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THREE-DIMENSIONAL INVISCID FLOW ABOUT
SUPERSONIC BLUNT CONES AT ANGLE OF ATTACK

I: Numerical Technique For The Three-Dimensional
Blunt-Body Problem

Prepared by
G. Moretti and G. Bleich
General Applied Science Laboratories, Inc.

SANDIA LABORATORIES

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G. Moretti

and

G. Bleich

General Applied Science Laboratories, Inc.
Merrick and Stewart Avenues
Westbury, Long Island, New York

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Contract Number 48-4014

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ABSTRACT

The three-dimensional flow field around blunted bodies traveling at supersonic speed is computed using a time-dependent technique. The problem is mathematically well posed, the technique is stable and its accuracy increases with the fineness of the mesh. Values at points within the shock layer are computed by a method closely related to the Lax-Wendroff technique. Values at shock points and body points are computed by a four-dimensional method of characteristics.

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FOREWORD

This report compiles the work of G. Moretti, M. J. Abbett, G. Bleich and R. Fort done under a Sandia contract (No. 48-4014). The aim of the contract was to develop a Fortran program which would determine the flow over a blunted cone in supersonic flow at an angle of attack. The contents of this report, developed in three sequential stages from February 1966 to July 1968, are presented here in a set of three volumes. Volume I, entitled "A Numerical Technique for the Three-Dimensional Blunt-Body Problem," develops the technique for handling three-dimensional flows. It includes a numerical model (cylindrical coordinates) for the nose region of a blunt body at an angle of attack. Upon using the resulting Fortran program it was found that the predicted pressure distributions were not accurate enough and the geometry of the downstream output surface was not easily adaptable to provide starting data for downstream supersonic solutions. In order to alleviate these problems, flow equations were re-derived using polar coordinates.

The reason for choosing the polar coordinate system is discussed in Volume II, "Improved Time-Dependent Techniques for the Blunt-Body Problem." A mathematical outline for this development is given. The resulting polar frame flow field does indeed improve the accuracy of the resulting flow properties and provides an output surface which can easily be used to start the downstream supersonic solution.

The development of the combined blunt body-afterbody solution is given in Volume III. This is entitled "Coupled Subsonic and Supersonic Programs for Inviscid Three-Dimensional Flow." In this volume the blunt body solution is linked with a solution for the three-dimensional supersonic afterbody flow. It will calculate the flow over a blunted cone at an angle of attack. The solution can be extended downstream to approximately 20 to 30 nose radii and takes 40 to 60 minutes of CDC 3600 computer time depending on the body geometry and mesh spacing.

Roger R. Eaton - 9341

SUMMARY

A technique for computing the ideal gas flow about a supersonic axisymmetric body at an angle of attack is developed. The system initially defined in a cylindrical coordinate system is transformed into a rectangular space for the time-dependent mathematical development. The rectangular space is bordered by the bow shock, the body, the bordering cylindrical surface and meridional planes $\phi = 0$ and 180 degrees. Calculations are begun by assuming a shock shape and calculating the flow across this shock layer by combined use of the Rankine-Hugoniot jump equations and a four dimensional method of characteristics scheme. The flow within the boundaries of the rectangular region are calculated by a time-dependent explicit Lax-Wendroff type numerical scheme. The dependent variables to be evaluated at $t_0 + \Delta t$ are expanded, about t_0 , into a truncated Taylor series which contains the second-order term. The body surface properties are also calculated using a method of characteristics scheme. The developed technique has the capability of calculating the shock layer over a wide range of geometries and free stream conditions, including Mach numbers larger than two and angles of attack from zero degrees to 35 degrees. Calculations can be carried downstream approximately two nose radii. A typical case requires approximately 50 minutes of CDC 3600 computer time.

TABLE OF CONTENTS

<u>SECTION</u>	<u>TITLE</u>	<u>PAGE</u>
I	INTRODUCTION	1
II	STATEMENT OF THE PROBLEM, NOTATION	3
III	INTERIOR POINTS NOT ON THE z -AXIS	7
IV	INTERIOR POINTS ON THE z -AXIS	15
V	SHOCK POINTS NOT ON THE z -AXIS	19
VI	SHOCK POINT ON THE z -AXIS	32
VII	BODY POINTS NOT ON THE z -AXIS	39
VIII	BODY POINT ON THE z -AXIS	46
IX	DETERMINATION OF THE STEP SIZE	49
X	OPERATING INSTRUCTIONS	50
XI	RESULTS AND DISCUSSION	57
	REFERENCES	61

I. INTRODUCTION

Few reports have been published so far on the three-dimensional blunt body problem, but not for lack of interest. The blunt body problem is a difficult one even in two dimensions and a number of different techniques have been suggested. Choosing one among them on the basis of an evaluation of advantages and disadvantages seems still to be a matter of personal taste.

Actual computations of three-dimensional blunt body flows have been presented in two papers, following two different techniques. The first¹ uses truncated series to iterate on an inverse computation. The second² uses a time-dependent technique.

Some of the shortcomings of both techniques for two-dimensional and axisymmetrical computations have been pointed

out by one of the present authors in a recent paper^{3,4} (Ref. 4 is an abridged version of Ref. 3). Such shortcomings are emphasized in the three-dimensional case. The time-dependent method, which is undoubtedly the safer, more accurate and logically simpler one, is unfortunately extremely time consuming. In the example shown in Ref. 2, 24,000 mesh points have been used. On the basis of our experience, we estimate that the running time on an IBM 7094 is about 40 hours.

The technique described in the present paper is the three-dimensional extension of the time-dependent technique of Refs. 3 and 4. Its main advantages with respect to Ref. 2 are:

(1) The body may have any (so far, axisymmetric) shape, whereas the body in Ref. 2 has to be a spherical sector,

(2) the shock wave is defined sharply,

(3) a reduction of 50 to 1 in the number of mesh points is achieved, and

(4) consequently, a reduction of more than 50 to 1 in running time is achieved. Note that the present program does not require additional tapes for temporary storage. As a matter of fact, typical problems have been solved in running times of about half an hour each.

II. STATEMENT OF THE PROBLEM. NOTATION

The flow of a perfect, inviscid gas with a constant ratio of specific heats, γ , around an axisymmetric blunt body is analyzed. The flow is assumed uniform and supersonic at infinity. The body axis is inclined with respect to the impinging flow.

Let z be the axis of symmetry of the body. Consider a cylindrical frame of reference, r, ϕ, z . At any point P a system of three unit vectors is defined:

$\hat{k}$ in the direction of the z -axis, pointing towards the body,

$\hat{i}$ in the direction of increasing r , and

$\hat{j} = \hat{k} \times \hat{i}$, that is, normal to the meridional plane through P , in the direction of increasing ϕ .

The components, u, v , and w of the velocity vector, $\vec{V}$ are defined as follows:

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k} \quad (1)$$

Let $\vec{V}_\infty$ be the velocity of the flow at infinity,

$$\vec{V}_\infty = u_\infty \hat{k}_1 \quad (2)$$

The meridional half-plane, $\phi = 0$ is defined by the unit vectors $\hat{k}$ and $\hat{k}_1$. Therefore, α being the angle of attack,

$$\hat{k}_1 = \sin \alpha \cos \hat{\varphi}_i - \sin \alpha \sin \hat{\varphi}_j + \cos \alpha \hat{k} \quad (3)$$

The body geometry is fixed and independent of φ , by assumption. Then, if r and z are the coordinates of a point of the body surface, the latter is described by

$$z - b(r) = 0 \quad (4)$$

where $b(r)$ is an assigned function.

The bow shock wave is not, in general, independent of φ and, in a time-dependent procedure, has to be assumed dependent on time, t . Such a wave can be described by

$$z - s(r, \varphi, t) = 0 \quad (5)$$

where $s(r, \varphi, t)$ is unknown and must be computed. If a steady state is reached asymptotically, the steady geometry of the shock is described by

$$z - s(r, \varphi, \infty) = 0 \quad (6)$$

The computation will be performed with non-dimensional quantities. Let $[p_\infty]$, $[\rho_\infty]$, $[u_0]$ be the dimensional pressure, density, and velocity of the free stream, $[p]$, $[\rho]$, $[V]$ the dimensional pressure, density, and (any) velocity at a point.

Let $[x]$ be any dimensional length, and $[r_0]$ a dimensional reference length. Let $[t]$ be the dimensional time. The following nondimensional quantities will be used:

$$\text{(pressure)} \quad p = [p]/[p_\infty] \quad (7)$$

$$\text{(density)} \quad \rho = [\rho]/[\rho_\infty] \quad (8)$$

$$P = \ln p \quad (9)$$

$$R = \ln \rho \quad (10)$$

$$\text{(entropy)} \quad S = P - \gamma R \quad (11)$$

$$\text{(velocity)} \quad v = [V]/\sqrt{[p_\infty]/[\rho_\infty]} \quad (12)$$

$$\text{(length)} \quad x = [x]/[r_0] \quad (13)$$

$$\text{(time)} \quad t = [t][\sqrt{[p_\infty]/[\rho_\infty]}/r_0] \quad (14)$$

If M_∞ is the free stream Mach number,

$$[u_0] = M_\infty \sqrt{\gamma [p_\infty]/[\rho_\infty]}$$

and then

$$u_0 = M_\infty \sqrt{\gamma} \quad (15)$$

In nondimensional form the equations of motion are:

$$R_t + uR_r + \frac{v}{r} R_\phi + wR_z + u_r + \frac{1}{r} v_\phi + w_z + \frac{u}{r} = 0 \quad (16)$$

$$u_t + uu_r + \frac{v}{r} u_\phi + wu_z - \frac{v^2}{r} + \frac{p}{\rho} p_r = 0 \quad (17)$$

$$v_t + uv_r + \frac{v}{r} v_\phi + wv_z + \frac{uv}{r} + \frac{1}{r} \frac{p}{\rho} p_\phi = 0 \quad (18)$$

$$w_t + uw_r + \frac{v}{r} w_\phi + ww_z + \frac{p}{\rho} p_z = 0 \quad (19)$$

$$S_t + uS_r + \frac{v}{r} S_\phi + wS_z = 0 \quad (20)$$

where subscripts indicate differentiation.

The problem will be solved in a three-dimensional region bounded by the body surface, the bow shock and a circular cylinder, $r = r_1$. Equations (16)-(20) will be used inside such a region. A special treatment will be given to shock points and to body points.

Note that, since the body is axially symmetric and the free stream is parallel to the $\phi = 0, \phi = \pi$ plane, such a plane is a plane of symmetry for the flow field, which can be analyzed in one half of the above region, for example the one defined by $0 \leq \phi \leq \pi$.

III. INTERIOR POINTS NOT ON THE z-AXIS

At each stage of the computation, a fixed number of equally spaced meridional half-planes is considered, including the $\varphi = 0$ half-plane and the $\varphi = \pi$ half-plane. For convenience, such a number is taken as odd, so that the $\varphi = \pi/2$ half-plane is part of the set.

In each meridional half-plane, a fixed number of equally spaced straight lines, parallel to the z-axis, is considered, including the z-axis itself and the $r = r_1$ line.

On each $r = \text{constant}$ line, a fixed number of equally spaced points is considered between the shock and the body. Note that the points are equally spaced on each $r = \text{constant}$ line, but the spacing differs from line to line, in general. In addition, the spacing changes in time since the shock shape is unsteady.

Now, let δ be the difference between the z-coordinate of a shock point and the z-coordinate of a body point on a given $r = \text{constant}$ line of a given $\varphi = \text{constant}$ half-plane:

$$\delta = \delta(r, \varphi, t) = s(r, \varphi, t) - b(r) \quad (21)$$

Let us introduce a new set of variables, ζ, Y, X, T , related to z, r, φ, t as follows:

$$\left\{ \begin{array}{l} \zeta = \frac{z-b}{\delta} \\ Y = r \\ X = \varphi \\ T = t \end{array} \right. \quad \left\{ \begin{array}{l} z = b + \zeta \delta \\ r = Y \\ \varphi = X \\ t = T \end{array} \right. \quad (22)$$

The region to be computed is then mapped onto a right parallelepiped bounded by the planes:

$$\begin{array}{ll} \zeta = 0 & (\text{body}) \\ \zeta = 1 & (\text{shock}) \\ Y = 0 & (z\text{-axis}) \\ Y = r_1 & (\text{external cylinder}) \\ X = 0 & (\varphi = 0 \text{ plane}) \\ X = \pi & (\varphi = \pi \text{ plane}) \end{array}$$

The z -axis, which is a singular line in the cylindrical frame, is obviously mapped onto a plane and needs a special treatment.

We give now a set of definitions and basic relations to be used in the course of the analysis (here $b' = b_r$ and f is any function of r , φ , z , and t).

$$\left. \begin{aligned}
 A &= \frac{v}{Y} \\
 C &= b'(\zeta - 1) - \zeta s_r \\
 B &= [w + uC - \zeta(s_t + As_\varphi)]/\delta \\
 E &= C/\delta \\
 F &= \zeta s_\varphi/Y\delta \\
 G &= p/\rho
 \end{aligned} \right\} (23)$$

$$\left. \begin{aligned}
 \delta_r &= s_r - b' \\
 \delta_\varphi &= s_\varphi \\
 \delta_t &= s_t
 \end{aligned} \right\} (24)$$

$$\left. \begin{aligned}
 \zeta_z &= 1/\delta \\
 \zeta_r &= E \\
 \zeta_\varphi &= -\zeta s_\varphi/\delta \\
 \zeta_t &= -\zeta s_t/\delta
 \end{aligned} \right\} (25)$$

$$\left. \begin{aligned}
 f_t &= f_T - (\zeta s_t/\delta) f_\zeta \\
 f_r &= f_Y + E f_\zeta \\
 f_\varphi &= f_X - (\zeta s_\varphi/\delta) f_\zeta \\
 f_z &= (1/\delta) f_{mb} \\
 f_t + u f_r + \frac{v}{r} f_\varphi + w f_z &= f_T + u f_Y + A f_X + B f_\zeta
 \end{aligned} \right\} (26)$$

The equations of motion are then transformed as follows:

$$R_T + uR_Y + AR_X + BR_\zeta + u_Y + Eu_\zeta + v_X/Y - Fv_\zeta + w_\zeta/\delta + u/Y = 0 \quad (27)$$

$$u_T + uu_Y + Au_X + Bu_\zeta - Av + GP_Y + GEP_\zeta = 0 \quad (28)$$

$$v_T + uv_Y + Av_X + Bv_\zeta + Au + GP_X/Y - GFP_\zeta = 0 \quad (29)$$

$$w_T + uw_Y + Aw_X + Bw_\zeta + GP_\zeta/\delta = 0 \quad (30)$$

$$S_T + uS_Y + AS_X + BS_\zeta = 0 \quad (31)$$

We will also need the following set of partial derivatives:

$$\left. \begin{aligned} A_X &= v_X/Y \\ C_X &= -\zeta s_{r\varphi} \\ B_X &= [w_X + u_X C + u C_X - \zeta(s_{t\varphi} + v_X s_{\varphi}/Y + A s_{\varphi\varphi})]/\delta - B s_{\varphi}/\delta \\ E_X &= (C_X - E s_{\varphi})/\delta \\ F_X &= (\zeta s_{\varphi\varphi}/Y - F s_{\varphi})/\delta \\ G_X &= G(P_X - R_X) \end{aligned} \right\} \quad (32)$$

$$\begin{aligned}
A_Y &= (v_Y - A)/Y \\
C_Y &= b''(\zeta - 1) - \zeta s_{rr} \\
B_Y &= [w_Y + u_Y C + u_C Y - \zeta(s_{tr} + v_Y s_\varphi/Y + A s_{\varphi r} \\
&\quad - A s_{\varphi/Y})]/\delta - B(s_r - b')/\delta \\
E_Y &= [(C_Y - E(s_r - b'))]/\delta \\
F_Y &= \zeta s_{\varphi r}/Y\delta - F/Y - F(s_r - b')/\delta \\
G_Y &= G(P_Y - R_Y)
\end{aligned}
\tag{33}$$

$$\begin{aligned}
A_\zeta &= v_\zeta/Y \\
C_\zeta &= b' - s_r \\
B_\zeta &= [w_\zeta + u_\zeta C + u_C \zeta - s_t - A s_\varphi - \zeta v_\zeta s_\varphi/Y]/\delta \\
E_\zeta &= C_\zeta/\delta \\
F_\zeta &= s_\varphi/Y\delta \\
G_\zeta &= G(P_\zeta - R_\zeta)
\end{aligned}
\tag{34}$$

$$\begin{aligned}
A_T &= v_T/Y \\
C_T &= -\zeta s_{rt} \\
B_T &= [w_T + u_T C + u_C T - \zeta(s_{tt} + v_T s_\varphi/Y + A s_{\varphi t})]/\delta - B s_t/\delta \\
E_T &= (C_T - E s_t)/\delta \\
F_T &= (\zeta s_{\varphi t}/Y - F s_t)/\delta \\
G_T &= G(P_T - R_T)
\end{aligned}
\tag{35}$$

Note that, as far as s is concerned, $s_t = s_T$, $s_\phi = s_X$, $s_r = s_Y$, etc. since s does not depend on ζ .

To compute the new value (at $t = t_0 + \Delta t$) of a function f ($f = R, u, v, w$, or S) at an interior point defined by $\zeta = \zeta_0$, $X = X_0$, $Y = Y_0$ (not on the z -axis) the following formula is used:

$$f(t_0 + \Delta t) = f(t_0) + f_T \Delta t + f_{TT} \Delta t^2 / 2 \quad (36)$$

where $f(t_0)$, f_T , and f_{TT} are evaluated at $t = t_0$ and at the point defined by $\zeta = \zeta_0$, $X = X_0$, and $Y = Y_0$. Note that the "new" point will be physically shifted along the $r = \text{constant}$ line with respect to the "initial" point if the shock shape is changed from $t = t_0$ to $t = t_0 + \Delta t$. In fact, the z -coordinate of the point will change if the ζ -coordinate is kept constant and s changes. However, the shifting in one time step is extremely small and tends to zero when the shock becomes steady.

In order to compute f_T and f_{TT} , a set of first and second derivatives with respect to the space coordinates is prepared. For example, referring to Fig. 1,

$$f_{\zeta}(n, m, \ell) = [f(n+1, m, \ell) - f(n-1, m, \ell)] / 2\Delta\zeta \quad (37)$$

$$f_{\zeta\zeta}(n, m, \ell) = [f(n+1, m, \ell) + f(n-1, m, \ell) - 2f(n, m, \ell)] / \Delta\zeta^2 \quad (38)$$

$$f_{\zeta Y}(n, m, \ell) = [f(n+1, m+1, \ell) - f(n+1, m-1, \ell) - f(n-1, m+1, \ell) + f(n-1, m-1, \ell)] / 4\Delta\zeta \Delta Y \quad (39)$$

FIG. 1

The values of the quantities in Eqs. (23), (32), (33), and (34) are also computed. Then, the first derivatives, f_T , are obtained from Eqs. (27)-(31). Such equations are now differentiated with respect to X , Y , and ζ . They can be solved for f_{TX} , f_{TY} , and $f_{T\zeta}$.

If Eqs. (27)-(31) are then differentiated with respect to T , the values of f_{TT} are expressed by quantities which are all known.

Note that b' and b'' are geometrical values which can be computed once and for all. At a given time, knowing the shape of the shock, s_r , s_ϕ , $s_{r\phi}$, s_{rr} , $s_{\phi\phi}$ are easily determined. The time derivative, s_t , is determined at every step through the analysis of the shock. Consequently, s_{tr} and $s_{t\phi}$ can be evaluated. The second derivative, s_{tt} is obtained as

$$s_{tt}(r, \phi, t) = [s_t(r, \phi, t) - s_t(r, \phi, t - \Delta t)] / \Delta t \quad (40)$$

At all points on the $\phi = 0$ and $\phi = \pi$ half planes, except on the z -axis, suitable symmetry conditions are used. Note that s , u , w , R , P , and S are even functions of ϕ and consequently of X , whereas v is an odd function of X and identically zero at all points of the symmetry plane.

IV. INTERIOR POINTS ON THE z-AXIS

The points on the z-axis appear in every meridional half-plane but need be computed only once, for example in the $\varphi = 0$ half-plane. Note that at a point on the z-axis a scalar quantity is independent of φ , and so is w . However $u = U \cos \varphi$, $v = -U \sin \varphi$ (Fig. 2) where U is u in the $\varphi = 0$ half-plane.

FIG. 2

Then

$$u_{\varphi} = -U \sin \varphi$$

$$v_{\varphi} = -U \cos \varphi$$

$$w_{\varphi} = 0, R_{\varphi} = 0, P_{\varphi} = 0, S_{\varphi} = 0.$$

The equations of motion (16)-(20) are still valid on any meridional half-plane for r tending to zero, with some obvious modifications. In Eqs. (16), (18), (19), and (20)

$$\lim_{r \rightarrow 0} \frac{v}{r} f_{\varphi} = -U \sin \varphi \cdot f_{r\varphi}$$

where f is, in turn, R , P , w , and S . In Eqs. (16) and (18),

$$\lim_{r \rightarrow 0} (v_{\varphi} + u)/r = (v_{\varphi} + u)_{mb}$$

since $v_\phi + u$ tends to zero with r , as shown above.

In Eq. (17),

$$\lim_{r \rightarrow 0} (u_\phi - v)/r = (u_\phi - v)_r$$

since $u_\phi - v$ also tends to zero with r .

In the $\phi = 0$ half-plane, in particular, where $v = 0$, the equations become

$$R_t + uR_r + wR_z + 2u_r + w_z + v_{\phi r} = 0 \quad (41)$$

$$u_t + uu_r + wu_z + \frac{p}{\rho} P_r = 0 \quad (42)$$

$$v_t = 0 \quad (43)$$

$$w_t + uw_r + ww_z + \frac{p}{\rho} P_z = 0 \quad (44)$$

$$S_t + uS_r + wS_z = 0 \quad (45)$$

In such a plane and on the z -axis,

$$\left. \begin{aligned} b' &= 0 \\ C &= -\zeta_{s_r} \\ B &= (w + uC - \zeta_{s_t})/\delta \\ A &= 0 \end{aligned} \right\} \quad (46)$$

Using Eqs. (26), and defining

$$H = v_{\phi r} \quad (47)$$

Eqs. (41)-(45) become

$$R_T + uR_Y + BR_\zeta + 2u_Y + 2Eu_\zeta + w_\zeta/\delta + H = 0 \quad (48)$$

$$u_T + uu_Y + Bu_\zeta + GP_Y + GEP_\zeta = 0 \quad (49)$$

$$v_T = 0 \quad (50)$$

$$w_T + uw_Y + Bw_\zeta + GP_\zeta/\delta = 0 \quad (51)$$

$$S_T + uS_Y + BS_\zeta = 0 \quad (52)$$

To compute H, use Eqs. (47) and (26) and keep only the non-zero terms. It follows that

$$H = v_{XY} + Ev_{X\zeta} \quad (53)$$

At this stage, Eqs. (48)-(52) can be differentiated with respect to Y, ζ , and T for the purpose explained in Section III.

Note that

$$\begin{aligned} C_Y &= b''(\zeta - 1) - \zeta s_{rr} \\ B_Y &= (w_Y + u_Y C + u C_Y - \zeta s_{tr})/\delta - Bs_r/\delta \\ E_Y &= (C_Y - Es_r)/\delta \\ G_Y &= G(P_Y - R_Y) \end{aligned} \quad (54)$$

$$\begin{aligned}
C_{\zeta} &= -s_r \\
B_{\zeta} &= (w_{\zeta} + u_{\zeta}C + uC_{\zeta} - s_t)/\delta \\
E_{\zeta} &= C_{\zeta}/\delta \\
G_{\zeta} &= G(P_{\zeta} - R_{\zeta})
\end{aligned} \tag{55}$$

$$\begin{aligned}
C_T &= -\zeta s_{rt} \\
B_T &= (w_T + u_T C + uC_T - \zeta s_{tt})/\delta - Bs_t/\delta \\
E_T &= (C_T - Es_t)/\delta \\
G_T &= G(P_T - R_T)
\end{aligned} \tag{56}$$

For convenience (to avoid third derivatives), H_Y , H_{ζ} , and H_T will be computed as finite differences.

V. SHOCK POINTS NOT ON THE z-AXIS

From Eqs. (1), (2), and (3), the components of the free stream velocity in the $\hat{i}, \hat{j}, \hat{k}$ frame at a point Q on the shock are

$$\left. \begin{aligned} u_{\infty} &= u_0 \sin \alpha \cos \varphi \\ v_{\infty} &= -u_0 \sin \alpha \sin \varphi \\ w_{\infty} &= u_0 \cos \alpha \end{aligned} \right\} \quad (57)$$

Since the shock is a moving surface, we can define the velocity of each point Q in a simple, convenient way. For example, we can say that each point Q of the shock moves in a direction parallel to the z-axis. The velocity vector, $\vec{W}$, of a point Q is then obviously

$$\vec{W} = W\hat{k} = s_t \hat{k} \quad (58)$$

The components of the free stream velocity relative to the shock, $\vec{v}_{\infty}^{\text{rel}}$, are

$$\left. \begin{aligned} u_{\infty}^{\text{rel}} &= u_{\infty} \\ v_{\infty}^{\text{rel}} &= v_{\infty} \\ w_{\infty}^{\text{rel}} &= w_{\infty} - W \end{aligned} \right\} \quad (59)$$

From Eq. (5) it follows that the unit vector normal to the shock, $\hat{N}$, is defined by

$$\hat{N} = (-s_r \hat{i} - \frac{1}{r} s_\varphi \hat{j} + \hat{k})/\nu \quad (60)$$

where

$$\nu = \sqrt{1 + s_r^2 + s_\varphi^2/r^2} \quad (61)$$

The component of the free stream velocity relative to the shock and normal to it, $v_{n\infty}^{rel}$, is

$$v_{n\infty}^{rel} = v_{\infty}^{rel} \cdot \hat{N} = (-u_{\infty} s_r - v_{\infty} s_\varphi/r + w_{\infty} - W)/\nu \quad (62)$$

With

$$M_{n\infty}^{rel} = v_{n\infty}^{rel}/\sqrt{\gamma} \quad (63)$$

the normal component of the velocity behind the shock, relative to it, v_n^{rel} is determined by the Rankine-Hugoniot equation,

$$v_n^{rel} = v_{n\infty}^{rel} \frac{1 + \frac{\gamma-1}{2} M_{n\infty}^2}{\frac{\gamma+1}{2} M_{n\infty}^2} \quad (64)$$

Consequently, if ρ , p , R , P are values behind the shock,

$$\rho = v_{n\infty}^{\text{rel}} / v_n^{\text{rel}} \quad \left. \vphantom{\rho} \right\} \quad (65)$$

$$R = \ln \rho$$

$$p = \frac{(\gamma+1)\rho - (\gamma-1)}{\gamma+1 - (\gamma-1)\rho} \quad \left. \vphantom{p} \right\} \quad (66)$$

$$P = \ln P$$

The velocity vector tangent to the shock, $\vec{v}_t$, is left unchanged across the shock. Such a vector is

$$\vec{v}_t = \vec{v}_\infty - v_{n\infty} \hat{N} \quad (67)$$

Therefore, the (absolute) velocity of a particle behind the shock is

$$\begin{aligned} \vec{V} &= v_n \hat{N} + \vec{v}_t = \vec{v}_\infty - (v_{n\infty} - v_n) \hat{N} \\ &= \vec{v}_\infty - (v_{n\infty}^{\text{rel}} - v_n^{\text{rel}}) \hat{N} \end{aligned} \quad (68)$$

or, by components,

$$\begin{aligned} u &= u_\infty + (v_{n\infty}^{\text{rel}} - v_n^{\text{rel}}) s_r / v \\ v &= v_\infty + (v_{n\infty}^{\text{rel}} - v_n^{\text{rel}}) s_\phi / r v \\ w &= w_\infty - (v_{n\infty}^{\text{rel}} - v_n^{\text{rel}}) / v \end{aligned} \quad \left. \vphantom{u} \right\} \quad (69)$$

Eqs. (61)-(66) and (69) define the conditions behind the shock, once W is known. The additional information needed to determine W is obtained from the compatibility equation along a characteristic line ending where Q is at $t = t_0 + \Delta t$ and contained in a plane defined by $\hat{N}$ and the time axis. Such a line comes from a point, A^* , located in the interior of the shock layer, along $\hat{N}$, at $t = t_0$. Once A^* is known, the initial conditions at A^* can be determined from the neighboring points by interpolation.

In order to obtain the location of A^* and the compatibility equation, consider a Cartesian frame, $\hat{N}, \hat{\tau}, \hat{b}$ where

$$\hat{\tau} = \hat{N} \times \hat{j} / |\hat{N} \times \hat{j}| \quad (70)$$

$$\hat{b} = \hat{N} \times \hat{\tau} \quad (71)$$

Note that in Eqs. (70) and (71) $\hat{N}$ and $\hat{j}$ are defined at a point Q and remain unchanged as long as Q is not changed.

Equation (60) can be supplemented with the following relations between $\hat{\tau}$ and $\hat{b}$ and the polar frame:

$$\hat{\tau} = (-\hat{i} - s_r \hat{k}) / \nu_1, \quad \nu_1 = \sqrt{1 + s_r^2} \quad (72)$$

$$\hat{b} = [s_r s_\phi / r \hat{i} - (1 + s_r^2) \hat{j} - s_\phi / r \hat{k}] / \nu_1 \quad (73)$$

Let ξ, η, ω be the coordinates and $\tilde{u}, \tilde{v}, \tilde{w}$ the velocity components in the $\hat{N}, \hat{\tau}, \hat{b}$ frame, in that order. The origin of the frame is at Q. The equations of motion are

$$R_t + \tilde{u}R_\xi + \tilde{v}R_\eta + \tilde{w}R_\omega + \tilde{u}_\xi + \tilde{v}_\eta + \tilde{w}_\omega = 0 \quad (74)$$

$$\tilde{u}_t + \tilde{u}\tilde{u}_\xi + \tilde{v}\tilde{u}_\eta + \tilde{w}\tilde{u}_\omega + \frac{p}{\rho} P_\xi = 0 \quad (75)$$

$$\tilde{v}_t + \tilde{u}\tilde{v}_\xi + \tilde{v}\tilde{v}_\eta + \tilde{w}\tilde{v}_\omega + \frac{p}{\rho} P_\eta = 0 \quad (76)$$

$$\tilde{w}_t + \tilde{u}\tilde{w}_\xi + \tilde{v}\tilde{w}_\eta + \tilde{w}\tilde{w}_\omega + \frac{p}{\rho} P_\omega = 0 \quad (77)$$

$$S_t + \tilde{u}S_\xi + \tilde{v}S_\eta + \tilde{w}S_\omega = 0 \quad (78)$$

From (74), (78), and (11) we obtain

$$P_t + \tilde{u}P_\xi + \tilde{v}P_\eta + \tilde{w}P_\omega + \gamma(\tilde{u}_\xi + \tilde{v}_\eta + \tilde{w}_\omega) = 0 \quad (79)$$

To find the characteristics in the (ξ, t) plane, we rewrite (75) and (79) as follows:

$$\left. \begin{aligned} \tilde{u}_t + \tilde{u}\tilde{u}_\xi + \frac{p}{\rho} P_\xi &= -\tilde{v}\tilde{u}_\eta - \tilde{w}\tilde{u}_\omega \\ \gamma\tilde{u}_\xi + P_t + \tilde{u}P_\xi &= -\tilde{v}P_\eta - \tilde{w}P_\omega - \gamma(\tilde{v}_\eta + \tilde{w}_\omega) \end{aligned} \right\} \quad (80)$$

Let $\lambda = \frac{dx}{dt}$ be the slope of a characteristic. It is easily found that

$$\lambda^I = \tilde{u} + a \quad (81)$$

$$\lambda^{II} = \tilde{u} - a \quad (82)$$

where a is the speed of sound, $a^2 = \gamma p / \rho$.

The corresponding compatibility equations are

$$\left(\frac{d\tilde{u}}{dt} \right)_I + \frac{a}{\gamma} \left(\frac{dP}{dt} \right)_I = - (\tilde{v}u_\eta + \tilde{w}u_\omega) - a(\tilde{v}P_\eta/\gamma + \tilde{w}P_\omega/\gamma + \tilde{v}_\eta + \tilde{w}_\omega) \quad (83)$$

$$\left(\frac{d\tilde{u}}{dt} \right)_{II} - \frac{a}{\gamma} \left(\frac{dP}{dt} \right)_{II} = - (\tilde{v}u_\eta + \tilde{w}u_\omega) + a(\tilde{v}P_\eta/\gamma + \tilde{w}P_\omega/\gamma + \tilde{v}_\eta + \tilde{w}_\omega) \quad (84)$$

In this case, the characteristic of the second family must be used. The compatibility equation between the initial point, A^* , and point Q reads

$$\begin{aligned} \tilde{u} - \tilde{u}_* - aP/\gamma + aP_*/\gamma &= \sigma \cdot \Delta t \\ \sigma &= - (\tilde{v}u_\eta + \tilde{w}u_\omega) + a(\tilde{v}P_\eta/\gamma + \tilde{w}P_\omega/\gamma + \tilde{v}_\eta + \tilde{w}_\omega) \end{aligned} \quad (85)$$

where $\tilde{u}_*$ and P_* are values at A_* , u and P are values at Q , a is an average between the speed of sound at A_* and Q , and σ may be assumed as constant along the characteristic (equal to its value at A_*).

The coordinate, ξ of A_* is given by

$$\xi = \left| \tilde{u} - a \right| \cdot \Delta t \quad (86)$$

Point A_* must now be determined with reference to the (X, Y, ζ) frame, in order to perform the interpolations and differentiations which allow σ to be computed.

The procedure is slightly cumbersome. It is convenient to start in a general way, by considering any point A, not necessarily on the ξ -axis. In what follows, $\hat{i}_A, \hat{j}_A, \hat{k}$ is the polar frame related to point A, whereas $\hat{i}, \hat{j}, \hat{k}$ is the polar frame related to point Q. Similarly, r_A, ϕ_A, z_A are the polar coordinates of point A and r, ϕ, s are the polar coordinates of point Q. If A is A^* , we will write r_* instead of r_{A^*} , etc.

Since

$$\begin{aligned} A - Q &= \xi \hat{N} + \eta \hat{r} + w \hat{b} \\ &= r_A \hat{i}_A - r \hat{i} + (z_A - s) \hat{k} \end{aligned} \quad (87)$$

and since (Fig. 3)

$$\left. \begin{aligned} \hat{i}_A \cdot \hat{i} &= \cos (\phi_A - \phi) \\ \hat{i}_A \cdot \hat{j} &= \sin (\phi_A - \phi) \end{aligned} \right\} \quad (88)$$

In what follows, we will need the derivatives:

$$\left. \begin{aligned} r_{\eta A} &= - (\omega s_r s_\varphi / r \nu \nu_1 + r - \xi s_r / \nu - \eta / \nu_1) / r_A \nu_1 \\ \varphi_{\eta A} &= \sin(\varphi_A - \varphi) \cos(\varphi_A - \varphi) / \nu_1 (\omega s_r s_\varphi / r \nu \nu_1 + r - \xi s_r / \nu - \eta / \nu_1) \\ z_{\eta A} &= - s_r / \nu_1 \end{aligned} \right\} (91)$$

$$\left. \begin{aligned} r_{\omega A} &= \{ (s_r s_\varphi / r \nu \nu_1) (\omega s_r s_\varphi / r \nu \nu_1 + r - \xi s_r / \nu - \eta / \nu_1) \\ &\quad + [(1 + s_r^2) / \nu \nu_1] [\xi s_\varphi / r \nu + \omega (1 + s_r^2) / \nu \nu_1] \} / r_A \\ \varphi_{\omega A} &= - \cos^2(\varphi_A - \varphi) \{ (1 + s_r^2) / \nu \nu_1 + \tan(\varphi_A - \varphi) s_r s_\varphi / r \nu \nu_1 \} / \\ &\quad (\omega s_r s_\varphi / r \nu \nu_1 + r - \xi s_r / \nu - \eta / \nu_1) \\ z_{\omega A} &= - s_\varphi / r \nu \nu_1 \end{aligned} \right\} (92)$$

The preceding equations take a simpler form at point A* located on the ξ -axis:

$$\left. \begin{aligned} r_*^2 &= (r - \xi s_r / \nu)^2 + (\xi s_\varphi / r \nu)^2 \\ \tan(\varphi_* - \varphi) &= - \frac{\xi s_\varphi / r \nu}{r - \xi s_r / \nu} \\ z_* &= s + \xi / \nu \end{aligned} \right\} (93)$$

$$\begin{aligned}
r_{\eta*} &= - (r - \xi s_r / \nu) / r_* \nu_1 \\
\varphi_{\eta*} &= \sin (\varphi_* - \varphi) \cos (\varphi_* - \varphi) / \nu_1 (r - \xi s_r / \nu) \\
z_{\eta*} &= - s_r / \nu_1
\end{aligned}
\tag{94}$$

$$\begin{aligned}
r_{\omega*} &= \{ (s_r s_\varphi / r \nu \nu_1) (r - \xi s_r / \nu) + [(1 + s_r^2) / \nu \nu_1] (\xi s_\varphi / r \nu) \} / r_* \\
\varphi_{\omega*} &= - \cos^2 (\varphi_* - \varphi) \{ (1 + s_r^2) / \nu \nu_1 + \tan (\varphi_* - \varphi) s_r s_\varphi / r \nu \nu_1 \} / \\
&\quad (r - \xi s_r / \nu) \\
z_{\omega*} &= - s_\varphi / r \nu \nu_1
\end{aligned}
\tag{95}$$

From (22),

$$\begin{aligned}
Y_* &= r_* \\
X_* &= \varphi_*
\end{aligned}
\tag{96}$$

Knowing r_* , $b_* = b(r_*)$ is determined and, knowing r_* and φ_* , $s_* = s(r_*, \varphi_*, t_0)$ is determined. Then $\delta_* = s_* - b_*$ and

$$\zeta_* = (z_* - b_*) / \delta_* \tag{97}$$

In this way, point A_* is found in the (X, Y, ζ) frame. Note that

$$1 - \Delta \zeta < \zeta_* < 1$$

As far as X and Y are concerned, there are four possible locations for A^* (Fig. 4). Cases like A_1^* and A_2^* occur if $\varphi_* > \varphi$, cases like A_1^* and A_3^* occur if $r_* > r$. Accordingly, the interpolations and differentiations must be performed in different cells.

Once the cell where A^* is located is defined, u_* , v_* , w_* , P_* , and R_* are computed by interpolation. Then $a_* = \sqrt{\gamma p_* / \rho_*}$. The derivatives, u_X , u_Y , u_ζ , v_X , v_Y , v_ζ , w_X , w_Y , w_ζ , P_X , P_Y , P_ζ can also be computed at A^* .

Using Eqs. (26) u_r , u_φ , u_z , v_r , v_φ , v_z , w_r , w_φ , w_z , P_r , P_φ , P_z at A^* are obtained. To compute the velocity components and their derivatives in the (ξ, η, ω) frame, note that

$$\begin{aligned} \bar{V}_* &= \tilde{u}_* \hat{N} + \tilde{v}_* \hat{r} + \tilde{w}_* \hat{b} \\ &= u_* \hat{i}_* + v_* \hat{j}_* + w_* \hat{k} \end{aligned} \quad (98)$$

Considering that

$$\left. \begin{aligned} \hat{i}_* &= \cos(\varphi_* - \varphi) \hat{i} + \sin(\varphi_* - \varphi) \hat{j} \\ \hat{j}_* &= -\sin(\varphi_* - \varphi) \hat{i} + \cos(\varphi_* - \varphi) \hat{j} \end{aligned} \right\} \quad (99)$$

and using Eqs. (60), (72), and (73), it follows that

FIG. 4

$$\left.
\begin{aligned}
\tilde{u}_* &= -U_* s_r / \nu - V_* s_\phi / r \nu + w_* / \nu \\
\tilde{v}_* &= -U_* / \nu_1 - w_* s_r / \nu_1 \\
\tilde{w}_* &= U_* s_r s_\phi / r \nu \nu_1 - V_* (1 + s_r^2) / \nu \nu_1 - w_* s_\phi / r \nu \nu_1
\end{aligned}
\right\} (100)$$

where

$$\left.
\begin{aligned}
U_* &= u_* \cos(\varphi_* - \varphi) - v_* \sin(\varphi_* - \varphi) \\
V_* &= u_* \sin(\varphi_* - \varphi) + v_* \cos(\varphi_* - \varphi)
\end{aligned}
\right\} (101)$$

The derivatives of $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ with respect to r , φ , and z at A^* are obtained by differentiating Eqs. (100) and (101). Note that only the starred terms are variable. Now, f being $\tilde{u}_*$, $\tilde{v}_*$, $\tilde{w}_*$, or P_* , we can compute

$$\left.
\begin{aligned}
f_{\eta} &= f_{r\eta_*} + f_{\varphi\eta_*} + f_{z\eta_*} \\
f_{\omega} &= f_{r\omega_*} + f_{\varphi\omega_*} + f_{z\omega_*}
\end{aligned}
\right\} (102)$$

using the results of Eqs. (94) and (95).

Then $\tilde{u}$ at the shock point, Q , is evaluated from Eqs. (85). Obviously, $\tilde{u}$ should equal V_n , where

$$V_n = V_n^{\text{rel}} + W/\nu \quad (103)$$

and V_n^{rel} is the one defined in Eq. (64). Since this can happen only for one value of W , an iteration procedure can be used to solve the problem.

VI. SHOCK POINT ON THE z-AXIS

For the shock point on the z-axis the procedure described in Section V has to be modified slightly. As for the interior points on the z-axis, the shock point will be considered as belonging to the symmetry plane. Equations (57) then become

$$\left. \begin{aligned} u_{\infty} &= u_0 \sin \alpha \\ v_{\infty} &= 0 \\ w_{\infty} &= u_0 \cos \alpha \end{aligned} \right\} \quad (104)$$

Eq. (60) becomes

$$\hat{N} = (-s_r \hat{i} + \hat{k})/\nu \quad (105)$$

where

$$\nu = \sqrt{1 + s_r^2} \quad (106)$$

Eq. (62) becomes

$$v_{n_{\infty}}^{\text{rel}} = (-u_{\infty} s_r + w_{\infty} - W)/\nu \quad (107)$$

The second of Eqs. (69) is simply

$$v = 0 \quad (108)$$

Eqs. (71) and (73) become

$$\hat{b} = -\hat{j} \quad (109)$$

Eqs. (87)-(95) become (note that $r = 0$):

$$\hat{\xi}_N + \hat{\eta}_T - \hat{\omega}_J = r_A \hat{i}_A + (z_A - s) \hat{k} \quad (110)$$

$$\left. \begin{aligned} \hat{i}_A \cdot \hat{i} &= \cos \varphi_A \\ \hat{i}_A \cdot \hat{j} &= \sin \varphi_A \end{aligned} \right\} \quad (111)$$

$$\left. \begin{aligned} -\xi_{s_r}/\nu - \eta/\nu &= r_A \cos \varphi_A \\ -\omega &= r_A \sin \varphi_A \\ \xi/\nu - \eta_{s_r}/\nu &= z_A - s \end{aligned} \right\} \quad (112)$$

$$\left. \begin{aligned} r_A^2 &= [\xi_{s_r}/\nu + \eta/\nu]^2 + \omega^2 \\ \tan \varphi_A &= \frac{\omega}{\xi_{s_r}/\nu + \eta/\nu} \\ z_A &= s + \xi/\nu - \eta_{s_r}/\nu \end{aligned} \right\} \quad (113)$$

$$\left. \begin{aligned} r_{\eta A} &= (\xi_{s_r}/\nu + \eta/\nu)/r_A \nu \\ \varphi_{\eta A} &= -\sin \varphi_A \cos \varphi_A / \nu (\xi_{s_r}/\nu + \eta/\nu) \\ z_{\eta A} &= -s_r/\nu \end{aligned} \right\} \quad (114)$$

$$r_{\omega A} = \omega/r_A$$

$$\varphi_{\omega A} = \cos^2 \varphi_A / (\xi s_r / \nu + \eta / \nu) \quad (115)$$

$$z_{\omega A} = 0$$

$$r_* = \xi |s_r| / \nu$$

$$\varphi_* = \begin{cases} 0 & \text{if } s_r < 0 \\ \pi & \text{if } s_r > 0 \end{cases}$$

$$z_* = s + \xi / \nu$$

(116)

In the following Eqs. (117) and (118), s_r is computed in the $\varphi = 0$ half-plane if $\varphi_* = 0$ and is computed in the $\varphi = \pi$ half-plane if $\varphi_* = \pi$. It is, consequently, always negative.

$$r_{\eta_*} = \xi s_r / r_* \nu^2 = -1/\nu$$

$$\varphi_{\eta_*} = 0$$

$$z_{\eta_*} = -s_r / \nu$$

$$\left. \begin{array}{l} r_{\eta_*} = \xi s_r / r_* \nu^2 = -1/\nu \\ \varphi_{\eta_*} = 0 \\ z_{\eta_*} = -s_r / \nu \end{array} \right\} (s_r \neq 0) \quad (117)$$

$$\left. \begin{aligned} r_{w*} &= 0 \\ \phi_{w*} &= \nu/\xi s_r = -1/r_* \\ z_{w*} &= 0 \end{aligned} \right\} (s_r \neq 0) \quad (118)$$

The interpolation of values at A^* is performed in the $\phi = 0$ half-plane if $\phi_* = 0$, in the $\phi = \pi$ half-plane if $\phi_* = \pi$, and along the z -axis if $s_r = 0$. The derivatives with respect to X , Y , and ξ are computed accordingly, if $s_r \neq 0$. Then Eqs. (26) are used to obtain the derivatives with respect to r , ϕ , and z . Note that

$$\begin{aligned} \vec{V}_* &= \tilde{u}_* \hat{N} + \tilde{v}_* \hat{\tau} - \tilde{w}_* \hat{j} \\ &= u_* \hat{i}_* + v_* \hat{j}_* + w_* \hat{k} \end{aligned} \quad (119)$$

and since

$$\begin{aligned} \hat{i}_* &= \hat{i} \\ \hat{j}_* &= \hat{j} \end{aligned} \quad (120)$$

using Eqs. (105), (72), and (109), we obtain

$$\left. \begin{aligned} \tilde{u}_* &= -u_* s_r / \nu + w_* / \nu \\ \tilde{v}_* &= -u_* / \nu - w_* s_r / \nu \\ \tilde{w}_* &= 0 \end{aligned} \right\} \quad (121)$$

where again s_r is computed as for Eqs. (117) and (118).

The derivatives of $\tilde{u}_*$ and $\tilde{v}_*$ with respect to r , φ and z are obtained by formally differentiating Eq. (121). However, note that

$$\tilde{w}_{*\psi} = - (v_{*\varphi} + u_*) \quad (122)$$

as it can be obtained from particularizing the general expression of $\tilde{w}_{A\varphi}$.

In the even more special case in which $s_r = 0$, r_* is equal to zero and ν is equal to 1. Then $\hat{N} = \hat{k}$ and $\hat{\tau}$ is opposite to the $\hat{i}$ vector of the $\varphi = 0$ half-plane. It follows that

$$\left. \begin{aligned} r_{\eta_*} &= -1 \\ z_{\eta_*} &= 0 \end{aligned} \right\} (s_r = 0) \quad (123)$$

$$\left. \begin{aligned} r_{\omega_*} &= 0 \\ z_{\omega_*} &= 0 \end{aligned} \right\} (s_r = 0) \quad (124)$$

$$\left. \begin{aligned} \tilde{u}_* &= w_* \\ \tilde{v}_* &= -u_* \\ \tilde{w}_* &= 0 \end{aligned} \right\} (s_r = 0) \quad (125)$$

the value of u_* being the one in the $\varphi = 0$ half-plane. In this case, one can compute directly

$$\left. \begin{aligned} \tilde{u}_{*\eta} &= w_{*\eta} = -w_{*r} \\ \tilde{v}_{*\eta} &= -u_{*\eta} = u_{*r} \\ \tilde{w}_{*\eta} &= 0 \end{aligned} \right\} (126)$$

$$\left. \begin{aligned} \tilde{u}_{*\omega} &= w_{*\omega} = -\bar{w}_{*r} \\ \tilde{v}_{*\omega} &= -\bar{v}_{*r} \\ \tilde{w}_{*\omega} &= \bar{u}_{*r} \end{aligned} \right\} (127)$$

the terms with a bar meaning quantities measured in the $\varphi = \pi/2$ half-plane (see Fig. 5).

FIG. 5

VII. BODY POINTS NOT ON THE z-AXIS

From Eq. (4) it follows that the unit vector normal to the body, $\hat{N}$, oriented towards the body, is defined by

$$\hat{N} = (-b'\hat{i} + \hat{k})/\nu \quad (128)$$

where

$$\nu = \sqrt{1 + b'^2} \quad (129)$$

The velocity component normal to the body is zero at a body point, Q. The pertinent information from inside the shock layer is obtained along a characteristic line ending where Q is at $t = t_0 + \Delta t$ and contained in a plane defined by $\hat{N}$ and the time axis. Such a line comes from a point, A^* , located in the interior of the shock layer, along $\hat{N}$, at $t = t_0$. Once A^* is known, the initial conditions at A^* can be determined from the neighboring points by interpolation.

In order to obtain the location of A^* and the compatibility equations, consider a Cartesian frame, $\hat{N}, \hat{\tau}, \hat{b}$ where

$$\hat{\tau} = \hat{N} \times \hat{j} = (-\hat{i} - b'\hat{k})/\nu \quad (130)$$

$$\hat{b} = \hat{N} \times \hat{\tau} = -\hat{j} \quad (131)$$

Let ξ , η , ω be the coordinates and $\tilde{u}$, $\tilde{v}$, $\tilde{w}$ the velocity components in the $\hat{N}$, $\hat{\tau}$, $\hat{b}$ frame, in that order. The origin of the frame is at Q. The equations of motion are Eqs. (74)-(78). Eqs. (79)-(84) follow. In this case, the characteristic of the first family must be used. The compatibility equation between the initial point, A^* , and point Q reads

$$-\tilde{u}_* + aP/\gamma - aP_*/\gamma = \sigma \cdot \Delta t \quad (132)$$

$$\sigma = -(\tilde{v}\tilde{u}_\eta + \tilde{w}\tilde{u}_\omega) - a(\tilde{v}P_\eta/\gamma + \tilde{w}P_\omega/\gamma + \tilde{v}_\eta + \tilde{w}_\omega). \quad (133)$$

where $\tilde{u}_*$ and P_* are values at A_* , P is a value at Q (where $\tilde{u} = 0$), a is an average between the speed of sound at A_* and Q, and σ may be assumed as constant along the characteristic (equal to its value at A_*).

The coordinate, ξ of A_* is given by

$$\xi = -(\tilde{u} + a) \cdot \Delta t \quad (134)$$

Using the same symbol conventions as in Section IV, Eqs. (87) and (88) still hold with b instead of s . Then from (128), (130), and (131) it follows that

$$\begin{aligned}
- \xi b' / \nu - \eta / \nu &= r_A \cos (\varphi_A - \varphi) - r \\
- \omega &= r_A \sin (\varphi_A - \varphi) \\
\xi / \nu - \eta b' / \nu &= z_A - b
\end{aligned}
\quad \left. \vphantom{\begin{aligned} - \xi b' / \nu - \eta / \nu &= r_A \cos (\varphi_A - \varphi) - r \\ - \omega &= r_A \sin (\varphi_A - \varphi) \\ \xi / \nu - \eta b' / \nu &= z_A - b \end{aligned}} \right\} (135)$$

Consequently,

$$\begin{aligned}
r_A^2 &= (r - \xi b' / \nu - \eta / \nu)^2 + \omega^2 \\
\tan (\varphi_A - \varphi) &= \frac{\omega}{\xi b' / \nu + \eta / \nu - r} \\
z_A &= b + \xi / \nu - \eta b' / \nu
\end{aligned}
\quad \left. \vphantom{\begin{aligned} r_A^2 &= (r - \xi b' / \nu - \eta / \nu)^2 + \omega^2 \\ \tan (\varphi_A - \varphi) &= \frac{\omega}{\xi b' / \nu + \eta / \nu - r} \\ z_A &= b + \xi / \nu - \eta b' / \nu \end{aligned}} \right\} (136)$$

$$\begin{aligned}
r_{\eta A} &= - (r - \xi b' / \nu - \eta / \nu) / r_A \nu \\
\varphi_{\eta A} &= \sin (\varphi_A - \varphi) \cos (\varphi_A - \varphi) / \nu (r - \xi b' / \nu - \eta / \nu) \\
z_{\eta A} &= - b' / \nu
\end{aligned}
\quad \left. \vphantom{\begin{aligned} r_{\eta A} &= - (r - \xi b' / \nu - \eta / \nu) / r_A \nu \\ \varphi_{\eta A} &= \sin (\varphi_A - \varphi) \cos (\varphi_A - \varphi) / \nu (r - \xi b' / \nu - \eta / \nu) \\ z_{\eta A} &= - b' / \nu \end{aligned}} \right\} (137)$$

$$\begin{aligned}
r_{\omega A} &= \omega / r_A \\
\varphi_{\omega A} &= - \cos^2 (\varphi_A - \varphi) / (r - \xi b' / \nu - \eta / \nu) \\
z_{\omega A} &= 0
\end{aligned}
\quad \left. \vphantom{\begin{aligned} r_{\omega A} &= \omega / r_A \\ \varphi_{\omega A} &= - \cos^2 (\varphi_A - \varphi) / (r - \xi b' / \nu - \eta / \nu) \\ z_{\omega A} &= 0 \end{aligned}} \right\} (138)$$

At point A*, located on the ξ -axis:

$$\begin{aligned}
r_* &= r - \xi b' / \nu \\
\varphi_* &= \varphi \\
z_* &= b + \xi / \nu
\end{aligned}
\quad \left. \vphantom{\begin{aligned} r_* &= r - \xi b' / \nu \\ \varphi_* &= \varphi \\ z_* &= b + \xi / \nu \end{aligned}} \right\} (139)$$

$$\left. \begin{aligned} r_{\eta*} &= -1/\nu \\ \varphi_{\eta*} &= 0 \\ z_{\eta*} &= -b'/\nu \end{aligned} \right\} (140)$$

$$\left. \begin{aligned} r_{\omega*} &= 0 \\ \varphi_{\omega*} &= -1/r_* \\ z_{\omega*} &= 0 \end{aligned} \right\} (141)$$

The X, Y, ξ coordinates of point A^* are found as explained in Section IV. Note that

$$0 < \xi_* < \Delta \xi$$

Since $X_* = X$, $b' > 0$, and $\xi < 0$, it follows that

$$Y < Y_* < Y + \Delta Y$$

To compute the velocity components and their derivatives in the (ξ, η, ω) frame, note that

$$\begin{aligned} \vec{V}_* &= \tilde{u}_* \hat{N} + \tilde{v}_* \hat{\tau} - \tilde{w}_* \hat{j} \\ &= u_* \hat{i} + v_* \hat{j} + w_* \hat{k} \end{aligned} \quad (142)$$

Then

$$\left. \begin{aligned} \tilde{u}_* &= -u_* b' / \nu + w_* / \nu \\ \tilde{v}_* &= -u_* / \nu - w_* b' / \nu \\ \tilde{w}_* &= -v_* \end{aligned} \right\} (143)$$

The derivatives of $\tilde{u}_*$, $\tilde{v}_*$, and $\tilde{w}_*$ with respect to r and z can be obtained by formal differentiation of Eqs. (143).

However,

$$\left. \begin{aligned} \tilde{u}_{*\varphi} &= - (u_{*\varphi} - v_*) b' / \nu + w_{*\varphi} / \nu \\ \tilde{v}_{*\varphi} &= - (u_{*\varphi} - v_*) / \nu - w_{*\varphi} b' / \nu \end{aligned} \right\} (144)$$

The pressure at Q is determined by Eqs. (132) and (133).

To evaluate the entropy, the streamline passing through Q at $t = t_0 + \Delta t$ and its initial point, $\bar{A}$, at $t = t_0$ are determined. Since

$$\bar{A} = Q - \vec{V} \Delta t \quad (145)$$

where $\vec{V}$ is the velocity at Q and at $t = t_0 + \Delta t$ (approximately), it follows that

$$\begin{aligned}
\bar{r} \cos (\bar{\varphi} - \varphi) &= -u \Delta t + r \\
\bar{r} \sin (\bar{\varphi} - \varphi) &= -v \Delta t \\
\bar{z} &= -w \Delta t + z
\end{aligned} \tag{146}$$

where values at $\bar{A}$ are marked by a bar.

From (146):

$$\begin{aligned}
\bar{r} &= \sqrt{(r - u \Delta t)^2 + v^2 \Delta t^2} \\
\bar{\varphi} &= \varphi + \arctan \frac{v \Delta t}{u \Delta t - r}
\end{aligned} \tag{147}$$

The value of $\bar{z}$ is better obtained as

$$\bar{z} = b(\bar{r}) \tag{148}$$

The coordinate, $\bar{\xi}$, of $\bar{A}$ is always zero. As far as $\bar{X}$ and $\bar{Y}$ are concerned, there are four possibilities, according to $\bar{r}$ and $\bar{\varphi}$ being less or greater than r and φ respectively (Fig. 6).

The interpolation to get the value of S at $\bar{A}$ must be carried on accordingly.

Then, from (20), it follows that S at Q and at $t = t_0 + \Delta t$ equals $\bar{S}$, and R at Q and at $t = t_0 + \Delta t$ is obtained from (11).

FIG. 6

In order to determine the velocity at Q , it is convenient to use Eqs. (76) and (77). Since $\tilde{u} = 0$ on the body, we obtain

$$\tilde{v}_t = - [\tilde{v}\tilde{v}_\eta + \tilde{w}\tilde{v}_\omega + (p/\rho)P_\eta] \quad (149)$$

$$\tilde{w}_t = - [\tilde{v}\tilde{w}_\eta + \tilde{w}\tilde{w}_\omega + (p/\rho)P_\omega] \quad (150)$$

Then the counterparts of Eqs. (143) on the body are used:

$$\begin{aligned} 0 &= -ub' + w \\ \tilde{v} &= -u - wb' \\ \tilde{w} &= -v \end{aligned} \quad (151)$$

to obtain u , v , and w .

VIII. BODY POINT ON THE z-AXIS

For the body point on the z-axis, $b' = 0$ and

$$\hat{N} = \hat{k} \quad (152)$$

$$\nu = 1 \quad (153)$$

$$\hat{\tau} = -\hat{i} \quad (154)$$

$$\hat{b} = -\hat{j} \quad (155)$$

where $\hat{i}$ and $\hat{j}$ refer to the $\varphi = 0$ half-plane. Then

$$\left. \begin{aligned} r_* &= 0 \\ \varphi_* &= 0 \\ z_* &= b + \xi \end{aligned} \right\} \quad (156)$$

where ξ is still given by (134). The interpolations to find the values of u , v , w , P , and R at A^* are to be performed with respect to ξ only. Note that

$$\left. \begin{aligned} \tilde{u}_* &= w_* \\ \tilde{v}_* &= -u_* \\ \tilde{w}_* &= 0 \end{aligned} \right\} \quad (157)$$

Figure 5 is still valid for this case, and so are the derivatives, given by (126) and (127).

Since the velocity at Q can only have one component, u , two cases must be considered, according to u being greater

or less than zero. If u is less than zero, point $\bar{A}$ is above Q in the $\varphi = 0$ half-plane,

$$\left. \begin{aligned} \bar{r} &= -u \Delta t \\ \bar{\varphi} &= 0 \\ \bar{z} &= b(\bar{r}) \end{aligned} \right\} \quad (158)$$

If u is greater than zero, point $\bar{A}$ is above Q in the $\varphi = \pi$ half-plane,

$$\left. \begin{aligned} \bar{r} &= u \Delta t \\ \bar{\varphi} &= \pi \\ \bar{z} &= b(\bar{r}) \end{aligned} \right\} \quad (159)$$

The interpolations are performed accordingly. The values of $\tilde{v}_\omega$ and $\tilde{w}_\omega$ to be used in Eqs. (149) and (150) are computed, according to Fig. 7, as

$$\tilde{v}_\omega = - (v_r)_{\varphi=\pi/2} \quad (160)$$

$$\tilde{w}_\omega = - (u_r)_{\varphi=\pi/2} \quad (161)$$

IX. DETERMINATION OF THE STEP SIZE

By experimenting, it has been found that the computation is stable if the step size keeps the point to be computed within the Mach cone issuing from the four neighboring points. Therefore, the step size, Δt , is taken as the minimum of the values

$$\Delta t = \frac{\Delta s}{1.5(q + a)} \quad (162)$$

computed at each mesh point (including the boundary points), where

$$\Delta s = \min (\Delta r, \Delta z, r \Delta \phi)$$

$$q = \sqrt{u^2 + v^2 + w^2}$$

The step size formula is actually coded with a factor (STAB) affecting the right-hand side of (162). Usually, such a factor is taken equal to unity. Other factors can be used to test the stability of the computation.

X. OPERATING INSTRUCTIONS

The program is written in FROTRAN IV. It consists of a main program and 6 subroutines, as follows:

<u>NAME</u>	<u>TASK</u>
2H (MAIN PROGRAM)	Input - Definition of initial conditions. Main loop for the computation of each step.
LAURA	Computation of shock points.
FLORA	Computation of inner points.
EVA	Computation of body points.
INTER	Interpolation for A [*] point in shock subroutine.
OUTPUT	Output.
REST	A subroutine called at the end of this problem. Presently, it is a dummy subroutine. Eventually, it can be used for calling further extensions (for example, a supersonic shock layer computation).

The dimensions of the symbols are such that one can use, at most,

6 points from the shock to the body (z-direction)

11 points from the centerline to the outer cylinder

(r-direction)

9 meridional half-planes (ϕ -direction)

Presently, the program and the common storage fill all the capacity of the IBM 7094 machine. If a bigger machine is used,

and more points are to be computed, the dimensions may be changed as follows:

Change all 7 into number of points from the shock and to the body + 1

Change all 12 into number of points from the centerline to the outer cylinder + 1

Change all 11 into number of meridional half-planes + 2

The number of meridional half-planes (including the $\varphi = 0$ and the $\varphi = \pi$ half-planes) must be odd.

The only 7 which is not to be changed is the one for the symbol EPS.

A. Input Instructions

The input data are punched on 3 main cards, followed by as many cards as necessary to define the body geometry (presently, 4 cards are needed; their number may increase if the 12 above is increased to more than 14).

Card #1 (Format 1415)

NRUN = run number

MONTH	}	date
MDAY		
MYEAR		

NA number of intervals between shock
 and body

 MA number of intervals between center-
 line and outer cylinder

 LA number of intervals between meridional
 half-planes

 KA total number of time steps

 JA number of steps between outputs

 LD an initial guess to determine the
 location of the sonic surface on the
 body (as a rule, let $LD = MA/2$)

 LB LB = 1 let some data be output at each
 step (step number, time, minimum
 standoff distance, maximum and minimum
 shock velocity, range of shock velocities,
 maximum pressure on the body)

 LB = 0 bypasses this output

Ignore symbols KC, KD, KE

Card #2 (Format 7E10.4)

EPS(1) tolerance for iteration in shock
 computation (usually equal to $1.0E-4$)

 EPS(2) tolerance for iteration in body com-
 putation (usually equal to $1.0E-4$)

Card #3 (Format 7E10.4)

ACH free stream Mach number

 GAMMA ratio of specific heats

STAB a stability factor (see Section IX; the factor is usually taken equal to 1.0)

DY mesh size in the r-direction. The r-coordinate of the limiting cylinder (called r_1 in Section II) equals $DY \cdot MA$

ALPHA angle of attack

SHAPE a constant (SHAPE) used to determine the initial shock shape.

$$(Y_{SHOCK})_M = (Y_{SHOCK})_{z \text{ axis}} + (SHAPE) (.047) M_\infty Y_M^2.$$
 Usually, $SHAPE=1$ but for very blunt bodies $SHAPE < 1$ to insure the shock and body not intersecting

Cards for body geometry (Format 7E10.4)

First card: $b(\Delta Y)$ (b is defined by (4))
 $b(0)$
 $b(\Delta Y)$
 $b(2\Delta Y)$
 $b(3\Delta Y)$
 $b(4\Delta Y)$
 $b(5\Delta Y)$

[Next cards similarly, until $b(MA \cdot \Delta Y)$ is reached.]

Next card: $-b'(\Delta Y)$ ($b' = db/dr$)
 $b'(0) = 0.0$
 $b'(\Delta Y)$
 $b'(2\Delta Y)$
 $b'(3\Delta Y)$
 $b'(4\Delta Y)$
 $b'(5\Delta Y)$

[Next cards similarly, until $b'(MA \cdot \Delta Y)$ is reached.]

B. Output

The first page repeats the input data. The theoretical stagnation pressure is also output.

Each output consists of a heading containing the step number, the latest value of Δt at the location of the point where such Δt is the minimum, and the time value at that step.

The meridional planes are numbered from 2 to LA+2, 2 being the $\phi = 0$ plane, LA+2 the $\phi = \pi$ plane, and the other being equally spaced in ϕ .

At each meridional plane, the shock velocity (s_t) and the values of s_r and s_ϕ (called SHR and SHP respectively) are output. Note, in general, that the numbering of the $r = \text{constant}$ lines is such that 2 means centerline ($r = 0$), 3 means $r = \Delta r$, 4 means $r = 2\Delta r$ etc.

Then each line between body and shock is output. Here again, line 2 means the body, and the last line means the shock. The intermediate lines are orderly arranged from the body to the shock. The geometry of such lines is given by the column of values labeled Z. The other columns are:

P	values of p/p_∞
RHO	values of ρ/ρ_∞
U	values of $u/\sqrt{p_\infty/\rho_\infty}$

V	values of $v/\sqrt{p_\infty/\rho_\infty}$
W	values of $w/\sqrt{p_\infty/\rho_\infty}$
S	values of S [defined by (11)]
M	Mach numbers
DH/HST	values of $\left[H - \frac{\gamma}{\gamma-1} \frac{p}{\rho} - \frac{1}{2} (u^2 + v^2 + w^2) \right] / H$ where H is the total enthalpy of the free stream.

C. Checks

1. In a true steady state, the stagnation pressure must equal the theoretical value given in the first page of outputs. Make a plot of pressure distribution on the body at plane 2 and plane LA+2 and compare the maximum pressure from the plot with the theoretical value.

2. The shock velocities should decrease to values of the order of 10^{-3} times the free stream velocity.

3. The values in the last column should decrease to at least 10^{-2} (this means that the total enthalpy (from the computed "steady state" differs from the exact value by less than 1%).

4. The Mach numbers are computed using p , ρ , and the velocity components. At a steady state on the body they should match the values given by

$$M = \left\{ \frac{2}{\gamma-1} \left[\left(\frac{p_t}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \quad (163)$$

Since the pressure computation is generally the most accurate, any departure of the computed M from the values of (163) gives a measure of the accuracy of the velocity components on the body.

XI. Results and Discussion

The following cases were run:

- 1) Sphere-cone at $\alpha=0^\circ$, $M_\infty=4$
- 2) Sphere-cone at $\alpha=10^\circ$, $M_\infty=4$
- 3) Ellipsoid ($a/b=1.5$) at $\alpha=20^\circ$, $M_\infty=8$
- 4) Ellipsoid ($a/b=1.5$) at $\alpha=30^\circ$, $M_\infty=8$
- 5) Ellipsoid ($a/b=2$) at $\alpha=20^\circ$, $M_\infty=8$.

The first two cases obviously were run to check the program.

In the first case, only 3 meridional planes were considered. Due to the absence of incidence, the three of them gave the same output. The output turned out to be the same as from another existing program for axisymmetric flow.

In the second case, the incidence has no effect on the flow field around the sphere since the sonic surface does not reach the cone and the sphere is axisymmetric about any axis through its center. However, the computation is performed with full usage of three-dimensional equations. The results are, as expected, different from one meridional plane to another but when transformed to a cylindrical frame with the axis along the V_∞ vector they became the same as in the previous case.

The other three cases are fully developed three-dimensional flows. The pertinent results are summarized in Figs. 8 to 24.

Fig. 8 shows how the standoff distance and the stagnation pressure develop in time for cases 3 and 4. The agreement between the computed stagnation pressure and its theoretical value is very good.

Figs. 9 and 17 show the development of the shock wave from an assumed axisymmetric shape to the final shape.

Figs. 10 and 18 show the Mach number distribution on the shock at its intersection with the symmetry plane. Note that the theoretical value for normal shock is consistent with the values of M at the computed points.

Figs. 11 and 19 show a similar distribution on the body. The stagnation point is thus easily detected.

Figs. 12 and 20 show the final section of the shock with the symmetry plane, the location of the stagnation point and the sonic lines.

Figs. 13 and 21 give the whole distribution of Mach numbers on the body.

Figs. 14 and 22 give a similar distribution of pressures. Note the perfect matching of the computed stagnation pressure with the theoretical value.

Figs. 15 and 23 show lines of constant Mach number on the body, as seen from the front.

Figs. 16 and 24 show the steady state shock shapes as functions of ϕ .

The last case is more difficult than the others because of the higher bluntness of the body.

Three runs have been made, starting with different initial shock shapes. All of them settle down to a configuration which has wiggles in the leeward side of the shock. It is interesting to see that the results are the same in all three cases. Evidently, there are difficulties in handling the leeward side of the shock layer which are related to the nature of the expanding flow and are emphasized when the bluntness of the body is too large.

For practical purposes, the shock shape can be smoothed down from the computed results (see Fig. 25) and the pertinent values of the physical parameters can be recomputed.

However, since the problem asks for some further investigation on the nature of the equations and the results may clarify an entire category of numerical techniques, we shall dedicate a special report to it. Any practical suggestion resulting from such an analysis will be incorporated into the present program as soon as possible.

Just prior to issuing this report, a Russian paper⁵ was brought to our attention. In this paper an approximate method for computing three-dimensional blunt body flows is presented. We made a

further computation of the ellipsoid ($a/b=1.5$) case at $\alpha=15^\circ$ and $M_\infty=3$, to compare with a figure from Ref. 5. The results are shown in Fig. 26. The agreement is excellent. Note that at $M_\infty=3$, the different shape of the body affects the sonic line in the windward side.

REFERENCES

1. R. J. Swigart, "A Theory of Asymmetric Hypersonic Blunt-Body Flows," AIAA J. 1, 1034-1042 (1963).
2. I. O. Bohachevsky and R. E. Mates, "A Direct Method for Calculation of the Flow About an Axisymmetric Blunt Body at Angle of Attack," AIAA J. 4, 776-782 (1966).
3. G. Moretti and M. Abbett, "A Fast, Direct, and Accurate Technique for the Blunt Body Problem," GASL TR-583, Westbury, N.Y. (January 31, 1966), AD 478119.
4. G. Moretti and M. Abbett, "A Time Dependent Computational Method for Blunt Body Flows," AIAA J. 4, 2136-2141, (1966).
5. G. F. Telenin and G. P. Tinyakov, "Method of Computing Three-Dimensional Flows Past Bodies After Shockwave Passage," Dokl. A. N. SSSR, Gidromekhan. 154, 1056-1058 (1964) (NASA TT F-8682).

FIG. 8 STAGNATION PRESSURE AND STANDOFF DISTANCE

FIG. 9 SHOCK SHAPE IN SYMMETRY PLANE AS A FUNCTION OF TIME

FIG. 10 MACH NUMBER DISTRIBUTION
ON INTERSECTION OF SHOCK
AND SYMMETRY PLANE

FIG. 11 MACH NUMBER DISTRIBUTION ON
INTERSECTION OF BODY AND
SYMMETRY PLANE

FIG. 12 SHOCK, SONIC LINES AND STAGNATION POINT IN SYMMETRY PLANE

FIG. 13 MACH NUMBER DISTRIBUTION ON THE BODY

FIG. 14 BODY PRESSURE DISTRIBUTION

FIG. 15 M -CONSTANT LINES ON THE BODY - FRONT VIEW

FIG. 16 SHOCK SHAPE AS A FUNCTION OF ϕ

FIG. 17 SHOCK SHAPE IN SYMMETRY PLANE AS A FUNCTION OF TIME

FIG.18 MACH NUMBER DISTRIBUTION ON INTERSECTION OF SHOCK AND SYMMETRY PLANE

FIG.19 MACH NUMBER DISTRIBUTION ON INTERSECTION OF BODY AND SYMMETRY PLANE

FIG. 20 SHOCK, SONIC LINES AND STAGNATION POINT IN SYMMETRY PLANE

FIG. 21 MACH NUMBER DISTRIBUTION ON THE BODY

FIG.22 BODY PRESSURE DISTRIBUTION

FIG. 23 M -CONSTANT LINES ON THE BODY-FRONT VIEW

FIG. 24 SHOCK SHAPE AS A FUNCTION OF ϕ

FIG. 25 ELLIPSOID ($a/b=2$) AT $M_\infty=8$, SHOCK SHAPE

FIG. 26 COMPARISON OF RESULTS BETWEEN PRESENT TECHNIQUE AND REF. 5

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