago we would have said that the two bumps in the poor-resolution spectrum are the $2p_{3/2}$ and $2p_{1/2}$ single-particle states which are spread out over an MeV or so among the levels of the nucleus. That this is not the case is clear from the upper part of the figure where the $\frac{3}{2}$ states and $\frac{1}{2}$ states are shown to be spread out by much more than 1 MeV. Making reasonable assumptions about the states with weak unmeasured spins, one can compute the average energy or "center of gravity" of each spin state and its width or second moment. These, and Gaussian curves of equivalent width, are shown in Fig. 16 from which it is clear that the spreading of these single-particle levels is in fact quite appreciable. In fact, the spreading is large compared with the spin-orbit splitting. This indicates that interactions other than the spin-orbit one are of at least comparable magnitude.

Fig. 15. Upper curve: Spectroscopic factors for states seen in the $\text{Fe}^{54}(\text{d}, \text{p})\text{Fe}^{55}$ reaction with $\ell_n = 1$. Spin assignments are indicated where known; parentheses are used to show the ones that are guessed. Lower curve: Spectrum that would be observed for these states if one were to look at them with 1-MeV resolution.

Fig. 16. The centers of gravity and widths of the $2p_{3/2}$ and $2p_{1/2}$ strength distributions as calculated from the data given in the top part of Fig. 14. Gaussian curves of equivalent widths are plotted.

* * *

W. C. PARKINSON, University of Michigan: It seems to me that this recently observed effect which you have just described, if confirmed in the tests to which it will be put, will represent one of the most important developments for nuclear spectroscopy since the original Butler theory. In particular, in the higher mass regions of the periodic table where the level density is high, determining the spins of states by the old techniques of measuring polarization and $(d, p\gamma)$ correlations would be an endless job. The question now arises: Is this a real effect? I hope so, but I prefer to adopt a conservative view on this for the time being. I remember in the early days of polarization measurements (1952 and 1953), the $(j \pm \frac{1}{2})$ rule seemed to be valid; but there was one exception. I note today that you say there is also one exception already observed for the j -dependence effect.

N. S. WALL, Massachusetts Institute of Technology: I have two questions. First, do you have a simple physical picture to tell us what is going on, i. e. , an explanation in terms of some absorption or cutoff? The other question is a little more specific. A cutoff was used in Rost's calculations that you showed us. Does the cutoff used in these calculations have any effect on the predicted minimum for the $\frac{1}{2}^-$ state?

SCHIFFER: The answer to both questions is "No." The "simple" pictures that have been suggested do not seem obvious to me, at least. It is a question of what one means by "simple." As to the second question, the qualitative features of the results are independent of whether the cutoff is included or not, although quantitatively the fit is slightly worse when the cutoff is used.

B. L. COHEN, University of Pittsburgh: I think John and Lin are very much to be congratulated for finding this fine new method. It is really very exciting. However, I feel that I must say something in defense of our

method. You may be right in saying that there is disagreement in two cases; but to say that our method is based on a model is misleading. It is based on two assumptions. One is that a lower energy state fills before a higher one does. A ball rolls down a hill for the same reason—and I never saw one roll up! The other assumption is that it is easier to take things out of a full basket than out of an empty one. Thus I think our method is physically very reasonable; and it has worked in at least 50 or 100 cases. Except in cases in which we could not measure the (d,t) reaction and guessed the wrong values, I do not know of any exceptions. Perhaps you can cite some.

SCHIFFER: One case is the first excited state of Fe^{55} . We are quite sure now that this has spin $\frac{1}{2}$, but the ratio of the ground state to the first excited state in the stripping reaction from Fe^{54} is roughly the same as in the pickup reaction from Fe^{56} . You, and other people, guessed that $J = \frac{3}{2}$ for both states; but in fact the first excited state is $J = \frac{1}{2}$. For $\ell = 1$, where the spin-orbit splitting is small, it might happen that the $p_{1/2}$ shell is filling at roughly the same rate as the $p_{3/2}$ shell. Although these spin estimates are reasonable in the absence of other evidence, I think they cannot be relied on absolutely. They would probably be more reliable for states with higher ℓ and therefore larger spin-orbit splittings. My point is that if one really wants to be certain of the spin of every state, one has to be a little cautious in applying this method.

(Since the two following contributed papers are also on the J dependence of angular distributions from direct reactions, discussion of this aspect of the paper is deferred until the end of the paper by Sherr, Rost, and Rickey, p. 309 of these Proceedings.)

SPIN DEPENDENCE OF (d,t) $\ell_n=1$ ANGULAR DISTRIBUTIONS
IN Ni AND Fe[†]

R. H. Fulmer and W. W. Daehnick*

University of Pittsburgh, Pittsburgh 13, Pennsylvania

Introduction

It has recently been reported that, for 10-MeV incident deuterons, the (d,p) reaction on medium-weight nuclei can be used to distinguish between $p_{1/2}$ and $p_{3/2}$ states of the "stripped" neutron.¹ The distinction between $p_{1/2}$ and $p_{3/2}$ transfers is made possible by a pronounced difference, for the two cases, in the angular distributions at large scattering angles ($\theta > 90^\circ$). The same effect has subsequently been noticed for an incident-deuteron energy of 12.0 MeV.²

The present paper investigates, in medium-weight nuclei, the extent of the above effect for the (d,t) reaction, in which a neutron is "picked up" by the incident deuteron rather than "stripped" from it. Several $\ell_n=1$ reactions proceeding to states of known spin are studied, and the shapes of the corresponding angular distributions are determined.

[†]Contributed paper. Presented by R. H. Fulmer. (Not included in Vol. I. Contributed Papers.)

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¹L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. Letters 12, 108 (1964).

²R. H. Fulmer and R. E. Sass (to be published).

Experimental Procedure

Foil targets of Fe^{56} , Ni^{60} , and Ni^{64} about 1 mg/cm^2 thick were bombarded with 14.7-MeV deuterons from the University of Pittsburgh cyclotron. The outgoing particles were detected by a system described in detail elsewhere.³ Briefly, it consisted of two telescopically-mounted solid-state detectors of the silicon surface-barrier type. The detectors formed an $E, dE/dx$ system which distinguished particles according to the value of their mass m multiplied by the square of their charge Z . The energy and the particle-analyzer pulses were fed into a two-dimensional 4096-channel pulse-height analyzer whose data display was a three-dimensional matrix whose axes were energy, $\sim mZ^2$, and intensity. Values of amplifier gains and biases were such that data from (d, d') and (d, t) reactions were recorded simultaneously. The energy resolution width for the detected tritons varied from 80 keV to 200 keV depending on the scattering angle and the thickness of the target.

In an alternative detecting system⁴ used to collect one set of data, tritons from the (d, t) reaction were passed through a momentum-analyzing magnetic spectrograph and detected by nuclear emulsion plates located in the focal plane of the spectrograph.

Results and Discussion

Angular distributions of (d, t) cross sections in the angular range 70° to 150° , which were recorded for a total of three $p_{1/2}$ levels

³ R. H. Fulmer and W. W. Daehnick (to be published).

⁴ B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 120, 698 (1962).

Fig. 1. Angular distributions for $p_{3/2}$ and $p_{1/2}$ levels in the reaction $\text{Ni}^{60}(\text{d},\text{t})\text{Ni}^{59}$. Spins and excitation energies in the key refer to levels formed in Ni^{59} .

Fig. 2. Angular distributions for $p_{3/2}$ and $p_{1/2}$ levels in the reaction $\text{Ni}^{64}(\text{d},\text{t})\text{Ni}^{63}$. Spins and excitation energies in the key refer to levels formed in Ni^{63} .

and three $p_{3/2}$ levels in Ni^{59} and Ni^{63} , are shown in Figs. 1 and 2. As the figures indicate, the angular distributions leading to $p_{1/2}$ and $p_{3/2}$ levels show pronounced differences at large scattering angles. Figure 3 illustrates these differences, for two levels observed in the reactions $\text{Ni}^{64}(\text{d},\text{t})\text{Ni}^{63}$, in the form of a ratio of $p_{3/2}$ to $p_{1/2}$ cross sections, where the ratio has been set equal to unity at a scattering angle of 70° . Below this angle, $p_{1/2}$ and $p_{3/2}$ angular distributions seem to have the same

Fig. 3. Ratio of the cross section for the 0.53-MeV level to that for the 1.01-MeV level in Ni^{63} . The ratio has been adjusted to be unity at a scattering angle of 70° .

shape,³ although from Fig. 3 we recognize a strong J dependence at large angles (which is possibly due to the spin-orbit part of the nuclear force).

From Figs. 1, 2, and 3, one test of the spin of a transferred p neutron is afforded by comparing the relative differential cross section at 70° with that at 90° . The ratio of cross sections should be

$$\sigma(70^\circ)/\sigma(90^\circ) \approx \begin{cases} 1.5 \text{ (spin } \frac{3}{2} \text{)}, \\ 3.5 \text{ (spin } \frac{1}{2} \text{)}. \end{cases} \quad (1)$$

Numerical values of this ratio are listed in Table I for the six cases shown in Figs. 1 and 2 and for four additional cases in which the spin transfer is known.⁵⁻⁹ In all ten cases the ratio of the cross sections agrees, via Eq. (1), with the known spin transfer.

⁵ Nuclear Data Sheets, compiled by K. Way et al. (Printing and Publishing Office, National Academy of Sciences—National Research Council, NRC 59-3-125, Washington 25, D. C.).

⁶ D. S. Gemmell, L. L. Lee, Jr., A. Marinov, and J. P. Schiffer, Bull. Am. Phys. Soc. 8, 523 (1963).

⁷ G. A. Bartholomew and M. R. Gunye, Bull. Am. Phys. Soc. 8, 367 (1963).

⁸ R. H. Fulmer and A. L. McCarthy, Phys. Rev. 131, 2133 (1963).

⁹ R. E. Coté, H. E. Jackson, L. L. Lee, Jr., and J. P. Schiffer, Bull. Am. Phys. Soc. 7, 551 (1962); Phys. Rev. 135, B52 (1964).

TABLE I. Ratio of the cross section at a laboratory scattering angle of 70° to that at 90° .

Reaction	Final state		$\sigma(70^\circ)/\sigma(90^\circ)$	Reference (for spin)
	Energy (MeV)	Spin		
$\text{Fe}^{56}(\text{d}, \text{t})\text{Fe}^{55}$	0.0	$3/2^-$	1.75 ± 0.19	5
	0.42	$1/2^-$	4.13 ± 0.91	6
$\text{Ni}^{60}(\text{d}, \text{t})\text{Ni}^{59}$	0.0	$3/2^-$	1.32 ± 0.06	7
	0.47	$1/2^-$	3.76 ± 0.56	7
	0.89	$3/2^-$	1.30 ± 0.20	7
	1.31	$1/2^-$	3.12 ± 0.60	7
$\text{Ni}^{64}(\text{d}, \text{t})\text{Ni}^{63}$	0.0	$1/2^-$	2.26 ± 0.18^a	8
	0.16	$3/2^-$	1.25 ± 0.05^a	8, 9
	0.53	$3/2^-$	1.72 ± 0.22	8, 9
	1.01	$1/2^-$	3.67 ± 0.48	8, 9

^aRatio of cross section at 74° to that at 88° .

Acknowledgments

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* * *

(Since the following contributed paper is also on the J dependence of angular distributions from direct reactions, the discussion of this paper is deferred until the end of the paper by Sherr, Rost, and Rickey, p. 309 of these Proceedings.)

J DEPENDENCE OF THE ANGULAR DISTRIBUTIONS
FROM (p, d) REACTIONS AT 28 MEV^{*†}

R. Sherr and E. Rost

Palmer Physical Laboratory, Princeton University
Princeton, New Jersey

and

M. E. Rickey

Department of Physics and Astrophysics, University of Colorado
Boulder, Colorado

Lee and Schiffer¹ have recently found a marked dependence of the angular distributions in the (d, p) reaction on the spin of the final state following capture of an $\ell=1$ neutron by a spin-zero target. Using 8- to 12-MeV deuterons and targets ranging from Ca to Ni, they found that $\sigma(\theta)$ for $J=\frac{1}{2}$ final states exhibited a very deep minimum at some angle beyond 90° . The cross sections in the angular region of interest were 2 to 5% of the forward-angle maxima. In an investigation of the (p, d) reaction at 28 MeV bombarding energy on elements ranging from Ti through Ni, we have found a strong dependence of the shape of the forward-angle maxima on the J of the final state following pickup of an $\ell=3$ neutron from a

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[†]Contributed paper presented by R. Sherr. (Not in Vol. I. Contributed Papers.) At the suggestion of the speaker, the actual talk has been replaced by the paper taken verbatim from Phys. Rev. Letters 12, 420 (13 April 1964), with the permission of the editor of this journal.

¹L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. Letters 12, 108 (1964).

Fig. 1. Deuteron spectra from the (p,d) reaction at 28 MeV. The known excitation energies for each residual nucleus are shown. J^π is given only for states discussed in the text.

spin-zero target. Since the presently observed effect occurs where the cross sections are large, it will also be useful as a spectroscopic tool for states whose cross sections are too small to permit observation of the Lee-Schiffer effect. Present indications are that the two effects have different origins, the latter (at large angles) being due to spin-orbit coupling in the entrance and/or exit channels, while the former (at forward angles) is due to a spin-orbit effect on the wave function of the picked-up neutron.

The spectra of deuterons from isotopic targets of Fe⁵⁶, Fe⁵⁸, Ni⁵⁸, Ni⁶⁰, and Ni⁶² are shown in Fig. 1. These spectra were obtained with the 28-MeV proton beam of the University of Colorado cyclotron. An E-ΔE telescope of solid state detectors and a multiplying

circuit were used to identify and measure the deuteron energies. The energy levels for Ni^{57} are those obtained in the present measurements; for the other nuclei, the previously known levels are listed.^{2,3} Spin assignments are shown only for those states showing $\ell=3$ distributions in (d,p) measurements,³⁻⁵ in the (p,d) reaction at 17.0 and 18.5 MeV,⁶ and in the current investigation. The relative cross sections^{3,4} in the (d,p) reaction are shown by the horizontal bars; spin $\frac{5}{2}^-$ is assigned to the strongest (as well as lowest) state. (The 136-keV state of Fe^{57} and the 68-keV state of Ni^{61} have previously been assigned² spin $\frac{5}{2}^-$.) In the (p,d) reaction, the strongest yield is expected for pickup of a neutron from the presumably filled $1f_{7/2}$ shell; these states will have a relatively feeble yield compared with that for the major $1f_{5/2}$ state in the (d,p) reaction. Thus states weakly excited in the (d,p) reaction and strongly excited in the (p,d) are assigned⁶ spin $\frac{7}{2}^-$.

The angular distributions for the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states are shown in Fig. 2(a). The distributions for the $\frac{7}{2}^-$ states are similar to those we have observed for states of known spin $\frac{7}{2}^-$ in $\text{Ti}^{47,48}$ and Cr^{51} . [The experimental energy of the $\frac{7}{2}^-$ peak in Fe^{55} is 1.38 MeV; Whitten⁷ has found in (p,d) measurements at 17.5 MeV that the 1.41 and 1.32 states

²K. Way et al., Nuclear Data Sheets, National Academy of Sciences — National Research Council (unpublished).

³Massachusetts Institute of Technology Laboratory for Nuclear Science Progress Reports: A. Sperduto, May 1962, p. 66; A. Sperduto and J. Rapaport, November 1961, p. 134; K. Moriyasu, J. L. Adams, and H. A. Enge, May 1962, p. 63; H. Enge and R. A. Fisher, May 1959, p. 124 (unpublished).

⁴B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 126, 698 (1962); R. H. Fulmer and A. L. McCarthy, Phys. Rev. 131, 2133 (1963); R. H. Fulmer, A. L. McCarthy, B. L. Cohen, and R. Middleton, Phys. Rev. 133, B955 (1964).

⁵K. Ramavataram, Phys. Rev. 132, 2255 (1963).

⁶J. C. Legg and E. Rost, Phys. Rev. (to be published).

⁷C. A. Whitten, Jr. (private communication).

Fig. 2. (a) Relative angular distributions for $\ell=3$ pickup to $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states for the various residual nuclei. (b) Relative angular distributions for $\ell=1$ pickup. (The curves shown are lines connecting the experimental points.)

have $\ell=3$ distributions with a cross section ratio of ~ 3.5 to 1.] From Fig. 1 it is evident that the energy resolution (~ 120 keV) of the present measurements was inadequate to resolve some of these states, particularly in the case of the 68 keV state of Ni^{61} . Some assurance that our decomposition of the experimental data is reasonably good is given by the angular distributions for the other members of the doublets. The latter should have $\ell=1$ distributions^{3,4,6} and are shown in Fig. 2(b) together with other $\ell=1$ distributions for comparison.

The differences in $\sigma(\theta)$ for $J = \frac{5}{2}$ and $J = \frac{7}{2}$ are very marked. The peaks for the $\frac{5}{2}^-$ states are at a smaller angle in each case and the fall-off with increasing angle is steeper and deeper. This effect is not due to the difference in Q value for the pairs of states, as can be seen by the essential identity of $\sigma(\theta)$ (relative) for the 1.38- and 2.91-MeV states of Fe^{55} in Fig. 2(a). The $\ell=3$ distributions for higher states show a slight increase of peak angle with excitation energy, but the rate of shift is too small to account for the observed shift for the $\frac{5}{2}^-$ states.

Fig. 3. Comparison of experiment and distorted-wave calculations. The heavy curves show the relative cross sections for $f_{5/2}$ and $f_{7/2}$ pickup, while the light curve for $\frac{7}{2}^-$ has been normalized ($\times 1.9$) to the same maximum value as for $\frac{5}{2}^-$ to exhibit more clearly the difference in $\sigma(\theta)$ for the two cases. The corresponding experimental points have been normalized to the theoretical curves.

The J dependence of $\sigma(\theta)$ can be investigated with a distorted-wave calculation where a spin-orbit term $V_1 \vec{l} \cdot \vec{s}$ is added to the optical potentials and also to the potential which describes the wave function of the picked-up neutron. The forward-angle behavior of $\sigma(\theta)$ was found to be insensitive to the optical potential parameters which were taken from elastic scattering data.⁸ The neutron wave function is customarily described by adjusting the potential well in order to give the actual neutron separation energy. This method fails in our case since the separation energies for $J = \frac{5}{2}$ and $\frac{7}{2}$ are very similar. However, the effect is easily explained if we take the neutron well from the simple jj shell model and ignore the incorrect asymptotic form of the neutron wave function.

Figure 3 shows the results of a distorted-wave calculation of $\text{Fe}^{56}(p,d)\text{Fe}^{55}$ reaction leading to the 1.38-MeV ($\frac{7}{2}^-$) and 0.93-MeV ($\frac{5}{2}^-$) states of Fe^{55} . The calculations assume 6 MeV splitting between the shell model $f_{7/2}$ and $f_{5/2}$ levels. The smaller peak angle and larger

⁸C. M. Perey and F. G. Perey, Phys. Rev. **132**, 755 (1963). The proton optical parameters were obtained by fitting 28-MeV Ni^{58} elastic data using a search program written by F. G. Perey.

fall-off for $J = \frac{5}{2}$ can be understood in the smaller binding (or equivalently larger "effective" radius) of the $f_{5/2}$ neutron. The effect on the relative strengths is also striking, the peak cross section for $J = \frac{5}{2}$ being almost a factor of two greater than the peak for $J = \frac{7}{2}$. If the shell-model wave functions are reliable for extracting relative magnitudes, some doubt is then cast on the extraction of spectroscopic information for different orbitals presently given in the literature.

Calculations using shell-model wave functions for $^{56}\text{Fe}(p,d)^{55}\text{Fe}$ at 17.5 MeV or for $^{54}\text{Fe}(d,p)^{55}\text{Fe}$ at 12 MeV indicate a smaller effect on the $\ell=3$ cross sections at forward angles than in the present case. However, the existing data are not of sufficient accuracy to check the small but observable shift predicted by these calculations.

The $\ell=1$ distributions shown in Fig. 2(b) are all similar. Several distributions for $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states¹ have been compared, but no unambiguous difference was found, although there is a slight indication that the 25° minimum may be relatively deeper for $\frac{1}{2}^-$ than for $\frac{3}{2}^-$. More detailed angular distributions will be required to establish the difference that it is now safe to conclude must exist. If the j dependence of the angular distributions arises from the neutron spin-orbit interaction, one would expect a smaller effect for $\ell = 1$ than for $\ell = 3$ at forward angles. By the same argument, larger effects are to be expected for pickup of neutrons from higher ℓ states.

We wish to thank Dr. J. P. Schiffer and Professor E. Kashy for stimulating and illuminating discussions. We are greatly indebted to C. Hoot and B. Bardin for their assistance in taking the data and to E. R. Martin and G. H. McCall for their aid in analyzing the spectra.

* * *

(This discussion is not limited to this paper but also includes discussion of the two preceding papers, as noted above, because of the close relation between them.)

N. K. GLENDENNING, Lawrence Radiation Laboratory: I have just a word of caution on the paper by Schiffer. At the place where the effect is observed in the angular distribution, the cross section is 1—2 orders of magnitude below the peak cross section. The observed difference between the angular distributions of spin-orbit partners therefore occurs at a place where other perturbing phenomena can make their presence felt. Such perturbations could be, for example, compound-nucleus contributions and the scattering coming from the interaction between the outgoing proton and the nuclear core, both of which are neglected in the distorted-wave method. In other words, the effect that Schiffer and Lee see in these nuclei may, in other cases, be perturbed by effects unrelated to the spin-orbit coupling.

D. KURATH, Argonne National Laboratory: I would like to ask Dr. Rost a question. He has attributed a 6-MeV difference to the spin-orbit interaction. How large is V_0 in comparison with V_1 ?

E. ROST, Princeton University: The 6-MeV was taken from the approximate splitting for the f nucleons in calcium. The depth of the central well is about 52 MeV, the depth that gives approximately the correct separation energy for the $\frac{7}{2}$ state. One imagines the $\frac{5}{2}$ state to have been split into many parts, one of which happens to lie right next to the ground state. This is the state that is adjusted to have the so-called model asymptotic form.

A. K. KERMAN, Massachusetts Institute of Technology: In connection with the remark that Sherr and Glendenning made about the

cross section being small (almost down to zero on the scale used), especially for $\ell = 1$, I wonder about the size of the compound-nucleus effect. Is it still down in the noise or is it comparable? Does anyone know the answer?

J. P. SCHIFFER, Argonne National Laboratory: I think no one has a definite answer. One can only say that it is an empirical effect and that the difference is real. The compound-nucleus effect is presumably small. After all, very many channels are open to both protons and neutrons in these reactions and the cross section for any one channel is going to be very small in comparison with the direct reactions. Quite possibly this could account for the fact that the effect is inverted in Ca^{43} . This effect seems to work for all cases in which the stripping cross sections are appreciable.

G. H. RAWITSCHER, Yale University: Dr. Schiffer, when you measured the position of the dip at 120° , did the angular position of the dip change with the energy of the incident particle? There is also a related question, directed perhaps to Dr. Rost. If one introduces an $\vec{l} \cdot \vec{s}$ potential, is there a simple intuitive explanation for finding the particular dip at around 120° , apparently at remarkably close to the same angle for all nuclei?

SCHIFFER: Experimentally the energy dependence, if any, is small. As to the dependence on A, the experimental position of the minimum does shift from one nucleus to another; there does seem to be a systematic trend.

ROST: There are two effects here. The one that John Schiffer mentions is at the back angles and its magnitude is about 2% of the forward maximum. To the best of my knowledge, it is something that comes out of the calculation and is not understood in terms of a simple physical model.

The effect that Prof. Sherr mentioned (the shift of the first maximum of the $J=\frac{5}{2}^-$ state to small angles and its rapid falloff with angle) can be understood easily in terms of the fact that this wave function is less tightly bound. Consequently it is farther outside, and therefore the angular distribution will be pushed in and will also be sharper.

K. R. GREIDER, Yale University: It is interesting that the same deep minimum observed by Lee and Schiffer for the $\ell=1$, $j=\frac{1}{2}$ case is also seen in $\ell=0$ stripping reactions and at about the same angles, namely from 100° to 140° . It shows up clearly in the data of Kuehner, Almqvist, and Bromley⁹ for the (d,p) reaction on silicon. This is an $\ell=0$ reaction, and the two residual states have the same total spin, $j = \frac{1}{2}$. One wonders if the deep minima represent a more general effect than the present $\ell=1$ data indicate.

J. G. CRAMER, JR., Indiana University: As I understand it, there are two spin-orbit effects, one between the neutron and the core and the other between the proton and the target. If the spin-orbit potential between the proton and the target were turned off, would the minimum go away?

ROST: Schiffer's backward-angle minimum depends upon the proton spin-orbit potential. Without this, it goes away completely. The effect at forward angles is due to the neutron binding energy alone. Spin-orbit effects do not need to be included to get the effect.

CRAMER: For the (d,p) reaction, do you have any predictions about how the proton polarization behaves in the angular region of this minimum?

ROST: No.

R. H. FULMER, University of Pittsburgh: Does Dr. Sherr have any experimental evidence that the cross sections of the $f_{5/2}$ and $f_{7/2}$ states differ by a factor of two?

R. SHERR: That is a difficult question to answer because there is no case in which we are certain of the population numbers. We use the distorted-wave calculations to get spectroscopic factors and therefore

⁹ J. A. Kuehner, E. Almqvist, and D. A. Bromley, Nucl. Phys. 21, 555 (1960).

the populations. To answer your question, we would have to have independent information about the population numbers.

D. R. INGLIS, Argonne National Laboratory: Dr. Rost's remark that the effect goes away if you turn off the proton spin-orbit coupling encourages me to introduce a model discussion. It is partially in answer to questions raised by Glendenning and others, particularly his remark that we have to be careful because the intensity in this backward direction is so low that small effects can become important. I suggest that it is just for this reason that this characteristic effect, sensitive to spin-orbit coupling, may appear.

In discussing a model, I do not want to suggest that this is the last word or that it gives the only explanation. I think it perhaps turns our thoughts in a useful direction, and I hope ideas may develop from the discussion. As a model for a (d,p) reaction in the forward direction, we think of the nucleus with a neutron and proton coming in from the left as a deuteron. A few protons come out in a generally backward direction; but most of the intensity is forward. We think of the stripping process taking place primarily near the front surface. Some may prefer to think of it as taking place throughout the nuclear volume, but I prefer to associate the breakup of the deuteron with the rather sudden jump in potential at the front face. We have a space wave function of roughly Gaussian form and a similar wave function in momentum space. The latter has a low intensity in the high-backward-momentum tail, particularly for sufficiently high backward internal momentum to overcome the forward translational momentum of the deuteron and give protons going backward.

The larger number of forward protons go on through and are partially absorbed. This reduces their intensity. The intensity is again reduced in a partial reflection on the far face. The reflected wave then comes back and competes with the other weak wave. The two

low-intensity processes can easily compete in the backward direction. Question: Can either or both of these contributions be sensitive to the spin-orbit coupling? At the point of reflection, the sign of the spin-orbit coupling of the proton is relevant, even though the signature of the final state is the sign of the spin-orbit coupling of the neutron. Both particles are created at the split-up of the deuteron; and presumably the deuteron is small enough that, in the most probable event, the proton and neutron go by on the same side of the center of the nucleus so that both have the same direction of orbital angular momentum. The triplet structure of the deuteron gives them the same spin direction so they have the same sign of spin-orbit coupling. The proton spin-orbit coupling at the point of reflection, being proportional to $r^{-1}(\partial V/\partial r)\vec{l} \cdot \vec{s}$, may modify the reflecting potential jump either in a way to superimpose a dip on it or to superimpose a protruding bump on it. From the theory of reflection from a potential jump, in which an incoming and a reflected wave are joined to form a transmitted wave, it seems very plausible that the magnitude of the reflected wave can be quite sensitive to the shape of the potential, i. e., to the presence of a dip or a bump. In this way the competition between the two waves could depend characteristically on the sign of the spin-orbit coupling. Of course, a lot of further details about the phase differences of these two waves will help to determine whether their interference will be of the observed destructive type or not.

RAWITSCHER: I have not thought at all deeply about such questions, but I would like to return to the possibility that the backward angle where the minimum occurs is insensitive to the energy. If that should be the case, then perhaps an explanation of the dip in terms of a potential would be hard to understand. The reason is that such a model would have in it the product of a radius times the wave number. The dip should therefore depend on both the nuclear size and the energy. However, if some

particular angular-momentum state is emphasized, then its angular dependence would be independent of the energy. In that case it might be very attractive to think of it in that way. I realize this is all very tentative since one does not know enough about the experimental details.

H. A. ENGE, Massachusetts Institute of Technology: I would think it would be possible to test this model to some extent by varying the W in the potential because the effect should be very sensitive to W . The mean free path of the proton through the nucleus, of course, is very sensitive to W , so in the calculations the dip should be very sensitive to W .

N. AUSTERN, University of Pittsburgh: I would like to ask Dr. Johnson whether his perturbation-theory investigations are related to the Inglis model.

R. C. JOHNSON, University of Wisconsin: By treating the deuteron and proton spin-orbit forces (V_s) by perturbation theory, I have so far only been able to confirm what Dr. Rost said earlier about the exact DWBA calculations. It can be shown that the (d,p) cross section can be written in the form

$$\sigma_{\ell j}(\theta) = (2j+1) A_{j\ell}(\theta) + (-1)^{j+\ell-\frac{1}{2}} B_{j\ell}(\theta).$$

To first order in V_s , $(2j+1) A_{j\ell}(\theta)$ is the cross section that obtains in the absence of spin-dependent forces. The dependence of $\sigma_{\ell j}$ on V_s arises purely through $B_{j\ell}$. $A_{j\ell}$ and $B_{j\ell}$ depends on j only through the radial parts of the neutron wave function. At forward angles, presumably $A \gg B$ and the j dependence of $\sigma_{\ell j}(\theta)$ arises from the neutron wave function. At backward angles A and B become comparable in magnitude; and the presence of the factor $(-1)^{j+\ell-\frac{1}{2}} B_{j\ell}(\theta)$, which arises only if $V_s \neq 0$, can produce the type of j dependence observed.

B. L. COHEN, University of Pittsburgh: Rost's suggestion (presented by Sherr) that one should use the location of the single-particle level rather than the Q value in distorted-wave calculations should not be taken at all lightly. It has a very serious effect on a lot of things. For example, in using (d,p) reactions to study states in a shell that is almost full, the nuclear levels are in the opposite order from that of the single-particle levels. The effects in this case can be really tremendous. Another case is pickup reactions on a shell that is just beginning to fill. Austern suggested this sort of thing a couple of years ago. A lot of results that look good now would not look good if you do this. I do not consider this to be evidence that Rost's suggestion absolutely cannot be, but certainly I think there should be some question marks put on it. I would appreciate any guidance some of the theoreticians here could give in this matter.

AUSTERN: I made some attempt last summer to treat this question quantitatively. Although the attempt did not go far because I did not anticipate so much interest in it, I can make a brief general statement about it. The point is that in the phenomenological procedures that have normally been used, the shape of the bound-state wave function is tied to the Q of the reaction. One wants to be able to criticize these procedures by asking if the shape of a component of the final-state wave function which is picked out in a stripping experiment really is tied to the over-all energy of that wave function. We are looking for a component of a state which also has other components, and we are attempting to measure the shape of one factor in that component by looking at the energy of the entire wave function. If there aren't very many components, then the tie between the energy and the shape of that component might be fairly close. On the other hand, if the state has many components, then presumably the shape of any one of them has very little to do with the energy of the state as a whole. Thus there is the possibility of having one answer in one circumstance

and another answer in another circumstance. On the whole, the form factors used in the stripping calculation are not to be divorced from the spectroscopy of the bound states themselves; the two are to be considered in conjunction. That is perhaps the principal lesson that we have to go away with.

THE FORMAL THEORY OF DIRECT-INTERACTION
PROCESSES AND SIMPLE APPROXIMATION METHODS*

K. R. Greider

Yale University, New Haven, Connecticut

One of the main impressions obtained from the papers presented during this conference is that an apparent dichotomy exists between the simple one-parameter models and the many-parameter DWBA methods in the analysis of experimental data. Because of the great utility of simple models in both data analysis and in gaining physical insight, it is worthwhile to find the reasons for their successes. To this end the work of both Blair and Austern has helped a great deal in non-rearrangement inelastic processes. However, an equally good understanding of rearrangement scattering, such as stripping reactions, is not available at present. What follows is an attempt to justify in a formal way the usefulness and applicability of simple few-parameter models, such as the Butler model and the Dar model as well as a new one-parameter model described below. If these simple models are properly understood it may be possible to improve on them and make them even more useful than heretofore, even in precise analyses of direct-interaction experiments.

A proper understanding is obtained only by starting from basic principles or from some "exact" formulation of the problem, and deriving the simple theory by applying well defined and well understood approximation schemes to the exact formalism. The physical insight

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obtained by this procedure can then lead to significant improvements without destroying the theory's great virtue—simplicity.

As a specific case, consider the deuteron stripping reaction. We adopt the usual point of view that this process can be approximated in terms of a three-body reaction involving the neutron, the proton, and the initial nucleus. The initial-state deuteron is bound by the neutron-proton potential V_{np} , and the final-state nucleus has a neutron bound to the nuclear "core" via V_n . The proton scatters from the core in both initial and final states because of the proton-core potential V_p . Several exact derivations of the transition matrix element have already been obtained¹ and the usual formulation used by DWBA adherents is²

$$T_{if} = \langle \chi_p^{(-)} | V_{np} | \psi_d^{(+)} \rangle + T' . \quad (1)$$

The first term of Eq. (1) describes an incoming wave of deuterons interacting via all the potentials (V_p , V_n , and V_{np}) in $\psi_d^{(+)}$. The stripping

¹ B. A. Lippmann, Phys. Rev. 102, 254 (1956). For a more complete list of references, see T. -Y. Wu and T. Ohmura, Quantum Theory of Scattering (Prentice-Hall, Inc., Englewood Cliffs, 1962), pp. 332, 397.

² If the elastic function $\chi_p^{(-)}$ is defined in terms of a proton optical potential U_p , diagonal in the energy, then T' will consist of a term involving $(V_p - U_p)$. An explicit form for T_{if} (and T') in the case of the analogous pick-up reaction is

$$T_{if} = \langle \psi_d^{(-)} | V_{np} | \chi_p^{(+)} \rangle + \langle \phi_d | V_p - U_p + V_n - V_{np} | \chi_p^{(+)} \rangle ,$$

where ψ_d has the same meaning as in the text, $\chi_p^{(+)}$ is the solution of the Schroedinger equation with the optical potential U_p , and ϕ_d is the asymptotic plane-wave state of deuterons.

"mechanism" is given by the factor V_{np} , and the scattering of the final-state proton on the residual nucleus is described by $\chi_p^{(-)}$. The use of partially interacting states (such as $\chi_p^{(-)}$ above) is described more fully by Gell-Mann and Goldberger.³ In the DWBA, the states $\psi_d^{(+)}$ and $\chi_p^{(-)}$ are approximated by optical-model functions that describe the elastic initial-state deuteron scattering by the target and the elastic final-state proton scattering by the residual nucleus, respectively. The second term T' has to our knowledge never been calculated in any approximation. We do not write T' explicitly, since its actual form depends on how the $\chi_p^{(-)}$ is defined.² In general it involves the inelastic parts of the V_p potential as well as differences between other potentials and is consequently difficult to calculate accurately. It seems reasonable that no theory of stripping can be believed, however, until contributions from this term are either shown to be small enough that they would not change the accuracy claimed for the first term, or are calculated and added to the first term.

There is a way out of this problem, and this way also relates to the partial justification of the simple one-parameter models mentioned earlier. In a recent article⁴ on the formalism appropriate to heavy-ion transfer reactions, a different representation for the matrix element of Eq. (1) is derived, namely

$$T_{if} = \langle \chi_{np}^{(-)} | t_p' | \chi_n^{(+)} \rangle + \langle \phi_f | V_{np} | \chi_n^{(+)} \rangle. \quad (2)$$

Equation (2) is derived from the same formal considerations as Eq. (1), and is also an exact form of the matrix element. If all potentials were

³ M. Gell-Mann and M. L. Goldberger, Phys. Rev. 91, 398 (1953).

⁴ K. R. Greider, Phys. Rev. 133, B1483 (1964).

known, Eq. (2) would yield results identical to Eq. (1). However, the exact potentials are not known, and consequently the formally identical expressions will not obtain identical results when approximation schemes are introduced. One must choose the formalism on its ability to provide a framework for accurate calculations, and we contend that Eq. (2) is superior to Eq. (1) in providing such a framework.

In Eq. (2), $\chi_n^{(+)}$ describes an incoming wave of deuterons in which only the neutron interacts with the target,⁵ ϕ_f is a plane-wave asymptotic state of an outgoing proton and the residual nucleus, the function $\chi_{np}^{(-)}$ describes the final-state scattering of the proton by the neutron that is captured, and t_p' is (approximately) the t matrix for the scattering of protons on the initial nucleus.

There are several distinct advantages of the formulation of Eq. (2) over Eq. (1) as an exact matrix element. First, both terms of Eq. (2) may be calculated consistently within the framework of the same approximation. Once an approximation is chosen for $\chi_n^{(+)}$ and the neutron elastic scattering is determined, the proton t matrix may be obtained directly from this solution. (Coulomb corrections may be incorporated also if necessary.) Such a procedure neglects the inelastic and the off-energy-shell contributions to t_p , but in the spirit of the usual optical-model methods the elastic scattering should serve as a good first approximation for the reaction process. The same consistency is not possible in calculating the second term of Eq. (1); one is instead required to know all the potentials rather accurately (including their phases) in order to obtain believable results.

⁵ A similar expression involving the initial-state neutron scattering was derived by N. C. Francis and K. M. Watson, Phys. Rev. 93, 313 (1954).

A second advantage of Eq. (2) over Eq. (1) is that both terms in the former equation lead to quite similar angular distributions; calculation of only the second term gives a good approximation to the form of the entire matrix element, but this is not the case in Eq. (1). Furthermore, the first term may often be smaller than the second. Insofar as the V_{np} interaction can be neglected in the final-state function $\chi_{np}^{(-)}$, or at least insofar as that interaction does not significantly increase the over-all magnitude of the first term, the absorption effects of the proton interactions (due to the imaginary part of the proton optical potential) should ordinarily make this term smaller than the second. Comparison of the magnitude of V_{np} in the second term with a reasonable value of t_p taken from a diffraction model yields a value of the order of 0.04 for the ratio for the magnitude of t_p to that of V_{np} . However, this is not the whole story since t_p is usually associated with a somewhat larger radius than V_{np} , and the ratio consequently should be somewhat larger.

Even if the second term is not considerably larger than the first, the former describes most of the physics of the reaction. The primary reaction mechanism that obtains the familiar Butler curves which allow identification of the orbital angular momentum L of the captured neutron is due to the simple overlap of the bound neutron final-state wave function with its initial-state scattering function. The second term describes just this overlap along with secondary effects due to the initial binding of the neutron in the deuteron. This same overlap occurs also in the first term; and if Coulomb effects are ignored, the proton scattering described by t_p' in the first term does not lead to significantly different distributions not already predicted by the second. The proton t matrix is folded into the distribution obtained from the second term and serves mainly to change the absolute magnitude and to smooth out the distribution. Although it is undoubtedly well known to those long practiced in DWBA calculations, it does not seem to be well recognized in the

literature that just the overlap in the neutron channel determines most of the observed experimental effects. The initial-state neutron-scattering wave function is the only logically tenable candidate to produce the required cutoff in the neutron coordinate in the Butler model. Arguments that this cutoff arises from channels other than the neutron-scattering channel are not consistent, since they require that an effect in the neutron coordinate space be provided by something occurring in another coordinate space, such as in the relative coordinate that describes the internal deuteron motion.⁶ The Butler radial cutoff can be understood naturally and logically when interpreted as a scattering approximation in the neutron channel, and does not necessitate recourse to less tenable ad hoc assumptions.

A third advantage of the formulation in Eq. (2) is that it already naturally describes cases in which there is a strong Coulomb field between the scattered proton and the core. In fact, the formalism was derived specifically to describe heavy-ion neutron-transfer reactions⁴ for which the first term of Eq. (2) dominates. Thus in calculating both terms of Eq. (2), one has a more general and useful approach to a wider range of nucleon transfer or stripping problems. Furthermore it should be quite reasonable to use the normal distorted-wave optical-model approximations in this formulation much the same as is currently done in evaluating the first term of Eq. (1).

A fourth (calculational) advantage of Eq. (2) relates to the fact that the second term describes scattering only in the neutron channel.

⁶ See, for example, S. T. Butler and O. H. Hittmair, Nuclear Stripping Reactions (John Wiley and Sons, New York, 1957), especially pp. 7, 42, and 52. See also Wu and Ohmura, Ref. 1, Eqs. (V4) and (V7).

Insofar as the first term can be reasonably neglected in a first-order theory of deuteron stripping, an enormous simplification of the problem is then obtained. The second term describes the neutron-target scattering interaction in the initial state and a plane-wave final state. Consequently this matrix element is similar to the usual Born-approximation result and is factorable into two integrals (if DWBA methods are assumed) because all proton interactions are absent. As in the usual Butler theory, one integral is over the relative neutron-proton coordinate while the other is over the neutron center-of-mass coordinate. As a result it is not difficult to perform relatively simple calculations without resorting to the usual zero-range approximation for the internal deuteron structure.

The second half of this talk describes a simple one-parameter stripping theory derived from just the second term of Eq. (2). If absorptive effects are important in the neutron-scattering channel, then it is reasonable to let the neutron-scattering wave function vanish in the region $r_n < R$; this rather simple condition obtains just the Butler cut-off theory. However, one can go further and can improve considerably on the Butler results by requiring also an outgoing scattered wave in this channel. Since the integral over the neutron coordinate is evaluated in the region just outside the nuclear boundary or potential, the asymptotic form of the scattered wave should be accurate. The approximation becomes

$$\Phi_n^{(+)} = \left[e^{i\vec{k}_n \cdot \vec{r}_n} + \sum_{\ell} a_{\ell} h_{\ell}^{(1)}(k_n r_n) P_{\ell}(\theta_n) \right] \Theta(r_n - R), \quad (3)$$

and the second term of Eq. (2) becomes

$$T_{if} \approx \langle \phi_f | V_{np} | \chi_n^{(+)} \rangle = T_{\text{Butler}} + T_{\text{scatt}}. \quad (4)$$

In Eq. (3), $\Phi_n^{(+)}$ is the neutron-scattering wave function from which the function $\chi_n^{(+)}$ required in Eq. (2) may be calculated by taking account of contributions to $\vec{k}_n$ from both the internal deuteron momenta and the deuteron center-of-mass momentum. The theta function $\Theta(r_n - R)$ in Eq. (3) vanishes for $r_n < R$ and is unity elsewhere. The choice of a_ℓ determines the scattered wave; its value may be obtained from models consistent with the radial cutoff. The principal results of the Butler theory are exactly reproduced if all the a_ℓ vanish (no scattered wave); a higher order approximation is obtained, for instance, by using the Blair sharp-cut-off model

$$a_\ell = \begin{cases} (2\ell + 1) & \text{for } \ell \leq k_n R \\ 0 & \text{for } \ell > k_n R \end{cases} \quad (5)$$

or by using other modified Blair models. Another possible way to describe $\Phi_n^{(+)}$ is in terms of boundary conditions near the nuclear radius, which reproduce diffraction scattering. Austern has described one such model,⁷ and it may be feasible to use others to describe the scattered wave.

Calculations of Eq. (4) by use of Eq. (5) show that one main effect of the scattered wave is to reduce the magnitude of the Butler term at forward angles. In fact, it can be shown that at $\theta = 0$ the real part of T_{scatt} always has the opposite sign from, and is nearly equal in magnitude to, the cut-off plane-wave contribution T_{Butler} . These calculations consequently give reduced widths much more in agreement with the usual DWBA computer results; and the differential cross sections show the general characteristics of Butler distributions, but display finite minima because of the presence of the imaginary part of T_{scatt} . In

⁷N. Austern, Ann. Phys. (N. Y.) 15, 299 (1961).

addition, the required cut-off radius is about 10% smaller than that of the Butler theory; and because of the reduced magnitude of the cross section at $\theta = 0$, the forward-to-backward cross-section ratio is in more reasonable agreement with experiment.

Moreover for the $L = 0$ case that we have investigated, there is an unusual effect which seems to be quite general in that a cancellation of terms occurs at large angles ($\theta \gtrsim 90^\circ$); over a fairly large angular region, T_{Butler} nearly cancels the real part of T_{scatt} . If the imaginary part of T_{scatt} has a zero in the same region, the angular distributions show a very deep minimum compared with the minima at both smaller and larger angles. Since this effect always seems to be present in our one-parameter model we suspect it may be a general effect and may be present also in stripping reactions for $L > 0$. There is considerable experimental verification for such deep minima at angles greater than 90° for $L = 0$ stripping,⁸ and the same effect may be in part responsible for the deep minima observed in the $L=1$ $J=\frac{1}{2}$ stripping distributions discussed earlier in this session by Schiffer—although proton interactions (spin-orbit scattering) are undoubtedly instrumental in obtaining the difference between the $J=\frac{3}{2}$ and $J=\frac{1}{2}$ distributions. It is significant that the one-parameter model described above predicts these deep minima and also gives more reasonable values of reduced widths and radii than the Butler model; this fact should lead to more general serious consideration of other few-parameter models in the future. Such models not only allow a clear understanding of the physical process, but also they may be able to predict many of the quantitative details of stripping and other direct reactions to an accuracy heretofore not possible in few-parameter theories.

⁸J. A. Kuehner, E. Almqvist, and D. A. Bromley, Nucl. Phys. 21, 555 (1960).

* * *

H. E. GOVE, University of Rochester: Would that explain the difference between the angular distributions for $j = \frac{3}{2}$ and $j = \frac{1}{2}$?

GREIDER: No, it would not in this first-order model. No spin-orbit effects are included here as yet. All I am saying is that there are large dips in the region $100^\circ < \theta < 140^\circ$ that can be understood in a simple way, and this is why the $\ell=0$ case I mentioned earlier is interesting. For this case there is no spin difference; yet the minimum exists.

A. K. KERMAN, Massachusetts Institute of Technology: With regard to the general theory of stripping, I am filled with admiration for the results that one can get with the distorted-wave codes. However, I think that a lot of theoretical work remains to be done before we really understand the stripping theory and that the kind of work that Greider has been reporting on is probably one of the things one should work on.

In particular, I still would hesitate to believe the absolute spectroscopic factors that people have been getting. If they are right, one should understand why. One should especially try to understand the point that Dr. Sherr raised about the neutron binding energy. This obviously introduces an uncertainty into all of these spectroscopic factors—possibly as much as a factor of 2, as I understand it.

As a possible way of exploring this effect, I would like to suggest doing stripping reactions below the Coulomb barrier, where the asymptotic form of the neutron wave function is well known in terms of the binding energy. As one gets closer and closer to the top of the Coulomb barrier, the stripping occurs more and more inside so that other effects begin to come in. Perhaps a theoretical and experimental collaboration to explore how the distorted-wave theory works as the top of the Coulomb barrier is approached would elucidate some of these points.

GREIDER: I agree completely with what Art Kerman said. The same is true for low-energy particle transfer in heavy-ion reactions. More complications arise, but the general idea is the same. At low enough energies, only the asymptotic tail of the bound-state wave function is required. I think that Jack McIntyre may have some comments on this.

J. A. McINTYRE, Texas A & M University: Experiments on neutron transfer below the Coulomb barrier can now give absolute reduced widths. In principle, such experiments get around the problems of what is going on inside the nucleus since there are no adjustable parameters describing the nuclear system. In a typical experiment, two nitrogen nuclei go by each other but stay out of range of the nuclear forces; in the transfer process a neutron tunnels from one potential well to the other. Such a transfer gives a definite reduced width that is independent of the DWB parameters.

N. AUSTERN, University of Pittsburgh: I would like to ask a question about the function $\chi_n^{(+)}$. What happened to the internal wave function of the deuteron?

GREIDER: It is included in the χ_n function. This wave function would be similar to that of the DWBA except that the whole deuteron does not scatter in the incident channel; only the neutron scatters. The scattering wave function is a solution of the wave equation in which the neutron scatters via V_n . At infinity it is asymptotic to an incoming free deuteron. In the approximation used in the calculation, the motion of the proton remains unaltered except that the proton is tied to the neutron in the deuteron and hence reflects the effects of the neutron scattering.

SESSION E

WEDNESDAY MORNING

11 MARCH 1964

CHAIRMAN: R. SHERR

SCIENTIFIC SECRETARIES

T. H. BRAID

D. S. GEMMELL

P. P. SINGH

NUCLEAR-STRUCTURE INFORMATION FROM MANY-PARTICLE TRANSFER REACTIONS*

Benjamin F. Bayman

Palmer Physical Laboratory, Princeton University
Princeton, New Jersey

My assignment is to discuss the type of nuclear-structure information that can be gleaned from reactions involving the transfer of several nucleons. In particular, what do they teach us that we cannot learn more easily from the better-understood one-particle transfer process? The most systematic way to treat this subject is in terms of the kind of coherence that each reaction involves. A very clear and penetrating exposition of this theme is given by Norman Austern in his Czechoslovak lectures.¹

To be specific, let us compare the processes (p,d), (p,t), and (p,a). For the process A(p,d)B, the differential cross section is proportional to something like

$$\sum_j \frac{1}{2\ell + 1} \sum_M \left| \sum_n S_{B;n\ell j}^A \int \Phi^{*(-)}(\vec{k}_d, \vec{r}) u_{n\ell j}(r) Y_M^\ell(\hat{r}) \Phi^{(+)}(\vec{k}_p, \vec{r}) d\vec{r} \right|^2 \quad (1)$$

$(\ell = j \pm \frac{1}{2})$

in the zero-range DWBA theory. The spectroscopic amplitude $S_{B;n\ell j}^A$ is defined by the fractional-parentage expansion

*This work was supported by the U. S. Atomic Energy Commission.

¹N. Austern, Selected Topics in Nuclear Theory (International Atomic Energy Agency, Vienna, 1963).

$$\Psi_{M_A}^{J_A(A)} = \sum_{B, n\ell j} S_{B; n\ell j}^A \left[\psi_{B(B)}^{J_B} \psi^{n\ell j} \right]_{M_A}^{J_A}. \quad (2)$$

The point we need to note is that expression (1) is incoherent in the orbital and total angular momenta removed from the target. This is a consequence of the assumption that the interaction responsible for the process is spin-independent, and occurs only between the incident proton and the picked-up neutron. Note that there is coherence in n , the principal quantum number of the picked-up neutron. Since the shell-model states $(n \ell j)$ and $(n+1 \ell j)$ are two major shells apart, they are not expected to be simultaneously active to an appreciable extent, so that the form in which (1) is usually used is

$$\sum_{n\ell j} \frac{[S_{B; n\ell j}^A]^2}{2\ell + 1} \sum_M \left| \int \Phi^{*(-)}(\vec{k}_d, \vec{r}) u_{n\ell j}(r) Y_M^\ell(\hat{r}) \Phi^{(+)}(\vec{k}_p, \vec{r}) d\vec{r} \right|^2. \quad (3)$$

To the extent that it makes sense to speak of $u_{n\ell j}(r)$ as characteristic of the single-particle state $n\ell j$ alone, one may regard expression (3) as effecting a factorization of the process into a part containing the nuclear-structure information and a part involving the reaction dynamics.

In the case of the (p, t) reaction, the corresponding expressions would be

$$\begin{aligned} & \sum_I \frac{1}{2I + 1} \sum_M \left| \int \Phi^{*(-)}(\vec{k}_t, \vec{r}_t) \Theta_0^{*0} V \right. \\ & \times \sum_{\substack{n_1 \ell_1 j_1 \\ n_2 \ell_2 j_2}} S^A(B; n_1 \ell_1 j_1, n_2 \ell_2 j_2; I) [(\ell_1 \frac{1}{2})_{j_1} (\ell_2 \frac{1}{2})_{j_2} | (\ell_1 \ell_2)_I (\frac{1}{2} \frac{1}{2})_0]_I \\ & \times u_{n_1 \ell_1 j_1}^{(r_1)} u_{n_2 \ell_2 j_2}^{(r_2)} [Y^{\ell_1}(\hat{r}_1) Y^{\ell_2}(\hat{r}_2)]_M^I \Phi^{(+)}(\vec{k}_p, \vec{r}_p) d\vec{r}_1 d\vec{r}_2 d\vec{r}_p \Big|^2, \end{aligned} \quad (1')$$

$$\Psi_{M_A}^{J_A(A)} = \sum_{\substack{n_1 \ell_1 j_1 \\ n_2 \ell_2 j_2 \\ B, I}} S^A(B; n_1 \ell_1 j_1, n_2 \ell_2 j_2; I) \left\{ \Psi_{M_A}^{J_B(B)} [\psi^{n_1 \ell_1 j_1} \psi^{n_2 \ell_2 j_2}]^I \right\}^{J_A(A)}. \quad (2')$$

Here V is the interaction, assumed spin-independent and involving only the three nucleons in the triton, and Θ_0^0 is the orbital part of the internal triton wave function, assumed permutation symmetric and spherically symmetric. The LS-jj transformation amplitude expresses the requirement that the picked-up neutrons be in a singlet state, since they are in a singlet state in the assumed permutation-symmetric triton and the interaction is assumed to be spin-independent. As before, the process is incoherent in the transferred total angular momentum, but it is coherent in the single-particle states of the picked-up neutrons. If the target neutrons can make use of their freedom to scatter into different single-particle states to produce a spatial correlation similar to that of the neutrons in the triton, so much the better for the (p,t) reaction.

Conversely, in the (t,p) reaction the states populated in the daughter nucleus are those that look like the target plus a closely correlated pair of neutrons. The (p,d) reaction will not distinguish between the states $a(j_1^2)_0^0 \pm b(j_2^2)_0^0$. For both signs, the j_1 and j_2 spectroscopic factors would be $2a^2$ and $2b^2$, respectively. But expression (1') says that the cross section is linear in the j_1 and j_2 spectroscopic amplitudes. Thus if a and b are comparable in magnitude, the (p,t) cross sections from these states [or the (t,p) cross sections to them] will be strongly affected by the relative sign of the two components.

This point has been stressed by Mottelson,² Austern,¹ Yoshida,³ and others. Yoshida has calculated the effect on the (p,t)

² B. R. Mottelson (private communication to N. M. Hintz).

³ S. Yoshida, Nucl. Phys. 33, 685 (1962).

reaction of the sort of coherence produced by a pairing force. He uses an approximation to the integral appearing in (1'). To my knowledge, this program of using the coherence in the (p,t) process to learn about the relative phases of the different single-particle components in the wave function has not really been applied quantitatively. Perhaps the simplest place for a quantitative study would be the calcium isotopes. A series of single-particle-transfer reactions would yield information on the single-particle compositions, i. e., how many $1f_{7/2}$, $2p_{3/2}$, $1d_{3/2}$, etc., particles each has in its ground state. Then a series of $I=0$ ground-state-to-ground-state (p,t) reactions would probably determine uniquely (or at least to a great extent) the relative phases of these different single-particle components in the ground states.

When we turn to the (p, α) reaction we encounter still greater complexity, since now we have coherence in the $n\ell j$'s of the three transferred nucleons, and in (say) the total angular momentum of the two neutrons. To simplify the discussion, let us focus our attention on the latter type of coherence, by assuming that all the nucleons are picked up from the same shell. If one assumes that the matrix element is dominated by the region of configuration space in which the incident proton is at the mass center of the three picked-up nucleons, the cross section can be expressed in terms of something proportional to

$$\sum_{J \atop (L=J \pm \frac{1}{2})} \frac{\sum_M |C^{LM}|^2}{2J+1} \left\{ \sum_I S_{B;IJ}^A (jj\frac{1}{2} - \frac{1}{2} | I0) (Ij0\frac{1}{2} | J\frac{1}{2}) \right\}^2, \quad (1'')$$

where, as before,

$$\Psi_{M_A}^{J_A(A)} = \sum_{B;IJ} S_{B;IJ}^A \left[\Psi_{M_B}^{J_B(B)} \left\{ [\psi^{\ell j}(n_1) \psi^{\ell j}(n_2)]^I \psi^{\ell j}(p) \right\}^J \right] M_A^{J_A}. \quad (2'')$$

The C^{LM} is a standard DWBA integral of a form factor times Y_M^L times distorted waves. The form factor is in essence the radial part of the wave function of the mass center of the picked-up nucleons, when their relative motion in the target is as it will be in the outgoing α . It is most easily obtained by assuming oscillator wave functions and performing a Mang transformation.⁴ Expression (1'') and Eq. (2'') clearly exhibit the coherence in the possible angular momenta of the picked-up neutrons.

Let us try out (1'') and (2'') in the simplest cases, namely those in which the target or daughter nuclei have angular momenta zero. Then only one angular-momentum transfer will occur. In the four cases I will discuss at first, we have transfers between $\frac{7}{2}^-$ and 0^+ states, so that $J = \frac{7}{2}$ and (by parity conservation, on the assumption of an even-parity alpha) $L = 3$. Figure 1 compares the experimental data and theory for the targets Sc^{45} , Ti^{48} , V^{51} , and Fe^{54} . The experiments were done by Funsten, Roberson, and Sherr on the Princeton cyclotron, and the curves were calculated by Ernest Rost and myself, the $S_{B;IJ}^A$ values used being those calculated as in (2'') from the wave functions of McCullen, Bayman, and Zamick⁵ based on the assumption of a pure $1f_{7/2}$ description of these nuclei. We do not have a normalized theory, and are operating on the assumption that much of the nuclear-structure information will express itself in terms of the relative cross sections, so that we may profitably try to understand these at this time. Thus all the (p, α) calculations I will show have a single normalization factor, chosen arbitrarily to match the experimental and theoretical $\text{Sc}^{45}(\frac{7}{2}^-)$ to $\text{Ca}^{42}(0^+)$ transition with 17.5-MeV incident protons. The optical parameters for the proton and alpha were chosen, as far as possible, to fit the corresponding elastic scattering.

⁴H. Mang, University of California, Lawrence Radiation Laboratory Report UCRL-8931.

⁵J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964).

Fig. 1. Comparison of calculated differential cross sections with experiment for the reactions $\text{Sc}^{45}(\text{p}, \alpha)\text{Ca}^{42}$, $\text{Ti}^{48}(\text{p}, \alpha)\text{Sc}^{45}$, $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$, $\text{Fe}^{54}(\text{p}, \alpha)\text{Mn}^{51}$, between the lowest $\frac{7}{2}^-$ and 0^+ states of the target and daughter nuclei. The incident proton energy is 17.5 MeV. The calculated curves have been normalized to produce the best fit to the absolute magnitude of the $\text{Sc}^{45}(\text{p}, \alpha)\text{Ca}^{42}$ cross section.

always gives $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$ the bigger cross section. These data thus favor the pick-up mechanism. More direct evidence supporting pickup is presented in the paper of Cerny and Pehl.⁶

It is seen that the fit to the angular distributions is not too bad; in fact, it could be called pretty good for $\text{Sc}^{45}(\text{p}, \alpha)\text{Ca}^{42}$ and $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$ [but pretty poor for $\text{Ti}^{48}(\text{p}, \alpha)\text{Sc}^{45}$]. I have remarked that these are all L=3 transitions. The differences in shape are largely due to the differences in Q value, so that the angular distributions for odd-A targets (small positive Q) are similar as are the angular distributions for even-A targets (small negative Q). Notice that once the over-all scale factor is chosen, the relative magnitudes are given fairly well. It is of some interest to remark that this pick-up theory agrees with experiment in giving the ground-state transition of $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$ a lower cross section than that of $\text{Sc}^{45}(\text{p}, \alpha)\text{Ca}^{42}$. I have calculated the consequences of a knock-out model using a variety of wave functions, and find that this

⁶J. Cerny and R. H. Pehl, Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 208.

Let us now turn to $\text{Sc}^{45}(\text{p}, \alpha)\text{Ca}^{42}(2^+)$. Here we connect $\frac{7}{2}^-$ to 2^+ with $J = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$ and thus with $L = 1, 3, \text{ and } 5$. The strengths of these contributions are determined by the spectroscopic amplitudes, as exhibited by expression (1''). Again, it is important that the expression in parentheses in (1'') involves a coherent linear combination of the spectroscopic amplitudes determined from the Sc^{45} wave function, so that we have some hope of learning about relative phases. Braid and Zeidman⁷ have done the experiment $\text{Sc}^{45}(\text{d}, \text{He}^3)\text{Ca}^{44}$, plucking out the 21st proton. They find a ground-state spectroscopic factor about 3 times that for the 2^+ state so that little strength is left for other states in Ca^{44} . This teaches us a great deal about the Sc^{45} ground state. It enables us to test the McCullen, Bayman, and Zamick wave function⁵ for this state, namely

$$\begin{aligned} \Psi_M^{7/2}(\text{Sc}^{45}) = & + 0.821 \left[\psi \left(\frac{7}{2}^4 00 \right) \psi \left(\frac{7}{2} \right) \right]_M^{7/2} + 0.543 \left[\psi \left(\frac{7}{2}^4 22 \right) \psi \left(\frac{7}{2} \right) \right]_M^{7/2} \\ & + 0.085 \left[\psi \left(\frac{7}{2}^4 24 \right) \psi \left(\frac{7}{2} \right) \right]_M^{7/2} - 0.013 \left[\psi \left(\frac{7}{2}^4 26 \right) \psi \left(\frac{7}{2} \right) \right]_M^{7/2} \\ & + 0.043 \left[\psi \left(\frac{7}{2}^4 42 \right) \psi \left(\frac{7}{2} \right) \right]_M^{7/2} - 0.146 \left[\psi \left(\frac{7}{2}^4 44 \right) \psi \left(\frac{7}{2} \right) \right]_M^{7/2} \\ & - 0.012 \left[\psi \left(\frac{7}{2}^4 45 \right) \psi \left(\frac{7}{2} \right) \right]_M^{7/2}. \end{aligned} \quad (4)$$

According to this wave function, the spectroscopic factors for the excitation of the ground state and first excited state in Ca^{44} are $(0.821)^2 = 0.675$ and $(0.543)^2 = 0.294$, in the ratio of 2.3. This is not 3, yet is not so far away as to make us very unhappy. But this one-particle-transfer reaction does not test the relative signs of the corresponding components in the wave function.

⁷T. H. Braid and B. Zeidman (private communication).

Fig. 2. Comparison of calculated differential cross sections with experiment for the $\text{Sc}^{45}(\text{p}, \alpha)\text{Ca}^{42}$ reactions induced by 17.5-MeV protons and leading to the ground state and first excited state of Ca^{42} . The upper curve corresponds to the ground-state transition; the two lower curves correspond to two different choices for the excited-state wave function. The normalization is the same as in Fig. 1.

Since the (p, α) reaction is coherent in these amplitudes, it does test the relative signs, as is shown in Fig. 2. The lower curve is the $\text{Sc}^{45}(\text{p}, \alpha)\text{Ca}^{42}(2^+)$ angular distribution calculated with the wave function (4), and the middle curve is obtained by replacing the 0.543 in Eq. (4) by -0.543. As expected, there is a sizeable effect on the angular distribution. The experimental points, again measured by Funsten, Roberson, and Sherr,⁸ seem to favor a relative plus sign,⁹ but it is clear that (4) is not yet adequate to give a complete explanation of the (p, α) data. But this example illustrates the point I wish to make. The (p, α) cross section generally depends in too complicated a fashion on the wave functions to enable us to infer, unambiguously, characteristics of the wave function from the experimental data. But if we first have 1-particle transfer data to give us the amplitudes of the various components, then

⁸H. O. Funsten, N. R. Roberson, and R. Sherr (private communication).

⁹The $\text{Ca}^{45} \rightarrow \text{Sc}^{45}$ log ft value is also sensitive to the relative phases of the components in Eq. (4). The experimental log ft value also favors the relative plus sign between the first two components.

accurate angular distributions for the (p, α) reaction can teach us a great deal about relative signs.

Funsten, Roberson, and Sherr also have angular distributions for the transitions to the 4^+ and 6^+ states of Ca^{42} . These will involve $L=7$ and $L=9$ components, which we have not yet calculated. They provide a further test of the Sc^{45} ground state. If we succeed in concocting a wave function similar to Eq. (4) and consistent with the (d, He^3) data, the magnetic moment, and the ft value, as well as all the (p, α) data, we will feel pretty confident that we have a good representation of this state. Note that we are putting all the complication of the (p, α) problem into the Sc^{45} wave function, assuming the Ca^{42} states are simple $\left(\frac{7}{2}^{\pm}\right)$ $I = 0, 2, 4, 6$. This may not be justified, but you have to start somewhere.

Figure 3 shows an angular distribution of the $\text{Fe}^{54}(\text{p}, \alpha)\text{Mn}^{51}$ $\left(\frac{7}{2}^{-}\right)$ transition measured at Boulder by Bardin, Rickey, and Sherr.¹⁰ The incident-proton energy is 28 MeV. The cross section is diminished somewhat, both in experiment and theory. (This figure uses the same normalization as Fig. 1.) Figure 4 shows the experimental and calculated spectra at 30° . Note that the transition to the $\frac{5}{2}^{-}$ ground state is weak

Fig. 3. Comparison of the calculated differential cross section with experiment for the $\text{Fe}^{54}(\text{p}, \alpha)\text{Mn}^{51}$ reaction induced by 28-MeV protons and leading to the first excited state $\left(\frac{7}{2}^{-}\right)$ of Mn^{51} at 0.24 MeV. The normalization of the calculated curve is the same as in Fig. 1.

¹⁰ B. M. Bardin, M. E. Rickey, and R. Sherr (private communication).

Fig. 4. Comparison between experiment and the calculated 30° differential cross sections for the excitation of states in Mn^{51} by the $\text{Fe}^{54}(p, \alpha)\text{Mn}^{51}$ reaction induced by 28-MeV protons. The vertical lines occur at the Mn^{51} excitation energies calculated by McCullen, Bayman, and Zamick.⁵ The lengths of these lines represent the calculated 30° differential cross sections to the correspond-

ing state, with the same normalization as was used in Fig. 1. The continuous curve is the experimental excitation function.

both in experiment and theory. The calculated spike at 4.4 MeV and the experimental peak at 4.3 MeV both refer to the $\text{Mn}^{51} \frac{7}{2}^-$ excited state which is the isobaric analogue of the Cr^{51} ground state. All the targets investigated with 28-MeV protons seem to excite the corresponding analogue states, and the theory also predicts strong excitation. Incidentally, this example shows another advantage of a many-particle-transfer process, namely, it often provides level spectra of nuclei which are otherwise difficult to reach.

* * *

R. R. CARLSON, State University of Iowa: You talked about (p, d) and (p, t) reactions in the case of light outgoing particles. You know the wave functions for the triton and deuteron very well. But in the case of the heavier particles such as Li^6 ions, do you assume that you know the wave function just as well?

BAYMAN: In our calculations, the wave function put in for the alpha particle in the case of the (p, α) reaction, or for the triton in the case of the (p, t) reaction, was not the best wave function (in its full complexity) that people have obtained by variational calculations. Instead one assumes that the internal wave functions of the outgoing tritons or alpha particles

are simple harmonic-oscillator eigenstates of reasonable size and with appropriate spin functions. If one can do that with the alpha particle, one can do much the same thing with Li^6 .

CARLSON: Some curious experimental results show up in the (Li^6, α) reaction, for instance. The angular distributions are very similar to those from (He^3, p) , in which the two particles are radically different. The angular distributions have very large peak-to-valley ratios; but they do not seem to depend at all on the character of the bombarding particles.

BAYMAN: Perhaps we can come back to this point after Rickey compares the $(\text{He}^4, \text{He}^6)$ reaction with the (p, t) process. I think that will be relevant.

R. H. DAVIS, Florida State University: You point out the advantage of using single-nucleon transfer to unscramble the interference for more complex nucleon transfers. Is there a similar advantage in the data from the inverse reaction, e. g., for the (α, p) reaction as compared with the (p, α) ?

BAYMAN: No. The (α, p) reaction will have the same sort of coherence as the (p, α) does. One gets the same sort of information from (α, p) as one does from (p, α) .

B. L. COHEN, University of Pittsburgh: I always had the impression that the ground state of an even-even nucleus had all positive signs in the wave function. In fact, that was the reason it was the ground state. It seems to me that this (p, t) experiment you are talking about determines nothing more than that. Am I right?

BAYMAN: What we really know about the ground state of an even-even nucleus is that the angular momentum is zero. Each pair of neutrons must be coupled to angular momentum zero; but that does not a priori tell the relative phases. Whether it is a positive or a negative sign in the ground state depends very sensitively on the kind of interaction. Now if

the interaction is taken to be a pairing force (a very reasonable assumption), then the proper choice of phases will give a positive sign. If the interaction were different from a pairing force, then the sign might be negative. I cannot tell you exactly what sort of interaction to concoct in order to get a minus sign, but I dare say a repulsive interaction (or even some reasonable sort of attractive interaction) might do so. Therefore, if we are completely convinced that the pairing type of picture is valid, then we are equally convinced that the wave function should have all signs plus. In this case, then, you might say that it is not necessary to do the experiment to establish that all the signs are positive; but if we want to check that assumption, then the experiment is needed.

COHEN: Wouldn't these forces that would give minus signs also destroy our understanding of why ground states are always 0^+ ?

BAYMAN: Not necessarily.

N. M. HINTZ, University of Minnesota: This is an addendum to the first comment. Aside from giving the signs of the U's and V's, (p,t) reactions are very sensitive to weak admixtures. You can pick up weak components in the wave function because they interfere with the strong components, thus giving more sensitivity than single-nucleon transfer reactions.

J. J. GRIFFIN, Los Alamos Scientific Laboratory: Are there any qualitative yes-no questions that one might ask from data on, say, the (p,t) and (p,d) reactions? In the simple case in which one has these coefficients, all but one is small. Can one get any intuitive feeling for that fact by just looking at the angular distribution—or is life just too complicated for that kind of a question to be appropriate?

BAYMAN: The fact is that life seems to be a lot simpler than would be expected from the full gory detail of those formulas that I showed. Particularly for the (p,t) process, for which there is coherence in the $n_1 \ell_1 j_1$

and $n_2 \ell_2 j_2$, it would not immediately occur to me from just looking at that formula that the shape would always be the same so long as the total angular momentum of the neutrons picked up is I ; the shape is buried in a very complicated way in that full coherent expression. I think Norton Hintz will show, as has also been shown in Boulder, that there is a very characteristic $I=0$ shape which seems to be independent of whether all of the particles are being removed from the same shell or whether they have been removed from different shells. I do not understand why this should be the case; perhaps someone else does have some idea about it.

In addition to this simple feature, one can make other simple estimates. For example, if we assume that particles do stay in the same shell, say in the $f_{7/2}$ shell, then for the even- A targets the ground-state-to-ground-state transitions would be from angular momentum zero to angular momentum zero. In this case, we do not need to know any more about the wave function because the angular momentum determines the state for these ground states. Then as you go through the shell you see a characteristic peaking and then a falling-off. This has been seen by the group at Oak Ridge in the case of zirconium isotopes.

S. MESHKOV, U. S. National Bureau of Standards: Can you say something about a less simple case in which you don't have just $f_{7/2}$? A well-known problem is the second 0^+ state in Ca^{42} . Is there any way to get some handle on the components of that system from, say, Sc^{45} ?

BAYMAN: A straightforward way of going about it is to make some sort of ansatz about what you think is a reasonable representation of this state and then test the hypothesis. I do not know of any other way of just taking the raw data and working out the wave functions.

You can make qualitative estimates of this sort in the (t,p) reaction, for example, in which the states being populated look like the target nucleus plus two correlated neutrons. If such a state is seen to be

very strongly populated, then you tend to say that this is the character of this particular state. I do not anticipate that the (p, α) reaction will tell us very much about that 0^+ state until we have information about that state from other data.

A. G. BLAIR, Los Alamos Scientific Laboratory: Will you bring me up to date on reaction mechanisms—in two or three well chosen sentences, perhaps? For example, in the (p, α) reaction, doesn't one have to consider the possibility of knockout? Why does it behave as a pickup reaction only? After all, there should be a lot of alpha particles in the nuclei that you are talking about.

BAYMAN: I think it is fair to say (and I think Rubby Sherr would agree with me) that when we started considering the (p, α) process $3\frac{1}{2}$ years ago, we started with the prejudice that the (p, α) proceeded by knockout. The investigation was started to prove that Bernie Cohen was wrong in saying that it was a pickup process; pickup was just forced upon us by the data—especially by such things as $V^{51}(p, \alpha)$ and $Sc^{45}(p, \alpha)$.

It is not easy to say why this should be. One approach is in terms of a very short-range picture. Only two protons can be put at the same point: one with spin up and one with spin down. Only two protons are there; then when another proton comes along it cannot get very close because it is a third proton and it would violate the exclusion principle if it came too close. Of course, the alpha particle is not a point particle, and the interaction ranges are not actually that small. But it may well be that something such as this is keeping the knockout process down as compared with the pickup.

B. G. HARVEY, Lawrence Radiation Laboratory: With regard to this knockout process, I think that we know something about the scattering of protons from helium and that $(\alpha, 2\alpha)$ experiments, for example, tell us something about the number of alpha particles in the surface—probably not very accurately. When you put these bits of information together,

what do you get? Does it really appear that the alpha particle couldn't be knocked out, or hasn't this been tried?

BAYMAN: I am not completely convinced that the interpretation of the results is completely unambiguous in this way. I have not tried to do what you are suggesting.

N. AUSTERN, University of Pittsburgh: I would like to subscribe to Ben Bayman's last remark; I am not convinced by the alpha correlation experiments either.

I also want to comment on the question of the distinction between pickup and knockout, which I think is more a distinction of words than of physics. There are the nuclear particles which are capable of participating in the process, there are the projectile particles, and there are the interactions among them. If the wave functions are proper wave functions, i. e., if the wave functions are symmetrized, then pickup and knockout are included automatically. The question of their relative importance is the question of exchange amplitudes. As usual, these things are not fully checked by calculations, but our sentiments are that exchange amplitudes are small. Those are what we call the "knockout amplitudes." It is not that these two processes are mutually exclusive; they certainly are both present. The fact that the events seem to follow one rather than the other is a description of what the wave function is like.

R. D. LAWSON, Argonne National Laboratory: With regard to Bernie Cohen's question about the relative sign of A and B in the ground-state wave function, I think that if you take the ordinary central force (not a pairing force, but an ordinary delta-function or finite-range central force) and calculate the off-diagonal matrix element between one pair of particles of even parity coupled to angular momentum zero and another pair of particles of odd parity coupled to angular momentum zero, then the relative

sign will ordinarily turn out to be opposite to that obtained from the diagonal matrix elements. This would imply, I think, that if j_1 in your example is an even-parity level and j_2 is an odd-parity level, then A and B will have opposite signs. I think this means that there certainly is no necessity that the signs be all positive in the ground-state wave function.

BAYMAN: I think that Bernie Cohen's implication would be that in such a case you should choose the phases of your states a little differently. A pairing force is constructed in such a way that its relative phases should look like a delta force. There have been many phase conventions, but I think that the situation in pairing-type calculations is that the relative phases of the different components in the basic pair are the same as that given by the delta force. I think Ray Sorensen can check me on that.

R. A. SORENSEN, Carnegie Institute of Technology: That is correct.

A STUDY OF THE NUCLEAR REACTIONS $\text{Mg}^{26}(\text{p}, \text{t})\text{Mg}^{24}$
 AND $\text{Mg}^{26}(\text{He}^4, \text{He}^6)\text{Mg}^{24} \dagger^*$

M. E. Rickey, $\dagger$ H. E. Wegner, and K. W. Jones

Brookhaven National Laboratory, Upton, L.I., New York

and

B. M. Bardin, C. Hoot, and M. Kondo

University of Colorado, Boulder, Colorado

Abstract of Report in "Vol. I. Contributed Papers" $\dagger$

A comparison of spectra and angular distributions is made for the two-neutron pickup reactions $\text{Mg}^{26}(\text{p}, \text{t})\text{Mg}^{24}$ at $E_p = 28$ MeV and $\text{Mg}^{26}(\text{He}^4, \text{He}^6)\text{Mg}^{24}$ at $E_\alpha = 40$ MeV. The energies are such that the ground-state pickup momentum transfers at 15° lab are equal; this permits a study of the relative excitations for transitions to the discrete state observed. Comparisons are made for the ground state (0^+), and the 1.37-MeV (2^+), 4.12—4.23-MeV ($4^+ - 2^+$), 5.22-MeV (3^+), and 6-MeV (4^+) states. Unresolved structure at higher energies is compared in terms of its gross properties. During the course of the measurements, particles with higher charge were also observed, consisting of Li^6 , Li^7 , and perhaps Be^7 . The $\text{Li}^6 - \text{Li}^7$ yield was considerably stronger than the He^6 yield, and the energy distribution showed marked structure.

$\dagger$ Work performed under the auspices of the U. S. Atomic Energy Commission.

* Contributed paper. Presented by M. E. Rickey.

$\dagger$ On leave 1963-1964 from the University of Colorado.

$\dagger$ M. E. Rickey, H. E. Wegner, K. W. Jones, B. M. Bardin, C. Hoot, and M. Kondo, in Argonne National Laboratory Report ANL-6848, March 1964, p. 217.

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C. D. ZAFIRATOS, Los Alamos Scientific Laboratory: What cross section did you find (say, in the forward peak) for the (α, He^6) reaction to the ground state?

RICKEY: We found $10 \mu\text{b/sr}$.

ZAFIRATOS: In the $\text{Al}^{27}(\alpha, \text{Li}^6)$ or $\text{Al}^{27}(\alpha, \text{Li}^7)$ reaction (we could not separate the two), the peak cross section for the ground-state transition is about $20 \mu\text{b/sr}$ at a forward angle. This is certainly a very nearby case in which the cross section for the formation of Li^6 or Li^7 seems to be roughly equivalent to that which you observed for He^6 in this reaction. What particles did you say had 100 times the intensity of the He^6 ? Was it just the particles with $Z = 3$, or was it everything above He^6 ?

RICKEY: It was everything higher than He^6 . For instance, Be^7 could be there.

ZAFIRATOS: That is why I wonder if something else may have been contributing to this strikingly large ratio you observed.

RICKEY: That was true in the last spectrum shown. However, in the earlier work for which no figure is presented, we were using a 25-micron detector that would specifically pick out mass-3 fragments, and it was quite clear there also. This group with mass 6 and 7 and charge 3 was very strong relative to the He^6 group.

J. CERNY, Lawrence Radiation Laboratory, Berkeley: This is in response to your comment about the relative sizes of the cross sections for the (p, He^3) and (p, t) reactions. For $T=0$ even-even target nuclei, it turns out for analogue final states that, as long as the outgoing particles have high enough energies, the (p, t) cross section is twice the (p, He^3) . For an even-odd target nucleus, the ratio may be 1:1. Experimentally there is no factor of several hundred or anything like that.

DEUTERON-TRANSFER REACTIONS*

D. Allan Bromley

Yale University, New Haven, Connecticut

I. Introduction

During the past decade a substantial fraction of all nuclear reaction work has been concentrated on the single-nucleon-transfer situation; coupled with this rapidly increasing output of experimental data has been the evolution of relatively sophisticated analysis techniques. These data have provided a major source of information on single-particle aspects of nuclear structure, on parentage questions, and most recently on the pairing and collective-motion behaviour of nuclear systems.

It is somewhat surprising that, in contrast, the multi-nucleon-transfer reactions have received relatively little study. This paper will briefly review some of the currently available information on only one specific type of two-nucleon transfer, namely, that involving transfer of a neutron-proton pair, i. e., a deuteron in either singlet or triplet configurations.

Table I presents a listing of reactions of this type involving available light projectiles together with representative Q values for targets in the range from He^4 to Ca^{40} . A number of reasons for the dearth of experimental data until very recently are obvious from the table. In cases of complex reaction products, recently developed high-resolution particle-identification systems are essential; until recently, inadequate beam

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TABLE I. Deuteron-transfer reactions.

Reaction	"Average" Q value ^a (MeV)
1. A(n,t)B	-10
2. (t,n)	+ 9
3. (p,He ³)	- 9
4. (He ³ ,p)	+11
5. (d,α)	+ 9
6. (α,d)	- 9
7. (α,Li ⁶)	-15
8. (Li ⁶ ,α)	+13
9. (B ¹¹ ,Be ⁹)	+ 1

^a"Average" Q values listed here-in were obtained as an average of actual or calculated Q values for accessible targets in the range from He⁴ to Ca⁴⁰.

energy resolutions were available for precision studies of the negative-Q reactions; and finally, beams of the heavier projectile (i. e., Li⁶ and B¹¹) of adequate quality and energy have only recently become available.

Essentially no work of spectroscopic nature has been reported on the (n,t) or (t,n) reactions; although quite extensive measurements are available on (He³,p) reactions,¹ isolated data on the inverse reaction have only recently become available.^{2,3} The (d,α) and (α,d) reactions have received extensive study in the energy ranges $E_d \leq 20$ MeV and $E_\alpha \leq 50$ MeV. (Li⁶,α) studies at low energies ($E_{Li^6} \leq 10$ MeV) are

¹D. A. Bromley and E. Almquist, Reports on Progress in Physics 23, 544 (1959); see also Chalk River Report CRP 881, 1959 (unpublished).

²J. Cerny and R. H. Pehl, Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 208.

³P. E. Cavanaugh, C. F. Coleman, G. A. Gard, and D. A. Boyce, PLA Progress Report, NIRL/R/60, Rutherford High Energy Laboratory, Didcot, Berks.

in progress in many laboratories⁴; at higher energies ($30 \leq E_{\text{Li}^6} \leq 60$ MeV) available evidence suggests⁵ that dissociation of the projectile, rather than transfer, completely dominates the reaction, largely precluding spectroscopic studies. Spectroscopic measurements on the ($\text{B}^{11}, \text{Be}^9$) reaction⁶ have required boron beams of adequate quality which have become available from the heavy-ion linear accelerators. It represents a unique reaction in the sense that it avoids the ambiguity characteristic of heavy-ion transfers where both the residual nucleus and product have low-lying excited states. In such situations it is not a priori possible to assign an observed Q value and corresponding excitation to one or both nuclei; fortunately Be^9 is unstable against neutron emission in all of its excited states and thus only unexcited Be^9 products survive the target-to-detector flight time and are registered. Structure in their spectrum necessarily reflects excitations in the conjugate product.

Studies on this latter reaction clearly require that the Be^9 be experimentally separable from a wide variety of other products. Identification systems based on $(dE/dx) \times E$ techniques are in widespread

⁴ For detailed references see G. C. Morrison, Direct Interactions and Nuclear Reaction Mechanisms, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, New York, 1963), p. 878.

⁵ R. Ollerhead, C. Chasman, and D. A. Bromley, Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, 1963, edited by A. Ghiorso, H. E. Conzett, and R. M. Diamond (University of California Press, 1963); Phys. Rev. (in press).

⁶ M. W. Sachs, C. Chasman, and D. A. Bromley, Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, 1963, edited by A. Ghiorso, H. E. Conzett, and R. M. Diamond (University of California Press, 1963); Phys. Rev. (in press).

use for study of the hydrogen and helium isotopes⁷⁻¹¹ and more recently for the lithium isotopes.^{12,13} For our work with the Yale HILAC we have concentrated on extending this approach to heavier systems¹⁴⁻¹⁶ and at present can reliably separate the oxygen isotopes and, with some ambiguity, those of fluorine. Figure 1 is typical of the isotopic separation achieved using an analog pulse multiplier; significantly better resolution has been obtained through direct use of our multiparameter analyzer on the two detector outputs.

Extremely arbitrary criteria have been applied in selecting data for inclusion in this paper in an attempt to emphasize areas where the deuteron-transfer reaction may be able to provide new and important information on nuclear structure; extensive radiochemical studies, particularly on (α ,pn) and (α ,d) reactions, for example, have been excluded entirely and no attempt has been made to assemble a complete bibliography.

⁷R. H. Stokes, Rev. Sci. Instr. 31, 768 (1960).

⁸F. Aschenbrenner, Phys. Rev. 98, 657 (1955).

⁹J. C. Keck, Phys. Rev. 85, 410 (1952).

¹⁰B. Wolfe, A. Silverman, and J. W. De Wire, Rev. Sci. Instr. 26, 504 (1955).

¹¹H. E. Wegner, Rev. Sci. Instr. 23, 271 (1962).

¹²H. E. Wegner (private communication, 1964).

¹³C. D. Zafiratos, doctoral dissertation, University of Washington, Seattle, 1962 (unpublished).

¹⁴D. A. Bromley, M. W. Sachs, and C. E. Anderson, in Symposium on Nuclear Instruments, Harwell, England, 1961, edited by J. B. Birks (Heywood & Co., Ltd., London, 1962), p. 67.

¹⁵C. E. Anderson, D. A. Bromley, and M. W. Sachs, Nucl. Instr. Methods 13, 238 (1961).

¹⁶M. W. Sachs, C. Chasman, J. P. Allen, J. Poth, and D. A. Bromley, Nucl. Instr. Methods (to be published).

Although we concentrate here on the spectroscopic rather than the mechanism aspects of the reactions, a basic question concerning the reaction mechanisms must be answered: Do we understand the mechanisms well enough to extract unambiguous spectroscopic information from the experimental results?

In the following sections we shall attempt to answer this question and then examine a number of the areas to which deuteron-stripping reactions can make important contributions.

II. Two-Nucleon-Transfer Reactions

Selection Rules

Much of the interest in the two-nucleon-transfer reactions, as direct spectroscopic probes, stems from the selection rules on angular momentum and parity in the reaction

$$J_i^{\pi_i} + (\ell_n, \ell_p)^{\pi_d} \rightarrow J_f^{\pi_f},$$

namely

$$\overline{J}_f = \overline{J}_i + \overline{L} + \overline{S},$$

$$\overline{L} = \overline{\ell}_p + \overline{\ell}_n,$$

$$\overline{S} = \overline{s}_p + \overline{s}_n,$$

where the ℓ_p and ℓ_n are defined relative to the capturing target core. It should be noted that as long as other than S-wave capture is involved for both nucleons, more than a single value of L is involved. Moreover if $\pi_d = (-1)^{\ell_p + \ell_n}$, there is no a priori reason why both even and odd values of L could not contribute. This situation has been clarified by Glendenning¹⁷

¹⁷N. K. Glendenning, Nucl. Phys. 29, 109 (1962).

Fig. 1. (A) Characteristic hyperbolic loci obtained by plotting dE/dx vs E for reaction products from $C^{12} + C^{12}$ interactions at an incident energy of 126 MeV. This illustration is a direct photograph of a plotting-oscilloscope screen with a superimposed raster. Reading from the top, the hyperbolic loci correspond to C^{13} , C^{12} , C^{11} , B^{11} , B^{10} , Be^{10} (weak), Be^9 , Be^7 , Li^7 , Li^6 , and alpha particles. In the latter case the semiconductor E detector was not adequately thick to stop the most energetic

who notes that if a plane wave representation of the incident beam is assumed and if L is decomposed as

$$\overline{L} = \overline{L}_{c.m.} + \overline{L}_{rel},$$

with components referring respectively to the orbital momentum of the center of mass of the pair relative to the target and relative to one another, then the parity of the captured pair is $(-1)^{\overline{L}_{c.m.} + \overline{L}_{rel}}$. In the hydrogen and helium projectiles, the transferred pairs all have $\overline{L}_{rel} = 0$; so also is the case in Li^6 if, as now appears well founded, we assume an $(\alpha+d)$ cluster

alpha particles and the characteristic fold-back of the hyperbolic locus results. Hydrogenic ions fall below the axes chosen here and oxygen and heavier nuclei have been removed by absorption. Structure is clearly evident in several of the loci shown here corresponding to excitation of isolated residual states. (B) Multiplier spectrum obtained by analog multiplication of dE/dx and E signals in the above reaction. (From Sachs, Chasman, and Bromley, Ref. 6.)

configuration. In the B^{11} projectile, the situation is much less clear and more data are required to elucidate it.

To the extent that we can ignore other than S-wave relative angular momenta, it follows that

$$\pi_f = \pi_i (-1)^{\ell_p + \ell_n} = \pi_i (-1)^L$$

so that L is restricted to either odd or even values.

Fig. 2. Proton angular distributions from the $O^{16}(He^3, p)F^{18}$ reaction leading to the 0^+ , $T=1$ and 3^+ , $T=0$ states at 1.038 and 0.934 MeV, respectively, in F^{18} . Data points are from Hinds and Middleton (Ref. 19); theoretical curves are from Newns PWBA cut-off theory (Ref. 22).

As first noted by Hinds and Newns,¹⁸ the correlation between spin and isobaric spin in the two-nucleon system results in the following selection rules for the indicated reactions and their inverses:

$$\begin{aligned}
 S = 0, 1 & \quad \text{for } (He^3, p), (t, n), (B^{11}, Be^9); \\
 S = 0 & \quad \text{for } (He^3, n), (t, p); \\
 S = 1 & \quad \text{for } (\alpha, d), (Li^6, \alpha).
 \end{aligned}$$

It follows clearly from these selection rules that, for example, the $O^{16}(He^3, p)F^{18}$ reaction involving transfer of a singlet 3S_0 deuteron can populate only natural-parity states with $T = 1$ in F^{18} whereas that involving transfer of a triplet 3S_1 deuteron can populate all states in F^{18} with $T = 0$ excepting those with $J^\pi = 0^+$. Figure 2 illustrates this from the work on

¹⁸S. Hinds and H. C. Newns (private communication to R. Middleton, 1961); see reference.

this reaction by Hinds and Middleton¹⁹ at Aldermaston. In the reaction to the 0^+ , $T=1$ state at 1.038 MeV, an $L=0$ transfer of a ${}^{31}\text{S}_0$ deuteron is involved whereas in that to the 3^+ , $T=0$ state at 0.934 MeV, an $L=2$ transfer of a ${}^{13}\text{S}_1$ deuteron is indicated.

Unfortunately, the deuteron-transfer reactions do not provide as direct a probe for spin and parity assignments as do those characterized by $S = 0$ only, in the above listing, where $J_f = L$ for 0^+ targets. On the other hand, the possible contribution of differing L values can provide a measure of the different configurational amplitudes in the final state, as will be discussed later, and on the coupling scheme itself.

Spectroscopic studies with these reactions fall naturally into two classes: those directed toward determination of quantum numbers (e.g., J , π , T) and, to a lesser extent, the gross wave-function characteristics through determination of differential cross sections, and those directed toward determination of more detailed structural information concerning the states involved through determination of absolute cross sections.

It should be emphasized that the above selection rules are not mechanism independent and further measurements are necessary to support the approximations which are implicit in them. The same is true in the case of the numerous plane-wave and very recent distorted-wave Born-approximation calculations of the reaction cross sections to which we shall refer.

Plane-Wave Theory

Bayman²⁰ has discussed these calculations in the preceding paper. However, for completeness we shall summarize a number of the

¹⁹ S. Hinds and R. Middleton, Proc. Phys. Soc. (London) 74, 762 (1959).

²⁰ B. Bayman, these Proceedings, p. 335.

results here. Apart from specific details (e. g., El Nadi²¹ does not include conservation of angular momentum and parity automatically, Newns²² does not consider explicit nuclear-structure information, Rustgi²³ considers the possibility of spin flip of one transferred nucleon, etc.), all the PWBA calculations predict an angular distribution of the form

$$\frac{d\sigma}{d\Omega} = F \sum_L |A_{Lj_L}(qR)|^2,$$

where F is an effective damping factor proportional to the momentum transform of the projectile and product wave functions and the A_L are structure-dependent coefficients weighting the different L contributions permitted by the selection rules. This is in marked contrast to the situation in single-nucleon stripping where only the lowest allowed L value has been found to contribute significantly to the reaction.

This expression is valid for stripping, knockout, and exchange-stripping reactions providing that the momentum transfer $\bar{q}$ is redefined²⁴ as follows for the reaction $M_T(M_I, M_P)M_R$:

Stripping:
$$\bar{q}_S = k_I - \frac{M_T}{M_R} \cdot k_P;$$

²¹M. El Nadi, Phys. Rev. 119, 242 (1960); Nucl. Phys. 28, 331 (1961); Proc. Phys. Soc. (London) A70, 62 (1957); Phys. Rev. 120, 1360 (1960).

²²H. C. Newns, Proc. Phys. Soc. (London) 76, 489 (1961).

²³M. L. Rustgi, Nucl. Phys. 25, 169 (1961).

²⁴M. K. Banerjee, in Nuclear Spectroscopy, Part B, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1960), p. 695.

Knockout:
$$\bar{q}_{\text{KO}} = \left(1 - \frac{M_{\text{P}}}{M_{\text{T}}}\right) \bar{k}_{\text{I}} - \left(1 - \frac{M_{\text{I}}}{M_{\text{R}}}\right) \bar{k}_{\text{P}} ;$$

Exchange stripping:
$$\bar{q}_{\text{ES}} = \bar{k}_{\text{I}} + \frac{M_{\text{I}}}{M_{\text{R}}} \cdot \bar{k}_{\text{P}} .$$

It follows from these expressions that if $M_{\text{T}} \gg M_{\text{P}}$ and $M_{\text{R}} \gg M_{\text{I}}$, as is the case except in reactions involving the lightest nuclei or in transfer reactions involving more complex projectiles and products [e.g., $(\text{B}^{11}, \text{Be}^9)$], then $\bar{q}_{\text{KO}} \approx \bar{q}_{\text{S}}$ and slight changes in the interaction radius R , appearing in the argument of the spherical Bessel function, will preclude determination of the mechanism from fitting the angular distributions alone.

As an example of a situation in which $q_{\text{S}} \neq q_{\text{KO}}$, Zeidman and Yntema²⁵ have studied the $\text{Li}^6(\alpha, \text{d})\text{Be}^8$ reaction at 42 MeV and have found that whereas a qualitative fit to their data could be obtained with a stripping mechanism, none was possible assuming knockout. Cohen and Rubin²⁶ have also presented a number of arguments against significant knockout contributions to either one- or two-nucleon-transfer reactions.

One of the few pieces of evidence which might suggest such a mechanism comes from studies of (d, α) reactions on C^{12} , N^{14} , O^{16} , F^{19} , Ne^{20} , Mg^{24} , Al^{27} , P^{31} , and S^{32} targets carried out at $E_{\text{d}} = 14.7$ MeV by Yanabu²⁷ and by Takamatsu.²⁸ Ground-state transition cross sections on the alpha-particle (4n) targets were consistently and markedly higher than those on non-alpha-particle (non-4n) targets. Even here, however, it is possible to invoke an argument, reminiscent of the Huby triangle rule²⁹

²⁵ B. Zeidman and J. L. Yntema, Nucl. Phys. 12, 298 (1959).

²⁶ B. L. Cohen and A. G. Rubin, Phys. Rev. 114, 1143 (1959).

²⁷ T. Yanabu, J. Phys. Soc. (Japan) 16, 2118 (1961).

²⁸ K. Takamatsu, J. Phys. Soc. (Japan) 17, 896 (1962).

²⁹ R. Huby, Proc. Roy. Soc. (London) 215A, 385 (1952).

in single-nucleon stripping, which may explain these (d, α) results as suggested by Pehl.³⁰ A similar argument will be invoked later in connection with selective population of two-nucleon states.

Assuming primarily a surface interaction, only partial waves in the entrance and exit channel in the vicinity of $L_d = k_d R$ and $L_a = k_a R$, respectively, will be expected to contribute strongly. Since the angular momentum transferred to the core in the transfer is given by

$$\underline{L} = \frac{M_R}{M_T} \cdot \underline{L}_d - \underline{L}_a ,$$

if $\ell_p + \ell_n \leq L'$, say (the momentum carried by the two nucleons), it is clear that if $L > L'$ the reaction will be inhibited by this angular-momentum mismatch to an extent dependent upon the degree of this inequality.* Since (d, α) reactions on 4n nuclei characteristically have lower Q values than those on non-4n nuclei, part of the observed difference may be explained without invoking knockout. Additional evidence supporting this hypothesis follows from the observation that the cross sections decrease with increasing A (hence R) leading to increased mismatch.

In the more complex transfer cases [e.g., (B^{11} , Be^9)], the knockout mechanism is expected a priori to have low probability reflecting the complexity and relatively low binding energies of the products involved.

³⁰ R. H. Pehl, doctoral dissertation, University of California, Lawrence Radiation Laboratory Report UCRL-10993, February 1964.

*Note added in proof: This momentum mismatch inhibition argument may be treated in a much more formal fashion assuming, in particular, a direct mechanism but with almost identical results. I am indebted to K. R. Greider for discussions on this formal treatment which will be published elsewhere.

Fig. 3. Deuteron angular distributions from the $C^{12}(\alpha, d)N^{14}$ g.s. reaction. In each curve, A represents the experimental data of Harvey and Cerny (Ref. 31); in the upper figure, the theoretical curve B was calculated from the Butler equations with $L = 2$, $R_0 = 6$ fm; in the lower figure, curve B was calculated from the Glendenning theory (Ref. 17) using $j_n = j_p = \frac{1}{2}$ and $R_0 = 4.7$ fm.

Among the approximations implicit in the PWBA treatments are (a) that the capture of the two-nucleon cluster, treated as an inert lump, occurs at the interaction surface (point capture); (b) that the target nucleus remains inert through the reaction and establishes the shell-model states into which the nucleons are captured; and (c) that plane-wave states are assumed for both entrance and exit channels. If, in addition, the internal structure of the incident and product particles is ignored, then the damping factor F degenerates to a constant as has been the case in many of the PWBA analyses reported to date.

Figure 2 illustrates typical fits to forward-angle data in the $O^{16}(He^3, p)F^{18}$ reaction using the Newns PWBA point-contact theory.²² Figures 3 and 4 show Harvey and Cerny's³¹ comparisons of angular distributions from the $C^{12}(\alpha, d)N^{14}$ reactions to the 1^+ ground and second excited states of N^{14} , with calculations using both the Butler³² and

³¹ B. G. Harvey and J. Cerny, Phys. Rev. **120**, 2162 (1960).

³² S. T. Butler, Nuclear Stripping Reactions (John Wiley and Sons, Inc., New York, 1957).

Fig. 4. Deuteron angular distributions leading to the 3.95-MeV state in N^{14} in the $C^{12}(\alpha, d)N^{14}$ reaction. In each case, curve A gives the experimental data of Harvey and Cerny (Ref. 31). In the left figure, curve B was calculated from the Butler equations with $L = 2$, $R_0 = 5.5$ fm; in the right figure, curve B was calculated from the Glendenning equations (Ref. 17) with $j_n = j_p = \frac{1}{2}$ and $R_0 = 4.3$ fm.

Glendenning¹⁷ formalism. Both give reasonable fits and, if anything, the simple Butler fits are somewhat preferable. Figure 5, also taken from Cerny,³³ shows similar fits to the $Li^6(\alpha, d)Be^8$ data. As indicated, the interaction radius required to obtain the fit to the rather anomalous distribution to the 2.90-MeV Be^8 state is only 2.1 fm suggesting that the fit itself is fortuitous.

While these data are typical of many of the higher energy deuteron-transfer distributions to states of other than pure two-particle parentage, which we shall consider in a later section, the situation is not without surprises. Figure 6 compares angular distributions from the $N^{14}(\alpha, Li^6)C^{12}$ measurements of Zafiratos³⁴ and from the $N^{14}(d, \alpha)C^{12}$ measurements of Fischer and Fischer,³⁵ plotted as functions of

³³ J. Cerny, B. G. Harvey, and R. H. Pehl, Nucl. Phys. 29, 120 (1962).

³⁴ C. D. Zafiratos, doctoral dissertation, University of Washington, Seattle, 1962 (unpublished); private communication, 1964.

³⁵ G. E. Fischer and V. K. Fischer, Phys. Rev. 114, 533 (1959).

Fig. 5. Deuteron angular distributions in the $\text{Li}^6(\alpha, d)\text{Be}^8$ reaction as measured by Cerny and Harvey (Ref. 31). In the upper figure a Butler fit using $L = 0$, $R_0 = 6.9$ fm is shown; in the middle figure a Glendenning fit using $j_n = j_p = \frac{3}{2}$ and $R_0 = 7.6$ fm is shown; and in the lower figure again a Glendenning fit with $j_p = j_n = \frac{3}{2}$ but $R_0 = 2.1$ fm is shown.

Fig. 6. Comparison of the angular distributions from the $N^{14}(\alpha, Li^6)C^{12}$ and $N^{14}(d, \alpha)C^{12}$ reactions, as studied by Zafiratos (Ref. 34) at 42 MeV and by Fischer and Fischer (Ref. 35) at 15 MeV, both presented as functions of the momentum transfer assuming in each case a deuteron-pickup reaction.

momentum transfer on the assumption of a pickup mechanism in both cases. The striking discrepancies here point to a much more complex mechanism and similar systematic comparisons would be of considerable importance in permitting isolation of the salient differences.

Reaction Mechanisms at Low Energy

At lower incident energies, the mechanism appears to depend very strongly on the individual reaction, its Q value, and the Coulomb and centrifugal barriers involved. Excitation functions characteristically show structure indicating the importance of compound amplitudes, although even under these conditions the angular distributions suggest dominant direct amplitudes. Figure 7 presents typical $O^{16}(d, \alpha)N^{14}$ g.s. excitation curves, as measured by Baumgartner and Fulbright,³⁶ and Fig. 8 shows similar data for the $B^{10}(\alpha, d)C^{12}$ reaction

³⁶E. Baumgartner and H. W. Fulbright, Phys. Rev. 107, 219 (1957).

Fig. 7. Excitation curves for the $O^{16}(d,\alpha)N^{14}$ g.s. reactions as measured by Baumgartner and Fulbright (Ref. 36). The peaks at deuteron energies of about 3.85 and 4.00 MeV correspond to excitations of 10.95 and 11.09 MeV in F^{18} .

Fig. 8. Excitation curves for the $B^{10}(\alpha, d)C^{12}$ g.s. reaction as measured by Ono and Kuroda (Ref. 37).

measured by Ono and Kuroda.³⁷ Figure 9 is a typical angular distribution measured by the latter authors at $E_\alpha = 3.68$ MeV; the large backward peak has been interpreted by them as reflecting dominant exchange-stripping amplitudes.

Inglis³⁸ has interpreted the striking difference between the angular distributions to the ground and second excited states of N^{14} in the $C^{12}(Li^6, \alpha)N^{14}$ reaction shown in Fig. 10, from Pham-Dinh-Lien and Marquez,³⁹ as reflecting the fact that these states are almost pure 3D and 3S , respectively. At low incident Li^6 energies, only an exchange or

Fig. 9. Deuteron angular distribution in the $B^{10}(\alpha, d)C^{12}$ g.s. reaction measured at $E_\alpha = 3.68$ MeV by Ono and Kuroda (Ref. 37).

³⁷K. Ono and K. Kuroda, Phys. Rev. 117, 214 (1960).

³⁸D. R. Inglis, Phys. Rev. 126, 1789 (1962).

³⁹Pham-Dinh-Lien and L. Marquez, Nucl. Phys. 33, 202 (1962).

Fig. 10. Angular distributions of alpha particles leading to the ground state and 3.95-MeV state of N^{14} from the reaction $C^{12}(Li^6, \alpha)N^{14}$ as studied by Pham-Dinh-Lien and Marquez (Ref. 39) at 2 MeV.

heavy-particle stripping mechanism satisfies orbital angular-momentum requirements. The argument is closely tied to the strong $(\alpha + d)$ cluster structure of Li^6 and is thus peculiar to the Li^6 reactions.

Figure 11 shows results of Hobbie and Forbes,⁴⁰ at higher energies, involving these same states. The qualitative similarity of all these angular distributions and the relatively slow variation with energy are also characteristic of a direct mechanism.

Although a priori it would be anticipated that Li^6 -induced reactions would provide a unique probe for the determination of otherwise inaccessible alpha-particle and deuteron reduced widths of bound residual states, it appears that the Li^6 cluster structure will limit the usefulness of this technique. In measurements below, and in the vicinity of, the

⁴⁰R. K. Hobbie and F. F. Forbes, Phys. Rev. 126, 2137 (1962).

Fig. 11. Comparison of the angular distributions of alpha particles from the $C^{12}(Li^6, \alpha)N^{14}$ reaction to the 1^+ , $T=0$, 3D ground state (left) and the 1^+ , $T=0$, 3S state at $E_x = 3.95$ MeV in N^{14} . The Li^6 energies are as indicated on the curves. (From Hobbie and Forbes, Ref. 40.)

Coulomb barrier energy, carried out at Chicago,⁴¹ Minnesota,⁴⁰ Iowa,⁴² and Yale,⁴³ striking evidence was obtained for almost pure direct-reaction amplitudes.

Reaction Mechanism at Intermediate and High Energies

In going to much higher energies to avoid distortion effects, in analogy with earlier single-nucleon stripping, Ollerhead, Chasman, and Bromley⁵ find that a direct dissociation mechanism completely dominates the interaction, with extremely weak population of residual final states from which the desired information might be obtained. This is illustrated in rather striking fashion in Fig. 12, which compares alpha-particle spectra from low⁴⁰ and high energy⁵ Li^6 bombardment of C^{12} . The broad structure in the high-energy alpha spectrum has been shown⁵ to result from direct dissociation of the Li^6 in the interaction.

Measurements by Morrison,^{4,44} for $E_{\text{Li}^6} \approx 10$ MeV, show angular distributions which vary markedly with energy and exit channel indicating a complex reaction mechanism, again precluding simple extraction of spectroscopic data and suggesting that at the lower energies,

⁴¹S. K. Allison, Proceedings of the Second Conference on Reactions Between Complex Nuclei, Gatlinburg, Tennessee, May 2—4, 1960, edited by Alexander Zucker, F. T. Howard, and E. C. Halbert (John Wiley and Sons, Inc., New York, 1960), p. 253; G. C. Morrison and M. N. Huberman, *ibid.*, p. 246.

⁴²R. R. Carlson and E. Norbeck, Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, California, 14—18 April 1963, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), p. 181.

⁴³D. A. Bromley, A. R. Quinton, L. C. Northcliffe, R. W. Ollerhead, and K. Nagatani, Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961, edited by J. B. Birks (Heywood & Co., Ltd., London, 1962), p. 597.

⁴⁴G. C. Morrison (private communication, 1963).

Fig. 12. Comparison of alpha-particle energy spectra from the $C^{12}(Li^6, \alpha)N^{14}$ reaction as measured at low and high energies by Hobbie and Forbes (Ref. 40) and by Ollerhead, Chasman, and Bromley (Ref. 5), respectively. The large peak in the high-energy spectrum at $E_\alpha \approx 40 \text{ MeV}$ has been shown to correspond to direct dissociation of the incident Li^6 projectile.

the apparent direct mechanisms reflected the extreme cluster structure and the effectiveness of barriers in inhibiting compound system formation.

Examination of Li^6 -induced reactions in the energy range from 15 to 30 MeV would be highly desirable in order to establish, if anywhere, a region where these reactions might serve as spectroscopic probes.

Figures 13 and 14 show results of systematic studies on the (d, α) reactions on C^{12} and O^{16} , for $15 \leq E_d \leq 20 \text{ MeV}$, as reported by Yanabu *et al.*⁴⁵ Strong backward and forward peaking is taken as evidence for a direct mechanism and attempts were made to fit the forward and

⁴⁵T. Yanabu, S. Yamashita, T. Nakamura, K. Takamatsu, A. Masaike, S. Kabigi, D. C. Nguyen, and K. Takimato, J. Phys. Soc. (Japan) **18**, 747 (1963).

Fig. 13. Alpha particle angular distributions from the $C^{12}(d, \alpha)B^{10}$ reaction measured at intermediate energies by Yanabu et al. (Ref. 45).

Fig. 14. Alpha particle angular distributions from the $O^{16}(d, \alpha)N^{14}$ reaction measured at intermediate energies by Yanabu et al. (Ref. 45).

Fig. 15. Deuteron angular distributions from the $\text{Li}^6(\alpha, d)\text{Be}^8$ reaction as measured in the range $20 \leq E_\alpha \leq 25$ MeV by Demortier and Deconninck (Ref. 46).

backward regions independently using pickup and exchange-pickup formalisms.

Figure 15 shows similar studies on the $\text{Li}^6(\alpha, d)\text{Be}^8$ reaction by Demortier and Deconninck.⁴⁶ It would be particularly desirable to subject all of these data to proper distorted-wave analyses to determine whether here, as in single-nucleon stripping, distortion effects can frequently reproduce strong backward peaking without requiring recourse to the heavy-particle or exchange-stripping mechanisms.

Only two measurements of particle-gamma angular correlations following a deuteron-transfer reaction have been reported as yet. Both the $\text{O}^{16}(\text{He}^3, p\gamma)\text{F}^{18*}$ reaction via the $3^+ 0.934$ -MeV state, studied at $E_{\text{He}^3} \approx 10$ MeV by Hinds and Hinds,⁴⁷ and the $\text{B}^{10}(\text{He}^3, p\gamma)\text{C}^{12*}$ reaction via the $2^+ 4.43$ -MeV state, studied at $E_{\text{He}^3} = 4.50$ MeV by Wolicki, Holmgren, and Johnston,⁴⁸ suggest that the dominant reaction mechanism involves direct transfer of the neutron-proton pair as a lump. However, additional interfering amplitudes destroy the simple correlation symmetry which would be the signature of a pure direct mechanism.

⁴⁶ G. Demortier and G. Deconninck, Phys. Letters (in press); and private communication.

⁴⁷ S. Hinds and B. M. Hinds, Nucl. Phys. **48**, 690 (1963).

⁴⁸ E. A. Wolicki, H. D. Holmgren, and R. L. Johnston, Proceedings of the Rutherford Jubilee Conference, Manchester, 1961, edited by J. B. Birks (Heywood & Co., Ltd., London, 1962), C5/23.

Distorted-Wave Theories

More recently Glendenning,¹⁷ and Rook and Mitra⁴⁹ have formulated distorted-wave treatments of the two-nucleon-transfer situation which, by analogy with single-nucleon experience, might be anticipated to provide more reliable data extraction. Moreover, these treatments provide for the possibility that the two nucleons will not be captured into equivalent states and thus permit a more detailed probing of residual configurations.

In these treatments the differential cross section is given as

$$\frac{d\sigma}{d\Omega} = C \left| \sum_{\gamma} \beta_{\gamma LSJ} B_{\gamma L}^M \right|^2,$$

where C is a product of standard kinematic and statistical factors, $\beta_{\gamma LSJ}$ is a measure of the overlap between the initial and final states and thus contains the nuclear-structure information, $B_{\gamma L}^M$ is the transfer amplitude for transfer of the nucleon pair in the (L, M) state defined in analogy to the $B_{\ell m}$ in single-nucleon DWBA treatments,⁵⁰ and the sum over γ introduces a coherence over the various components of the state configurations involved.

The relative magnitudes of the overlap factors $\beta_{\gamma LSJ}$ may be estimated on the basis of any particular model and coupling scheme for comparison with experimental data. Table II due to Clement⁵¹ shows in

⁴⁹ J. R. Rook and D. Mitra, as quoted by R. Middleton, Ref. 52.

⁵⁰ R. Huby, M. V. Refal, G. R. Satchler, Nucl. Phys. 9, 94—107 (1958).

⁵¹ C. F. Clement, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 457.

TABLE II. Comparison of amplitude ratios in deuteron-transfer reactions.

Reaction	jj	R_{theor}	LS
		I. C. ($a/K = 5$)	
$\text{Li}^7(n, t)\text{He}^5$	1		0.32
$\text{Li}^7(d, \alpha)\text{He}^5$	1—8		2.1
$\text{B}^{11}(\text{He}^3, p)\text{C}^{13}$	2		2.42
$\text{B}^{11}(\alpha, d)\text{C}^{13}$	1.2		1.51
$\text{C}^{13}(\text{He}^3, p)\text{N}^{15}$	2	0.96	0.32
$\text{C}^{13}(\alpha, d)\text{N}^{15}$	8	3.47	1.28

striking fashion how the ratio $R = \beta_2/\beta_0$, calculated on the assumption of shell-model wave functions, would be expected to vary as a function of reaction type and coupling scheme. It is somewhat surprising that the difference between the (α, d) and the (He^3, p) reactions, reflecting the fact that only $T=0$ amplitudes contribute in the former, appears to depend upon the individual nuclear species involved.

Comparison of these predictions, if the basic assumptions in the analyses can be substantiated, will be expected to provide crucial information on the detailed wave-function configurations involved.

Figure 16 shows typical $\text{N}^{14}(\alpha, \text{Li}^6)\text{C}^{12}$ curves as measured by Zafiratos³⁴ together with a simple PWBA calculation demonstrating the effects of superposing various L components. It would appear hazardous to draw conclusions from such PWBA analyses without having further experimental evidence supporting its reliability for extraction of the different L components.

Unfortunately, at present, results of DWBA analyses on only seven reaction channels are available from the work of Pehl³⁰ (using a Glendenning code) on (d, α) reactions on Li^6 , N^{14} , and O^{16} with 24-MeV

Fig. 16. Li^6 angular distributions to the ground and 9.43-MeV state of C^{12} from the reaction $N^{14}(\alpha, Li^6)C^{12}$ studied at 42 MeV by Zafiratos (Ref. 34), together with PWBA fits using the indicated parameters.

Fig. 17. Alpha-particle angular distributions leading to the ground and 4.43-MeV state in C^{12} from the $N^{14}(d, \alpha)C^{12}$ reaction studied by Pehl (Ref. 30) with 15-MeV deuterons, together with DWBA fits obtained using a Glendenning code and pure $L=2$ transitions.

deuterons and of Zafiratos³⁴ (using a Gibbs-Tobocman code) on (α, Li^6) reactions on C^{12} and N^{14} . In all these cases the "best" fit was found, for reasonable optical model parameters, with $R = \infty$, i. e., for pure $L=2$ transitions.

Figure 17 shows Pehl's³⁰ results for the $\text{N}^{14}(\text{d}, \alpha)\text{C}^{12}$ reactions to the ground and 4.43-MeV states in C^{12} while Fig. 18 shows those of Zafiratos³⁴ for the $\text{C}^{12}(\alpha, \text{Li}^6)\text{B}^{10}$ and $\text{N}^{14}(\alpha, \text{Li}^6)$ ground-state reactions. Particularly the latter figure illustrates that these best fits characteristically fail to fit the most forward peak in addition to predicting an incorrect period for the diffraction oscillations. In the latter case, however, the DWBA predictions are in reasonable accord with the measured cross section magnitudes; in the former, absolute cross sections were not calculated and the experimental and calculated curves have been arbitrarily normalized. Clearly more DWBA studies of this type are badly needed to permit identification of the source of the discrepancies.

Middleton⁵² has noted that although the current DWBA codes permit calculations wherein stripping or pickup of individual nucleons occurs from different configurations, the fits obtained to angular distributions will not in general provide a sensitive measure of the higher configurations since the transfer amplitudes decrease by at least an order of magnitude for each unit increase in the ℓ_p or ℓ_n value involved. In consequence, relatively large admixtures of f capture may be present in a dominant p capture, for example, without having any discernible effect on the predicted cross sections. Only when the weak admixture is a lower orbital does this approach show any promise; while this is of

⁵²R. Middleton, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 435.

Fig. 18. Li^6 angular distributions from the $C^{12}(\alpha, Li^6)B^{10}$ (upper graph) and $N^{14}(\alpha, Li^6)C^{12}$ (lower graph) reactions studied at 42 MeV by Zafiratos (Ref. 34). The curves are DWBA fits obtained using a Gibbs-Tobocman code.

relatively little use in light nuclei it will become increasingly important as more precision studies are carried out in the f shell and beyond.

Henley and Yu⁵³ have recently completed a comprehensive examination of two-nucleon stripping including and comparing PWBA, DWBA, and a diffraction model—with particular reference to the (He^3, n) reaction.

Summary

Although it is very probably premature to draw any general conclusions regarding the relevant mechanisms in the two-nucleon-transfer reaction, the available information suggests the following.

A. In reactions induced by mass-3 and mass-4 projectiles:

(a) At low energies (< 15 MeV), both direct and compound-system amplitudes are important. Pickup amplitudes are much more probable than knockout; although strong backward peaking has been attributed to exchange or heavy-particle stripping, it will be essential to carry out DWBA fits to these data before significant conclusions can be drawn regarding these phenomena in other than special cases. (b) At higher energies (> 15 MeV), characteristic direct-interaction diffraction patterns are observed leading to states other than those having essentially pure two-particle character where an exponential distribution is found.

B. In reactions induced by heavier projectiles (e. g., Li^6 and B^{11}), the apparent cluster structure of the projectiles plays a dominant role, particularly in the case of Li^6 and to a lesser extent in B^{11} . A surface mechanism is strongly indicated. (a) At incident energies below or comparable to the barrier, Li^6 reactions proceed through a direct transfer; as the energy is increased (≈ 10 MeV) strong interference between direct and compound amplitudes is observed. At still higher energies

⁵³E. M. Henley and D. U. L. Yu (private communication, 1964; and to be published).

(> 30 MeV) a direct dissociation of the projectile occurs with very high probability and greatly decreases the yield of spectroscopic information. (b) Although only very limited data are available as yet on (B^{11}, Be^9) reactions at high energy (≈ 110 MeV), they suggest that the transferred neutron-proton pair is in a $^{13}S_1$ configuration only. It is not yet known why the $^{31}S_0$ configuration, which is not inhibited by any selection rule, should not be equally probable.

III. Pairing-Force and Quadrupole-Force Studies

Systematic studies on (d, α) reactions with 15-MeV deuterons on heavy targets ranging from Ni to Pb have been reported by Mead and Cohen.⁵⁴ Figure 19 reproduces the energy spectra obtained and Fig. 20 the observed dependence of these spectra on scattering angle. Mead and Cohen have presented arguments for a pickup rather than a knockout process in these data, characterized by weak population of the residual ground states and strong population of a group of states at an excitation of some 5—8 MeV in all nuclei studied.

In attempting to understand such behavior, Yoshida⁵⁵ has examined the two-nucleon-transfer reaction using a pairing-interaction model and has found that collective enhancement (by large factors, analogous to that observed in electromagnetic transitions and inelastic scattering) is indeed possible through coherent addition of paired-nucleon

⁵⁴J. B. Mead and B. L. Cohen, Phys. Rev. 125, 947 (1962); Phys. Rev. Letters 5, 105 (1960).

⁵⁵S. Yoshida, Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961, edited by J. B. Birks (Heywood & Co., Ltd., London, 1961), p. 609; and to be published.

Fig. 19. Alpha-particle energy spectra from 15-MeV deuteron bombardment of the indicated target nuclei. (From Mead and Cohen, Ref. 54.)

Fig. 20. Angular dependence of alpha-particle energy spectra from 15-MeV deuteron bombardment of the indicated targets. (From Mead and Cohen, Ref. 54.)

contributions. Yoshida gives explicit expressions for transition strengths between many quasiparticle configurations.

It will be of very great interest to search for such enhancements over a wide range of A to provide further insight into the pairing and quadrupole-force behavior.

In order to do so, with some confidence in the formalism developed, it is highly desirable that this formalism should be "calibrated" by application to transfer reactions linking states with known structure.

Fig. 21. Deuteron angular distributions from the $\text{Ca}^{40}(\text{d}, \alpha)\text{K}^{38}$ reaction studied with 15-MeV deuterons by Micheletti and Mukherjee (Ref. 56). The solid curves were calculated by Lin and Yoshida (Ref. 57) who assumed a pairing model and DWBA formalism. The experimental points for $J = 1$ (left graph) have been multiplied by 100 and those for $J = 2$ (right graph) by 200 for presentation here.

To this end, Micheletti and Mukherjee⁵⁶ at Pittsburgh have studied the $\text{Ca}^{40}(\text{d}, \alpha)\text{K}^{38}$ reaction involving several excited states of K^{38} .

Lin and Yoshida⁵⁷ have examined the results, using, however, a PWBA treatment and K^{38} wave functions calculated by Arima *et al.*,⁵⁸ for two-hole configurations in the sd shell. With model parameters adjusted to give a reasonable fit to the known K^{38} excitation spectrum, results such as those shown in Fig. 21 were obtained. The

⁵⁶S. Micheletti and P. Mukherjee (private communication, 1962).

⁵⁷C. L. Lin and S. Yoshida, University of Tokyo Report TVEP-63-4, 1963; and to be published.

⁵⁸T. Inoue, T. Sebe, H. Hagiwara, and A. Arima (to be published).

experimental data have been multiplied by factors of 100 and 200 for presentation in these figures; this provides a measure of the degree to which the calculated cross sections exceed those measured. Such an overestimate is characteristic of a PWBA treatment without cutoff, however, and it clearly would be desirable to repeat the analysis of these data with a DWBA code.

Austern⁵⁹ is currently engaged in such a program, using K^{38} and Ca^{40} wave functions calculated by J. P. Elliott, analogous to his earlier two-particle calculations at the low end of the sd shell.⁶⁰

When such "calibration" analyses are completed, it may be anticipated that the two-nucleon-transfer reactions will provide a very important addition to existing techniques for studying pairing and quadrupole force phenomena.

IV. Spectroscopic Studies

A number of spectroscopic studies have already been mentioned herein. These have included the most obvious measurements directed toward determination of J^π assignments through observation of the gross character of the angular distributions. Except for cases such as (He^3, n) and (t, p) with $S = 0$, the two-nucleon transfers are inherently less suited to such studies than single-nucleon transfers because of the L ambiguity: a large number of assignments have been made however.

⁵⁹N. Austern, E. Baranger, R. Drisko, and E. Sanderson, Report to the N S F from the Physics Department, University of Pittsburgh, November 6, 1963.

⁶⁰J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) A229, 536 (1955).

Assignment of isobaric-spin quantum numbers through comparison of (α, d) and (He^3, p) reactions, for example, has provided considerable new information. Such a comparison in future might be expected to provide more fundamental insight into the neutron-proton forces and correlations since the (α, d) [or (d, α) etc.] reactions involved only $S=1$ correlations whereas the (He^3, p) [or (p, He^3) etc.] reactions involve both $S=0$ and $S=1$ correlations. Further discussion of these topics will be deferred to a later section of this paper.

Shell-Model Studies

One of the most obvious studies, using two-nucleon-transfer reactions, involves a search for specific shell effects when the transferred nucleons enter, or are stripped from, different shells. At the moment this question remains completely open. Yntema and O'Neill⁶¹ found no significant differences in (d, α) reactions on Ti^{48} and Ti^{50} when the added neutron enters an sd or an f orbit, respectively. It is conceivable that this may reflect the presence of a small p admixture in the predominantly f subshell, as noted above in connection with the DWBA calculations permitting independent nucleon orbitals.

In contrast to these results on titanium, Micheletti and Mead⁶² have been able to account for their (d, α) data on Zr^{90} and on Pb^{208} only by involving independent pairing forces for neutrons and protons in different orbital states.

More data of this type would provide a means for systematic isolation and examination of the (n, n) , (p, p) , and (n, p) forces operating in

⁶¹J. L. Yntema and W. J. O'Neill, Proceedings of the Conference on Direct Reactions and Nuclear Reaction Mechanisms, Padua, Italy, 1962 (Gordon and Breach, Science Publishers, New York, 1963), p. 1084.

⁶²S. Micheletti and J. B. Mead, Nucl. Phys. 37, 201 (1962).

different shell-model states and thus provide crucial data on the pairing and residual forces central to the generalized shell model.^{63,64}

We have already referred to the calculations of Yoshida⁵⁵ and the direct measure of pairing-force amplitudes which the two-nucleon reactions can provide as well as to the detailed information concerning coupling schemes and specific model predictions obtainable from experimental evaluation of the β_2/β_0 and perhaps β_4/β_0 or β_4/β_2 ratios appearing in the formulations of the two-nucleon-transfer theory by Glendenning¹⁷ and others.

Extensive examinations of (a,d) and (d,a) reactions in light nuclei, in terms of an assumed intermediate-coupling shell-model description of the final states, have been carried out particularly by Cerny,⁶⁵ by Pehl,³⁰ and Zafiratos.³⁴ This has led to greatly increased knowledge, particularly concerning states of higher excitation on which little detailed structural information was previously available, and has been of great value in checking and calibrating the detailed shell-model calculations now available in the p and sd shells.

As an example of such studies we shall refer to the work of Pehl³⁰ and others on N^{14} , both because extensive work has been done on this nucleus, using a variety of reactions, and because we shall return to N^{14} shortly in connection with studies on higher pure two-nucleon configurations.

⁶³ R. Mottelson, in Nuclear Spectroscopy, edited by G. Racah (Academic Press, Inc., New York, 1962), p. 44.

⁶⁴ G. E. Brown, in Nuclear Physics, edited by V. F. Weisskopf (Academic Press, Inc., New York, 1963), p. 99.

⁶⁵ J. Cerny, doctoral dissertation, University of California, Lawrence Radiation Laboratory Report UCRL-9714, May 1961 (unpublished).

Table III reproduces Table VIII of Ref. 30 and summarizes the (α, d) , (d, α) , and (α, α') data, obtained at Berkeley, leading to states of N^{14} . The dominant configurations are those from the calculations of True,⁶⁶ of Talmi and Unna,⁶⁷ and of Warburton and Pinkston.⁶⁸

The first striking observation on these reactions concerns their high degree of selectivity; the number of populated states is relatively small. From the general reaction characteristics it might be assumed a priori that the $C^{12}(\alpha, d)N^{14}$ reaction would selectively populate N^{14} states having strong parentage based on an unexcited C^{12} core $[(s_{1/2})^4(p_{3/2})^8]$ with two extra-core valence-nucleon states. Similarly in the $O^{16}(\alpha, d)N^{14}$ reaction, it would be expected that states corresponding to two holes in the O^{16} structure $[(s_{1/2})^4(p_{3/2})^8(p_{1/2})^4]$ would be preferentially populated. In particular, no strong population of N^{14} states having significant $2s_{1/2}$ or $1d_{5/2}$ amplitudes would be expected since these are not present in the O^{16} target. The inelastic alpha-particle scattering, on the other hand, has been shown to preferentially excite single-particle states in light nuclei and would thus be anticipated to show single-nucleon excitations from the dominant $[(s_{1/2})^4(p_{3/2})^8(p_{1/2})^2] N^{14}$ ground-state configuration.

In general, as indicated in Table III, quite acceptable agreement is attained between the shell-model assignments listed and the experimental observations. For example, the 3.95- and 7.03-MeV two-hole states were not strongly populated; the observed population of the 3.95-MeV state could occur through the weak $[(s_{1/2})^4(p_{3/2})^6(p_{1/2})^2]$ component of the C^{12} ground state, as given by Kurath for example.

⁶⁶W. W. True, Phys. Rev. 130, 1530 (1963).

⁶⁷I. Talmi and I. Unna, Ann. Revs. Nucl. Sci. 10, 353 (1960).

⁶⁸E. K. Warburton and W. T. Pinkston, Phys. Rev. 118, 733 (1960).

TABLE III. Shell-model configurations and relative cross sections for formation of N^{14} levels.

Energy (MeV)	Level		Dominant configuration ^a	Relative cross sections		
	J ^π	T		(a, d) ^b	(a, a')	(d, a)
0	1 ⁺	0	(p _{1/2}) ²	1	-	1
2.31	0 ⁺	1	(p _{1/2}) ²	c	c	c
3.95	1 ⁺	0	(p _{3/2}) ⁻¹ (p _{1/2}) ⁻¹	0.2	Strong	1.60
4.91	0 ⁻	0	p _{1/2} ^s _{1/2}	1	Strong	0.72
5.10	2 ⁻	0	p _{1/2} ^d _{5/2}			
5.69	1 ⁻	0	p _{1/2} ^s _{1/2}	0.4	Strong	0.94
5.83	3 ⁻	0	p _{1/2} ^d _{5/2}			
6.05			?	c	c	c
6.21	1 ⁺	0	(s _{1/2}) ²	0.7	Very weak	0.72
6.44	3 ⁺	0	s _{1/2} ^d _{5/2}			
7.03	2 ⁺	0	(p _{3/2}) ⁻¹ (p _{1/2}) ⁻¹	Weak	Strong	1.19
7.40			?	c	c	c
7.60			?	c	c	c
7.97	2 ⁻	0	p _{1/2} ^d _{3/2}	Weak	Weak	Weak
8.06	1 ⁻	1	p _{1/2} ^s _{1/2}	c	c	c
8.47		0	?	Fairly strong	Weak	He ³ contaminant
8.63	0 ⁺	1	(s _{1/2}) ²	c	c	c
8.71	0 ⁻	1	p _{1/2} ^s _{1/2}	c	c	c
8.91	3 ⁻	1	p _{1/2} ^d _{5/2}	c	c	c
8.99	1 ⁺	(0)	?	1.5	c	c
9.00	5 ⁺	0	(d _{5/2}) ²			
9.17	2 ⁺	1	(s, d) + (p _{3/2}) ⁻¹ (p _{1/2}) ⁻¹	c	c	c
9.41	1 ⁻		p _{1/2} ^d _{3/2}	Fairly strong	Weak	Fairly strong
9.51	2 ⁻	1	p _{1/2} ^d _{5/2}	c	He ³ contaminant	c
9.71	1 ⁺		(d _{5/2}) ²	Weak	He ³ contaminant	Very weak
10.09	1 ⁺	0	s _{1/2} ^d _{5/2}	Fairly strong	c	Weak
10.22	1 ⁻		?	d	c	Weak
10.42	2 ⁺	1	s _{1/2} ^d _{5/2}	d	c	c

^aFrom Refs. 66—68.^bNumerical values for E_{inc} = 48 MeV.^cNot observed.^dNot studied.

Pehl³⁰ has noted that the 7.97-MeV state should have been populated strongly if the indicated configuration is correct and suggests the possibility that this should be assigned rather to the previously unobserved state at 8.47 MeV which is strongly populated. We shall later note qualitative reasons why the 9.71-MeV state, for example, having a dominant $(d_{5/2})^2$ configuration, might not be expected to be strongly populated in this reaction.

In the (α, α') reaction, on the other hand, all states corresponding to possible single-particle excitations are observed with strong yield except that at 7.97 MeV, which was populated considerably less than 50% as much as the new 8.47-MeV state. This low yield supports the above tentative change in assignment. States corresponding to double excitations were suppressed by at least an order of magnitude in the (α, α') measurements.

The fact that the 3.95- and 7.03-MeV states are more strongly populated than the ground state in the $O^{16}(d, \alpha)N^{14}$ reaction reflects the greater number of $(p_{3/2})$ nucleons available, hence the greater probability of $(p_{3/2}, p_{1/2})$ rather than $(p_{1/2})^2$ transfers. The relatively large yield to states at 4.91, 5.10, 5.69, 5.83, 6.21, and 6.44 MeV, however, would require, in addition to removal of two nucleons, the promotion of one or more from the p to the sd shell. That this excitation occurs in so many cases lends weight to the alternative possibility that the O^{16} ground state is not a pure closed shell but rather contains a significant $[p^{-2}(s_{1/2})^2 + p^{-2}s_{1/2}d_{5/2} + p^{-2}(d_{5/2})^2]$ component. To the extent that jj coupling holds in O^{16} , population of the $(d_{5/2})^2 5^+$ level at 9.0 MeV in the (d, α) reaction would require a four-hole O^{16} ground-state component $[p^{-4}(d_{5/2})^4]$; consequently it is not surprising that such population should not have been observed.

Although, as indicated above, the available evidence weighs against important knockout amplitudes in the deuteron-transfer reactions, if subsequent studies should show that the O^{16} ground state is a purer

$[(s_{1/2})^4 (p_{3/2})^8 (p_{1/2})^4]$ configuration than the above arguments would require, the knockout mechanism provides the only apparent alternative explanation of these data. In this, an alpha particle would be ejected bodily from the O^{16} ground state and one or both of the transferred nucleons would be captured into the sd shell; the fact that the 5^+ level at 9.0 MeV is not populated does not preclude such a mechanism since angular-momentum effects would strongly inhibit such a high-spin knockout transition. It is clear that similar systematic reaction comparisons leading to other nuclei can provide extensive information on the correctness of model assignments as well as on the reaction mechanisms themselves.

Cluster-Model Studies

In many of the analyses of deuteron-transfer reactions thus far reported, evidence for a binary cluster structure (involving a deuteron as one member) has been adduced for one or more of the reactant nuclei. In the simplest instance, this would be anticipated to reflect itself in augmented yields involving such configurations.

After DWBA analyses of his experimental data on (α, Li^6) reactions at ≈ 40 MeV, Zafiratos³⁴ concludes that even after removing the usual spin multiplicity factors (which we shall consider in a later section), the $N^{14}(\alpha, Li^6)C^{12}$ reaction is enhanced by a factor of nine over the $C^{12}(\alpha, Li^6)B^{10}$ reaction. This suggests significant $(C^{12} + d)$ cluster amplitudes in the N^{14} ground state, as indeed was suggested previously on the basis of gamma-radiation observations on the $C^{12}(N^{14}, d)Mg^{24}$ reaction by Bromley, Kuehner, and Almqvist.

In studies on the $Li^6(Li^6, \alpha)Be^8$ reaction at ≈ 2 MeV, Coste and Marquez⁶⁹ obtained the alpha-particle spectrum shown in Fig. 22 and

⁶⁹ M. Coste and L. Marquez, *Compt. rend.* 254, 1768 (1962).

Fig. 22. Alpha-particle energy spectrum from the $\text{Li}^6(\text{Li}^6, \alpha)\text{Be}^8$ reaction with 2-MeV Li^6 ions. The indicated excitation energies are for states in Be^8 . (From Coste and Marquez, Ref. 69.)

state. We propose to examine the $\text{Li}^6(\text{B}^{11}, \text{Be}^9)\text{Be}^8$ reaction in this region. However, the energy definitions of existing high-energy boron beams are not adequate to resolve this excitation discrepancy. These latter studies are intended to search for the second strong $(\text{Li}^6 + d)$ cluster state in Be^8 , predicted by Temmer⁷⁰ at an excitation of ≈ 22.15 MeV. Lemeille, Manesse, Marquez, and Saunier⁷² have reported measurements on the

suggested that the large peak corresponding to an excitation of ≈ 20.4 MeV in Be^8 might correspond to formation of a strong $(\text{Li}^6 + d)$ cluster state, suggested by Temmer⁷⁰ from observation on the $\text{Li}^7(p, \alpha)\text{He}^4$ reaction. That such a cluster-transfer mechanism was indeed operational was here demonstrated by Allison, Kamegai, and Morrison⁷¹ in a beautiful series of double-coincidence measurements on the product alpha particles. These latter authors quote an excitation

energy of 20.95 ± 0.3 MeV for the Be^{8*} state. It will be important to re-examine the discrepancy between this value and that of 19.96 MeV quoted by Temmer,⁷⁰ to establish

whether these are in fact the same

⁷⁰G. M. Temmer, Bull. Am. Phys. Soc. 7, 59 (1962).

⁷¹S. K. Allison, M. Kamegai, and G. C. Morrison, Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, 1963, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), p. 155.

⁷²C. Lemeille, D. Manesse, L. Marquez, and N. Saunier, J. Phys. Rad. (in press).

$\text{Li}^6(\text{Li}^6, \alpha)\text{Be}^8$ reaction at 5 MeV, in which no evidence for this state was found at an excitation below 23 MeV. However, examination of this region of excitation, at this bombarding energy, encountered serious experimental difficulties and repetition or extension of these measurements at higher energies would be very desirable.

Very similar data have been obtained by Garin and Marquez⁷³ on the $\text{Li}^7(\text{Li}^6, \alpha)\text{Be}^9$ reaction at 2 MeV leading to a suggested state at an excitation of ≈ 12.5 MeV in Be^9 having a strong $(\text{Li}^7 + d)$ -cluster parentage.

Zafiratos³⁴ has presented a systematic analysis of (α, d) reactions at 42 MeV on Be^9 , B^{10} , B^{11} , and N^{14} and of (α, Li^6) reactions at the same energy on N^{14} and C^{12} within the same cluster-model framework.

Figure 23 shows, albeit with perilously little data as yet, the striking decrease in the (α, Li^6) cross sections, observed by Zafiratos,³⁴ as a function of the atomic number of the target. This would, of course, be consistent with the concept of a direct deuteron-cluster transfer and the rapidly decreasing probability of significant deuteron-cluster amplitude in the target ground states, particularly as the neutron and proton shells fill out of phase. It would be very desirable to have further points on such a curve extrapolating it to higher values of A .

Fig. 23. Qualitative behavior of (α, Li^6) reactions with increasing target atomic number. Data as indicated from Zafiratos (Ref. 34) and from Rickey *et al.* (Ref. 74).

⁷³ A. Garin and L. Marquez, J. Phys. Rad. (in press).

Fig. 24. Q values for giant excitations observed with (α, d) reactions on p-, sd-, and f-shell nuclei. (From Harvey, Rivet, Cerny, Pehl, and Haag, Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 182.)

Also shown on this figure are equivalent data of Rickey *et al.*⁷⁴ on the $\text{Mg}^{26}(\alpha, \text{Li}^6)\text{Na}^{24}$ and $\text{Mg}^{26}(\alpha, \text{He}^6)\text{Mg}^{24}$ reactions, illustrating the very strong relative suppression of the latter reaction.

Garvey⁷⁵ has suggested that this inhibition may well reflect the $T=1$ structure of He^6 and the correspondingly reduced probability (in consequence of the Pauli principle) of finding a closely space-correlated neutron pair in the target nucleus for removal in an effective point-transfer process. Such comparisons can clearly provide important information on such correlations, at least in the nuclear surfaces.

Giant Excitations — Two-Nucleon States

Typical oscillatory angular distributions from the $\text{C}^{12}(\alpha, d)\text{N}^{14}$ reaction are shown in Figs. 3 and 4. In pioneering studies on this reaction, Harvey *et al.*⁷⁶ observed that "giant excitations" were also present in the deuteron spectra with yields perhaps an order of magnitude higher than all others. Moreover, it was found that these transitions showed exponential angular distributions with no evidence for diffraction-like oscillation. Figure 24 shows the Q values obtained for the giant

⁷⁴M. Rickey, H. Wegner, K. W. Jones, B. M. Bardin, C. Hoot, and M. Kondo, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 217.

⁷⁵G. T. Garvey (private communication, 1964).

⁷⁶B. G. Harvey, J. Cerny, R. H. Pehl, and E. Rivet, *Nucl. Phys.* **39**, 160 (1962).

excitations in these and more recent studies, as functions of the atomic number, again suggesting a systematic behavior.

Figure 25 compares the energy spectra measured in the (α, d) ,⁷⁶ (He^3, p) ,⁷⁷ and $(\text{B}^{11}, \text{Be}^9)$ reactions⁶ on O^{16} at the indicated energies and angles, converted to a F^{18} excitation scale. The "giant excitation" in this case is assumed, from systematics, to correspond to the known $5^+ (d_{5/2})^2$ state at 1.1 MeV in F^{18} although energy resolution has been inadequate, in all cases, to resolve the closely-spaced quadruplet of levels at this excitation. The greater complexity of the (He^3, p) spectrum illustrates the fact that much less stringent selection rules operate than in the (α, d) reactions. It is not obvious why the $(\text{B}^{11}, \text{Be}^9)$ spectra, both here and in the following figure, appear more similar to the (α, d) than to the (He^3, p) spectra.

Figure 26 shows corresponding data leading to excitations in N^{14} where the "giant excitations" appear at ≈ 9 and 13 MeV. An obvious extension of the previously-mentioned momentum-mismatch hypothesis is of particular relevance here as shown schematically in Fig. 27. Since the "carrier" nuclei (in particular H^2 and Be^9) are very loosely bound, it would not be anticipated that either would survive close interaction with the target during the deuteron transfer.⁷⁸ This leads to the hypothesis that the strongest transfer reactions would be those involving minimal disturbance of the C^{12} core and a simple delivery of the two nucleons to the surface of this core. At the energies of 10 MeV/nucleon, involved in all these measurements, the orbital angular momentum per nucleon in such a surface mechanism is ≈ 2 , suggesting that the preferred

⁷⁷As quoted in Ref. 76. I am indebted to Dr. E. Rivet for copies of the spectra in question.

⁷⁸D. A. Bromley, Nucl. Instr. Methods (Proceedings of the Third International Accelerator Conference) (in press).

Fig. 25. Comparison of spectra of deuteron-transfer reaction products leading to states in F^{18} . The (He^3, p) and (α, d) reaction data are those of Harvey *et al.* (Ref. 76) at Berkeley; the (B^{11}, Be^9) reaction data are those of Sachs *et al.* (Ref. 16) at Yale. These data illustrate the striking difference between the (He^3, p) and (α, d) reactions expected at least qualitatively from selection-rule considerations.

Fig. 26. Comparison of spectra of deuteron transfer reaction products leading to N^{14} . The (He^3, p) and (α, d) data are those of Harvey *et al.* (Ref. 76) at Berkeley; the (B^{11}, Be^9) data are those of Sachs *et al.* (Ref. 16) at Yale, and the higher excitation (α, d) measurements are those of Nagatani and Bromley (Ref. 80) at Yale. Only in the lower spectrum is the experimental resolution adequate to resolve individual states. The (He^3, p) data show characteristic population of the 0^+ , 2.31-MeV, $T=1$, first excited state of N^{14} which is not inhibited by selection rules as it is in the (α, d) reaction. The lower spectrum shows the weak population observed in the (α, d) reaction, presumably via isobaric-spin mixing in the F^{18} compound states.

Fig. 27. Schematic representation of the (α, d) and (B^{11}, Be^9) deuteron-transfer reactions on C^{12} .

configurations would be those involving $C_{g.s.}^{12} + d_J^2$. Moreover, since the nucleons in the simplest collision picture preferentially move in parallel orbits, states involving $L = \ell_p + \ell_n = 4$ would be expected to be favored, thus selecting particular members of the J multiplet corresponding to each of the above configurations $(d_{5/2})^2$, $(d_{3/2}, d_{5/2})$, and $(d_{3/2})^2$. Simple arguments based on the O^{17} spin-orbit splitting suggest that the

centroids of these multiplets should be separated by some 5 MeV, with the $(d_{5/2})_J^2$ multiplet lying lowest.

Table IV presents the LS configurations for these multiplets as calculated using transformation coefficients of Kennedy and Cliff⁷⁹ and lists the excitations and T values predicted by the intermediate-coupling calculations of True⁶⁶ for states with these dominant components.

From this table, and comparison with similar data in neighboring nuclei typified by that of Fig. 25, it appears reasonable to identify the 9.0-MeV giant excitation as corresponding to a $(d_{5/2})^2$, $J^\pi = 5^+$ state. Lacking (α, d) reaction data at the higher excitations when the (B^{11}, Be^9) measurements were obtained, Sachs, Chasman, and Bromley⁶ tentatively identified the 13-MeV giant excitation as corresponding to a 4^+ , $T=1$ member of this same multiplet, $(d_{5/2})^2$, predicted by True⁶⁶ at 12.33 MeV, since this state also possesses a substantial G-wave component and since $^{31}S_0$ deuteron transfer was not expected to be inhibited in this reaction.

⁷⁹ J. M. Kennedy and M. J. Cliff, Chalk River Report CRT-609, August 1955 (unpublished).

TABLE IV. LS representations of $(d)^2$ jj configurations.^a

Configuration	J	LS representation	T	$E_x (N^{14})^b$
$(d_{5/2})^2$	5	1.00 3G	0	9.32
	4	0.447 1G + 0.894 3F	1	12.33
	3	- 0.151 3G + 0.600 1F + 0.786 3D	0	11.60
	2	- 0.268 3F + 0.693 1D + 0.669 3P	1	12.31
	1	- 0.400 3D + 0.748 1P + 0.529 3S	0	9.92
	0	- 0.632 3P + 0.775 1S	1	10.93
$(d_{5/2}, d_{3/2})$	4	0.632 1G + 0.707 3G - 0.316 3F	0	13.82
			1	15.95
	3	-0.278 3G + 0.707 3F + 0.490 1F - 0.428 3D	0	15.76
			1	17.55
	2	- 0.358 3F + 0.707 3D + 0.346 1D - 0.502 3P	0	14.28
			1	17.22
$(d_{3/2})^2$	3	0.907 3G + 0.400 1F - 0.313 3D	0	19.46
			1	21.65
	1	0.748 3D + 0.600 1P - 0.283 3S	0	20.57
			1	20.86

^aFrom J. M. Kennedy and M. J. Cliff, Chalk River Report CRT-609, August 1955.

^bFrom W. W. True, Phys. Rev. 130, 1530 (1963).

Very recently, however, Nagatani and Bromley⁸⁰ have extended the (α, d) reaction study at 42 MeV to include this region of excitation and have found the results shown in Fig. 26. As shown, strong population of the 13-MeV state does occur. This precludes a $T=1$ assignment and suggests that the state involved is the 4^+ , $T=0$ member of the $(d_{3/2}, d_{5/2})$ multiplet predicted by True at 13.82 MeV. The shaded peak, when compared with the data of Fig. 24, may well be the $(f_{7/2})^2$ peak in N^{14} since it falls roughly on the $(f_{7/2})^2$ curve of Harvey et al.⁷⁶

Glendenning⁸¹ has recently carried out detailed calculations of the (α, d) overlap integrals required to predict the transfer-reaction cross sections, using True⁶⁶ wave functions, and has indeed predicted that the excitation of this state should be comparable to that of the $(d_{5/2})^2$, 5^+ state. Similar calculations are not yet available for the $T=1$ states but would be of great interest in possibly leading to an understanding of the absence of such excitations in the experimental data.

Similarly, the smaller peak in the Be^9 spectrum corresponding to an excitation of ≈ 19 MeV may well be the $J^\pi = 3^+$ member of the $(d_{3/2})^2$ configuration, predicted by True at 19.46 MeV.

An experiment of particular interest, which is not feasible with the HILAC because of duty-cycle limitations, would involve a coincidence correlation study of the decay of these "giant" states—hopefully to establish their assignments unambiguously. Boron beams from the large tandem accelerators will make such measurements feasible.

Following a suggestion of Levinson,⁸² we have carried out a detailed search for possible signatures of these two-particle states in

⁸⁰ K. Nagatani and D. A. Bromley, abstract contributed to these Proceedings, p. 423.

⁸¹ N. K. Glendenning, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 188.

⁸² C. Levinson (private communication, 1964).

all available data relevant to this excitation region in N^{14} —including (d,n), (d,p), (d,d), and (d, α) studies on C^{12} , (p, γ) and (p,p) studies on C^{13} , and (α ,p) and (α ,n) studies on B^{10} . While it was not anticipated that the higher spin-multiplet members would show up (because of centrifugal barrier factors), it was hoped that particularly in the $C^{12} + d$ reactions some evidence for enhanced participation of lower spin-multiplet members might appear, perhaps as dominant compound-system amplitudes in the immediate energy region spanned by the two-particle compound state. No unambiguous evidence of this type was found.

Detailed study of the decay of these two-particle states may, however, provide a particularly favorable situation for examining the interference between direct and compound reaction amplitudes. Tobocman⁸³ is currently engaged in a theoretical examination of this problem.

Figure 28 shows the angular distributions measured for the 9.0-MeV excitation in N^{14} in the (α ,d) and (B^{11} , Be^9) reactions as functions of both angle and of momentum transfer assuming a stripping mechanism. The similarity of these results confirms the stripping mechanism assumed for the (α ,d) reaction since knockout of Be^9 would be expected to be of negligible importance. This presentation emphasizes one of the advantages of the heavier projectiles in the deuteron-transfer reactions, namely that they provide information over a greater range of momentum transfers in convenient angular ranges.

Very preliminary attempts to fit these angular distributions⁸⁴ using a Greider core-scattering model⁸⁵ have resulted in reasonable

⁸³ W. Tobocman (private communication, 1964).

⁸⁴ M. W. Sachs, doctoral dissertation, Yale University, December 1963 (unpublished); Phys. Rev. (to be published).

⁸⁵ K. R. Greider, in Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, 1963, edited by A. Ghiorso, H. E. Conzett, and R. M. Diamond (University of California Press, 1963), p. 148.

Fig. 28. Angular distributions of deuterons and Be^9 leading to the 9.0-MeV giant excitation in N^{14} shown as functions both of angle and of momentum transfer (assuming a stripping mechanism) for both reactions measured at ≈ 10 MeV/nucleon. This figure is characteristic of all the giant excitations and illustrates the utility of the heavy-ion reactions in examining the deuteron-transfer reactions under conditions of higher momentum transfer.

qualitative agreement only on the assumption that very high linear momentum components corresponding to $\sim 50\text{--}75$ MeV, or very restricted spatial regions (≤ 1 fm), are involved. The former energies are much in excess of any reasonably expected to participate while the latter assumption, which appears most reasonable, would be entirely consistent with our earlier assumption of a sharply defined surface mechanism. Much more work remains to be done before this mechanism can be considered to be understood, however.

V. Isobaric-Spin Studies

Isobaric-Spin Channel Strengths

As noted previously, the transferred deuteron may be in a $^{31}S_0$ or $^{31}S_1$ configuration in a general reaction. However, the unique correlation implied between T and J results in these transfer reactions constituting an effective probe for isobaric-spin phenomena in light nuclei. In particular, while the $(He^3, p)(t, n)$, and (B^{11}, Be^9) reactions, for example, have two incoherent isobaric-spin exit channels ($T = 0, 1$), the (α, d) and (Li^6, α) reactions have only the $T = 0$ channel. The same is clearly true of the inverse reactions in each case. By comparing (He^3, p) and (α, d) reactions linking the same initial and final states, it will thus be possible to determine the relative channel strengths. Data at appropriate energies to permit extraction of such information are not yet available. However, in the analogous comparison of (He^3, p) and (He^3, n) reactions (the latter having only the $T = 1$ channel) by Fulbright *et al.*,⁸⁶ agreement with the channel-strength ratio of 2, obtained from the simple isobaric-spin Clebsch-Gordan coefficients, was observed.

Studies of Isobaric-Spin Purity

Extensive use of the $T=0$ reactions to probe the isobaric-spin purity of nuclear levels has been reported, particularly with reference to the population of the lowest $T=1$ state in light odd-odd nuclei. Reactions leading to the 0^+ , $T=1$ state in N^{14} at 2.31 MeV have received

⁸⁶H. W. Fulbright, W. P. Alford, O. M. Bilaniuk, V. K. Deshpande, and J. W. Verba, University of Rochester Report NYO-10034, February 1962.

Fig. 29. Ratios of the reaction yield in $\Delta T=1$ and $\Delta T=0$ reaction channels in (α, d) , (d, α) , and (Li^6, α) reactions leading to the ground state and first excited state of N^{14} as functions of the corresponding F^{18} compound-system excitation. The shaded areas represent Wilkinson's estimate (Ref. 93) of the boundaries of the region of strong isobaric mixing in this nucleus. (Slightly abridged from Cerny et al., Ref. 94.)

most study. A compilation of the relevant data^{31,35,40,65,87-94} is given in Fig. 29, slightly abridged from a similar figure published by Cerny et al.⁹⁴

To the extent that these reactions involve compound states of F^{18} , it is clear that population of the 2.31-MeV state can proceed only through isotopic-spin impurities in one or all of the initial, compound, and final states. Excellent evidence has been adduced indicating that these impurities are extremely small for low-lying states for which $\Gamma \ll D^{91,92,93}$ since the overlap of adjacent states of like spin and parity, but opposite

⁸⁷A. Ashmore and R. J. Raffle, Proc. Phys. Soc. (London) 64A, 754 (1951); 65A, 296 (1952).

⁸⁸R. J. Van de Graaff, A. Sperduto, W. W. Buechner, and H. A. Enge, Phys. Rev. 86, 966 (1952).

⁸⁹C. Hu, J. Phys. Soc. (Japan) 15, 1741 (1960).

⁹⁰F. Pellegrini, Nucl. Phys. 24, 372 (1961).

⁹¹W. M. Macdonald, in Nuclear Spectroscopy, Part B, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1960), p. 932.

⁹²A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).

⁹³D. H. Wilkinson, Phil. Mag., Ser. VIII, 1, 379 (1956).

⁹⁴J. Cerny, R. H. Pehl, E. Rivet, and B. G. Harvey, Phys. Letters 1, 67 (1963).

T , is too small to permit mixing via a Coulomb perturbation. Similarly, at high excitations where $\Gamma \gg D$, the correspondingly short compound lifetime might be expected to preclude significant mixing even with appreciable overlap.^{92,93} At an intermediate excitation, however, with intermediate overlap and lifetime, Coulomb forces would be expected to produce significant T mixing and thus to allow the above reaction. The shaded regions of Fig. 29 denote Wilkinson's estimates of the boundaries of this intermediate excitation region. As indicated, the experimental data thus far available are consistent with these qualitative ideas with the possible suggestion that the excitation region in which high T mixing occurs is somewhat higher and more restricted than has been anticipated.

Similar systematic studies leading to the 0^+ states at 3.56, 1.74, and 1.085 MeV in Li^6 , B^{10} , and F^{18} , respectively, would be of considerable interest in establishing the characteristics of such mixing. As first emphasized by Hashimoto and Alford,⁹⁵ however, it should be noted that in (d, α) reactions (for example) leading to such 0^+ states, only compound states of natural parity contribute to the reaction. Since these constitute, on the average, one third of those populated by deuterons incident on an even-even target, a corresponding approximate factor-of-3 suppression in addition to that from a $(2J_f+1)$ final-state rule (see below) might be anticipated.

Direct-reaction amplitudes can contribute to these $0^+ T=1$ final states only through violation of angular-momentum and parity conservation, or through spin-flip of one of the transferred nucleons, in addition to isobaric impurities in the initial and/or final states. In consequence, it would be expected that these amplitudes would be very strongly inhibited.

⁹⁵ Y. Hashimoto and W. P. Alford, Phys. Rev. 116, 981 (1959).

On the contrary, direct amplitudes are known to play an important role in other than these 0^+ exit channels; in the case of the $O^{16}(d,\alpha)N^{14}$ reaction with 19-MeV deuterons, Fischer and Fischer³⁵ have estimated that roughly 90% of the observed cross section must be attributed to direct processes. Since correction has not been made for such contributions in Fig. 29, it is clear that the data presented underestimate the degree of isobaric mixing present. Information leading to a reliable removal of the direct amplitudes in such studies would lead to greatly improved measures of isospin mixing.

In the $C^{12}(B^{11}, Be^9)N^{14}$ reaction^{17,83} as studied at 115 MeV, the fact that none of the $T=1$ states in N^{14} appear to be populated with significant yield suggests that transfer of the $^{31}S_0$ configuration is markedly suppressed relative to that of the $^{13}S_1$ configuration. There are no obvious reasons for this preference. Moreover, in the (He^3, p) reaction both $T=0$ and $T=1$ states are populated with roughly equal probability.¹ More extensive measurements involving the heavier projectiles are required to establish whether this is other than an isolated phenomenon peculiar to N^{14} .

In order to reduce the ambiguity in these studies, it would clearly be desirable to examine other than 0^+ , $T=0$ to 0^+ , $T=1$ transitions. Pehl³⁰ has reported on studies of the $C^{12}(d,\alpha)B^{10*}$ reaction to the 2^+ , $T=1$ and 3^- , $T=1$ states at 8.89 MeV in B^{10} . Unfortunately a number of alpha-particle-producing, multi-body, breakup reactions compete to give a background continuum in the region of interest and the $C^{12}(d, He^3)B^{11}$ reaction provides overlapping He^3 groups which Pehl's equipment did not resolve completely. Within the limits of error, no population of the $T=1$ states was observed.

It would be of considerable importance to re-examine this and similar situations under improved experimental conditions to provide

a more stringent measure of the isobaric-spin mixing involved in these reactions.

Schier *et al.*⁹⁶ have reported relatively strong population of the 15.1-MeV 1^+ , $T=1$ state in C^{12} in the $N^{14}(d,\alpha)C^{12*}$ reaction implying strong isobaric-spin mixing in O^{16} and examination of the equivalent $B^{10}(Li^6,\alpha)C^{12*}$ reaction would be of interest. From the available data, it is not yet clear to what extent the alpha breakup and He^3 contamination in the $N^{14}(d,\alpha)C^{12*}$ reaction may have been excluded in Schier's work since Pehl was unable to extract data on this particular state from his spectra. Comparison of the isobaric-spin mixing in the O^{16} and F^{18} systems would be of interest in emphasizing the effects of the characteristically higher level density in the odd-odd nucleus.

The larger tandem accelerators, soon to become available, will be particularly suited to these measurements because of their broader energy range and greater beam species flexibility.

Of particular interest in examining isobaric-spin behavior are the $\Delta T=1$ transitions exemplified by (t,p) or (He^3,n) , particularly in cases involving $T=1$ target nuclei and population of $T=2$ residual states which are otherwise extremely difficult to examine. Population of such states will permit much more extensive examination of effective Coulomb energies in nuclear matter and more stringent tests of the isobaric-spin selection rules⁹⁷⁻⁹⁹ applicable to electromagnetic transitions than are now convenient.

⁹⁶W. A. Schier, C. P. Browne, and I. F. Wright, Bull. Am. Phys. Soc. 6, 416 (1961).

⁹⁷G. Morpurgo, Phys. Rev. 110, 721 (1958).

⁹⁸E. K. Warburton, Phys. Rev. Letters 1, 68 (1958).

⁹⁹M. Gell-Mann and V. L. Telegdi, Phys. Rev. 91, 169 (1953).

VI. Application of the $2J_f + 1$ Rule

Recent interest, both experimental and theoretical, has centered on the dependence of reaction cross sections [in particular, those for (d,a) reactions] upon the angular momenta of the residual nuclear states when these cross sections are averaged over a range of bombarding energy adequate to destroy any special correlation between the compound and final states. It has been shown by MacDonald¹⁰⁰ and Ericson¹⁰¹ that these averaged cross sections are proportional to $(2J_f + 1)$ providing that: (a) a compound-system mechanism is involved, (b) the exit-channel energy is adequate to avoid inhibition of any partial wave consistent with the angular momenta J_c and J_f of the compound and final states involved, and (c) the entrance channel is such that a large number of compound states are populated and adequate angular momentum is available to permit $J_c > J_f$.

Corroborative experimental data have been published by Hinds, Middleton, and Litherland¹⁰² and by Colli et al.¹⁰³ Figure 30 shows more recent data of Hansen et al.¹⁰⁴ on the $Mg^{25}(d,a)Na^{23}$ and $Al^{27}(d,a)Mg^{25}$ reactions which provide further qualitative support for the

¹⁰⁰ N. MacDonald, Nucl. Phys. 33, 110 (1962).

¹⁰¹ T. Ericson, Advances in Physics 9, 425 (1960).

¹⁰² S. Hinds, R. Middleton, and A. E. Litherland, in Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961, edited by J. B. Birks (Heywood & Co., Ltd., London, 1962), p. 305.

¹⁰³ L. Colli, I. Iori, M. G. Marcazzan, and M. Milazzo, Nucl. Phys. 43, 529 (1963).

¹⁰⁴ O. Hansen, E. Koltay, N. Lund, and B. S. Madsen (private communication to C. K. Bockelmann; and to be published).

Fig. 30. Experimental test of the $(2J_f + 1)$ rule in the (d, α) reactions on Mg^{25} and Al^{27} as measured by Hansen *et al.* (Ref. 104). The points represent cross-section measurements for states of known spin; the horizontal lines those for states of previously undetermined spin. The insert in the lower figure gives the known previously assumed spins and the deduced spins for states in Na^{23} labeled in order of increasing excitation. The energy range in each case is that over which the cross sections have been averaged to obtain the data shown. Differential cross sections in each case have been integrated over approximately $30^\circ \leq \theta \leq 160^\circ$.

rule, and assuming its validity, provides tentative spin assignments for seven excited states in Na^{23} in the excitation range below 4.8 MeV. Such data are of vital importance to the examination of the applicability of the wide variety of models currently fashionable in the description of sd-shell nuclei.

If further similar measurements continue to support the $(2J_f + 1)$ rule, it will provide an extremely convenient technique for specifying, or at least limiting, such spin assignments in a wide variety of reactions.

This $(2J_f + 1)$ rule for cross sections averaged over energy and angle should not be confused with the factor $[(2J_f + 1)/(2J_i + 1)]$ which appears in the differential cross section expression for a direct stripping reaction or the corresponding $[(2i_f + 1)/(2i_i + 1)]$ factor appearing in a direct pickup reaction, where i_i and i_f represent the spins of the entrance and exit-channel projectiles, respectively.

It will be of considerable interest to examine this rule at higher incident energies, where other evidence suggests a dominant direct-reaction mechanism, in order to establish whether the first proviso noted above is a necessary one. If it should develop that it is not, this technique will be of much greater utility. At present the success of this rule subject to this stipulation is somewhat embarrassing because even at these energies other data mentioned above suggest that direct amplitudes are far from negligible.

VII. Detailed Balancing in Nuclear Reactions and Time-Reversal Invariance

With the availability of more data on inverse deuteron-transfer reaction pairs, and rapidly improving understanding of the

Fig. 31. Angular distributions for the inverse reactions $C^{12}(\alpha, d)N^{14}$ and $N^{14}(d, \alpha)C^{12}$ at matched energies. The curves have been arbitrarily normalized for best fit. (From Bodansky *et al.*, Ref. 106.)

nuclear behavior leading to particular angular distributions, it becomes desirable to consider the possibility of examining detailed balancing as applied to nuclear reactions and their inverses, as suggested by Henley and Jacobson,¹⁰⁵ to test for time-reversal invariance in strong interactions.

Figure 31 shows one of the earliest examinations of this particular problem by Bodansky *et al.*¹⁰⁶ The distributions shown were normalized, arbitrarily, to give best fit; and lacking a detailed theory for the reactions, these authors applied the approximate criteria given by Henley and Jacobson¹⁰⁵ to conclude that the time-reversal noninvariant component of the nuclear Hamiltonian is probably less than 3%. In this work the deuteron and alpha-particle energies were adjusted to satisfy the equation

¹⁰⁵ E. M. Henley and B. A. Jacobson, *Phys. Rev.* **108**, 502 (1957); **113**, 225 (1959).

¹⁰⁶ D. Bodansky, S. F. Eccles, G. W. Farwell, M. E. Rickey, and P. C. Robison, *Phys. Rev. Letters* **2**, 101 (1959).

Fig. 32. Comparison of angular distributions of deuterons and alpha particles from the $O^{16}(d,\alpha)N^{14}$ reaction at $E_d = 23.8$ MeV and the $N^{14}(\alpha,d)O^{16}$ reaction at $E_\alpha = 46.5$ and 42 MeV. The F^{18} excitation energies corresponding to these energies are 28.6, 40.5, and 37.1 MeV, respectively. Within the framework of a plane-wave theory these figures (plotted as functions of momentum transfer) would be predicted to be identical. (From Pehl, Ref. 30.)

$$\left(\frac{d\sigma}{d\Omega}\right)_{d,a} = \frac{(2J_a + 1)(2J_A + 1)}{(2J_d + 1)(2J_{A+2} + 1)} \cdot \frac{(p_a)^2}{(p_d)^2} \left(\frac{d\sigma}{d\Omega}\right)_{a,d}.$$

Legg¹⁰⁷ has shown, however, that within a PWBA treatment of the reactions, only the momentum transfer q need be matched, and for a given q

$$\left(\frac{d\sigma}{d\Omega}\right)_{d,a} = \frac{(2J_a + 1)(2J_A + 1)}{(2J_d + 1)(2J_{A+2} + 1)} \cdot \frac{(p_a)}{(p_d)_{d,a}} \cdot \frac{(p_a)}{(p_d)_{a,d}} \left(\frac{d\sigma}{d\Omega}\right)_{a,d}.$$

In consequence, by varying the angle of observation, a comparison may be made for equal q , even if the more stringent energy-matching conditions of the first equation above are not satisfied. Figure 32 from Pehl³⁰ shows a comparison of the angular distributions obtained with 23.8-MeV deuterons [$N^{14}(\alpha,d)O^{16}$], and with 46.5- and 42.0-MeV alpha particles [$N^{14}(\alpha,d)O^{16}$]. The corresponding compound-system excitations are 28.6, 40.5, and 37.1 MeV, respectively, so that the energy-matching conditions are far from satisfied. The extent to which agreement was obtained here when the experimental cross sections were multiplied by

¹⁰⁷J. C. Legg, Phys. Rev. 129, 272 (1963).

the appropriate kinematic factors (from the above equation) suggests considerable validity for the plane-wave stripping treatments but does not permit as stringent a test of time-reversal invariance because of the more obvious mechanism and model dependence.

VIII. Conclusions

Deuteron-transfer reactions have only recently begun to receive extensive study and have been demonstrated to be highly selective in populating residual states; available evidence suggests that they form effective probes for two-nucleon giant excitations in nuclei. As such, in analogy with studies on single-nucleon excitations in the past, they hold promise of greatly extending contemporary knowledge of nuclear structure and in particular of pairing and collective-motion phenomena and the generalized shell model. In light nuclei they provide information on deuteron clustering phenomena.

In reaction-mechanism studies they show promise of providing a particularly direct examination of the detailed interference between direct and compound-system amplitudes.

They provide an effective approach to the examination of isobaric-spin phenomena and nucleon-nucleon spin-correlation effects. As more information becomes available, in contrast to single-nucleon transfers which provide information only on the magnitudes of fractional-parentage coefficients, two-nucleon transfer reactions can provide a measure of the relative phases as well.

As in the past, much more effort will be required to establish the reliability of the methods used to extract nuclear data from the experimental measurements as in the case of the DWBA calculations for example.

At best, the field has been scarcely opened as yet, particularly at the higher energies where interpretation may be more direct and where the more interesting higher final-state excitations become accessible. There is much yet to be done.

IX. Acknowledgements

It is a pleasure to acknowledge my special indebtedness to Drs. J. Cerny, G. T. Garvey, N. K. Glendenning, K. R. Greider, B. G. Harvey, E. Rivet, H. E. Wegner, and C. D. Zafiratos for providing me with information prior to its publication and for permission to include it herein.

* * *

M. E. RICKEY, University of Colorado: Has anyone done any (α, Li^6) work in which the Li^6 is seen by virtue of the different Q as it comes out in the T=1 state? If so, can this be interpreted as the pickup of a singlet deuteron and as such provide a method for measurement of the reaction amplitude for pickup of a singlet deuteron from the target?

BROMLEY: At the moment, I know of no such data in which evidence for the T=1 state, i.e., the analogue of the He^6 ground state, appears. It would be particularly interesting in combination with your data on the (α, Li^6) reactions.

B. G. HARVEY, Lawrence Radiation Laboratory: With regard to this momentum mismatch, I would like to mention a recent observation by Ernest Rivet¹⁰⁸ at Berkeley. He studied the $\text{O}^{16}(\text{He}^3, \text{p})\text{F}^{18}$ reaction

¹⁰⁸E. Rivet, University of California Lawrence Radiation Laboratory Report UCRL-11341 (1964).

to the 5^+ state in F^{18} at 1.1 MeV, which would have to be $L = 4$. He saw this state very strongly in this (He^3, p) reaction and also in the corresponding (α, d) reaction. One would expect that reducing the energy of the incident alpha could achieve a momentum transfer which could no longer match $L = 4$, so the cross section should fall off. But in doing some calculations based on the concept that the momentum transfer takes place entirely in a surface interaction, we found that the momentum transfer in the (α, d) reaction is almost entirely determined by the Q of the reaction—and Q is strongly negative. Even at very small particle energies, therefore, it is impossible to get much below $L = 4$.

Hinds and Middleton¹⁹ have reported some work with 5-MeV and 9-MeV He^3 beams, and have given angular distributions for the protons from the $O^{16}(He^3, p)F^{18}$ reaction to the 1.1-MeV level. At each of these energies there is a single maximum which occurs at exactly the angle for which $kR = 4$, on the assumption that the radius R of interaction is just the nuclear radius. If this is found to be true in additional cases, then one has a very nice L meter. One would simply vary the energy of the He^3 in such reactions and find the angle at which the cross section reaches its peak. Then a simple semiclassical argument apparently can lead directly to a measurement of the angular-momentum transfer. It appears that $L = 4$ could certainly be distinguished from $L = 5$; in other words, the technique has a resolution width of less than one unit of angular momentum.

BROMLEY: This confirms in striking fashion my earlier remark that this could be a very important tool in sorting out angular-momentum transfer in these direct reactions, since the reactions themselves seem to be much simpler than we had ever anticipated.

R. SHERR, Princeton University: In the $f_{7/2}$ shell, we talk about transferring two nucleons. In the lighter nuclei, you talk about deuteron transfer. Are these just different words for identical descriptions of the reactions?

BROMLEY: "Deuteron" is a fine generic term for a neutron-proton pair but is not intended to have any further significance. There are two points to be made here however. The first is that with the exception of the very few (He^3, p) reactions, which may be special cases since only three particles are involved in the projectile, we see no evidence for transfer of the neutron-proton pair in a $T=1$ configuration. In the (α, d) , (d, α) , and (Li^6, α) reactions, of course, $T=1$ transfer is impossible providing that T is a good quantum number; in the $(\text{B}^{11}, \text{Be}^9)$ reaction, however, we should have the "deuteron," in both singlet and triplet configurations, transferred with equal ease. The strange thing is that the available work on all these more complex reactions shows no evidence whatever that the neutron-proton pair is ever transferred in the singlet deuteron configuration. We do not understand this as yet and certainly additional work is required to demonstrate whether the effect is fortuitous or whether it reflects some more basic facet of the reaction mechanism.

The other point concerns what happens when the (d, α) reaction is used to put two nucleons into different shells. In the work of Yntema and O'Neill on Ti^{48} and $\text{Ti}^{50,61}$ it does not appear to make much difference whether two nucleons transferred in the (d, α) reaction come from the same or from different shells. On the other hand, to fit their data on zirconium and on lead, Micheletti and Mead⁶² found that they had to assume different pairing forces for neutrons and for protons in different orbital states. This would clearly suggest a mechanism of considerably greater complexity than that involving a simple deuteron transfer. I would hope that by obtaining more systematic information on the transfer reactions involving not only "deuterons" but also neutron and proton pairs, it might be possible to obtain relatively direct information on the two-nucleon forces. In this sense I think that there is no significant difference between these studies and those which you have reported in the $f_{7/2}$ shell.

G. H. RAWITSCHER, Yale University: The angular distribution of the Be^9 from the $(\text{B}^{11}, \text{Be}^9)$ reaction (Fig. 28) fell off exponentially with angle; it had no wiggles. As was mentioned in the talk, this must have something to do with the fact that the reaction proceeds through a surface interaction. To understand the angular distribution in a rough intuitive fashion, one represents the Be^9 emission amplitude as a sum of Legendre polynomials with coefficients that are large only near a certain value L' which corresponds to a surface reaction. For the large momenta of the Be^9 at the energies under consideration, L' is about 30. The resulting cross section varies almost exponentially with angle at c.m. angles larger than $20-30^\circ$, but has interference wiggles at smaller angles. In this case the expected interference structure, which is indeed observed for the reaction involving lighter nuclei, is shifted to small angles and is not seen—even though the reaction may have occurred at the surface.

BROMLEY: I do not think that this can be the complete explanation because the angular distributions for the (α, d) reactions were identical to those for the $(\text{B}^{11}, \text{Be}^9)$ reactions leading to the two-particle states although the Be^9 distributions to other types of states are strikingly different. Something more specific of a spectroscopic nature must be involved. What you mention is very similar to an argument which has been advanced by Wall to explain large-angle oscillations in elastic alpha-particle scattering angular distributions.

SHERR: Have distorted-wave calculations been done for the (α, d) reactions?

BROMLEY: They have been done for some (α, d) reactions where characteristic diffraction structure appeared in the angular distributions and several typical examples of such fits will be included herein (Figs. 17 and 18). No such fits have been attempted as yet in either the (α, d) or $(\text{B}^{11}, \text{Be}^9)$ reaction cases leading to the two-particle states since these

exponential angular distributions appeared too pathological for any simple DWBA code. I do not believe that any of the DWBA specialists have tried these exponential cases yet. However, Dr. Cerny may have begun to do so at Berkeley.

J. CERNY, Lawrence Radiation Laboratory: We have such calculations in progress.

G. M. TEMMER, Rutgers University: I would like to point out the existence of a possible tool for forcing singlet deuterons into nuclei by doing (α, np) reactions in which the neutron and proton are in the singlet state, as demonstrated (for example) by measuring their correlation in angle and energy. In this way one can be sure that the deuteron inserted into the nucleus will consist of a neutron-proton pair coupled to zero spin. Incidentally, the isotopic-spin transfer in this case will be unity and makes it possible to excite $T=2$ states when using $T=1$ targets.

K. H. BHATT, Oak Ridge National Laboratory: From the point of view of shell-model calculations, it is very desirable¹⁰⁹ to have the experimental two-body interaction energies. These are sometimes available for two protons and two neutrons in the lower orbit outside a closed shell. The corresponding information for the proton-neutron energies is rarely available. Direct reactions involving two-nucleon transfer might be of great help in determining these energies, not only for nucleons in the lowest orbits but also in excited orbits outside a closed shell.

BROMLEY: First of all, I agree completely with you regarding the desirability of such data. In terms of getting relevant results, two things deserve mention. First, the over-all amount of data available is very small—still not anywhere near enough to permit extraction of

¹⁰⁹ See, for example, I. Talmi, Rev. Mod. Phys. 34, 704 (1962); J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964); K. H. Bhatt and J. B. Ball, Nucl. Phys. (to be published).

proton-neutron interactions with any degree of certainty. We do not yet have enough confidence in the analysis techniques to really believe any interaction data which might be extracted. It is clear that if we continue these studies on reactions which involve two nucleons, and systematically go through shells, as the neutron-proton ones referred to earlier, we will pick up the neutron-neutron and proton-proton interactions as well. I think that this may be as reliable a way as any to get these interactions. Such measurements unquestionably have to be done; they have not been done yet.

BHATT: Perhaps the small splitting of the multiplet of energy levels arising from residual interactions between a proton and a neutron outside a closed shell may involve a problem of resolution. Would a (He^3, p) reaction be better than an (α, d) as far as resolution is concerned? It seems that (He^3, p) might be better suited than (α, d) to the determination of the p-n interaction energies.

BROMLEY: I think that the problem of resolution can be solved completely for all these reactions when the newer machines become operational. The more serious problem may lie in getting our theoretical friends to give us a better understanding of how to extract meaningful results on the nucleon-nucleon forces from the experimental data. This is still a very hazy area.

SHERR: I must say that I am still worried about stuffing deuterons into things. I can put golf balls into my pocket, but with a football I would have trouble.

BROMLEY: You should have big pockets!

EVIDENCE FOR A HIGHLY EXCITED $(d_{3/2}, d_{5/2})$ CONFIGURATION IN $N^{14} \dagger$

K. Nagatani and D. A. Bromley

Yale University, New Haven, Connecticut

Measurements on the $C^{12}(\alpha, d)N^{14}$ and $C^{12}(B^{11}, Be^9)N^{14}$ reactions^{1,2} have shown strong selective population of a probable 5^+ , $T=0$, $(d_{5/2})^2$ state at ~ 9 MeV in N^{14} . A second strong excitation at ~ 13 MeV in N^{14} was observed in the latter reaction² and tentatively identified as the 4^+ , $T=1$, $(d_{5/2})^2$ state predicted by True³ at 12.33 MeV. Re-examination of the former reaction with 42-MeV alpha particles, however, has also shown strong population of this higher state; this suggests that it is the 4^+ , $T=0$, $(d_{3/2}, d_{5/2})$ state predicted by True to be at 13.82 MeV and by Glendenning⁴ to have high population probability in this reaction. Advantages of the (α, d) and (B^{11}, Be^9) reactions as spectroscopic probes will be discussed.

[†]This does not appear in Vol. I. Contributed Papers, but is presented as abstract E8 in the report on the Symposium in the Bulletin of the American Physical Society.

¹B. G. Harvey, J. Cerny, R. H. Pehl, and E. Rivet, Nucl. Phys. 39, 160 (1962).

²M. Sachs, C. Chasman, and D. A. Bromley, Direct Interactions and Nuclear Reaction Mechanisms (Gordon and Breach, Science Publishers, New York, 1963), p. 987.

³W. W. True, Phys. Rev. 130, 1530 (1963).

⁴N. K. Glendenning, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 188.

PAIRWISE STRIPPING AND PICKUP REACTIONS

N. M. Hintz

University of Minnesota, Minneapolis, Minnesota

Reactions involving the transfer of two identical nucleons such as (p,t), (He^3 ,n), etc., have certain features which make them especially interesting for the study of nuclear structure. First of all, one can reach relatively inaccessible nuclei such as O^{14} , Ca^{50} , and Ni^{56} . An example of this has been shown in the work of Cerny and Pehl.¹ One can also reach levels with isotopic spin two units greater than that of the ground state. Figure 1 is a slightly doctored version of the energy-level

Fig. 1. Lowest energy levels of specified T in the mass-16 system, the stated energies being based on the energy of the ground state of the $T=T_z$ member. This diagram illustrates the use of the (p,t) reaction to excite states with $T = T_z + 2$.

¹J. Cerny and R. H. Pehl, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 208.

diagram in the paper of Garvey and Bayman.² This figure shows that since both the proton and triton have an isotopic spin of $\frac{1}{2}$, the $O^{18}(p,t)O^{16}$ reaction enables us to reach $T=2$ states of O^{16} , among which is the analogue of the ground state of C^{16} and an excited state of N^{16} . One can also locate the position of "single pair" levels as shown in Fig. 3 of the paper of Harvey et al.³ where the A dependences of maximum- J states of the $(d_{5/2})^2$ and $(f_{7/2})^2$ configurations are plotted.

From the (p,t) reaction one can also determine the angular momentum coupling of the transferred pair. Figure 2 shows some recent work by Rickey et al.⁴ at Colorado. For even-even nuclei, the selection rules are such that if the transition goes to the ground state, one knows that one must pick up a pair of neutrons coupled to zero angular momentum. One sees in the $Ni^{58}(p,t)Ni^{56}$ reaction leading to Ni^{56} (which is a relatively inaccessible nucleus) that by far the strongest transition is to the ground state. This is also true for titanium(Ti^{48}). This shows the general tendency for the coupling in the ground states to be something like that predicted by pairing or seniority.

The other point I wanted to illustrate from the angular distributions seen at the top of Fig. 2 was that the maxima and minima all come at very nearly the same angle for the $L=0$ ground-state transitions.

²G. T. Garvey and B. F. Bayman, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 125.

³B. G. Harvey, E. Rivet, J. Cerny, R. H. Pehl, and J. Haag, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 182.

⁴M. E. Rickey (private communication).

Fig. 2. Upper part: Angular distributions of (p,t) reactions leading to the ground states and first excited states of the indicated Ti, Fe, and Ni isotopes. Lower part: Spectra of tritons from the (p,t) reactions on Ti^{48} and Ni^{58} . The proton energy was 28 MeV in each case.

This shows an insensitivity to the mass number and Q value. The situation is the same for the $L=2$ transitions shown later.

Finally, the (p,t) transitions are going to be enhanced by collective effects, especially those of the kind produced by nucleon pairing. If one has a really good theory for the reaction, including both finite-range interactions and a finite-size triton, one might learn something about the short-range spatial correlations between the particles of the nucleus.

Table I presents the selection rules for the (p,t) reaction. I will use J , L , and S without any indices to represent the total, orbital, and spin angular momentum of the transferred pair, and J_i and J_f for the

TABLE I. Selection rules for the (p,t) reaction, the process being pictured as

$\vec{J} = \vec{L} + \vec{S}$

$\vec{L} = \vec{l}_1 + \vec{l}_2$

Exact	Approximate
$ J_i - J_f \leq J \leq J_i + J_f $	$S = 0 \quad \text{so} \quad J = L$
$\Delta\pi = (-1)^{\ell_1 + \ell_2}$	$\Delta\pi = (-1)^L$
$\Delta\nu = 0, \pm 2$	$(\Delta J)_{i \rightarrow f} = L$
$T = 1$	

initial and final total angular momentum of the nuclear states. On the left of Table I are the exact selection rules, based on a one-step direct-interaction model, in which the initial and final nuclear states must be connected by the J of the pair. The parity change is $(-1)^{\ell_1 + \ell_2}$, where ℓ_1 and ℓ_2 are the orbital states from which the pair is transferred.

The isotopic-spin selection rule, $\Delta T = 0, \pm 1$, follows from the fact that $T = 1$ for the transferred neutron pair. In the (p, He^3) reaction both $T = 0$ and $T = 1$ are allowed. As a consequence, the ratio of (p, t) to (p, He^3) cross sections to analog states in neighboring nuclei are given by simple expressions involving only vector coupling coefficients. This is shown in Fig. 3, taken from the contribution of Cerny and Pehl,¹

Fig. 3. Angular distributions for the $O^{16}(p,t)O^{14}$ [g. s., 0^+ , $T=1$] and $O^{16}(p,He^3)N^{14*}$ [2.31 MeV, 0^+ , $T=1$] transitions. The $O^{16}(p,He^3)N^{14*}$ data have been multiplied by 2, the ratio predicted for the cross sections if charge independence is assumed in the reaction mechanism and in the nuclear states. The smooth line is an $L=0$ two-nucleon DWBA fit to the (p,t) transition at an interaction radius of 6.2 fm.

where the data fit the theoretical ratio very well. This agreement indicates charge independence in the reaction mechanism and the nuclear states.

The parity selection rule $\Delta\pi = (-1)^{\ell_1 + \ell_2}$ says that to excite a state with opposite parity, you must pick up nucleons from different orbitals having opposite parity.

Finally, there is a seniority selection rule $\Delta\nu = 0, \pm 1$. This reflects the fact that one can break at most two pairs in picking up two particles.

On the right-hand side of Table I are some approximate selection rules. The first one, $L = J$ is based on the space symmetry of the triton wave function (and therefore $S = 0$) and is the most important. It should be about 96% good. Thus the J of the transferred pair is equal to its total orbital angular momentum. Since the angular distributions are characterized by the L of the transferred pair, the angular distributions give directly the J to which the pair has been coupled. The first selection rule becomes particularly simple in the case of even-even targets, since

the J_f of the final state will then be equal to L . Thus there is no ambiguity in the angular momentum assignments for even-even targets as is the case for single-nucleon transfer.

A weaker selection rule is based on the fact that the neutrons in the triton are mostly in a relative S state. It then follows that $\Delta\pi = (-1)^L$. This should be somewhat weaker than the $S=0$ rule. If one sees an unnatural parity state, then it must go by either a violation of this latter selection rule or by a violation of the $J=L$ selection rule. For example, the 3^+ state seen in Mg^{24} by Rickey et al.⁵ must go by violation of one or the other of these selection rules. By measuring the angular distribution it should be possible to distinguish which.

Finally, the relationship between the spectroscopic amplitude, the spectroscopic factor, and the cross section is illustrated by the expression for the cross section for transfer of two nucleons in the (p,t) reaction in the DWBA, namely

$$\sigma(\theta) = \sum_L \left| \sum_{\substack{\ell_1, j_1 \\ \ell_2, j_2}} S_L^{1/2}(\ell_1, j_1, \ell_2, j_2) F_L(\theta, \ell_1, j_1, \ell_2, j_2) \right|^2. \quad (1)$$

The first factor is the spectroscopic amplitude $S_L^{1/2}$ (defined below). This is multiplied by the factor F_L which depends on distortion effects, the energies, and the Q value. It contains the angular distribution. The point has been made several times that in the (p,t) reaction, and likewise in the (He^3, n) or the like, the indicated summation is a coherent sum over all of the occupied orbital states of the transferred pair.

⁵M. E. Rickey, H. E. Wegner, K. W. Jones, B. M. Bardin, C. Hoot, and M. Kondo, Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, March 1964, p. 217.

A second point is that the F_L contain not only the distortion effects, but also the bound-state wave functions and will therefore be twice as sensitive to the assumed binding energies for the single-particle states—since the F_L contain products of the (ℓ_1, j_1) orbital with the (ℓ_2, j_2) orbital. That is, the F_L contain two single-particle orbitals instead of only one as is the case in a (d, p) reaction.

The spectroscopic amplitude, $S_L^{1/2}$ is defined as the overlap between the final state [with a pair of particles created in states (ℓ_1, j_1) and (ℓ_2, j_2) coupled to L] and the initial wave function. For nonequivalent particles, it is given by

$$S_L^{1/2} = \langle 0 | \Psi^\dagger(\alpha, J_i, M_i) \{ c [a_{\ell_1 j_1}^\dagger a_{\ell_2 j_2}^\dagger]^L \Psi(\beta J_f) \}_{J_i} | 0 \rangle . \quad (2)$$

Under the special circumstance that the two nucleons are picked up from a single pair of orbital states, such as the experiments that we will discuss later in the $f_{7/2}$ shell, one can factor out the spectroscopic amplitude from the sum in Eq. (1) and talk about a spectroscopic factor. Ordinarily, however, one must calculate the coherent sum over many orbitals.

Let us next consider the spectroscopic factor (or amplitude) calculated from various models. First of all, the very simplest picture to take in transferring a pair of neutrons is to entirely neglect the protons and assume that the neutrons are in the configuration $(j)^n$ with seniority coupling. The resulting spectroscopic factors have been calculated by Bayman.⁶ The spectroscopic factors S_0 (for a transition between two states with seniority zero) and S_2 (for a transition from seniority 0 to

⁶B. F. Bayman (private communication). See also R. Sherr, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 1025.

seniority 2) in even-even nuclei are given by

$$S_0 = \frac{n}{2} \frac{(2j + 3 - n)}{2j + 1} \rightarrow \frac{n}{2} \frac{(2\Omega + 2 - n)}{2\Omega}, \quad \left(\begin{array}{c} \text{degenerate} \\ \text{limit} \end{array} \right) \quad (3)$$

$$S_2 = \frac{n(n - 2)(2J_f + 1)}{(2j - 1)(2j + 1)}, \quad (\Delta v = 2). \quad (4)$$

In Eq. (3) one sees that the $L=0$ spectroscopic factor increases linearly at the beginning of the shell as one begins to add particles, reaches a maximum as the shell is half filled, and then decreases at the end of the shell to the same value as at the beginning. This behavior reflects the fact that the Pauli principle forces the added nucleons to spend a good part of their time in paired states other than $J = 0$ as the shell fills, even though the state has seniority zero. On the other hand, the $L=2$ spectroscopic factor increases (roughly quadratically) as one goes through the shell.

In the case of a nucleus with a number of nearly degenerate levels, as is the case in the nickel-copper-zinc region where the $1f_{5/2}$ and the $2p$ levels are nearly degenerate, one can approximate the results of the pairing theory by replacing $(j + \frac{1}{2})$ by the total pair degeneracy $\Omega = \sum_j (j + \frac{1}{2})$. We call this the degenerate model.

When the cross section is obtained by use of the pairing wave functions, the exact expression for transitions between two states with seniority zero is

$$\sigma = \left| \sum_j (2j + 1)^{1/2} U_j V_j F_0(j, \theta) \right|^2, \quad (5)$$

where the V_j are the occupation amplitudes of the parent nucleus and the U_j are the unoccupation amplitudes of the daughter. The coherent sum over j can give rise to a collective enhancement of the cross section. The factor F_0 , which depends on j , is the factor that contains the radial

4

wave functions and the distortion effects. In an approximate treatment of this equation, Yoshida⁷ shows that F_0 can be replaced by $(2j + 1)^{1/2}$ times a factor that does not depend on the j of the occupied orbital, i. e.,

$$F_0(j, \theta) \approx (2j + 1)^{1/2} g(\theta). \quad (6)$$

Finally, in the case of a transition from a zero-quasiparticle to a two-quasiparticle state specified by $(j_1, \ell_1), (j_2, \ell_2)$, the sum reduces to a single term. The cross section then is

$$\sigma = \left| V_{j_1} V_{j_2} F_J(j_1, j_2, \theta) \right|^2.$$

The spectroscopic amplitude is $V_{j_1} V_{j_2}$ and again is multiplied by the reaction factor. If the final state is a vibrational state that involves several quasiparticles, then one would sum over all the possible two-quasiparticle states $(j_1, \ell_1) (j_2, \ell_2)$ which make up the vibrational state. In that case, one could get the same collective enhancement in the two-quasiparticle transitions as one gets in the $\nu=0 \rightarrow \nu=0$ transitions.

Table II shows the kind of collective enhancement that one can get from pairing correlations in the Yoshida formula. If the two extra neutrons in Ni^{58} are put in the $2p_{3/2}$ shell and coupled to zero, the resulting cross section is proportional to 2. If the two neutrons are put in the $f_{5/2}$ shell, then the cross section is proportional to 3—a little more, but not much. On the other hand, if the two neutrons are distributed with the kind of relative probability predicted by Kisslinger and Sorensen⁸ throughout the f , p , and g shells, then there is a large enhancement due to

⁷S. Yoshida, Nucl. Phys. 33, 685 (1962).

⁸L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 9 (1960).

TABLE II. Collective enhancement from pairing correlations in the Yoshida formula for the case of the $\text{Ni}^{58}(\text{p},\text{t})\text{Ni}^{56}$ reaction to the $L=0$ ground state.

Configuration	σ
$(\text{p}_{3/2})_0^2$	2.0
$(\text{f}_{5/2})_0^2$	3.0
$0.83(\text{p}_{3/2})_0^2 + 0.50(\text{f}_{5/2})_0^2$ $+ 0.18(\text{p}_{1/2})_0^2 + 0.16(\text{g}_{9/2})_0^2$	7.3

the fact that the particles can correlate much more strongly in the highly mixed configuration characteristic of the pairing theory.

The next data were taken at Minnesota by Bassani, Maxwell, Reynolds, Kavaloski, and Hintz.⁹ Figure 4 shows a spectrum for $\text{Ni}^{60}(\text{p},\text{t})\text{Ni}^{58}$ at 40 MeV. The characteristic feature here is the very strong $L=0^+$ ground-state transition. It has been reduced by a factor of five on the graph. A moderately strong 2^+ transition can also be seen. The other thing I want to point out in Fig. 4 is that the total strength to the group of states which are sometimes described as 2-phonon is somewhat less than the total strength in the first 2^+ state. Finally, the 3^- state, which is known to be at about 4.1 MeV, is apparently quite weakly excited—as would be expected if it were a particle-hole state with a particle in the 1g shell.

⁹G. Bassani, N. M. Hintz, and C. D. Kavaloski, Phys. Rev. (in press); G. Bassani, J. R. Maxwell, G. Reynolds, and N. M. Hintz, in Proceedings of the International Conference of Nuclear Physics, Paris, 1964.

Fig. 4. Spectrum of tritons at 20° from the $\text{Ni}^{60}(p,t)\text{Ni}^{58}$ reaction at 40 MeV. The positions expected for the first 2^+ , two-phonon triplet (0^+ , 2^+ , 4^+), and the collective octupole state (3^-) are shown.

In the analysis of these experimental data, rather than use the BBA (Bad Born Approximation, as introduced the first day by Norman Austern), we use the NBA (No Born Approximation) in the spirit of Bernie Cohen. In the comparison with theory, we simply take our integrated cross sections and talk about them as if they were spectroscopic factors. This obviously is an inadequate analysis, but it is the best we have at present. This must be borne in mind for all of the data points which I show from now on; they are simply integrated cross sections. Then in going through the shells we will look for systematic effects in the $L=0$ and in the $L=2$ "spectroscopic factors."

Part of our justification for this method is shown in Fig. 5. Here, as was seen also in Fig. 2, the angular distributions are very insensitive to both the Q value and the A of the target. Figure 5 shows $L=0$ and $L=2$ angular distributions for Ni^{60} and Ti^{48} . The two $L=0$ angular distributions have their maxima at an angle of $20^\circ \pm 1^\circ$ and their shapes are nearly identical, with the same ratio of maxima to minima. The angular distributions for the first 2^+ states ($L=2$) for Ni^{60} and Ti^{48} are

Fig. 5. Angular distributions for the lowest $L=0$ and $L=2$ transitions for the $\text{Ti}^{48}(p,t)\text{Ti}^{46}$ and $\text{Ni}^{60}(p,t)\text{Ni}^{58}$ reactions at 40 MeV.

Fig. 6. Ground-state ($L=0$) and first-excited-state ($L=2$) integrated (p,t) cross sections (solid circles) and theoretical spectroscopic factors for $f_{7/2}$ -shell nuclei. For V^{51} , the lowest $L=2$ group (which contains several states) is shown. Theoretical spectroscopic factors (MBZ) are shown as open squares, connected with short dotted lines to experimental points wherever uncertainties of association can arise.

The dashed lines give the prediction for pure neutron seniority coupling. All spectroscopic factors are normalized to the data at Ca^{44} ($L=0$) or Ca^{48} ($L=2$).

also very similar. Thus we simply take an integrated cross section over the observed angular region which contains the forward peak and call that a spectroscopic factor.

I would now like to discuss the data on the $f_{7/2}$ shell which are shown in Fig. 6. On the left-hand side of the figure I have plotted the cross sections for the $L=0$ (ground state) transitions and on the right, the cross sections for the $L=2$ (first excited state) transitions. The black points, connected by solid lines for isotopic sequences, are the experimental points. To provide an absolute normalization for the $f_{7/2}$ spectroscopic factors we have included some recent results on the calcium isotopes. Calcium should have close to a pure $(f_{7/2})^n$ configuration for the neutrons and a closed shell for the protons.

The simple neutron seniority formulas of Eqs. (3) and (4) are shown by dashed lines, normalized to Ca^{44} for the $L=0$ plot and to Ca^{48} for the $L=2$ plot. The open squares are the spectroscopic factors from the "exact" shell-model calculations of McCullen, Bayman, and Zamick¹⁰ (MBZ) discussed in an earlier paper. The short curved lines show which experimental point a calculated spectroscopic factor is

¹⁰ J. D. McCullen, B. F. Bayman, and L. Zamick (to be published).

supposed to fit. MBZ have assumed a pure $(f_{7/2})^n$ configuration for neutrons and protons and have taken the two-body matrix elements from the spectrum of Sc^{42} . Their calculations reduce to the simple neutron seniority predictions for the calcium isotopes, but predict considerable seniority mixing for the other nuclei. The theoretical $L=0$ spectroscopic factors are seen to be in rather good agreement with the data, although the calculated Ca^{42} and Cr^{52} points are a bit low and the Ca^{48} point too high.

The effect of protons in producing states of mixed neutron seniority is seen clearly in the reduction of the $L=0$ cross sections for the titanium isotopes below those for calcium.

The general over-all fit to the ground-state transitions in the $f_{7/2}$ shell leads us to believe that our method of substituting cross sections for spectroscopic factors is valid to about $\pm 20\%$.

The $L=2$ situation is not quite as good. With the normalization of the $L=2$ spectroscopic factors to the data at Ca^{48} (the only $L=2$ currently available for the calcium isotopes), the remaining $L=2$ predictions of MBZ are systematically low, except for Fe^{54} . This systematic discrepancy indicates a failure of the pure $(f_{7/2})^n$ configuration to produce the full "collective enhancement" seen in the $L=2$ transitions. The discrepancy is especially great for Ti^{46} , indicating $2s$ - or $1d$ -hole admixtures in the 2^+ state of Ti^{44} , which could be excited by pickup of pairs in Ti^{46} from the s - d shell, thereby increasing the $L=2$ cross section.

Figure 7 shows all of the $1f_{7/2}$, $2p-1f_{5/2}$, and $1g_{9/2}$ nuclei, the $L=0$ spectroscopic factors being plotted against the target neutron number. On the left are the $f_{7/2}$ data that I have just discussed: titanium, chromium, vanadium, etc. On the right part of the figure are shown data on the iron, nickel, zinc, copper, and germanium isotopes. There are several comments to make on these data. I will discuss the nickel isotopes separately so let me by-pass them a moment.

The three levels

$2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ are nearly degenerate. Thus we have tried a fit with $\Omega = 6$ in the degenerate formula [the right-hand side of Eq. (3)]. The fact that the gross features of the data do agree with that indicates something like a

shell closure at the $f_{5/2}$ shell, at least for the nickel isotopes, and somewhat less so for the zinc isotopes. In the zinc and germanium isotopes, there is clearly

evidence for even more configuration mixing involving the $g_{9/2}$ shell since no minimum can be seen at $N = 40$ where the $f_{5/2}$ shell closes. Secondly, the fact that there is a decreasing cross

section in the zinc and germanium region as the $g_{9/2}$ shell fills indicates that as you begin to fill the proton shells, more $L=2$ pairs get mixed into the ground state, thus decreasing the $L=0$ cross section. In other words, bigger quadrupole fluctuations are occurring in the ground states. This reduces the $L=0$ contribution below what would be expected for seniority coupling of the neutrons in the $g_{9/2}$ shell.

Something similar is indicated by the fact that the odd-Z

copper and cobalt isotopes are low relative to the even-Z nickel isotopes. This indicates admixtures of 2^+ core states in the ground states of the nuclei.

Figure 8 compares the Ni^{58} , Ni^{60} , Ni^{62} , and Ni^{64} data

with pairing theory. The data points for the $L=0$ cross sections are the

Fig. 7. Ground-state ($L=0$) integrated (p,t) cross sections (circles) and theoretical spectroscopic factors (dashed lines) for $f_{7/2}$ and $2p-1f_{5/2}-1g_{9/2}$ nuclei. Odd- Z nuclei are shown as crossed circles. Theoretical spectroscopic factors for the $f_{7/2}$ -shell nuclei are calculated from the seniority formula; those for the $2p-1f_{5/2}$ nuclei are obtained from the degenerate model with $\Omega = 6$.

Fig. 8. Ground-state ($L=0$) integrated (p, t) cross sections for the nickel isotopes (solid and open circles) compared with various theoretical predictions for the cross sections. (See text for explanation.)

solid black circles. However, I have replotted the data, changing the normalization (open circles) to emphasize an important point that I want to make in relation to absolute spectroscopic factors. It is important not only to have good relative spectroscopic factors, which we really do not have yet, but also to have good absolute spectroscopic factors because the interpretation of the data will depend on whether it is normalized according to the upper or lower set of points. The solid line in Fig. 8 shows the degenerate model with $\Omega = 6$. The dashed line is the pairing formula of Yoshida,⁷ discussed earlier, in which the radial integrals are treated approximately. We have taken the U 's and V 's from Kisslinger and Sorenson.⁸ If you use the experimental U 's and V 's from single-nucleon-transfer experiments, the results are very much the same.

Now to return to the question of normalization. If the upper set of points is correct, then the pairing theory indicates too little cross section for Ni^{58} and Ni^{60} . This would indicate an excess of 0^+ coupled pairs in their ground states, probably pair excitations from the $f_{7/2}$ shell. The presence of these excited pairs would then increase the $L=0$ pickup cross section at the beginning of the shell. On the other hand, if you believe that the data should be normalized so that the fit is good for Ni^{58} and Ni^{60} , then the two higher nickel isotopes (Ni^{62} and Ni^{64}) are quite low as compared with the pairing theory. This could indicate some replacement of zero-coupled pairs by 2^+ coupled pairs in the Ni^{62} and Ni^{64} ground states. Another way to produce the latter discrepancy is to

have the $1g_{9/2}$ level treated incorrectly in the Yoshida formula. The levels that contribute in this region are the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$, all of which have negative parity, and the $1g_{9/2}$ which has positive parity. As you go to the heavier nickel isotopes, you start getting a large contribution to $L=0$ pickup from the $g_{9/2}$ level. This level causes the pairing prediction to shoot up between Ni^{60} and Ni^{62} . If you arbitrarily reverse the sign of the occupation amplitude for the $g_{9/2}$ level, then you get the curve shown by the coarse dashed line—which looks very much like the experimental curve if I normalize it as shown. Finally, the disagreement with the pairing theory in Fig. 8 may be due to the fact that in the Yoshida formula the radial integrals have not been treated adequately. I am told that Drisko now has a code which does this much more correctly.

It should also be remarked that in comparing the cross sections with theory for a sequence of isotopes, the neutron separation energy changes monotonically as you go through the sequence. The data have not been corrected for that since nobody knows how to do it in the right way. If you use the experimental separation energies, that would make the disagreement with theory worse. If you use a constant well depth for the single-particle states but let the radius vary as $A^{1/3}$, then that will tend to move the high- A points up and give better agreement with pairing or seniority.

Figure 9 shows some data in two isotopic sequences, Cd and Sn. Here the $L=0$ ground-state-to-ground-state transitions are plotted against the number of neutrons in the target. The circles are the tin isotopes. Recently we have remeasured the $L=0$ cross sections for the tin isotopes with different targets and have found disagreement with the data shown in Fig. 9. Until this discrepancy is understood, the discussion on the tin isotopes should be taken lightly. The crosses are some measurements made last week on four of the cadmium isotopes; the curves are from pairing theory. Although the normalization of the experimental

Fig. 9. Ground-state ($L=0$) integrated (p,t) cross sections for the Sn (dotted circles) and Cd (crossed circles) isotopes compared with predictions of the pairing theory. (See text for explanation.) The data for the Sn isotopes should be regarded as somewhat tentative.

curve to the theoretical curves is arbitrary, the relative normalization of the cadmium to the tin is correct to within about 10%. Once again one sees that as soon as you open up the proton shell in going from Sn to Cd, the $L=0$ strength decreases because you are now apparently mixing some 2^+ pairs into the ground state that were not present in the tin isotopes.

Also, as in the $1f-2p$ shell data, we are too high at the beginning of the shell as compared with theory (or too low at the end of the shell). The reasons for this can be the same as before. The excess $L=0$ strength at the beginning of the shell could be due to the fact that we really should include some excited neutron pairs from the lower "closed" shells in the ground states. This would be the explanation if you believe the low- A isotopes are high in cross section relative to theory. On the other hand, the rise that comes in the theoretical cross sections for large A comes from the one odd-parity level that is participating in this group of tin isotopes, namely the $h_{11/2}$. Thus the situation could be the same as for the $g_{9/2}$ level in the Ni isotopes.

The theoretical curves shown are all calculated from the Yoshida formula by use of either the U_j and V_j obtained experimentally from Cohen and co-workers or from the recent pairing-theory calculations by Arvieu.¹¹ The bottom theoretical curve is calculated by Rasmussen.¹²

¹¹ R. Arvieu, thesis, University of Paris, 1962; *Annales de Physique* **8**, 407 (1963).

¹² J. O. Rasmussen (private communication). See also *Nucl. Phys.* **44**, 93 (1963).

Fig. 10. First-excited-state ($L=2$) integrated (p,t) cross sections for $f_{7/2}$ and $2p-1f_{5/2}$ shell nuclei (dotted circles). For the odd- Z nuclei the lowest $L=2$ group, which may contain several states, is shown (crossed circles). The dashed lines show the behavior of simple neutron seniority spectroscopic factors.

He tried to improve the Yoshida formula by weighting each of the $U_j V_j$ factors in Eq. (5) by the squared amplitude of the single-particle bound-state wave function, evaluated at the nuclear surface, thus doing a somewhat better job of treating the dependence on the radial wave functions from the states with various values of j .

It is really crucial to a final interpretation of these results to do a better job of treating the radial integrals, and also, of course, to do the full DWBA calculation in which attempts are made to take proper account of the binding-energy effects mentioned above and also of the effects of the finite size of the triton.

Figure 10 shows the rest of the $L=2$ data. On the left are shown the $f_{7/2}$ -shell data which I have already discussed. Shown on the right are the data on the $2p$ and $1f_{5/2}$ shells. The rapidly rising dashed line is again the simple degenerate model with seniority coupling with $\Omega \geq 4$. (The shape of the $L=2$ curve is independent of Ω .) However, the experimental points do not rise as rapidly as expected. Furthermore, the cross sections for the Zn isotopes are considerably lower than those for Ni with the same neutron number. The dashed line gives in essence the total transition strength to the seniority-two states on the assumption that the ground state is a state of neutron seniority zero. If the seniority-two strength is not all contained in the first 2^+ state, then we would expect the cross section to the first 2^+ state to be reduced. If that is the case, we should see some of the seniority-two strength higher up.

Fig. 11. Spectrum of tritons from $\text{Zn}^{66}(\text{p}, \text{t})\text{Zn}^{64}$ at 10° and 20° at 40 MeV. The group at 2.7 MeV in Zn^{64} appears to be a second strong $L=2$ transition.

Some indication of this is seen in Fig. 11, which shows the Zn^{66} spectra taken at two angles. The upper curve shows the spectra taken at 20° and the lower curve the spectra at 10° . At 20° the $L=0$ cross sections are at a maximum; at 10° the $L=2$ are at their maximum. The first excited state is known to be 2^+ . Evidently there is a strong $L=2$ group, appearing at about three times the excitation energy of the first 2^+ state, which carries a large part of the seniority-two strength. This is evidently due to the fact that with protons outside the closed shell, neutron seniority is

no longer a good quantum number, as we saw was also the case for the $f_{7/2}$ shell.

I will conclude by saying that the general features of the $f_{7/2}$ shell data for the $L=0$ transitions are well described by the shell-model calculation of McCullen, Bayman, and Zamick,¹⁰ but the $L=2$ transitions are not so well described. Also the pairing theory either underestimates the contribution from the closed shells, or possibly overestimates the contribution from the high- j states of odd parity in the $2p-1f_{5/2}-1g_{9/2}$ data and in the tin isotopes.

* * *

R. SHERR, Princeton University: Your method of the NBA may be paraphrased to say it is better to do something than to do nothing. It

is well known, I think, from these distorted-wave calculations that shapes can be insensitive, but the absolute cross section can be sensitive. Do you have any evidence to indicate that ignoring the Q dependence could cause you to be off by a factor of two?

HINTZ: Just before coming, Peter Gould from Minnesota ran a series of DWBA calculations on the tin isotopes. If we keep the well depth for the single-particle states the same and change the radius by $A^{1/3}$, which is not much of a change, then the binding-energy dependence through the shell is only about 10%. However, if you let the binding energy change by a couple of MeV out of 8 MeV, which is roughly what the Q value does, then there is a huge change—a factor of two or more. That is one of the annoying things. One is really going to have to know how to handle the binding-energy problem in order to extract meaningful spectroscopic factors from the data.

B. L. COHEN, University of Pittsburgh: I think you should not be too discouraged about the disagreement with the results of (d, p) reactions. The states that your calculations are most sensitive to are the $h_{11/2}$ in tin and the $g_{9/2}$ in nickel; and those are the ones in which we do the poorest job. I hope no one is judging our work by the accuracy of our results for the $h_{11/2}$ and $g_{9/2}$ states.

To be sure that no rumors get started, we are among JULIE's and SALLY's best customers.

HINTZ: We have the equivalent of JULIE and SALLY, but that is not good enough for two-nucleon-transfer analysis.

N. K. GLENDENNING, Lawrence Radiation Laboratory: In your analysis you have used Yoshida's formulae for the nuclear-structure factors.¹³ That is correct as far as it goes. However, beside the factor which Yoshida calls $B(J; j_1 j_2)$, there are two other factors which are important. With

¹³ S. Yoshida, Nucl. Phys. 33, 685 (1962).

reference to my contribution to this meeting,¹⁴ of the three parts making up the structure factor G , Yoshida's method yields only the parentage factor β . The others do not appear because of the approximations: a point triton is assumed, and the long-wavelength approximation is applied to the plane-wave method that he employs. As I implied, this does not compromise his results for the parentage factor B . However, the other factors should be retained for a detailed comparison with experiment.

HINTZ: When you write nice papers on deuteron transfer, why don't you bother to write the equations for transfer of an identical pair as well?

J. S. BLAIR, University of Washington: I have an experimental question concerning your last figure (Fig. 11) in which you showed the spectrum of the reaction from Zn^{66} to Zn^{64} . I seem to recall there is a 3^- state in Zn^{64} at about 3 MeV. Could that be confused with the second 2^+ state which you showed?

HINTZ: The answer is that we do not know. We know only the 0^+ , 2^+ , and 4^+ angular distributions—they are quite different. But the characteristic ratio of the 10° to 20° cross sections for the 2^+ states may accidentally hold for the 3^- . We don't know since we have not resolved a known 3^- state so as to get its angular distribution. We hope to be able to do that in the future.

N. AUSTERN, University of Pittsburgh: Regarding your approximate parity selection rule on the coupled L , you indicated that one of the reasons the approximation might be bad is because of the properties of the triton; but another reason why the approximation is capable of breaking down is because of the finite-range effects. This has been known since

¹⁴N. K. Glendenning, in Nuclear Spectroscopy with Direct Reactions. I. Contributed Papers, Argonne National Laboratory Report ANL-6848, p. 188.

Levinson and Banerjee,¹⁵ and it has been emphasized again recently in some more recent papers.¹⁶

HINTZ: I got nearly all of my discussion of spectroscopic factors from Bayman. I was under the impression that this was independent of whether you took finite-range forces or not. Maybe Ben would comment on that.

B. F. BAYMAN, Princeton University: As I understand it (Norman Glendenning can correct me), if you want to get that $\Delta\pi=(-1)^L$ rule you have to assume that the two picked-up neutrons are in a relative s state. This will come about if you assume that the two neutrons are picked up at the same place (the point-triton approximation). Otherwise you can assume that they are in a relative s state in the outgoing triton, and that the interaction that picks them up is such as not to change the orbital angular momentum. Thus, they are only picked up in the s state and the $\Delta\pi=(-1)^L$ selection rule follows.

E. M. HENLEY, University of Washington: That is essentially correct, except that as long as the interaction is not with the center of mass of the two nucleons, but rather with each nucleon that gets picked up, you can obtain contributions from other s states.¹⁷ I have one other remark, and that concerns the angular distribution. I think that it is a hangover from plane-wave theories that one makes comparisons with fixed momentum-transfer values. In particular, that is true with some of the figures that Dr. Bromley showed concerning (α, He^6) and (p, t) comparisons. I think that if you take distortions into account, it is not

¹⁵C. A. Levinson and M. K. Banerjee, *Ann. Phys. (N. Y.)* 2, 471 (1957); 3, 67 (1958).

¹⁶For example, see N. Austern, R. M. Drisko, M. L. Halbert, and G. R. Satchler, *Phys. Rev.* 133, 3 (1964).

¹⁷E. M. Henley and D. U. L. Yu, *Phys. Rev.* 133, B1445 (1964); *ibid.* (to be published).

relevant to compare curves with fixed values of momentum transfer. In particular, if you have a simple diffraction picture as advertised by Lipkin earlier and as also investigated by Yu and me,¹⁷ then it turns out this is not the correct comparison to make. One further advantage of this simple diffraction picture is that we find that it does predict the expected large cross sections to final angular-momentum states that are preferred by the phasing of the incident and outgoing waves (i.e., to states for which $|k_f - k_i|R = J_f$ if the initial nuclear state has spin $J_i = 0$).

D. R. INGLIS, Argonne National Laboratory: Dr. Hintz, did you think that the interesting experimental discrepancy that you showed in the cross sections of the sequence of tin isotopes seemed to be larger than expected at the beginning of the shell?

HINTZ: It was either larger at the beginning or smaller at the end.

INGLIS: This might arise from a systematic mistake in the radial integrals at the beginning of the shell as a result of using oscillator wave functions where the shell is not yet very well formed. Actually, there is systematically more penetration of the particle wave functions out into the classically forbidden region. The tails found in nature are stronger than the ones we calculate in this way.

HINTZ: Until Drisko finishes his good calculation, I don't think we will know the answer to that. Presumably what Rasmussen did in weighting the various U's and V's with a radial function evaluated at a large radius should do a better job than the Yoshida formula; but actually the worst curve was the Rasmussen curve. It was very low at the beginning of the shell, lower than the straight Yoshida formula.

SESSION F

WEDNESDAY AFTERNOON

11 MARCH 1964

CHAIRMAN: S. K. ALLISON

SCIENTIFIC SECRETARIES

D. KURATH

A. ELWYN

G. R. RINGO

INTERMEDIATE STRUCTURE AND DIRECT REACTIONS

A. K. Kerman

Massachusetts Institute of Technology, Cambridge, Massachusetts

(No manuscript received.)

* * *

K. K. SETH, Northwestern University: Firstly, I want to mention that we have measured average neutron total cross sections in a large number of nuclei from copper to uranium and have indeed found pronounced structure of the intermediate-resonance type. For reasons of penetrability alone, only s- or p-wave resonances are possible in most of these cases. The structure is genuine because the average spacing of compound nuclear levels is of the order of a few electron volts, our resolution width is about 10 keV, and the width of the structure is between 50 and 100 keV. This structure cannot be the Ericson fluctuations since they can be observed only when the resolution width is less than Γ . We, on the other hand, have a resolution width a thousand times Γ . Also, for us $\Gamma \leq D$. This is not true for the σ_t measurements between 3 and 8 MeV that you referred to, since $\Gamma > D$ in that energy range.

Now I want to mention that what surprises me most about these intermediate resonances is that they show up in the total-cross-section measurements of Foster et al. in spite of the fact that many, many angular momenta must be participating at this excitation (up to 16 MeV in this case) and in spite of the fact that the targets were natural elements with many isotopes present. Why doesn't all the intermediate resonance structure get washed out?

KERMAN: I wish I knew! But the fact that they are there certainly implies something very interesting in this connection. Of course, there is the whole matter of fluctuation in the cross sections. Some of the "intermediate structure" might conceivably be fluctuations, but this seems hard to believe because the fluctuation theory is based on nuclear dynamics; and in a complicated nucleus the fluctuation in the total cross section should wash out.

SETH: My argument about their being washed out applies equally well whether the structure originates as intermediate resonances or as fluctuations.

KERMAN: It is a very interesting problem, but I don't know the answer.

SETH: I have a second question which refers to the fact that when giant resonances are considered from the point of view of Lane, Thomas, and Wigner, then the single-particle weighting introduces a sort of clustering of many-particle compound-nucleus resonances with a particular angular momentum. If now we consider these intermediate resonances weighting the many-particle compound-nucleus resonances corresponding to their own characteristic widths, wouldn't this correspond to some kind of clustering of resonances with large widths? This would imply that the values of Γ_n for the i th and $(i + 1)$ th resonances would be correlated.

KERMAN: That agrees with my understanding of it. According to the Lane-Thomas-Wigner picture, a bump in a region where the compound-nucleus resonances do not overlap indicates a bump in the strength function.

N. M. HINTZ, University of Minnesota: What would the angular distribution look like if one of the two particles went back out and left the nucleus in a vibrational state? Is this sort of thing included in a normal DWBA calculation?

KERMAN: The DWBA calculations that include the vibrational degree of freedom, especially the coupled equations used by some people, would include that effect.

R. E. SEGEL, Argonne National Laboratory: We have made a rather extensive study of the $\text{Al}(p, \gamma)$ reaction through the giant-resonance region. The structure in the cross section has a characteristic width of 30—50 keV. Are there any experimental characteristics that would aid us in deciding whether we are observing intermediate structure or fluctuations?

KERMAN: The only experimental characteristic that I have been able to put my finger on is a similarity to a direct reaction. In other words, if you come in with a nucleon and sit on top of an intermediate structure bump, something different should happen in the outgoing channels for particles coming out, namely, the particle coming out should preferentially leave certain kinds of excitations in the target. That is the only thing that I have really been able to put my finger on. The system is complicated enough. It is almost complicated enough to do statistics on; so that is about the only thing.

With regard to your data, if Si^{28} is a deformed nucleus as it is believed to be, this fact would imply that these structures would be more split up in it than in calcium or some other nucleus that is closer to a closed shell. Thus your observations may very well be consistent with intermediate structure. On the other hand, they may also be fluctuations. This may be an example in which the number of bumps predicted on the basis of general considerations and the number expected from particular considerations come out the same. It will be difficult to tell which it is in this particular nucleus, but I think (p, p') experiments and (p, n) experiments on those bumps would help.

SEGEL: If the (p, γ) reaction yields two gamma rays, one to the ground state and one to the first excited state, wouldn't cross correlations

between the two be expected if the structure is not due to Ericson fluctuations?

KERMAN: There might be correlations, but they are not guaranteed. Their occurrence depends on gamma-ray physics, and this introduces another variable into the problem. If you find any strong correlations, then you are not observing ordinary fluctuations. If you don't find them, then I don't know.

A. I. YAVIN, University of Illinois: I want to ask a similar question except that it refers not only to gammas but to all kinds of particles. If you do (p, p') , (p, n) , (p, γ) , (p, α) , and so on, would you expect correlation between the fluctuations? What different behaviors would you expect? Are there any experimental data which compare (p, γ) , (p, α) , (p, p') , and (p, n) reactions in this region?

KERMAN: I do not know of any good case, although there may be some tentative ones. However, I think that in (p, p') or (p, n) reactions one should look for some correlation in the spectrum of the final particle as a function of the energy of the incident particle. If you find no correlation at all, then I do not understand it on the basis of intermediate structure.

E. P. WIGNER, Princeton University: If you interpret these puzzling fluctuations as giant resonances, there are certain characteristics that might possibly give experimental clues. In particular, one should look to see whether a peak in the structure represents an increase in the density of the microscopic levels, or whether it increases their depth or their width. There may be a combination of such effects; or, finally, the background cross section, i. e., the cross section between the microscopic levels, may have a maximum. If we have to do with a giant resonance, there would be an increase in the widths of the levels. If it is an interference with a peripheral reaction, as is quite possible, it would show up principally in the background. Hence, I wonder if these

fluctuations could not be investigated in an energy region in which the maxima in the cross section can still be resolved into microscopic resonances so that their density, their widths, and the effect of background can be determined.

KERMAN: In the region in which the levels are not overlapping, there is no fluctuation theory. Where there is a clumping of levels, I think that the widths have to be wider than average. At the moment I see no alternative to describing this as a giant resonance. I understand your point about the background, and it certainly is a possibility. But on the picture I am advocating, it is the widths that would be larger, though somewhat narrower than in a regular giant resonance; the density would be the same.

WIGNER: May I make another point which may be conceptually of some importance. The ordinary theory of giant resonances would give a maximum in the width only if the states have an admixture of single-particle excitation. What your picture implies, then, is that there is a small maximum in the single-particle excited state also where there are three-particle excitations.

KERMAN: That is certainly the way I understand it.

S. FALLIEROS, Massachusetts Institute of Technology: In connection with the giant resonances, isn't it true that neither in the case of the dipole states nor in the case of the analog states do we know of any single-particle resonance in the neighborhood?

KERMAN: I believe that is correct, and I thank you for making this rather important point. These kinds of things occur even away from any single-particle giant resonance. In some sense which I won't be specific about, they are modulated by the ordinary shape resonance; but they can occur both in the minima and at the maxima.

The analogue states are not single-particle excitations, they are three-quasiparticle states. As far as is known, there is no single-particle resonance in that neighborhood.

SETH: I think I have a partial answer to Dr. Wigner's question. To examine the intermediate resonances, we took a number of well-resolved resonances in neutron cross sections; then by hand we averaged them to bring out this feature. From this it is obvious that the greater amount of structure is due to an increase in the widths rather than to the decrease in spacing in any particular energy region.

A second answer that follows from this work at low energy (i.e., from about 2 to about 600 keV) is that there are no competing reactions, so that the effect cannot be attributed to them. It is a genuine feature that the widths suddenly become larger in the bumps. This shows up specifically in regions of large p-wave strength function (e.g., near niobium, molybdenum, and so on) and also at neodymium and the deformed nuclei at which the s-wave strength function is large.

SEGEL: Experimentally, the opposite seems to be true in the (p, γ) reaction. As far as I can see, at low bombarding energies individual levels are seen; but as the energy is raised the levels start to overlap and the structure gets to look like Ericson fluctuations. However, both the level widths and the angular distributions remain rather constant and appear to be independent of the character of the structure.

KERMAN: I think both experimentalists and theorists will have an exciting time as they try to unravel this whole business. It certainly is very, very interesting.

N. AUSTERN, University of Pittsburgh: I would like to ask about the theoretical background against which discussions of this kind are made. I suppose the basic spectrum of the nucleus is the spectrum of configurations of the shell model, with no residual two-body interactions among the particles. We know the distribution in energy of these shell-model states. Then when we turn on the residual interactions, in the fashion you describe, we get nuclear states which are linear combinations of the shell-model states; the spectrum of states is thereby

rearranged, but the total number of states is not changed. Is it your opinion that turning on the residual interactions causes gross changes in the energy-averaged structure of the spectrum or is the basic structure of the shell-model spectrum still preserved after all these things have been done?

KERMAN: Well, I hope it is preserved. I think that the experiments that find single-particle states with strength functions about 1 MeV wide at excitations of around 6 MeV should give us courage to look for these other things at higher energies. But I think we will not understand them if they are too far from the original shell-model picture. Even diagonalizing million-by-million matrices will not enable us to understand everything. (Ten years ago, when a problem required diagonalizing a 7×7 matrix one said, "I don't understand it." Now we accept 20×20 matrices because we have done it more often and are more familiar with it.) But if all of the levels shift radically and the final spectrum bears no resemblance to the initial one, then I think it would be virtually impossible to interpret. Of course, some special states will always shift radically.

My last comment is a more general one from a more fundamental point of view, namely that of starting from a real two-body force and calculating nuclear physics. I believe that the average potential we are talking about is the Hartree-Fock potential (if it can be shown to exist) coming from a reasonable two-body force. But that, of course, is speculation at the moment.

DIRECT-REACTION CONTRIBUTIONS TO FLUCTUATING REACTION CROSS SECTIONS^{*†}

G. M. Temmer, B. Teitelman, and F. G. Fender

Rutgers, The State University, New Brunswick, New Jersey

Abstract of Report in "Vol. I. Contributed Papers"[†]

We have analyzed excitation curves for several (p, α) reactions leading to several residual states,¹ using the prescriptions of Ericson and Stephen.² At excitations near 20 MeV and at 90°, the fluctuations about the local average are surprisingly well accounted for by the theory without direct-reaction amplitude (pure statistical regime). The strong dependence on the number of degrees of freedom N is clearly demonstrated between $N = 2$ and 72, using target nuclei between N^{15} and Co^{59} . The sensitivity to the admixture of a nonfluctuating component has been computed (Fortran program) for all N , using the theory of Stephen.² Low- N reactions are preferable in deciding relative contributions of direct and compound-nucleus amplitudes. For N^{15} , data ranging from 20° to 160° can follow the triton pick-up contribution systematically (question of heavy-particle stripping). The coherence energy Γ at ~ 20 MeV decreases monotonically from ~ 230 keV for O^{16} to $\lesssim 13$ keV for Ni^{60} . The roles of resolution, measurement interval, and the properties of the residual states are discussed. Cross correlations between channels are available.

^{*}Supported in part by the National Science Foundation.

[†]Contributed paper. Presented by G. M. Temmer.

[†]G. M. Temmer, B. Teitelman, and F. G. Fender, in Argonne National Laboratory Report ANL-6848, March 1964, p. 228. The text that follows the abstract is in amplification of this earlier report.

¹K. L. Warsh, G. M. Temmer, and H. R. Blieden, Phys. Rev. 131, 1690 (1963); G. Roy, Ph. D. dissertation, Florida State University, 1963 (unpublished); G. Roy and G. M. Temmer, Bull. Am. Phys. Soc. 8, 12 (1963) and to be published, K. L. Warsh, G. M. Temmer, and H. R. Blieden, Nucl. Phys. 46, 45 (1963), 44, 329 (1963); K. L. Warsh, Ph. D. dissertation, Florida State University, 1963 (unpublished); R. G. Davies, M. S. thesis, Florida State University, 1963 (unpublished).

²T. Ericson, Phys. Letters 4, 258 (1963), R. O. Stephen (unpublished).

Fig. 1. Artist's conception of a nuclear cross section measured with high resolution.

My first statement was going to be that the subject of fluctuations is a controversial one. Clearly I no longer need to make this statement. Also, I hope that you do not expect me to be a valiant and perfect defender of the cause of fluctuations of the other variety (not the Kerman variety).

I am somewhat of an innocent bystander. We have many data. Until I came to this meeting, I have had very little opportunity to discuss with theorists what their general feeling is about these fluctuations. I will try to present the bare-bone experimental facts. I will make a few general remarks and hope it can be done in about twelve minutes. If not, I will stop anyway.

The reason that this subject is complicated is, of course, that it has to do with the interior of the nucleus—which is admittedly an unpopular region, even in conferences of this type. But eventually one has to face it. There is no question about all this having to do with the interior of the nucleus; to a large extent, I think, all of us realize that there the problem lies.

I am now going to draw a symbolic picture (Fig. 1) which will, I hope, satisfy everyone. There are vested interests in this field. About three classes of people are trying to see at least three different kinds of things in the same data. This does not happen very often in physics, but it happens here. The picture is supposed to symbolize three processes that may be going on.

Now, some people would look at this and say: "It is obviously smoothly varying with energy. This is the zero level. There

is a little bit of structure on top, but it is mainly a direct reaction." Other people would look at this and say: "There are tremendous wiggles here, or fluctuations, or ups and downs, or what-have-you, and there is strictly a lot of statistical business going on." There are still other people who would say: "There is a modulation which interests me. Look at this! Do you see that slight variation on top of the wiggles? That's what is really interesting!" Then there are some people who see further modulations of the latter. I am thinking first of the giant resonances; the "doorway" states would then be modulations on top of that modulation, carrying with them the fluctuations which sit on top of the "dc" component.

Well, this is the general problem we are facing. Arthur Kerman has made a very beautiful introduction for me, and has practically said everything. I agree completely with what he says. I have to; I really have no basis for disagreeing.

We all know that smoothly varying things are direct, and wiggly things are compound. Now, we do not have a Unified Theory that combines all aspects; therefore we have to deal with extremes, as we always do. In other words, we are perhaps five years ahead of the time when we will understand these things. The best we can do is to apply the best theories we have; and perhaps the best we have, in the statistical field, are the theories of Ericson, Brink, Satchler, Stephen, and a number of other people who are getting into this business. These theories combine the two extremes, i. e., take cognizance of two facts, namely that cross sections are wiggly, and that they lie perhaps quite a bit above the abscissa at the same time. So we have some direct and some compound processes.

I am going to neglect everything that has to do with giant resonances—i. e., optical resonances or giant resonances of the giant-dipole type, any level correlations that are not included in the general statistical description, isobaric analogue states, and all the other pitfalls

which (if they occur in the midst of where you are measuring fluctuations) could cause you trouble, although perhaps not as much as some people think.

Now, as Kerman said, perhaps in some nucleus, at some energy, he might be lucky enough to see what he wants to see. Similarly we felt that perhaps at some energy, in some nucleus, we will find what we want to see.

I think that there are some lucky regions in the world, some fortunate circumstances that we do not understand, where conditions seem to apply such that at the present time, at the present level of misunderstanding, we may know what we are doing.

People who have 10—12-MeV high-resolution machines — and you know who they are — do experiments, and they necessarily see the fluctuations. This is how it all started, of course. They are told: "You cannot do direct-reaction nuclear spectroscopy! Look how things are fluctuating!" But now take people who have 20-MeV particles; nobody asks them if they have fluctuations. They cannot measure them since their machines have fixed energy and poor resolution! So they extract spectroscopic factors which everybody is pleased about. Somewhere between 11—12 and 18—20 MeV, something catastrophic must be happening. The new generation of unspecified accelerators (be they linear, circular, or spiral) will, of course, investigate these problems.

We have done as much as we can with some data which were obtained over the last three years at Florida State University by a succession of Ph.D. candidates and one Master's candidate, namely, Doctors G. Roy, K. L. Warsh, H. R. Blieden, and Mr. R. G. Davies. The subject is vast. I wish I could do what Kerman did, namely not show a single slide but just walk up and down. That is elegant, but I am afraid I cannot do it. I will have to show slides containing facts. First I will simply show a few curves which contain what we believe are fluctuations.

The first one (Fig. 2) is for $F^{19}(p, \alpha_0)O^{16}$ at 70° , as a function of proton energy over the range from 3–12 MeV, corresponding to compound-nucleus excitations in the region of 20 MeV.

The next figure (Fig. 3) shows an α spectrum for $Al^{27}(p, \alpha)Mg^{24}$ which allows you to look at various final exit channels, namely states having spins 0, 2, 3, and 4.

I hope you can fill in some of the details. These are all (p, α) reactions on odd-mass nuclei. They yield 16 or 17 residual states. We can do autocorrelations and cross correlations and anything else that anybody wants—and we are willing to do it, as long as it is at 90° . I want you to note here very briefly that $Al^{27}(p, \alpha_0)Mg^{24}$, meaning the ground state of magnesium remains behind, has wiggles which almost touch the x axis (Fig. 4). I will show this more quantitatively in a minute. Figure 5

Fig. 2. Excitation function of $F^{19}(p, \alpha_0)O^{16}$ at 70° (lab) between 3.3- and 12.2-MeV proton energy.

Fig. 3. Pulse-height distribution of alpha particles from the reaction $Al^{27}(p, \alpha)Mg^{24}$ observed at $E_p = 10$ MeV and $\theta_\alpha = 90^\circ$.

Fig. 4. Excitation curve for the reaction $\text{Al}^{27}(p, \alpha_0)\text{Mg}^{24}$ observed at 90° to the beam.

Fig. 5. Excitation curve for the reaction $\text{Al}^{27}(p, \alpha_1)\text{Mg}^{24}$ observed at 90° to the beam.

Fig. 6. Alpha-particle spectrum from $\text{Co}^{59}(\text{p}, \alpha)\text{Fe}^{56}$ at $E_p = 10.0 \text{ MeV}$ and $\theta = 90^\circ$.

shows the $\text{Al}^{27}(\text{p}, \alpha_1)$ reaction which, I want you to notice, has more of a "dc" component; this does not necessarily mean that it has a direct-reaction component in it. We first have to understand completely what the pure statistical-model predictions are in general. For instance, this kind of a "dc" behavior, with no valleys touching the x axis and no very high peaks, looks like direct reaction. It could be; you cannot tell from looking at just one curve. One has to make a systematic study for many levels. In our case it simply means more degrees of freedom N .

Figure 6 shows the heaviest nucleus we have examined, Co^{59} . We have looked at the first four residual states of Fe^{56} . Figure 7 shows how these excitation curves look. It is incredible that anybody had the patience to measure this and repeat it time and again. The entire region covered is only 0.5 MeV from end to end, and there are three excited states and a ground state available. Much of this has not yet been analyzed completely.

A formula, coherently combining a completely statistical amplitude as far as fluctuations are concerned and a direct amplitude which does not vary with energy, is

Fig. 7. Excitation curves for the ground state and first three excited states for the reaction $\text{Co}^{59}(\text{p}, \alpha)\text{Fe}^{56}$. The proton energy range was 10.0—10.5 MeV, corresponding to excitation at about 20 MeV in Ni^{60} .

$$P_N(y, y_D) = N \frac{y^{N-1}}{1 - y_D} \left[\exp \frac{-N(y + y_D)}{1 - y_D} \right] \frac{J_{N-1} \left(\frac{2iN \sqrt{yy_D}}{1 - y_D} \right)}{(i \sqrt{yy_D})^{N-1}}, \quad (1)$$

where

$$y = \frac{\sigma(\theta)}{\langle \sigma(\theta) \rangle}, \quad y_D = \frac{\sigma_D(\theta)}{\langle \sigma(\theta) \rangle},$$

and J_{N-1} is the Bessel function of $(N - 1)$ st order and imaginary argument. Special cases are

$$P_1(y, y_D) = \left[\exp \frac{-(y + y_D)}{1 - y_D} \right] J_0 \left(\frac{2i \sqrt{yy_D}}{1 - y_D} \right) \quad \text{for } N = 1$$

and

$$P_N(y, 0) = \frac{N(Ny)^{N-1}}{(N - 1)!} e^{-Ny} \quad \text{for } y_D = 0.$$

This most general expression is due to R. O. Stephen of Oxford; any bright young theoretician can work out the same formula, I understand. In Eq. (1), N is the number of independent contributing channels so that $2N$ is the number of degrees of freedom. I use this simplified formula for N , which is not always correct, however; N always represents an upper limit. It can be calculated better, but we had neither the time nor the inclination to do it. If we are using this formula, it contains all of the spin weights involved in the reaction (for projectile, target, outgoing particle, and residual nucleus). It is a number, and we do not vary that number. We take it as it comes.

Figure 8 shows plots of this expression. Note an important point here. Namely, you see at first sight that top and bottom portions

Fig. 8. Plot of Stephen's formula for probability distributions of cross sections. Upper curves: $\sigma_{DI} = 0$, and $N = 1, 2, 4, 10, 20, 36$, and 72 . Lower curves: $N = 1$, and $y_D = 0, 0.50, 0.80, 0.90, 0.95$, and 0.98 .

Along the ordinate, I am plotting the probability of an occurrence as a function of $\sigma/\langle \sigma \rangle$. It is always plotted in the same way. The parameter varying from one curve to the next is the amount of the direct component in units of the average cross section. These are all well-defined concepts and are available in the literature.

The case of $N = 1$ (e.g., all spins equal to 0), which has been much studied by Leachman et al. at Copenhagen and by Almqvist et al. at Chalk River, very nicely bears out not only the predicted e^{-y} curve, which it should be, but one or two others as well for low values of N .

of the figure contain similar families of curves. However, the parameter that is varied in the upper curves is very different from the parameter that is varied below. No direct reaction is assumed above. We have a purely statistical régime. All that is changed is the number N , the number of final exit channels available which are mutually uncorrelated and equally likely—multivariate in nature, if you want a big word. The lower curves, which look quite similar, are for fixed N ($N = 1$), and we vary the amount of direct interaction in terms of the parameter $y_D = \sigma_D/\langle \sigma \rangle$. In the lower part of the figure we show curves for 98%, 95%, 90%, 80%, 50%, and 0% direct component.

Along the abscissa we plot the cross section in units of the average cross

I would just like to show you how things go with N up to 72, and how N alone is sometimes not enough to account for the shapes (you need to invoke also some direct component). This, I think, is the proper and (if this model holds at all) the most sophisticated way of determining how much direct and how much compound-nucleus reaction takes place. It supersedes all other arm-waving methods such as saying that what is isotropic is compound-nucleus, or that which has fore-and-aft symmetry is compound-nucleus, or the like.

You will see Fig. 9 in the Physical Review Letters.³ I will not take too much time with it. The curves in all cases are taken from the families in Fig. 8. As a first try, no direct reaction is assumed in any of these. Let us see if things are statistical. We are measuring at 90° . The histograms are obtained by counting the excursions from the mean, as illustrated in the upper left; we are simply calculating the higher moments of the distribution, but I prefer to see it in a more pictorial way. All that changes between the different curves is the spin in the final state. Here N is 54, 42, 30, and 6. There is no normalization except that the total probability (area) must be equal to unity. We claim that this is quite a reasonable agreement. In fact, there is a 0^+ state way up here (6th excited state). We did not even know at first that it was 0^+ . It gives a much broader distribution, as it should for $N = 6$.

Figure 10 shows what happens when you go to really high values of N , in cobalt. You get extremely sharp distributions. The averaging is a long story which takes two hours by itself to discuss. I can only say that there is a great problem about averaging procedure, of course. That is where some real problems lie. In the lower part of Fig. 10 are two different averaging intervals (7—9 MeV, and 10.0—10.5 MeV, respectively) yielding comparable results.

³G. M. Temmer, Phys. Rev. Letters 12, 330 (1964).

Fig. 9. (a) Illustration of fluctuation analysis by obtaining histogram of distribution of cross section σ about the average $\langle \sigma \rangle$. Averaging interval ΔE . Vertical numbers indicate points per bin. $\Delta E \gg \Gamma \gg D$. (b) through (f). Probability distributions for $Al^{27}(p, \alpha)Mg^{24}$ at 90° (lab). Proton bombarding energy interval 9.40 to 10.90 MeV, corresponding to excitations in the compound nucleus Si^{28} between 20.6 and 22.1 MeV. Solid histograms for bins of 0.1, dotted histograms for bins of 0.2 in $y = \sigma / \langle \sigma \rangle$. Solid curves are theoretical expressions for pure compound-nucleus formation and appropriate values of N . Numbers of bits are indicated on the figures and represent independent measurements. Small arrows indicate predicted peak positions. Theoretical and experimental distributions normalized to area unity. (b) Ground state of Mg^{24} , $I' = 0$, $N = 6$; (c) first excited state of Mg^{24} (1.368 MeV), $I' = 2$, $N = 30$; (d) fourth excited state of Mg^{24} (5.224 MeV), $I' = 3$, $N = 42$; (e) fifth excited state of Mg^{24} (6.005 MeV), $I' = 4$, $N = 54$; (f) sixth excited state of Mg^{24} (6.432 MeV), $I' = 0$, $N = 6$.

Fig. 10. Probability distributions for $\text{Co}^{59}(p, \alpha)\text{Fe}^{56}$ at 90° (lab). Proton bombarding energy interval corresponds to excitations in the compound nucleus Ni^{60} between 19.5 and 20.0 MeV for (a), (b), and (c), and between 16.5 and 18.7 MeV for (d). (a) Ground state of Fe^{56} , $I' = 0$, $N = 8$; (b) first excited state of Fe^{56} (0.895 MeV), $I' = 2$, $N = 40$; (c) and (d) second excited state of Fe^{56} (2.085 MeV), $I' = 4$, $N = 72$; (c) and (d) differ only in the energy interval (see above), and in the grid size ϵ (see text).

Figure 11 shows the most remarkable agreement of all, for Na^{23} . We averaged everything from 3 to 12 MeV. We have 250 bits of information. These bits do not represent counting statistics; they refer to the number of measurements. For the proper $N = 4$, the agreement is as good as any histogram ought to be.

Figure 12 shows an autocorrelation function for F^{19} . I assume that everybody knows how an autocorrelation function is computed. It determines coherence; one finds out how far away in energy you can move before you forget where you were. It should fall off for larger values of the step size ϵ you use in the autocorrelations. The half-width Γ is usually called the "coherence width" or some average width of the compound states involved. It turns out to be about 200 keV in the case of

Fig. 11. Probability distribution for $\text{Na}^{23}(p, \alpha_0)\text{Ne}^{20}$ at $\theta = 90^\circ$ for $3.175 \leq E_p \leq 12.150$ MeV. The histogram represents 251 measured points; the curve is the theoretical prediction for $N = 4$ and no direct interaction.

Fig. 12. Autocorrelation function for $\text{F}^{19}(p, \alpha_0)\text{O}^{16}$; $\Gamma = 230$ keV. The intercept for $\epsilon = 0$ is 0.033. For no direct interaction this value would be 0.50. The oscillation about 0 is a typical finite-sample effect.

Ne²⁰. You will note that the value for zero step size is 0.03—not 0.5 as it would be if the process were purely statistical. After you put in the number of degrees of freedom, namely 2 in my language (although some would call it four), you would still need 96% direct interaction in the case of fluorine.

Table I summarizes the Γ values, which are of great interest. There is a clear monotonic decrease of Γ with A ; in fact Γ varies roughly as A^{-2} . We always try to keep the measuring step size ϵ smaller than Γ . One knows how to cope with these problems. An important consideration is that the important number $\Delta E/\Gamma$, ΔE being the averaging interval, varies from 25 to 35 in all of these cases. This is the sampling parameter which tells you whether or not you have enough deals of the cards to get good statistics.

Figure 13 shows the theoretical curves for $N = 2$ once again for varying amounts of direct component. Figure 14, which is our pride and joy, shows the fluorine data which for two years we have attempted to interpret by direct-reaction mechanisms. We were very happy to find this result. There is no direct admixture in the curve labeled 0. The histogram gives the experimental data, and we have plotted the curves for $N = 2$. We cannot tamper with that number N . One finds that between 90 and 95% (closer to 95%) direct component is required to explain the data; in other words, perhaps we are justified in using a direct-reaction interpretation.

Now, this is really all I have time for. There is much philosophy that can be presented. I cannot do justice to the many people who work in this field, not even to myself.

TABLE I. Summary of data and results.

Reaction	Compound nucleus	Excitation (MeV)	N	$\bar{\epsilon}$ (keV)	Γ (keV) (half- width)	Γ (keV) (peak- counting)
$N^{15}(p, \alpha_0)C^{12}$	O^{16}	18.7—23.8	2	42	250 (60°)	230
$F^{19}(p, \alpha_0)O^{16}$	Ne^{20}	18.5—24.2	2	25	200 (70°)	160
$Na^{23}(p, \alpha_0)Ne^{20}$	Mg^{24}	19.4—23.3	4	48	135 (90°)	110
$Na^{23}(p, \alpha_1)Ne^{20}$	Mg^{24}	19.4—23.3	20	48	335	
$Na^{23}(p, \alpha_2)Ne^{20}$	Mg^{24}	19.4—23.3	36	48	195	
$Na^{23}(p, \alpha_3)Ne^{20}$	Mg^{24}	19.4—23.3	20	48	180	
$Al^{27}(p, \alpha_0)Mg^{24}$	Si^{28}	20.7—22.1	6	38	85 (90°)	65
$Al^{27}(p, \alpha_1)Mg^{24}$	Si^{28}	20.7—22.1	30	38	35	
$Al^{27}(p, \alpha_4)Mg^{24}$	Si^{28}	20.7—22.1	42	38	35	
$Al^{27}(p, \alpha_5)Mg^{24}$	Si^{28}	20.7—22.1	54	38	35	
$Al^{27}(p, \alpha_6)Mg^{24}$	Si^{28}	20.7—22.1	6	38	60	
$Co^{59}(p, \alpha_0)Fe^{56}$	Ni^{60}	19.5—20.0	8	10.4	5 (90°)	14
$Co^{59}(p, \alpha_1)Fe^{56}$	Ni^{60}	19.5—20.0	40	10.4	6	
$Co^{59}(p, \alpha_2)Fe^{56}$	Ni^{60}	19.5—20.0	72	10.4	7	

Fig. 13. Theoretical probability distributions for $N = 2$ and the same values of y_D as in the lower part of Fig. 8.

Fig. 14. Probability distribution for $F^{19}(p, \alpha_0)O^{16}$ at 70° (lab). Proton bombarding energy interval corresponds to excitations in the compound nucleus Ne^{20} between 18.5 and 24.2 MeV. Solid curves labeled $y_D = 0, 0.90$, and 0.95 correspond to no direct contribution, 90% direct, and 95% direct contribution, respectively. $I = \frac{1}{2}$, $I' = 0$, $N = 2$.

* * *

D. W. LANG, Argonne National Laboratory: Was the correlation analysis carried far enough to see the intermediate structure that Kerman⁴ was talking about?

TEMMER: The measurements have been in progress yesterday and today and I don't have the results yet. (Note added in proof: This turned out not to be significant in view of finite-sample effects.)

A. I. YAVIN, University of Illinois: Did you get Segel's data⁵ or did Segel get your data? And have the two sets of data been put together to see whether or not there is any correlation between the (p, α) and the (p, γ) reactions on aluminum?

TEMMER: No, we did not get together yet. Our processes are quite different and require different treatments.

YAVIN: It would be very interesting to see them on the same graph.

TEMMER: They will be put together soon. (Note added in proof: There is no significant correlation in the structure of these two reactions.)

J. P. BONDORF, Nordisk Institut for Teoretisk Atomfysik, Copenhagen: I shall make a comment on your analysis. The distribution function is very insensitive to the ratio of DI to CN when this ratio is less than one and therefore the experimentally determined ratio is uncertain in such cases. Hence, I suggest a Hauser-Feshbach calculation of the absolute CN cross section as an independent check of the CN cross section obtained from the fluctuation analysis.

TEMMER: I agree.

⁴A. K. Kerman, in these Proceedings, p. 449.

⁵P. P. Singh, R. E. Segel, L. Meyer-Schützmeister, S. S. Hanna, and R. G. Allas, Proceedings of the International Conference on Nuclear Physics, Paris, July 1964 (to be published).

P. A. MOLDAUER, Argonne National Laboratory: I think that one significant assumption underlying the fluctuation theory used by Dr. Temmer is the lack of correlations between the amplitudes corresponding to emission into different channels and between amplitudes belonging to different resonance terms. In Dr. Kerman's model,⁴ these amplitudes are correlated and it is these correlations that give rise to the intermediate-structure peaks. In that sense there is a conflict between Kerman's predictions and Temmer's interpretation of the fluctuation data.

It does not seem to be known at present how to determine the extent and type of amplitude correlations from data on cross-section fluctuations because no cross-section autocorrelations or cross-correlations or distributions appear to have been calculated except under the assumption of completely uncorrelated amplitudes. While it is therefore not at all clear that the presently available data exclude the possibility of amplitude correlations, it does seem that the presence of such correlations would affect the interpretation of these data as regards total widths, ratios of compound to direct cross section, effective channel numbers, etc.

Thus the important physical information that is really needed concerns amplitude correlations. As Prof. Wigner suggested, it would be nice if one could find isolated resonances for which one could examine reaction amplitudes. Unfortunately, there are very few available reactions among isolated resonances; this is one difficulty in following that suggestion.

One thing puzzles me about the formula that you wrote down: it depends on only one parameter. This would not appear to be the case for an arbitrary reaction since the number of exit channels, the number of entrance channels, and especially the number of total-angular-momentum states through which the reaction proceeds would be important. Thus at least three parameters ought to be involved. Possibly this particular reaction was simple enough that the number of parameters reduced to one.

TEMMER: I will reply to your first point. I understand from Dr. Böhning, who in turn has heard from Dr. Brink, that even if there are cross correlations you may still be in a statistical régime.

MOLDAUER: I agree. But the formula would not come out the same.

TEMMER: That is certainly true; but a cross correlation can be computed from the data without having any theory in mind at all. I wish I had the numbers—but they will not be available until tomorrow. [Note added in proof: The cross correlations turned out to be insignificantly small, except in $\text{Na}^{23}(\text{p}, \alpha)\text{Ne}^{20}$.]

BONDORF: A comment on Dr. Moldauer's last remark. The effective number of independent channels N in the differential cross section is approximately equal to half the number of combinations of magnetic quantum numbers for incoming and outgoing particles and does not depend on the number of different total angular momenta of the compound nucleus.

L. ZAMICK, Princeton University: I would like to ask Prof. Kerman a question. Professor Temmer has shown us that the level width decreases as the mass number increases. In your paper⁶ in the Physical Review Letters, wasn't one of your main points that the level width is independent of the mass number for your particular type of resonance?

A. K. KERMAN, Massachusetts Institute of Technology: I think I touched on that point in my talk. I think that the level width should go down faster for the genuine fluctuations than for the intermediate structure; and my feeling is that Dr. Temmer's rate of descent is too high for the intermediate structure. He probably is not seeing fluctuations of the kind we are talking about.

⁶A. K. Kerman, L. S. Rodberg, and J. E. Young, Phys. Rev. Letters 11, 422 (1963).

TEMMER: I figured out on the back of an envelope that our Γ varies as A^{-2} . Is that all right?

KERMAN: It looks too narrow in nickel to satisfy me in lead. If it decreases that fast, I suspect it will be down to something like 0.1 keV in lead. Isn't that right?

TEMMER: It would be negative.

KERMAN: I would like to make a general remark related to Moldauer's remark. I do not believe there is any conflict between Dr. Temmer's mechanism and mine; I feel that they are complementary. Sometimes the discussions refer to exactly the same data, at other times to different analyses of similar data. It seems quite certain that the statistical analysis can be modified with appropriate direct terms, and perhaps intermediate terms or the like, so that the whole thing can be analyzed and picked apart.

TEMMER: I likewise do not want to give the impression that these two mechanisms are in conflict.

THE (p,t), (p,He³), AND (p,α) REACTIONS INDUCED IN O¹⁶
BY 43.7-MEV PROTONS*†

Joseph Cerny and Richard H. Pehl

Department of Chemistry and Lawrence Radiation Laboratory
University of California, Berkeley, California

Abstract of Report in "Vol. I. Contributed Papers" ¹

Development of a new particle-identifier² has permitted simultaneous measurement of proton-induced two-nucleon and three-nucleon pickup reactions on O¹⁶. A comparison of transitions to analog final states was possible through observation of the O¹⁶(p,t)O¹⁴ g.s. and O¹⁶(p,He³)N¹⁴* (2.31-MeV) reactions. Virtually identical angular distributions were obtained with a (p,t)/(p,He³) cross-section ratio of two. The final states below 9 MeV excitation strongly populated in the (p,He³) and (p,α) reactions were solely those of p¹⁰ configuration in N¹⁴ and p⁹ configuration in N¹³, respectively, as would be expected from a pickup mechanism. A knockout mechanism would predict population of these pⁿ states and states corresponding to a pⁿ⁻¹ configuration plus a nucleon in the 1d 2s shell; no such latter states were significantly populated. The O¹⁶(p,t)O¹⁴ reaction strongly populated the ground state and two states at approximately 6.3 and 7.5 MeV excitation. These excited states can be assigned as the isobaric analogs of the [(s,d) + p_{3/2}⁻¹ p_{1/2}⁻¹], 2⁺, T=1 states of N¹⁴ at 9.17 and 10.42 MeV excitation.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

† Contributed paper. Presented by Joseph Cerny.

¹ J. Cerny and R. H. Pehl, in Argonne National Laboratory Report ANL-6848, March 1964, p. 208.

² F. A. Goulding, Lawrence Radiation Laboratory, Berkeley (to be published).

HIGH-ENERGY REACTIONS AND NUCLEAR STRUCTURE

H. J. Lipkin

Weizmann Institute of Science, Rehovoth, IsraelI. High-Energy Structure and Nuclear Reactions

I am afraid that introducing me as "the author of a Madison Avenue survey of nuclear models who happens to know some physics" is an excellent introduction to what I am about to say. Although I may have done some work in physics at some time, none of it is really very relevant to this conference, and I still wonder at times why I was invited here. The Madison Avenue part is also appropriate because my main function here seems to be selling one thing or another which has nothing to do with any work I have done.

Tomorrow many of you will be visiting Argonne and seeing the high-energy accelerator and the High Energy Physics building. Perhaps it has occurred to you that there is a big building full of many people who do research and call themselves physicists, but who have probably never heard of the distorted-wave Born approximation. To them it probably seems equally peculiar that you have a conference here with a room full of people who have never heard of the f_0 particle. I think that this is really a situation that should be improved. If there is one message that I am trying to sell in this talk, it is that people who know high-energy physics and people who know low-energy physics should talk to one another. They have a lot in common, even though they may not know it. For example, a few years ago people working in nuclear structure happened (for some peculiar reason) to talk to people working in superconductivity and many exciting things came out.

These days many high-energy physicists are busy investigating spectroscopy with direct reactions. If we look at what they are doing, that is what we would call it. They do not know they are doing this because they have another language. Most elementary-particle physicists have finally realized that they are not dealing with a few elementary building blocks of nature. The things they have been calling elementary particles are a large number of quantum states of something (it is not clear of what). There is a big job of finding out the properties of these states, their quantum numbers, how they interact, and so forth. In other words, they are involved in spectroscopy.

They study these states by doing reactions with accelerators. Many of these reactions are very complicated. There are a few types of reactions which are analyzed by use of a simple model called the "peripheral model." When you look at the peripheral model, it is what we call direct reactions. On the other hand, they have, as I have said, never heard of a distorted-wave Born approximation, so they really don't know the best way to handle these reactions. There is a lot that could be useful both to people here and to people in high energy if they would get together.

Just to help you get started, I should like to introduce you to a little bit of the language. Let me first describe a stripping experiment in the language of high-energy physics. Before a high-energy physicist thinks, he draws a Feynman diagram [Fig. 1(a)]. We start out with a deuteron and a nucleus of mass A . The deuteron breaks up into a neutron and a proton. The proton goes out, the neutron goes across, and another nucleus having mass $A + 1$ comes out. You can draw in some arrows to indicate directions.

The next thing they do with the Feynman diagram, is to turn the arrows around in all directions. If you look at this backwards,

as in Fig. 1(b), you get a (p,d) reaction. This is perfectly okay with us, too. But they do not stop there; they keep on going. They reverse only the direction of the neutron arrow, and it becomes a neutron going backwards in time, an antineutron, as in Fig. 1(c). This describes the reaction in which the nucleus of mass A turns into a nucleus of mass $A + 1$ by emitting an antineutron, and the antineutron annihilates the neutron in the deuteron to give you a proton. This is a perfectly good reaction if you are up at high enough energies. Of course, at the energies at which most people work, this would be called the "nonphysical region."

The next thing that you can do is to look at the Feynman diagram sideways, as in Fig. 1(d). This is sometimes called the "cross channel." There are also the famous letters s and t which appear everywhere in elementary-particle papers these days. The so-called s channel and t channel are marked in Figs. 1(c) and 1(d). If you are looking in the s channel, s is the total energy and t is related to the momentum transfer. If you look at it the other way, the two are interchanged. Figure 1(d) shows a reaction in which a deuteron and an antiproton come together and make a neutron. The neutron then breaks up into a nucleus and an antinucleus. In other words, $d + \bar{p} \rightarrow n \rightarrow \bar{A} + (A + 1)$. This example is very amusing because the reaction clearly goes through a "compound

Fig. 1. A high-energy physicist's way of visualizing direct reactions. The Feynman diagrams shown represent (a) a (d,p) stripping reaction, (b) a (p,d) pickup reaction, (c) a (d,p) reaction proceeding by an antineutron transfer, and (d) the "cross channel" reaction $d + \bar{p} \rightarrow n \rightarrow \bar{A} + (A + 1)$.

Fig. 2. High-energy physicist's view of the distorted-wave Born approximation for a (d, p) reaction. The blobs represent elastic scattering.

state," namely the neutron. This illustrates the effect of the point of view: a direct reaction in one channel is a compound-nucleus reaction in the cross channel. These descriptions sound very funny to a nuclear physicist because he is not accustomed to high enough energies to make nucleons go into antinucleons, etc.

In the elementary-particle reactions, these transformations are very important because one looks at nonphysical values of these parameters and does analytic continuation in the complex plane. If you look at the stripping reaction in the cross channel, it is simpler because there is one compound state, and this produces a singularity (pole) in the amplitude. Using these analytic properties, you can calculate all kinds of things.

This picture, of course, would make Norman Austern extremely unhappy because it is nothing more than the plane-wave Born approximation. We have to fix this up. It can be done very easily as shown in Fig. 2. The blobs represent elastic scattering. This is now the distorted-wave Born approximation because we have included the elastic scattering in the entrance and exit channels.

Now that we have learned all the okay words for elementary-particle reactions, we can take a look at the analogous reactions for elementary particles. Consider the reaction $K + N \rightarrow K^* + N^*$, where K is a K meson, N is a nucleon, and K^* and N^* are K - π and N - π resonances, respectively. These reactions are believed to go by the exchange of a π meson, as shown in Fig. 3.

Experiments on high-energy reactions are similar to experiments on nuclear reactions. You measure angular distributions

and polarizations. But the difficulty is reversed: with particles it is often much easier to measure the polarization than it is to measure the angular distribution. This is because these measurements are usually done by looking at bubble chamber pictures, where you have only a few events and it is hard to get statistics for the angular distribution. On the other hand, a single event means much more than the kind of event that you people are used to. That is why the Ω^- can be determined by one event. An analogous measurement with standard counting techniques would be something like a sextuple coincidence with time-of-flight, delay, and energy measurements in all channels. This is the kind of information you get from one picture. It turns out that by analyzing a certain number of pictures it is very easy to establish the polarization, but you need many more events in order to get the angular distribution.

In many of these so-called peripheral reactions, the polarization data came out first and were fitted very well by the calculations with a model of one-meson exchange. Then when enough data became available to see the angular distributions, they did not agree with the predictions of the model. This caused people to worry. They started modifying the diagram (Fig. 3) by putting in form factors, replacing the exchanged pion by some heavier meson which had not yet been discovered, etc. The one thing that did not occur to them was to do a distorted-wave calculation. This, of course, is obvious to anybody in this room because these are all strongly interacting particles, and you would not expect plane waves to work. All kinds of things happen to these particles, and there certainly should be considerable distortion.

Fig. 3. Elementary-particle analogue of the (d,p) reaction. The reaction $K + N \rightarrow K^* + N^*$ proceeds by exchange of a π meson.

The question of formulating a distorted-wave model for this case is not so easy. Things have to be relativistic, and you may worry about whether or not it satisfies the Mandelstam representation. You may do it with a dispersion formulation. You may need elastic scattering from experiments, and no one is ever going to scatter a K^* against an N^* . These are interesting problems that people should look into, and I leave this to you as a challenge. Go and do something about it!

Arnon Dar¹⁻⁴ at the Weizmann Institute used a simple black-sphere diffraction model to try to treat these reactions and a similar approach was used by Durand and Chiu.⁵ Dar takes the results that a high-energy physicist gets with a normal calculation and sees that absorption reduces the contribution of the low partial waves. So he does something like expanding the standard results in partial waves, throwing out the contribution of the low partial waves up to a certain point (which he uses as a parameter), and fits the data. I will show a few examples to indicate that this is quite successful and leaves some hope that a one-pion exchange or a one-meson exchange will fit the data. It is certainly good enough as a first step. In the present state of the published data, it is probably the best that one can do; but as more data are accumulated people should start looking at it and getting better models. I think people here have had more experience in doing this than anyone else. I give this to you as a challenge.

¹ A. Dar, Phys. Letters 7, 339 (1963).

² A. Dar, M. Kugler, Y. Dothan, and S. Nussinov, Phys. Rev. Letters 12, 82 (1964).

³ A. Dar and W. Tobocman, Phys. Rev. Letters (in press).

⁴ A. Dar, Phys. Rev. Letters 13, 91 (1964).

⁵ L. Durand and Y. T. Chiu, Phys. Rev. Letters 12, 399 (1964).

Fig. 4. Comparison of the experimental and theoretical curves for the reaction $K^+ + p \rightarrow K^* + N^*$. The experimental data were taken from Goldhaber *et al.* [e.g., G. Goldhaber, W. Chinowsky, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Letters 6, 62 (1963)]. The calculation of the one-pion exchange curve is due to Berman as quoted in this reference. The coupling constants were determined from the decay of K^+ and N^* . $g^2(K^+, \pi, K^*)/4\pi \sim 1.8$, $g^2(p, \pi, N^*)/4\pi \sim 24.0$. (From Ref. 4.)

Figure 4 shows the angular distribution for the reaction $K^+ + p \rightarrow K^* + N^*$ at 1.96 GeV, taken from a Berkeley report. The simple one-pion-exchange model gives the upper curve. The curves marked B and C are obtained by putting in arbitrary form factors with no theoretical justification to fit the data. Dar has punched out the low partial waves and uses two parameters: a normalization to fit the peak, and the radius which tells him how far he punches out things. As you see he has quite a good fit.

Figure 5 shows the reaction $\bar{p} + p \rightarrow \bar{\Lambda} + \Lambda$. The curve used to fit is K^* exchange. The natural thing to use is a K exchange, but this is even worse than the K^* . In this case, there is a scarcity of data; but it is quite clear there is some kind of a diffraction structure, and

Fig. 5. Comparison of the experimental and theoretical curves for the reaction $\bar{p} + p \rightarrow \bar{\Lambda} + \Lambda$. The experimental data are due to Baltay et al. [C. Baltay et al., Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 233]. The theoretical curves were calculated for the lower momentum and have been normalized to give correctly the number of events in the first interval in the experimental histogram. (From Ref. 4.)

Dar's curve more or less fits it. The other lines are histograms computed from the curve in order to get a better comparison with the data.

Similar results have been obtained for the reactions⁴

$$\bar{p} + p \rightarrow Y^* + \bar{\Sigma}, \quad (\text{Fig. 6})$$

$$\pi^- + p \rightarrow \rho^0 + n, \quad (\text{Fig. 7})$$

$$\bar{p} + p \rightarrow \bar{Y}^* + \Sigma,$$

$$\pi^- + p \rightarrow \rho^0 + n,$$

$$\pi^- + p \rightarrow \rho^- + p, \quad (\text{Fig. 8})$$

$$\pi^- + p \rightarrow f^0 + n. \quad (\text{Fig. 9})$$

I hope that this gives you something to think about and perhaps some ideas of things to do.

Fig. 6. Comparison of the experimental and theoretical curves for the reaction $\bar{p} + p \rightarrow Y_1^{*+} + \bar{\Sigma}^-$.

Fig. 7. Comparison of the experimental and theoretical curves for the reaction $\pi^- + p \rightarrow \rho^0 + n$.

Fig. 8. Comparison of the experimental and theoretical curves for the reaction $\pi^- + p \rightarrow \rho^- + p$.

Fig. 9. Comparison of the experimental and theoretical curves for the reaction $\pi^- + p \rightarrow f^0 + n$.

II. High-Energy Reactions and the Mössbauer Effect

The title of this talk is "High-Energy Reactions and Nuclear Structure." Up to this point, I seem to have been talking about high-energy structure and nuclear reactions. The two titles are probably related by a unitary transformation, but I should probably also say something about high-energy reactions and nuclear structure.

The reason that I was asked to give this talk is that I have done no work whatsoever in the field of high-energy reactions and nuclear structure. The organizing committee of the conference felt that they could not get a balanced survey of the field from someone actively working in it, since it is always easier for such a person to talk about his own work rather than to learn about the contributions of others. The organizing committee has made the mistake of assuming that I would not be able to do this, having done no work in the field. However, since it is still easier to talk about my own work even though it is completely unrelated to the subject of the talk, I have naturally chosen this alternative. I shall say a few words about the Mössbauer effect, about which I may claim to know something, rather than about high-energy reactions and nuclear structure about which I know very little. But first, as a gesture to the organizing committee, I have to say something about high-energy reactions and establish some relation between what I am about to say and the title of the talk.

The first reaction of almost everybody to this subject is to question the use of high-energy techniques to study what is basically a low-energy phenomenon. We are dealing with a system of particles bound together by forces leading to a very rich spectrum of energy levels in the low-energy region. One should naturally use particles of comparable energy to probe this structure and look at all these excitations. The use

of a projectile having an energy so much higher than that required to produce the low-lying excitations will certainly produce something very complicated and difficult to unravel. This argument may seem reasonable, but it is interesting to note that one could use exactly the same argument against the use of very "high energy" x rays to study the structure of a system whose characteristic excitations lie in a very, very much lower energy range, namely a crystal. Thus the study of nuclear structure by use of high-energy machinery might be called "x-ray crystallography of nuclei."

Now that I have used the words "x rays," "crystals," and "nuclei," I have an excuse for talking about the Mössbauer effect. In the Mössbauer effect, a nucleus bound in a crystal emits a gamma ray and the recoil required by momentum conservation is miraculously absorbed by the whole crystal instead of only by the single nucleus. Further examination of this process reveals that this momentum transfer is not really so miraculous after all. From the point of view of the whole crystal, the emission of the gamma ray is a "direct reaction" on one nucleus, and this reaction can be either elastic or inelastic. The energy spectrum of the emitted gamma rays consists of an elastic peak and a background due to inelastic transitions. The elastic peak is what is commonly called the Mössbauer gamma ray.

It is at first sight surprising that the elastic peak should be so large in the cases in which the Mössbauer effect is observed. Again, closer investigation reveals that it is not so surprising after all. The nucleus emitting the gamma ray is bound and therefore does not have a well defined momentum. There is a certain momentum spread in the wave function describing the state of this nucleus. If this spread is large compared with the recoil momentum required by the emission of the gamma ray, the nucleus can easily absorb the recoil momentum and

remain in the same quantum state. On the other hand if the recoil momentum is large compared with the momentum spread of the nuclear motion, then it is very difficult for the nucleus to absorb this momentum without also changing its quantum state. Consequently the probability of an inelastic transition is much higher than that of an elastic transition.

This qualitative argument is expressed quantitatively by the familiar result that the probability of a transition from an initial state $|n\rangle$ of the crystal to a final state $|n'\rangle$ as a result of the emission of a gamma ray of momentum $\hbar\mathbf{q}$ is proportional to the square of the matrix element⁶

$$M_{nn'} = \langle n' | \exp(-i\vec{q} \cdot \vec{x}) | n \rangle, \quad (1)$$

where $\vec{x}$ is the coordinate of the nucleus emitting the gamma ray. This result has a simple physical interpretation. If the initial state $|n\rangle$ is multiplied by a factor $\exp(-i\vec{q} \cdot \vec{x})$, the result is a state which is the same as the initial state $|n\rangle$ except that the momentum distribution for the nucleus at $\vec{x}$ has been shifted by an amount $\vec{q}$. We then take the overlap between this state having a shifted momentum and the state $|n'\rangle$ to get the transition matrix element.

This description is common to a large number of physical phenomena characterized by a momentum transfer to a particle bound in some system. The transition matrix element is found to separate into two factors. One factor describes the particular elementary process in the absence of binding forces. The other factor describes the effect of the binding and depends only on the initial and final wave functions of the bound system and on the momentum transfer. The momentum-transfer factor is known as the Debye-Waller factor, the structure factor, the form factor,

⁶H. Frauenfelder, The Mössbauer Effect (W. A. Benjamin, Inc., New York, 1962).

and an assortment of other names. Each name is known to the specialists working on one particular type of reaction and is generally unknown to other specialists working on slightly different reactions; and the different groups have no idea that the other groups are really talking about the same thing.

Such momentum-transfer processes are used for studying nuclear structure. In this case, we have an elementary process or "direct reaction" involving a particular nucleon in a nucleus. Momentum is transferred to that nucleon but it is not knocked out of the nucleus and the whole nucleus does not fly apart; it remains bound and the recoil is miraculously transferred to the whole nucleus (either elastically or inelastically). In all of these processes, momentum-transfer matrix elements of type (1) play a crucial role and give information about nuclear structure.

Let us now examine the kind of information that is obtainable from processes describable by matrix elements of the type (1). Consider first the "long-wavelength limit." Here the value of the momentum transfer $\vec{q}$ is so small that $\vec{q} \cdot \vec{x}$ is a small quantity for all values of $\vec{x}$ less than the nuclear radius. It is then useful to expand the exponential in powers of the argument and to assume that the dominant contribution to the matrix element comes from the first nonvanishing term in the expansion. We then have the customary classification of transitions into allowed and forbidden with varying degrees of forbiddenness.

One example of such a momentum-transfer process is nuclear beta decay. A nucleon in the nucleus emits an electron and a neutrino and the recoil is taken up by the whole nucleus. In the Fermi theory of beta decay, a momentum-transfer matrix element of type (1) appears and is called "the nuclear matrix element." Beta transitions are treated in the long-wavelength limit by use of the expansion of the exponential. An allowed Fermi transition is one for which the constant term

dominates the expansion and the transition depends directly on the overlap of the initial-state and final-state wave functions. From the expansion of the matrix element (1), one easily sees how to obtain a description of forbidden beta decays. The existence of Gamow-Teller as well as Fermi transitions reminds us that the matrix element (1) has to be generalized slightly to include the possibility of nucleon spin flip in the transition and also to include relativistic effects which are neglected in the simple Mössbauer argument used to obtain the matrix element (1).

Another example of a momentum-transfer process is the emission or absorption of electromagnetic radiation by a nucleon in a nucleus. Here the generalization of the matrix element (1) to include spin effects leads to two types of transitions: electric and magnetic. The expansion of the exponential factor in spherical harmonics gives the familiar multipole expansion. In the long-wavelength limit, a given transition is dominated by the lowest multipoles allowed by conservation laws.

Let us now consider the case of large momentum transfer. In this case the expansion of the exponential no longer determines the dominant term in a given transition; many terms or multipoles can contribute to it. We immediately see that a high-energy reaction of this type may give information similar to that from corresponding low-energy reactions—only much more so. For a given transition, the contributions of a number of different multipoles can be studied at higher energies where the effects of the higher multipoles are not masked by the presence of the stronger lower multipoles. The most promising technique for studying such multipoles is by high-energy electron scattering. There, by varying both the energy of the incident electron beam and the momentum transfer (or the angle of scattering and the momentum transfer) one obtains a wealth of information. Techniques for improving experimental resolution and disentangling troublesome effects such as bremsstrahlung

are developing rapidly, and a number of electron accelerators in the appropriate energy range should be coming into operation soon. Thus one can expect considerable progress along these lines in the near future.

The first results obtained by the electron-scattering technique was the determination of the nuclear radius and charge distribution. This is easily understood from the matrix element (1). When we consider that the electron can be scattered by any one of the protons in the nucleus, we see that the matrix element should really be summed over all the protons. If we consider the matrix element for elastic scattering

$$M_{nn} = \langle n | \sum_p \exp(-i\vec{q} \cdot \vec{x}_p) | n \rangle = \int \sum_p \rho_p(\vec{x}) \exp(-i\vec{q} \cdot \vec{x}_p) d\vec{x}. \quad (2)$$

This is just the Fourier transform of the proton density $\rho_p(\vec{x})$. Thus measurement of the electron scattering cross section as a function of momentum transfer gives the Fourier transform of the charge density. The electron-scattering people call this the "electric form factor" of the nucleus.

An additional type of information is found by noting that a cross section is always proportional to a squared matrix element

$$\begin{aligned} |M_{nn'}|^2 &= |\langle n' | \sum_i \exp(-i\vec{q} \cdot \vec{x}_i) | n \rangle|^2 \\ &= \langle n | \sum_i \exp(i\vec{q} \cdot \vec{x}_i) | n' \rangle \langle n' | \sum_j \exp(-i\vec{q} \cdot \vec{x}_j) | n \rangle. \end{aligned} \quad (3)$$

If we now sum this result over all final states $|n'\rangle$ we obtain an interesting sum rule

$$\sum_{n'} |M_{nn'}|^2 = \sum_i \sum_j \langle n | \exp[i\vec{q} \cdot (\vec{x}_i - \vec{x}_j)] | n \rangle. \quad (4)$$

This is some function of the distance between pairs of particles and is a component of the Fourier transform of the correlation function for the pair. Fancier sum rules are obtainable by a standard form of trickery in which time-dependent Heisenberg operators suddenly appear out of nowhere. For example, consider the expression

$$\begin{aligned}
 \sum_{n'} |M_{nn'}|^2 \exp[i(E_n - E_{n'})s] &= \sum_i \sum_j \langle n | \exp(iE_n s) \exp(i\vec{q} \cdot \vec{x}_i) \\
 &\quad \times \exp(-iE_{n'} s) | n' \rangle \langle n' | \exp(-i\vec{q} \cdot \vec{x}_j) | n \rangle \\
 &= \sum_i \sum_j \langle n | \exp(iHs) \exp(i\vec{q} \cdot \vec{x}_i) \exp(-iHs) | n' \rangle \langle n' | \exp(-i\vec{q} \cdot \vec{x}_j) | n \rangle \\
 &= \sum_i \sum_j \langle n | \exp[i\vec{q} \cdot \vec{x}_i(s)] \exp[-i\vec{q} \cdot \vec{x}_j(0)] | n \rangle, \tag{5}
 \end{aligned}$$

where

$$\vec{x}_i(s) = e^{iHs} \vec{x}_i e^{-iHs}. \tag{6}$$

The quantity $\vec{x}_i(s)$ is just a Heisenberg operator at a time $t = s/\hbar$. One therefore obtains a sort of two-body space-time correlation function involving the correlation between finding one particle at a given point at a certain time and finding another particle at a different point at a different time.

The study of these two-particle correlations is of great interest in nuclear structure because people would like to know something about the departure of the real nuclear wave function from an independent-particle or shell-model wave function when two particles are close together. From nucleon-nucleon scattering data, we know that there are repulsive cores and that something must happen when the particles are very close together. We would like to find experiments to tell us what happens when nucleons get close together in finite nuclei. People hope to learn this from such experiments as high-energy electron scattering. So far, the

experiments have not been too informative; the interesting conclusions have been lost in other effects. Some people say that more precise experiments are possible and will give correlation information. Other people say that it will not help, because things are complicated and the correlations are not really being measured.

Since I do not know anything about this argument, I will say a few more words about the Mössbauer effect. I am reminded very much of a question people raised about the momentum transfer in the Mössbauer effect: Does the recoil happen suddenly or does it happen over a time equal to the lifetime of the gamma ray? People can argue this question for a long time; but when you try to settle the argument by proposing a suitable experiment, you find that you cannot do it without violating the uncertainty principle. The contradictions in the argument involve quantities that cannot be measured. In discussing nuclear correlations, we must remember that a nucleus is a quantum mechanical system. You cannot give a classical space-time description of exactly where a certain particle is at a given time and what happens just before or just after the virtual photon is absorbed by the nucleon. If you write down the right formulas and employ some nuclear model, you will get sensible answers; but it is risky to interpret these Heisenberg operators as classical events happening at different times. You are on shaky ground when you use a classical billiard-ball picture in which one nucleon scatters the electron, then the electron gets away, and finally the momentum is classically transferred to the rest of the nucleus. You are neglecting the wave properties of the nucleon.

High-energy electron scattering is a particularly attractive tool for studying nuclear structure because it avoids some of the principal complications of direct nuclear reactions and of ordinary photonuclear reactions. The electromagnetic interaction is weak and in addition is well known in this energy range. This eliminates the difficulties in unscrambling

the reaction mechanism and nuclear structure, complicated final-state interactions, distorted waves, etc. The only complication is that of bremsstrahlung. The advantage over conventional photonuclear investigations is that the use of virtual photons which are off the energy shell allows simultaneous and independent variation of the energy and the momentum transfer. This gives new insight into such problems as the giant resonance in O^{16} which has been beaten to death by conventional photonuclear means.

One can also consider "high-energy beta decays" in considering muon capture in complex nuclei.⁷ Here the momentum carried by the neutrino and absorbed by the nuclear recoil is sufficiently large so that one needs the full matrix element (1) and not simply the leading term as in beta decay. It turns out that there is a strong transition probability to states in the giant-resonance region and one sees the "isobaric analog" of the giant dipole resonance—i. e. , another member of the $T=1$ isobaric multiplet including the giant-resonance state. The transition probability again involves a "form factor." Thus we find that electron scattering and μ capture are natural extensions of photonuclear reactions involving respectively virtual photons and charged particles which add a nuclear charge exchange to the absorption process.

The Mössbauer effect has given us a general picture of the kind of information obtainable from weak, high-energy reactions on nuclei. The kind of reactions more closely related to those discussed in this conference are those involving strongly interacting particles, either high-energy nucleons or mesons. For these reactions we are faced with the characteristic dilemma of direct-interaction physics: Which comes first,

⁷L. L. Foldy and J. D. Walecka, "Muon Capture in Nuclei," Congrès International de Physique Nucléaire, Paris, 2—8 July 1964, Paper 5/C55.

the nuclear structure or the reaction mechanism; or which comes first, the theory or the experiment? One begins with a naive picture of a reaction mechanism such as that in the now famous $(p, 2p)$ experiments in which a nucleon is knocked out of an inner shell in the nucleus and all distorted waves and final-state interactions are ignored. The beautiful experimental results obtained indicate that there is something in the naive picture. However, if one is interested in pushing things further, one clearly needs a better theory, just as a better theory than the Butler theory was necessary for a more detailed understanding of stripping. However, such a better theory is not easily obtained from first principles and theorists are guided by experimental data which they try to fit. On the other hand, experimentalists, having a very wide range of experimental quantities to measure, need some theoretical guidance or intuition in order to know what kind of experimental information will be useful. The precise direction which such work should take is certainly beyond my capacity to judge. I shall only venture to conjecture that if a number of good people, both theorists and experimentalists, get into this field and do some work, some very interesting things will come out.

On the other hand, one often hears the following argument. High-energy reactions between strongly interacting particles are not simple and a lot of work has to be done in order to understand the reaction mechanism. But after all this is done, what will you learn about nuclear structure? The same sort of thing that you'll learn anyway by direct-interaction experiments at conventional energies. So why bother developing a new and expensive technique which doesn't offer much more than the techniques we already have? Since I have no good answer to this argument, I shall again say something about the Mössbauer effect.

When the Mössbauer effect was first discovered, it was generally considered to be an interesting but rather useless curiosity. It could not be used in any application where other conventional techniques

could not be used more easily. For example, in the study of internal fields in solids the Mössbauer effect would not be able to measure anything that could not in principle be measured with nuclear magnetic resonance. The fallacy in this argument is that measurements are not made "in principle"—they are made in the laboratory. There are certain materials which, for down-to-earth practical reasons cannot easily be investigated in the laboratory with nuclear magnetic resonance, but are very easily investigated with the Mössbauer effect. Since one of these materials happens to be ferromagnetic iron, the internal field at an iron nucleus was first measured by the Mössbauer effect and not by NMR. So perhaps, for similar reasons, one should be skeptical of arguments that a tandem Van de Graaff will give all the information about nuclear structure that can be obtained with a higher-energy machine. There may be a particular nuclear level of great interest to theorists which cannot be reached by conventional reactions because no one knows how to make a suitable target, whereas a high-energy reaction can reach this level from an easily produced target.

An amusing example along these lines is the study of the $J=1^+$, $T=0$ level in C^{12} at 12.71 MeV by inelastic proton scattering. The transition matrix element which determines the cross section for this reaction is the same as the magnetic-dipole matrix element in ordinary electromagnetic transitions. This looks like studying a simple electromagnetic transition the hard way. However, it turns out that magnetic-dipole $\Delta T=0$ radiative transitions are weak because the neutron and proton contributions cancel. You cannot study such transitions easily by electromagnetic techniques and high-energy inelastic proton scattering has turned out to be a very effective way to investigate these interesting transitions.

One general class of experiments which are particularly suited for high-energy techniques are what I call "Fractional Parentage

Experiments." A particle is knocked out of the nucleus, hopefully without disturbing the rest of the nucleus, and we learn something about those states of the residual nucleus that can be made by plucking a particle out of the target. The $(p, 2p)$ reactions are excellent examples of this. We can also consider reactions in which two particles are knocked out of the nucleus, again hopefully leaving the rest of the nucleus alone. There are also charge-exchange and double-charge-exchange reactions in which the charge of one or two nucleons in a nucleus is changed, again hopefully without otherwise disturbing the nucleus. With these experiments there always come the questions: "Can one really get away with this and not disturb the rest of the nucleus?" "What about final-state interactions?" Etc.

Since I cannot answer these questions, I again talk about the Mössbauer effect. There, to produce a transition involving one member of a bound system with a minimum disturbance of the bound system, we must make the momentum transfer small so that it is easy for the whole system to absorb the recoil. This suggests that fractional-parentage experiments are best done at low momentum transfer. This has been the case in the $(p, 2p)$ experiments where the observations are made just at the particular angles at which the momentum transfer to the residual nucleus is very small. Another important feature of this type of experiment is that the region of low momentum transfer to the residual nucleus corresponds to a very small portion of the phase space for the outgoing particles. One therefore is looking for an effect which, neglecting final-state interactions, should appear as a peak concentrated in a very small region of phase space. The total cross section for such an effect may be small compared with that of other reactions, but it sticks up above the background because of its concentration in a small region of phase space. Final-state interactions might be expected to scatter particles out of this region, thereby reducing the peak, or to spread out the peak and change its shape. They might even cause the peak to disappear

entirely. However, one would not expect final-state interactions to create a spurious peak in this small region of phase space. Thus if a peak is observed at all, its existence tells interesting information about nuclear structure, even though finer details of the angular distributions may require extensive analysis of final-state interactions in order to be properly interpreted.

It is interesting to do one little calculation relating the momentum transfer to the energy transfer in a particular reaction. Suppose that the momentum of the incident particle is $\vec{p}_i$ and that it has mass M and that the momentum transfer is $\vec{q}$. The energy transfer is then

$$\Delta E = \frac{(\vec{p}_i + \vec{q})^2 - p_i^2}{2M} = \frac{2\vec{p}_i \cdot \vec{q} + q^2}{2M} . \quad (7)$$

If we are interested in exciting a particular level, the energy transfer ΔE is given. To get a low momentum transfer q for a fixed energy transfer ΔE , we should use a high value of the incident momentum p_i . Thus, high incident energies lead to lower momentum transfers for a given excitation and may give a cleaner fractional-parentage experiment.

In discussing the Mössbauer effect, I seem to have found some general types of high-energy experiments that might give useful information about nuclear structure. These are listed in Table I.

* * *

M. E. RICKEY, University of Colorado: I do not want to ask about the Mössbauer effect, but I would like to ask a question about Dar's work which you discussed earlier.⁸ With respect to the fit to the $C^{12}(\alpha, p)N^{15}$

⁸H. J. Lipkin, in these Proceedings, p. 20.

TABLE I. Types of high-energy experiments.

1. Nuclear x-ray crystallography	(Nuclear density distribution and correlations)
2. Extended multipole spectroscopy	(Higher multipole contributions)
3. Extended photonuclear reactions	(Virtual photons and charge exchange)
4. Conventional spectroscopic studies	(Investigations for which low-energy techniques are unfeasible for experimental reasons)
5. Fractional-parentage experiments	

ground-state reaction at 30.5 MeV, does he have data going to larger angles? There is a very large backward peak at that energy. It would be very interesting to see if the very precise fit of these data predicts or can be made to fit the large backward-angle maximum. In addition, in some very pretty work done in Japan,⁹ they look at the angular distribution as a function of energy. I wonder if Dar applies his model to a situation in which there is a lot of information on the energy variation and the like.

LIPKIN: I do not know. I think that it is surprising that Dar has obtained such good fits on the reactions he has looked at. He certainly should do much more; and he plans to do it as soon as he has the time.

⁹I. Nonaka, H. Yamaguchi, T. Mikumo, I. Umeda, T. Tabata, and S. Hitaka, J. Phys. Soc. Japan 14, 1260 (1959).

ERRATA

in

Vol. I. Contributed Papers

ANL-6848

p. 2, line 2 of text:

· · · of calculations in · · ·
 $\Delta\Delta$

p. 38:

The author's name is Fernando B. Morinigo
 Δ

p. 56, line 10:

· · · to determine band structure from the energy separation · · ·
 $\Delta\Delta$

p. 61, line 2: Replace "=" by "-"

· · · $[I(I + 1) - I(K + 1)]$ · · ·
 Δ

p. 110, second line of Eq. (3):

$$\times \left\{ \begin{matrix} \ell_h & \ell_p & \ell \\ j_p & j_h & \frac{1}{2} \end{matrix} \right\} \left(\begin{matrix} \ell_h & \ell & \ell_p \\ 0 & 0 & 0 \end{matrix} \right)$$

$\Delta \qquad \qquad \Delta$

p. 113, Fig. 2, label of dashed curve: Should be

3.84 MeV
 $\Delta\Delta$

p. 113, line 5 after figure caption:

· · · an intensity of 0.19 relative · · ·
 $\Delta\Delta$

p. 113, line 7 after figure caption:

· · · This intensity is not in very good · · ·
 Δ

p. 128:

Second line of equation in bottom half of page:

$$\sum_L \left(\frac{7^{-2}}{2} L; \frac{7^2}{2} L \middle| \right\} \frac{7^8}{2} 0 \Bigg) \left\{ \cdot \cdot \cdot \right.$$

Fourth line of same equation:

$$\sum_L \left(\frac{7^{-2}}{2} L; \frac{7^2}{2} L \middle| \right\} \frac{7^8}{2} 0 \Bigg) C_{0L}^{aL}$$

p. 129:

First line of equation near top of page:

$$S_{aI} = \frac{8 \cdot 7}{2} \left(\frac{7^{-2}}{2} I; \frac{7^2}{2} I \middle| \right\} \frac{7^8}{2} 0 \Bigg)^2 \begin{bmatrix} C_{0I}^{aI} \end{bmatrix}$$

p. 189, second line of Eq. (4):

$$= \frac{[\cdot \cdot \cdot]^{1/2}}{\cdot \cdot \cdot \Delta} \left(\frac{\cdot \cdot \cdot}{\cdot \cdot \cdot} \right)^{3/2} \left(\frac{\cdot \cdot \cdot}{\cdot \cdot \cdot} \right)^{n-1}_{\Delta}, \quad (4)$$

p. 199, line 5 from bottom:

· · · H. W. Fulbright, W. Parker Alford, O. M. Bilaniuk, · · ·
 $\Delta \Delta \Delta$

PARTICIPANTS AT THE SYMPOSIUM ON NUCLEAR
SPECTROSCOPY WITH DIRECT REACTIONS

Fay Ajzenberg-Selove
Physics Department
Haverford College
Haverford, Pennsylvania

W. P. Alford
Physics Department
University of Rochester
Rochester 27, New York

R. G. Allas
D203-G153
Argonne National Laboratory
Argonne, Illinois

Samuel K. Allison
University of Chicago
Enrico Fermi Institute
Chicago 37, Illinois

Jonas Alster
Department of Physics
University of Washington
Seattle 5, Washington

Ralph D. Amato
Department of Physics
University of Pennsylvania
Philadelphia 4, Pennsylvania

Kenneth Albert Amos
Department of Physics
University of California
Davis, California 95616

John D. Anderson
University of California
Lawrence Radiation Laboratory
Livermore, California

Dale D. Armstrong
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

Norman Austern
Physics Department
University of Pittsburgh
Pittsburgh 13, Pennsylvania

Peter Axel
University of Illinois
Urbana, Illinois

R. Bahnsen
State University of Iowa
Iowa City, Iowa

James B. Ball
Bldg. 9201-2, Y-12
Physics Division
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37831

M. K. Banerjee
University of Rochester
Rochester, New York

Elizabeth Baranger
Physics Department
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

Michel Baranger
Physics Department
Carnegie Institute of Technology
Pittsburgh, Pennsylvania 15213

Peter D. Barnes
Department of Physics
Yale University
New Haven 11, Connecticut

Norton Baron
National Aeronautics and Space
Administration
Lewis Research Center
2100 Brookpark Road
Cleveland 35, Ohio

Roger C. Barrett
Department of Physics
Columbia University
538 West 120 Street
New York 27, New York

J. F. Barry
D208-C105
Argonne National Laboratory
Argonne, Illinois

H. H. Barschall
Physics Department
University of Wisconsin
Madison 6, Wisconsin

Giancarlo Bassani
Department of Physics
University of Minnesota
Minneapolis, Minnesota

Benjamin Bayman
Physics Department
Princeton University
Princeton, New Jersey

Thomas A. Belote
High Voltage Laboratory
Bldg. 46
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Raymond Benenson
The City College of New York
139 Street and Convent Avenue
New York 31, New York

E. Berkowitz
Physics Department
State University of Iowa
Iowa City, Iowa

A. Bernstein
Department of Physics
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts

Kumar H. Bhatt
Oak Ridge National Laboratory
Oak Ridge, Tennessee

Felton W. Bingham
Physics Department
University of Illinois
Urbana, Illinois

Allen G. Blair
Los Alamos Scientific Laboratory
P. O. Box 1663
Los Alamos, New Mexico 87544

John S. Blair
Department of Physics
University of Washington
Seattle 5, Washington

C. H. Blanchard
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

Lawrence Blau
Physics Department
Rochester University
Rochester 20, New York

Claude Bloch
Palmer Physical Laboratory
Princeton University
Princeton, New Jersey

Henry G. Blosser
Cyclotron Laboratory
Michigan State University
East Lansing, Michigan

Richard A. Blue
Van de Graaff Laboratory
Ohio State University
1302 Kinnear Road
Columbus 12, Ohio

Charles K. Bockelman
Department of Physics
Yale University
New Haven 11, Connecticut

Arnold Bodmer
D203-B241
Argonne National Laboratory
Argonne, Illinois

Manfred Böhning
Institut für Theoretische Physik
Philosophenweg 16
University of Heidelberg
Heidelberg, Germany

L. M. Bollinger
D203-E113
Argonne National Laboratory
Argonne, Illinois

J. P. Bondorf
Nordisk Institut for Teoretisk
Atomfysik
Blegdamsvej 17
Copenhagen, Denmark

Thomas H. Braid
D203-G149
Argonne National Laboratory
Argonne, Illinois

Dermott Bredin
Physics Department
Rutgers University
New Brunswick, New Jersey

D. Allan Bromley
Physics Department
Yale University
New Haven 11, Connecticut

Cornelius P. Browne
Department of Physics
University of Notre Dame
Notre Dame, Indiana

Morton K. Brussel
Physics Department
University of Illinois
Urbana, Illinois

Andre Bussiere
University of California
Berkeley 4, California

Richard R. Carlson
Physics Department
State University of Iowa
Iowa City, Iowa

Raymon T. Carpenter
Department of Physics and
Astronomy
State University of Iowa
Iowa City, Iowa

Joseph Cerny
Bldg. 70A, Chemistry Division
Lawrence Radiation Laboratory
University of California
Berkeley 4, California

Paul R. Chagnon
Physics Department
University of Notre Dame
South Bend, Indiana

R. Chaminade
C.E.N. Saclay
B.P. No. 2
Gif-sur-Yvette, France

William J. Childs
D203-F109
Argonne National Laboratory
Argonne, Illinois

John W. Clark
Washington University
St. Louis 30, Missouri

Calvin M. Class
Physics Department
Rice University
P.O. Box 1892
Houston, Texas 77001

H. Gunther Clausnitzer
University of Erlangen
Erlangen, Germany

Douglas Cline
Department of Physics
University of Rochester
Rochester 20, New York

Bernard L. Cohen
Physics Department
University of Pittsburgh
Pittsburgh 13, Pennsylvania

Stanley Cohen
D203-B265
Argonne National Laboratory
Argonne, Illinois

Richard C. Cox
2223 Ridge Avenue
Evanston, Illinois

John G. Cramer, Jr.
Physics Department
Swain Hall West
Indiana University
Bloomington, Indiana

B. T. Darling
Faculte de Sciences
Department of Physics
Laval University
Quebec, Quebec, Canada

Robert H. Davis
Physics Department
Florida State University
Tallahassee, Florida

Robert B. Day
Box 1663
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

George T. Dell
Physics Department
Purdue University
Lafayette, Indiana

Louis J. Denes
Physics Department
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

V. K. Deshpande
Department of Physics and
Astronomy
University of Rochester
Rochester, New York 14627

Herbert Diamond
D200-F121
Argonne National Laboratory
Argonne, Illinois

Darcy Dillenburg
Department of Physics
Carnegie Institute of Technology
Schenley Park
Pittsburgh 13, Pennsylvania

Ghias Ud Din
Rice University
Houston, Texas

Timothy R. Donoghue
Van de Graaff Laboratory
Ohio State University
1302 Kinnear Road
Columbus 12, Ohio

Ross A. Douglas
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

Darrell Drake
Department of Physics
University of Illinois
Urbana, Illinois

Richard M. Drisko
Physics Department
University of Pittsburgh
Pittsburgh, Pennsylvania

Steve Edwards, Jr.
Department of Physics
Florida State University
Tallahassee, Florida 32306

Dieter von Ehrenstein
D203-L115A
Argonne National Laboratory
Argonne, Illinois

William W. Eidson
Physics Department
Indiana University
Bloomington, Indiana

L. R. B. Elton
Battersea College of Technology
London, S.W. 11
England

Alexander Elwyn
D203-G133
Argonne National Laboratory
Argonne, Illinois

Pieter M. Endt
Physics Laboratory
State University
Bijlhouwerstraat 6,
Utrecht, Netherlands

H. A. Enge
Physics Department, Bldg. 46
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts

John R. Erskine
D203-L115A
Argonne National Laboratory
Argonne, Illinois

S. Fallieros
Department of Physics
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts

Paul Fields
D200-M169
Argonne National Laboratory
Argonne, Illinois

Roger W. Finlay
Department of Physics
Ohio University
Athens, Ohio

R. A. Fisher
Department of Physics
Northwestern University
Evanston, Illinois

N. R. Fletcher
Department of Physics
Florida State University
Tallahassee, Florida

Edward Flynn
Los Alamos Scientific Laboratory
University of California
Los Alamos, New Mexico

John M. Fowler
Department of Physics
Washington University
St. Louis 30, Missouri

J. D. Fox
Department of Physics
Florida State University
Tallahassee, Florida 32306

J. B. French
Physics Department
University of Rochester
Rochester 20, New York

Arnold Friedman
D200-M167
Argonne National Laboratory
Argonne, Illinois

C. B. Fulmer
Physics Division
Bldg. 9201-2, Y-12
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37831

Richard H. Fulmer
Sarah Mellon Scaife Radiation
Laboratory
University of Pittsburgh
Pittsburgh 13, Pennsylvania

Fletcher Gabbard
Department of Physics
University of Kentucky
Lexington, Kentucky 40506

Donald I. Garber
Case Institute of Technology
Cleveland, Ohio

Gerald T. Garvey
Palmer Physical Laboratory
Princeton University
Princeton, New Jersey

J. Gastebois
Centre d'Etudes Nucleaires de
Saclay
B. P. No. 2, Gif-sur-Yvette
France

Laurie Gauvin
Faculty of Sciences
Physics Department
Laval University
Quebec 10, Quebec, Canada

D. S. Gemmell
D203-G137
Argonne National Laboratory
Argonne, Illinois

Faison P. Gibson
Physics Division
P. O. Box X
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37831

J. E. Gindler
D200-E109
Argonne National Laboratory
Argonne, Illinois

Norman K. Glendenning
Lawrence Radiation Laboratory
University of California
Berkeley 4, California

R. N. Glover
Bldg. N62
U.K. Atomic Energy Authority
A.W.R.E., Aldermaston
Berkshire, England

Paul Goldhammer
Department of Physics
University of Nebraska
Lincoln, Nebraska

Charles D. Goodman
Bldg. 9201-2, Y-12
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37831

Leonard S. Goodman
D203-F117
Argonne National Laboratory
Argonne, Illinois

M. M. Gordon
Michigan State University
East Lansing, Michigan

Peter Gould
School of Physics
Institute of Technology
University of Minnesota
Minneapolis 14, Minnesota

Harry E. Gove
Physics Department
University of Rochester
Rochester 20, New York

Hans G. Graetzer
Department of Physics
University of Colorado
Boulder, Colorado

Arnfinn Graue
Physics Department
University of Bergen
Bergen, Norway
(On leave at M. I. T.)

L. L. Green
Chadwick Physics Laboratory
University of Liverpool
Liverpool 13, England

G. W. Greenlees
Physics Department
University of Birmingham
Birmingham 15, England

Reginald C. Greenwood
Physics Department
I.I. T. Research Institute
Chicago 16, Illinois

K. R. Greider
Physics Department
Yale University
217 Prospect Street
New Haven, Connecticut

James J. Griffin
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

W. Gruebler
Department of Physics
The University of Wisconsin
Madison 6, Wisconsin

Eugene Guth
Oak Ridge National Laboratory
Oak Ridge, Tennessee

William Haeberli

Department of Physics
University of Wisconsin
Madison 6, Wisconsin

Joseph C. Hafele

Nuclear Radiations Laboratory
Department of Physics
University of Illinois
Urbana, Illinois

M. E. Haglund

Physics Department
Florida State University
Tallahassee, Florida

Frank R. Haig

Department of Physics
Wheeling College
Wheeling, West Virginia 26003

Morton Hamermesh

10-D101
Argonne National Laboratory
Argonne, Illinois

Stanley S. Hanna

Physics Department
Stanford University
Stanford, California

H. S. Hans

D203-F113
Argonne National Laboratory
Argonne, Illinois

Gerald Hardie

I.I.T. Research Institute
3300 S. Federal Street
Chicago, Illinois

Gale I. Harris

Aeronautical Research Laboratory
Wright-Patterson A.F.B.
Dayton 31, Ohio

S. M. Harris

Department of Physics
Columbia University
New York 27, New York

Bernard G. Harvey

Lawrence Radiation Laboratory
University of California
Berkeley 4, California

M. Harvey

Chalk River Nuclear Laboratories
Chalk River, Ontario, Canada

Herschel J. Hausman

Van de Graaff Laboratory
Ohio State University
1302 Kinnear Road
Columbus 12, Ohio

D. Heikkinen

State University of Iowa
Iowa City, Iowa

E. M. Henley

Department of Physics
University of Washington
Seattle 5, Washington

Hans J. Hennecke

5973 Chatsworth Drive
Dayton, Ohio

Caroline L. Herzenberg

I.I.T. Research Institute
3300 S. Federal Street
Chicago, Illinois

Gerhardt Heymann

Room 44-120
Laboratory for Nuclear Science
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts

R. Hiddleston
State University of Iowa
Iowa City, Iowa

Norton M. Hintz
School of Physics
University of Minnesota
Minneapolis, Minnesota

B. Hird
Physics Department
University of Manitoba
Winnipeg 19, Canada

P. E. Hodgson
Clarendon Laboratory
Parks Road
University of Oxford
Oxford, England

Robert E. Holland
D203-G121
Argonne National Laboratory
Argonne, Illinois

H. D. Holmgren
Physics Department
University of Maryland
College Park, Maryland

John R. Huizenga
D200-E113
Argonne National Laboratory
Argonne, Illinois

Richard Humes
Department of Physics
Ohio State University
1302 Kinnear Road
Columbus 12, Ohio

Douglas L. Humphrey
Department of Physics
Ohio University
Athens, Ohio

Peter J. Iano
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

D. R. Inglis
D203-B218
Argonne National Laboratory
Argonne, Illinois

Eugene V. Ivash
Physics Department
University of Texas
Austin, Texas

Daphne F. Jackson
Department of Physics
University of Washington
Seattle 5, Washington

Harold E. Jackson, Jr.
D203-F153
Argonne National Laboratory
Argonne, Illinois

K. P. Jackson
Physics Department
University of Toronto
Toronto 5, Ontario, Canada

Arthur H. Jaffey
D200-E181
Argonne National Laboratory
Argonne, Illinois

Ronald C. Johnson
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

William P. Johnson
Michigan State University
East Lansing, Michigan

R. K. Jolly
Physics Department
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

A. D. W. Jones
Department of Nuclear Physics
Nuclear Physics Laboratory
Oxford University
21 Banbury Road
Oxford, England

G. A. Jones
A. E. R. E. Harwell
Didcot, Berkshire
England

K. W. Jones
Department of Physics
Brookhaven National Laboratory
Upton, L.I., New York

Edwin Kashy
Physics Department
Princeton University
Princeton, New Jersey

Charles D. Kavaloski
Department of Physics
University of Washington
Seattle 5, Washington

M. Kawai
Department of Physics
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

Charles Kelsey
Department of Physics
The University of Wisconsin
Madison 6, Wisconsin

A. K. Kerman
Room 26-411, Physics Department
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts

K. Kibler
State University of Iowa
Iowa City, Iowa

Y. E. Kim
Bell Telephone Laboratories
Murray Hill, New Jersey

B. B. Kinsey
Department of Physics
University of Texas
Austin, Texas

Ernest D. Klema
Department of Engineering Sciences
Technological Institute
Northwestern University
Evanston, Illinois

Alvin R. Knudson
U. S. Naval Research Laboratory
Washington 25, D. C.

S. Kobayashi
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

Richard Koshel
36 Briarwood
Athens, Ohio

V. E. Krohn
D203-E137
Argonne National Laboratory
Argonne, Illinois

Krishna Kumar
Michigan State University
East Lansing, Michigan

Thomas Tzu Szn Kuo
University of Pittsburgh
Pittsburgh, Pennsylvania 15213

Dieter Kurath
D203-B246
Argonne National Laboratory
Argonne, Illinois

Alexander Lande
Department of Physics
Princeton University
Princeton, New Jersey

Raymond O. Lane
D203-G105
Argonne National Laboratory
Argonne, Illinois

Donald W. Lang
D203-B241
Argonne National Laboratory
Argonne, Illinois

Alexander Langsdorf, Jr.
D203-G113
Argonne National Laboratory
Argonne, Illinois

R. D. Lawson
D203-B222
Argonne National Laboratory
Argonne, Illinois

R. B. Leachman
Los Alamos Scientific Laboratory
University of California
Los Alamos, New Mexico

Linwood L. Lee
D203-G117
Argonne National Laboratory
Argonne, Illinois

James C. Legg
Department of Physics
Rice University
P.O. Box 1892
Houston 1, Texas 77001

Rene Levesque
Department of Physics
University of Montreal
Case Postale 6128
Montreal, Quebec, Canada

Frank S. Levin
Department of Physics
Brookhaven National Laboratory
Upton, L.I., New York

Leon J. Lidofsky
Room 246, S.W. Mudd Bldg.
Physics Department
Columbia University
New York 27, New York

Gerard P. Lietz
University of Notre Dame
South Bend, Indiana

John S. Lilley
Nuclear Physics Laboratory
Cyclotron Bldg.
University of Washington
Seattle 5, Washington

E. K. Lin
Department of Physics
University of Pittsburgh
Pittsburgh 13, Pennsylvania

K. H. Lindenberg
Hahn-Meitner Institut für Kernphysik
Glienicke Str. 100
Berlin 39, W. Germany

Harry J. Lipkin
Physics Department
Weizmann Institute of Science
Rehovoth, Israel

M. A. K. Lodhi
Department of Physics
Texas Technological College
P. O. Box 4180
Lubbock 9, Texas

Edward Ludwig
Physics Department
Rutgers University
New Brunswick, New Jersey

Frank J. Lynch
D203-F113
Argonne National Laboratory
Argonne, Illinois

Brian E. F. Macefield
Department of Nuclear Physics
Nuclear Physics Laboratory
University of Oxford
21 Banbury Road
Oxford, England

Malcolm H. Macfarlane
D203-B210
Argonne National Laboratory
Argonne, Illinois

Ronald Malmin
D203-G129
Argonne National Laboratory
Argonne, Illinois

M. Mancusi
State University of Iowa
Iowa City, Iowa

Amnon Marinov
D203-G117
Argonne National Laboratory
Argonne, Illinois

George Matois
University of Notre Dame
South Bend, Indiana

Donald R. Maxson
Physics Department
Brown University
Providence, Rhode Island

Bob Maxwell
Department of Physics
University of Minnesota
Minneapolis, Minnesota

C. Mayer-Borick
Department of Physics
Florida State University
Tallahassee, Florida

A. Lance McCarthy
Radiation Laboratory
University of Pittsburgh
Pittsburgh 13, Pennsylvania

I. E. McCarthy
Physics Department
University of California
Davis, California

J. A. McCray
Physics Department
University of California
Davis, California

John D. McCullen
Department of Physics
Princeton University
Princeton, New Jersey

R. McGrath
State University of Iowa
Iowa City, Iowa

John A. McIntyre
Cyclotron Institute
Texas A and M University
College Station, Texas

K. W. McVoy
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

R. Mendelson
State University of Iowa
Iowa City, Iowa

Sydney Meshkov
Radiation Division
National Bureau of Standards
Washington 25, D.C. 20234

Luise Meyer-Schützmeister
D203-G141
Argonne National Laboratory
Argonne, Illinois

R. K. Mohindra
Bartol Research Foundation of the
Franklin Institute
Swarthmore, Pennsylvania

P. A. Moldauer
D208-C121
Argonne National Laboratory
Argonne, Illinois

James E. Monahan
D203-B245
Argonne National Laboratory
Argonne, Illinois

F. Paul Mooring
D203-L013
Argonne National Laboratory
Argonne, Illinois

Fernando B. Morinigo
W. K. Kellogg Radiation
Laboratory
California Institute of Technology
Pasadena, California

Richard D. Murphy
Department of Physics
Brown University
Providence, Rhode Island 02912

M. A. Nagarajan
Department of Physics
Case Institute of Technology
University Circle
Cleveland 6, Ohio

Ove Nathan
Institute for Theoretical Physics
Blegdamsvej 17
Copenhagen, Denmark

J. H. Neiler
Oak Ridge National Laboratory
853 W. Outer Drive
Oak Ridge, Tennessee

Eugene Newman
Electronuclear Research Division
Oak Ridge National Laboratory
Oak Ridge, Tennessee

H. C. Newns
Department of Theoretical Physics
University of Liverpool
Liverpool, England

W. C. Parkinson
Physics Department
University of Michigan
Ann Arbor, Michigan

Edwin Norbeck
State University of Iowa
Department of Physics
Iowa City, Iowa

Francis G. Perey
Neutron Physics Division
Oak Ridge National Laboratory
P. O. Box X
Oak Ridge, Tennessee

Rolf Nordhagen
Institute of Physics
University of Oslo
Blindern, Norway

Gilbert J. Perlow
D203-H113
Argonne National Laboratory
Argonne, Illinois

K. -E. Nystén
D203-G121
Argonne National Laboratory
Argonne, Illinois

Paul J. Persiani
D208-W201
Argonne National Laboratory
Argonne, Illinois

Jean M. O'Dell
2226 Ohio
Lawrence, Kansas

Lucy Wu Person
D362
Argonne National Laboratory
Argonne, Illinois

J. C. Overley
Nuclear Structure Laboratory
Yale University
51 Sachem Street
New Haven 11, Connecticut

Murray Peshkin
D203-B226
Argonne National Laboratory
Argonne, Illinois

S. P. Pandya
Physical Research Laboratory
Ahmedabad, India
(On leave at Argonne National
Laboratory)

R. J. Philpott
University of Pittsburgh
Pittsburgh, Pennsylvania

John A. Panontin
D200-E169
Argonne National Laboratory
Argonne, Illinois

W. T. Pinkston
Bartol Research Foundation
of the Franklin Institute
Swarthmore, Pennsylvania

J. Y. Park
North Carolina State of the
University of North Carolina
Raleigh, North Carolina

H. S. Plendl
Department of Physics
Florida State University
Tallahassee, Florida 32306

W. D. Ploughe
Van de Graaff Laboratory
The Ohio State University
1302 Kinnear Road
Columbus 12, Ohio

Ray Poore
University of North Carolina
Chapel Hill, North Carolina

Carl H. Poppe
University of Minnesota
Minneapolis, Minnesota

A. Prakash
Radiation Laboratory
University of Pittsburgh
Pittsburgh 13, Pennsylvania

Francis W. Prosser, Jr.
Physics Department
University of Kansas
Lawrence, Kansas

C. D. Pruett
409 Nicholson
Joliet, Illinois

Arthur R. Quinton
Department of Physics
University of Florida
Gainesville, Florida

Sol Raboy
D203-H105
Argonne National Laboratory
Argonne, Illinois

Keith F. Ratcliff
University of Pittsburgh
Pittsburgh, Pennsylvania

George H. Rawitscher
Physics Department
Yale University
217 Prospect Street
New Haven, Connecticut

B. James Raz
Physics Department
State University of New York
Stony Brook, L.I., New York

Mannque Rho
Lawrence Radiation Laboratory
University of California
Berkeley 4, California

H. T. Richards
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

Martin E. Rickey
Physics Department
University of Colorado
Boulder, Colorado
(On leave at Brookhaven National
Laboratory)

Rogers C. Ritter
Department of Physics
University of Virginia
Charlottesville, Virginia

N. R. Roberson
Nuclear Structure Laboratory
Department of Physics
Duke University
Durham, North Carolina

Russell L. Robinson
Bldg. 5500
Oak Ridge National Laboratory
P. O. Box X
Oak Ridge, Tennessee

D. Robson
Department of Physics
Florida State University
Tallahassee, Florida

William S. Rodney
National Science Foundation
Washington, D.C.

Norbert Rosenzweig
D203-B261
Argonne National Laboratory
Argonne, Illinois

Baruch Rosner
Department of Physics
University of Pittsburgh
Pittsburgh 13, Pennsylvania

Ernest Rost
Palmer Physical Laboratory
Princeton University
Princeton, New Jersey

Gerald Roy
Rice University
Houston, Texas

Balder Sahai
University of Chicago
Chicago, Illinois

Claude St. Pierre
Department de Physique
Faculte des Sciences
Laval University
Quebec, Quebec, Canada

Jurg X. Saladin
Physics Department
University of Pittsburgh
Pittsburgh, Pennsylvania

E. A. Sanderson
University of Pittsburgh
Pittsburgh, Pennsylvania

G. R. Satchler
Physics Division
Oak Ridge National Laboratory
P. O. Box X
Oak Ridge, Tennessee

Jean Saudinos
C.E.N. Saclay, B. P. No. 2
Gif-sur-Yvette, France

John Schiffer
D203-G109
Argonne National Laboratory
Argonne, Illinois

Ulrich Schmidt-Rohr
Max-Planck Institute für
Kernphysik
Heidelberg, West Germany

Edward J. Schneid
Radiation Laboratory
University of Pittsburgh
Pittsburgh 13, Pennsylvania

John Jay Schwartz
Physics Department
University of Rochester
Rochester, New York

R. E. Segel
D203-F129
Argonne National Laboratory
Argonne, Illinois

K. K. Seth
Northwestern University
Evanston, Illinois

Richard Seyler
Department of Physics
Ohio State University
1302 Kinnear Road
Columbus 12, Ohio

Rubby Sherr
Palmer Physical Laboratory
Princeton University
Princeton, New Jersey

Franklin B. Shull
Department of Physics
Washington University
St. Louis 30, Missouri

R. H. Siemssen
D203-G105
Argonne National Laboratory
Argonne, Illinois

Edward A. Silverstein
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

P. P. Singh
D203-G137
Argonne National Laboratory
Argonne, Illinois

Donald Slanina
Michigan State University
East Lansing, Michigan

William R. Smith
Bldg. 5500
Oak Ridge National Laboratory
Oak Ridge, Tennessee

Robert Smither
D203-F137
Argonne National Laboratory
Argonne, Illinois

R. A. Sorensen
Physics Department
Carnegie Institute of Technology
Pittsburgh, Pennsylvania

Daniel Sperber
I.I.T. Research Institute
Chicago 16, Illinois

Anthony Sperduto
Room 58-007
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts

Brajesh B. Srivastava
Purdue University
Lafayette, Indiana

K. G. Standing
Physics Department
University of Manitoba
Winnipeg, Manitoba
Canada

A. F. Stehney
D362-E325
Argonne National Laboratory
Argonne, Illinois

Nelson Stein
Nuclear Physics Laboratory
Cyclotron Bldg.
University of Washington
Seattle 5, Washington

Ellis P. Steinberg
D362-E301
Argonne National Laboratory
Argonne, Illinois

- William M. Stewart
N. A. S. A.
Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135
- E. N. Strait, Jr.
Physics Department
Northwestern University
Evanston, Illinois
- David C. Sutton
Department of Physics
University of Illinois
Urbana, Illinois
- Arata Suzul
Nuclear Research Center
Carnegie Institute of Technology
Saxonburg, Pennsylvania
- Taro Tamura
Physics Division
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37831
- G. M. Temmer
Physics Department
Rutgers University
New Brunswick, New Jersey 08903
- V. K. Thankappan
Physics Department
University of California
Davis, California 95616
- G. D. Thaxton
University of North Carolina
Chapel Hill, North Carolina
- George Thomas
D203-F153
Argonne National Laboratory
Argonne, Illinois
- M. Throop
State University of Iowa
Iowa City, Iowa
- Francis E. Throw
D203-F169
Argonne National Laboratory
Argonne, Illinois
- D. R. Tilley
Nuclear Structure Laboratory
Department of Physics
Duke University
Durham, North Carolina
- Zeev Vager
D203-G117
Argonne National Laboratory
Argonne, Illinois
- C. Van der Leun
Physics Laboratory
State University
Bijlhouwerstraat 6
Utrecht, Netherlands
- R. Venter
Department of Physics
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts
- Margaret A. Waggoner
University of Maryland
College Park, Maryland
- N. S. Wall
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts
- Richard L. Walter
Nuclear Structure Laboratory
Duke University
Durham, North Carolina 27706

Kenneth Warsh
Lockheed Aircraft Corporation
Atlanta, Georgia

Jesse L. Weil
Department of Physics
University of Kentucky
Lexington, Kentucky 40506

Robert B. Weinberg
Pegram Nuclear Physics Laboratories
Columbia University
538 West 120 Street
New York 27, New York

Jerome Wesolowski
Lawrence Radiation Laboratory
Livermore, California

Richard W. West
Department of Physics and Astronomy
University of Rochester
River Campus Station
Rochester, New York 14627

C. A. Wiedner
Massachusetts Institute of
Technology
Cambridge 39, Massachusetts

W. J. Wieseahn
University of Toronto
Toronto, Ontario, Canada

Eugene P. Wigner
Palmer Physical Laboratory
Princeton University
Princeton, New Jersey

Charles L. Wiley
Purdue University
Lafayette, Indiana

John H. Williams
Physics Department
University of Minnesota
Minneapolis 14, Minnesota

John G. Wills
Department of Physics
Indiana University
Bloomington, Indiana

Fred L. Wilson
University of Kansas
Lawrence, Kansas

Robert J. Wilson
Physics Department
Washington University
St. Louis 30, Missouri

Hubert Winkler
California Institute of Technology
Pasadena, California

Rolf G. Winter
Accelerator Bldg.
Nuclear Physics Laboratories
Pennsylvania State University
University Park, Pennsylvania 16802

E. A. Wolicki
Nucleonics Division
U. S. Naval Research Laboratory
Washington 25, D. C.

D. Worth
Department of Physics
University of Wisconsin
Madison 6, Wisconsin

Avivi I. Yavin
Physics Department
University of Illinois
Urbana, Illinois

- J. L. Yntema
D203-F177
Argonne National Laboratory
Argonne, Illinois
- Y. Yoshizawa
Indiana University
Bloomington, Indiana
- James E. Young
University of California
Los Alamos Scientific Laboratory
P. O. Box 1663
Los Alamos, New Mexico 87544
- Chris D. Zafiratos
Los Alamos Scientific Laboratory
Los Alamos, New Mexico
- Larry Zamick
Department of Physics
Princeton University
Princeton, New Jersey
- H. Dieter Zeh
Institut für Theoretische Physik
Philosophenweg 16
University of Heidelberg
Heidelberg, Germany
- Benjamin Zeidman
D203-G149
Argonne National Laboratory
Argonne, Illinois
- Robert Zimmerman
Department of Physics
Case Institute of Technology
University Circle
Cleveland 6, Ohio
- Wolfgang Zuckschwerdt
Purdue University
Lafayette, Indiana

INDEX OF CONTRIBUTORS

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