UNCLASSIFIED

AECU-3607(Vol. I)

REACTORS—GENERAL

UNITED STATES ATOMIC ENERGY COMMISSION

**FINAL PHYSICS REPORT FOR THE
ENGINEERING TEST REACTOR**

By
I. B. Wolfe

June 25, 1956

Atomic Power Equipment Department
General Electric Company
Schenectady, New York

Technical Information Service Extension, Oak Ridge, Tenn.

UNCLASSIFIED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

This is Volume I of two volumes, pgs. 1-284.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.

This report has been reproduced directly from the best available copy.

Printed in USA. This report consists of 2 volumes, total price \$7.00. Available from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

AECU-3607

Volume I

FINAL PHYSICS REPORT

for the

ENGINEERING TEST REACTOR

June 25, 1956

AUTHORS

Volume I B. Wolfe

**Volume II W. Cegelski
 W. Machell**

Prepared by

**Atomic Power Equipment Department
General Electric Company**

for

**Kaiser Engineers
Division of Henry J. Kaiser Company**

for

**Idaho Operations Office
U. S. Atomic Energy Commission
Idaho Falls, Idaho**

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

CORRECTIONS
TO THE
FINAL PHYSICS REPORT
FOR THE
ENGINEERING TEST REACTOR

Corrections to Volume I.

Summary - The major correction of a non-typographical nature is found on Page A7-4. The macroscopic activation cross sections of oxygen in water are all a factor of two too high. Because of this error, all N^{16} activities quoted in the physics report are too high by a factor of two.

ERRATA SHEET
FOR THE
ETR FINAL PHYSICS REPORT

Page 1-5

Fluxes quoted as time averages over cycle are actually fluxes at the end of the cycle.

Page 1-7 Initial loading with no burnable poison.

Case II - 9.7 Kg instead of 8.7 Kg

Page 3-2 Next to last line: Section II D not Section II C

Page 3-4 Line 8 - Asymmetry not symmetry

Page 4-2 Line 2 - Figure 4-4 not Figure 4-2

Figure 4.4 Solid line should intersect last boundary between Pb, Fe and concrete at a value of 1.8×10^{-3} watts/cc.

Figure 4.4 is the midplane heating.

Page 5-5 Line 6 Figure 2-4 not Figure 2-7

Page 6-8 Equation (2) $\mathcal{S}/\nu$ should be $-\mathcal{S}/\nu$

Page 6-9 Footnote $\mathcal{S}/\nu$ should be $-\mathcal{S}/\nu$

Page 7-1 Line 10 Appendix IV not Appendix III

Page 7-2 Equation 5 $\mathcal{S}\Sigma_{25}$ should be $-\mathcal{S}\Sigma_{25}$

Equation 6 $\mathcal{S}\Sigma_f$ should be $-\mathcal{S}\Sigma_f$

Equation 7 $\mathcal{S}\nu$ should be $-\mathcal{S}\nu$

Page 7-3 Equation 9 $\mathcal{S}\nu$ should be $-\mathcal{S}\nu$

Page 7-4 Equation 10 $\mathcal{S}\nu$ should be $-\mathcal{S}\nu$

Figure 7-6 Ordinate should be $-\mathcal{S}\nu/\nu$ instead of $\mathcal{S}\nu$

Page 7-8 Equation 15 $\mathcal{S}\nu$ should be $-\mathcal{S}\nu$

Equation 16 ϕ_0 should be ϕ_2

Figure 7-8 Ordinate should be $-\mathcal{S}\nu/\nu$ instead of $\mathcal{S}\nu/\nu$

Page 7-9 Equation (19) .029 should be -.029

Page 7-11 Equation (22) $\mathcal{S}\sigma$ should be $\mathcal{S}\nu$

ERRATA SHEET (Continued)

Page 7-13 Sixth line from bottom. Figure 7-13 should be Figure 7-12

Page 7-14 Second paragraph, first sentence should read:

One can also compute the running cycle when there is no burnable poison present.

(Figure 7-16 does not show running cycle)

Page 7-15 Equation (23) $\int \nu$ should be $-\int \nu$

Page 7-16 Line 5 246 gms should be 239 gms
2.80 gms should be 2.62 gms

Line 6 19.2 Kg should be 18.6 Kg

Second line from bottom - Three days should be four days

Page 8-4 Second sentence - Appendix IV should be Appendix III

Third equation from bottom - denominator should be multiplied by Σ_{25}

Second equation from bottom - Integrand should be multiplied by $C(t)$

Page 8-5 Line 8 .032t should be .029t, t in days

Line 9 "per second" should be omitted.

Equation (4) should be $\int k/k = -0.14T$, T in hours

Page 9-6 Third line from top - "less" should be "more"

Page 10-4 Ninth line from top - 6.7×10^{-6} should be -6.7×10^{-6}

Page 10-6,7 The parameters listed in the table, starting at the bottom of 10-6 and continuing at the top of 10-7 are for the initial start-up core instead of the core with experiments. The corrections as indicated in Table II, page 1-4, are:

-3.7×10^{-4} should be -3.1×10^{-4}
 -2.5×10^{-3} should be -3.2×10^{-3}
 -4.7×10^{-9} should be -2.7×10^{-9}
 -4.3×10^{-8} should be -2.6×10^{-8}
 -3.8×10^{-4} should be -2.0×10^{-4}

Figure 13-1 .016 Cadmium sheet should be .040
1/4" Aluminum outside wall should be 1/16"

Page 13-2 Third, fourth and fifth equations - " $k-1$ " should be $k(k-1)$

Page 13-4 Third equation from bottom should be multiplication sign after $\frac{0.869}{0.132}$

Page A1-9 Second Table Title "2 Mev Gamma Flux" should be "1 Mev Gamma Flux"

ERRATA SHEET (Continued)

Page A1-10 Column 7 1.60×10^{-4} should be 1.16×10^{-3}
 2.50×10^{-5} should be 1.8×10^{-4}

Page A1-11 Column 7 2.96×10^{-4} should be 1.3×10^{-3}

Figures A4-2, A4-3, A4-4

Days of operation should be relabeled to agree with Figure A4-1; i.e.,
5 days should be 4.25 days.

Page A6-7 The first sentence is not true in general. The boundary condition on a region is given in terms of D_1 . Thus, the reactivity is insensitive to a change in D_1 if the change in the boundary condition does not produce a large effect. This will be the case when the leakage from the reactor is not large.

Page A7-4 Second half of page - For $E = 11 \text{ Mev}$ σ activation should be .005 instead of .050.

All $\sum_a$ should be divided by 2. This error will cause all the N^{16} activities to be a factor of 2 high.

ACKNOWLEDGEMENTS

The study of the physics of the Engineering Test Reactor was a task to which a number of people, in addition to the authors, contributed. Preliminary physics analysis of the ETR was carried out by Dr. John Weil together with Mr. R. Meyerand and Mr. D. Steward. After this preliminary stage, the work was taken over by the authors of this report who had a considerable amount of aid from a large number of physicists in the General Electric Physics Training Program. Those physicists on the General Electric program who made significant contributions to this study are listed below:

D. Gavin
M. Chimera
R. Chrenko
S. Glickstein
C. Griffin
P. Hess
L. North
L. Orr
A. Schweber
V. Stratton
J. Thorson
A. Watson

In the latter phases of the shielding analysis Dr. R. Haling and Mr. M. Weiss aided in evaluating some of the techniques used and in checking numerical results.

Mr. J. Arterburn was in charge of the detailed planning of the experimental program of the RMF tests and, in addition, aided in almost all the other aspects of the physics analysis. Throughout the analysis, Dr. J. Weil acted as a consultant.

In addition to the General Electric Atomic Power Equipment Department personnel enumerated above, this work has profited greatly from the aid of people in other organizations. Dr. R. Stark of KAPL and Dr. B. Cuthill of the David Taylor Model Basin were kind enough to supply computer codes and considerable information on the use of them.

Personnel of the Phillips Petroleum Company were very generous with ideas and suggestions helpful to this analysis. In particular, Dr. D. deBoisblanc was most helpful in presenting ideas on the control problem in the ETR.

Dr. E. Fast of the Phillips Petroleum Company was in charge of the experimental RMF tests which aided greatly in evaluating the analytical techniques used in the physics analysis. Other members of the Phillips Petroleum Company were called on frequently to relate their experiences with the MTR.

It is a pleasure to acknowledge the help provided by Mr. W. Dodson of Kaiser Engineers whose frequent suggestions and valuable comments were of considerable aid throughout the course of this work.

The analysis has also profited greatly from the help and understanding of Mrs. L. Wolfe.

B. Wolfe

FOREWORD

This report is a summary of the physics design work performed on the Engineering Test Reactor. The ETR presents computational difficulties not found in other reactors because of the large number of experimental holes in the core. The physics of the ETR depends strongly upon the contents of the in-core experimental facilities. In order to properly evaluate the reactor, taking into account the experiments in the core, multi-region, two-dimensional calculations are required. These calculations require the use of a large computer such as the Remington Rand Univac and are complex and expensive enough to warrant a five-stage program:

1. In the early stages of design, only preliminary two-dimensional calculations were performed in order to obtain a rough idea of the general behavior of the reactor and its critical mass with tentative experiments in place.
2. A large amount of work was carried out in which the reactor was approximated as one with a uniform homogeneous core. With this model, detailed studies were carried out to investigate the feasibility and to obtain general design data on such points as the design and properties of the gray and black control rods, the design of the beryllium reflector, gamma and neutron heating, the use of burnable poisons, etc. In performing these calculations, use was made of the IBM 650 PROD code obtained from KAPL.
3. With stages 1 and 2 carried out, two-dimensional calculations of the core at start-up conditions were performed on the Univac computer.
4. Detailed two-dimensional calculations of the properties of the ETR with a proposed first set of experiments in place were carried out.
5. A series of nuclear tests were performed at the reactivity measurements facility at the MTR site in order to confirm the validity of the analytical techniques in physics analysis.

In performing the two-dimensional Univac calculations, the MUG code developed by KAPL and the Cuthill code developed at the David Taylor Model Basin were utilized.

In order to reduce as much as possible the cost of the physics analysis and in order to obtain information otherwise unavailable, considerable use was

FOREWORD (cont)

made of perturbation techniques. These perturbation computations were initially performed using the one-dimensional approximation and were extended to two dimensions in stages 3 and 4. To do this a method of obtaining adjoint fluxes, using available reactor computer codes, was developed.

In order to reduce the size of this report, only the physics which bears on the final design of the ETR is summarized. The preliminary studies are reviewed in the Physics Progress Report of December 1, 1955. This volume, together with the Physics Progress Report, represents a complete account of the studies undertaken, methods used, and results obtained in the physics work on the ETR.

B. Wolfe
Physicist in charge of ETR Physics

J. Barnard
Project Manager, ETR Project

TABLE OF CONTENTS

PART I - ETR FINAL PHYSICS REPORT

<u>Section</u>		<u>Page</u>	
I	General Information	1-1	1
II	Flux in the ETR	2-1	9
III	Gamma Heating in the Core	3-1	30
IV	Gamma Heating External to the Core	4-1	39
V	Study of the ETR Reflector	5-1	46
VI	The Control System	6-1	66
VII	Burnup in the ETR	7-1	95
VIII	The Xenon Problem in the ETR	8-1	129
IX	Startup of the ETR and Withdrawal Speed of Control Rods .	9-1	138
X	Temperature and Void Coefficients in the ETR	10-1	149
XI	Neutron Lifetime	11-1	157
XII	The Fuel Element Transfer Cask	12-1	160
XIII	The Fuel Element Storage Rack	13-1	164
XIV	Startup Source Strength in the ETR	14-1	170
XV	Experimental Verification of Calculational Techniques . .	15-1	179
XVI	APPENDICES	16-1	202

I. GENERAL INFORMATION

A. Function of the ETR

The ETR is a high thermal flux reactor which uses fully enriched uranium as fuel. The function of the reactor is to provide neutron flux to a number of in-pile experimental facilities. A list of the key design parameters for the ETR follows.

B. Design Parameters for the Engineering Test Reactor

TABLE I

1. Power generation (heat) exclusive of experimental facilities	175 MW
2. Approximate thermal flux	Of the order of 3 to 5×10^{14} nv; depending on experimental loading
3. Maximum heat flux	1,180,000 BTU/hr/sq ft
4. Average heat flux	460,000 BTU/hr/sq ft
5. Coolant	light water
6. Coolant flow	(approx) 44,400 gpm
7. Moderator	light water
8. Fuel	Al clad U^{235} with burnable boron poison
9. Control	2 regulating rods 12 grey rods 4 black rods
10. Reflector	4 1/2 inches Be followed by Al
11. Major experimental facilities (each completely surrounded by fuel).	
one 9-by-9 inch through facility	
one 6-by-9 inch through facility	

TABLE I (CONT'D)

11. (Cont'd)

three 6-by-6 inch through facilities

four 3-by-3 inch facilities

In addition, experimental holes are provided in the reflector.

The layout of the core is largely determined by the space requirements of the experimental facilities. Although the specific arrangement of the facilities can be varied, the over-all core size is determined by the requirement that these facilities be surrounded on all sides by fuel. A number of core configurations were studied to determine the optimum arrangement. In order to provide adequate space for control rods and to allow for sufficient strength in the core support grid, the asymmetrical core cross section shown in Figure 1-1 was chosen.

The fuel elements for this core were specified at the start as 19-plate aluminum-uranium assemblies. This choice was made in order to use as much as possible of the existing MTR fuel element technology. Since the ETR is expected to go into operation during March 1957, it was deemed inadvisable to depend on any fuel element type which required appreciable development.

However, because of experimental demands, the ETR fuel elements are 36 inches in active length as compared to the 24-inch active length of the MTR fuel elements. The core configuration shown in Figure 1-1 contains 65 fuel element positions, some of which are occupied by the control rods as indicated.

C. Power Level of the ETR

Reference to Figure 1-1 shows that the core arrangement in the ETR consists basically of a 10 by 10 rectangular grid. Of the 96 unit spaces available, aside from the four corner reflector positions, 49 are occupied by fuel elements, 16 by control-rod fuel-element combinations, and 31 by experimental facilities. It is therefore meaningless to speak of the critical fuel loading of the 65 fuel elements or the flux attained at a given power level, unless the contents of the 31 percent of the core containing experimental facilities are specified. This point was realized early in the conceptual design of the ETR. At that time it was decided to set the power level of the ETR at a figure determined by heat transfer considerations. The figure arrived at in this way was a power level of 175 MW of heat generation in the portion of the core excluding the experimental facilities. The actual flux in a hole depends, of course, on the contents of the hole. The flux is relatively high when the hole contains fuel and a good moderator, and in the extreme case, is very low when a poison such as boron is present. In addition, the flux in one experimental hole depends on the loading in the other holes. Thus, the presence of highly absorbing material in one hole raises the critical mass of the reactor and at a given power level reduces the average flux in the reactor. The logic of setting the power level of the reactor on the basis of heat transfer considerations is evident, since under the most adverse circumstances the maximum flux attainable within present technological limitations will be provided.

D. Summary of the Important Physics Data for the ETR

The basic core mesh of the ETR consists of a square array of 100 3 by 3-inch units. Reference to Figure 1-1 shows that 31 of these units are occupied by experimental facilities. The properties of the core will of course be strongly dependent on the loading of these experimental holes. Two experimental loadings have been investigated in considerable detail. These are:

1. The first startup core in which all experimental holes contain aluminum filler pieces, and
2. The core with experiments as proposed on January 30, 1956, in the experimental holes. The data on the experiments was obtained from the users of the experimental facilities.

A summary of the important physics data for the two cases is outlined in the following. Other core loadings are also discussed in the body of the report.

E. Physics Parameters of ETR

TABLE II

	Case I (Initial startup) Al in experimental facilities	Case II Experiments in facilities
Uranium (235) in Fuel Section (including control rods)		
Startup	14.1 kg	13.2 kg
End of Cycle	9.5 kg	9.0 kg
Total uranium in core at startup	14.1 kg	18.6 kg

TABLE II (CONT'D)

Physics Parameters of ETR (Cont'd)	<u>Case I</u>	<u>Case II</u>
Loading per fuel element		
Uranium	255 g	239 g
Boron	1.55 g	2.62 g
Loading of fuel section of control rods		
Uranium	103 g	97 g
Boron	0.63 g	1.07 g
Peak to average power over whole core	3.86	2.88
Peak to average power in fuel section	2.35	2.37
Average thermal flux in experiments		
Startup	2.9×10^{14} nv	2.3×10^{14} nv
Time average over cycle	4.4×10^{14} nv	3.5×10^{14} nv
Average thermal flux in fuel section		
Startup	3.3×10^{14} nv	3.1×10^{14} nv
Time average over cycle	5.0×10^{14} nv	4.7×10^{14} nv
Average total flux in experiments		
Startup	18×10^{14} nv	27×10^{14} nv
Time average over cycle	20×10^{14} nv	28×10^{14} nv
Average total flux in fuel section		
Startup	20×10^{14} nv	20×10^{14} nv
Time average over cycle	22×10^{14} nv	22×10^{14} nv

TABLE II (CONT'D)

Physics Parameters of ETR (Cont'd)	<u>Case I</u>	<u>Case II</u>
Void coefficient in fuel section (120 F)		
Start of cycle	-2.5×10^{-3} k per % void in water	-3.2×10^{-3}
Temperature coefficient in fuel section (120 F)		
Start of cycle	-3.7×10^{-4} k/°C	-3.1×10^{-4}
Temperature coefficient in fuel section (68 F) (no xenon)		
Startup	-3.8×10^{-4} k/°C	-2.0×10^{-4}
Void coefficient at core hot spot	-4.3×10^{-8} k per % void in water/cc	-2.6×10^{-8}
Temperature coefficient at core hot spot	-4.7×10^{-9} k/°C/cc	-2.7×10^{-9}
Neutron lifetime	8.1×10^{-5} sec	5.0×10^{-5} sec
Temperature coefficient in WAPD 6 by 6 hole		-8.1×10^{-5} k/°C
Temperature coefficient in ANP 9 by 9 hole		-1.4×10^{-5} k/°C
Void coefficient in WAPD 6 by 6 hole		-5.6×10^{-5} k per % void in water

TABLE II (CONT'D)

Physics Parameters of ETR (Cont'd)	<u>Case I</u>	<u>Case II</u>
Effect of flooding ANP 9 by 9 hole		+6.5% k
Effect of flooding ORNL 6 by 9 hole		+~0% k
Length of running cycle (k excess = 0 at the end)	20 days	20 days
Excess k at startup	11%	11%
k required to reach operating temperature	0.85%	0.89%
k required for equilibrium xenon	2.9%	3.2%
k required for burnup effects over the cycle	7%	7%
Time available for restart after shutdown:		
Start of cycle	26 minutes	21 minutes
After 10 days	18 minutes	22 minutes
After 15 days	7 minutes	12 minutes
After 20 days	0	0
Initial loading in fuel section with no burnable poison	10.9 kg	8.7 kg
Running cycle with no burnable poison	7.5 days	4.0 days
Fuel inventory at end of cycle	9.2 kg	8.8 kg

II. FLUX IN THE ETR

A. General Considerations

Because the ETR core contains such a large percentage of experimental space, no precise flux figures can be quoted without stating the loadings of the experimental holes. However, rough estimates of the magnitude of the flux in the fuel section can be obtained on the basis of the power level and approximate fuel loading in the fuel section.

1. Thermal Flux

The ETR will be operated so that there is a power generation of 175 MW in the fuel section of the core. This corresponds to a power generation of 510 watts per cc in the fuel section. The power per cc is proportional to the number of fissions per cc, and the number of fissions per cc is related to the average thermal flux and the effective macroscopic fission cross section. A power generation of 510 watts per cc is equivalent to 1.6×10^{13} fissions per cc per second. The number of fissions per cc per sec is equal to the average thermal flux multiplied by the macroscopic fission cross section. Thus,

$$1.6 \times 10^{13} = \sum_f \phi_{th} \quad (1)$$

As will be discussed shortly the loading of the fuel section is expected to be in the neighborhood of 14 kg of U^{235} . This loading is characterized by an average $\sum_f$ of about 0.05, so that an average flux somewhat over 3×10^{14} nv is expected in the fuel section at startup. At a power level of 175 MW, 4.5 kg of U^{235} will be burned over the 20-day running cycle

of the ETR. Thus at the end of the cycle Σ_f will have decreased by a third, so that ϕ_{th} will increase 50 percent. The thermal flux in the fuel section averaged against time will be close to 4×10^{14} nv.

2. Above-thermal Flux

The average above-thermal (fast) flux can also be estimated from simple considerations. The average source of fast neutrons in the fuel section is $2.46 \times 1.6 \times 10^{13} = 3.9 \times 10^{13}$ neutrons/cc/second. This must equal the average number of neutrons removed per cc per second. If 20 percent is allowed for fast leakage, then

$$3.9 \times 10^{13} = 1.2 \phi_f \Sigma_{S.D.} \quad (2)$$

The slowing down cross section, $\Sigma_{S.D.}$, for the ETR fuel section is 0.021, so that the above-thermal flux will be about 1.6×10^{15} nv. The above-thermal flux will be fairly constant over the running cycle.

3. Flux above 1 Mev

In estimating the average flux above 1 Mev the following assumptions are used:

- a. The Watt Spectrum* represents the high energy fission spectrum,
- b. A collision with an hydrogen atom, or an inelastic collision with an aluminum atom, reduces the neutron energy below 1 Mev. The inelastic cross section for aluminum is taken as 0.5 barns,
- c. 20 percent of the high energy neutrons are assumed to leak from the fuel section.

* AECD-3645, The Reactor Handbook, Vol. I, p. 621, U. S. Government Printing Office, Washington, D. C. , 1955.

The high energy flux at energy E must therefore satisfy the balance equation

$$1.2 \phi(E) \Sigma_r(E) = W(E) \times 3.9 \times 10^{13} \quad (3)$$

where the removal cross section, $\Sigma_r(E)$, is the macroscopic cross section for a collision with an hydrogen atom plus the cross section for an inelastic collision with aluminum

$W(E)$ is the Watt Spectrum normalized so that $\int_0^{\infty} W(E) dE = 1$

The fission neutron source is 3.9×10^{13} neutrons/cc/sec. The flux above 1 Mev is obtained by numerically integrating (3)[†] as follows:

$$\phi_{>1 \text{ Mev}} = \int_{1 \text{ Mev}}^{\infty} \phi(E) dE = \int_{1 \text{ Mev}}^{\infty} \frac{W(E)}{\Sigma_r(E)} \times \frac{3.9 \times 10^{13}}{1.2} dE$$

$$\phi_{>1 \text{ Mev}} = 1.98 \times 10^{14} \text{ nv}$$

It should be stressed again that the estimates made above will be sensitive to the experimental loadings. The flux in an experimental hole will of course be strongly influenced by the contents of the hole. In addition, the fuel loading depends strongly on the contents of the experimental holes.

If the contents of the holes were less poisonous than in the cases considered, the fuel loading would decrease and the thermal flux increase.

[†] The hydrogen cross sections were taken from BNL 325, Hughes and Harvey, Neutron Cross Sections, U.S. Government Printing Office, Washington, D.C. 1955.

B. Particular Experimental Loadings

In order to examine the characteristics of the ETR under conditions approximating its normal mode of operation, a number of cores have been examined with particular experimental loadings.

At the initial startup of the ETR it is presently planned to place aluminum filler pieces in the experimental facilities. The startup core with the filler pieces in place has been investigated in some detail. The other cores investigated all contained experiments which were specified by the prospective users of the ETR. Although the experiments which actually are placed in the ETR will probably be considerably modified over those used in these investigations, the calculations do provide a picture of the ETR under conditions of fairly realistic operation.

The material presented here is the result of calculations which were performed on the UNIVAC electronic computer. These calculations were carried out in two dimensions (a constant leakage cross section accounting for leakage in the vertical direction) and either two or three neutron groups were used.

C. Startup Core; Aluminum Filler Pieces in the Experimental Facilities

Figure 2-1 is a diagram which shows the core arrangement used for the startup core calculations. It should be noted that in addition to the fuel elements and the aluminum filler pieces the core contains the fuel sections of 13 control rods and the absorber section of three "grey rods".

LEGEND

Al Aluminum
 Be Beryllium
 F Fuel
 R_A Rod (Absorber)
 R_F Rod (Fuel)
 X Experimental
 Hole

CORE SCHEMATIC
 SHOWING MATERIALS IN INITIAL START-UP CORE

Figure 2-2 is a plot of the thermal flux in the reactor midplane at the start of the cycle with the loading shown in Figure 2-1. This plot shows that the presence of the aluminum produces a considerable effect on the flux distribution. However, exterior to the core proper the flux distribution is symmetrical about the core. Both the thermal and fast flux are made to go to zero at the outer boundary shown, so that the gradient in the aluminum is increased somewhat over actual conditions.

The control rods will be discussed in a separate section, but it should be noted here that external to the control rod region there is no large flux perturbation. The flux tends to peak somewhat inside the control rod fuel section where the absorption is less than in a standard fuel rod, and to be depressed in the absorber section.

Figure 2-3 is a plot of the fast flux in the startup core. It is characteristic of the above thermal flux, with its longer interaction distance, to have a fairly smooth distribution.

The total loading of this core is 14.1 kg of U^{235} . The average thermal flux in the fuel section is 3.3×10^{14} nv at startup, and the average above thermal flux is 1.7×10^{15} nv. The detailed loadings of the fuel and control elements are shown in Table II. Table III is a compilation of the cross sections used in the various regions of the reactor.

MIDPLANE THERMAL FLUX ϕ_2 ($\text{nv} \times 10^{14}$) IN INITIAL START-UP CORE
(Aluminum in Experimental Holes)

TABLE III

Cross Sections at 50°C for Startup Core

	D_1	D_2	Σ_a	$\Sigma_{S.D.}$	Σ_f
Fuel	1.307	0.2590	0.0972 0.1002(Xe)	0.02159	0.06006
Control Rod Fuel Section	1.370	0.351	0.05335	0.0181	0.02438
Be	0.600	0.8850	0.00643	0.00151	---
Al	2.54	1.49	0.01183	0.00168	---

D. Core with Experiments as of January 15, 1956

Figure 2-4 is a plot of the thermal flux in the ETR midplane when it is loaded with experiments. Because the computer program used for this calculation was limited in the number of different regions which could be accommodated, the fuel section was made homogeneous; no control rods were included. The data on the experimental loadings was provided by the first set of experimenters scheduled to use the ETR.

As expected, the contents of the holes produce large effects on the thermal flux distribution. In particular, the large 9 by 9 hole, which is heavily loaded with fuel, tends to peak the flux in its vicinity. This is because the 9 by 9 hole is not as well moderated as the fuel section and the thermal flux in the hole interior decreases. Figure 2-5 is a plot of the above-thermal flux

Figure 2-5

in this core, and again it is to be noted that the fast flux distribution is considerably less irregular than the thermal flux distribution.

The total fuel loading in this core is 19.2 kg, with 5.6 kg of fuel in the experiments and 13.6 kg in the fuel section. The increase in loading over the core with the aluminum filler pieces is due largely to the absorption of neutrons in the structural material of the experiments.

The average thermal flux in the fuel section at the start of the cycle is 3.1×10^{14} nv, while the flux averaged over all experiments is 2.3×10^{14} nv. The average above-thermal flux in the fuel section at startup is 1.4×10^{15} nv. The average above-thermal flux in the experiments is 2.1×10^{15} nv. The increase of fast flux in the experiments over that in the fuel section is caused by the relatively poorer moderation in some of the experimental holes.

Detailed loadings of the fuel and control rod sections of this core are shown in Table II. Information concerning the contents of the experimental holes is classified and therefore not included here.

E. Other Loadings

In June of 1955 General Electric received tentative specifications for the first set of proposed experiments. In order to evaluate the effect of the experimental loadings on the ETR, calculations were performed on the G-E UNIVAC at Louisville, Kentucky, using a two-dimensional code (the MUG code). The MUG code was the only two-dimensional computer code available at this time, but unfortunately it is applicable only to cylindrical (r, θ) geom-

etry. The ETR configuration is plainly more suitable for representation in rectangular co-ordinates. However, in order to obtain results quickly, the ETR was "mocked up" as closely as possible in an r, Θ , geometry. Figure 2-6 compares the actual ETR midplane configuration (dashed lines) with the representation used for the calculations (solid lines). Because of the limitations of the computer program, the mock-up is taken as symmetrical about the base line.

In the mock-up of Figure 2-6 the ratio of area for the experimental holes to the area of the rest of the core is 24 percent. In the actual ETR core, considered as a 10 by 10 element grid, 35 percent of the core does not contain fuel elements. Thus the core considered is equivalent to the case where the loading of 11 of the 35 experimental positions is the same as for a standard fuel element.

It is plain from the foregoing that the mock-up is only an approximation of the ETR. Nevertheless, the salient features and approximate dimensions of the reactor have been retained, and it is believed that the results give a realistic picture of the general nature of the ETR with the proposed experiments in place.

The calculations on this core were performed using three neutron groups; the thermal group, the epithermal group with energies between 10 Kev and thermal energies, and the fast group of neutrons with energies above 10 Kev. Figures 2-7 to 2-9 show the flux distribution obtained for this core. Figure 2-10 is a plot of the power generation in the core at midplane. Again it is to be

Figure 2-6

Figure 2-8

Figure 2-9

03607

noted that the unequal loadings of the experimental holes have a marked effect on the flux distribution. The 9 by 9 hole contains a higher density of fissionable material, and the other holes a lower density, than is present in the core proper. This unequal loading manifests itself by tilting the flux distribution so that the flux decreases as distance from the 9 by 9 hole increases. Inside the 9 by 9 hole the fast flux peaks above that of the surrounding regions, the epithermal flux remains about the same, while the thermal flux drops below that of the surrounding regions. If more moderator were added to the experiment, it is expected that the fast flux would decrease and the thermal flux increase.

The loading of this core was 18.9 kg of U^{235} total, with 2.2 kg of fuel in the experimental holes shown (including the fuel in the holes in the other half of the reactor). The average thermal flux in the fuel section at the start of the cycle is 3.3×10^{14} nv, while the average thermal flux in the experiments is 1.6×10^{14} nv. This decrease in flux in the experiments over the other two cases illustrates the fact that the flux in the experiments is strongly dependent on the contents of the experimental holes. The average fast flux in the fuel section is 1.4×10^{15} nv and that in the experiments is 1.5×10^{15} nv.

F. Vertical Distribution of Flux

The vertical flux distribution has been obtained by approximating the ETR in slab geometry. Above the core there was assumed to be aluminum-water mixture with an aluminum to water ratio of 1:3. The core contained standard fuel elements with a loading of 185 g of U^{235} and 1.5 g of boron per

fuel element. Below the core the steel grid plate was approximated as a slab of 65 percent steel and 35 percent water, followed by water.

The flux plots obtained with this slab geometry are shown in Figure 2-11. The flux has been normalized to a power generation of 175 MW in 65 fuel elements, although it is primarily the shape of the flux distribution which is of interest, since in the actual case the magnitude of the flux will vary point-to-point in the horizontal plane.

The results shown in Figure 2-11 indicate that the peak-to-average flux in the vertical direction is 1.38. The calculation has been repeated with other loadings and the shape of the flux distribution does not appear sensitive to the fuel content. For the design considerations of the ETR a peak-to-average value of 1.4 has been used for the vertical flux distribution.

G. Hot Spot Factor for the ETR

It is to be expected that the flux distribution, and thus the hot spot factor, will be sensitive to the loading of the experimental holes. The hot spot factor has been determined for the three cases described in this section.

1. Startup Core

In the startup core the peak-to-average power generation at the start of the cycle with all but two grey control rod fuel sections out of the core is calculated as 2.30, which includes the factor of 1.4 for the vertical distribution. With seven grey control rod fuel sections in the core, the calculated hot spot factor is 2.35. Since each control rod generates on the average about 40 percent of the power of a fuel rod, the peak power

VERTICAL FLUX IN ETR

Figure 2-11

Distance from Core Center in Cm

generation in the two cases is about the same. The hot spot in this case occurred immediately outside of the control rod fuel section. (See Figure 2-2.)

2. Core with Experiments as of January, 1956

The hot spot factor was calculated to be 2.37 in the fuel section of the core. As explained previously, the core section in this case was made homogeneous, and the effect of the individual control rods could not be seen.

3. Core with Experiments as of June, 1956

This core has not been broken up as much as the actual ETR core. As a consequence the hot spot factor is reduced to 1.96 in the fuel section.

The hot spot factors quoted above give the peak-to-average power generation in the fuel section of the core, exclusive of the experimental holes. In the calculations presented here, no allowance has been made for such factors as uneven flow through the fuel plates, non-uniformities in the fuel plates, etc., which may tend to increase the hot spot factor. Also, no allowance has been made for the possibility of arranging the control rods in a fashion which would increase the hot spot factor. The control rods can be used to decrease the hot spot factor, but, as will be discussed in a later section, some arrangements can increase the hot spot factor. Finally, it should be pointed out that the hot spot factors quoted above are for the core at the start of the cycle. As the fuel is burned, the hot spot factor will decrease where the flux is highest.

The ETR has been designed with an assumed hot spot factor of 2.5 in the fuel section.

III. GAMMA HEATING IN THE CORE

When a substance is placed in the reactor core it is heated predominately by the absorption of gamma rays. This gamma heating can be computed easily if simplifying assumptions are used. Assume that the reactor is large in dimensions compared to a relaxation length for gamma absorption (9 cm for 1 Mev gammas), and that the gamma production is uniform throughout the core. If a region is chosen in the middle of the reactor, at equilibrium the total gamma power produced in the region must equal the gamma power absorbed, since the leakage out of the region is compensated for by an influx from the (infinite) surroundings. About six percent of the total fission energy produced is in gamma ray energy. At a total power of 175 MW in 65 fuel elements this amounts to 30 watts per cc. If an average thermal flux of 5×10^{14} nv is assumed, there will be an additional five watts per cc produced by neutron absorption in the aluminum and water. Thus, at equilibrium, 35 watts per cc must be absorbed in the interior of the core. The core consists of 60 percent water and 40 percent aluminum by volume. The total mass per cc is, therefore, $0.4(2.7) + 0.6 = 1.7$ g. This leads to a total gamma heating of 21 watts per g of which about 65 percent is due to gamma rays below two Mev.

The discussion above concerns the gamma heating in the middle of a large reactor operating at uniform power density. In the ETR the power density changes with position. Thus, at the midplane the average power density is 1.4 times the average power density over the whole core. The gamma heating in this plane would then approach 1.4 times the values indicated above, or approximately 29 watts per g. Because of the finite relaxation length of the gamma

rays, there will be some smearing-out of the peaking so that the actual average heating in the midplane would be below 29 watts per g. Similarly, with the model used above the gamma heating at the edge of the core would be expected to be one half the heating in the middle of the core. However, since the power falls off at the edge of the core, the gamma heating will be reduced.

According to Figure 3-1, which is normalized to an average value of 100, the power generation at the edge of the core is 70 percent of the average power generation. Thus the gamma heating at the core edge should be roughly $0.70 \times 0.5 = 0.35$ of the average gamma heating. This would amount to 7.4 watts per g average gamma heating at the core edge, and 10.3 watts per g at the midplane.

The above analysis is of course not strictly applicable to the ETR, because the in-core experimental facilities produce considerable effects on the flux distribution. Nevertheless, the general conclusions have been substantiated by more detailed calculations performed where the effects of the experiments were considered.

For this computation the experimental holes were assumed to be loaded with experiments typical of the first set of experiments expected in the ETR. X-Y UNIVAC calculations were performed in which the flux and power distribution in the core was obtained. The flux distributions are shown in Figures 2-4 and 2-5 and the core is discussed in Section II. C. The data were then handled in the following manner:

POWER DISTRIBUTION VS REFLECTOR COMPOSITION

Figure 3-1

1. It was assumed that 12.2 Mev per fission went into gamma-ray production. This includes prompt gammas, fission product gammas, and capture gammas in U^{235} .
2. Gamma production from neutron capture, excluding U^{235} capture, was computed at points throughout the core by multiplying the thermal flux by the macroscopic capture cross section of each material at the point in question. The capture gamma energy production was obtained by multiplying the gammas produced per second by the energy per gamma characteristic of the material in which the neutron capture occurred.
3. As was pointed out above, roughly 65 percent of the gamma energy in the fuel region is due to gamma rays of an average energy of 1 Mev. It was assumed that the average gamma energy absorption at a point was equal to the average gamma energy production in the region around the point, within one mean free path for one Mev gamma rays. Thus, in the fuel section, where the mean free path for one Mev gamma rays is nine centimeters, the gamma absorption at a point was taken equal to the average gamma production in a circle of nine centimeter radius with the point as center. For a point in an experimental hole, the circle was constructed with a radius characteristic of the mean free path of the materials in the hole. For a point on the edge of a hole the radius of the arc in the hole was taken equal to the mean free path in the hole, and the radius of the arc in the fuel section was taken as nine centimeters.
4. The structure within an experimental hole was ignored. The materials within the hole were assumed to be homogeneous within the hole volume.

A plot of the gamma heating computed as described above is shown in Figure 3-2. Figure 3-3 shows the vertical distribution of the gamma heating under the same

GAMMA HEATING ($\frac{\text{WATTS}}{\text{GRAM}}$) IN THE E.T.R. CORE DUE TO
RADIATION FROM FISSION AND CAPTURE GAMMA
RAYS (AT THE MIDPLANE)

GAMMA HEATING - NUMBERS ARE WATTS/GRAM

Figure 3-2

conditions as outlined above and using the plot of Figure 2-11 for the vertical distribution of thermal flux.

It can be seen in Figure 3-2 that the experiments produce large effects on the gamma heating distribution. At the midplane, the maximum gamma heating is 45 watts per g near the 9 by 9 hole, although the average gamma heating in the core interior is close to the 29 watts per g expected on the basis of the simple arguments stated at the start of the section. The gamma heating at the core edges show a distinct symmetry due to the presence of the experiments. The maximum gamma heating around the core edge is almost 20 watts per g. However, in regions away from the large 9 by 9 hole, the heating at the edge is not far from the 10 watts per g expected from the simple considerations.

The figures presented here indicate that gamma heating will present serious difficulties for the in-core experimenter. Discussion with proposed users of the experimental facilities indicates that the heating is so large that it is not feasible in many cases to design the experiments for this heating with large built-in safety factors. In the analysis presented above the hot spot in the core was close to the 9 by 9 hole. However, if for instance the experiment in this hole is replaced with a highly absorbing, low fuel content experiment, the hot spot will shift. In such a case, the gamma heating in the other holes may increase markedly. Similarly, if the grey rods are put in the core near the large 9 by 9 hole, this will also shift the gamma heating distribution. The gamma heating will also be sensitive to the facility loadings.

Discussion with the experimenters indicates that in practice the experiments will not be as heavily loaded with fuel as for the case calculated. This will aid to some extent by reducing the gamma source in the hole interior and also in reducing the power generation around the hole. Unfortunately the flux will also be reduced somewhat, although the gamma heating in the experiments may be reduced below the peak values indicated in Figure 3-2.

To avoid the possibility of experiment failures due to gamma heating, the experiments must be reliably instrumented and the reactor must be operated so that temperatures and stresses do not exceed safe limits.

Detailed sample calculations of gamma heating in the ETR core are presented in Appendix II.

IV. GAMMA HEATING EXTERNAL TO THE CORE

To obtain information about the region external to the core in the horizontal plane, the ETR was mocked up as a cylinder of 40 cm in radius, surrounded by a reflector and thermal shields of thicknesses equal to those proposed for use in the actual ETR. Inside the core the buckling in the vertical direction was made equal to the buckling of the ETR as determined in Appendix VI. External to the core the buckling in the vertical direction was taken as zero. This has the effect of producing a $1/R$ dependence of the flux, instead of a $1/R^2$ dependence, and thus overestimates the flux at large distances. For use in the heating calculations, the ETR was assumed to be operating at a total power level of 270 MW in the core, including the experimental facilities. This corresponds to a power generation of 175 MW in the 65 fuel elements and 95 MW in the experiments. The experiments will actually produce considerably less power than 95 MW. Thus, the neutron fluxes computed on this basis are expected to be somewhat higher than will be met in practice. External to the reactor the neutron spectrum is different than it is in the core, and not well known. Consequently, the effective cross sections in these regions are in doubt. In the calculations the cross sections for the various materials external to the core were computed assuming that the neutron spectrum was the same as in the core. Therefore, the flux values at large distances from the core should not be considered as accurate.

Figure 4-1 is a plot of the flux computed in the manner described above. From these plots and the slowing-down cross sections in the various materials, the

MIDPLANE THERMAL FLUX

heating due to neutrons slowing down can be computed. This heating is shown in Figure 4-2 by the dashed line. From the flux plots and the capture cross section the gamma ray production external to the core can be computed. At any point, the gamma flux is the sum of the contributions from the gammas produced in the core and those produced in regions external to the core. The gamma flux from the core region was computed by assuming a total power generation of 270 MW distributed uniformly in the core. As mentioned above, this is an overestimate. Also, since the experiments and the leakage from the core cause the power to go down at the core edges, and since the main contribution to the external gamma flux comes from the core edges, the calculations are expected to yield conservative results. The details of the calculations of gamma flux are given in Appendix I. The only gamma rays which are important are the two Mev capture gammas from water, the seven Mev gammas from iron and aluminum capture, and the gamma rays of one Mev average energy from fission. §

Figure 4-3 is a plot of the isotropic gamma flux external to the core. The values were computed at each boundary only, and the computed points joined by straight lines. The flux values include contributions due to degraded photons as well as undegraded photons. The flux is defined so that when the values are multiplied by the various energy-absorption coefficients in the different regions the heating per unit volume is obtained. The dark solid lines in Fig-

§ The gamma spectrum due to fission (including prompt gammas, fission product gammas, and uranium capture gammas) was taken from work by Motz as reported by Stephenson in Introduction to Nuclear Engineering, p. 209, McGraw Hill, New York, 1954.

MIDPLANE ABOVE-THERMAL FLUX

GAMMA FLUX EXTERNAL TO CORE AT MIDPLANE

Figure 4-3

Distance from Core Center in Cm

ure 4-4 show the total heating in the midplane due to neutron slowing-down and gamma ray heating of one, two, and seven Mev gammas. The light solid lines are the gamma heating alone. At the edge of the pressure vessel about four fifths of the heating is due to seven Mev gamma production in the aluminum and water-iron regions. The curves of Figures 4-1 to 4-4 contain a factor of 1.4 to account for the hot spot factor in the vertical direction. At large distances from the core this hot spot factor will not be large so this again produces an overestimate.

It should be clear from the above discussion that these curves are expected to give overestimates of the gamma heating. Thus the cooling required external to the core can be designed on conservative grounds. A more precise analysis is difficult in the case of the ETR, because close to the core the flux distribution is distorted by the experiments, while at large distances the neutron spectrum is in doubt. However, reasonably accurate estimates can be obtained for regions close to the core by scaling the curves of Figures 4-1 to 4-3 to the more precisely determined values in Figure 3-2. However, Figure 3-2 itself is only valid for the particular experimental loadings considered.

The heating in the bottom grid plate was computed in a manner identical to that described above. The total heating at the top of the grid plate in the center of the steel is 70 watts per cc, while at the bottom the heating is 1 watt per cc. The heating at the top is due mostly to gamma-rays originating in the core, whereas at the bottom the heating is due to the capture gammas produced in the grid plate.

HEATING IN HORIZONTAL DIRECTION

Figure 4-4

Distance from Core Center in Cm

V. STUDY OF THE ETR REFLECTOR

The ETR reflector consists of a beryllium slab in contact with the core, followed by aluminum elements which can be removed to insert experiments. This reflector performs three functions:

1. It reduces the critical mass of the reactor and therefore increases the average flux at a given power level.
2. It flattens the flux distributions in the reactor core and reduces the hot spot factor, permitting the reactor to run at a higher power level than would be the case if there were no reflectors. The utility of the reactor for experimental purposes is enhanced since the reflector tends to raise the flux in the low flux regions near the core perimeter.
3. It provides a buffer between the experimental holes in the aluminum and the reactor core. Therefore, these holes can be utilized without greatly affecting the reactor core. For instance, experiments can be performed in them where large amounts of highly absorbing materials are involved.

The effect of placing the beryllium slab before the aluminum section of the reflector was studied with three-group calculations on the 650 computer. The three neutron groups studied were the thermal, epithermal, with neutron energies between thermal and 10 Kev; and fast, with neutron energies above 10 Kev. The core was approximated as a slab 30 inches thick in the Z direction, and with buckling in the X and Y directions chosen to equal the buckling in the actual ETR. The flux and power distributions and change of k_{eff} were studied as a function of beryllium thickness. One hundred cm of aluminum were assumed to follow the beryllium in all cases. The flux is made to go to zero following the aluminum.

Figure 5-1 is a plot of the thermal flux distributions as a function of beryllium thickness. The beneficial effects of flux flattening produced by the beryllium are apparent from Figure 5-1. It should be noted that although the flux in the beryllium reflector is an increasing function of total beryllium thickness, the flux in the aluminum slowly decreases as the beryllium thickness is increased. Figure 5-2 is a plot of the hot spot factor, Figure 5-3 a graph of k_{eff} , and Figure 5-4 a plot of the ratio of maximum to minimum power generation in the core as a function of beryllium thickness. These plots show that the beneficial effects of the beryllium reflector vary from a rapid rate of increase at small thicknesses to a slow rate of increase at large thicknesses. The curves are somewhat unrealistic in that the core is assumed to be uniform, whereas the actual ETR core has a large number of experimental holes. In the ETR flux distribution section (II), it was shown that the loading of these experimental holes was a major determinant of the hot spot factor and flux distribution. Curves of Figures 5-1 to 5-4 indicate that, for a uniform core, a beryllium thickness of between 4 1/2 and 8 inches is desirable. Since the effects of the experimental holes will negate to some extent the effects of the beryllium, a thickness of 4 1/2 inches has been chosen for the ETR. Figure 5-6 is a plot of the thermal flux in the core and beryllium reflector. It was obtained with the ETR approximated as a uniform cylinder 41 cm in radius surrounded by 4 1/2 inches beryllium and 100 cm of aluminum.

The beryllium reflector will be cooled with water which flows through holes drilled in the reflector. Thus, the reflector will not be pure beryllium but will be a mixture of beryllium and water. To study the effect of the water

THERMAL FLUX DISTRIBUTION VS Be THICKNESS

POWER $\frac{\text{MAX}}{\text{AVERAGE}}$
VS
Be THICKNESS

Figure 5-2

Figure 5-4

THERMAL FLUX-REACTOR WITH NO REFLECTOR

Figure 5-6

cooling, the composition of the beryllium was varied from 100 percent beryllium, to 50 percent beryllium - 50 percent water, to 100 percent water. Figures 5-7 to 5-10 show the power, thermal flux, epithermal flux, and fast flux distributions as a function of the reflector composition. As the amount of water is increased the flux peak in the reflector, the hot spot factor, and the critical mass increase, but the flux in the aluminum decreases. The effects are not large, however, so that the effect of the cooling water will not be detrimental to the reactor operation. For experimental work in the beryllium reflector itself the presence of the water is helpful, since it increases the flux peaking in the reflector. The behavior of the 100 percent water reflector compared to the 100 percent beryllium reflector is interesting because the water peaks the flux more than the beryllium. This is due to the fact that the square root of the age and the diffusion length in water are small compared to 4 1/2 inches, while in beryllium these quantities are larger than 4 1/2 inches. Thus, the 4 1/2 inches of water approach the effect of an infinite water reflector, but over 10 inches of beryllium are required to approximate an infinite beryllium reflector (as can be seen in Figure 5-1). At the present time, it is believed that the cooling of the beryllium reflector will require about five percent water, by volume, in the beryllium reflector. Figure 5-11 is a plot of the flux distribution with a reflector composed of 95 percent beryllium and five percent water, followed by 100 cm of aluminum. A power distribution plot for this case is shown in Figure 5-12.

An interesting variation of the reflector design is to follow the beryllium slab with a slab of water.* The large slowing-down cross section of the water

* This scheme was suggested by Mr. S. Untermyer.

POWER DISTRIBUTION VS REFLECTOR COMPOSITION

Figure 5-7

Figure 5-9

Figure 5-11

would then thermalize the fast flux, and the thermal neutrons could leak back to the core through the beryllium. The effect of such a reflector is shown in Figure 5-13, where a 4 1/2-inch thickness of beryllium is followed by 4 1/2 inches of water and 100 cm of aluminum. The thermal spike in the reflector has been improved considerably over the case in which no water was included. The k_{eff} has increased and the hot spot factor has been reduced slightly. However, the effect of the short diffusion length in water causes a rapid fall-off of thermal flux in the reflector after the spike. Consequently, the flux in the aluminum is considerably reduced as opposed to the case in which no water followed the beryllium. This reduction in flux in the aluminum can be minimized by reducing the thickness of the water. Figure 5-14 shows the thermal flux distribution when a two-inch thickness of water follows the beryllium. The increase in k_{eff} and reduction in the hot spot factor are still present, but the reduction of flux in the aluminum is now a smaller effect.

The studies of the ETR reflector are summarized in part in Figure 5-15, where k_{eff} is shown as a function of the reflector composition. The solid curve is of no significance, but is drawn to help locate the points. The studies indicate that the choice of 4 1/2 inches of beryllium preceding the experimental space in the aluminum reflector represent a good compromise between cost and the attainment of optimum flux in the reactor core and reflector. The cost of the reflector can be decreased by replacing the beryllium with water. However, such a change would increase the critical mass of the reactor by about 10 percent and the loading of the fuel rods by about 15 percent, thus decreasing the flux in the reactor and also in the aluminum reflector. The addition of a water

Figure 5-14

blanket behind the reflector decreases the critical mass by about $2\frac{1}{2}$ percent and fuel loading by about $3\frac{1}{2}$ percent, which increases the flux in the core. However, the flux in the aluminum is reduced. As has been mentioned previously, the loading of the in-core experimental facilities will tend to negate the effect of the reflector in flattening the flux distribution and reducing the critical mass (see Figure 2-7). The decrease in flux in the aluminum produced by the water blanket will be present regardless of the experimental loadings. Therefore, the addition of the water blanket does not seem warranted.

On the basis of the considerations described in this section, the reflector of the ETR has been set at $4\frac{1}{2}$ inches of beryllium surrounding the core, followed by the aluminum pieces in which experiments can be placed.

VI. THE CONTROL SYSTEM

A. General

The control system for the ETR must perform a number of functions. It must provide operating control of the reactor including compensation for temperature changes and for transient xenon and samarium poisoning. In addition, the control system is called upon to maintain criticality as the fuel burns out and as fission products accumulate. It is not expected that a large amount of control will be needed to compensate for changes in experimental loading during any one fuel cycle. Rather, it is expected that major changes in reactivity caused by experimental changes will be compensated by changing the core loading. This is consistent with the expected type of experiment to be run in the ETR in which fuel samples of various types will be irradiated in the experimental loops for long periods of time. An additional requirement imposed by the experimenters is that the control system should distort the reactor flux as little as possible. In particular, it is hoped that the severe distortion of flux observed in the MTR due to the use of cadmium shim-rod control will be avoided.

To fulfill these requirements the ETR has a control system relying upon the motion of control rods mechanically inserted into the core or removed from it. In view of the short time schedule for design and construction of the ETR, the use of a mechanical control system was deemed desirable since considerable experience with such systems has been gained on other reactors.

The control rods in the ETR will consist of four conventional black rods, such as are used in the MTR, and 12 grey rods. Both black and grey rods are part of a two-section control element, with a fuel section beneath the grey (or black) section

in the element. The grey rods are designed so that the capture and scattering cross sections in the rod match closely the cross sections in a standard fuel element. However, the grey rods will contain no fissionable material. Thus the major effect of the insertion of a grey rod into a region is the removal of the source of fast neutrons from that region. Because of the diffusion of neutrons from other regions, the flux distortion produced by the insertion of a grey rod is not expected to be serious. Calculations indicate that the flux will be reduced by only 10 to 20 percent in the immediate vicinity of the grey rod. The use of grey rods, therefore, meets the objective of having the control system exert only small perturbations on the flux distribution in the reactor core. The 12 grey rods together are worth about 12 percent in k , or an average Δk of about one percent per control rod. During the running cycle of the ETR the black rods, worth about 3.5 percent in k each, are completely out of the core and used only as safety devices to shut down the reactor in case it reaches an unsafe operating point.

The running cycle of the ETR is expected to be about 20 days and to result in about 33 percent average burnup of fuel. The grey rods must be able to compensate for this fuel burnup. To minimize the effect of fuel burnup on reactivity the ETR uses a burnable poison which reduces the total excursion in k due to burnup. When the fuel elements are fabricated a small amount of boron will be included. The amount of boron in the rod is adjusted so that when the reactor is operating the burnup of boron compensates for the burnup of the U^{235} . While perfect compensation cannot be achieved a considerable reduction in the amount of control required by other means can be obtained. In particular, it is anticipated that the use of burnable poisons in the ETR fuel element will reduce the change in reactivity produced by fuel burnup from about 14 percent to about six percent.

The control provided by the grey rods must compensate for the changes in k during the running cycle which are roughly as follows:

Xe	0.03
Other fission products	0.01
Temp	0.01
Poison mismatch	<u>0.06</u>
	0.11 k

The grey rods produce about a 12 percent effect on k . Therefore, if the cycle is started with only one grey rod absorber section in the core after operating temperature and xenon equilibrium are reached, there will be six percent excess k available for burnup effects and adjustments in the operation of the reactor. The change in reactivity with time due to burnup depends upon the amount of burnable poison used in the reactor and will be discussed in a later section. However, regardless of the boron content, if the reactor is shut down in the middle of a run, there will not be enough excess k available to override the xenon buildup, except in the first few minutes. As a result, in the case of a shutdown the reactor must either start up immediately, be reloaded, or remain shut down for two days while the xenon decays.

In addition to the control rods discussed above there are two regulating rods in the ETR each worth less than 0.5 percent in k . Only one regulating rod functions at a time, the other being used as a spare. The regulating rod is a high-speed servo-controlled rod and will automatically compensate for small changes in reactivity. These rods will be small black rods placed in the reflector, and will have no fuel section.

B. Preliminary Investigations

In order to investigate the properties of the proposed grey rods and compare them with conventional black rods, a number of computations have been performed on the 650 computer. In these calculations the ETR was approximated as a cylinder. Because of the limitation of the 650 three-group code, the effect of control rods was studied at the central position of the cylindrical reactor. The results of these calculations were then used, together with perturbation techniques, to find the worth of the rods in off-center positions. On the basis of these calculations the grey rods were designed to have an average worth of about one percent in k . Finally, the results were checked using the X-Y code on the UNIVAC where the effect of off-center rods could be observed directly.

Figure 6-1 is a plot of the relative power distribution in the cylindrical reactor with a 3.7-cm radius grey control rod inserted. In all of these preliminary grey rod studies, the cross sections of the grey rod are the same as those in the fuel elements, except that the fission cross section is zero. It should be noted from Figure 6-1 that the effect of inserting a grey rod into the high flux region of the core is to flatten the power distribution and reduce the hot spot factor. The average power in Figure 6-1 is normalized to 100, so that insertion of the grey rod has reduced the hot spot factor from 1.41 to 1.30. This has been done without serious distortion of the flux. Figures 6-2, 6-3, and 6-4 are plots of the thermal, epithermal, and fast flux* with and without the control rod inserted. The most seriously distorted flux is the fast flux shown in Figure 6-4. Even in this case,

* Epithermal flux has neutron energies between thermal and 10 Kev, and fast flux has neutron energies greater than 10 Kev.

POWER DISTRIBUTION WITH
ONE GREY CONTROL ROD INSERTED

Figure 6-2

ONE GREY CONTROL ROD-EPITHERMAL FLUX

Figure 6-4

immediately outside the control rod the flux has been reduced by only 17 percent. The effect of a grey rod can be compared to that of a black rod. Figure 6-5 is a plot of the power distribution produced when a black rod is inserted into the center of the core. Again, the hot spot factor can be reduced by placing the rod in a high flux region. However, Figure 6-6 indicates that this is accomplished only at the cost of serious flux distortion in the reactor.

The Δk produced by the insertion of a single grey rod in the core center is 0.015, while that produced by the insertion of a black rod is 0.06. A study was made to determine the shadowing effect produced on one rod by the insertion of another. In the case of grey rods, since the flux distortion is small, the shadowing effect would be expected to be small. However, in the case of the black rods the flux distortion is high, and shadowing effects are known to be significant. These points are brought out by the information displayed in Figure 6-7, where the effect of four grey rods is compared to that produced by four black rods. The four rods are approximated by doubling the radius of the single rod of Figures 6-1 to 6-6. The Δk produced by the four grey rods is 0.05 in k , as compared to 0.06 expected on the basis of one rod producing 0.015 Δk . This small difference is due in part to the fact that the outer regions of the control rod are in regions of less importance than the central region. In the case of the black rod, the Δk produced by four rods is only nine percent, as compared to six percent produced by one rod. The shadowing effect here is plain. Again, the high flux distortion produced by the black rods can be compared to the relatively small effect on the flux of the grey rods.

The studies on shadowing seriously overestimate the effects, since in practice no two control rods will be immediately adjacent to each other. Therefore, it can be

Figure 6-5

concluded that in actual operation of the ETR the grey rods will exert little or no shadowing effects on each other. Similarly, the four black rods will be worth considerably more than nine percent in k , as is indicated by UNIVAC calculations described in another section.

The observations describing the lack of flux distortion produced by the grey rods are in agreement with studies made using the MUG code on the UNIVAC. In this case, a grey control rod was inserted in position 22, 11. (See Figure 2-6.) The Δk produced was about one percent, and the flux distortion was 20 percent for the fast flux immediately outside the control rod, but less than 15 percent for the thermal and epithermal fluxes. The disturbance produced by the grey rods was only a few percent three inches away from the rod.

The ETR will have a total of 12 grey rods. The control system will be flexible enough so that these rods may each be inserted to any extent desired. One method of operating the grey rods is to have 11 rods arranged such that some of them are all the way in the core, and the rest of them all the way out of the core. The remaining grey rod is placed so that the regulating rod is in the center of its region of travel. An advantage of this method of operation is that the rods which are inserted all the way in may be inserted in high flux regions. They thus will tend to flatten the flux distribution, reduce the hot spot factor, and reduce uneven burn-out of the fuel elements. In Figure 6-7 the shadowing effects and flux distortion in such a method of operation have been shown to be small.

Another method of operating the grey control rods is to operate them ganged, with all the rods inserted the same distance into the core, so that the regulating rod is in the center of its region of travel. This method of operation corresponds to the

THERMAL FLUX WITH FOUR GREY OR BLACK RODS INSERTED

present procedure on the MTR, where black rods are used for control. Figure 6-8 compares the flux distribution obtained when the ganged grey rods are inserted one third of the way into the core to the normal flux distribution with the grey rods removed. This plot was obtained by reducing the fission cross section by 20 percent in the region assumed to be occupied by the grey rods. All other cross sections, slowing down, leakage, and total absorption were kept constant. It can be seen from Figure 6-8 that the flux in the region occupied by the grey rods at the top of the reactor has been reduced by more than a factor of two over the corresponding region at the bottom of the reactor. Similarly from Figure 6-9, which shows the power distribution, it can be seen that the hot spot factor has increased from 1.3 to over 1.5 (again the power distribution is normalized to an average of 100 over the reactor).

C. Design of the Grey Rods

The calculations described above indicate that when a 3.7-cm diameter grey rod is placed in the center of the core, k_{eff} is reduced by 0.015, and $\delta k/k$ is -1.33 percent. In order to evaluate the strength of the rod in off-center positions, perturbation techniques were used. A discussion of the perturbation methods used for the design of the ETR is given in Appendix IV. According to perturbation theory the change in ν , the number of neutrons per fission required to make the reactor critical, is related to a change in fission cross section by

$$\delta \nu = \frac{-\nu \int \Phi^+ \delta J \Phi d\tau}{\int \Phi^+ J \Phi d\tau} \quad (1)$$

THERMAL FLUX—12 GANGED CONTROL RODS

Figure 6-9

The integration takes place over the whole reactor.

The calculations described here are three-group calculations, where fissions only occur in the third (thermal) group. Therefore, in the notation of (1)

$$J = \begin{pmatrix} 0 & 0 & \Sigma_f \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

so that in the grey rod absorber where $\Sigma_f = 0$,

$$\sigma J = - \begin{pmatrix} 0 & 0 & \Sigma_f \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The vector $\underline{\Phi}$ is the flux vector $\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix}$ and the vector $\underline{\Phi}^+$ is the adjoint flux

$$\text{vector } \begin{pmatrix} \Phi_1^+ \\ \Phi_2^+ \\ \Phi_3^+ \end{pmatrix}$$

$k_{\text{eff}} = \frac{2.5}{\nu}$, and Σ_f is constant over the core.

$$\text{Thus } \frac{\sigma k_{\text{eff}}}{k_{\text{eff}}} = \frac{\sigma k}{k} = \frac{\sigma \nu}{\nu} = - \frac{\Phi_1^+ \Phi_3 A}{\int \Phi_1^+ \Phi_3 d\tau} \quad (2)$$

Where A is the area of the grey rod and $\Phi_1^+ \Phi_3$ of the numerator is evaluated at the grey rod position. Figures 6-10 to 6-12 show the fluxes and adjoint fluxes over the reactor.

Figure 6-10

FAST NEUTRON FLUX AND ADJOINT FLUX

FLUX AND ADJOINT FLUX USED TO CALCULATE GREY ROD EFFECTIVENESS AS A FUNCTION OF POSITION

Figure 6-12

When the right side of (2) is evaluated at the center of the core, and A is taken as the area of a 3.7-cm radius fuel rod. The $\delta k/k$ is -1.55 percent. This is 17 percent higher than the value obtained by direct calculation on the computer and reflects the fact that ϕ is reduced at the position of the grey rod.* Figure 6-13 shows the product of $\phi_1 + \phi_3$ over the core. According to perturbation theory, the curve of Figure 6-13 can be considered as a curve of the relative worth of a grey rod as a function of core position when the grey rod absorber section differs from its fuel section in Σ_f only. Actually the rod as designed differs somewhat in water-to-metal ratio and also in absorption cross section. However, since the flux and adjoint flux in the different groups have roughly the same shape in the central region of the core, Figure 6-13 can be used as an approximation of the relative worth of a grey rod, regardless of small changes in other quantities besides Σ_f .

The discussion so far has assumed that the grey rod fuel section and the fuel rods would both be loaded with the same amount of fuel and boron. However, it is possible for a grey rod fuel section to be kept out of the core during most of the ETR running cycle and then be inserted. About 33 percent of the fuel in a fuel rod is burned during the ETR running cycle, so that a new element inserted at the end of the cycle would generate about 50 percent more power per fuel plate than the surrounding elements. Since the ETR is already operating at extremely high heat

* The more exact form of equation (1) is

$$\frac{\delta \nu}{\nu} = \frac{\int \Phi^+ \delta J \Phi' d\tau}{\int \Phi^+ J \Phi' d\tau}$$

where Φ' is the perturbed flux.

Figure 6-13

fluxes, such a condition must be avoided. In order to prevent such an occurrence the grey rod fuel section was reduced in fuel content per fuel plate to 70 percent of that of a standard element. The boron content has likewise been reduced, since calculations similar to those above indicated that if the fuel and boron were both decreased the change in control rod effectiveness would be small. Since the fuel plate area in the grey rod section is only 58 percent of the area of a standard rod, the total fuel in the fuel section will be $0.70 \times 0.58 = 40.6$ percent of the fuel in a fuel rod.

On the basis of a preliminary analysis, using the perturbation results described above, a tentative grey control rod design was obtained. A cross-section of this rod is shown in Figure 6-14. In order to evaluate the rod's performance the calculations in the cylindrical geometry were repeated using two groups, taking into account all of the important properties of the proposed rod as well as the structure of the guide tube and the control rod.

Figure 6-15 shows the thermal flux in the control rod region with the fuel section in the core, and the thermal flux with the absorber section in the core. When the absorber section is in the core there is a considerable flux peaking, since the interior of the rod is mostly water. Similarly, there is some peaking even when the fuel section is in the core, because its absorption cross section is lower than that in the surrounding fuel. However, at the end of the cycle when the fuel and boron in the core have been reduced, an unburned control rod fuel section placed in the core produces a flux depression (see Figure 6-16). In any case, the flux distortion produced exterior to the guide tube by the presence of the rod is small. The calculation yields a control rod effectiveness at the core center of 1.27 percent $\delta k/k$.

GREY ROD

Figure 6-14

FIGURE 6-15

The average position of the control rods is about two thirds of the way between the middle and the edge of the core. * Reference to Figure 6-13 indicates that at this position the rod will be worth 0.64 percent. The desired effectiveness is an average of one percent per rod, so this analysis would lead us to increase the strength of the rod. However, the calculation was based on a uniform cylindrical approximation of the ETR, which is equivalent to assuming that the ETR has 100 fuel elements instead of forty nine fuel sections and 16 control rods. Equation (1), which is valid when there is a change in $\sum_f$, indicates that this approximation will underestimate the control rod effectiveness, since the denominator will be larger than for the actual ETR core. The denominator (1) is contained in the perturbation expressions for the effect of changes in $\sum_a$, $\sum_{SD}$, D_1 , D_2 , and $\sum_f$. Thus, the conclusion that the rod worth is underestimated in the cylindrical approximation is generally valid. As a first estimate, the worth of the rod would be expected to be greater than indicated by the ratio of 100:65. In this case the control rod discussed above would have an average worth in the ETR of about one percent.

Since it appeared that the tentative rod design might have close to the desired effectiveness in the ETR core, UNIVAC calculations were performed to check the rod effectiveness in the two-dimensional case. The initial startup core of Figure 2-1 was used. Two calculations were performed. First, the arrangement of Figure 2-1 was used. Secondly, the three control rod absorber sections in the core were replaced with the control rod fuel sections. The three positions

* A more precise calculation of the average position of the grey rods does not seem warranted, since the presence of experiments in the ETR will change the relative worth of the various rods.

shown were chosen because together they represented what would be expected from three "average rods". The calculation yielded an average worth for the three rods of 1.1 percent each. Thus, it appears that the rod of Figure 6-14 will come close to providing the desired average of one percent effect on k per rod.

D. Design of the Black Rods

The work of Section B indicates that a 3.7-cm radius black rod at the core center would have about a six percent effect on k. In these calculations the black rod was simulated by making the thermal diffusion coefficient small, and the thermal absorption coefficient as large as the computer program (the PROD code) would allow. The proper boundary condition at a black rod boundary is of course given in terms of the logarithmic slope of the flux curve at the rod surface. Such a boundary condition was not available in the one-dimensional computer code, so the results of the calculation must be considered only as approximate. In order to get a more accurate estimate of black rod worth, the control rod fuel sections at positions (21, 19), (19, 27), (27, 23), and (31, 29) of Figure 2-3 were replaced by black rods. An X-Y UNIVAC calculation was performed and compared to the previous calculation with the fuel section in place. The X-Y code permits the use of a current boundary condition at a black rod surface. The logarithmic slope at the surface was set so that $\frac{-1}{\phi_{th}} \frac{d\phi_{th}}{dn} = \frac{1}{0.71 \lambda_{tr}}$, where

$\frac{d\phi}{dn}$ is the inward normal derivative of the thermal flux at the rod boundary and

λ_{tr} is the transport mean free path in the core. The calculation indicates an effectiveness of 14.4 percent in k for the four control rods together.

E. Design of the Regulating Rods

The use of perturbation theory in evaluating the behavior of a black rod is questionable, because the effects on flux distribution cannot be considered as small perturbations. However, the worth of the regulating rod is not critical, if it is under 0.5 percent and in the neighborhood of 0.2 to 0.5 percent. Therefore, in calculating the effectiveness of a regulating rod the curve of Figure 6-13 was used as a guide in evaluating the relative effectiveness of a black rod as a function of position. The four black rods have an average worth of 3.6 percent each, and an average position about half way between the core center and the core edge. From Figure 6-13 the relative worth of a rod in this position is 0.7 versus 0.21 at the reflector boundary. The regulating rod is a cylinder, 1 7/8 inches in diameter, with no fuel section, whereas a black rod is 2 1/2 inches square and when inserted replaces a fuel section. The calculations of Section B indicate that 25 percent of the black rod effectiveness is due to the removal of the fuel section. Thus the effectiveness of the absorber section alone is 3.2 percent in k. Therefore the regulating rod worth should be about

$$3.2\% \times \frac{0.21}{0.70} \times \frac{\pi \times (1.875)^2}{4 \times (2.5)^2} = 0.42 \text{ percent in k.}$$

VII. BURNUP IN THE ETR*

A. The Change of Reactivity with Time

According to perturbation theory, the change in ν , the number of neutrons per fission required for criticality, produced by a change in the reactor parameters can be written

$$\delta \nu = \frac{\int \bar{\Phi}^+ \delta K \bar{\Phi}' - \nu' \int \bar{\Phi}^+ \delta J \bar{\Phi}'}{\int \bar{\Phi}^+ J \bar{\Phi}'} \quad (1)$$

where the notation is the same as that in Appendix IV, and the integration is over all space. Expression (1) is exact. In this treatment we will make the assumption that the shape of the flux in the reactor does not change during the running cycle, but that only the ratio of the thermal to the above-thermal flux changes during the cycle. This assumption is discussed in Appendix III. The treatment will consider the changes produced by

1. burnup of fuel
2. burnup of boron
3. buildup of xenon
4. buildup of fission products other than xenon.

1. Burnup of Fuel

Let Σ_{25} be the macroscopic absorption cross section of the fuel and σ_{25} be the microscopic absorption cross section of the fuel. Then the number of atoms of fuel burned per second per cc at a point in the core is

$$\frac{dN}{dt} = - \phi \Sigma_{25} \quad (2)$$

* See Appendix III for a description of the change of flux with time.

where ϕ_2 is the thermal flux and N is the number of U^{235} atoms per cc.

Then, since $N\sigma = \Sigma$, expression (2) can be written

$$\frac{1}{\Sigma_{25}} \frac{d\Sigma_{25}}{dt} = -\sigma_{25} \phi_2 \quad (3)$$

and (3) can be integrated to give

$$\Sigma_{25} = \Sigma_{25}^0 e^{-\int \sigma_{25} \phi_2 dt} \quad (4)$$

The change in Σ_{25} is then

$$\Delta \Sigma_{25} = \Sigma_{25}^0 - \Sigma_{25} = \Sigma_{25}^0 (1 - e^{-\int \sigma_{25} \phi_2 dt}) \quad (5)$$

Similarly,

$$\Delta \Sigma_f = \Sigma_f^0 (1 - e^{-\int \sigma_{25} \phi_2 dt}) \quad (6)$$

Substituting (5) and (6) into (1) yields the change in ν due to the burnup of fuel

$$\Delta \nu = \frac{\int \phi_2^+ \Sigma_{25}^0 (1 - e^{-\int \sigma_{25} \phi_2 dt}) \phi_2 - \nu \int \phi_1^+ \Sigma_f^0 (1 - e^{-\int \sigma_{25} \phi_2 dt}) \phi_2}{\int \phi_1^+ \Sigma_f^0 \phi_2} \quad (7)$$

In obtaining (7) ϕ_2 has been replaced by ϕ_2 , since the shape of ϕ_2 is assumed

not to change. Although the shape of ϕ_2 does not change, its magnitude does.

Therefore, $\phi_2(t)$ can be written

$$\phi_2(t) = \phi_2^c(t) \quad (8)$$

where ϕ_2 is the initial flux at $t = 0$, and $c(t)$ is dependent only on time and not on position. Using (8), expression (7) can be rewritten

$$\begin{aligned} \mathcal{V} = & \frac{\int \phi_2^+ \sum_{25}^o (1 - e^{-\phi_2 \sigma_{25} \int c dt}) \phi_2}{\int \phi_1^+ \sum_f^o \phi_2} \\ & - \frac{\mathcal{V} \int \phi_1^+ \sum_f^o (1 - e^{-\phi_2 \sigma_{25} \int c dt}) \phi_2}{\int \phi_1^+ \sum_f^o \phi_2} \end{aligned} \quad (9)$$

In order to use (9) the adjoint flux ϕ_1^+ and ϕ_2^+ must be known. These were obtained in the cases to be discussed using the method described in Appendix IV.

Figures 7-1 and 7-2 show respectively the adjoint thermal flux ϕ_2^+ and the adjoint

above-thermal flux ϕ_1^+ in the reactor midplane for the case of the initial startup

core with aluminum filler pieces in the experimental holes. Figures 7-3 and 7-4 show the adjoint thermal flux and the adjoint above-thermal flux for the core with experiments as of January 1956.

MIDPLANE ADJOINT FLUX ϕ_2^+ ($\text{nv} \times 10^{14}$) IN INITIAL START-UP CORE
(Aluminum in Experimental Holes)

Figure 7-2

Figure 7-4

In order to evaluate the integrals of (9) the vertical distribution of the flux and adjoint flux must be known. Figure 7-5 is a plot of the vertical thermal flux distribution obtained as described in Section II, Flux in the ETR. To make the integrations of (9) more amenable to calculation, the curve of Figure 7-5 was approximated by the straight-line sections shown. Both the straight line figure and the actual flux plot have the same area under the curve. All the flux and adjoint flux distributions in the vertical direction were approximated by the straight-line sections of Figure 7-5.

Since the vertical and horizontal components of the flux are considered separable, the vertical integration in (9) can be performed immediately. With $\Phi(x,y,z) = \Phi(x,y)(z + 0.5)$, for $0 \leq z \leq 1$, (9) becomes

$$\begin{aligned}
 \mathcal{J} = & \frac{2 \int \Phi_2^+ \sum_{25}^0 (1 - e^{-\Phi_2 \sigma_{25} (z + 0.5) \int c dt}) \Phi_2 (z + 0.5)^2 dx dy dz}{\int \Phi_1^+ \sum_f^0 \Phi_2 (z + 0.5)^2 dx dy dz} \\
 & - 2 \frac{\int \Phi_1^+ \sum_f^0 (1 - e^{-\sigma_{25} \Phi_2 (z + 0.5) \int c dt}) \Phi_2 (z + 0.5)^2 dx dy dz}{\int \Phi_1^+ \sum_f^0 \Phi_2 (z + 0.5)^2 dx dy dz}
 \end{aligned} \tag{10}$$

Where the z integration is to be carried between $z = 0$ and $z = 1$, and the fluxes and adjoint fluxes are to be considered functions of x and y alone. The factor of 2 in (10) arises because the z integration is performed only from $z = 0$ to $z = 1$;

the contribution from $z = 1$ to $z = 2$ is of course equal to the contribution from $z = 0$ to $z = 1$.

For simplicity, consider the first integral in the numerator of (10)

$$2 \int_0^1 \int_{x,y} \phi_2^+ \sum_{25}^0 (1 - e^{-\phi_2 \sigma_{25}(z+0.5) \int c dt}) \phi_2 (z+0.5)^2 dx dy dz$$

Substitute $R = z + 0.5$

$$= 2 \int_{x,y} \phi_2^+ \phi_2 \left[\frac{3 \cdot 25}{3} + R^3 e^{-aR} \left\{ \frac{a^2 R^2 + 2aR + 2}{a^3 R^3} \right\} \right] \bigg/_{R=0.5}^{R=1.5} dx dy \quad (11)$$

where $a = \phi_2 \sigma_{25} \int c dt$.

Now ϕ_2^+ and ϕ_2 are known as a function of position. Therefore, the above in-

tegral can be evaluated for a given $\int c dt$. To do this a curve of the quantity in brackets was drawn as a function of a . Then, for a given $\int c dt$ and ϕ_2 , the quantity in brackets was read from the curve and multiplied by $\phi_2^+ \phi_2$. The

integrand was then evaluated point by point, and the integral evaluated numerically by adding the contributions from the various points. In this way expres-

sion (9) was computed as a function of $\int_0^t c dt$. Figure 7-6 is a plot of $-\frac{dV}{V}$

obtained from (9) for the initial startup core as a function of $\int c dt$ in days. In order for the above results to be of use, the quantity $\int c dt$ must be known as a function of actual days of full-power operation.

In Appendix III it is shown that for fuel burnup of the order of 50 percent or less

$\int c \, dt$ may be represented as

$$\int c \, dt = \frac{K - \sqrt{K^2 - 4KBt}}{2B} \quad (12)$$

where

$$c = \frac{K}{\sqrt{K^2 - 4KBt}} \quad (12a)$$

and

$$K = \int \phi_2 \sum_f^0 dV$$

$$B = \frac{1}{2} \int \sigma \phi_2^2 \sum_f^0 dV$$

In the startup core with no power generation in the experimental facilities

$$K = 3.1 \times 10^{10} \times 175 \times 10^6 = 5.42 \times 10^{18} \text{ fissions/sec}$$

B can be evaluated numerically from the flux plots of Figures 2-2 and 7-5, and was computed to be 5.37×10^{11} .

Figure 7-7 is a plot of t vs $\int c \, dt$ as obtained from expression (12). Using Figure 7-7, the reactivity curve of (6) can be drawn with time as the abscissa. This is shown in Figure 7-8.

It should be noted that the change of reactivity due to the change in $\sum_{25}$ is a linear function of time. This is a consequence of the fact that $B/K \approx \frac{1}{2} \sigma \phi_0$.

The average thermal flux in the reactor is 3.3×10^{14} and $\sigma_{25} = 572 \times 10^{-24} \text{ cm}^2$, so $\frac{1}{2} \sigma_{25} \bar{\phi} = 9.6 \times 10^{-8}$. But $B/K = 9.9 \times 10^{-8}$, and thus

PLOT OF $\int c dt$ vs t

FIGURE 7-7

$B/K \approx 1/2 \sigma_{25} \bar{\Phi}$. It is shown in Appendix III that when $B/K \approx 1/2 \sigma \bar{\Phi}$, then to the same order of accuracy as (12)

$$\int c dt = - \frac{\ln}{\sigma \bar{\Phi}} (1 - \sigma \bar{\Phi} t) \quad (13)$$

Expression (12) is obtained from an expansion in which terms of higher order than $(\sigma \bar{\Phi} t)^2$ are dropped, and expression (13) is exact for the case where the flux is independent of position. Without taking higher order terms in the expansion leading to (12), it cannot be determined whether (12) or (13) is more correct. For small $\sigma \bar{\Phi} t$ the two relations are equivalent, and in the range used for these calculations the two expressions give results which are not appreciably different.

We have been dealing with a term of the form

$$\int \Sigma_{25_0} \phi_2^+ (1 - e^{-\phi_2 \sigma_{25} \int c dt}) \phi_2 dV \quad (14)$$

For $B/K \approx 1/2 \sigma \bar{\Phi} t$ expression (13) is valid. Substituting (13) into (14) yields

$$\begin{aligned} \int \Sigma_{25} \phi_2^+ (1 - e^{-\ln (1 - \sigma \bar{\Phi} t)}) \phi_2 dV \\ = \sigma t \bar{\Phi} \int \Sigma_{25} \phi_2^+ \phi_2 dV \end{aligned}$$

so that $\int k/k$ should vary linearly with time as observed.

The second term in expression (9) can now be evaluated in an exactly analogous manner to that described above for the first term. Figure 7-9 is a plot of the total change of $\int k/k$ with time due to the burnup of U^{235} .

2. Burnup of Boron

The effect of the burnup of boron is calculated exactly as described for U^{235} , except of course that the boron cross sections are used and not the U^{235} cross sections. Thus for boron

$$\delta \nu = \frac{\int \phi_2^+ \sum_B^0 (1 - e^{-\sigma_B \phi_2 \int c dt}) \phi_2 d\tau}{\int \phi_1^+ \sum_f \phi_2 d\tau} \quad (15)$$

The relation between $\int c dt$ and t is obtained as before from Figure 7-8. Figure 7-10 is a plot of $\int k/k$ as a function of time due to the burnup of boron.

3. Buildup of Xenon

It is shown in Reference 1* that at equilibrium the macroscopic xenon absorption cross section can be written as

$$\frac{\delta \Sigma_x}{\Sigma_{25}} = \frac{\sigma_2 (\gamma_1 + \gamma_2) \sum_f \phi_2}{(\lambda_2 + \sigma_2 \phi_0) \Sigma_{25}} \quad (16)$$

where

ϕ_2 = the thermal flux

λ_1 = the decay constant of iodine 135

γ_1 = the fractional yield of iodine 135 per fission = 0.056

λ_2 = the decay constant of xenon 135 = 2.5×10^{-5} sec

σ_2 = the microscopic absorption cross section of xenon 135 =
 $2.7 \times 10^{-18} \text{ cm}^2$

* Reference 1, Glasstone and Edlund, Elements of Nuclear Reactor Theory, Van Nostrand, New York, 1954.

$\frac{\Delta \rho}{\rho}$ DUE TO BORON BURNUP

FIGURE 7-10

γ_2 = the fractional yield of xenon 135 per fission = 0.003

Σ_{25} = the uranium macroscopic absorption cross section

Σ_f = the macroscopic fission cross section

For a flux of the order of 4×10^{14} , expression (16) yields

$$\frac{\delta \Sigma_x}{\Sigma_{25}} = 0.049 \quad \text{or} \quad \delta \Sigma_x = 0.049 \Sigma_{25} \quad (17)$$

with a very weak dependence on flux.

In (16) and (17) $\delta \Sigma_x$ is used instead of Σ_x , since any formation of xenon represents a change from the initial conditions.

From (1) therefore

$$\delta \nu_{\text{xenon equilibrium}} = \frac{\int 0.049 \Sigma_{25} \phi_2^+ \phi_2}{\int \Sigma_f \phi_1^+ \phi_2} \quad (18)$$

When (18) is evaluated for the initial startup core

$$-\frac{\delta \nu}{\nu} = \frac{\delta k}{k} = 0.029 \quad (19)$$

Incidentally, if the treatment of Reference 1 for a non-reflected core were used, $\delta k/k$ due to equilibrium xenon would be evaluated as 0.034.

In obtaining (19) the initial value of Σ_{25} was used in (18). However, due to fuel burnup, Σ_{25} changes over the cycle and so therefore will the reactivity effect of the xenon. The change in this effect is

$$\Delta \rho_{\text{xenon equilibrium}} = \frac{0.049 \int \sigma_{25} \phi_2^+ \phi_2}{\int \sum_f \phi_1^+ \phi_2} \quad (20)$$

Thus the change in the reactivity effect of the xenon is just equal to 0.049 times the change in reactivity produced by the change in the absorption cross section of U^{235} . This latter quantity was evaluated as a function of time in Section VII A.1, Burnup of Fuel.

Figure 7-11 is a plot of the buildup of xenon from startup to its equilibrium value. This curve is plotted from equation (11.56.1) of Reference 1 for a flux of 4×10^{14} . It can be seen that it takes over 12 hours for the xenon to get close to its equilibrium value. Thus in the first day of operation the reactivity decrease produced by the xenon will approach the value in (19), and then during the running cycle the xenon effect will decrease as given by (20).

The reactivity effect produced by xenon is shown in Figure 7-12, and the effect of fuel and boron burnup is also shown. Figure 7-12 also shows the effect of fission product formation other than xenon, and this is discussed below.

4. Buildup of Fission Products other than Xenon

The buildup of fission products other than xenon in highly enriched reactors has been studied in detail by Deutsch.* Figure 7-13 is a plot of Deutsch's data adapted for the ETR. In computing the fission product effects as a function of time, Deutsch's curve was approximated by the initial step and the straight line as shown in Figure 7-13.

* R. Deutsch, Fission Product Buildup in Fully Enriched Reactors, KAPL Memo R.W.D.-10, January 5, 1956.

TIME BUILDUP OF XENON

FIGURE 7-11

$\frac{\Delta k}{k}$ DUE TO BURNUP IN STARTUP CORE

FIGURE 7-12

TOTAL FISSION PRODUCTS (LESS XENON)

FIGURE 713

$$\delta \Sigma_{fp} = 0.000934 + 0.038 \delta \Sigma_{25} \quad (21)$$

Then the effect of the fission products on reactivity is given by

$$\delta \rho = \frac{\int \phi_2^+ \delta \Sigma_{fp} \phi_2 d\tau}{\int \phi_1^+ \Sigma_f \phi_2 d\tau} \quad (22)$$

The effect of the first term in (21) can be evaluated immediately. The effect of the second term as a function of time can be obtained (as with the xenon) by multiplying by 0.038 the results obtained in Section VII. A. 1. for the effect of the change in Σ_{25} on reactivity.

The final results can be brought closer to reality by starting the curve of $\delta k/k$ at zero for zero time and then joining it to the curve for the results of expression (22) by a smooth curve. The two curves are joined at the time when the average change in Σ_{25} due to burnup is about 0.006.

Figure 7-12 is a plot showing the changes in reactivity as a function of time in the initial startup core due to the four effects discussed above. Also shown is the drop in reactivity (0.85 percent) caused by the initial rise in temperature when the ETR is first started up. This is discussed in Section X.

Figure 7-14 is a plot of the excess k available during the ETR running cycle. The curve is plotted assuming at startup one grey rod absorber section is in the core, and there is therefore 11 percent excess k initially available. The

% EXCESS $\frac{\Delta K}{K}$ IN STARTUP CORE

FIGURE 7-14

reactor will run 20 days before its excess reactivity is reduced to zero, and this is exactly what is desired for the ETR. The curve of Figure 7-14 is of course no accident. The running cycle of the ETR can be adjusted within wide limits by changing the fuel and boron loading. The advantage of the methods used above is that a rational means is supplied which makes it possible to determine the loading required for a desired running cycle. This is discussed below.

B. The Use of Burnable Poisons in the ETR

The operation of the burnable poison, boron, in limiting the reactivity excursion due to burnup is clearly illustrated in Figure 7-12. It can be seen from this figure that the boron reduces the excursion in k by 8 percent at the end of the cycle, changing the effect of fuel burnup from a 15 percent to a 7 percent loss of k . Now it is plain from expression (15) that any reduction desired in the loss of k by fuel burnup can be achieved by increasing the initial content of boron, that is, making $\sum_B^0$ larger. According to (15) the boron effect is proportional to $\sum_B^0$. Of course, the difficulty is that an initial increase of boron calls for an initial increase of U^{235} in order to keep the reactor critical at startup. In calculating the problem the situation faced is that a change in boron requires a change in U^{235} , which in turn changes the effect of the boron, etc. To adequately account for this using standard techniques would require a large number of UNIVAC calculations which is an expensive investment in time and money. Fortunately, the methods developed above lead to a fairly simple means of solving the problem.

Again we make the assumption that the shape of the thermal flux does not change. Now examine the effect of increasing the boron content by the factor B . If the reactor were critical the increase in boron content would make it subcritical, so that the U^{235} content must be increased by a factor U to compensate for the boron. Then, from expression (15) it can be seen that the net result for a given $\int c \, dt$ is to multiply the effect of the boron by the factor B/U . On the other hand, since both the numerator and the denominator of (7) are multiplied by U , the effect of the U^{235} burnup for a given $\int c \, dt$ does not change. Similarly, since xenon and other fission-product effects were proportional to the first term in (7), these effects will also be unchanged for a given $\int c \, dt$.

In addition to these effects at a constant power level, the increase in U^{235} will change the thermal flux by the factor $1/U$, and $\int c \, dt$ will also be changed. Reference to either (12) or (13) shows that if $\int c \, dt = f(t)$ for a given ϕ_2 , then if ϕ_2 changes to ϕ_2/U , $\int c \, dt$ changes to $U f(t/U)$ and therefore $\phi_2 \int c \, dt = \phi_2 f(t)$ changes to $\phi_2 f(t/U)$. In other words, the time scale must be multiplied by the factor U in order to keep $\phi_2 \int c \, dt$ the same as before the change.

To sum up then, if the boron is increased by the factor B and the U^{235} by the factor U , in order to describe the new situation the time scale of Figure 7-13 must be multiplied by U and the values of the boron curve by B/U .

Using the methods described above, the running cycle shown by Figure 7-14 is obtained with an initial fuel element loading of 255 g of U^{235} and 1.55 g of boron per fuel element. The control rod fuel sections are loaded with 40 percent of this amount of U^{235} and boron. The cycle shown by Figure 7-14 requires the

least amount of U^{235} (and boron) to achieve a 21-day cycle. Figure 7-15 illustrates the running cycle with a loading of 311 g of U^{235} and 3.1 g of boron per fuel element, with 40 percent of the loadings per control rod fuel section. The cycle illustrated by Figure 7-15 has advantages in that a large amount of excess k is available for the whole 21-day cycle (and can of course run longer than 21 days). However, this cycle requires a high fuel element loading.

Figure 7-16 shows the running cycle expected when there is no burnable poison present. In this case the fuel loading is reduced to 224 g per fuel element, but the running cycle is only seven days.

At present it is planned to do the initial testing of the ETR with the aluminum filler pieces in the experimental holes. However, there is some possibility that one or more of the facility pipes may be in place at startup. To gain some knowledge of the effect of the facility pipes a 6 by 6-inch hole was assumed to contain a six-inch diameter, 1/4-inch thick, stainless steel pipe and to be otherwise filled with aluminum filler pieces. New cross sections for the 6 by 6 hole were computed, and these new cross sections were then assumed to apply to all of the facility holes. The additional U^{235} required to compensate for the presence of the stainless steel was then computed using perturbation theory (see expression (38), Appendix IV) and the running cycle computed as described above. The loading for the case of the facility tubes in place was 311 g of U^{235} and 1.55 g of boron per fuel element.

The running cycle with this loading is shown in Figure 7-16. These results indicate that the loading must be increased considerably to compensate for the absorp-

% EXCESS $\frac{\Delta k}{k}$ - STEEL PIPES IN STARTUP CORE (U^{235} INCREASED 22%)

FIGURE 7-16

tion in the pipes, and that with the above loading the running cycle will be only 18 days.

The discussion thus far has described the results obtained for the ETR core with filler pieces in the experimental holes. Similar core calculations were also performed with experiments as of January, 1956, in the core. Since it was known at the time of the computation that the experiments would be modified considerably, the analysis described above was also modified somewhat. In order to obtain the effects of U^{235} burnup the exponents in expression (7) were expanded, and only the most significant term was considered. Thus for small t

$$\rho_v = \frac{t \left[\int \phi_2^+ \sum_{25}^0 \phi_2^2 d\tau - \int \phi_1^+ \sum_f^0 \phi_2^2 d\tau \right]}{\int \phi_1^+ \sum_f^0 \phi_2} \quad (23)$$

This then gives the initial slope of reactivity with time. In performing the calculation it was assumed that the slope was constant. As shown above this assumption is equivalent to assuming that $B/K \approx 1/2 \overline{\phi} \sigma_{25}$, and in this

case $\int c dt = - \frac{\ln (1 - \sigma \overline{\phi} t)}{\sigma \overline{\phi}}$. Thus, the effect of the U^{235} burnup can

be handled. The initial slope of the curve for boron was computed by expanding the exponent in expression (15). The saturation value, ρ_{vs} , produced by the complete burn-up of boron was also obtained from expression (15).

$$\delta \nu_s = \frac{\int \phi_2^+ \delta \Sigma_a \phi_2 dT}{\int \phi_1^+ \Sigma_f \phi_2 dT} = \frac{\int \phi_2^+ \Sigma_B^0 \phi_2 dT}{\int \phi_1^+ \Sigma_f \phi_2 dT} \quad (24)$$

Let T be the time at which the straight line of (23) reached the value of (24).

Then the boron curve was assumed to have the shape $\delta \nu_B = \delta \nu_s (1 - e^{-t/T})$.

Using this calculation approach, the curves of Figure 7-17 were constructed for the case of the minimum fuel loading required for a 20-day cycle. The loading is 246 g of U^{235} and 2.80 g of boron per fuel rod, with 40 percent of this loading per control rod fuel section. The total mass of U^{235} in the core is 19.2 kg, and this reflects the large absorption effects of the structural materials in the holes. The large boron content of the fuel elements is required because the boron in the fuel section must compensate for fuel burnup in the fuel section as well as experimental holes.

In this core, with experiments as of January, 1956, the loading without boron is 162 g U^{235} per fuel element. The running cycle in this case is three days as shown by Figure 7-18.

% EXCESS $\frac{\Delta K}{K}$ IN CORE WITH EXPERIMENTS

FIGURE 7-17

VIII. THE XENON PROBLEM IN THE ETR

A. Statement of the Problem

During operation of the ETR, xenon (with a neutron absorption cross section of 3×10^6 barns) is formed as a product of the fission process. Although some xenon (an average of 0.003 atoms per fission) is produced directly in the fission process, most of the xenon (0.056 atoms per fission) is formed from the decay of iodine 135, which has a 6.7-hour half-life. When the reactor is started up, the concentration of xenon increases until an equilibrium value is reached, that is, the number of xenon atoms formed per second is equal to the number which are removed per second by neutron capture and radioactive decay. The effect of equilibrium xenon has been discussed in Section VII, where it was shown that the xenon produced about a three percent negative effect on reactivity. Thus, the main effect of the presence of equilibrium xenon is to require an initial excess k of three percent. However, in terms of reactor operation, equilibrium xenon does not present a serious problem.

When the reactor is shut down after running at power, the xenon concentration increases markedly since the iodine continues to decay. If the reactor is not quickly brought up to power again, the xenon will poison the reactor so that a restart will not be possible until after the xenon has decayed or until the reactor fuel is changed. The period between shutdown and the time when a restart becomes impossible depends upon the excess k available at shutdown and the speed of the control rods. If the reactivity insertion rate of the con-

trol rods is less than the reactivity poisoning rate introduced by the xenon buildup, it will never be possible to override the xenon after a shutdown. If the reactivity insertion rate of the rods is greater than the reactivity poisoning rate due to the xenon buildup, then the time available for a shutdown can be determined by equating the decrease in reactivity, which occurs before and during the withdrawal of the control rods, to the excess reactivity available in the core at the time of shutdown.

MTR experience indicates that shutdowns will not be uncommon during operation of the ETR. Efficient operation of the reactor therefore requires that the control rods be able to move as fast as possible, consistent with safety considerations. The determination of a safe control rod removal rate is discussed in Section IX. The following analysis of the xenon buildup problem indicates that the time available for a shutdown will vary between about 25 minutes in the early part of the running cycle to zero at the very end of the cycle when no excess k is in the reactor.

B. Xenon Buildup After a Shutdown

It can be shown that the poisoning, P , due to the buildup of xenon after a complete shutdown is*

$$P = \sigma_2 \phi_2 \frac{\sum_f}{\sum_{25}} \left[\frac{\gamma_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 T} - e^{-\lambda_2 T}) + \frac{\gamma_1 + \gamma_2}{\lambda_2 + \sigma_2 \phi_2} e^{-\lambda_2 T} \right] \quad (1)$$

* Glasstone and Edlund, The Elements of Nuclear Reactor Theory, Chapter 11.

where $P = \text{poisoning} = \frac{\sum_{\text{xenon}}}{\sum_{25}}$

T = time after shutdown

ϕ_2 = the steady-state flux before shutdown

λ_1 = the decay constant of iodine 135 = $2.9 \times 10^{-5} \text{ sec}^{-1}$

γ_1 = the fractional yield of iodine 135 per fission = 0.056

λ_2 = the decay constant of xenon-135 = $2.5 \times 10^{-5} \text{ sec}^{-1}$

σ_2 = the microscopic cross section for neutron capture in xenon-135 = $2.7 \times 10^{-18} \text{ cm}^2$

γ_2 = the fractional yield of xenon 135 per fission = 0.003

$\sum_f$ = the thermal neutron macroscopic fission cross section in neutron fuel

$\sum_{25}$ = the thermal neutron absorption cross section in the fuel

$$\frac{\sum_f}{\sum_{25}} = 0.84 \text{ for uranium}$$

For time periods less than a few hours, the exponents of (1) can be expanded in a power series. This leads to

$$P = \frac{\sum_x}{\sum_{25}} = \gamma_1 \sigma_2 \phi_2 \frac{\sum_f}{\sum_{25}} T \quad (2)$$

Expression (2) neglects the term $\sigma_2 \phi_2 \frac{\sum_f}{\sum_{25}} \frac{\gamma_1 + \gamma_2}{\lambda_2 + \sigma_2 \phi_2}$ which is the

equilibrium value of poisoning. Thus, (2) represents the excess poisoning which occurs after a shutdown. According to perturbation theory, the re-

activity effect of an increase in absorption is

$$\begin{aligned} \frac{\delta k}{k} &= - \frac{\delta \nu}{\nu} = \frac{- \int \phi_2^+ \delta \Sigma_a \phi_2 d\tau}{\nu \int \phi_1^+ \Sigma_f^0 \phi_2 d\tau} \\ &= \frac{- \int \phi_2^+ \delta \Sigma_{25} \phi_2 d\tau}{\nu \int \phi_1^+ \Sigma_f^0 \phi_2 d\tau} \end{aligned}$$

Again assuming that $\phi_2(x, y, z, t) = \phi_2(x, y, z) c(t)$ with $c(0) = 1$ (See Appendix IV)

$$\begin{aligned} \frac{\delta k}{k} &= \frac{- \gamma_1 \sigma_2 \Sigma_f T}{\nu} \cdot \frac{\int \phi_2^+ \Sigma_{25}^0(t) c(t) \phi_2^2 d\tau}{\int \phi_1^+ \Sigma_f^0 \phi_2 d\tau} \\ &= \frac{- \gamma_1 \sigma_2 \Sigma_f T}{\Sigma_{25}} \cdot \frac{\int \phi_2^+ \Sigma_{25}^0 e^{-\phi_2 \sigma_{25} \int_0^t c dt} \phi_2^2 d\tau}{\nu \int \phi_1^+ \Sigma_f^0 \phi_2 d\tau} \\ &= \frac{- \gamma_1 \sigma_2 \Sigma_f T}{\sigma_{25} \Sigma_{25}} \cdot \frac{d}{dt} \left[\frac{\int \phi_2^+ \Sigma_{25}^0 (1 - e^{-\phi_2 \sigma_{25} \int_0^t c dt}) \phi_2 d\tau}{\nu \int \phi_1^+ \Sigma_f^0 \phi_2 d\tau} \right] \quad (3) \end{aligned}$$

where $\underline{t}$ is the time of the running cycle before the shutdown, and T is time after the shutdown. The factor $\sum_{25}^0 e^{-\phi_2 \sigma_{25} \int_0^t c dt}$ is the value of $\sum_{25}$ at time $\underline{t}$ in terms of $\sum_{25}^0$, the initial value of $\sum_{25}$ at $t = 0$. The quantity in the brackets of (3) is just the $\delta k/k$ produced by the change of $\sum_{25}$ during the running cycle. This was computed in Section VII, Burnup in the ETR. Thus (3) can be readily evaluated as a function of time during the running cycle.

In the initial startup core the quantity in the brackets of (3) was found to be a linear function of time equal to $0.032 t$. Then for this case the decrease in reactivity per second at time T after a shutdown is

$$\frac{1}{k} \frac{dk}{dt} = -0.16 T \quad (4)$$

where $\underline{t}$ is the time of the running cycle before the shutdown.

Figure 7-14 of Section VII shows the excess reactivity available for the initial startup core during the running cycle of the ETR. Using Figure 7-14 and expression (4), the time required after a shutdown (T_M) for the reactivity to decrease to a value equal to the excess reactivity available can be determined as a function of t . Thus, $T_M(t)$ represents the maximum time available for a restart, assuming that the control rod insertion and withdrawal times are negligible. $T_M(t)$ is shown in Figure 8-1.

It can be seen from Figure 8-1 that in the event of a scram the operators have little time to locate the cause of the scram, correct it, and then restart before xenon buildup occurs. The time actually available is reduced from the time

Figure 8-1

available in Figure 8-1 by insertion and withdrawal times of the control rods. Thus, the necessity of making these times as small as possible consistent with safety considerations is obvious. As is explained in Section IX, about two minutes is required to run the control rods down after a scram, and an additional four minutes is required to withdraw the rods. While the rundown is taking place the cause of the scram can be sought. Thus the two-minute rundown time does not interfere with the corrective action. The T_M of Figure 8-1 should therefore be reduced by four minutes to allow for the withdrawal time of the rods. In addition, if T_M is less than six minutes, a restart is not possible. This situation is illustrated in Figure 8-2.

During the running cycle the time available for a restart decreases from an initial value of 28 minutes at the start of the cycle. By the seventeenth day only the two-minute rundown time is available, after which withdrawal of the rods must immediately begin. After the seventeenth day a restart is not possible, and the core must be reloaded or a period of two days allowed for the xenon to decay.

Figure 8-3 shows the time available for a restart as a function of time during the running cycle for the core with experiments as of January, 1956. Figures 8-2 and 8-3 are based on the assumption that a full scram has occurred. If a partial scram takes place (only some rods are dropped), the time available for a restart will be increased.

Figure 8-2

Figure 8-3

IX. STARTUP OF THE ETR AND WITHDRAWAL SPEED OF CONTROL RODS.

The maximum withdrawal speed of the control rods is set so that the fuel plates in the ETR will not melt in the event of a startup accident. Suppose that the reactor is critical at startup and that the control rods are withdrawn at the maximum rate. The reactor flux will rise on a fast period, and if it is assumed that the period safety circuit does not function, the rods will continue to be withdrawn until a scram is initiated by one or all of the three flux limit safety channels. These channels are set at a level above the normal flux, and when the flux exceeds this level a scram is initiated.

The sensing device in the safety circuit is an ionization chamber. The delay time from the generation of a scram signal to the time when the control rods are free to drop (under gravitational and water flow forces) is estimated to be 11 milliseconds. This includes a one millisecond electronic delay and a 10 millisecond mechanical delay in the rod release mechanism. In order to allow for some variation in the release mechanisms, all the calculations described in this section have been performed assuming a 25 millisecond delay. After the control rods start to drop into the core, the flux will continue to rise for a short period and will then drop sharply.

It has been shown by Newson* that if a reactor starts up in a prompt critical state and the control rods are withdrawn at a constant rate, then the following expression holds:

* H. W. Newson, The Control Problem in Piles Capable of Very Short Periods, Mon P-271, 1947.

$$\Theta > \sqrt{\frac{L}{2R \ln N_1/N_s}} \quad (1)$$

where L = mean lifetime of a neutron in the reactor

R = rate of increase of k

N_1 = neutron level at which the safety devices operate

Θ = the period when the level N_1 is reached

N_s = neutron level at startup

After the level N_1 is reached at time T_1 , there will be a delay of time $(T_2 - T_1)$ before the control rods are free to drop. During this time the flux will rise to a level N_2 such that

$$\frac{N_2}{N_1} < e^{\frac{T_2 - T_1}{\Theta}} \quad (2)$$

After the level N_2 is reached the control rods will fall into the core under gravitational and water flow forces. However, the flux will continue to rise for a short period, and Newson shows that under gravitational forces alone the maximum flux reached ($N_{\max}$) will be such that

$$\ln \frac{N_{\max}}{N_2} < 6.0 \times 10^{-2} \left[\ln \frac{N_1}{N_s} \right]^{3/4} \left[\frac{x^2 \Delta k}{LD^3} \right]^{1/4} \quad (3)$$

where $N_{\max}$ = highest neutron level reached

N_1 = neutron level at which the safety devices separate

N_s = neutron level at startup

X = total travel of rods in cm

ΔK = total δk in rods being withdrawn (in these calculations Δk is taken as 0.16 which corresponds approximately to the removal of the four black rods)*

D = total time to withdraw rods

L = mean lifetime of a neutron in the reactor

The time when this maximum is reached ($T_{\max}$) is shown by Newson to satisfy the expression

$$(T_{\max} - T_2) < \left[\frac{L}{g \ominus S} \right]^{1/2} \quad (4)$$

where $S = 2\Delta k/x$

and g = gravitational acceleration = 980 cm/sec²

With this information a conservative estimate of the power overshoot during a startup accident can be made. The reactor is assumed to be on a period $\ominus$ when the flux reaches the level N_1 . With this period the flux rises during the time $T_2 - T_1$. After time T_2 the flux rises to $N_{\max}$ in time $T_{\max} - T_2$. At time T_1 the flux N_1 is already above the normal operating level of the ETR. Thus the overshoot curve should be extended back from the level N_1 to the operating level N_0 , again assuming a period of $\ominus$ during the time $T_1 - T_0$. The above procedure provides a picture of the rise of power above the operating level. The fall-off of power can be obtained in the following way. Newson

* The calculations of Section VI show that Δk is 14.4 percent in the startup core. Thus, the results derived will slightly overestimate the overshoot.

shows that the change in neutron density (N_1) can be represented by

$$\frac{1}{N} \frac{dN}{dt} = \frac{k-1-B}{L} + \frac{1}{N} \sum_i \frac{C_i}{\tau_i}$$

where k = the effective multiplication constant

B = the fraction of delayed neutrons

$\frac{1}{N} \sum_i \frac{C_i}{\tau_i}$ = the contribution from delayed neutrons

L = mean lifetime of a neutron in the reactor

at $T_{\max}$

$$\frac{1}{N} \frac{dN}{dt} = 0$$

at time t , after $T_{\max}$

$$\frac{1}{N} \frac{dN}{dt} = \frac{k(t) - k(T_{\max})}{L}$$

Here we neglect the change in the delayed neutron contribution during the short periods of interest. At time T_2 the rods started to drop. Assuming the rods drop with gravitational acceleration alone (ignoring the hydraulic forces), then

$$k(t) - k(T_{\max}) = -S \left[\frac{1}{2} g (t - T_{\max})^2 + V_m (t - T_{\max}) \right] \quad (7)$$

where $V_m = g (T_{\max} - T_2)$

g = gravitational constant

$S = \frac{\delta k}{\delta x}$; assumed constant

In obtaining (3) Newson assumed that $S = \frac{2\Delta k}{x}$ in order to obtain the largest estimate of $N_{\max}$. It will therefore be a conservative estimate if at this point we take $S = \frac{\Delta k}{\Delta x}$.

If expression (6) is integrated from $T_{\max}$ to t , the fall-off of N with time can be obtained. The calculations described here were performed using the following values:

$$L = 5 \times 10^{-5} \text{ seconds}$$

$$\frac{N_1}{N_s} = 10^{10}$$

$$T_2 - T_1 = 25 \text{ milliseconds}$$

$$x = 90 \text{ cm}$$

$$\Delta K = 0.16$$

The calculations were carried out for three values of Θ :

$$\Theta = 0.100 \text{ second}$$

$$\Theta = 0.050 \text{ second}$$

$$\Theta = 0.020 \text{ second}$$

The results of these calculations are shown in Figures 9-1, 9-2, and 9-3, where N_1/N_0 has been taken as 1.5. The scram level set at 1.5 times full power corresponds to present operation on the MTR.

The plots of the power overshoot have been analyzed from a heat transfer viewpoint to determine whether or not the fuel plates will melt. These heat transfer calculations proceeded on the conservative assumption that the reactor was at operating temperature during startup. Such a condition might

THE POWER OVERSHOOT DURING
A STARTUP ACCIDENT

Figure 9-1

THE POWER OVERSHOOT DURING A STARTUP ACCIDENT

Figure 9-2

THE POWER OVERSHOOT DURING
A STARTUP ACCIDENT

Figure 9-3

occur if the reactor was started up, brought up to temperature, shut down, and then started up again. The heat transfer calculations indicated that a period of about 60 milliseconds or less could be tolerated with $N_1/N_0 = 1.4$.

For a Θ of 65 milliseconds, expression (1) can be inverted to give the rate of reactivity increase, R. Using $L = 5 \times 10^{-5}$ and $N_1/N_S = 10^{10}$, a value for R of 0.00026 is obtained. For ganged withdrawal of the black rods

$$R = \frac{2\Delta k}{\Delta x} \times \text{withdrawal speed}$$

$$R = \frac{2 \times 0.16}{90} \times \text{withdrawal speed}$$

then,

$$\begin{aligned} \text{withdrawal speed} &= \frac{90}{2 \times 0.16} \times 0.00026 \\ &= 0.073 \text{ cm/sec} \end{aligned}$$

Thus 1200 seconds, or 20 minutes, will be required to remove the black rods, and an additional 20 minutes to remove the grey rods. Reference to Section VIII, The Xenon Problem in the ETR, indicates that such a slow withdrawal speed will not permit a restart when the reactor scrams. This situation results in very inefficient operation of the reactor. The solution* adopted for the ETR is to have a startup scram level set at 10 percent of full power. The control system is arranged such that the scram level can be raised to 1.4 of normal power when the reactor is safely brought to a power level somewhat below 10 percent of full power. In this way, if a startup accident

* This solution was suggested by D. Deboisblanc of Philips Petroleum Company.

occurs when the reactor is on a fast period, the reactor will scram at 10 per cent of full power. A large overshoot can then be tolerated, since a factor of 10 increase in power is required before the normal operating power is reached.

After reaching the 10 percent power level safely the reactor can proceed to full power, since the scram level is moved up to 1.4 full power. The control rod speed is set so that if the reactor becomes prompt critical at 10 percent of full power and the rods are withdrawn at the maximum rate, the reactor will be on a safe period if a scram is reached at 1.4 full power. The rod speed so determined is 16 inches per minute.

With 0.16 k in the ganged black rods moving at 16 inches per minute, the maximum increase in reactivity is

$$\begin{aligned} R &= 2 \times \frac{0.16 \text{ k}}{90 \text{ cm}} \times \frac{16 \times 2.5 \text{ cm/sec}}{60} \\ &= 2.4 \times 10^{-3} \text{ } \delta\text{k/sec} \end{aligned} \quad (8)$$

If (8) is substituted into (1) with $L = 5 \times 10^{-5}$ and $N_1/N_s = 14$, the period obtained is

$$\Theta > \sqrt{\frac{5 \times 10^{-5}}{2 \times 2.4 \times 10^{-3} \times \ln 14}} = 63 \text{ milliseconds}$$

It has previously been stated that such a period is safe. On the other hand, if a scram occurs at the 10 percent power level the period becomes

$$\Theta > \sqrt{\frac{5 \times 10^{-5}}{2 \times 2.4 \times 10^{-3} \times \ln 10^{10}}} = 21 \text{ milliseconds}$$

Reference to Figure 9-1 shows that for a 20 millisecond period the overshoot above the scram level is $18.4/1.5 = 12.3$. Thus, during a startup accident from zero power, the reactor power will rise to a peak of 1.23 of normal power level, but will remain above the normal power level for only 30 milliseconds. Such an overshoot is safe.

X. TEMPERATURE AND VOID COEFFICIENTS IN THE ETR

General Considerations

The ETR is a reactor which differs from reactors previously built in that it has a large number of in-core experimental facilities. The experiments of course represent potential sources of trouble, since by their nature they include components whose properties are not fully known. Because the ETR is an experimental reactor of this type, it is important that in addition to having adequate scram devices the reactor be stable and self-limiting in the event of a power excursion.

The temperature and void coefficients of the reactor are of particular interest. It is of course extremely desirable that these coefficients be negative.

It was the desire for information concerning these coefficients that led to the development of the perturbation treatments which have been used so extensively throughout the analysis of the ETR. The temperature and void coefficients are dependent on the geometry of the reactor, and therefore require a two-dimensional calculation in order to obtain meaningful results. The initial attempt at the calculation of the temperature coefficient consisted of performing a UNIVAC calculation with all cross sections computed for a temperature of 50°C. The calculation was then repeated with cross sections computed at 70°C. The difficulty encountered in this attempt was due to the high accuracy required in the UNIVAC calculation. As will be shown, the ETR has a temperature coefficient in the order of 10^{-4} δ k per

degree centigrade. Thus, in a temperature change of 20 degrees there will be a change in k of about 0.2 percent. This means that in order to obtain meaningful results the calculations must be accurate to five figures. The multigroup computer codes available for two-dimensional calculations solve the diffusion equations by means of iterative procedures. The calculation is repeated many times and the results of the successive calculations asymptotically approach the true answer. When the calculation described above was attempted, the convergence of the successive results was so slow that the computer time required to obtain the necessary accuracy would have resulted in a prohibitive cost.

The perturbation techniques make it possible to directly relate the change in reactivity to the change in a reactor parameter. In this manner, the necessity of taking the difference between two very large numbers in order to obtain a number of small magnitude has been eliminated. In addition, one set of UNIVAC calculations is all that is required to compute the effect of any small change from the initial conditions.

B. Computations

According to perturbation theory the change in reactivity due to a change in reactor parameters can be written in matrix form as

$$\delta \rho = \frac{\int \Phi^+ \delta K \Phi d\tau - \nu \int \Phi^+ \delta J \Phi d\tau}{\int \Phi^+ J \Phi d\tau} \quad (1)$$

where the notation is the same as in Appendix IV. In a two-group calculation expression (1) becomes

$$\begin{aligned}
 \mathcal{V} = \int & \left[\mathcal{D}_1 (-\phi_1^+ \nabla^2 \phi_1) + \mathcal{L}_{SD} (\phi_1^+ \phi_1 - \phi_2^+ \phi_1) \right. \\
 & + \mathcal{D}_2 (-\phi_2^+ \nabla^2 \phi_2) + \mathcal{L}_a (\phi_2^+ \phi_2) \\
 & \left. - \mathcal{V} \mathcal{L}_f (\phi_1^+ \phi_2) + \mathcal{B}^2 (\mathcal{D}_2 \phi_2^+ \phi_2 + \mathcal{D}_1 \phi_1^+ \phi_1) \right] d\tau \\
 & \int \phi_1^+ \mathcal{L}_f \phi_2 d\tau
 \end{aligned} \tag{2}$$

where the integration is over all space.

The fluxes used in (2) are solutions of the equations

$$\mathcal{D}_1 \nabla^2 \phi_1 - \mathcal{L}_{SD} \phi_1 + \mathcal{V} \mathcal{L}_f \phi_2 = 0 \tag{3a}$$

$$\mathcal{D}_2 \nabla^2 \phi_2 - \mathcal{L}_a \phi_2 + \mathcal{L}_{SD} \phi_1 = 0 \tag{3b}$$

Expression (3a) can be solved for $\nabla^2 \phi_1$ to yield

$$\nabla^2 \phi_1 = \frac{\mathcal{L}_{SD} \phi_1 - \mathcal{V} \mathcal{L}_f \phi_2}{\mathcal{D}_1} \tag{4a}$$

and similarly, using (3b)

$$\nabla^2 \phi_2 = \frac{\mathcal{L}_a \phi_2 - \mathcal{L}_{SD} \phi_1}{\mathcal{D}_2} \tag{4b}$$

Expressions (4a) and (4b) can be used in (2) for $\nabla^2 \phi_1$ and $\nabla^2 \phi_2$ respectively. Since the flux and adjoint flux are known from the UNIVAC calculations, it is only necessary to compute the various changes of cross sections and integrate (2) numerically. The calculation of the change in cross sections is discussed in Appendix VI.

Using expression (2), in the fuel section of the initial startup core the temperature coefficient was computed to be -2.7×10^{-4} k/°C and the void coefficient -2.8×10^{-3} k/% void in H₂O. In the control rods the temperature coefficient was computed to be 6.7×10^{-6} k/°C.

The temperature coefficients computed above are for a one degree temperature rise in both the water and metal in the fuel region. During a fast power rise, the metal in the fuel element may increase in temperature before the water is heated significantly. The temperature coefficient for an increase in metal temperature alone is -2.0×10^{-5} k/°C.

The coefficients quoted above are for full power operation at the start of the running cycle after the xenon has reached its equilibrium value. This was the condition under which the fluxes and adjoint fluxes were obtained on the UNIVAC. The void coefficient at startup when the reactor is at 20 C and the xenon concentration zero can be computed by calculating the changes in cross section at 20 C and using expression (2). However, in order to compute the temperature coefficient under these conditions, the effect of the

xenon must be removed.* Before xenon buildup the reactor contains more absorber and less uranium than after xenon buildup, since the grey absorber sections are moved out of the core to compensate for the xenon buildup. Both the nickel in the absorber section and the uranium in the fuel section of the rod have closer to a $\frac{1}{\sqrt{E}}$ energy dependent absorption cross section than xenon. In order to keep the reactor critical when the xenon builds up, absorber must be removed and uranium inserted so that (approximately)

$$\Sigma_x = \delta \Sigma_a + \nu \delta \Sigma_f \quad (5)$$

Where $\delta \Sigma_a$ represents the absorber cross section removed and $\delta \Sigma_f$ fission cross section added to compensate for the xenon buildup represented by the cross section Σ_x . The right side of (5) can be considered a fictitious absorption cross section, Σ' , just equal to the equilibrium xenon cross section, but having an $\frac{1}{\sqrt{E}}$ energy dependence. Thus, in obtaining the temperature coefficient before xenon buildup the contribution to $\delta \Sigma_a$ from the xenon should be replaced with Σ' . Since the xenon cross section falls less rapidly with energy than $\frac{1}{\sqrt{E}}$, the net effect will be to make the temperature coefficient less negative.

Using the methods described above the temperature coefficient at cold start-up in the initial startup core was computed to be -3.8×10^{-4} k/ $^{\circ}$ C. It is pos-

* Goertzel and Oppenheim, Temperature Dependence of Xenon Cross Section, Y-691, ORNL, 1953.

sible to have an over-all negative temperature or void coefficient, and at the same time have the coefficient positive over a region of the core. In order to determine the distribution of the void coefficient in the core, the numerator of (2) was evaluated point-by-point in the core. Figure 10-1 is a plot of the void coefficient in a core with experiments very similar to those specified by the experimenters as of January, 1956. For this calculation the ETR was "mocked-up" in cylindrical co-ordinates. This was one of the first two-dimensional perturbation calculations performed, and at the time the XY code was not available. It can be seen in Figure 10-1 that the void coefficient is most negative at the core hot spot, a very satisfactory situation from a safety standpoint. A calculation to determine the distribution of the void and temperature coefficients was not repeated in the XY geometry. In the XY geometry the coefficients were calculated at the core hot spot.

At operating temperature with equilibrium xenon, the void coefficient at the core hot spot was computed to be -3.6×10^{-8} k/% void in H_2O/cc and the temperature coefficient -2.5×10^{-9} k/ $^{\circ}C/cc$.

The values previously quoted for temperature and void coefficients have been for the initial startup core with aluminum filler pieces in the experimental holes. The corresponding values for the core with experiments as of January, 1956, are listed below.

Start of cycle at operating temperature and xenon equilibrium

Over-all temperature coefficient in fuel section
of core

-3.7×10^{-4} k/ $^{\circ}C$

VOID COEFFICIENT IN REACTOR MIDPLANE PER PERCENT VOID IN WATER PER CC ($-\frac{\partial k}{\partial K} \times 10^{-8}$)

Figure 10-1

Start of cycle at operating temperature and xenon equilibrium (Cont'd)

Over-all void coefficient in fuel section
of core

$$-2.5 \times 10^{-3} \text{ k/\% void in } \text{H}_2\text{O}$$

Temperature coefficient at core hot spot

$$-4.7 \times 10^{-9} \text{ k/}^\circ\text{C/cc}$$

Void coefficient at core hot spot

$$-4.3 \times 10^{-8} \text{ k/\% void in } \text{H}_2\text{O/cc}$$

Cold startup and no xenon

Over-all temperature coefficient in fuel
section

$$-3.8 \times 10^{-4} \text{ k/}^\circ\text{C}$$

In addition, studies were made of the effect of changes in some of the experiments at operating temperature.

Temperature coefficient in 9 by 9 ANP hole $-1.4 \times 10^{-5} \text{ k/}^\circ\text{C}$

Temperature coefficient in 6 by 6
WAPD hole

$$-8.1 \times 10^{-5} \text{ k/}^\circ\text{C}$$

Void coefficient in 6 by 6 WAPD hole

$$-5.6 \times 10^{-5} \text{ k/\% void in } \text{H}_2\text{O}$$

Effect of flooding 9 by 9 ANP hole

$$+6.5 \text{ \% k}$$

Effect of flooding 6 by 9 ORNL hole

$$0 \text{ \% k}$$

Some of the experiments above operate at temperatures in excess of the fuel section temperature. In these cases the neutron temperature was assumed to be that of the water in the fuel section of the core.

XI. THE NEUTRON LIFETIME

In previous work we have been concerned with the flux. The flux is the product of the neutron density, n , and the average group velocity, v , so that

$\Phi_1 = n_1 v_1$. Instead of writing the reactor equations in terms of Φ_1 , and Φ_2 , they can just as easily be written in terms of n_1 and n_2 .

In this case the velocity can be contained in the matrices K and J , and the equations can be solved for the neutron densities. Thus the new equations become

$$J^* N - K^* N = 0 \quad (1)$$

where $N = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$

$$J^* = \begin{pmatrix} 0 & v_2 \Sigma_{f2} \\ 0 & 0 \end{pmatrix}$$

$$K^* = \begin{pmatrix} v_1 (-\nabla \cdot D_1 \nabla + \Sigma_{r1}) & 0 \\ v_1 \Sigma_{SD1} & v_2 (-\nabla \cdot D_1 \nabla + \Sigma_{r2}) \end{pmatrix} \quad (2)$$

of course,

$$J^* N = J \Phi \quad (3)$$

$$K^* N = K \Phi$$

and

$$N = \begin{pmatrix} \Phi_1 / v_1 \\ \Phi_2 / v_2 \end{pmatrix}$$

The matrices K^* and J^* are obtained from K and J by multiplying the two columns of K and J by v_1 and v_2 respectively. Similarly, the adjoint matrices K^{*+} and J^{*+} can be obtained by multiplying the two rows of K^+ and J^+ by v_1 and v_2 respectively. The adjoint equations represented in matrix form as

$$\nu J^{*+} N^+ - K^{*+} N^+ = 0 \quad (4)$$

are:

$$\begin{aligned} \nu_2 \nabla \cdot D_2 \nabla N_2^+ - \nu_2 \sum r_2 N_2^+ + \nu_2 \sum f_2 N_1^+ &= 0 \\ \nu_1 \nabla \cdot D_1 \nabla N_1^+ - \nu_1 \sum r_1 N_1^+ + \nu_1 \sum_{SD_1} N_2^+ &= 0 \end{aligned} \quad (5)$$

It is plain that the velocities can be left out of the (5) without effecting the content of the equations. In this case (5) becomes identical to the adjoint equation written in terms of flux, so we have

$$N^+ = \Phi^+ \quad (6)$$

It can be shown[†] that the neutron lifetime, L , can be represented as

$$L = \frac{\int N^+ N d\tau}{\int N^+ K^* N d\tau} \quad (7)$$

where the integration occurs over the whole reactor. With the help of expressions (3) and (6), this can be rewritten as

[†] The Reactor Handbook, Vol. 1, Ch. 1.6.

$$L = \frac{\int N^+ N d\tau}{\int \Phi_K^+ \Phi d\tau} = \frac{\int N^+ N d\tau}{\nu \int \Phi_J^+ \Phi d\tau}$$

$$L = \frac{\int \frac{\Phi_1^+ \Phi_1}{v_1} + \frac{\Phi_2^+ \Phi_2}{v_2}}{\int \sum_f \Phi_1^+ \Phi_2 d\tau} d\tau \quad (8)$$

The magnitudes of $\Phi_1^+ \Phi_1$ and $\Phi_2^+ \Phi_2$ are about equal throughout the reactor, but $1/v_1$ is negligible compared to $1/v_2$. Thus, very closely

$$L = \frac{\frac{1}{v_2} \int \Phi_2^+ \Phi_2 d\tau}{\nu \int \sum_f \Phi_1^+ \Phi_2 d\tau} \quad (9)$$

The integrals in (9) can be evaluated numerically from the flux and adjoint flux plots obtained from the UNIVAC calculations. In evaluating (9) the integration was performed only in the horizontal plane. This is equivalent to taking the vertical distributions of all flux and adjoint flux as being identical, and going to zero at the core boundary. Such a procedure underestimates the neutron lifetime somewhat (of the order of 10 percent), since above and below the core $\sum_f$ is zero while $\Phi_2^+ \Phi_2$ is not zero.

The neutron lifetime for the critical startup core was calculated to be 8.1×10^{-5} seconds, and the lifetime for the core with experiments as of January, 1956, was calculated to be 5.0×10^{-5} seconds. In all calculations involving the neutron lifetime the smaller value was used.

XII. THE FUEL ELEMENT TRANSFER CASK

A diagram of the fuel element transfer cask is shown in Figure 12-1. The analysis to follow shows that the cask will be subcritical when loaded with fuel elements each containing less than 800 g of U^{235} .

Consider a 36-inch vertical section containing fuel rods. Assume that all positions, including the central position, contain ETR fuel rods. Then, in addition to the U^{235} , the materials within (and including) the stainless steel framework are:

Stainless steel	=	534 cubic inches
Aluminum	=	1,166 cubic inches
Water	=	<u>1,968</u> cubic inches
Total volume	=	3,668 cubic inches
	=	6.02×10^4 cc

The absorption cross section of these materials is calculated below. (Twenty-degree cross sections uncorrected for the Boltzman distribution are used.

The results will depend upon the ratio of cross sections, and the small corrections required for non- $1/v$ behavior will not be significant in the analysis.)

	<u>V.F.</u>	<u>Σ_a</u>	<u>V.F. Σ_a</u>
H ₂ O	0.537	0.0223	0.01195
Al	0.318	0.0127	0.00404
S.S.	<u>0.145</u>	<u>0.2521</u>	<u>0.03650</u>
		Σ_a Total	0.05249

ETR FUEL ELEMENT TRANSFER CASK

(All plates are stainless steel - dimensions as illustrated)

Figure 12-1

If the Uranium cross section is Σ_{25} , then

$$k_{\infty} = 2.1 \frac{\Sigma_{25}}{\Sigma_{25} + 0.05249}$$

The metal occupies 46.3 percent of the volume of the section of the cask considered. Reference to Figure A5-3 indicates that the age in this mixture will be about 57 sq cm. The dimensions in the horizontal plane of the section considered are 10.1 inches by 10.1 inches. If we assume a reflector savings of 7.5 cm (approximately the savings expected from an infinite water reflector), then the effective dimensions are 40.6 cm by 40.6 cm. The buckling in the horizontal plane is therefore

$$B^2 = 2 \left(\frac{\pi}{40.6} \right)^2 = 1.2 \times 10^{-2}.$$

If the thermal leakage is neglected, the critical equation is

$$1 = \frac{K}{1 + B^2 \tau} = \frac{2.1 \Sigma_{25} / \Sigma_{25} + 0.052}{1.68}$$

This can be solved for the critical value of Σ_{25} .

$$\Sigma_{25} = \frac{1.68 \times 0.052}{0.4} = 0.218.$$

$$\text{But } \Sigma_{25} = \frac{(\text{mass/fuel element}) (\# \text{ of fuel elements}) \times \sigma_{25} \times N_0}{\text{volume of cask} \times \text{atomic weight of } U^{235}}$$

where

$$\sigma_{25} = \text{the microscopic absorption cross section} = 698 \times 10^{-24} \text{ cm}^2$$

$$N_0 = \text{Avagadro's number} = 6.02 \times 10^{23}$$

Thus, the mass per fuel element required for criticality is

$$\begin{aligned}\text{Critical mass/fuel rod} &= \frac{(0.218) (6.02 \times 10^4) (235)}{9 \times 6.02 \times 10^{23} \times 698 \times 10^{-24}} \\ &= 810 \text{ g U}^{235}/\text{fuel element}\end{aligned}$$

The calculation is conservative, since it neglects leakage in the vertical direction and underestimates leakage in the horizontal plane. More important, the calculation assumes that the central position contains a fuel rod. Actually, the central position contains a boron-steel frame and will be sealed off so that a fuel element cannot be inserted.

The cask, therefore, will be safe for any fuel element loading now anticipated.

XIII. THE FUEL ELEMENT STORAGE RACK

A diagram of the fuel element storage rack is shown in Figure 13-1. The analysis which follows shows that when fully loaded with fuel rods the rack is subcritical if each fuel rod contains less than 700 g of U^{235} .

The two-group critical diffusion equations[†] are

$$\nabla^2 \phi_1 - \frac{1}{\tau} \phi_1 + \frac{k \sum_a}{D_1} \phi_2 = 0 \quad (1)$$

$$\nabla^2 \phi_2 - \frac{1}{L^2} \phi_2 + \frac{\sum SD}{D_2} \phi_1 = 0 \quad (2)$$

where k is $\nu \sum_f / \sum_a$, and the other notation is standard.

To be conservative, leakage from the rack was neglected. This is equivalent to making the rack extend an infinite distance in all directions. Consider a unit cell of the rack. In a two-group diffusion analysis the boundary conditions are (a) the thermal flux must be zero at the cadmium walls of the cell and (b) (for reasons of symmetry) the derivative of the fast flux must be zero at the cell walls.

In order to make the mathematics practicable, the unit cell was approximated as a cylinder of cross-sectional area equal to the 4 by 4-inch area of a cell of the rack. The radius of this cylinder is therefore 2.3 inches, or 5.7 cm.

The solutions of equations (1) and (2) in cylindrical geometry are*

* Glasstone and Edlund, *The Elements of Reactor Theory*, p. 242, Van Nostrand, New York, 1952.

† This type of approach was suggested to the author by Dr. R. Soberman.

ETR FUEL ELEMENT STORAGE RACK

(Scale $1/4'' = 1''$)

Distance between compartment centers is 4"

Compartment separators and inside wall
(shaded area)

-0.016 Cadmium sheet
clad with $1/16''$ Aluminum
sheet

Outside wall

- $1/4''$ Aluminum

$$\phi_1 = A_1 J_0(\mu r) + B_1 I_0(\nu r) \quad (3)$$

$$\phi_2 = A_2 J_0(\mu r) + B_2 I_0(\nu r) \quad (4)$$

where J_0 is the zero order Bessel function of the first kind, and I_0 is the zero order Modified Bessel function of the first kind. μ and ν are defined by the following equations

$$\mu^2 = \frac{1}{2} \left[-\left(\frac{1}{\tau} + \frac{1}{L^2}\right) + \left[\left(\frac{1}{\tau} + \frac{1}{L^2}\right)^2 + \frac{4k-1}{\tau L^2} \right]^{1/2} \right]$$

$$-\nu^2 = \frac{1}{2} \left[-\left(\frac{1}{\tau} + \frac{1}{L^2}\right) - \left[\left(\frac{1}{\tau} + \frac{1}{L^2}\right)^2 + \frac{4k-1}{\tau L^2} \right]^{1/2} \right]$$

Since $\frac{4k-1}{L^2} \ll \left(\frac{1}{\tau} + \frac{1}{L^2}\right)^2$ the term under the square root sign can be expanded in a Taylor series. Therefore to first order in $\frac{\frac{k-1}{\tau L^2}}{\left(\frac{1}{\tau} + \frac{1}{L^2}\right)}$

$$\mu^2 = \frac{k-1}{\tau^2 + L^2} \quad (5)$$

$$+\nu^2 = \frac{1}{\tau} + \frac{1}{L^2} \quad (6)$$

If (3) and (4) are substituted in (1) the following is obtained

$$\left[-\left(\mu^2 + \frac{1}{\tau}\right) A_1 + \frac{k \sum a}{D_1} A_2 \right] J_0(\mu r) + \left[(\nu^2 - \frac{1}{\tau}) B_1 + \frac{k \sum a}{D_1} B_2 \right] I_0(\nu r) = 0 \quad (7)$$

Expression (7) must be true for all $\underline{r}$, so each of the terms in the brackets must be zero.

Therefore

$$\frac{-(\mu^2 + \frac{1}{\tau}) A_1}{(\nu^2 - \frac{1}{\tau}) B_1} = \frac{A_2}{B_2} \quad (8)$$

The boundary conditions applied to the solutions of (1) and (2) give

$$A_2 J_0(\mu R) + B_2 I_0(\nu R) = 0 \quad (9)$$

$$-\mu A_1 J_1(\mu R) + \nu B_1 I_1(\nu R) = 0 \quad (10)$$

where

J_1 is the first order Bessel function of the first kind

and I_1 is the first order Modified Bessel function of the first kind

R is the cell radius, 5.7 cm.

If (10) is divided by (9)

$$-\mu \frac{A_1}{A_2} \frac{J_1(\mu R)}{J_0(\mu R)} = \nu \frac{B_1}{B_2} \frac{I_1(\nu R)}{I_0(\nu R)} \quad (11)$$

If (8) is substituted in (11) the resulting critical equation is

$$\frac{J_1(\mu R)}{J_0(\mu R)} = \frac{\nu}{\mu} \frac{(\mu^2 + \frac{1}{\tau})}{(\nu^2 - \frac{1}{\tau})} \frac{I_1(\nu R)}{I_0(\nu R)} \quad (12)$$

Now $\nu R > 5$, and in this case $\frac{I_1(\nu R)}{I_0(\nu R)} \approx 1$ (to high accuracy).

Thus, the critical equation becomes

$$\frac{J_1(\mu R)}{J_0(\mu R)} = \frac{\nu}{\mu} \frac{(\mu^2 + 1/\tau)}{(\nu^2 - 1/\tau)} \quad (13)$$

Equation (13) relates the cell radius R required for criticality to the materials in the cell. Since R is known, the equation can be used to determine the uranium content of the cell required to make the infinite assembly critical.

This can be done numerically.

If the contents of the cell are assumed to be homogeneous within, then in the cell containing an ETR fuel element the aluminum-to-water ratio is

$$M/W = 0.290.$$

Thus,

$$T = 45$$

$$D_1 = 1.241$$

$$D_2 = 0.2035$$

If there are 700 g of U^{235} per fuel element, then

$$\Sigma_a = 0.1488$$

$$\Sigma_f = 0.1091$$

$$\mu = 0.1317$$

$$\nu = 0.869$$

$$\frac{\nu (\mu^2 + 1/T)}{\mu (\nu^2 - 1/T)} = \frac{0.869}{0.132}$$

$$\frac{(0.0174 + 0.0222)}{(0.753 - 0.022)} = 0.357$$

$$J_1(\mu R) = 0.3174$$

$$\frac{J_1}{J_0} = 0.357$$

$$J_0(\mu R) = 0.8900$$

$$J_0$$

Thus, we conclude that the rack is safe for handling fuel elements containing up to 700 g of U^{235} .

The calculation is conservative since we have neglected leakage, flux depressions in the element (since the cell contents were homogeneous), and the presence of boron in the fuel element.

XIV. STARTUP SOURCE STRENGTH IN THE ETR

A. Photoneutron Production in the Beryllium Reflector from the Gamma Activity of a Fuel Element.

Assume a fuel element adjacent to the beryllium reflector (see diagram).

Then, midway along the length of the reflector, the average path length ($\bar{P}$) for the gammas through the beryllium is

$$\bar{P} = T \frac{\int_0^{1.2} \frac{d\theta}{\cos \theta}}{\int_0^{1.2} d\theta} = \frac{T}{1.2} \left[\frac{1}{2} \ln \left(\frac{1 + \sin \theta}{1 - \sin \theta} \right) \right]_0^{1.2}$$

$$\bar{P} = \frac{T}{2.4} \ln \left(\frac{1 + \sin 1.2}{1 - \sin 1.2} \right) \quad (1)$$

$$\bar{P} = \frac{11.4}{2.4} \ln 28.5 = 4.75 \text{ (3.35)}$$

$$\bar{P} = 15.8 \text{ cm}$$

where $\Theta = 1.2$ radians

T = thickness of beryllium reflector

The radius of a cylinder with the same volume as one fuel element is determined by:

$$\pi r^2 = \left[(3.04) (2.54) \right]^2 \text{ cm}^2$$

$$r = \frac{(3.04) (2.54)}{\sqrt{\pi}} = 4.37 \text{ cm}$$

The surface area of this cylinder is

$$\text{Area} = 2\pi rh = 2.5 \times 10^3 \text{ cm}^2$$

The self absorption coefficient of gammas in the fuel element (60% H_2O , 40% Al) is

$$\mu_s = \mu_{\text{H}_2\text{O}} (\text{V. F. H}_2\text{O}) + \mu_{\text{Al}} (\text{V. F. Al}) \quad (2)$$

AVERAGE ATTENUATION OF IMPORTANT GAMMAS IN THE FUEL ELEMENT

$\frac{\text{Mev}}{\gamma}$	$\mu_{\text{H}_2\text{O}} (\text{cm}^{-1})$	$\mu_{\text{Al}} (\text{cm}^{-1})$	$\mu_s (\text{cm}^{-1})$	μ_s^z	$e^{-\mu_s^z}$
1.65	5.4×10^{-2}	1.3×10^{-1}	8.4×10^{-2}	0.22	0.80
2.10	4.8×10^{-2}	1.1×10^{-1}	7.3×10^{-2}	0.20	0.82
2.55	4.3×10^{-2}	1.0×10^{-1}	6.6×10^{-2}	0.18	0.84

Here z is the effective attenuation distance of a cylindrical source. The average attenuation of gamma flux from geometrical considerations is

$$\frac{r_1}{r_1 + R} = \frac{4.37}{4.37 + 15.8} = 0.216$$

where r_1 = radius of equivalent cylinder in cm

R = average path length through beryllium in cm

AVERAGE ATTENUATION OF IMPORTANT GAMMAS IN THE BERYLLIUM

$\frac{\text{Mev}}{\gamma}$	Average path Length $\bar{P}$ (cm)	μ	$\mu \bar{P}$	$e^{-\mu \bar{P}}$
1.65	15.8	8×10^{-2}	1.26	0.284
2.10	15.8	7×10^{-2}	1.11	0.330
2.55	15.8	6.2×10^{-2}	0.98	0.376

For 1000 hours operation, the fission product gamma spectrum for gammas above the photoneutron threshold of Be was obtained (APEX 176).

FISSION PRODUCT GAMMA SPECTRUM ABOVE BERYLLIUM PHOTONEUTRON THRESHOLD FOR 1000 HOURS OPERATION

$\frac{\text{Mev}}{\gamma}$	$\frac{\text{Mev}}{\text{sec-watt}}$	$\frac{\text{Watts}}{\text{element}}$	$\frac{\text{Mev}}{\text{sec}}$	$\frac{\text{photons}^*}{\text{cm}^2\text{-sec}}$ (emitted from fuel element)	$\frac{\text{ave photons}^{**}}{\text{cm}^2\text{-sec}}$ (in Be)
1.65	2.9×10^9	2.7×10^6	7.8×10^{15}	1.5×10^{12}	9.2×10^{10}
2.10	7.7×10^7	2.7×10^6	2.1×10^{14}	3.3×10^{10}	2.4×10^9
2.55	1.9×10^8	2.7×10^6	5.1×10^{14}	6.7×10^{10}	5.4×10^9

* Found by multiplying Mev/sec by $e^{-\mu z}$ and dividing by the surface area of the equivalent cylinder and the Mev/photon.

** Found by multiplying gamma flux emitted from the element by the attenuation caused by the geometry and absorption in the beryllium.

$$\bar{\Phi} = 1 \times 10^{11} \frac{\text{photons}}{\text{cm}^2\text{-sec}}$$

The volume of beryllium corresponding to the average path length is

$$\bar{V} = (15.8) 36 (2.54) 38 = 5.5 \times 10^4 \text{ cc} \quad (3)$$

The number of neutrons produced per second is

$$\begin{aligned} \Sigma \bar{\Phi}_\gamma \bar{V} &= (5 \times 10^{-28}) (6.02 \times 10^{23}) \left(\frac{1.84}{9} \times 10^{11}\right) (5.5 \times 10^4) \quad (4) \\ &= 3.4 \times 10^{11} \text{ n/sec} \end{aligned}$$

using $5 \times 10^{-28} \text{ cm}^2$ as the approximate cross section for the photoneutron production in the beryllium.

If it is assumed that 50 percent of these neutrons entering the core are distributed uniformly and that there is a multiplication factor of 5, the effective operating power for the core is approximately

$$\frac{\frac{3.4}{2} \times 10^{11}}{5.3 \times 10^5} \times \frac{5}{1.46 \times 10^5} = 11 \text{ watts}$$

B. Neutron Source

The startup neutron source is an antimony source producing photoneutrons by gamma absorption in beryllium. The source considered is similar to the antimony gamma source designed to fit into a beryllium A-piece of the MTR. The magnesium-antimony alloy contains approximately 0.36 g per cc of antimony. The source can be activated in the MTR.

The 60-day gamma activity of Sb^{124} is produced by neutron activation of Sb^{123} . This occurs as 43 percent of natural antimony and has a neutron absorption cross section of 2.5 barns. The activity present after 60 days irradiation in a thermal neutron flux of 10^{14} is

$$\begin{aligned} N\lambda &= \sum_a \phi (1 - e^{-\lambda t}) \\ &= \frac{6.02 \times 10^{23}}{123} (0.36) (2.5 \times 10^{-24}) (0.43 \times 10^{14}) (0.5) \quad (5) \\ &= 9.5 \times 10^{10} \text{ dis/sec/cc} \end{aligned}$$

Approximately 50 percent of the disintegrations give off 1.69 Mev gammas, and four percent yield 2.06 Mev. Thus the unit volume source is

$$4.8 \times 10^{10} \text{ photons/sec/cc of 1.69 Mev}$$

$$3.8 \times 10^9 \text{ photons/sec/cc of 2.04 Mev}$$

The volume of the source is 32.3 cc, so that the total source is

$$1.55 \times 10^{12} \text{ photons/sec of 1.69 Mev}$$

$$1.23 \times 10^{11} \text{ photons/sec of 2.04 Mev}$$

Total = 1.67×10^{12} photons/sec, which corresponds to a 45-curie source of photons with energy greater than 1.69 Mev.

Since this source will be located in the beryllium reflector, 11.4 cm thick, it was assumed that the source was positioned on the axis of a beryllium cylinder 11.4 cm in diameter. If it is also assumed that all the gammas emerge radially from the source, the average gamma flux through the beryllium is defined by

$$\Sigma \Phi_{\gamma} V = \int_{r_1}^R \Sigma \Phi_{\gamma}(r) h 2\pi r \, dr$$

$$\Phi = \frac{\int_{r_1}^R \Phi_{\gamma}(r) 2\pi r \, dr}{\int_{r_1}^R 2\pi r \, dr} \quad (6)$$

where absorption is neglected.

$$\Phi_{\gamma}(r) = \Phi_{\gamma_1} \frac{r_1}{r}$$

$$\Phi_{\gamma} = \frac{\Phi_{\gamma_1} r_1 \int_{r_1}^R dr}{\int_{r_1}^R r \, dr} = \frac{2 \Phi_{\gamma_1} r_1}{R + r_1} \quad (7)$$

where

r_1 = radius of antimony rod = 0.476 cm

R = radius of outer surface of Be = 5.7 cm

h = height of Be cylinder = 18 in.

Φ_{γ_1} = gamma flux at radius r_1

$$\Phi_1 = \frac{1.67 \times 10^{12}}{2\pi (0.476) 17.9 (2.54)} = 1.23 \times 10^{10} \frac{\text{photons}}{\text{cm}^2\text{-sec}}$$

$$\Phi_2 = \frac{2 (1.23 \times 10^{10}) (0.476)}{6.2} = 1.9 \times 10^9 \frac{\text{photons}}{\text{cm}^2\text{-sec}}$$

Using an approximate cross section for photoneutron production in the beryllium of $5 \times 10^{-28} \text{ cm}^2$

$$\Sigma = (5 \times 10^{-28}) \frac{6.02 \times 10^{23}}{9} (1.84) = 6.2 \times 10^{-5} \text{ cm}^{-1} \quad (8)$$

Then the total neutrons produced are

$$\Sigma \bar{\Phi}_2 V = (6.2 \times 10^{-5}) (1.9 \times 10^9) (4.6 \times 10^3) = 5.4 \times 10^8 \text{ n/sec} \quad (9)$$

If one half of these neutrons enter the core and are distributed uniformly throughout the core with a multiplication factor of 5, the number of neutrons per cc per sec is

$$\frac{2.7 \times 10^8}{5.3 \times 10^5} \times 5 = 2.6 \times 10^3$$

An operating core will produce

$$\frac{3.1 \times 10^{10}}{5.3 \times 10^5} \times 2.5 = 1.46 \times 10^5 \text{ source neutrons/cc/watt}$$

Therefore, this neutron source will produce a neutron flux equivalent to an approximate operating power of

$$\frac{2.6 \times 10^3}{1.46 \times 10^5} = 2 \times 10^{-2} \text{ watts.}$$

C. Photoneutrons from Induced Gamma Activity of Impurities in the Beryllium.

The beryllium reflector was assumed to contain 200 ppm manganese and 5 ppm sodium. The activation cross sections for these are $5.07 \times 10^{-5} \text{ cm}^{-1}$ and $1.18 \times 10^{-7} \text{ cm}^{-1}$, respectively. With an average reflector thermal neutron flux of $2.2 \times 10^{14} \text{ n/cm}^2\text{-sec}$, the respective yields are obtained at saturation.

$$Q_v = \sum \phi (0.125) = 1.39 \times 10^9 \quad 2.1 \text{ Mev photons/cc/sec}$$

$$Q_v = 2.6 \times 10^7 \quad 2.8 \text{ Mev photons/cc/sec}$$

The 2.1 Mev gammas are produced from 12.5 percent of the disintegrations of Mn^{56} and have a 2.6-hour half-life. The 2.8 Mev gammas are produced from 100 percent of the disintegrations of Na^{24} and have a 14.9-hour half-life.

The beryllium photoneutron cross sections, $\Sigma = N\sigma$, are shown below.

BERYLLIUM PHOTONEUTRON CROSS SECTIONS

E(Mev)	(cm^2)	N	$\Sigma (\text{cm}^{-1})$
2.8	7×10^{-28}	1.2×10^{23}	8.4×10^{-5}
2.1	4×10^{-28}	1.2×10^{23}	4.8×10^{-5}

If it is assumed that the gamma sources are uniformly distributed and that the absorption is linear with thickness (since $e^{-\Sigma x} \approx 1 - \Sigma x$ for small Σx), the number of gammas absorbed is roughly

$$\frac{Q_v t^2 \Sigma}{2} \quad (10)$$

and the number of neutrons entering the core is approximately

$$\frac{Q_v t^2 \Sigma}{4} \text{ n/cm}^2\text{-sec} \quad (11)$$

For $t = 11.4 \text{ cm}$

$$\frac{(1.39 \times 10^9) (1.31 \times 10^2) (4.8 \times 10^{-5})}{4} = 2.18 \times 10^6 \text{ n/cm}^2\text{-sec from the 2.1 Mev gammas}$$

$$\frac{(2.6 \times 10^7) (1.31 \times 10^2) (8.4 \times 10^{-5})}{4} = 7.14 \times 10^4 \text{ n/cm}^2\text{-sec from the 2.8 Mev gammas}$$

The total neutron flux entering the core at shutdown is $2.25 \times 10^6 \text{ n/cm}^2\text{-sec}$. An equivalent reactor power level is determined by assuming this flux to be constant over the entire core-reflector surface ($2.8 \times 10^4 \text{ cm}^2$) and assuming the neutrons to be distributed uniformly throughout the volume ($5.3 \times 10^5 \text{ cc}$). A multiplication factor of 5 is assumed in the core. At shutdown this power level will be approximately

$$\frac{(2.25 \times 10^6) (2.8 \times 10^4) 5}{(5.3 \times 10^5) (1.46 \times 10^5)} = 4 \text{ watts}$$

1.46×10^5 source neutrons per cc are produced per watt of operating power.

The power level will decay at the combined rate of the Mn (2.6-hr) and Na (14.9-hr) decay.

XV. EXPERIMENTAL VERIFICATION OF CALCULATIONAL TECHNIQUES

A. General Considerations

The running cycle of the ETR is only 20 days in length. Because of this the effect of a miscalculation in the initial loading of the ETR is not serious, provided the margin of error is small enough that minor modification of the core geometry can compensate for it.* If this is possible, the net effect will be one or two cycles of modified operation until fuel elements with the correct loadings can be obtained.

Since the effect of a miscalculation of critical mass can be so easily corrected, it was decided that a full scale critical experiment for the ETR would not be necessary. However, a considerable amount of the ETR design is based on the premise that the calculational techniques used will provide results not far from reality. In addition, the analysis of the hazards involved in the operation of the ETR was based to a large extent on the results of the Physics calculations presented in this report. For these reasons, a set of experiments was performed to check the calculation methods used for the Physics of the ETR.

The Physics calculations have relied primarily on a two-group theory. Such calculations have proved reliable in the past,† but have not been tested on a

* The aluminum filler pieces used in the startup core are designed so they can be removed and replaced by fuel elements in case the critical loading is underestimated. There will be "dummy" aluminum elements containing no fuel available to replace standard fuel elements in case the critical loading is overestimated.

† See for instance Research Reactors, p. 207, Selected Reference Material of the U.S. Atomic Energy Program, U.S. Government Printing Office, Washington, D.C., 1955.

core with a large volume of in-core space loaded differently from the fuel section. The tests were designed to check the calculations for such a situation.

The experimental work was performed at the Reactivity Measurements Facility (RMF), which is located at the end of the canal belonging to the Materials Test Reactor (MTR). The reactor is located at the A. E. C. Reactor Testing Station in Idaho. The tests were performed between March 7 and April 6, 1956, under the direction of Dr. E. Fast of the Phillips Petroleum Company. He was aided by Mr. J. O. Arterburn of the General Electric Company, who was responsible for the detailed design of the experiments.

The original concept of the tests was worked out by D. R. deBoisblanc and R. M. Jones of the Phillips Petroleum Company, R. V. Hurst of the Idaho Operations Office of the AEC, W. Dodson of the Kaiser Engineers, and J. O. Arterburn, J. W. Weil, and B. Wolfe of the Atomic Power Equipment Department of the General Electric Company. The final experimental program was set up by the ETR Physics Group of the General Electric Company. A complete report of the experimental program carried out during the tests has been issued by the Phillips Petroleum Company, * and therefore the experimental techniques used are not reported here in detail.

B. Description of the RMF Tests

The RMF is a low-power swimming pool reactor. The RMF grid plate is designed for a 6-by-7 lattice with spacing such that adjacent MTR-type fuel

* E. Fast and E. E. Burdick, ETR-Type Core Studies in the MTR-Reactivity Measurements Facility, PTR-69, Phillips Petroleum Co., May 18, 1956.

elements have a 1/4-inch space between them. In addition, between the second and third rows and the fourth and fifth rows of the lattice, an additional 1/4-inch space is provided for safety curtains which slide into this space. The presence of the water annuli around the fuel elements caused the only departure in the RMF calculations from the calculational techniques used in the ETR analysis. It has been pointed out by Warren Neyer* that the presence of the water annuli makes it incorrect to uniformly homogenize the metal and water mixture (as was done in the ETR calculations). The flux will peak in the water annuli and therefore reduce the thermal utilization below that computed for a homogenized core. Before this point was realized, UNIVAC calculations were performed for a core arrangement on which careful measurements had been performed by the Phillips Petroleum Co. This arrangement consisted of a 6-by-6 array of fuel elements (loaded with 168 g of U^{235}) with the central four elements removed to leave a water space. The core was water-reflected on all sides. The detailed loading of this core was supplied by Dr. D.R. deBoisblanc.

The UNIVAC calculations were performed by homogenizing the contents of the fuel section. The result of the calculation in addition to flux plots was a value of ν , the number of neutrons per fission required to make the reactor critical. The value obtained in this case was 2.37, which is in reasonable agreement with the experimental value of 2.46 ± 0.04 . However, after learning of Neyer's suggestion, the flux distribution in the unit cell was computed

* McMurray, Webster, Maisden, and Grimaud, Nuclear Calculations for the RMF with Spaced 200-gram Fuel Assemblies, IDO 16184, Phillips Petroleum Co., Sept. 9, 1954.

by means of a P3 calculation performed on the IBM 650 computer using a program developed by J. Weil. In this calculation the unit cell of the RMF was approximated in cylindrical geometry with the areas of each annular region made equal to the area of the corresponding section in the RMF cell. The results of the P3 calculation indicated that the thermal utilization used for the UNIVAC calculation was 3.5 percent too high. With a non-uniform core reflected as in the RMF, it is not valid to apply a thermal utilization correction directly to the results of the calculation. However, it does appear that correction would bring the calculated result into extremely good agreement with the measurements.

In performing the calculations on the RMF tests, the flux curve in the unit cell obtained above was used to compute average cross sections in the cell. With the exception of this averaging procedure, the calculation methods used for the RMF tests were identical to those used on the ETR.

In order to reduce the computer time required for the calculations (and thus the cost), the procedure adopted for the test program was to first perform the experimental work and obtain detailed loadings. These loadings were then used as input data for the calculations. The test of the calculation is then the agreement between the computed η and the value of 2.46 ± 0.04 .

In order to test the ETR calculations the RMF was arranged in a 4-by-5 fuel-element array surrounded by a single row of beryllium elements. The lattice arrangement is shown in Figure 15-1. As can be seen from the figure, four

MASS U-235
EXCESS REACTIVITY

2771.8 gm
0.21 %

CRITICAL LOADING

AIR TANK EMPTY - C

Reprinted through courtesy
of Phillips Petroleum Co.

Figure 15-1

fuel elements in the central region of the core were removed, and the vacant space used to simulate experiments of the type planned for the ETR. The slab beryllium reflector and the experimental hole occupying a large percentage of the core volume were of course designed to resemble features in the ETR. In addition, the experimental hole was removed from the beryllium by one row of fuel elements as is the case in the ETR.

A steel pipe with aluminum corner pieces, designed to duplicate pipes to be used in the ETR experiment holes, was placed inside the experimental space in the RMF. A diagram of this pipe is shown in Figure 15-2. The pipe was 18 feet long and extended approximately one foot above the water in the RMF pool. This pipe-length was chosen so that neutron streaming effects in the pipe would resemble those in an ETR pipe. In addition, the six-inch diameter of the pipe was such that at the midplane of the RMF the solid angle subtended by the pipe at the top of the RMF core was the same as the solid angle subtended by a nine-inch pipe in the ETR core. Figure 15-3 is a photograph showing the RMF arrangement during the test program outlined above.

The test program consisted of obtaining a critical loading of the RMF for several cases in which the contents of the in-core pipe were varied. The contents of the pipe were chosen to have features similar to those of some of the proposed experiments in the ETR. Flux measurements were made along the third row of the core so that the flux plots obtained from the calculations could be evaluated. The cases tested are described in the following.

Reprinted through courtesy
of Phillips Petroleum Co.

Figure 15-2

PHOTOGRAPH OF THE RMF
SHOWING A TYPICAL LOADING

Reprinted through courtesy
of Phillips Petroleum Co.

Case 1

Case 1 is illustrated in Figure 15-1. In this arrangement the pipe was left empty so that the calculation of neutron streaming effects could be evaluated. The effect of the streaming was computed by using a modification of a technique described by Wachspress.* In this case the computed value of λ was 2.369, or 3.7 percent below the measured value for λ of 2.46. However, for the loading computed the excess reactivity measured was 0.21 percent. Thus the computed result is 3.5 percent low, assuming that the correct value of λ is 2.46. Figure 15-4 is a plot of the flux distribution across row three, comparing the calculated and the experimental results. In comparing the calculated to the experimental results, it must be remembered that the contents of each fuel cell are homogenized (with proper weighting of the constituents) throughout the cell area. Therefore, the structure within a cell, and the flux peaking between the cells, are averaged out.

The calculated and experimental results shown in Figure 15-4 are in very good agreement, except in the region to the right of the air column where greater peaking was predicted than was observed. The total fuel in the core for this test was 2,772 g of U^{235} .

Case 2

Case 2 had special-type fuel elements in the experimental pipe and no moderator. This loading was made to simulate an ETR experimental hole highly loaded with

* E. Wachspress, Two-Dimensional, $r \cdot \infty$ Multigroup Calculations, KAPL 1172, August 15, 1954. See also Appendix VI.

RMF TEST - AIR IN EXPERIMENTAL PIPE

FIGURE 15-4

fuel but poorly moderated. The total fuel in the pipe during the test was 920 g of U^{235} . During the test a model of the ETR grey rod was inserted in the core in the third row, fifth column position. The fuel in the core for this run was 3,068 g of U^{235} and the total loading, core plus experiment, was 3,988 g. The β computed for the loading was 2.42 when the measured excess reactivity was 0.18 percent. Thus the calculated value is within 1.4 percent of the measured value of 2.46. Figure 15-5 compares the measured and computed flux distribution through row three. The high experimental point in the center of the grey rod is caused by the large water space in the middle of the rod and is in excellent agreement with the plot of Figure 6-15 in Section VI. Since the rod was homogenized in these calculations, the water peaking does not appear.

Case 3

Case 3 was identical to Case 2 in experimental loading, but the grey rod was removed from the core. In order to keep the reactor critical the fuel loading in the core had to be decreased, and the core poisoned with 22 40-mil cobalt wires. The arrangement is shown in Figure 15-6. For this run the fuel loading was 2,684 g U^{235} in the fuel section, and 920 g U^{235} in the experiment. The calculation for this case yielded a value for β of 2.74. A check on the input data to the problem was then made, and an error of a factor of ten found in the cobalt cross section. This error decreased the thermal utilization by a factor of 0.845 of the value which should have been calculated for the fuel section.

The average flux in the fuel section was about three times that in the experimental hole, while the fission cross section in the hole was 1.4 times that in

FIGURE 15-5

CIRCLED FIGURES DENOTE NUMBER OF COBALT WIRES
TOTAL NUMBER OF COBALT WIRES 105

MASS U-235 2683.9 gms. + FUEL SPECIMEN
EXCESS REACTIVITY 0.19 %

POISONED CRITICAL LOADING

FUEL IN AIR TANK

Reprinted through courtesy
of Phillips Petroleum Co.

Figure 15-6

the fuel section. If the various regions are weighted as the flux squared, then the computed ν should be changed as follows:*

$$\delta \nu = \frac{\int_{\text{fuel section}} \delta \Sigma_a \phi^2 d\tau}{\int_{\text{fuel section}} \Sigma_f \phi^2 d\tau + \int_{\text{experimental section}} \Sigma_f \phi^2 d\tau} \quad (1)$$

The denominator is equal to

$$\int_{\text{fuel section}} \left(\Sigma_f \phi^2 \right) + \frac{4}{16} (1.4 \Sigma_f \frac{\phi^2}{9}) d\tau \quad (2)$$

so that (1) becomes

$$\delta \nu = \frac{\delta \Sigma_a}{1.04 \Sigma_f} \text{ evaluated in the fuel section}$$

$$\delta \nu = \frac{-0.0149}{1.04 \times 0.053} - 0.27$$

The corrected value of ν is therefore $2.74 - 0.27 = 2.47$.

The flux squared weighting function used in this analysis is of course not correct, so the corrected value of ν obtained is not to be considered an accurate value. However, the correction does indicate that if it were not for the input error, the UNIVAC calculation would have again produced a result for ν at least within a few percent of 2.46. The error in the input should not

* See Appendix IV on Perturbation Theory.

affect the shape of the flux distribution appreciably. Figure 15-7 is a plot comparing the calculated and experimental flux distributions. Again, rather striking agreement is observed between the calculated and the observed results.

Case 4

An aluminum plug, four inches in diameter, was placed in the tube of Figure 15-8. The total mass of the U^{235} was 2,735 g and the excess reactivity observed with the aluminum in place was 0.49 percent. The calculated value of ρ was 2.41. Thus the calculation overestimated the reactivity by 1.5 percent. No experimental flux plots were obtained with this loading.

An attempt was also made to measure a temperature coefficient for the RMF. However, the facility is not equipped for this type of measurement and experimental difficulties did not permit the determination of a meaningful number, or even a definite assignment of whether the coefficient was positive or negative.

C. Summary

Under a variety of loadings for the core and the experimental hole, the calculational techniques used on the ETR yielded results which agreed within a few percent in reactivity to those obtained experimentally. A change of about 2.5 percent in U^{235} produces a one percent change in reactivity, so in each case the critical loading of the reactor would have been obtained to better than 10 percent.

Although the RMF was not an exact duplicate of the ETR core, the features which would be expected to present calculational difficulties were still present. These features included the large ratio of experimental facility area to core area, the presence of a slab beryllium reflector followed by water, the single row of fuel elements between the experimental hole and reflector, the presence of an in-core stainless steel pipe, and the streaming of neutrons through the void in the pipe. The tests were made under a number of conditions which required different fuel loadings. Thus, it is not very likely that the good agreement between the theory and the experiments was accidental.

Although the perturbation techniques were not checked experimentally, they are shortcuts which are equivalent to doing a complete calculation. Therefore, if the calculational methods used to get the flux distribution (and thus the adjoint flux distribution) are correct, then the perturbation calculations are also correct (provided the perturbations are small).

It then follows that the calculations for the ETR can be expected to be as close to reality as the calculations performed for the RMF tests.

D. RMF Test for Flux Peaking in the Grey Rod

Figure 6-15 of Section VI indicates that in the interior of the grey rod there will be considerable flux peaking, and the experimental points of Figure 15-5 verify this prediction. If this flux peaking extends below the grey section into the fuel section of the control rod, then the power generation at the top of the fuel section will be increased. Such a condition could be serious if the junction

between the absorber and fuel section is near the core hot spot. To check the effect of the flux peaking, a model of the proposed ETR grey rod was built which fitted into the RMF so that the fuel-absorber junction was at the core midplane.

To determine the possible neutron flux peaking in the region between the fuel and absorber sections of the model grey rod, the activities of gold wires were obtained for several positions. The wires were positioned parallel to the longitudinal axis of the control rod, with a reference point two inches below the top of the fuel plates in the fuel section (as shown in Figures 15-9 and 15-10). The end of the fuel plate was at the horizontal midplane of the core. The lateral positions of the wires are shown in Figure 15-11.

After being activated the wires were cut into 0.25-inch increments and counted on a proportional counter. The count rates (corresponding to the saturated activities and to the thermal flux) were plotted at the midpoint of the 0.25-inch increment, after being normalized by the reciprocal of the value for the first increment. The relative neutron flux distributions were obtained in this manner for the following conditions:

1. A hollow square aluminum spacer, providing 1.75 inches between the fuel and absorber sections of the control rod.
2. The same aluminum spacer as above, with five nickel plates inserted (see Figure 15-11).
3. A hollow square aluminum spacer, providing 0.875 inches between the fuel and absorber sections.

RELATIVE ACTIVITY OF Au WIRE

WIRE POSITION #200

FIGURE 15-9

RELATIVE ACTIVITY OF Au WIRE

WIRE POSITION #300

FIGURE 15-10

LATERAL POSITION OF GOLD WIRES

NI GREY ROD
(BOTTOM VIEW)

AL SPACER
(TOP VIEW)

FUEL SECTION
(TOP VIEW)

FOR DIMENSIONS SEE
ETR-5528-MTR 642-M-907
ETR-5528-MTR 642-M-905

FIGURE 15-11

A peaking effect of 40 to 50 percent occurred in the water volume at two inches from the reference point (near the mid-point between the fuel and the absorber when the large aluminum spacer was used). The value at the end of the fuel section was approximately 30 percent higher than the value at the reference point. An analysis of the relative values of flux at several points near the grey rod position, with and without the grey rod in place, indicated that the flux at the reference point was not affected appreciably by the presence of the grey rod. This means that the 30 percent peak occurred superimposed on the unperturbed longitudinal flux or power distribution.

The value at the end of the fuel section was reduced approximately 10 percent by the insertion of the nickel plates into the large spacer, or by moving the nickel rod nearer the fuel (see Figures 15-9 and 15-10). It is expected that further reduction in the value will be obtained by increasing the absorption cross section within the spacer region and/or by placing the absorber still nearer to the fuel.

Consistent with this premise, the design of the control rod has been modified to include boron-steel (one percent boron enriched to 93 percent B^{10}) plates, 0.5 inches long and 0.054 inches thick, placed at the end of each fuel plate in the control rod fuel section.

During the ETR startup tests it is planned to check the flux distribution with this new design to assure that the flux peaking has been reduced to tolerable limits.

APPENDIX I

SAMPLE CALCULATION FOR GAMMA HEATING EXTERNAL TO CORE

To determine the gamma heating external to the reactor core, the ETR was assumed to be operating at a total power of 175 MW in 65 fuel elements. This corresponds to

$$\frac{175 \times 10^6 \times 3.1 \times 10^{10}}{5.3 \times 10^3 \times 65} = 1.58 \times 10^{13} \text{ fissions/sec/cc}$$

For each fission there are on the average 0.1173 gamma rays produced in the 4 to 10 Mev range.[†] These include prompt gammas, U^{235} capture gammas, and fission product gammas. For purposes of this calculation these gammas were all assumed to be 7 Mev gamma rays. In addition to these gamma rays, there are 7 Mev gammas produced by neutron capture in the core aluminum. The effective macroscopic thermal cross section for neutron capture in the core aluminum is 0.00455.

Taking the average thermal flux as 5×10^{14} nv, the aluminum capture gamma production is 2.27×10^{12} gamma/cc/sec. This value is added to the value for production in the uranium to give a total figure of

$$4.13 \times 10^{12} \text{ gamma/cc/sec.}$$

The average over the midplane is (hot spot factor 1.4)

$$1.4 \times 4.13 \times 10^{12} \text{ gamma/cc/sec} = 5.7 \times 10^{12} \text{ gamma/cc/sec.}$$

The gamma flux seen by an isotropic detector[§] external to a cylindrical source is given by the expression

[†] See Stevenson, Introduction to Nuclear Engineering, p. 209.

[§] See Stevenson, Introduction to Nuclear Engineering, p. 182, for a discussion of the difference between an isotropic and a directional detector.

$$\phi = \frac{B Q_v C^2}{2(a+z)} F(\Theta, b)$$

where

$$Q_v = 5.7 \times 10^{12} \text{ gamma/cc/sec}$$

$$C = \text{radius of the cylinder} = 40 \text{ cm}$$

$$a = \text{distance from cylinder perimeter to point of interest}$$

$$z = \text{the effective self absorption distance}$$

$$B = \text{buildup factor}^\dagger$$

$$b = \text{the number of relaxation lengths to the point of interest.}$$

$$F(\Theta, b) = \int_0^\Theta e^{-b \sec \Theta} d\Theta$$

$$\Theta = \tan^{-1} \left[\frac{\frac{1}{2} h}{a + z} \right]$$

where

$$h = \text{height of core} = 3.0 \text{ ft.}$$

Consider the point at the boundary of the aluminum and beryllium (see figure A-1).

$$a = 11.4 \text{ cm}$$

$$z = 21.5$$

$$b = 1.33$$

[†] See APEX 176. Values for an isotropic point source in water were used.

MIDPLANE CROSS SECTION

Boundary

all dimensions are given in inches

Figure A1-1

$$\Theta = 54^{\circ}$$

$$F(\Theta, b) = 0.21$$

$$B = 1.5$$

$$\phi = \frac{5.7 \times 10^{12} \times 40^2 \times 1.5 \times 2.1 \times 10^{-1}}{65.4}$$

$$\phi = 4.4 \times 10^{13} \text{ gamma/cm}^2/\text{sec}$$

This is the effective gamma flux due to core radiation. In addition, there are 7 Mev gamma rays produced by capture in the material external to the core.

Consider the capture in the beryllium region of the reactor core. The average midplane neutron flux in the beryllium is 4.06×10^{14} and the thermal macroscopic capture cross section is 0.00151. The average beryllium capture gamma production is 6.12×10^{11} gamma/cc/sec. The beryllium region and the other regions external to the core were considered as infinite slabs, and the gamma flux contribution at an external point was computed from the expression

$$\phi = B \frac{Q_v}{2\mu_T} \left[F(b) - F(b_o) \right]$$

where

μ_T = total gamma absorption cross-section

b = number of relaxation lengths from front of slab to point of interest

b_o = number of relaxation lengths from rear of slab to point of interest.

The contribution from the beryllium region to the gamma flux at the aluminum-beryllium interface is

$$\phi = \frac{1.2 \times 6.12 \times 10^{11} \times 0.6}{2 (0.036)} = 6.12 \times 10^{12} \text{ gamma/cm}^2/\text{sec}$$

where

$$B = 1.2$$

$$\mu_T = 0.036$$

$$F(b) - F(b_0) = 0.6$$

Similarly the contribution to the gamma flux at the aluminum-beryllium interface from the other regions is:

Aluminum region	$\phi = 1.25 \times 10^{13} \text{ gamma/cm}^2/\text{sec}$
-----------------	--

Fe region (4)	$\phi = 4.52 \times 10^{10} \text{ gamma/cm}^2/\text{sec}.$
---------------	---

The contribution of 7 Mev gammas from the other regions is small compared to the contribution from the core, beryllium and aluminum. The total 7 Mev gamma flux at the beryllium-aluminum interface is the sum of the contributions enumerated above.

Thus

	4.4 x 10 ¹³
	0.612 x 10 ¹³
	1.25 x 10 ¹³
	0.004 x 10 ¹³
	<hr/>
ϕ_{Total}	6.27 x 10 ¹³

This is the effective 7 Mev gamma flux which would be seen by an isotropic detector at the beryllium-aluminum interface. This flux is defined so that the

heating per unit volume is the energy absorption coefficient multiplied by the flux. Thus in the beryllium, where the energy absorption coefficient is 0.024, the heating due to 7 Mev gamma rays at the interface is

$$0.024 \times 6.27 \times 10^{13} \times 7 \times 10^6 \times 1.6 \times 10^{-19} = 1.68 \text{ watts/cc}$$

When the contribution due to the 1 Mev and the 2 Mev gamma ray heating and the neutron heating is added to this, the total heating at the interface is found to be 10.9 watts/cc.

This calculation was repeated for each interface and the computed points at the interfaces were joined by straight lines on semi-log paper. The results are recorded in Figures 4-1 to 4-2, where neutron and gamma flux, and neutron and gamma heating are shown as functions of distance from the center of the core.

Figure A1-1 is a diagram of the shielding surrounding the ETR. Tables A-I and A-II show the source of gamma flux at each interface. The interface numbers correspond to the labeling of Figure A1-1. Table A-III is a summary of the various heat sources at each boundary. Tables A-I, A-II, and A-III are averaged over the reactor height and should be multiplied by 1.4 to obtain the mid-plane values.

TABLE A-I

7 MEV GAMMA FLUX

Source of radiation		BOUNDARY						
		1	2	3	4	5	6	7
1	Core	3.65×10^{13}	1.73×10^{13}	2.1×10^{12}	1.54×10^{12}	9.1×10^{11}	1.3×10^{10}	5.5×10^7
2	Be	4.04×10^{12}	4.38×10^{12}	2.83×10^{11}	1.74×10^{11}	1.82×10^{11}	3.52×10^9	
3	Al	4.08×10^{12}	8.94×10^{12}	8.15×10^{12}	4.55×10^{12}	4.06×10^{12}	4.63×10^{10}	6.85×10^7
4	Fe	2.5×10^{11}	3.23×10^{10}	7.9×10^{11}	6.6×10^{11}	3.74×10^{11}	3.71×10^9	
5	H ₂ O							
	Fe	2.45×10^{10}		4.07×10^{11}	4.95×10^{11}	1.6×10^{12}	2.11×10^{10}	
	H ₂ O							
	Fe				1.35×10^{11}	1.13×10^{11}	1.94×10^{10}	
6	H ₂ O							
	Fe					2.93×10^{10}	1.89×10^{10}	
	H ₂ O							
	Fe					6.44×10^9	8.53×10^{10}	
	H ₂ O							
	Fe						2.57×10^8	2.78×10^7
7	Air							
	Fe							
	Pb						6.4×10^8	3.26×10^8
	Fe							8.55×10^7

TABLE A-I (CONT'D)

Source of radiation	BOUNDARY						
	1	2	3	4	5	6	7
8 concrete							3.64×10^9
due to fission in core	3.0×10^{13}	1.42×10^{13}	1.74×10^{12}	1.27×10^{12}	7.5×10^{11}	1.07×10^{10}	4.54×10^7
Total	7.65×10^{13}	4.48×10^{13}	1.3×10^{13}	8.82×10^{12}	7.98×10^{12}	2.48×10^{11}	4.18×10^9

TABLE A-II

2 MEV GAMMA FLUX

Source of radiation	BOUNDARY						
	1	2	3	4	5	6	7
1	8.23×10^{13}	3.26×10^{13}	1.47×10^{12}	7.0×10^{11}	4.45×10^{11}	2.31×10^{10}	1.02×10^8
5(a)	3.72×10^{10}	6.8×10^{10}	3.4×10^{12}	5.6×10^{12}	5.36×10^{12}	1.89×10^{11}	4.52×10^8
(b)	2.28×10^8	5.4×10^8	1.05×10^{10}	1.3×10^{10}	2.35×10^{10}	2.03×10^9	2.42×10^7
(c)						3.42	1.0×10^7
(d)						9.8	2.63×10^7
(e)						1.1	1.97×10^7
due to fission in core	1.73×10^{14}	6.9×10^{13}	3.08×10^{12}	1.48×10^{12}	9.35×10^{11}	4.86×10^{10}	2.15×10^8
Total	2.55×10^{14}	1.02×10^{14}	7.96×10^{12}	7.79×10^{12}	6.66×10^{12}	2.87×10^{11}	8.52×10^8

due to fission
in core

2 MEV GAMMA FLUX

	1	2	3	4	5	6	7
Total	1.5×10^{15}	6.93×10^{14}	1.65×10^{13}	7.95×10^{12}	3.25×10^{12}	1.18×10^{10}	8.8×10^6

TABLE A-III

	BOUNDARY						
	1	2	3	4	5	6	7
Gamma Heating							
Watts/cc (left side of boundary)	17.1	7.33	1.09	2.48	2.34×10^{-1}	3.50×10^{-2}	1.60×10^{-4}
Watts/cc (right side of boundary)	15.7	12.6	3.67	2.85×10^{-1}	1.10	8.21×10^{-2}	
Watts/gram (left side of boundary)	10.2	4.00	4.04×10^{-1}	3.25×10^{-1}	2.34×10^{-1}	8.55×10^{-3}	2.50×10^{-5}
Watts/gram (right side of boundary)	8.55	4.65	4.84×10^{-1}	2.85×10^{-1}	2.7×10^{-1}	1.28×10^{-2}	
Neutron Heating							
Watts/cc (left side of boundary)	7.42	4.37×10^{-1}	2.68×10^{-2}	3.80×10^{-1}	8.22×10^{-2}	2.5×10^{-3}	1.36×10^{-4}

TABLE A-III (CONT'D)

	1	2	3	4	5	6	7
Neutron Heating (Cont'd)							
Watts/cc (right side of boundary)	2.12	9.47×10^{-2}	4.38×10^{-1}	5.92×10^{-1}	4.90×10^{-2}	5.75×10^{-4}	
Watts/gram (left side of boundary)	4.40	2.36×10^{-1}	9.88×10^{-3}	4.98×10^{-2}	8.22×10^{-2}	6.10×10^{-4}	2.64×10^{-5}
Watts/gram (right side of boundary)	1.15	3.5×10^{-2}	5.76×10^{-2}	5.92×10^{-1}	1.20×10^{-2}	9.00×10^{-5}	
Total Heating							
Watts/cc (left side of boundary)	24.5	7.77	1.12	2.86	3.16×10^{-1}	3.75×10^{-2}	2.96×10^{-4}

TABLE A-III (CONT'D)

	BOUNDARY						
	1	2	3	4	5	6	7
Total Heating (Cont'd)							
Watts/cc (right side of boundary)	17.8	12.7	4.10	8.77×10^{-1}	1.15	8.26×10^{-2}	
Watts/gram (left side of boundary)	14.6	4.24	4.13×10^{-1}	3.75×10^{-1}	3.16×10^{-1}	9.16×10^{-3}	5.14×10^{-5}
Watts/gram (right side of boundary)	9.7	4.69	5.41×10^{-1}	8.77×10^{-1}	2.82×10^{-1}	1.28×10^{-2}	

APPENDIX II

SAMPLE CALCULATION FOR GAMMA HEATING IN THE CORE

Consider the point located on column 28, row 28, just outside the 6 by 6 experimental hole (see Figure 2-2). Gamma rays emerging from this point will lose most of their energy in traveling their first mean free path. The heat produced by the gamma rays at this point is calculated assuming that the gamma absorption here is equal to the average gamma production in the region within one mean free path of this point.

The mean free path of a 1 Mev gamma ray in the fuel area of the core is a function of the linear absorption coefficient of the aluminum and water mixture at this energy.

$$\mu_{\text{Al}} = 0.165 \text{ cm}^{-1} \quad \text{Volume fraction} = 0.4$$

$$\mu_{\text{H}_2\text{O}} = 0.042 \text{ cm}^{-1} \quad \text{Volume fraction} = 0.6$$

$$\mu_{\text{T}} = \sum_i \mu_i V F_i$$

$$\mu_{\text{T}} = (0.4) (0.165) + (0.6) (0.042)$$

$$\mu_{\text{T}} = 0.091 \text{ cm}^{-1}$$

$$\text{but } \lambda = \frac{1}{\mu_{\text{T}}}$$

$$\text{Therefore } \lambda = 11.0 \text{ cm.}$$

In like manner the mean free path of a 1 Mev gamma ray in the experimental hole is:

$$\mu_{Al}^{VF_{Al}} = (0.25) (0.165) = 0.040$$

$$\mu_{H_2O}^{VF_{H_2O}} = (0.39) (0.070) = 0.027$$

$$\mu_{SS}^{VF_{SS}} = (0.07) (0.47) = 0.033$$

$$\mu_{Zr}^{VF_{Zr}} = (0.05) (0.37) = 0.018$$

$$\text{Others} = 0.008$$

Therefore

$$\mu_T = 0.126 \text{ cm}^{-1}$$

and

$$\lambda = 7.94 \text{ cm.}$$

Since the fission cross section in the fuel region differs from that in the experimental hole, the gamma production in each region must be calculated separately. The gamma producing region of the fuel is obtained by drawing an arc of a circle whose center is just outside the corner of the 6 by 6 experimental hole. The circle radius is equal to the mean free path of a 1 Mev gamma ray as calculated for this aluminum and water mixture. Continuing this circle inside the experimental hole with a radius equal to the mean free path for this mixture results in the total gamma producing region to be considered.

The average flux contained in the sector drawn inside the experimental hole was found to be 5.36×10^{14} neutrons/cm²/sec. The average flux in the remaining

region outside the experimental hole was found to be 5.50×10^{14} neutrons/cm²/sec.

Of the total gamma producing region, three quarters lies outside the experimental hole in the fuel region, while the other one quarter is contained in the experimental hole.

Therefore the expression for gamma heating due to fission may be expressed as follows:

$$\frac{\text{power}}{\text{volume}} = \frac{1}{4} \sum f_E \phi_E + \frac{3}{4} \sum f_F \phi_F$$

where subscript E represents experimental hole region and subscript F represents normal fuel region.

Since each fission gives off approximately 12.2 Mev, the complete expression is:

$$\frac{\text{absorbed power}}{\text{volume}} = 12.2 \left\{ \frac{\sum f_E \phi_E}{4} + \frac{3 \sum f_F \phi_F}{4} \right\} \frac{\text{Mev}}{\text{sec-cc}}$$

Expressed in watts per gram the same formula takes the form:

$$\frac{\text{absorbed power}}{\text{gram}} = \frac{(12.2)(1.6 \times 10^{-13})}{\sum_i \rho_i V F_i} \left\{ \frac{1}{4} \sum f_E \phi_E + \frac{3}{4} \sum f_F \phi_F \right\}$$

$$\frac{\text{absorbed power}}{\text{gram}} = \frac{(12.2)(1.6 \times 10^{-13})}{(0.4)(2.699) + (0.6)(1)} \left\{ \frac{(4.25 \times 10^{-2})(5.36 \times 10^{14})}{4} + 3 \frac{(4.189 \times 10^{-2})(5.5 \times 10^{14})}{4} \right\}$$

$$\frac{\text{absorbed power}}{\text{gram}} = 26.7 \text{ watts/gram.}$$

Gamma rays produced by the capture of neutrons by nuclei are given off with a total energy approximately equal to the average binding energy per nucleon of the atom. For convenience, it was assumed that gamma rays totaling 7 Mev were given off for each neutron absorbed by a heavy metal. A value of 2.2 Mev was employed for water capture gammas.

Thus the power absorbed per gram due to capture gamma rays can be expressed in the form

$$\frac{\text{absorbed power}}{\text{gram}} = \frac{\phi_i \sum_i \leq_{ai} V F_i E_i + \phi_j \sum_j \leq_{aj} V F_j E_j}{\sum_i \rho_i V F_i}$$

where

$$\begin{aligned} \sum_i \leq_{ai} V F_i E_i &= (0.0197)(0.6)(2.2) + (0.0113)(0.4)(7) \\ &= 5.76 \times 10^{-2} \text{ Mev/cm.} \end{aligned}$$

$$\sum_j \Sigma_{aj} V F_j E_j = (0.25) (0.0113) (7) + (0.07) (0.208) (7) \\ + (0.39) (0.0197) (2.2) + (0.13) (0.058) (7) \\ = 1.914 \times 10^{-1} \text{ Mev/cm.}$$

$$\sum_i \rho_i V F_i = (0.4) (2.699) + (0.6) (1) \\ = 1.70 \text{ grams/cc}$$

$$\text{and } \phi_i = 5.50 \times 10^{14} \text{ neutrons/cm}^2/\text{sec}$$

$$\phi_j = 5.36 \times 10^{14} \text{ neutrons/cm}^2/\text{sec.}$$

Thus absorbed power per gram is

$$\frac{(3/4) (5.5 \times 10^{14}) (5.76 \times 10^{-2}) + (1/4) (5.36 \times 10^{14}) (0.191)}{1.7}$$

$$\frac{\text{power}}{\text{gram}} = 2.91 \times 10^{13} \frac{\text{Mev}}{\text{gram-sec}}$$

or

$$\frac{\text{power}}{\text{gram}} = 4.66 \text{ watts/gram.}$$

Therefore, the total power absorbed per gram may be found by taking the sum of the power produced by fission gamma rays and capture gamma rays.

$$\frac{\text{Total power absorbed}}{\text{gram}} = \frac{\text{Power due to fission}}{\text{gram}} + \frac{\text{Power due to capture}}{\text{gram}}$$

$$= 26.7 + 4.66$$

$$= 31.4 \text{ watts/gram.}$$

APPENDIX III

THE CHANGE OF THERMAL FLUX WITH TIME

As the fuel in the ETR is used up the thermal absorption cross section in the core decreases, and thus the thermal flux will increase if the power level remains constant. Since the source of fission neutrons is constant at a constant power level and since the fast cross sections do not change due to fuel burnup, the above-thermal flux should not change in magnitude although the shape of the distribution may change.

In the following analysis it is assumed that the thermal flux distribution does not change in shape during the running cycle. This hypothesis is discussed in Appendix IV. The problem now is to determine the magnitude of the flux changes with time.

A. Flat Flux Distribution

The problem can be solved simply and exactly if the flux is independent of position. Since the power level remains constant, at any time

$$\sum_f \phi = \sum_f^0 \phi_0 \quad (1)$$

where $\sum_f^0$ is the initial fission cross section and ϕ_0 is the initial thermal flux.

Then

$$c = \frac{\phi}{\phi_0} = \frac{\sum_f^0}{\sum_f} \quad (2)$$

The change in the number of atoms of fuel per cc per sec is

$$-\frac{1}{\sigma_{25}} \frac{d\sum_{25}}{dt} = -\frac{dN}{dt} = \phi \sum_{25} \quad (3)$$

$$\begin{aligned} \text{or } d\phi_{25} &= -\sigma_{25} \phi_{25} dt \\ d\phi_f &= -\sigma_{25} \phi_{25} dt \end{aligned} \quad (4)$$

But from (2) $\phi_o \leq_f^o = \phi \leq_f$, so that

$$d\phi_f = -\sigma_{25} \phi_o \leq_f^o dt \quad (5)$$

Integrating (5) yields

$$\phi_f = \phi_{fo} - \sigma_{25} \phi_o \leq_f^o dt \quad (6)$$

Then putting (6) into (2),

$$c = \frac{1}{1 - \sigma_{25} \phi_o t} \quad (7)$$

In the work on burnup the integral of c with time will be required. This is easily obtained from (7)

$$\int_0^t c dt = -\frac{1}{\sigma_{25} \phi_o} \ln (1 - \sigma_{25} \phi_o t) \quad (8)$$

B. Non-uniform Flux Distribution

Since the reactor runs at constant power

$$\int \phi \leq_f dV = \int \phi_o \leq_{fo} dV = K = \text{constant}. \quad (9)$$

Integrating (4) with respect to time yields

$$\phi_f = \phi_f^o e^{-\int \sigma_{25} \phi dt} \quad (10)$$

Substitute (10) into (9) and let $\Phi = c\Phi_0$ to obtain

$$\int c \Phi_0 \sum_f^0 e^{-\sigma_{25} \Phi_0} \int c dt dV = K \quad (11)$$

This can be rewritten as

$$K = - \frac{1}{\sigma_{25}} \frac{d}{dt} \int \sum_{f_0} e^{-\sigma_{25} \Phi_0} \int c dt dV \quad (12)$$

$$K = - \frac{1}{\sigma_{25}} \frac{d}{dt} \int \sum_{f_0} \left\{ 1 - \sigma_{25} \Phi_0 \int c dt + \frac{1}{2!} \left[\sigma_{25} \Phi_0 \int c dt \right]^2 + \dots \right\} dV \quad (13)$$

$$\text{Let } x = \int_0^t c dt, \quad B = \frac{1}{2} \int \Phi_0^2 \sigma_{25} \sum_{f_0} dV$$

Then neglecting terms in the integrand of higher order than $(\sigma_{25} \Phi_0 x)^2$

$$\int K dt = Kt = Kx - x^2 B \quad (14)$$

Thus,

$$x = \int_0^t c dt = \frac{K \pm \sqrt{K^2 - 4KBt}}{2B} \quad (15)$$

Now for small t , c must equal 1 and thus x must equal t . This then dictates the use of the minus sign in (15).

$$\int_0^t c dt = \frac{K - \sqrt{K^2 - 4KBt}}{2B} \quad (16)$$

Differentiating (16) yields

$$c = \frac{K}{\sqrt{K^2 - 4KBt}} = \frac{1}{\sqrt{1 - 4B/K t}} \quad (17)$$

The expression (17) is only valid for $\phi_0 \sigma t \ll 1$.

On the other hand, the corresponding equation (7) is true for all t .

It is of interest to apply the method of Section B when there is a flat flux distribution. In this case

$$\begin{aligned} \frac{B}{K} &= \frac{\frac{1}{2} \int \phi_0^2 \sigma \Sigma_{f0} dv}{\int \phi_0 \Sigma_{f0} dv} \\ &= \frac{1}{2} \sigma \phi_0 \end{aligned}$$

Using this result in (17) the following is obtained:

$$c = \frac{1}{\sqrt{1 - 2 \sigma \phi_0 t}} \quad (18)$$

Figure A3-1 is a plot of (18) and (7) as a function of t .

For $\sigma_{25} \phi_0 t < 0.3$ the two plots are in good agreement. However, for

$\sigma_{25} \phi_0 t > 0.3$ the results diverge rapidly.

In the burnup calculations of Section VII the quantity c is not important, but rather the integral $\int c dt$ is used. Integrating (18) gives

$$\int c dt = \frac{1 - \sqrt{1 - 2 \sigma_{25} \phi_0 t}}{\sigma_{25} \phi_0} \quad (19)$$

and this should be compared to the exact result of equation (8).

Plots of equations (19) and (8) are shown in Figure A3-2. It can be seen that these plots agree for values of $\phi_0 \sigma_{25} t < 0.4$, or values of $\sigma_{25} \phi_0 \int c dt = 0.5$. This would correspond to an average fuel burnup of 40 percent.

Thus equation (16) should be fairly accurate over the running cycle of the ETR. As can be seen from Figure A3-2, the use of equation (16) will cause an over-estimate of the burnup rate.

It is interesting to note, as might be argued on physical grounds, that in any reactor running at constant power the thermal flux must initially increase at a rate greater than that indicated by (7). Equation (17), which is valid for small t , shows that $\underline{c}$ is a monotonically increasing function of B/K . B/K in turn depends upon the ratio $\frac{\int \phi^2 dV}{\int \phi dV}$. If use is made of Schwartz's inequality,

then it is easily shown that

$$\frac{\int \phi^2 dV}{\int \phi dV} \geq \frac{\int \phi dV}{\int dV} \quad (20)$$

The quantity on the right is just the average flux, $\bar{\phi}$. Thus $\frac{B}{K} \geq \frac{1}{2} \bar{\phi} \sigma$.

The equality is true if ϕ is independent of position, and this is not possible in an actual reactor. Thus in a reactor

$$\frac{dc}{dt} > \sigma_{25} \bar{\phi} \quad (21)$$

This is obtained by differentiating (7) or (17). For the ETR, with $\bar{\phi} \approx 3 \times 10^{14}$, $\sigma_{25} = 572 \times 10^{-24}$

$$\frac{dc}{dt} > 1.5 \text{ percent per day.}$$

FIGURE A3-2

APPENDIX IV

THE USE OF PERTURBATION THEORY IN THE DESIGN OF THE ETR

A. Introduction

A considerable amount of the information important to the design and evaluation of the characteristics of a reactor is dependent upon a knowledge of the effect on reactivity produced by a variation in one or more of the reactor parameters. For example, in order to determine the critical mass of the reactor, the effect of the fuel loading on reactivity must be known. Similarly, the calculations of void coefficients, temperature coefficients, fission product poisoning, the effect of fuel burnup, etc. are no more than the determination of the effect on reactivity produced by a particular change in the reactor.

Throughout this discussion the following definition of reactivity will be used.

Let ν_e be the number of neutrons per fission observed experimentally, i. e., for U^{235} $\nu_e = 2.46$, and let ν be the number of neutrons per fission required to make a reactor critical in a particular situation. Then, in the particular situation the reactivity is defined as $\frac{\nu_e}{\nu}$. Therefore, for a U^{235} reactor if $\nu < 2.46$, the reactivity is greater than one, and the reactor is supercritical. If $\nu > 2.46$, the reactor is subcritical.

This definition of reactivity is that commonly used to define k_{eff} ,* the effective multiplication constant. Another physical definition of reactivity is that in an actual reactor the reactivity is the ratio of the number of neutrons in one generation to the number in the preceeding generation.

* Glasstone and Edlund, The Elements of Nuclear Reactor Theory, Van Nostrand, New York, 1952.

To avoid confusion, the infinite multiplication factor will always be written k_{∞} , so the symbol k will refer only to the effective multiplication factor or reactivity. In this discussion the symbol k will be used for reactivity. The change in reactivity will be expressed in terms of the relative change in k , or $\delta k/k$. From this definition $\delta k/k$ is equal to $-\delta\rho/\rho$.

If it is possible to compute the critical mass of the reactor, i. e. determine the reactivity under a given set of conditions, then in principle, reactivity coefficients can also be determined by merely finding the cross sections at the new condition and recomputing the reactivity as before. Unfortunately, except in the simplest cases, the computation of reactivity is involved, and expensive, and the program outlined above requires a separate calculation for each change considered. In addition, some of the effects of interest are very small and require extremely high accuracy in the calculations. Consider for example a reactor with a temperature coefficient of $10^{-5} \delta k/k/^{\circ}\text{C}$. In order to calculate the effect of a 10 degree rise in temperature, the reactivity at the two temperatures must be computed to an accuracy much better than one hundredth of a percent. Although this accuracy is required, at present the best results obtained for such calculations agree with experimental values only to the order of one percent or so in reactivity and possibly to within a factor of two (or worse) of this in temperature coefficient. Finally, some of the effects of interest involve non-uniform changes in the reactor core. Available calculational techniques are able to accommodate such changes only with difficulty, if at all.

A means of circumventing the difficulties described above was first proposed by E. P. Wigner, who suggested that the perturbation methods used extensively in quantum theory* could be adopted for reactor calculations. A large amount of work has subsequently been performed using perturbation techniques. (Several calculations of this type are described in The Reactor Handbook.†)

The advantage of using perturbation theory is that the change in reactivity can be expressed directly in terms of the initial conditions of the reactor and the variations in reactor parameters. Thus a single calculation for the initial case permits the effect on reactivity produced by any number of small changes to be determined. In addition, since the change in reactivity is determined directly the calculation need only be carried to the accuracy required in βk .

Perturbation techniques have been relied on heavily in the analysis of ETR cores. The use of perturbation techniques has been a necessity as well as a convenience in the design of the ETR, since the cost of the two-dimensional UNIVAC calculations needed to otherwise obtain required information would have been prohibitive. To the author's knowledge the two-dimensional adjoint fluxes presented in this document are the first to have been obtained, and the analyses described in Sections VII to X represent the first extensive use of perturbation techniques where a two-dimensional treatment was required.

* See for example, Schiff, Quantum Mechanics, Chapter 7, McGraw-Hill Book Co., New York, 1949.

† The Reactor Handbook, Volume 1, AECD-3645, Government Printing Office, Washington, D. C., 1955.

B. Derivation of the Perturbation Formula

The results derived here are applicable to the solution of reactor diffusion problems, regardless of the number of neutron groups used in a multigroup approximation. However, with the exception of the one-dimensional calculations of Section VI, the work on the ETR has been restricted to two neutron groups. The perturbation techniques will be illustrated using a two-group theory.

The central problem in reactor core calculations is the solution of the eigenfunction problem, which represents the diffusion equations. The problem may be written as

$$K\Phi = \nu J\Phi \quad (1)$$

The lowest eigenvalue ν_0 together with its eigenfunction Φ_0 , are sought as a function of the operators K and J, which depend upon the physical properties of the reactor.

In two-group theory equation (1) is a representation of the following equations:

$$\begin{aligned} \nabla \cdot D_1 \nabla \phi_1 - \Sigma_{r1} \phi_1 + \nu \Sigma_f \phi_2 &= 0 \\ \nabla \cdot D_2 \nabla \phi_2 - \Sigma_{r2} \phi_2 + \Sigma_{SD} \phi_1 &= 0 \end{aligned} \quad (2)$$

where

D_n is the diffusion coefficient of group n

Σ_{rn} is the removal cross section of group n

Σ_{SD} is the slowing down cross section

Σ_f is the thermal fission cross section

ϕ_n is the flux in group n

The matrix operator K is

$$- \begin{pmatrix} \nabla \cdot D_1 \nabla - \Sigma_{r1} & 0 \\ \Sigma_{SD} & \nabla \cdot D_2 \nabla - \Sigma_{r2} \end{pmatrix}$$

The matrix operator J is

$$\begin{pmatrix} 0 & \Sigma_f \\ 0 & 0 \end{pmatrix}$$

and the flux vector Φ is $\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

The lowest eigenvalue, λ_0 , sought is the number of neutrons produced per fission which would be required to make the reactor critical.

In order to perform perturbation calculations, the adjoint flux Φ^+ must be found in addition to the flux Φ . The adjoint flux Φ^+ is the solution of the adjoint equation

$$K^+ \Phi^+ = \lambda J^+ \Phi^+ \quad (3)$$

In a multigroup treatment, where the flux goes to zero at the boundaries or has a given logarithmic derivation at the boundaries, the adjoint operators K^+ and J^+ are obtained from K and J by rotating the matrices K and J 180 degrees around the diagonal axis. Thus

$$K^+ = - \begin{pmatrix} \nabla \cdot D_1 \nabla - \sum_{r_1} & \sum_{SD} \\ 0 & \nabla \cdot D_2 \nabla - \sum_{r_2} \end{pmatrix}$$

$$J^+ = \begin{pmatrix} 0 & 0 \\ \sum_f & 0 \end{pmatrix}$$

As previously stated, in reactor calculations interest centers around the lowest eigenvalue, ν_0 , and the associated flux vector, Φ_0 . There are, in general, a large number of eigen values for equation (1), each associated with a particular flux vector. The eigenvalues are designated as $\nu_0, \nu_1, \nu_2, \nu_3 \dots$, and the associated eigenfunctions as $\Phi_0, \Phi_1, \Phi_2 \dots$.

It is easily shown that the eigenvalues of equations (3) and (1) are identical, so there is a flux vector, Φ_i , and an adjoint flux vector, Φ_i^+ , associated with the eigenvalue ν_i . The following relationships are also easily proven:

$$\int \Phi_i^+ J \Phi_j = \int \Phi_i^+ K \Phi_j = 0 \quad \text{if } \nu_i \neq \nu_j \quad (4)$$

$$\int X J Y = \int Y J^+ X \quad \int X K Y = \int Y K^+ X \quad (5)$$

for any vectors X and Y which satisfy the boundary conditions. In expressions (4) and (5) $\int \Phi_i^+ J \Phi_j$ is defined to mean that the operator J operates on Φ_j , and then the inner product is taken between the vectors Φ_i^+ and $J \Phi_j$. The re-

sult is to be integrated over all space within the boundaries of the problem.

(See page A4-22 for such an expression written in detail.)

Throughout the remaining discussion the assumption is made that the lowest eigenvalue, ν_0 , is non-degenerate, i. e., there is a unique flux vector, Φ_0 , associated with ν_0 . This is a reasonable assumption which states in physical terms that there is a unique flux distribution in a critical reactor.

At this point the development of the perturbation expression can begin. Assume that a reactor is characterized by K and J, so that in the critical condition

$$K \Phi_0 = \nu_0 J \Phi_0$$

where

Φ_0 is the flux in the reactor

and

ν_0 is the number of neutrons per fission.

Let some change occur in the reactor causing K and J to change

$$K \longrightarrow K + \lambda \delta K$$

$$J \longrightarrow J + \lambda \delta J$$

for $0 \leq \lambda \leq 1$, where λ is a parameter which determines the strength of the perturbation. The perturbed reactor will have an eigenvalue and an eigenfunction different from ν_0 and Φ_0 . It is assumed that the new eigenvalue, ν' can be expressed as a power series in λ

$$\nu' = \nu_0 + \lambda \nu_a + \lambda^2 \nu_b + \lambda^3 \nu_c + \dots \quad (6)$$

and similarly assumed that the new eigenfunction can be expanded in a power series in λ .

$$\Phi' = \Phi_0 + \lambda \psi_1 + \lambda^2 \psi_2 + \lambda^3 \psi_3 + \dots \quad (7)$$

The equation of the perturbed reactor becomes

$$\begin{aligned} (K + \lambda \delta K) (\Phi_0 + \lambda \psi_1 + \lambda^2 \psi_2 + \dots) \\ = (\nu + \lambda \nu_a + \lambda^2 \nu_b + \dots) (J + \lambda \delta J) (\Phi_0 + \lambda \psi_1 + \lambda^2 \psi_2 + \dots) \end{aligned} \quad (8)$$

If it is assumed that (8) is analytical in λ for $0 \leq \lambda \leq 1$, then the various orders of perturbation theory can be obtained by equating the coefficients of the various powers of λ in (8). Zero-order perturbation theory considers the coefficient of λ^0 , and in this case

$$K \Phi_0 = \nu J \Phi_0$$

(i. e., zero order theory concerns the unperturbed reactor.)

In first-order perturbation theory the coefficients of λ^1 are equated to give

$$K \psi_1 + \delta K \Phi_0 = \nu \delta J \Phi_0 + \nu_a J \Phi_0 + \nu J \psi_1 \quad (9)$$

If (9) is multiplied on the left by Φ_0^+ and integrated, then (10) is obtained.

$$\begin{aligned} \int \Phi_0^+ K \psi_1 - \nu \int \Phi_0^+ J \psi_1 + \int \Phi_0^+ \delta K \Phi_0 - \nu \int \Phi_0^+ \delta J \Phi_0 \\ = \nu_a \int \Phi_0^+ J \Phi_0 \end{aligned} \quad (10)$$

By using expressions (5) and (3), the first two terms of (10) can be written as

$$\int \Phi_o^+ K \psi_1 - \nu \int \Phi_o^+ J \psi_1 = \int \psi_1 (K^+ \Phi_o^+ - \nu J^+ \Phi_o^+) = 0$$

Therefore

$$\nu_a = \frac{\int \Phi_o^+ \mathcal{K} \Phi_o - \nu \int \Phi_o^+ \mathcal{J} \Phi_o}{\int \Phi_o^+ J \Phi_o} \quad (11)$$

$$\text{or } \lambda \nu_a = \frac{\int \Phi_o^+ \lambda \mathcal{K} \Phi_o - \nu \int \Phi_o^+ \lambda \mathcal{J} \Phi_o}{\int \Phi_o^+ J \Phi_o}$$

If λ is absorbed in the matrices $\mathcal{K}$ and $\mathcal{J}$, then (11) can be rewritten

$$\frac{\mathcal{K} \nu}{\nu} = \frac{\overline{\lambda} \nu_a}{\nu} = \frac{\int \Phi_o^+ \mathcal{K} \Phi_o - \nu \int \Phi_o^+ \mathcal{J} \Phi_o}{\nu \int \Phi_o^+ J \Phi_o} \quad (12)$$

Expression (12) is the desired relation, expressing the change in reactivity in terms of the initial conditions of the reactor and the perturbations $\mathcal{J}$ and $\mathcal{K}$.

The derivation of (12) shown above is not the easiest way of reaching the result.

In the Reactor Handbook, p. 471, it is shown very simply that

$$\mathcal{K} \nu = \frac{\int \Phi_o^+ \mathcal{K} \Phi' - \nu \int \Phi_o^+ \mathcal{J} \Phi'}{\int \Phi_o^+ J \Phi'} \quad (13)$$

where $\bar{\Phi}$ is the flux in the perturbed reactor

$$\text{and } \nu' = \nu_0 + \delta\nu$$

The result (13) is exact and agrees with (12) if the perturbations are small, i.e., $\delta\nu$ is small and $\bar{\Phi}'$ not very different from $\bar{\Phi}_0$.

In most work with perturbation techniques equation (12) is used, and the analysis proceeds no further. However, in the work on burnup in the ETR assumptions were made concerning the shape of the thermal flux as a function of time when the original conditions were perturbed. Some insight into how the flux changes can be obtained by continuing the perturbation analysis, and this is the reason for proceeding in the manner described above.

Before proceeding further, it is instructive to look at the reactor equation from a different viewpoint. Equation (1) can be rewritten

$$\begin{aligned} K\bar{\Phi} - \nu J\bar{\Phi} &= (K - \nu J)\bar{\Phi} \\ &= M\bar{\Phi} = 0 \end{aligned} \quad (14)$$

Now the eigenfunction equation can be written

$$M\bar{\Phi}_i = \alpha_i \bar{\Phi}_i \quad (15)$$

and equation (14) can be considered the defining equation for $\bar{\Phi}_0$, the eigen-

function belonging to the eigenvalue $\alpha_0 = 0$. With the exception of the eigenfunction belonging to α_0 , the eigenfunctions $\bar{\Phi}_i$ of (15) are not equal to the

$\bar{\Phi}_i$ previously discussed. It is easily shown that the equations analogous to

(4) and (5) are

$$\int \Phi_i^{'+} \Phi_j = 0 \quad (16)$$

$$\int \mathbf{X} \mathbf{M} \mathbf{Y} = \int \mathbf{Y} \mathbf{M}^+ \mathbf{X} \quad (17)$$

The eigenfunction Φ_i is arbitrarily normalized so that

$$\int \Phi_i^{'+} \Phi_i = 1 \quad (18)$$

A fundamental assumption of perturbation theory is that the eigenfunctions form a complete set. In this case, an arbitrary function can be expanded in a series of eigenfunctions. For example, the function ψ_1 of (8) can be expanded in the form

$$\psi_1 = a_0 \Phi_0' + a_1 \Phi_1' + a_2 \Phi_2' + \dots \quad (19)$$

If ψ_1 is known, the individual a_i can be determined by multiplying (19) on the left by $\Phi_i^{'+}$ and integrating. Then, using (16)

$$a_i = \int \Phi_i^{'+} \psi_1 \quad (20)$$

If (19) is substituted into (9) and the result multiplied by Φ_i' , (21) is obtained.

$$a_i = \frac{\int \Phi_i^{'+} \mathcal{L} K \Phi_0 - \nu \int \Phi_i^{'+} \mathcal{L} J \Phi_0 - \mathcal{L} \nu \int \Phi_i^{'+} J \Phi_0}{-\alpha_i} \quad (21)$$

Equation (21) is the first order result which describes the change in the initial flux distribution, Φ_0 , due to the perturbation. Equation (21) can be easily interpreted if the perturbed reactor equations are set down.

$$\left[(K + \delta K) - (\nu + \delta \nu) (J + \delta J) \right] \left[\Phi_0 + \delta \Phi \right] = 0 \quad (22)$$

Collecting terms

$$(K - \nu J) \delta \Phi = \delta K \Phi_0 - \nu \delta J \Phi_0 - \delta \nu J \Phi_0 \quad (23)$$

where terms of the order of $\delta \Phi \delta \nu$, $\delta \Phi \delta K$, $\delta \Phi \delta J$ have been neglected. If (23) is used in (21), then

$$\begin{aligned} a_i &= \frac{\int \Phi_i^{'+} (K - \nu J) \delta \Phi}{+\alpha_i} = \frac{\int \Phi_i^{'+} M \delta \Phi}{+\alpha_i} \\ &= \frac{\int \delta \Phi M^+ \Phi_i^{'+}}{+\alpha_i} \end{aligned} \quad (24)$$

and

$$a_i = \int \Phi_i^{'+} \delta \Phi \quad (25)$$

The a_i are therefore just the projection of $\delta \Phi$ on the eigenfunction $\Phi_i^{'+}$. This result is, of course, not strange. Since $\delta \Phi$ is equal to ψ_1 in first order perturbation theory, equations (22) to (25) have merely led us back to (20).

A comparison of (21) with (16) indicates that an a_i will be small, unless

1. α_i is small

or

2. Φ_i is close in shape to $[\delta K - \nu \delta J - \delta \nu J] \Phi_0$

The first possibility implies that there is another flux distribution besides

Φ_0 (with $\alpha_0 = 0$) which comes very close to satisfying the diffusion equations.

Therefore, it is possible for a small perturbation to shift the reactor into a mode such that the flux distribution has large components of this second distribution. It is pointed out below that this situation does not exist for reactors with uniform cores, where the flux distribution is close to the shape of a cosine curve. However, in the ETR it is possible that with certain experimental loadings this situation can exist.

The second possibility states that those eigenfunctions important in the expansion of $\delta \Phi$ will be close in shape to $[\delta K - \nu \delta J - \delta \nu J] \Phi_0$. In the

study of burnup in the ETR two types of perturbations arose. In the first type, Σ_f and Σ_a were changed uniformly. Therefore, the flux change, $\delta \Phi$, will have thermal and above-thermal components which are close in shape to the thermal component of Φ_0 . In the second type of perturbation, δK and δJ were proportional to the thermal flux.* Thus, $(\delta K - \nu \delta J - \delta \nu J) \Phi_0$ will have thermal and above-thermal components proportional to the square of the

* A perturbation of the Type $\sum (1 - e^{-\sigma \phi_2 t})$ can be considered as $\sum \sigma \phi_2 t$ for small $\sigma \phi_2 t$, and as $\sum$ for large $\sigma \phi_2 t$.

thermal component of $\bar{\Phi}_0$. Since the components of $\bar{\Phi}_0$ are positive everywhere, the square of the thermal component resembles in shape the thermal component itself. It can therefore be concluded that for this type of perturbation the components of $\delta\bar{\Phi}$ will resemble the thermal component of $\bar{\Phi}_0$. Thus, if there is no solution to (15) except $\bar{\Phi}_0$ with α_1 close to zero, then the shape of the flux change will be close to the shape of the thermal flux in the reactor.

The situation is illustrated by the results of a three-group, one-dimensional calculation. The three neutron groups considered were thermal neutrons, neutrons between 10 Kev and thermal energies, and neutrons above 10 Kev. In these calculations the core was assumed to be uniformly loaded at the start with 185 g of U^{235} and 1.25 g of boron per fuel element. The power distribution at startup was determined using this loading, and the reactor was assumed to operate with this power distribution for 4.25 days. The core was then divided into three concentric regions of equal annular widths, and the average fuel and poison burnup was computed separately in each region, assuming a power generation of 270 MW in the core (175 MW in 65 fuel rods). The flux distribution in this new core was then computed using the different loadings in each of the three annular regions. The procedure was repeated at 4.25-day intervals.

The results of these calculations are shown in Figures A4-1 to A4-4. Figure A4-1 shows the change in the power distribution in the core at the different intervals of time. The humps appearing in the curve are a result of approxi-

POWER DISTRIBUTION

Figure A4-1

THERMAL FLUX

EPITHERMAL NEUTRON FLUX

Figure A4-3

FAST NEUTRON FLUX

imating the core as three concentric annuli. More representative curves of actual conditions would be obtained if "average" lines were drawn through each curve, since the fuel and poison content of the core vary continuously. The "average" lines should follow the general tendency of the curves, but not the sharp humps. Figures A4-2 to A4-4 show the thermal, epithermal and fast flux distributions. The flux distribution which shows the most change is the thermal flux, but its shape changes only slightly. Thus, at startup the ratio of the flux at the core center to the flux at the core-beryllium interface is 2.08. After 17 days operation, the ratio has changed by only six percent, to 2.21. The epithermal and fast flux distributions do not change very much, but the general tendency is towards a flattening of the flux distribution.

Dr. Karl Cohen* has reported the results of similar calculations performed to predict the lifetime of low enriched reactors. In these (three-group) calculations it was also found that the thermal flux distribution did not vary appreciably from its initial configuration. In the case of a one-group approximation with a bare reactor, the calculation can be handled analytically. Cohen reports that the flux distribution is very similar to the original flux distribution, provided

that $\frac{\Delta k_{\infty \max}}{k_{\infty c} - 1} < 4$,[†] where $\Delta k_{\infty \max}$ is the maximum change in k_{∞} at any

point in the reactor, and $k_{\infty c}$ is the critical value of k_{∞} . This is a surprisingly weak condition.

* K. Cohen to B. Wolfe, Private Communication, March 3, 1956.

† A result similar to this can be obtained from the perturbation treatment described in this Appendix.

The cases discussed above are cases where the flux distributions have shapes similar to cosine or Bessel functions. In these simple cases the various eigenfunctions and eigenvalues can easily be obtained, and it can be verified that only α_0 is close to zero.

In the case of the actual ETR, there is no evidence available concerning the eigenvalues, other than the zero eigenvalue. In all of the work on burnup in the ETR the assumption was made that the other eigenvalues are not close to zero, and thus the thermal flux distribution does not change appreciably. In general, this assumption would be expected to lead to an overestimate of the burnup effects, since the assumption was made that the maximum burnup occurred at the place of highest thermal flux. In other words, the $\int K$ and $\int J$ of (12) are largest where Φ_0 is largest. If the flux distribution shifts after some burnup has occurred, the positions of greatest burnup will be in regions of lower relative thermal flux than they were previously. Thus (12) will overestimate the effect on k when the old flux distribution is used. It is therefore expected that the estimates in Section VII will somewhat underestimate the running cycle of the ETR.

C. Method Used to Obtain the Adjoint Flux

This discussion describes the method used in the ETR analysis to obtain the adjoint flux from a computer code set up to solve only the flux equations. The method is directly applicable to a multigroup analysis when fission occurs only in the last group. The scheme will be illustrated by reference to a three-group calculation in which fission occurs only in the third group, and fission neutrons enter only the first group.

In three groups, the diffusion equations to be solved are

$$\begin{aligned}
 \nabla \cdot D_1 \nabla \phi_1 - \sum_{r_1} \phi_1 + \nu \sum_{f_3} \phi_3 &= 0 \\
 \nabla \cdot D_2 \nabla \phi_2 - \sum_{r_2} \phi_2 + \sum_{SD_1} \phi_1 &= 0 \\
 \nabla \cdot D_3 \nabla \phi_3 - \sum_{r_3} \phi_3 + \sum_{SD_2} \phi_2 &= 0
 \end{aligned}
 \tag{26}$$

where D_n is the diffusion coefficient of group n

$\sum_{r_n}$ is the removal cross section of group n

$\sum_{SD_n}$ is the slowing down cross section of group n

$\sum_{f_n}$ is the fission cross section in group n

ϕ_n is the flux in group n

In matrix notation the equations (26) can be represented as

$$\nu J \bar{\Phi} - K \bar{\Phi} = 0 \tag{27}$$

where the vector $\bar{\Phi}$ is

$$\bar{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \tag{28}$$

the matrix J is

$$J = \begin{pmatrix} 0 & 0 & \sum_{f_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{29}$$

and the matrix K is

$$K = \begin{pmatrix} -\nabla \cdot D_1 \nabla + \sum_{r_1} & 0 & 0 \\ -\sum_{SD_1} & -\nabla \cdot D_2 \nabla + \sum_{r_2} & 0 \\ 0 & -\sum_{SD_2} & -\nabla \cdot D_3 \nabla + \sum_{r_3} \end{pmatrix} \tag{30}$$

In order to perform perturbation calculations, the adjoint flux $\bar{\Phi}^+$, which is the solution to equation (31), must be found in addition to the flux $\bar{\Phi}$.

$$\nu J^+ \bar{\Phi}^+ - K^+ \bar{\Phi}^+ = 0 \quad (31)$$

The matrix J^+ is

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Sigma_f & 0 & 0 \end{pmatrix} \quad (32)$$

and the matrix K^+ is

$$\begin{pmatrix} -\nabla \cdot D_1 \nabla + \Sigma_{r1} & -\Sigma_{SD1} & 0 \\ 0 & -\nabla \cdot D_2 \nabla + \Sigma_{r2} & -\Sigma_{SD2} \\ 0 & 0 & -\nabla \cdot D_3 \nabla + \Sigma_{r3} \end{pmatrix} \quad (33)$$

The set of equations represented by (31) can be written

$$\begin{aligned} \nabla \cdot D_3 \nabla \Phi_3^+ - \Sigma_{r3} \Phi_3^+ + \nu \Sigma_{f3} \Phi_1^+ &= 0 \\ \nabla \cdot D_2 \nabla \Phi_2^+ - \Sigma_{r2} \Phi_2^+ + \Sigma_{SD2} \Phi_3^+ &= 0 \\ \nabla \cdot D_1 \nabla \Phi_1^+ - \Sigma_{r1} \Phi_1^+ + \Sigma_{SD1} \Phi_2^+ &= 0 \end{aligned} \quad (34)$$

Let the quantities in (34) be relabeled in the following way:

OLD LABEL

Φ_3^+

Φ_2^+

NEW LABEL

Φ_1

Φ_2

OLD LABELNEW LABEL

ϕ_1^+

ϕ_3

$D_3, \sum_{r_3}$

$D_1, \sum_{r_1}$

$\sum_{f_3}$

$\sum_{f_3}$

$D_2, \sum_{r_2}$

$D_2, \sum_{r_2}$

$\sum_{SD_2}$

$\sum_{SD_1}$

$D_1, \sum_{r_1}$

$D_3, \sum_{r_3}$

$\sum_{SD_1}$

$\sum_{SD_2}$

When the quantities in (34) are relabeled it can be seen that the resulting equations are identical to those of (26). Therefore, if a machine code is available which can solve the reactor equations (26), the adjoint equations (34) can also be solved by means of the relabeling indicated above. A test of the procedure is that the value of ν obtained from the solution of (26) is the same as the value of ν obtained from the solution of (34).

This scheme has been used with the one-dimensional PROD code in three groups, and the two-dimensional MUG and Cuthill codes in two groups. In each case the ν of (34) equalled the ν of (26). The adjoint flux plots shown in this report were all obtained in the manner described above.

D. Expansion of the Perturbation Formula

The two-dimensional approximation to the diffusion equations is

$$\begin{aligned} D_1 \nabla^2 \phi_1 - D_1 B^2 \phi_1 - \Sigma_{SD} \phi_1 + \nu \Sigma_f \phi_2 &= 0 \\ D_2 \nabla^2 \phi_2 - D_2 B^2 \phi_2 - \Sigma_a \phi_2 + \Sigma_{SD} \phi_1 &= 0 \end{aligned} \quad (35)$$

In (35) the fluxes ϕ_1 and ϕ_2 are considered functions of x and y . The z dependence is taken into account by the term $DB^2\phi$, which represents the leakage of neutrons in the vertical direction from the horizontal plane of interest.

With the equations (35) the expanded form of (12) becomes:

$$\begin{aligned} \delta \nu = \int \bigg\{ & \delta D_1 \left[-\phi_1^+ \nabla^2 \phi_1 + B^2 \phi_1^+ \phi_1 \right] \\ & + \delta \Sigma_{SD} \left[\phi_1^+ \phi_1 - \phi_2^+ \phi_1 \right] \\ & + \delta D_2 \left[-\phi_2^+ \nabla^2 \phi_2 + B^2 \phi_2^+ \phi_2 \right] + \delta \Sigma_a \left[\phi_2^+ \phi_2 \right] \\ & + \delta B^2 \left[D_1 \phi_1^+ \phi_1 + D_2 \phi_2^+ \phi_2 \right] \\ & - \nu \delta \Sigma_f \left[\phi_1^+ \phi_2 \right] \bigg\} d\tau \\ & + \int \Sigma_f \phi_1^+ \phi_2 \, d\tau \end{aligned} \quad (36)$$

If (35) is solved for $\nabla^2 \Phi_1$ and $\nabla^2 \Phi_2$, then

$$\begin{aligned}\nabla^2 \Phi_1 &= B^2 \Phi_1 + \frac{\sum_{SD}}{D_1} \Phi_1 - \frac{\nu \sum_f}{D_1} \Phi_2 \\ \nabla^2 \Phi_2 &= B^2 \Phi_2 + \frac{\sum_a}{D_2} \Phi_2 - \frac{\sum_{SD}}{D_2} \Phi_1\end{aligned}\tag{37}$$

When (37) is substituted into (36) the result is

$$\begin{aligned}\delta \nu &= \int \left\{ \delta D_1 \left[-\Phi_1^+ \frac{\sum_{SD}}{D_1} \Phi_1 + \frac{\nu \sum_f}{D_1} \Phi_1^+ \Phi_2 \right] \right. \\ &\quad + \delta \sum_{SD} \left[\Phi_1^+ \Phi_1 - \Phi_2^+ \Phi_1 \right] \\ &\quad + \delta D_2 \left[-\frac{\sum_a}{D_2} \Phi_2^+ \Phi_2 + \frac{\sum_{SD}}{D_2} \Phi_2^+ \Phi_1 \right] \\ &\quad + \delta \sum_a \left[\Phi_2^+ \Phi_2 \right] + \delta B^2 \left[D_1 \Phi_1^+ \Phi_1 + D_2 \Phi_2^+ \Phi_2 \right] \\ &\quad \left. - \nu \delta \sum_f \left[\Phi_1^+ \Phi_2 \right] \right\} d\tau \\ &\quad + \int \sum_i \Phi_1^+ \Phi_2 d\tau\end{aligned}\tag{38}$$

Equation (38) is the expanded form of the perturbation equations which have been used in the analysis of the ETR.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

A. Introduction

A number of the ETR calculations required a knowledge of the age in a metal-water mixture, on which no experimental evidence exists. In the fuel element itself the problem is not serious, since measurements are available for aluminum-water mixtures close to the composition of the fuel element (60 percent water, 40 percent aluminum). However, in the ETR the aluminum filler pieces are 93 percent aluminum, 7 percent water, and the experimental holes when loaded with the various experiments contain a variety of metals in metal-to-water ratios on which no experimental evidence is available. In particular, no measurements are reported on iron-water mixtures.

A survey of the situation is presented in the Geneva paper by Wilkins, et al.¹ In this paper the meager experimental results available are compared to various theoretical calculations. It is plain from the comparison that the theoretical results are not to be trusted.

Taking into consideration the situation described above and the need for information on the age in metal-water mixtures, a phenomenological approach to the problem was adopted. The aim was not to predict the age from first principles, but rather to use the picture of the underlying physical processes to interpret available experimental data in order to obtain age values. By choosing two constants which appear in this treatment, the age is made to agree with experimental values for pure water and for an aluminum-water mixture having an aluminum-to-water ratio of 0.5. The results of this treatment agree very well with

1. Wilkins, Hellens, and Zweifel, Status of Experimental and Theoretical Information on Neutron Slowing-Down Distributions in Hydrogenous Materials, Geneva Conference, 8/P/597, 1955.

experimental data on both the other aluminum-water mixture and the two zirconium-water mixtures. A curve of the age in iron-water mixtures was also computed and appears reasonable, although as previously stated no experimental values are available for comparison.

B. Calculation of the Age in Metals

A difficulty encountered in the computation of the age in metals and metal-water mixtures was the uncertainty of how to treat the inelastic scattering from the metal nucleus. However, all the materials of interest here have the property that the first excited state is in the vicinity of 1 Mev.^{2, 3, 4} Thus, once a neutron is reduced below this energy no inelastic events can take place. The inelastic scattering threshold for aluminum, iron, and zirconium are respectively 0.85, 0.85 and 0.94 Mev. This inelastic scattering threshold suggests that the age calculation should be performed separately in two energy regions, that above the inelastic scattering threshold and that below the threshold.

Consider the region below the inelastic threshold. Neutrons enter this region in three ways; by direct formation in the fission process, by inelastic scattering from a higher energy, and by a succession of elastic collisions from a higher energy. This latter process makes a comparatively small contribution, compared to the effect of inelastic collisions. The shapes of the curves for

2. Hollander, Perlman, and Seabark, RMP 25, p. 469, 1953.

3. Kiehn and Goodman, Physics Review, Vol. 95, p. 989, 1954.

4. Guernsey and Wattenberg, Data for Zirconium, quoted in BNL 325.

inelastic cross section as a function of energy^{3,4} indicate that at threshold the cross section is zero, and that until 100 Kev or more above the threshold is reached the cross section does not become large. After the region of rise, the cross section remains fairly flat for several Mev or more. Thus the spectrum of inelastically scattered fission neutrons will be fairly smooth and roughly follow the shape of the fission spectrum. However, the spectrum is displaced by the energy loss of the inelastic event. Below 100 Kev or so the spectrum drops rapidly. The bulk of the inelastically scattered neutrons lie in the region between 200 Kev and one Mev.

Consider the case of a pure metal. Below the inelastic threshold, age theory is applicable for the metals of interest. Thus, the age to thermal energies for a neutron of $E < E_{in}$, where E_{in} is the threshold for inelastic scattering, is given by

$$\tau = \int_{E_{thermal}}^E \frac{D(E)}{\xi \Sigma_s} \frac{dE}{E} \quad (1)$$

In the region between 200 Kev and $1/40$ ev $\int_{1/40}^{2 \times 10^5} \frac{dE}{E} = 13.6$. In the region

between 1 Mev and 200 Kev $\int_{2 \times 10^5}^{10^6} \frac{dE}{E} = 1.6$, so that if nothing was known about

the neutron spectrum, except that the neutrons were between 200 Kev and 1 Mev, the maximum error in the age calculation would be about 12 percent. This error can be reduced by taking the average energy of the inelastic neutrons as 700 Kev on the basis of the above arguments.

Summarizing then, in the case of a metal the age of inelastically scattered neutrons can be computed using age theory and an average energy of 700 Kev.

For aluminum this gives $\tau_{<0.7} = 3,640 \text{ cm}^2$.

Now consider the region above 0.7 Mev, where inelastic events are prominent. In this region inelastic scattering measurements made with fission neutrons by Berster and Carter⁵ are used. In the case of aluminum, these investigators quote a cross section for inelastic scattering of fission neutrons to an energy below 0.7 Mev. The measured value is 0.37 barns. This corresponds to a macroscopic cross section for inelastic collisions of $\Sigma_i = 0.022 \text{ cm}^{-1}$. For the region above 0.7 Mev, this can be treated as a removal cross section. Thus the age in going from fission energy to below 0.7 Mev is given by

$$\tau_{>0.7} = \frac{D_{>0.7}}{\Sigma_i} \quad (2)$$

The diffusion coefficient for energies above 0.7 Mev can be obtained in the usual way from the formula

5. Berster and Carter, quoted in AECD 345, p. 112.

$$D_{>0.7} = \frac{\int_{0.7}^{\infty} D(E) \phi(E) dE}{\int_{0.7}^{\infty} \phi(E) dE} \quad (3)$$

In evaluating (1) and (3), the scattering cross sections were taken from BNL 325. In evaluating (2), $\phi(E)$ was taken from the spectrum measured in a water-aluminum reactor and quoted in IDO 16133. The results for aluminum are

$$D_{>0.7} = 1.8 \text{ cm}$$

$$\tau_{>0.7} = \frac{1.8}{0.022} = 82 \text{ cm}^2$$

Thus, the total age in aluminum is 3720 cm^2 . Analogous work for iron yields

$$\tau_{\text{Fe}} = 588 \text{ cm}^2$$

and for zirconium

$$\tau_{\text{Zr}} = 6,940 \text{ cm}^2.$$

C. Calculation of the Age in Metal-Water Mixtures

In computing the age in metal-water mixtures, the energy range was again broken up into regions above and below 0.7 Mev. In the region above 0.7 Mev, $D_{>0.7}$ was computed for water using the curve in IDO 16133 for $D_{\text{H}_2\text{O}}(E)$ and $\phi(E)$. A removal cross section, $\Sigma_{\text{H}_2\text{O}}$, is then defined so

that $\frac{D_{H_2O > 0.7}}{\sum_{H_2O}} = \tau_{> 0.7}$. Then, for a metal-water mixture, the age of fission

neutrons to 0.7 Mev is given by

$$\tau_{> 0.7} = \frac{\left(\frac{V.F. H_2O}{D_{H_2O > 0.7}} + \frac{V.F. M}{D_{M > 0.7}} \right)^{-1}}{\sum_M + \sum_{H_2O}} \quad (4)$$

where the subscript M refers to the metal. The total $D_{> 0.7}$ of the mixture, calculated as in the numerator, is not strictly correct, since the energy integration indicated by (3) should be performed. However, it was felt that this approximation was within the accuracy of the calculation.

For energies below 0.7 Mev, except for very high metal-to-water ratios, the energy loss in collisions with metal nuclei is insignificant compared to the losses incurred in collisions with hydrogen. In this region the physical definition of the age can be used, i. e., one sixth of the mean square slowing down distance. With this definition, the slowing down process can be taken as equivalent to a random walk process, and the effect the presence of the metal nuclei have on the age can be inferred. It is well known that in a random walk process the mean square distance travelled is proportional to the number of collisions multiplied by the square of the average collision length. Thus, in water

$$\tau_{H_2O} = C N_{H_2O} \lambda_{H_2O}^2$$

where

N_{H_2O} is the number of collisions required to slow down a neutron
in water

λ_{H_2O} is the path length between collisions

and

C is a proportionality constant.

If the nuclear effects of the metal nuclei were completely ignored, the presence of the metal would simply reduce the volume fraction of the water and thus increase λ . Equation (5) would then become

$$\tau = C N_{H_2O} \left(\frac{\lambda_{H_2O}}{V.F. H_2O} \right)^2$$

where V. F. H_2O is the volume fraction of the water in the metal-water mixture.

Actually, the metal nuclei perform another function in that they tend to keep a neutron in a given region by changing its direction of motion in a collision.

Therefore, in a metal-water mixture the number of collisions is greater than N_{H_2O} , but the mean path length between collisions is less than λ_{H_2O} . If the energy loss in a neutron-metal nucleus collision is neglected, it still takes N_{H_2O} hydrogen collisions to slow down the nucleus. The total number of collisions required is therefore

$$N = N_{H_2O} \frac{(V.F. H_2O \sum_{H_2O} + V.F. M \sum_M)}{V.F. H_2O \sum_{H_2O}}$$

where Σ_M refers to the metal, Σ_{H_2O} to the water, and

$$\frac{V.F._{H_2O} \Sigma_{H_2O} + V.F._M \Sigma_M}{V.F._{H_2O} \Sigma_{H_2O}} \text{ is the number of collisions required in the}$$

mixture in order to have one collision with the hydrogen. The average distance between a collision is

$$\lambda = \frac{1}{V.F._{H_2O} \Sigma_{H_2O} + V.F._M \Sigma_M} = \lambda_{H_2O} \left(\frac{\Sigma_{H_2O}}{V.F._{H_2O} \Sigma_{H_2O} + V.F._M \Sigma_M} \right)$$

Then τ for the mixture is equal to

$$\tau = CN \lambda^2 = CN_{H_2O} \lambda_{H_2O}^2 \frac{\Sigma_{H_2O}^2}{V.F._{H_2O} \Sigma_{H_2O} (V.F._{H_2O} \Sigma_{H_2O} + V.F._M \Sigma_M)}$$

$$\tau = \frac{\tau_{H_2O}}{V.F._{H_2O} (V.F._{H_2O} + V.F._M \Sigma_M / \Sigma_{H_2O})} \quad (7)$$

Thus, using (4) for $E > 0.7$ Mev and (7) for slowing down from 0.7 Mev to thermal energies, the total age is given by

$$\tau = \frac{\left(\frac{V.F._{H_2O}}{D_{H_2O > 0.7}} + \frac{V.F._M}{D_{M > 0.7}} \right)^{-1}}{\Sigma_M + \Sigma_{H_2O}} + \frac{\tau_{H_2O < 0.7}}{V.F._{H_2O} (V.F._{H_2O} + V.F._M \Sigma_M / \Sigma_{H_2O})} \quad (8)$$

In calculating (8) the quantity Σ_M / Σ_{H_2O} was obtained by averaging the cross sections in BNL 325 between 0.025 ev and 0.7 Mev. Only the hydrogen in the water was considered significant.

There are two unknown parameters in (8), $\tau_{H_2O < 0.7}$, the age from 0.7 Mev to thermal energies in water, and Σ_{H_2O} , the removal cross section in water for fission neutrons above 0.7 Mev. These were computed by making τ agree with the measured values in both pure water and in a water-aluminum mixture with a water to metal ratio of 0.5. In the former case τ is 31.4 cm^2 , and in the latter case $\tau = 51.6 \text{ cm}^2$. Solving (8) for $\tau_{H_2O < 0.7}$ and Σ_{H_2O} gives in these cases

$$\tau_{H_2O < 0.7} = 12.8 \text{ cm}^2 \quad (9)$$

$$\Sigma_{H_2O} = 0.113$$

With the values of (9), equation (8) can be used for any water to metal ratio. In the analysis above, the slowing down by elastic collisions with the metal nuclei was ignored. For very high metal to water ratios, this is of course not valid. However, the age for metals alone has previously been computed. Thus for very high metal to water ratios the curve obtained in (8) can be extended to meet the values obtained for the metals alone.

The results for the case of aluminum-water mixtures can be seen in Figure A5-1. It should be noted that the curve is in good agreement with the value

7. AECD 3645, p. 485-6.

Figure A5-1

for 33 percent aluminum in the mixture. Of course, it might be argued that almost any smooth curve through the other two experimental points would come close to the third point. More interesting is the excellent agreement with the experimental points in zirconium-water mixtures (Figure A5-2). No new information was added to the theory to obtain the zirconium-water curve. The values of (9) were used for $\tau_{\text{H}_2\text{O}} < 0.7$ and $\sum_{\text{H}_2\text{O}}$. Figure A5-3 shows the results for iron-water mixtures. As previously mentioned, there are no experimental results for these mixtures. In this curve the values obtained from (8) for high metal-to-water ratios are shown so the method used to extrapolate the curve to the 100 percent metal value can be seen.

D. Summary

A method for computing ages in metal-water mixtures has been presented. The results agree very well with the few experimental checks available in aluminum- and zirconium-water mixtures, and provide a convenient way of interpolating between the experimental points. The age in iron-water mixtures has been calculated but no experimental checks are available. A method for the calculation of the age in pure materials has been presented and applied to aluminum, iron, and zirconium. These latter results are estimated to be good to within about 10 percent.

In computing cross sections for the ETR the curves of Figures A5-1, A5-2, and A5-3 were used. When an experimental hole contained a number of metals and water, the curve for zirconium-water mixture was assumed to be valid for the mixture in question.

Figure A5-3

APPENDIX VI

THE COMPUTATION OF CROSS SECTIONS

A. Capture Cross Section

The capture cross sections used in the ETR analysis were obtained from References 1, 2, and 3. The macroscopic cross sections were calculated from the formula

$$\Sigma_a = \frac{\rho N_0 \sigma_a V \cdot F}{A} f(T) \quad (1)$$

where Σ_a = macroscopic capture cross section

ρ = density of the material

σ_a = microscopic capture cross section

N_0 = Avagadro's number

A = atomic weight of the material

$V \cdot F$ = volume fraction occupied by the material.

$f(T)$ is a factor which averages the cross section over the Maxwellian distribution at the operating temperature, T . Except for uranium and xenon, the capture cross sections of all materials were assumed to have a $\frac{1}{\sqrt{E}}$ energy

dependence. When σ_a is given at 20 C, $f(T) = \sqrt{\frac{\pi}{2}} \cdot \sqrt{\frac{293}{T}}$.

The uranium capture cross sections were obtained from BNL 325¹ at each energy of interest, and the "f" factor for correcting the non $\frac{1}{\sqrt{E}}$ nature of

1. BNL 325, Hughes and Harvey, Neutron Cross Sections, United States Government Printing Office, Washington, D. C., 1955.
2. The MTR. Handbook.
3. IDO 16133, Reactivity Effect of Reducing the Aluminum-Water Ratio in the MTR Core, Phillips Petroleum Co., Oct. 26, 1953.

uranium was used. Xenon cross sections averaged over the Maxwellian and the temperature dependence of these cross sections were obtained from Reference 4.

B. The Neutron Age

The neutron age was obtained as described in Appendix V. In the early phases of the work, the age in aluminum-water mixtures was obtained from the curve of Dismuke and Arnette, given in IDO 16133. Over the range used, the difference between the values in IDO 16133 and those in Figure A5-1 of Appendix V is not significant in the ETR calculations.

C. Diffusion Coefficients

With the exception of the slow diffusion coefficient for water, the slow diffusion coefficients were calculated from the scattering cross sections given in References 1, 2, and 3, using

$$\sum_{tr} = \sum_s (1 - 2/3 A) \quad (2)$$

$$\sum_s = \frac{\rho N_0 \sigma_s V. F.}{A} \quad (3)$$

where

$\sum_{tr}$ = the macroscopic transport cross section

$\sum_s$ = the macroscopic scattering cross section

σ_s = the microscopic scattering cross section

4. Goertzel and Oppenheim, Temperature Dependence of Xenon Cross Section, Y-691, ORNL.

and the other rotation is the same as for (1).

Then the slow diffusion coefficient, D_2 , is calculated from

$$D_2 = \frac{1}{3 \sum_{tr}} \quad (4)$$

For a mixture of materials,

$$\frac{1}{D_2} = \frac{V.F.1}{D_{2_1}} + \frac{V.F.2}{D_{2_2}} + \frac{V.F.3}{D_{2_3}} + \dots \quad (5)$$

$$\frac{1}{D_2} = 3 (V.F.1 \sum_{tr2_1} + V.F.2 \sum_{tr2_2} + V.F.3 \sum_{tr2_3} + \dots)$$

The slow diffusion coefficient for water was taken directly from IDO 16133.

It's value is 0.159.

The calculation of the fast diffusion coefficient is more difficult. It is usually computed from the formula⁵

$$D_1 = \frac{\int_{E_{th}}^{\infty} D(E) \phi(E) dE}{\int_{E_{th}}^{\infty} \phi(E) dE} \quad (6)$$

In the case of aluminum-water mixtures, (6) has been evaluated in IDO 16133 as a function of aluminum to water ratio, up to ratios of 0.7. The values in

5. Glasstone and Edlund, The Elements of Nuclear Reactor Theory, p. 229.

IDO 16133 were used where applicable. In addition, (6) was evaluated for nickel, iron, zirconium, and aluminum. In evaluating (6) the cross sections were taken from BNL 325.¹ Smooth "average" curves were drawn through the portions of the cross section curves exhibiting marked resonance effects, and the integration was performed using these smooth curves rather than integrating over the individual resonance structure. The curve of $\phi(E)$ from IDO 16133 was used in (6) for the energy dependence of the flux. The following values were obtained for D_1 .

D_1 nickel	= 3.28
D_1 iron	= 1.87
D_1 aluminum	= 2.81
D_1 zirconium	= 1.24

The fast diffusion coefficient for water was taken from IDO 16133. D_1 of water = 1.19.

To compute the D_1 of a mixture in an experimental hole, the equivalent of equation (5) was used.

$$\frac{1}{D_1} = \frac{V.F.1}{D_{11}} + \frac{V.F.2}{D_{12}} + \frac{V.F.3}{D_{13}} + \dots \quad (7)$$

Equation (7) is not strictly valid for the computation of the fast diffusion coefficient. However, D_1 enters the diffusion equations in two places, as the coefficient of ∇^2 and in the calculation of the slowing down cross section,

$\sum_{SD} = D_1/\tau$. In Appendix IV it is shown that a change in $\mathcal{V}$, produced by a change in D_1 and $\sum_{SD}$, can be written

$$\begin{aligned} \delta \mathcal{V} = & - \int \phi_1^+ \delta D_1 \nabla^2 \phi_1 d\tau + \int \delta \sum_{SD} \phi_1^+ \phi_1 d\tau \\ & - \int \delta \sum_{SD} \phi_2^+ \phi_1 d\tau \end{aligned} \quad (8)$$

where the integration occurs over all space. The equation satisfied by ϕ_2^+ is

$$D_1 \nabla^2 \phi_1^+ - \sum_{SD} \phi_1^+ + \sum_{SD} \phi_2^+ = 0 \quad (9)$$

$$\phi_2^+ = \frac{-D_1 \nabla^2 \phi_1^+}{\sum_{SD}} + \phi_1^+ \quad (10)$$

Substituting (10) into (8) yields

$$\delta \mathcal{V} = \int \phi_1^+ \nabla^2 \phi_1 (-\delta D_1 + \delta \sum_{SD} \frac{D_1}{\sum_{SD}}) d\tau \quad (11)$$

Therefore, if

$$\frac{\delta D_1}{D_1} = \frac{\delta \sum_{SD}}{\sum_{SD}} \quad (12)$$

there will be no change in $\mathcal{V}$. However, (12) is just the statement that

$\tau = D_1 / \sum_{SD}$ is constant. Thus, if τ is chosen for a given mixture, the reactivity is unchanged by small changes in D_1 , provided the slowing down cross section is chosen as $\sum_{SD} = D_1 / \tau$. Since $\sum_{SD}$ is defined this way in the two-group calculations, the results will be insensitive to the value of D_1 , provided D_1 is close to the true value.

D. The Slowing Down Cross Section

As explained above, $\sum_{SD}$ was obtained from

$$\sum_{SD} = D_1 / \tau \quad (13)$$

E. Leakage from Pipe with Voids

The ETR calculations were all performed in the horizontal plane. However, a leakage cross section was included in the calculations to account for neutron leakage in the vertical direction. This leakage cross section in group 1 was calculated in the usual way as $D_1 B^2$

$$\sum_{l_i} = D_1 B^2 \quad (14)$$

where D is the diffusion coefficient

and B^2 is the buckling in the vertical direction

The leakage term is included in the thermal and above-thermal groups, so the above-thermal flux equation is

$$D_1 \nabla^2 \phi_1 - \sum_{SD} \phi_1 - D_1 B^2 \phi_1 + \sum_f \phi_2 = 0 \quad (15)$$

and the thermal flux equation is

$$D_2 \nabla^2 \phi_2 - \sum_a \phi_2 - D_2 B^2 \phi_2 + \sum S D \phi_1 = 0 \quad (16)$$

In a void $D \rightarrow \infty$, so the use of (14) implies an infinite leakage.

The leakage from an air pipe in the ETR was calculated by adopting a method described by Wachspress⁶. In this scheme a fictitious value of D is calculated for a pipe filled with void such that DB^2 represents the neutron leakage. The fictitious D is given by

$$D = \frac{a}{2} \left(\ln \frac{H}{a} - 0.705 \right) \quad (17)$$

where

a = the radius of the pipe

and

H = the height of the core

In this case, $DB^2 \phi$ is then equal to the net neutrons/cc/sec streaming from the midplane. In some of the experimental holes in the ETR there is a pipe filled with void which has aluminum corner pieces attached to the outside. In this way the assembly will fill a square facility hole. The mesh used in the UNIVAC calculations was not small enough to take into account the structure in the facility hole, so average cross sections had to be determined. This was done as follows:

6. E. W. Wachspress, Two-Dimensional R - ∞ Multigroup Calculations, KAPL 1172, August 15, 1954.

1. In the tube filled with void, the leakage term was computed from (14) and (17). In the material of the pipe and corner pieces, the leakage was estimated by using the diffusion coefficients of the structure material. Then the total fast leakage in the vertical direction is given by

$$\text{leakage/cc/sec} = V \cdot F_1 D_{11} B^2 \phi + V \cdot F_2 D_{12} B^2 \phi + V \cdot F_3 D_{13} B^2 \phi$$

where the various diffusion coefficients are computed as described earlier. Thus the fast leakage cross section is

$$\Sigma_1 = V \cdot F_1 D_{11} B^2 + V \cdot F_2 D_{12} B^2 + V \cdot F_3 D_{13} B^2 + \dots \quad (18)$$

This leakage cross section was used in the diffusion equations.

2. The constants in (15) and (16), other than the leakage term, describe the diffusion of neutrons in the horizontal plane. Then, to compute these constants the materials in the facility tube were assumed homogenized, so for example

$$\frac{1}{D_1} = \frac{V \cdot F_1}{D_{11}} + \frac{V \cdot F_2}{D_{12}} + \frac{V \cdot F_3}{D_{13}} + \dots \quad (19)$$

$$\Sigma_a = V \cdot F_1 \Sigma_{a1} + V \cdot F_2 \Sigma_{a2} + V \cdot F_3 \Sigma_{a3} + \dots \quad (20)$$

Note that if the leakage cross section in (18) is defined as DB^2 , the D of (18) and that of (19) will be different. Equation (18) describes the leakage in the vertical direction from the various areas of the facility space. Equation (19) describes the materials seen by a neutron traveling through the space in the horizontal direction.

F. The Leakage Cross Section DB^2

The vertical buckling was obtained from the one-dimensional axial calculation described in Section II of this report. In this calculation the horizontal buckling was taken as 0.02678, a value obtained from a one-dimensional radial calculation. The total buckling is defined as

$$B_T^2 = \frac{k_\infty - 1}{\tau + L^2} \quad (21)$$

$k_\infty = \nu \frac{\sum f}{\sum a}$, where ν is obtained from the solution of the axial problem.

The value of k_∞ calculated was 1.2317. With $\tau + L^2 = 64.23$

$$B_T^2 = 0.0036081$$

where B_T^2 is the sum of the vertical and horizontal buckling. Then the vertical buckling is

$$B_V^2 = 0.0036081 - 0.002678$$

$$B_V^2 = 0.000930$$

This is the value of B_V^2 used for the ETR calculations.

APPENDIX VII

THE FORMATION OF N^{16} IN THE ETR REACTOR VESSEL

The N^{16} formation is calculated on the following assumptions:

1. Only virgin neutrons contribute to N^{16} formation.
2. There are no virgin neutron leaks from the reactor pressure vessel.
3. The Watt Spectrum* represents the fission neutron spectrum.

Let $W(E)$ = the Watt Spectrum normalized so that $\int W(E) dE = 1$

$\Sigma_a(E)$ = activation cross section for $O^{16}(n,p)N^{16}$

$\Sigma_s(E)$ = scattering cross section in the core.

We consider scattering only from hydrogen, and inelastic scattering from aluminum in computing $\Sigma_s(E)$. The microscopic cross section for inelastic scattering in aluminum is taken as 0.5 barns.† The hydrogen cross section is taken from reference 1‡ and the N^{16} activation cross section from reference 2.¶

Then if $\Phi_v(E)$ is the virgin flux, the number of virgin neutrons removed per second from the pressure vessel is

$$\text{number removed/sec} = \int_{V,E} \Phi_v(E) \left[\Sigma_s(E) + \Sigma_a(E) \right] dE dV$$

Then at equilibrium

$$\int_E N(E) dE = \int_{V,E} \Phi_v(E) \left[\Sigma_s(E) + \Sigma_a(E) \right] dE dV$$

* See AECD 3645, The Reactor Handbook, Vol. 1, p. 621.

† Kuhn and Goodman, Physical Review, Vol. 95, p. 989, 1954.

‡ Reference 1. BNL 325, Neutron Cross Section.

¶ Reference 2. H.C. Martin, Physical Review, Vol. 95, p. 989, 1954.

where $N(E) dE$ is the number of fission neutrons produced in the reactor in the energy range dE .

Assume that $\sum_a(E)$ and $\sum_s(E)$ are independent of position. This neglects the change of composition away from the core, but since most N^{16} production occurs in the core a negligible error should result. With this assumption the number of N^{16} formations per second can be computed:

$$\begin{aligned} N^{16} \text{ activations/sec} &= \int_{V,E} \phi_v(E) \sum_a dE dV \\ &= \int_E N(E) \frac{\sum_a(E)}{\sum_a(E) + \sum_s(E)} dE \\ &\approx \int_E N(E) \frac{\sum_a(E)}{\sum_s(E)} dE \end{aligned}$$

since $\sum_a(E) \ll \sum_s(E)$. $\sum_s(E)$ may be calculated as follows:

$\sum_s(E)$ due to aluminum, volume fraction = 0.4

$$\sum_{Al} = \frac{2.7 \times 6.02 \times 10^{23} \times 0.4 \times 0.5 \times 10^{-24}}{27}$$

$$\sum_{Al} = 0.012$$

$\sum_s(E)$ due to hydrogen in H_2O , volume fraction of H_2O = 0.6

$$\sum_H(E) = \sigma(E) \times \frac{2 \times 6.02 \times 10^{23} \times 0.6}{18}$$

$$\sum_H(E) = 4.0 \sigma(E) \times 10^{-2} \quad (\sigma \text{ in barns})$$

<u>E (Mev)</u>	<u>σ (barns)</u>	<u>$\sum_H$</u>	<u>$\sum_S(E) = \sum_H + \sum_{Al}$</u>
10	0.94	0.038	0.050
11	0.86	0.035	0.047
12	0.79	0.032	0.044
13	0.74	0.030	0.042
14	0.69	0.028	0.040
15	0.64	0.026	0.038
16	0.60	0.024	0.036
17	0.57	0.023	0.035
18	0.54	0.022	0.034
19	0.51	0.020	0.032
20	0.48	0.019	0.031

<u>E (Mev)</u>	<u>$\sum_S(E)$</u>	<u>$\sigma_{\text{activation}}$ (barns)</u>	<u>$\sum_a$</u>	<u>W(E)</u>	<u>$W(E) \frac{\sum_a(E)}{\sum_S(E)}$</u>
10	0.050	0	0		
11	0.047	0.050	0.0002	4.4×10^{-4}	1.9×10^{-6}
12	0.044	0.024	0.00096	2.0×10^{-4}	4.4×10^{-6}
13	0.042	0.070	0.0028	8.9×10^{-5}	5.9×10^{-6}
14	0.040	0.087	0.0038	4.0×10^{-5}	3.5×10^{-6}
15	0.038	0.075	0.0030	1.7×10^{-5}	1.3×10^{-6}
16	0.036	0.060	0.0024	9.8×10^{-6}	0.66×10^{-6}
17	0.035	0.050	0.0020	3.4×10^{-6}	0.20×10^{-6}
18	0.034	0.042	0.0017	1.5×10^{-6}	0.075×10^{-6}
19	0.032	0.030	0.0012	6.4×10^{-7}	0.024×10^{-6}
20	0.031				

The number of activations per fission neutron is

$$\int W(E) \frac{\sum_a(E)}{\sum_S(E)} dE$$

The integration can be evaluated numerically to give

$$\overset{0.9}{1.8} \times 10^{-5} \text{ activations/fission neutron}$$

The number of fission neutrons produced per second is

$$2.46 \times 3.1 \times 10^{10} \times 175 \times 10^6 = 1.35 \times 10^{19} \text{ neutrons/second}$$

Thus the total number of N^{16} activations per second is

$$\overset{0.9}{1.8} \times 10^{-5} \times 1.35 \times 10^{19} = \overset{1.2}{2.4} \times 10^{14} \text{ activations/second}$$

At equilibrium the number of N^{16} nuclei formed in the vessel must equal the number leaving the vessel.

The number of gallons of water leaving the vessel per minute is 44,370. This is equivalent to 2.8×10^6 cc/sec. Therefore, neglecting those nuclei which decay before leaving the vessel, the number of N^{16} nuclei per cc of water leaving the vessel is

$$N^{16} \text{ per cc} = \frac{2.4 \times 10^{14}}{2.8 \times 10^6} = 8.6 \times 10^7.$$

N^{16} has a half-life of 7.3 seconds. Thus, the activity of the water leaving the vessel is

$$\begin{aligned} \text{number of disintegrations/cc/sec} &= \frac{0.693}{7.3} \times 8.6 \times 10^7 \\ &= 8.2 \times 10^6 \end{aligned}$$

Eighty percent of these disintegrations result in 6.5 Mev gamma rays. Therefore, 6.6×10^6 6.5 Mev gammas are produced per sec cc in the water leaving the pressure vessel.