PARAMETER ESTIMATION OF MICROWAVE FILTERS

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Thesis prepared for the Degree of

MASTER SCIENCE

UNIVERSITY OF NORTH TEXAS

December 2015

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Sun, Shuo. *Parameter estimation of microwave filters*. Master of Science (Electrical Engineering), December 2015, 62 pp., 22 figures, 15 numbered references.

The focus of this thesis is on developing theories and techniques to extract lossy microwave filter parameters from data. In the literature, the Cauchy methods have been used to extract filters' characteristic polynomials from measured scattering parameters. These methods are described and some examples are constructed to test their performance. The results suggest that the Cauchy method does not work well when the Q factors representing the loss of filters are not even. Based on some prototype filters and the relationship between Q factors and the loss, we conduct preliminary studies on alternative representations of the characteristic polynomials. The parameters in these new models are extracted using the Levenberg–Marquardt algorithm to accurately estimate characteristic polynomials and the loss information.

ACKNOWLEGEMENTS

I would like to express my highest gratitude for my major advisor, Dr. Yan Wan, not only for her brilliant guidance and continuous support, but also for her warm heart and noble spirit. Her rigorous academic research style and striving for the best will always be my light in the life.

I also would like to express my gratitude for Dr. Xinrong Li and Dr. Hualiang Zhang for their supports as the committee members. They gave me a lot of helpful advices. I also express my gratitude for Dr. Shengli Fu for his helps and advices. And I express heartfelt thanks for my lab-mates, my friends and all the people who have helped me.

At last, I thank my family for their endless love and support, thank you.

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CHAPTER 1

Introduction

Microwave filters, especially band-pass (BP) filters, are widely used in communication systems (including transmitter and receiver components) to select and shape signals of particular frequency band of interest. The increasing requirements, such as higher working frequency and lower power consumption for the communication systems make the selectivity more restricted. The implementation of BP filter requires manual tuning during the practical design and implementation stages.

However, manual tuning highly depends on human experiences and is not scalable. Recent years, more attentions have been devoted to the computer-aided tuning (CAT) of microwave filters [1]. Through comparing the extracted parameters of the implemented filter and the original parameters of the designed filter, the difference between the implemented and theoretical designs can be revealed, which helps to tune each element of the filter. As such, the accuracy of filter parameter extraction is essential. The Cauchy method has been applied in extracting the characteristics polynomials of scattering parameters of the microwave filters [2], [3], [4], [5], based on the assumption that the filter is lossless or that all the resonators share the same quality factor (Q). This method does not work when each resonator has different loss. To address this issues, in [12], the coupling matrix (CM) is extracted in two steps: 1) applying the Cauchy method to roughly construct the coupling matrix, and 2) the loss information for the each resonator captured in the diagonal matrix is estimated. However, this method can not extract the quality factors accurately when the losses are uneven.

In this thesis, we explore possible methods to extract the coefficients quickly and accurately even when uneven losses exist. An enhanced Cauchy method and two optimization models are proposed. They all have good performance on extracting the characteristic polynomials. An optimization model based on the filter structure can also successfully estimate the loss information. Though a model for the high order case still needs to be further improved, the methods proposed in this thesis show a possible direction.

The thesis is organized as follows: In Chapter 2, theories and techniques for synthesizing microwave filter's are described, including the microwave resonator fundamentals, filter design techniques, network theory, etc. In Chapter 3, the Cauchy method for parameter extraction is described. A two-step optimization method using Cauchy method and CM is also proposed. Some examples are then illustrated to test the accuracy and efficiency of these techniques. Then the advantages and disadvantages of these current methods are also discussed. In Chapter 4, some new methods to extract the S-parameters from measured data of the filters with different unloaded quality factors are explored. Examples are also illustrated throughout this chapter to test and compare the results using different methods. In Chapter 5, we conclude the thesis and provide a brief discussion of future work.

CHAPTER 2

SYNTHESIS OF MICROWAVE FILTER

2.1 Introduction

In this chapter, the theories and techniques for synthesizing microwave filter's are described. The background knowledge of the microwave resonator, network theory, LP prototype filter design and LP to BP transform that have been well established in some classic textbooks are reviewed [6]. In addition, the procedures to construct a coupling matrix are described [7].

2.2 Microwave Resonators

Microwave resonators are widely used in amplifiers and microwave filters. Close to the resonant frequency, a resonator can be modeled as a RLC equivalent circuit. The basic properties and characteristics of the circuit are discussed here to ease understanding.

As is well known, the resonant circuits can be modeled in two forms: the series RLC circuit and the parallel RLC circuit. A typical series resonant circuit is shown in Fig. 2.1.

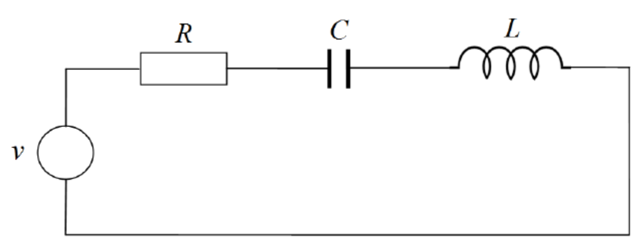


Figure 2.1 A series RLC resonant circuit

The input impedance of this circuit is:

$$Z_{in} = R + j\omega L + \frac{1}{j\omega C} \tag{2.1}$$

The resonant frequency is:

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{2.2}$$

An important parameter of the resonant circuit is the Q factor, or the quality factor, which indicates the loss of a circuit. Q factor is defined as:

$$Q = \omega \frac{\text{Average Energy Stored}}{\text{Average Energy Loss}} \tag{2.3}$$

According to the definition, the higher Q is, the lower loss rate the circuit is. It is worthy to point out that the loss of the circuit can be caused by many factors: radiation loss, conductor loss, etc. These can be captured by the resistance R in the prototype circuit. If an external network be connected to the resonator circuit, the loss will be higher. As such, to describe the properties of the circuit itself, another parameter is introduced. Q_0, the unloaded Q, defines the loss of the circuit itself only, ignoring the external sourcing/loading structures.

At the resonant frequency, the unloaded Q can be calculated as [6], page 272-278:

$$Q_0 = \omega_0 \frac{L}{R} = \frac{1}{\omega_0 RC} \tag{2.4}$$

It shows from Equation (2.4) that if the loss factor, R, increases, the unloaded Q will decrease.

A parallel RLC resonant circuit is shown in Figure (2.2)

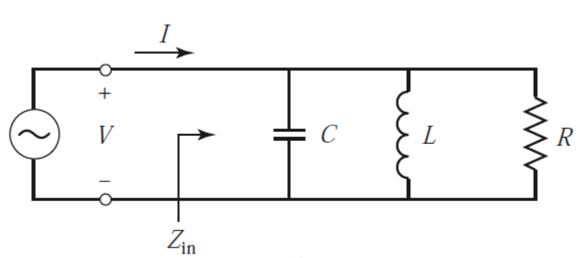


Figure 2.2 A parallel RLC resonant circuit [6]

The input impedance of the parallel circuit is:

$$Z_{in} = (\frac{1}{R} + \frac{1}{j\omega L} + j\omega C)^{-1} \quad (2.5)$$

The resonant frequency is the same as that in Equation (2.2): $\omega_0 = \frac{1}{\sqrt{LC}}$.

The input impedance of the parallel circuit will be:

$$Z_{in} = R \quad (2.6)$$

And the unloaded Q of the parallel circuit will be:

$$Q_0 = \frac{R}{\omega_0 L} = \omega_0 RC \quad (2.7)$$

2.3 Lossy filters and Q factor

As discussed above, the loss of the resonant filter at the resonant frequency can be captured by the unloaded Q factor, Q_0. But what if the frequency is not at the resonant frequency? In most practical BP filters, the passband is very small compared to the center frequency, and hence the working frequency is usually not far from the resonant frequency. As such, the properties of resonators are mostly concerned around the resonant frequency. Let the actual frequency, ω, be slightly different from the resonant frequency ω_0: $\omega = \omega_0 + \Delta\omega$, $\Delta\omega \ll \omega$, and $\omega^2 - \omega_0^2 = (\omega + \omega_0)(\omega - \omega_0) = \Delta\omega(2\omega - \Delta\omega)$, then the input impedance of the series circuit will be:

$$Z_{in} = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L\left(\frac{\omega^2 - \omega_0^2}{\omega^2}\right) = R + jL \cdot (2\omega - \Delta\omega)\Delta\omega/\omega \quad (2.8)$$

For $\Delta\omega \ll \omega$, $\Delta\omega(2\omega - \Delta\omega) \cong \Delta\omega \cdot 2\omega$, and from Equation (2.4), $L = Q_0 R/\omega_0$, and substitute $L = Q_0 R/\omega_0$ and $\Delta\omega(2\omega - \Delta\omega) \cong \Delta\omega \cdot 2\omega$ into Equation 2.8, the input impedance can be represented as:

$$Z_{in} \cong jL \cdot \Delta\omega \cdot \frac{2\omega}{\omega} = R + jL \cdot 2\Delta\omega = R + j\frac{2RQ_0\Delta\omega}{\omega_0} \quad 2.9)$$

Particularly, this series lossy resonator can be transformed to a lossless form, where the resonant frequency, ω_0, is replaced by an adjusted resonant frequency, ω_0':

$$\omega_0' = \omega_0(1 + \frac{j}{2Q_0}) \tag{2.10}$$

As the adjusted form is lossless, the input impedance will be:

$$Z_{in} = jL \cdot 2\Delta\omega = j2L(\omega - \omega_0') \tag{2.11}$$

Substitute Equation (2.10) into Equation (2.11), then the input impedance is:

$$Z_{in} = j2L(\omega - \omega_0') = j2L\left(\omega - \omega_0(1 + \frac{j}{2Q_0})\right) = \frac{\omega_0 L}{Q_0} + j2L(\omega - \omega_0) = R + jL \cdot 2\Delta\omega \tag{2.12}$$

Here the expressions of the input impedance in Equation (2.12) and Equation (2.9) are identical.

Now consider the case of a parallel circuit. Similarly, let $\omega = \omega_0 + \Delta\omega$, $\Delta\omega \ll \omega$, then the form of the input impedance of the parallel circuit, by applying the Taylor expansion, will be:

$$Z_{in} \cong (\frac{1}{R} + \frac{1-\frac{\Delta\omega}{\omega_0}}{j\omega_0 L} + j\omega_0 C + j\Delta\omega C)^{-1} = \frac{R}{1+2j\Delta\omega RC} = \frac{R}{1+2jQ_0\Delta\omega/\omega_0}. \tag{2.13}$$

Again, substitute Equations (2.7) & (2.10) into Equation (2.13), and make R to be infinite (lossless), then the input impedance expression will be simplified as:

$$Z_{in} = \frac{1}{j2C(\omega-\omega_0)} \tag{2.14}$$

It is noticed that in the practical cases, the Q factor can be included in the adjusted resonant frequency, and then the lossy resonators can be considered as lossless resonators, which provides great convenience. Furthermore, this effective resonant frequency contributes to the adjustment of the complex frequency domain and extraction of the lossy factors, which will be denoted later.

2.4 Network Analysis and S-parameter

In this part, the definition and calculation of scattering parameters and network theory are introduced.

Consider a generalized N-port microwave network shown in Figure 2.3,

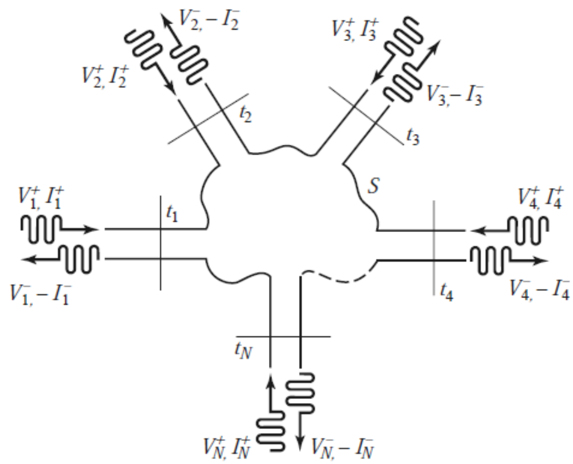


Figure 2.3 A generalized N-port microwave network, obtained from David. M. Pozar. *Microwave Engineering*, pp174

where V_n^+ is the incident voltage wave's amplitude to port n, and V_n^- is the voltage wave's amplitude from port n. Then the scattering matrix, usually called [S] matrix, is defined as:

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

And for each parameter of the scattering matrix can be defined as [15]:

$$S_{ij} = \frac{V_i^-}{V_j^+}\bigg|_{V_k^+=0 \text{ for } k\neq j} . \tag{2.15}$$

The voltage and current on each port can be written as:

$$V_i = V_i^+ + V_i^- \tag{2.16a}$$

$$I_i = Y_0(I_i^+ - I_i^-) \tag{2.16b}$$

From the two above equations,

$$V_i + Z_0 I_i = V_i^+ + V_i^- + V_i^+ - V_i^- = 2V_i^+ \tag{2.16c}$$

$$V_i - Z_0 I_i = V_i^+ + V_i^- - V_i^+ + V_i^- = 2V_i^- \tag{2.16d}$$

For a 2-port network with reciprocal structure, the S parameters can be written as:

$$S_{21} = \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0} \tag{2.17a}$$

$$S_{11} = \frac{V_1^-}{V_1^+}\bigg|_{V_2^+=0} \tag{2.17b}$$

The S parameters S_{21} and S_{11} are the transfer and reflection characteristics parameters of the 2-port network respectively.

For a nonreciprocal network, which means the impedance at each terminate is different, the normalized voltage wave, are determined as[13], page 23-30, [15], page 174-180:

$$S_{21} = \frac{V_2^-}{V_2^+}\frac{\sqrt{Z_{0,1}}}{\sqrt{Z_{0,2}}}\bigg|_{V_2^+=0} = \frac{2V_2}{V_1+Z_0 I_1}\frac{\sqrt{Z_{0,1}}}{\sqrt{Z_{0,2}}} \tag{2.17c}$$

$$S_{11} = \frac{V_1^-}{V_1^+}\frac{\sqrt{Z_{0,1}}}{\sqrt{Z_{0,1}}}\bigg|_{V_2^+=0} = \frac{V_1-Z_0 I_1}{V_1+Z_0 I_1} \tag{2.17d}$$

where $Z_{0,i}$ is the characteristic impedance at terminal i, Z_0 is the input impedance at the loading port[13], [15].

2.5 Design of Chebyshev Filter

Chebyshev filter is a widely used in practical microwave filters. In this session, its theoretical design and implementation are introduced.

2.5.1 Theoretical Design of Chebyshev Filter

The gain (or amplitude) response as a function of angular frequency of an N-order LP filter is [6], [8] :

$$G_n(\omega) = |H_n(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 T_n^2(\frac{\omega}{\omega_0})}} \quad (2.18)$$

where ϵ is the ripple factor, ω_0 is the cutoff frequency, and T_n is a Chebyshev polynomial of the nth-order.

Then the transfer function is given by

$$H(s) = \frac{1}{2^{n-1}\epsilon} \sum_{m=1}^{n} \frac{1}{(s-s_{pm}^{-})}, \quad (2.19)$$

where s_{pm}^{-} are only those poles with a negative real part for the poles of $G(\omega)$.

The ripple, determined by ϵ, is often given in dB: $\text{Ripple in dB} = 10\lg(1+\epsilon^2)$. For example, the ripple is 0.5dB when $\epsilon = 0.3493$.

2.5.2 LP Prototype Filter Design and LP to BP Transformation

Fig.2.4 shows the commonly used structures for LP prototype filter. The values of the elements, $g_0, g_1, \dots, g_{N+1}$, can be determined by the insertion loss method which is well established in [6].

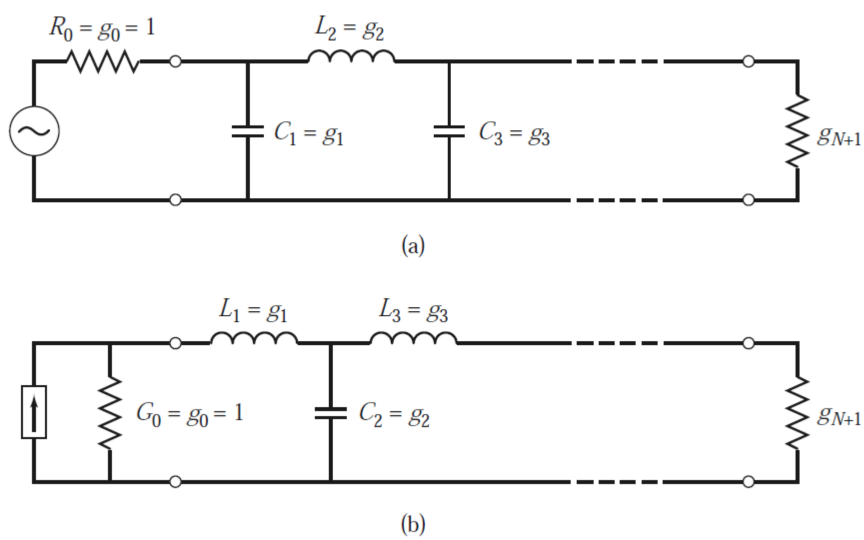


Fig. 2.4 Ladder structures for low-pass filters prototypes, derived from David. M. Pozar. *Microwave Engineering*, pp403

After the LP filter prototype is designed, the BP filter can be obtained in two steps: 1) impedance &.frequency scaling and 2) LP to BP transformation.

In the prototype design, the source resistance is unity. If the source resistance is R_0, the new LP filter's component values after impedance scaling can be obtained by multiplying or dividing all the component values of the prototype design.

The new filter's component values, after both impedance and frequency scaling, are [6]:

$$L'_k = \frac{R_0 L_k}{\omega_c}, \tag{2.20}$$

$$C'_k = \frac{C_k}{R_0 \omega_c}. \tag{2.21}$$

The design of LP prototype filter can then be transformed to obtain the BP filter. If ω_1 and ω_2 are the lower and upper frequencies of the passband, then the LP to BP transformation can be obtained after the frequency substitution:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{2.22}$$

where $\Delta = \frac{\omega_0}{\omega_2 - \omega_1}$ is the fractional bandwidth and $\omega_0 = \sqrt{\omega_1 \omega_2}$ is the center frequency of the passband, or the resonant frequency. Note that during the LP to BP frequency transformation, the series inductor, L_k, in the LP prototype are replaced by a series LC circuit; the shunt capacitor, C_k, in the LP prototype are replace by a shunt LC circuit.

After both impedance & frequency scaling and LP to BP frequency transformation, the new component values are:

$$L'_k = \frac{L_k R_0}{\omega_0 \Delta}, \quad C'_k = \frac{\Delta}{\omega_0 L_1 R_0} \text{ for series inductor} \tag{2.23}$$

and

$$L'_k = \frac{\Delta R_0}{\omega_0 C_k}, \; C'_k = \frac{C_k}{\omega_0 \Delta R_0} \quad \text{for shunt capacitor.} \tag{2.24}$$

The LP to BP transform will be:

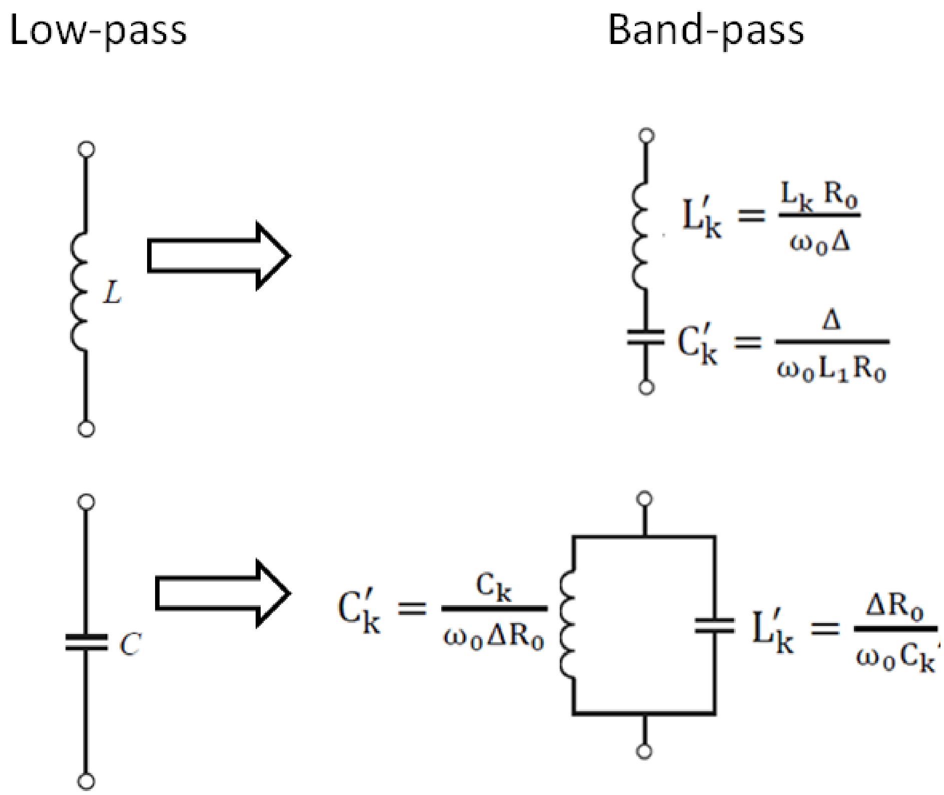


Fig.2.5 LP to BP transformation

2.5.3 Implementation Example

Here we use the 2-order Chebyshev filter as an example to illustrate the BP filter design.

According to Section 2.5.1, when n=2, ω_0=1, ripple= 0.5 dB, the gain response is:

$$G(\omega) = \frac{1}{\sqrt{1+\varepsilon^2[2\omega^2-1]^2}}, \tag{2.25}$$

and the transfer function is:

$$S_{21}(s) = \frac{1.4314}{s^2+1.4256s+1.5162} \tag{2.26}$$

Then it is easy to obtain the reflection function:

$$S_{11}(s) = \frac{s^2+0.5}{s^2+1.4256s+1.5162} \tag{2.27}$$

An ideal Chebyshev prototype filter can be implemented as a RLC circuit. The circuit implementation of this filter with unity input impedance is shown in Fig. (2.6):

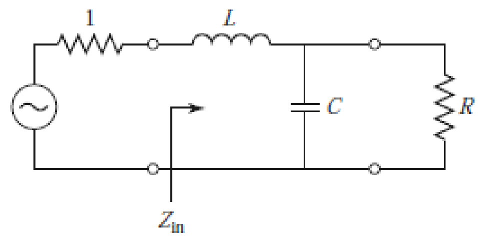


Fig. 2.6 2-order Chebushev filter prototype [6]

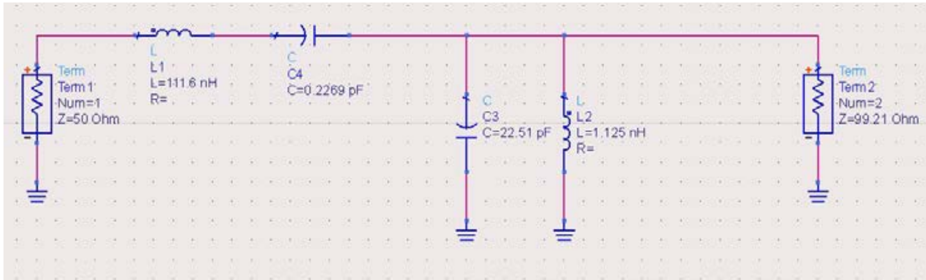
Then the S-parameters can be written as:

$$S_{11}(s) = \frac{Z_{0,1}-Z_{in}}{Z_{0,1}+Z_{in}} = \frac{R\|\left(\frac{1}{sC}\right)+sL-1}{R\|\left(\frac{1}{sC}\right)+sL+1} = \frac{s^2+\left(\frac{1}{RC}-\frac{1}{L}\right)s+\frac{1}{LC}(1-\frac{1}{R})}{s^2+\left(\frac{1}{RC}+\frac{1}{L}\right)s+\frac{1}{LC}(1+\frac{1}{R})} \tag{2.28a}$$

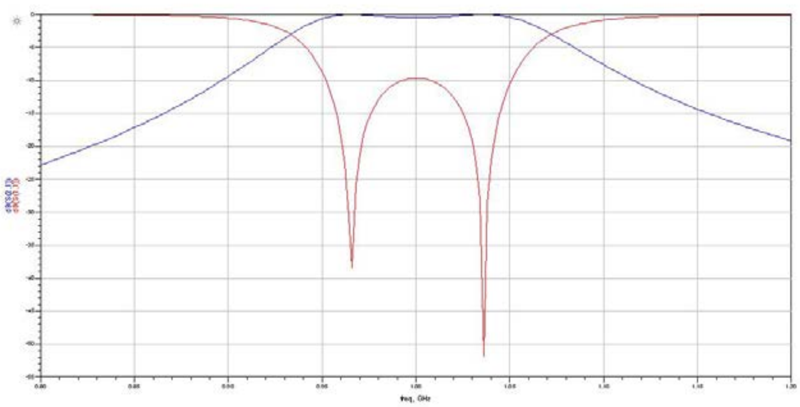
$$S_{21}(s) = \frac{2Z_{0,2}}{Z_{0,1}+Z_{in}}\frac{\sqrt{Z_{0,1}}}{\sqrt{Z_{0,2}}} = \frac{2\left(R\|\left(\frac{1}{sC}\right)\right)}{R\|\left(\frac{1}{sC}\right)+sL+1}\frac{\sqrt{Z_{0,1}}}{\sqrt{Z_{0,2}}} = \frac{2}{LCs^2+\left(C+\frac{L}{R}\right)s+\left(1+\frac{1}{R}\right)}\frac{\sqrt{Z_{0,1}}}{\sqrt{Z_{0,2}}} \tag{2.28b}$$

The elements' values can be calculated by comparing Equations (2.26), (2.27) , (2.28a) & (2.28b) as:$R = 1.9840557, L = 1.4028939, C = 0.7070839$. Then plug the RLC elements' values into (2.23) & (2.24), it can be seen that the implemented filter's reflection and transfer function $S_{11}(s)$ &$S_{21}(s)$ fulfill the design of the ideal filter.

Then a 2-order BP Chebyshev filter can be implemented in Advanced Design System (ADS). Here the BP filter is of a 0.1 GHz band-width and a resonant frequency at 1 GHz, the schematic and simulation result are shown below in Fig. 2.6.



(a)



(b)

Fig. 2.7 (a) Schematic of a 2-order BP Chebyshev filter (b) Simulated S-parameters responses of a 2-order BP Chebyshev filter

2.6 Coupling Matrix Synthesis

2.6.1 Low-pass prototype of a lossless coupled resonator filter

A general 2-port cross-coupled lossless network is shown below. All the resonators/cavities are tuned at the same normalized resonant frequency. The source impedance R_1 is connected to the port 1 and the load impedance R_N is connected to the port 2.

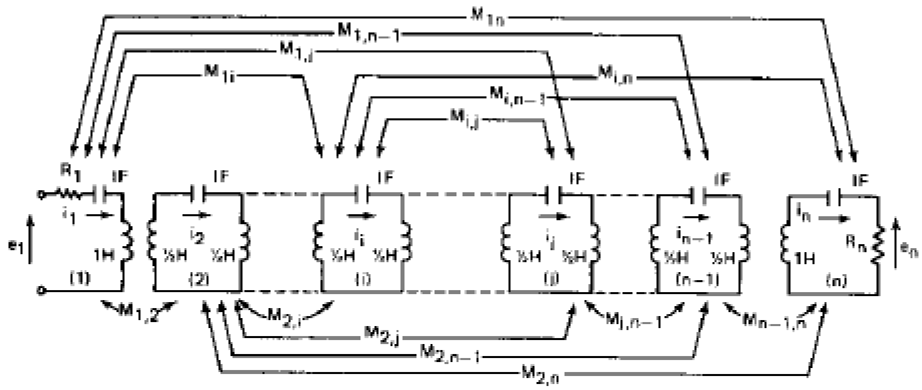


Figure 2.8 The general network of a two-port cross-coupled filter, derived from A. Atia and A. Williams, "New types of waveguide bandpass filters for satellite transponders," *Comsat Tech. Review*, vol. 1, no. 1, pp. 23, 1971.

Applying the Kirchhoff Circuit Laws in the different cavities, it's easy to demonstrate the equations as below [9]:

$$[R_1\delta_{1i} + R_N\delta_{Ni} + j\omega]I_i + j\sum_{\substack{k=1 \\ k\neq i}}^{N} M_{ik}I_k = e_1\delta_{1i}\,, \quad i = 1,2,3,\ldots,N \tag{2.29}$$

where I_i is the loop current in the ith cavity;δ_{ij} is the Kroneeker delta; e_1 is the input voltage (which is normalized to unity); M_{ij} is the coupling coefficient between the ith and the jth cavities. Note that the coupling coefficientsM_{ij} are all real and are independent of frequency. Equation (2.29) can be rewritten as:

$$j[M - j\cdot s\cdot I - jR]\cdot\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2.30}$$

$$j \cdot A \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2.31)$$

where $R_{N\times N} = diag[R_1, 0,0, \ldots ,0, R_N]$; $I_{N\times N}$is anidentity matrix; $M_{N\times N}$ is the coupling matrix which is reciprocal: $M_{ij} = M_{ji}$.

2.6.2 Construction of the admittance

Then the admittances of the network can be determined [10], [11]:

$$y_{21}(s) = \frac{i_n}{e_1}|_{R_1=R_2=0} = j[-M - \omega I]^{-1}_{N,1} \quad (2.32a)$$

$$y_{21}(s) = j \cdot [T \cdot \Lambda \cdot T^t - wI]^{-1}_{N,1} = j \cdot \sum_{k=1}^{N} \frac{T_{Nk}T_{1K}}{\omega - \lambda_k} \quad (2.32b)$$

$$y_{22}(s) = \frac{i_n}{e_1}|_{R_1=R_2=0} = j[-M - \omega I]^{-1}_{N,N} \quad (2.32c)$$

$$y_{22}(s) = j \cdot [T \cdot \Lambda \cdot T^t - wI]^{-1}_{N,N} = j \cdot \sum_{k=1}^{N} \frac{{T_{Nk}}^2}{\omega - \lambda_k} \quad (2.32d)$$

where $T \cdot \Lambda \cdot T^t$ is the eigen-decomposition of $-M$; $\Lambda = \mathrm{diag}[\lambda_1, \lambda_2, \lambda_{3,} \ldots, \lambda_N,]$, λ_i is the eigenvalue of $-M$.

The driving point impedance can be written as [10], [11]:

$$Z_{11}(s) = \frac{z_{11}[1/y_{22}+R_N]}{z_{22}+R_N} = \frac{z_{11}[1/y_{22}+1]}{z_{22}+1} \quad (2.33)$$

Also, it can be expressed as

$$Z_{11}(s) = \frac{1-S_{11}(s)}{1+S_{11}(s)} = \frac{E(s)+F(s)}{E(s)-F(s)} = \frac{m_1+n_1}{m_2+n_2} \quad (2.34)$$

where m_1, m_2, n_1, and n_2 are complex-even and complex-odd polynomials, respectively.

When N is even, from Equation (2.34) it can yield that

$$Z_{11}(s) = \frac{n_1(m_1/n_1+1)}{m_2+n_2} \quad (2.35)$$

Comparing Equation (2.33) and Equation (2.34), it can be found that

$$y_{22}(s) = \frac{n_1}{m_1} \tag{2.36}$$

As $y_{22}(s)$ and $y_{21}(s)$ share the same denominator, and the transmission zeros of $y_{21}(s)$are exactly the same as those of $S_{21}(s)$, then

$$y_{21}(s) = \frac{P(s)}{m_1} \tag{2.37}$$

Similarly, when N is odd, it can be obtained that

$$y_{22}(s) = \frac{m_1}{n_1} \tag{2.38}$$

$$y_{21}(s) = \frac{P(s)}{n_1} \tag{2.39}$$

Then, from (2.34), m_1 and n_1 can be constructed as:

$$m_1 = Re[e_s(0) + f_s(0)] \cdot s^0 + j \cdot Im[e_s(1) + f_s(1)] \cdot s^1 + Re[e_s(2) + f_s(2)] \cdot s^2 + \cdots\cdots \tag{2.40}$$

$$n_1 = j \cdot Im[e_s(0) + f_s(0)] \cdot s^0 + Re[e_s(1) + f_s(1)] \cdot s^1 + j \cdot Im[e_s(2) + f_s(2)] \cdot s^2 + \cdots\cdots \tag{2.41}$$

where $e_s(i)$ and $f_s(i)$, $i = 0, 1, 2, 3, \ldots, N$ are the coefficients of $E(s)$ and $F(s)$. These procedures guarantee that the coefficients of the highest order term s^N in $E(s)$ and $F(s)$ are all purely real.

After all the procedures above, the coupling matrix $M_{(N+2)\times(N+2)}$ can be then constructed as shown in Fig. 2.8:

	S	1	2	$\cdots$	k	$\cdots$	N	L
S		M_{S1}	M_{S2}	$\cdots$	M_{Sk}	$\cdots$	M_{SN}	M_{SL}
1	M_{1S}	M_{11}						M_{1L}
2	M_{2S}		M_{22}					M_{2L}
$\vdots$	$\vdots$			$\ddots$				$\vdots$
k	M_{kS}				M_{kk}			M_{kL}
$\vdots$	$\vdots$					$\ddots$		$\vdots$
N	M_{NS}						M_{NN}	M_{NL}
L	M_{LS}	M_{L1}	M_{L2}	$\cdots$	M_{Lk}	$\cdots$	M_{LN}	

Fig. 2.9 N+2 symmetric coupling matrix [11]

where S—Source, L—Load, $M_{SL} = M_{LS}$ are the Source-Load coupling coefficients, $M_{Lk} =$

$M_{kL} = T_{Nk}$,

$M_{Sk} = M_{kS} = T_{1k}$,

$M_{kk} = -\lambda_k$.

CHAPTER 3

PARAMETER EXTRACTION METHODS

3.1 Introduction

In this chapter, classical theories and some existing methods for parameter extraction are described. Some examples are illustrated in the end of this chapter to test the accuracy and efficiency of these techniques. Meanwhile, the advantages and disadvantages of these current methods are also be discussed.

3.2 Cauchy Method

Cauchy method, which is well established in [2], [3], [4], [5], is a widely used technique for parameter extraction. As discussed in Chapter 2, a two-port lossless network can be described by its scattering parameters $S_{11}(s)$and $S_{21}(s)$, whose three characteristic polynomials $F(s)$, $P(s)$, and $E(s)$ can completely determine a rational model for a LP prototype filter. The characteristic polynomials are:

$$S_{11}(s) = \frac{F(s)}{E(s)} = \frac{\sum_{k=0}^{n} a_{1k}(s)^k}{\sum_{k=0}^{n} b_k(s)^k} \tag{3.1}$$

$$S_{21}(s) = \frac{P(s)}{E(s)} = \frac{\sum_{k=0}^{nz} a_{2k}(s)^k}{\sum_{k=0}^{n} b_k(s)^k} \tag{3.2}$$

where $S_{11}(s)$ is the reflection function and $S_{21}(s)$ is the transmission function; n represents the order of the filter and n_z represents the number of finite transmission zeros (TZs); $s = j\Omega$ is the complex domain where Ω is the normalized frequency for the LP prototype. The relationship between the BP frequency f and the normalized frequency Ω is described as:

$$s = \frac{f_0}{BW}\left(\frac{f}{f_0} - \frac{f_0}{f}\right) \qquad (3.3)$$

where BW is the bandwidth of the band-pass filter; $f_0 = \omega_0/2\pi$ is the resonant frequency.

The Equations (3.1) and (3.2) can be formulated in the matrix form as:

$$[S_{21}V_n \quad -S_{11}V_{nz}]\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \qquad (3.4)$$

where $a_1 = [a_{1,0},\ a_{1,1}, \dots, a_{1,n}]^T$, $a_2 = [a_{2,0},\ a_{2,1}, \dots, a_{2,nz}]^T$ are the coefficients vectors; $S_{11} = diag\{S_{11}(s_i)\}, S_{21} = diag\{S_{21}(s_i)\}$, $i = 1,2, \dots Ns$, are the measured values at N_s different sampling frequency points; and V_m is an increasing-power m-order Vandermonde matrix whose size is $N_s \times (m+1)$ and elements are $v_{m,k} = (s_m)^{k-1}, k = 1, 2, \dots, i+1$.

In order to guarantee that the system matrix has a reasonable solution, N_s must be greater than or equal to $(n + n_z + 1)$. The coefficients of the numerators can be solved with TLS (total least square) method. Once the polynomials F(s) and P(s) have been computed, the poles (roots of E(s)) can be computed using the Feldkeller's equation, based on precondition that the filter is lossless:

$$F(s)F^*(-s) + P(s)P^*(-s) = E(s)E^*(-s) \qquad (3.5)$$

The roots of the LHS part of Equation (3.5) appear in pairs with opposite real parts. Selecting the roots with negative real part, the poles of the filter can be found. Then the coefficients of $E(s)$ can be determined from those selected poles. By this way, the characteristic polynomials, $F(s)$, $P(s)$ and $E(s)$, are all obtained.

3.3 Q Factors and The Adjustment of Complex Domain

It is noticed that Cauchy method requires a lossless network, which makes it hard to obtain the accurate extraction for the lossy filter. Some works have been done in the literature to address the problem [2], [3], [4], [10]. However, a unique transformation of s domain is established in [5], which can conclude the loss in the complex domain s'.

In Section 2.3, it is proved that for a resonant circuit, a lossy resonator can be modeled as a lossless resonator, after replacing the resonant frequency, ω_0, with a new complex resonant frequency $\omega_0\left(1+\frac{j}{2Q_0}\right)$, where Q_0 is the unloaded quality factor. Applying this result on the transformation from the BP domain to LP domain, the complex domain s for the LP prototype filter, can be replaced by a new complex domain s' [5]:

$$s' = \frac{f_0}{BW}\frac{1}{Q_0} + j\frac{f_0}{BW}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{3.6}$$

where BW is the bandwidth of the BP filter, f_0 is the resonant frequency. Note that the characteristic polynomials are in the same domain, as is based on the assumption that all the resonators are of the same Q_0. The unloaded factor Q_0 can be obtained by the best matching between measured and evaluated S_{11} values at the resonant frequency f_0 [5].

3.4 A Parameter Extraction Method with Coupling Matrix and Cauchy Method

A two-stage optimization method is established in [12]. As described in Section 2.6, the first step is to construct the source-load coupling matrix. Note that the coupling matrix is built based on the precondition that the filter is lossless. To guarantee this, the Cauchy method with the adjusted complex domain is applied first, then the coupling matrix, $M_{(N+2)\times(N+2)}$, is built from the characteristics polynomials extracted by the Cauchy method. To take the loss into account, let

$$A_{(N+2)\times(N+2)}(s) = [R - j \cdot M + G - s \cdot U] \tag{3.7}$$

where $G_{(N+2)\times(N+2)} = diag[0, G_1, G_2, \cdots, G_N, 0]$, $G_i = \frac{BW}{f_0} \cdot \frac{1}{Q_0}$, represents the loss, Q_0 is the common unloaded Q, $U_{(N+2)\times(N+2)} = diag[0, 1, 1, \cdots, 1,1,0]$, $R_{(N+2)\times(N+2)} = diag[1, 0, 0, \cdots, 0,0,1]$.

Then $S_{11}(s)$ and $S_{21}(s)$ are extracted [9], [12]:

$$S_{11}^{ext}(s) = 1 + 2j \cdot [A^{-1}]_{1,1} , \tag{3.8}$$

$$S_{21}^{ext}(s) = -2j \cdot [A^{-1}]_{N+2,1}. \tag{3.9}$$

Then in the second step a range of Q is set as ± 30% of the Q_0 value obtained in the first step and the new different Q_i values of each resonator can be calculated by minimizing the cost function:

$$Fun = \sum_{i=1}^{m} \left| S_{21}^{ext}(s_{(i)}) - S_{21}^{mea}(s_{(i)}) \right|^2 + \left| S_{11}^{ext}(s_{(i)}) - S_{11}^{mea}(s_{(i)}) \right|^2 \tag{3.10}$$

$S_{21}^{mea}(s_{(i)})$ and $S_{11}^{mea}(s_{(i)})$ are known values which are the measured S_{21} and S_{11} values at m different sampling $s_{(i)}$; $s_{(i)} = \mathrm{j}\Omega_{(i)}$ is the complex domain; $\Omega_{(i)}$ is the normalized sampling frequency for the LP prototype.

3.5 Examples and Analysis

In this section, several different filters will be illustrated to test the methods discussed above. The simulation results will be shown and the performances will be discussed. Furthermore, the disadvantages of each method will be analyzed, which motivate our new development.

3.5.1 Testing of Cauchy Method in Lossless Condition

First, the transfer function $S_{21}(s)$ for an ideal 2-order Chebyshev filter, with the ripple of 0.5 dB, was plotted in Section 2.5.3. Using the BP filter S-parameters simulated in ADS, and applying the Cauchy method in Matlab to extract the S-parameters, we obtain the transfer function plot using the extracted values. The comparison of the two plots are shown below:

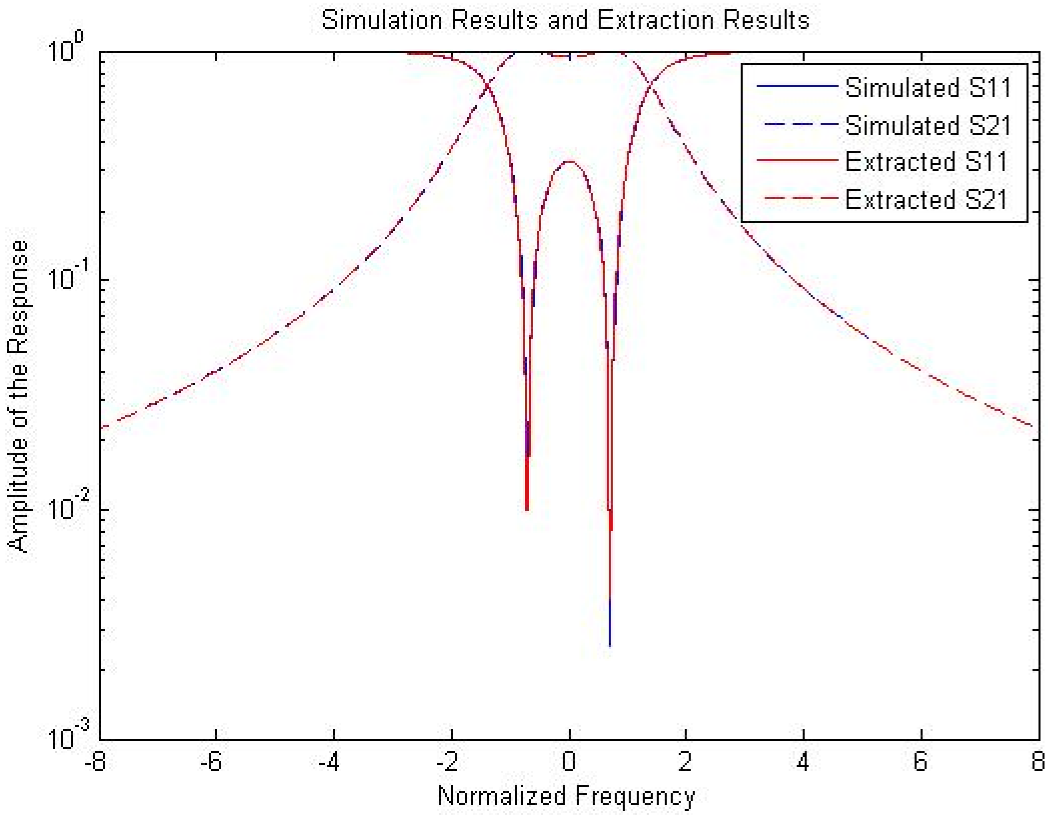


Fig. 3.1 Simulation and extraction results of a lossless BP filter.

The blue curves is simulated using the real S-parameters and the red curves are the simulated using the extracted S-parameters. The curves match very well, showing that the Cauchy method works well in the lossless case.

3.5.2 Testing of Cauchy Method in Lossy Filters

First, a lossy filter model is constructed. As discussed previously in Chapter 2, a BP filter can be constructed by the impedance scaling, frequency scaling and frequency transformation from the LP prototype. The loss, due to various reasons, can be modeled as a resistor in each resonator. And the resistors presenting the loss in the LP prototype can also be transformed into BP filters. For a resonator in the lossy BP filter, from Equations (2.4), (2.7), (2.23) & (2.24), the Q factor can be calculated in the equation as:

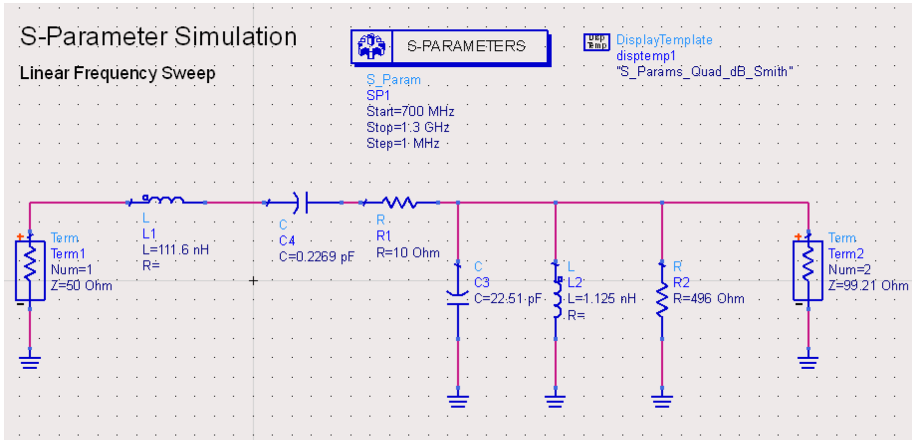
$$Q_k = \omega_0 r_k C^{'} = \omega_0 r_k \frac{C_k}{\Delta R_0} = \frac{C_k r_k}{\Delta R_0} \quad \text{for a shunt capacitor} \tag{2.11a}$$

and

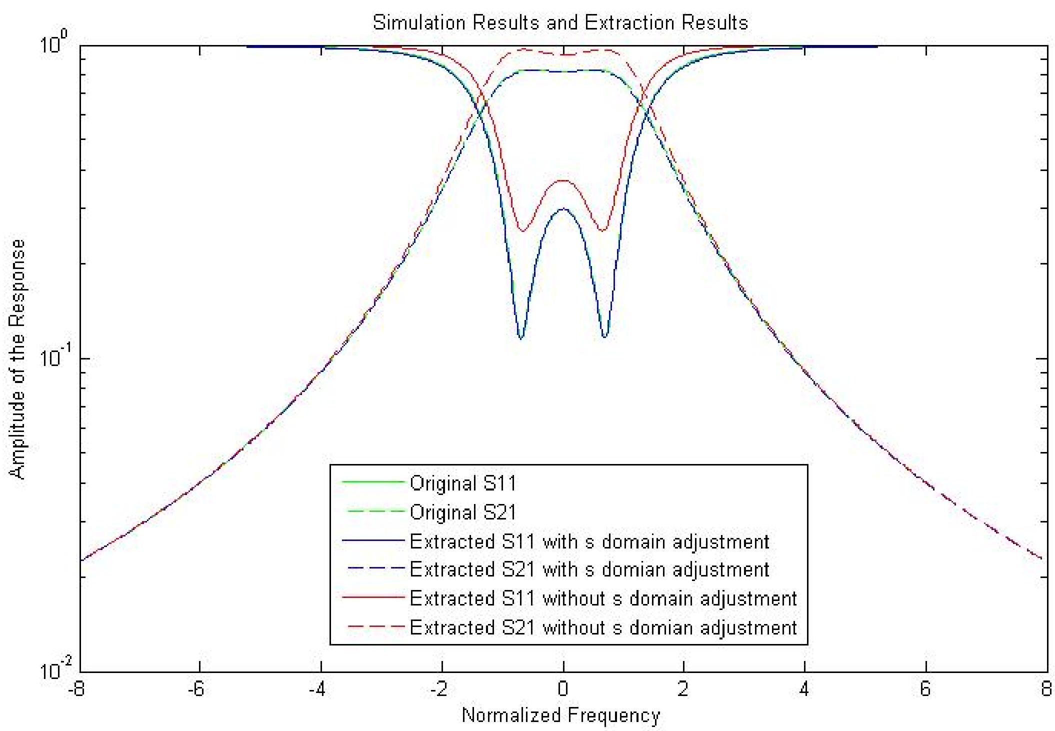
$$Q_k = \omega_0 \frac{L_k^{'}}{r_k} = \omega_0 \frac{L_k R_0}{r_k \Delta \omega_0} = \frac{L_k R_0}{\Delta R_k} \quad \text{for a series inductor} \tag{2.11b}$$

where $\mathrm{L_k}$ and $\mathrm{C_k}$ are the kth component's value of the LP prototype filter; the L_k' and $\mathrm{C_k'}$ are the components' values of the kth resonant cavity of the BP filter; Δ is the normalized bandwidth, which is defined as $\Delta = \frac{BW}{f_0}$; Q_k is the quality factor of the kth resonator.

Fig. 3.2 (a) shows the schematic of the 2-order band-pass Chebyshev filter designed based on the LP filter in Figure 2.6 (a). Figure 3.2 (b) shows the simulation response and the extracted results with and without adjustment of complex domain. The filter is of 0.5dB ripple, the resonant frequency, $f_0 = 1GHz$, and the bandwidth: $BW = 100MHz$. The two resonant cavities have the same quality factor: $Q_1 = Q_2 = 70.1$.



(a)

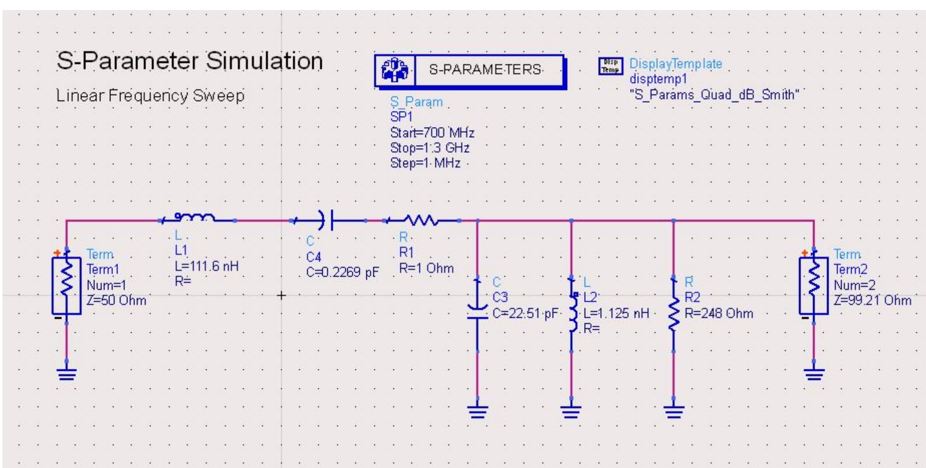


(b)

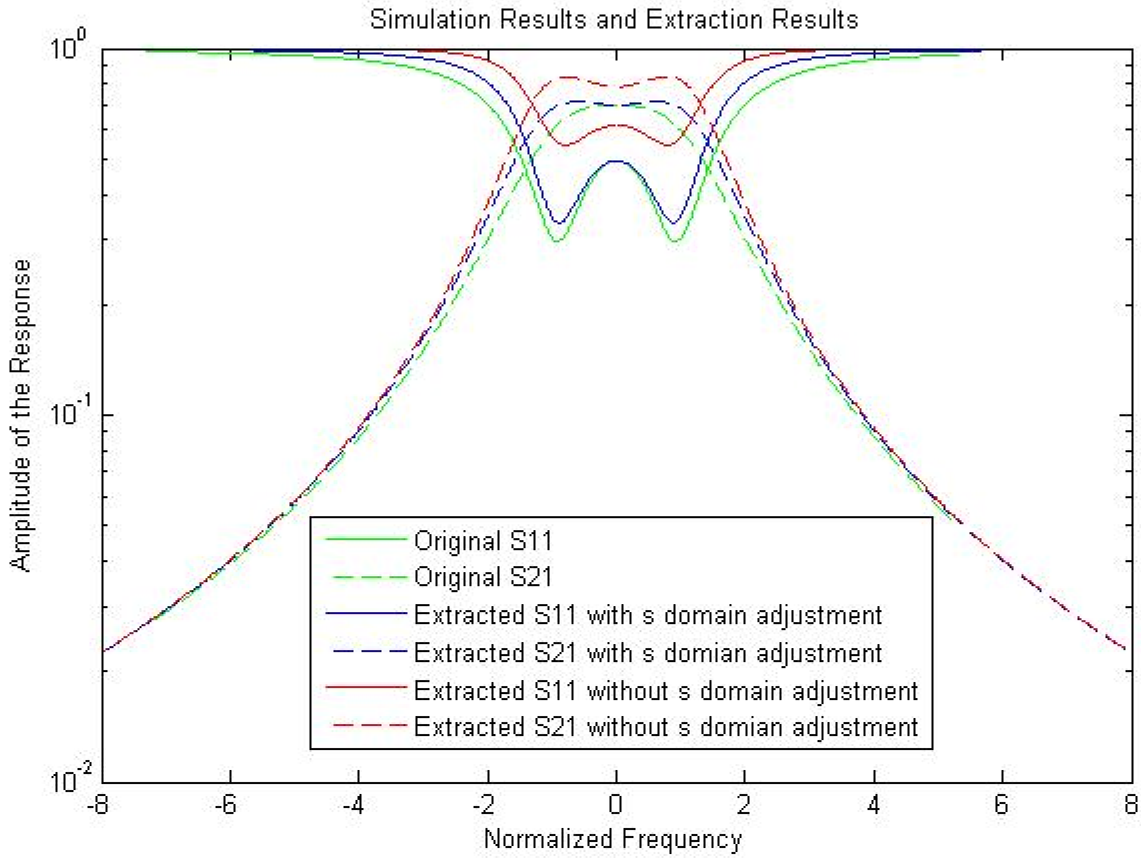
Fig. 3.2 (a) Schematic of the 2-order BP Chebyshev filter with the same Q factors (b) Measured and extracted results of the 2-order BP lossy filter.

The green curves shows the original measured S-parameters, and the blue curves and red curves are the extracted results with and without adjustment of s domain. It shows that the common Cauchy method fails to extract the loss of the filter, but after the adjustment of s domain, the Cauchy method is able to cover the loss and have a great match with the measured data.

Note that the extraction result above is based on the same unloaded Q. To further test the Cauchy method, an example of un-even unloaded Qs is illustrated here: one resonator's Q is 35 and the other one's Q is 701. To get the best match at the resonant frequency, the effective quality factor, Q=70, is used here to present the loss of the resonators. The schematic and extraction result of this example are shown below:



(a)



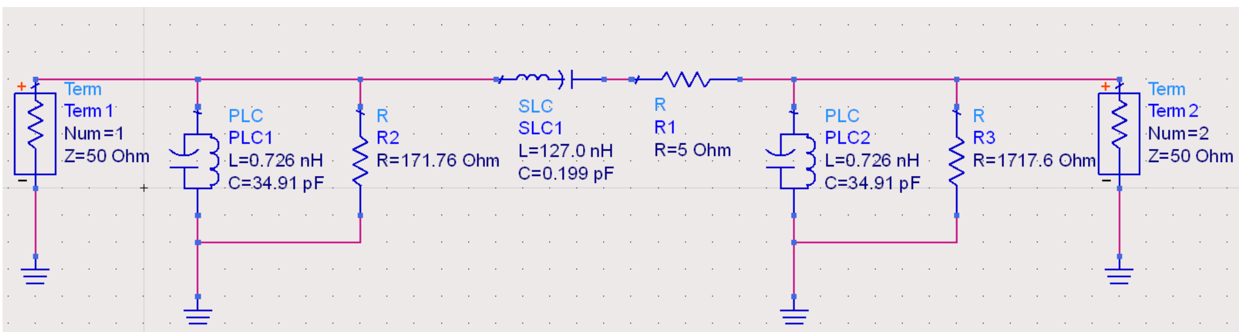
(b)

Fig. 3.3 (a) Schematic of the 2-order BP Chebyshev filter with the different Qs (b) Measured and extracted results of the 2-order BP lossy filter.

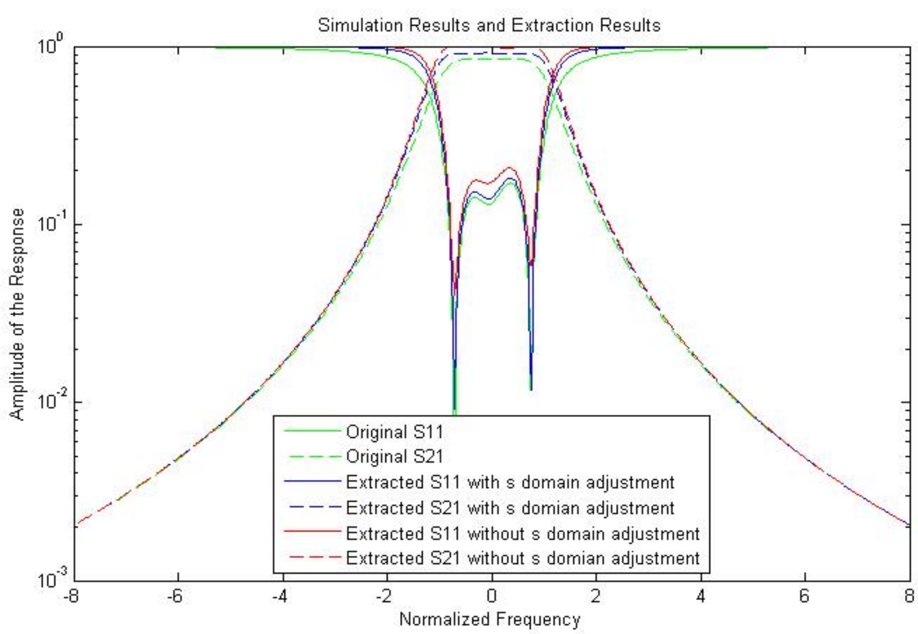
The green curves show the original measured S-parameters, and the blue curves and red curves are the extracted result with and without adjustment of s domain. The comparison shows that the original Cauchy method fails to extract the loss of the filter. After the adjustment in s domain, the result based on extracted parameters and the measured data are close, but with some mismatches.

To further test the Cauchy method, some higher order filter examples are illustrated below:

Fig. 3.4 (a) shows a 3-order BP filter with resonant frequency,$f_0 = 1GHz$, and the bandwidth:$BW = 100MHz$. The three resonant cavities have the different quality factors: $Q_1 = 55, Q_2 = 548, Q_3 = 548$. The effective common quality factor is simulated as 137. The schematic and extraction results are shown below:



(a)

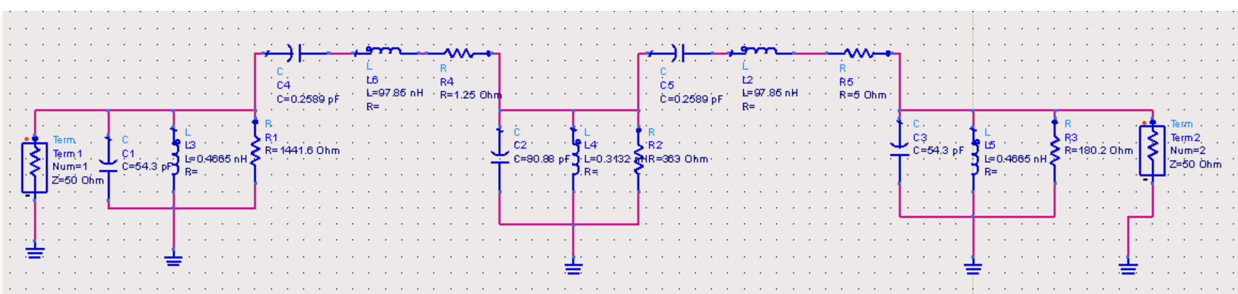


(b)

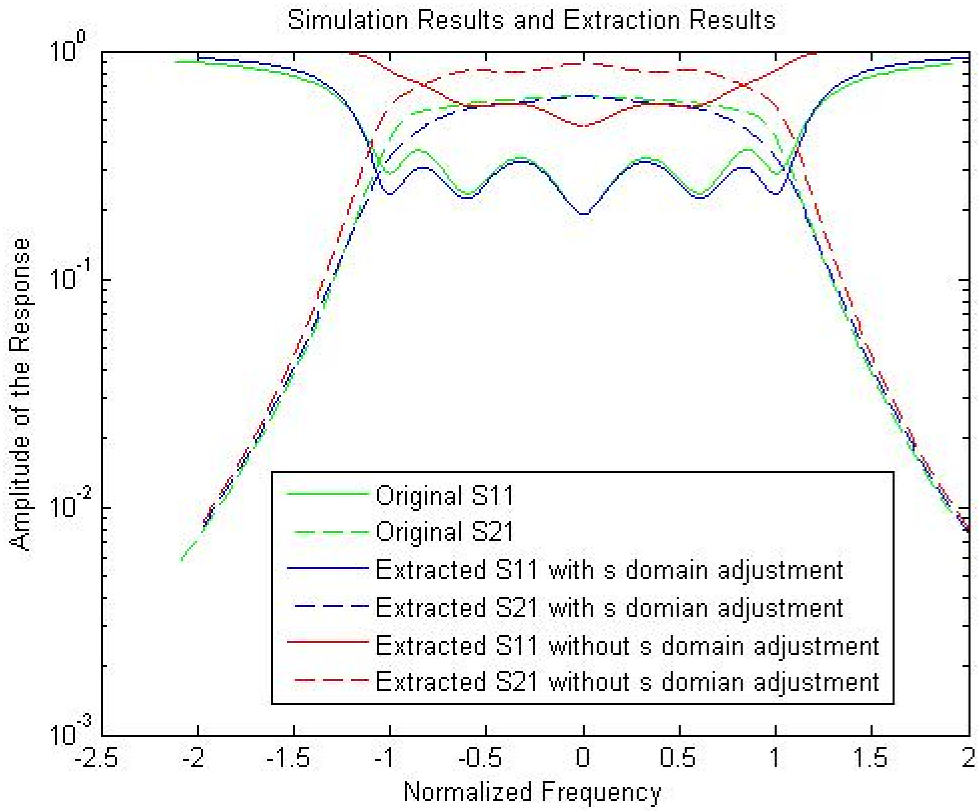
Fig. 3.4 (a) Schematic of the 3-order BP Chebyshev filter with the different Qs (b) Measured and extracted results of the 3-oder BP lossy filter.

In Fig. 3.4, the green curves show the original measured S-parameters, and the blue curves and red curves are the extracted results using the extracted methods, with and without adjustment of s domain.

Figure 3.5 (b) shows a 5-order BP filter with resonant frequency, $f_0 = 1GHz$, and the bandwidth: $BW = 100MHz$. The five resonant cavities have the different quality factors: $Q_1 = 31, Q_2 = 62, Q_3 = 185, Q_2 = 308, Q_1 = 532$. The effective common quality factor is simulated as 125.



(a)



(b)

Fig. 3.5 (a) Schematic of the 5-order BP Chebyshev filter with the different Qs (b) Measured and extracted results of the 5-oder BP lossy filter.

In Fig.3.5, the green curve is plotted using the original measured S-parameters, and the blue curves and red curves show results using the extracted parameters, with and without adjustment in the s domain.

The results from these additional experiments lead to a similar conclusion: 1) if the loss is not taken into account, the extraction result is always bad, 2) the Cauchy method works very well when the unloaded Qs are equal; and 3) the Cauchy method does not perform very well when the Qs are un-even; in particular, the more un-even the Q factors are, the worse extraction performance the Cauchy method will have.

3.5.3 Testing of the Method with Coupling Matrix and Cauchy Method

To compare deferent extraction methods, we use the same lossy used in Section 3.4.2. The first two examples are a 2-order BP Chebyshev filters with the same quality factors, $Q_1 = Q_2 = 70.1$; and a 2-order BP Chebyshev filters with the different quality factors $Q_1 = 35,\ Q_2 = 701$. The third example is a 3-order BP filter with five different quality factors: $Q_1 = 55, Q_2 = 548, Q_3 = 548$. The last one is a 5-order BP filter with five different quality factors: $Q_1 = 31, Q_2 = 62, Q_3 = 185, Q_4 = 308, Q_5 = 532$.

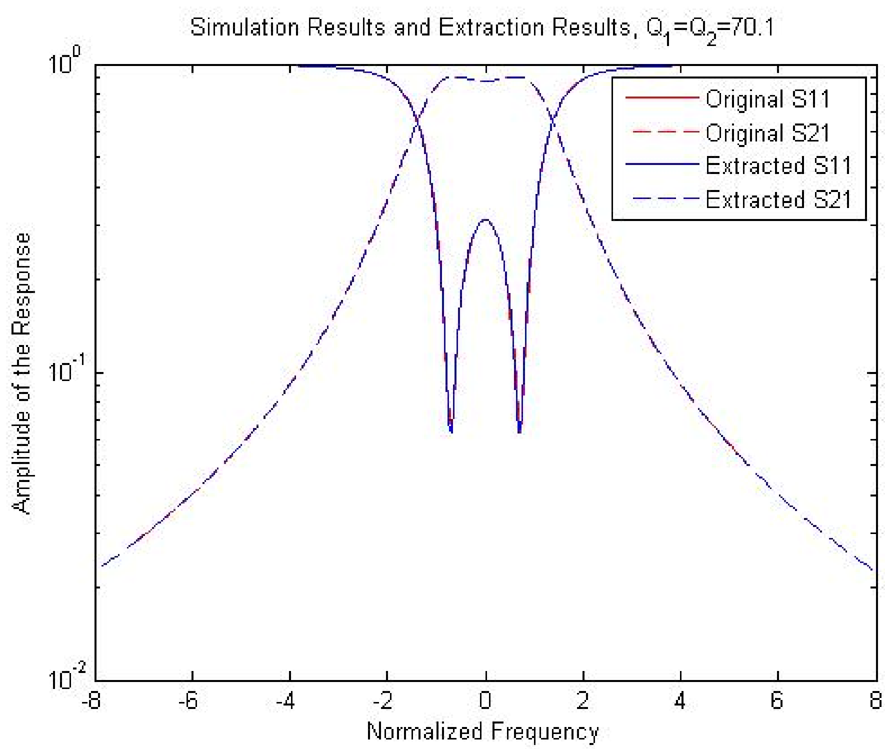


Fig. 3.6 Measured and extracted results of the 2-oder BP lossy filter with the same Qs, using the mixed methods.

The measured (red curves) and extracted (blue curves) curves match with each other very well, and the Q values estimated by the this method are $Q_1 = 68$, $Q_2 = 69$, which are close to the settled values. It indicates that this method works well in the case of similar Q values.

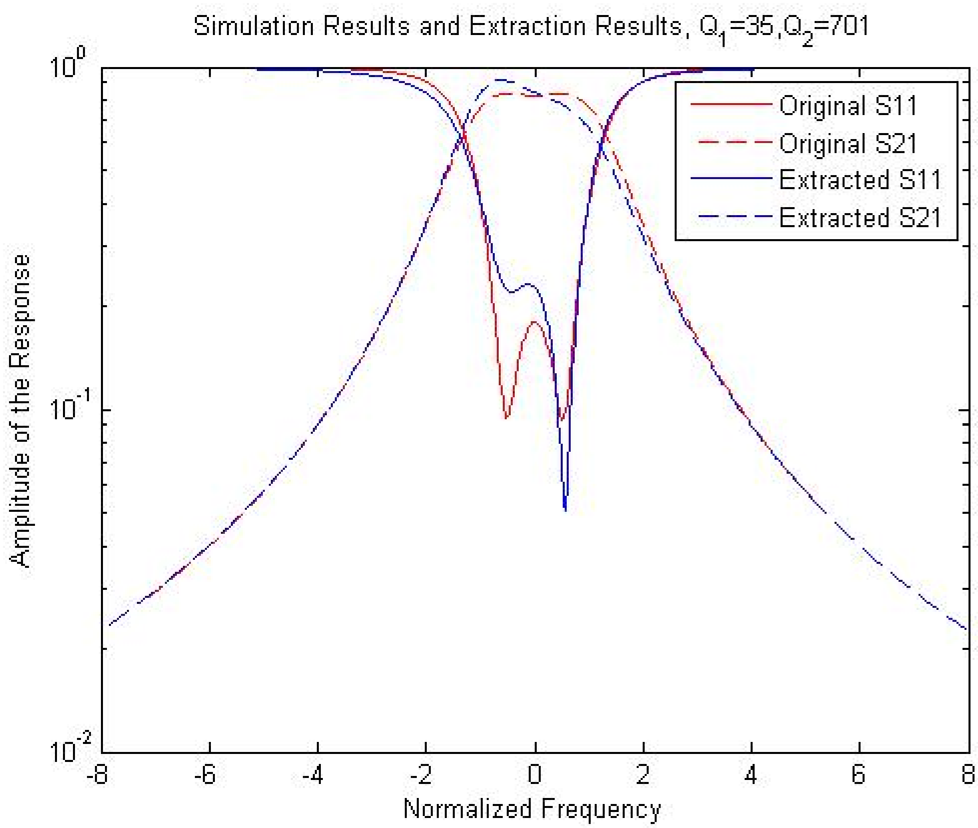


Fig. 3.7 Measured and extracted results of the 2-oder BP lossy filter with the different Qs, using the mixed methods.

The measured (red curves) and extracted (blue curves) curves have significant mismatch, and the Q values estimated by the this method are $Q_1 = 62$, $Q_2 = 234$, which are quite different from to the settled values, 35 and 701. These show that this method does not work well in the case of different Q values in a low-order filter.

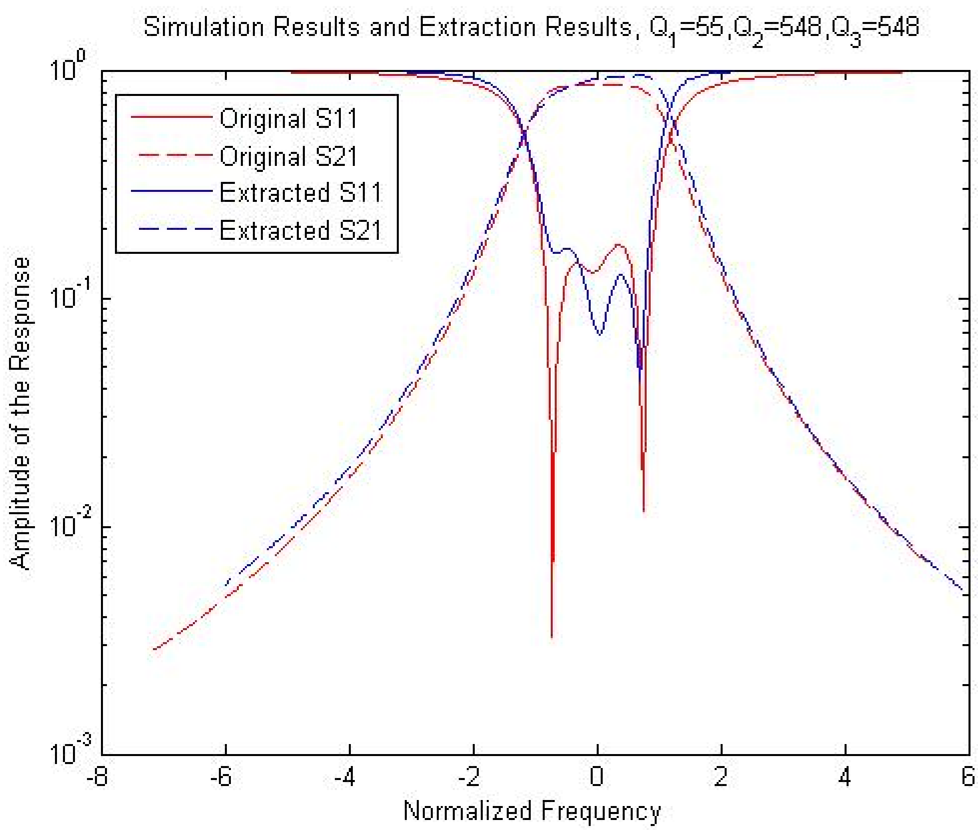


Fig. 3.8 Measured and extracted results of a 3-order BP lossy filter with different Qs, using the mixed methods.

The extracted curves (blue curves) also differ from the measured ones (red curves) significantly. The estimated Q values are $Q_1 = 104$, $Q_2 = 114, Q_3 = 132$, which are quite different from to the settled values, 35 and 701. These results also show that this method does not work well in the case of different Q values in a low-order filter.

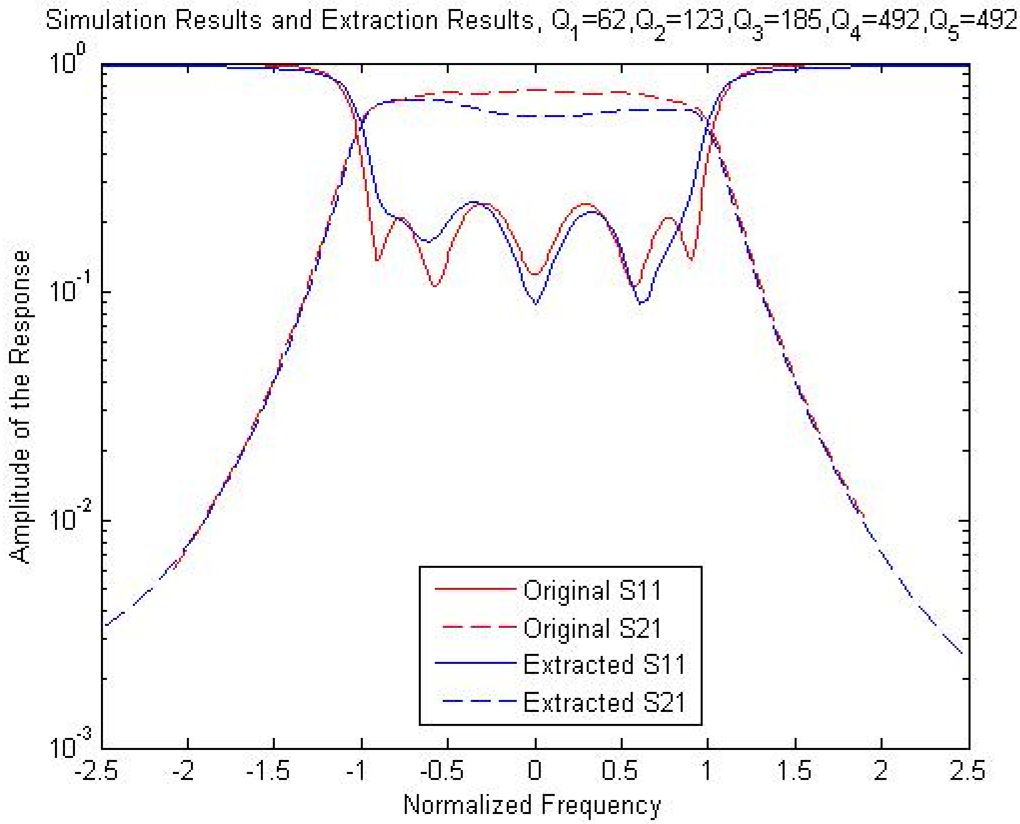


Fig. 3.9 Measured and extracted results of a 5-order BP lossy filter with different Qs, using the mixed methods

The extracted (blue curves) curves differ significantly from the measured ones (red curves). The estimated Q values are $Q_1 = 131, Q_2 = 142, Q_3 = 189, Q_4 = 168, Q_5 = 192$, which are quite different from to the original data. This comparison also shows that this method does not work well in the case of different Q values in a higher-order filter.

3.6 Analysis of the results

The Cauchy method is cost effective and has good performance only when the filter is lossless or lossy but with same or similar quality factors. However, when the resonators are of different quality factors, Cauchy method fails to get a good extraction.

In the 2-step extraction method, Cauchy method is applied first to generate the "lossless" characteristics polynomials and construct a coupling matrix, and then the loss can be represented in the diagonal terms. This method also works well in the case of lossy filters with same or similar Q factors, and provides a possible way to estimate the independent loss for each resonator. However, experiments show that it does not perform well in the cases of un-even quality factors. The reason for this might be that in the first step, the Cauchy method can not guarantee that the estimated parameters are a lossless filter's parameters. In other words, some information of the loss are reflected in the coupling matrix but not in the diagonal terms. The whole procedures need to be improved.

CHAPTER 4

A NEW OPTIMIZATION METHOD

4.1 Introduction

In this chapter, we explore some new methods to extract S-parameters from measured data of the filters with different unloaded quality factors. In Section 4.2, an enhanced Cauchy method is proposed first to accurately extract the unloaded Qs and characteristic polynomials. Then a formulation by re-adjusting the parameters to be estimated of the new Cauchy method is proposed in Section 4.3. To improve the performance, in Section 4.4, another formulation based on the relationship between the characteristics polynomials and the elements in the implemented circuit is proposed. Examples are illustrated throughout this section to test and compare the results using different methods.

4.2 An Enhanced Cauchy Method with Change in Characteristic Polynomials

4.2.1 Changes in the complex frequency domain

In the description of Cauchy method in Section 3.1, the scattering parameters $S_{11}(s)$ and $S_{21}(s)$ are expressed in terms of only one variable: s. To take the loss of the resonators into account, the complex domain s for the low-pass prototype filter is replaced by a new complex domain s':

$$s' = \frac{f_0}{BW}\frac{1}{Q_0} + j\frac{f_0}{BW}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{4.1}$$

As the new variable s' requires the same unloaded quality factor, all the resonators are assumed to have the same unloaded Q. So when the resonators are of different Qs, an effective or best approximated Q is used to conclude the loss. This method works well when the Qs are the same or very close. But for the case of un-even Qs, the accuracy is not very good.

Note that variable s' is related to unloaded quality factor of the resonator, so that for each resonator, its loss can be concluded in one variable s' with its own unloaded Q_i:

$$s'_i = \frac{f_0}{BW}\frac{1}{Q_i} + j\frac{f_0}{BW}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{4.2}$$

Then for an n-order filter, its scattering parameters can be represented as:

$$S_{11}(s_1, s_2, \dots, s_n) = \frac{F(s_1,s_2,\dots,s_n)}{E(s_1,s_2,\dots,s_n)} = \frac{\sum_{k=1}^{m_1} a_k (s_1)^{t_{1k}} (s_2)^{t_{2k}} \dots (s_n)^{t_{nk}}}{\sum_{k=1}^{m_3} b_k (s_1)^{t_{1k}} (s_2)^{t_{2k}} \dots (s_n)^{t_{nk}}} \tag{4.3.1}$$

$$S_{21}(s_1, s_2, \dots, s_n) = \frac{P(s_1,s_2,\dots,s_n)}{E(s_1,s_2,\dots,s_n)} = \frac{\sum_{k=1}^{m_2} c_k (s_1)^{t_{1k}} (s_2)^{t_{2k}} \dots (s_n)^{t_{nk}}}{\sum_{k=1}^{m_3} b_k (s_1)^{t_{1k}} (s_2)^{t_{2k}} \dots (s_n)^{t_{nk}}} \tag{4.3.2}$$

where

$$s_i = \frac{f_0}{BW}\frac{1}{Q_i} + j\frac{f_0}{BW}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{4.3.3}$$

is the new variable of S_{11} and S_{21};$F(s_1, s_2, \dots, s_n)$, $P(s_1, s_2, \dots, s_n)$ and $E(s_1, s_2, \dots, s_n)$ are characteristic polynomials with m_1,m_2 and m_3 different terms respectively; $t_{1k} + t_{2k} + \cdots + t_{nk} \le n$.

In this way, the loss of each resonator is accounted in the polynomials and all the information of the filter are included too. s_i can also be expressed in terms of s:

$$s_i = \frac{f_0}{BW}\frac{1}{Q_i} + j\frac{f_0}{BW}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = s + \frac{f_0}{BW}\frac{1}{Q_i} \tag{4.3.4}$$

In this way, the polynomials $F(s_1, s_2, \dots, s_n)$, $P(s_1, s_2, \dots, s_n)$ and $E(s_1, s_2, \dots, s_n)$ with multi variables can be converted into the presentation of $F(s)$, $P(s)$, and $E(s)$, with a single variable after all the coefficients are determined.

To get the best approximation of the loss, the quality factors can be determined by a recursive approach: 1) a common quality factor Q_0 can be obtained by the best matching of the S-parameter values at the resonant frequency f_0, 2) according the common quality factor, set a recurring range for each quality factor, and then get the best combination of the quality factors by the best matching of the overall curves. Then the coefficients of the filter can be obtained by Cauchy method established in [3].

The coefficients approximation approaches and results are discussed in the following using several different case studies.

4.2.2 Case Analysis of 2-order Chebyshev filter

For the 2-order Chebyshev filter, the scattering parameters are:

$$S_{11}(s_1, s_2) = \frac{F(s_1,s_2)}{E(s_1,s_2)} = \frac{a_1 s_1^2 + a_2 s_2^2 + a_3 s_1 s_2 + a_4 s_1 + a_5 s_2 + a_6}{b_1 s_1^2 + b_2 s_2^2 + b_3 s_1 s_2 + b_4 s_1 + b_5 s_2 + b_6} \tag{4.4.1}$$

$$S_{21}(s_1, s_2) = \frac{P(s_1,s_2)}{E(s_1,s_2)} = \frac{c_1}{b_1 s_1^2 + b_2 s_2^2 + b_3 s_1 s_2 + b_4 s_1 + b_5 s_2 + b_6} \tag{4.4.2}$$

These equations can be represented as:

$$[A_1 \quad -S_{11}A_1]\begin{bmatrix} a \\ b \end{bmatrix} = 0 \qquad [B \quad -S_{21}A_1]\begin{bmatrix} c \\ b \end{bmatrix} = 0 \tag{4.4.3}$$

where $$A_{1(m\times 6)} = \begin{bmatrix} {s_1}^2{}_{(1)} & {s_2}^2{}_{(1)} & s_{1(1)} \cdot s_{2(1)} & s_{1(1)} & s_{2(1)} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ {s_1}^2{}_{(m)} & {s_2}^2{}_{(m)} & s_{1(m)} \cdot s_{2(m)} & s_{1(m)} & s_{2(m)} & 1 \end{bmatrix},$$

$$B_{(m\times 1)} = [1, 1, \ \ldots, 1]^T,$$

$$S_{11(m\times m)} = diag[S_{11}(s_1(1), s_2(1)), S_{11}(s_1(2), s_2(2)), \ldots, S_{11}(s_1(m), s_2(m))],$$

$$S_{21(m\times m)} = diag[S_{21}(s_1(1), s_2(1)), S_{21}(s_1(2), s_2(2)), \ldots, S_{21}(s_1(m), s_2(m))],$$

$$a_{(6\times 1)} = [a_1, a_2, a_3, a_4, a_5, a_6]^T,$$

$$b_{(6\times 1)} = [b_1, b_2, b_3, b_4, b_5, b_6]^T,$$

$$c_{(1\times 1)} = [c_1]^T.$$

$S_{11}(s_1(i), s_2(i))$ and $S_{21}(s_1(i), s_2(i))$ are the measured scattering parameters at different sampling frequencies; a, b and c are the polynomial coefficients vectors.

Note that unlike the method in Section 3.1, where the polynomial coefficients of $F(s)$ and $P(s)$ are solved first and then the coefficients of $E(s)$ can be fixed, it is difficult to relate the coefficients of $F(s_1, s_2)$ and $P(s_1, s_2)$ to the coefficients of (s_1, s_2) . To solve this problem, the matrix equations in Equation (4.4.3) can be represented as:

$$\begin{bmatrix} A_{1(m\times 6)} & 0_{(m\times 1)} & -S_{11}A_1 \\ 0_{(m\times 6)} & B_{(m\times 1)} & -S_{21}A_1 \end{bmatrix} \begin{bmatrix} a \\ c \\ b \end{bmatrix} = 0 \tag{4.4.4}$$

Then the complex coefficients a, b, c in system (4.4.4) can be solved by the method presented in [3] with least square method (TLS) and singular value decomposition (SVD) at one time. To guarantee that Equation (4.4.4) has reasonable solutions, m must be greater or equal to (6+1+6=) 13.

For the schematic in Fig. 2.3, a 2-order Chebyshev filter with different quality factors of 35 and 701, 301 different sampling frequencies between 0.7GHz and 1.3 GHz are selected to generate the measured data. Then approach the procedures described above, the approximated coefficients vectors are:

$$a = [0.1103, -0.0822 + 0.0160i, -0.0281 - 0.0160i, -0.5283 + 0.3782i, 0.5805 - 0.3826i, -0.1515 + 0.1027i]^T;$$

$$b = [-0.0045 + 0.0181i, -0.0164 - 0.0041i, 0.0210 - 0.0140i, 0.1391 + 0.0111i, -0.1359 - 0.0051i, 0.0381 + 0.0017i]^T;$$

$$c = [-2.2500e - 014, -1.2023e - 014i]^T.$$

And the quality factors are approximated as 35 and 706.Then the relationships between s_1, s_2 and s, which are revealed by Equation (4.3.4), are:

$$s_1 = s + \frac{1}{3.5},\ s_2 = s + \frac{1}{70.5} \tag{4.4.5}$$

Convert the multi-variable polynomials into single-variable polynomials, by substituting Equation (4.4.5) and the coefficient values into Equation (4.4.1) & (4.4.2), the characteristic polynomials are extracted as:

$$F(s) = (1.0045 - 0.0031i) \cdot s^2 + (0.3088 - 0.0054i) \cdot s + (0.3105 - 0.0054i)$$

$$P(s) = (1.4360 + 0.0010i)$$

$$E(s) = 1 \cdot s^2 + (1.7339 - 0.0027i) \cdot s + (1.7455 - 0.0074i) \tag{4.4.6}$$

Then the extracted responses can be plotted:

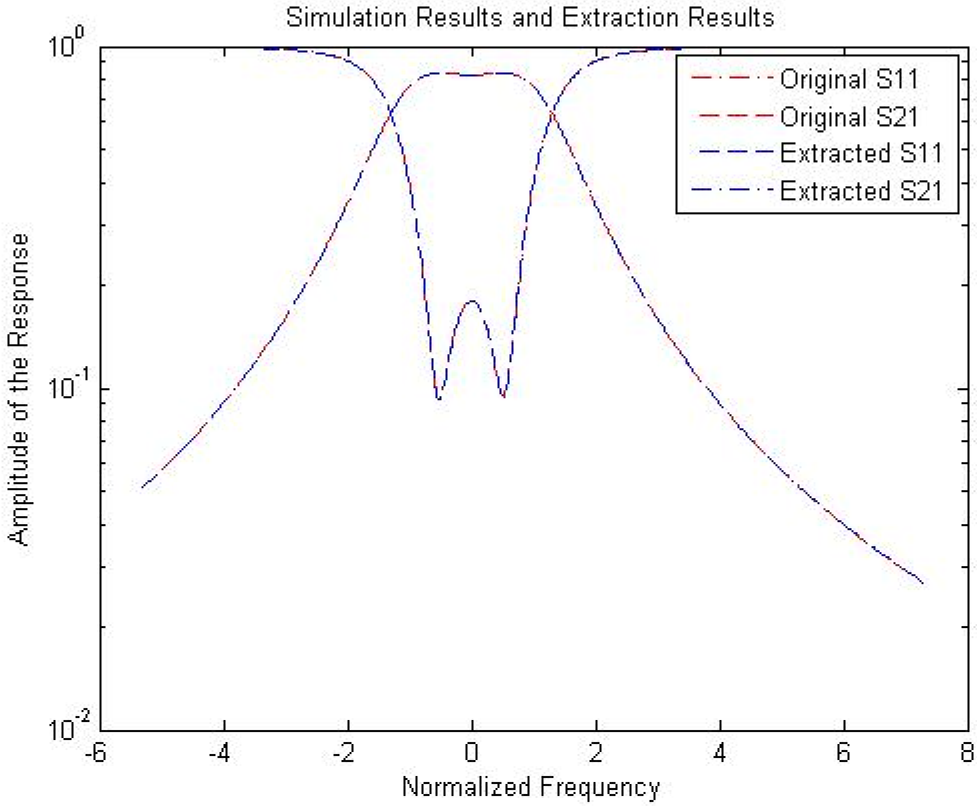


Fig. 4.1 Measured and Extracted Responses of a 2-order lossy filter, using the enhanced Cauchy method

The red curves are the measured data, and the blue curves are the extracted responses. It can be seen that the performance has been improved significantly comparing with that of the former Cauchy method. At the same time, the quality factors, which represent the loss, are approximated to be close to the real values.

4.2.3 Case Analysis of 3-order Chebyshev filter

For the 3-order Chebyshev filter, the scattering parameters are in the form of:

$$S_{11}(s_1, s_2, s_3) = \frac{F(s_1,s_2,s_3)}{E(s_1,s_2,s_3)}$$

$$= \frac{a_1 s_1^3 + a_2 s_2^3 + a_3 s_3^3 + a_4 {s_1}^2 s_2 + a_5 {s_1}^2 s_3 + a_6 {s_2}^2 s_1 + a_7 {s_2}^2 s_3 + a_8 {s_3}^2 s_1 + a_9 3^2 s_2 + a_{10} s_1 s_2 s_3 + a_{11} s_1^2 + a_{12} s_2^2 + a_{13} s_3^2 + a_{14} s_1 s_2 + a_{15} s_1 s_3 + a_{16} s_2 s_3 + a_{17} s_1 + a_{18} s_2 + a_{19} s_3 + a_{20}}{b_1 s_1^3 + b_2 s_2^3 + b_3 s_3^3 + b_4 {s_1}^2 s_2 + b_5 {s_1}^2 s_3 + b_6 {s_2}^2 s_1 + b_7 {s_2}^2 s_3 + b_8 {s_3}^2 s_1 + b_9 3^2 s_2 + b_{10} s_1 s_2 s_3 + b_{11} s_1^2 + b s_2^2 + b_{13} s_3^2 + b_{14} s_1 s_2 + b_{15} s_1 s_3 + b_{16} s_2 s_3 + b_{17} s_1 + b_{18} s_2 + b_{19} s_3 + b_{20}}$$

(4.5.1)

$$S_{21}(s_1, s_2, s_3) = \frac{P(s_1,s_2,s_3)}{E(s_1,s_2,s_3)}$$

$$= \frac{c_1}{b_1 s_1^3 + b_2 s_2^3 + b_3 s_3^3 + b_4 {s_1}^2 s_2 + b_5 {s_1}^2 s_3 + b_6 {s_2}^2 s_1 + b_7 {s_2}^2 s_3 + b_8 {s_3}^2 s_1 + b_9 3^2 s_2 + b_{10} s_1 s_2 s_3 + b_{11} s_1^2 + b s_2^2 + b_{13} s_3^2 + b_{14} s_1 s_2 + b_{15} s_1 s_3 + b_{16} s_2 s_3 + b_{17} s_1 + b_{18} s_2 + b_{19} s_3 + b_{20}}$$

(4.5.2)

Similarly, a matrix equation can be formed as:

$$\begin{bmatrix} A_{2(m\times 20)} & 0_{(m\times 1)} & -S_{11}A_2 \\ 0_{(m\times 20)} & B_{(m\times 1)} & -S_{21}A_2 \end{bmatrix} \begin{bmatrix} a \\ c \\ b \end{bmatrix} = 0 \tag{4.5.3}$$

where

$$A_{2(m\times 20)} = \begin{bmatrix} {s_1}^3{}_{(1)} & {s_2}^3{}_{(1)} & {s_3}^3{}_{(1)} & {s_1}^2{}_{(1)} \cdot s_{2(1)} & \cdots & s_{1(1)} & s_{2(1)} & s_{3(1)} & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ {s_1}^2{}_{(m)} & {s_2}^2{}_{(m)} & {s_3}^3{}_{(m)} & {s_1}^2{}_{(m)} \cdot s_{2(m)} & \cdots & s_{1(m)} & s_{4(m)} & s_{3(m)} & 1 \end{bmatrix},$$

$$B_{(m\times 1)} = [1, 1, \ \dots, 1]^T,$$

$$a_{(20\times 1)} = [a_1, a_2, \dots, a_{20}]^T,$$

$$b_{(20\times 1)} = [b_1, b_2, \dots, b_{20}]^T,$$

$c_{(1\times1)} = [c_1]^T.$

The quality factors are approximated as 56, 467 and 491, then the relationship between s_1, s_2, s_3 and s are:

$$s_1 = s + \frac{1}{5.6}, s_2 = s + \frac{1}{4.67}, s_3 = s + \frac{1}{4.91} \tag{4.5.4}$$

Applying the same technique presented in Section 4.2.1, the characteristic polynomials can be approximated:

$$F(s) = 1 \cdot s^3 + (0.3050 + 0.0014i) \cdot s^2 + (0.5391 + 0.0071i) \cdot s + (0.0.1582 - 0.0170i)$$

$$P(s) = (1.0416 + 0.0003i)$$

$$E(s) = 1 \cdot s^3 + (2.1293 + 0.0121i) \cdot s^2 + (2.2751 + 0.0360i) \cdot s + (1.2221 + 0.0196i) \tag{4.5.5}$$

The curves of measured and extracted scattering parameters are shown in Fig. 4.2:

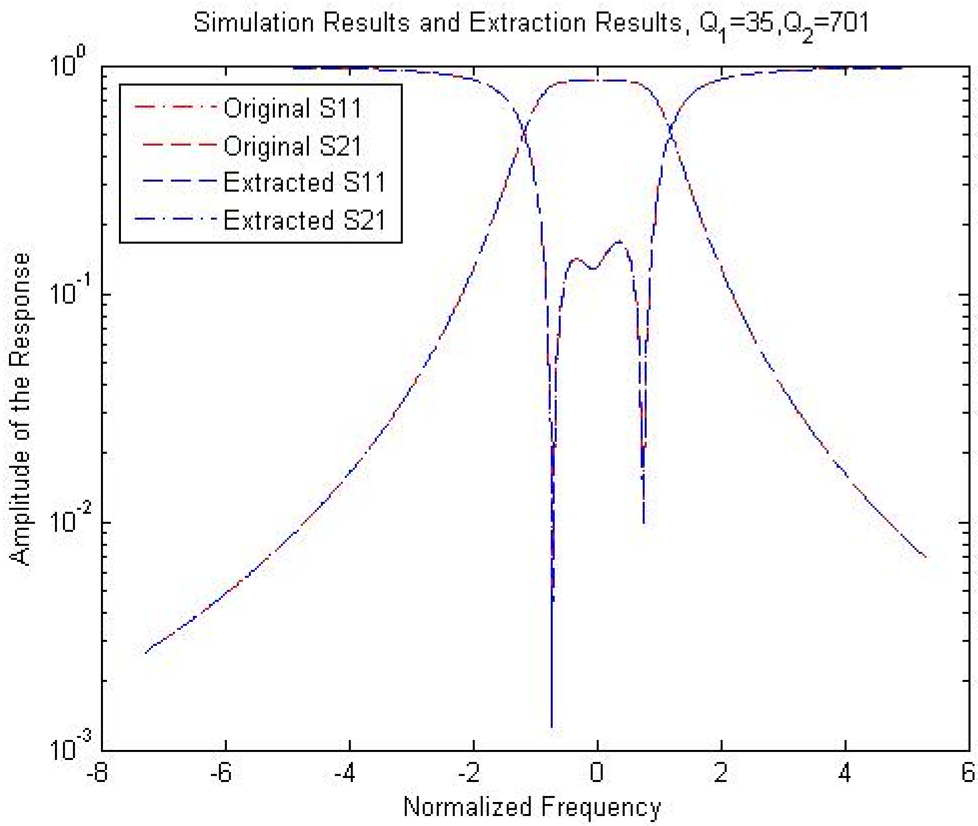


Fig. 4.2 Measured and Extracted Responses For the 3-Order Filter, using the enhanced Cauchy method

The red curves are the measured responses, the blue curves are the extracted responses using enhanced Cauchy method. The curves also have a very good matching. For the quality factors, the estimated values are close to the original values, but the differences are larger comparing to those in the 2-order case.

4.2.4 Cases of high order filters

From the above examples, it can be seen that the enhanced Cauchy method has good performance as reflected in the well matched curves. However, the accuracy is associated with a cost in the increasing quantity of polynomials' terms. In fact, if the number of the filter's order increases to 5, the number of the polynomial $E(s_1, s_2, \dots, s_n)$'s order will reach to 252. It becomes difficult to implement the method for higher order filters. At the same time, the recursive approach used to extract the quality factors becomes more and more time consuming as the order increases. As such, this method seems to be only suitable for low order filters.

4.3 Another model for parameter extraction

4.3.1 Parameter Re-arrangement of the 2-Order Chebyshev Filter

For the 2-order Chebyshev filter, the scattering parameters are:

$$S_{11}(s_1, s_2) = \frac{F(s_1,s_2)}{E(s_1,s_2)} = \frac{a_1 s_1^2 + a_2 s_2^2 + a_3 s_1 s_2 + a_4 s_1 + a_5 s_2 + a_6}{b_1 s_1^2 + b_2 s_2^2 + b_3 s_1 s_2 + b_4 s_1 + b_5 s_2 + b_6} \quad (4.6.1)$$

$$S_{21}(s_1, s_2) = \frac{P(s_1,s_2)}{E(s_1,s_2)} = \frac{c_1}{b_1 s_1^2 + b_2 s_2^2 + b_3 s_1 s_2 + b_4 s_1 + b_5 s_2 + b_6} \quad (4.6.2)$$

Plug the equations $s_1 = s + \sigma_1 = s + \frac{f_0}{BW}\frac{1}{Q_1}$, $s_2 = s + \sigma_2 = s + \frac{f_0}{BW}\frac{1}{Q_2}$ into Equation (4.6.1) and Equation (4.6.2), the characteristic polynomials can be rewritten as:

$$F(s) = (a_1 + a_2 + a_3)s^2 + (2a_1\sigma_1 + 2a_2\sigma_2 + a_3\sigma_1 + a_3\sigma_2 + a_4 + a_5)s + (a_1\sigma_1^2 + a_2\sigma_2^2 + a_3\sigma_1\sigma_2 + a_4\sigma_1 + a_5\sigma_2 + a_6)$$

$$E(s) = (b_1 + b_2 + b_3)s^2 + (2b\sigma_1 + 2b_2\sigma_2 + b\sigma_1 + b_3\sigma_2 + b_4 + b_5)s + (b_1\sigma_1^2 + b_2\sigma_2^2 + b_3\sigma_1\sigma_2 + b_4\sigma_1 + b\sigma_2 + b_6)$$

$$P(s) = c_1 \tag{4.7.1}$$

Then the S-parameters can be rewritten as:

$$S_{11}(s) = \frac{F(s)}{E(s)}$$

$$= \frac{a_1(s+\sigma_1)^2 + a_2(s+\sigma_2)^2 + a_3(s+\sigma_1)(s+\sigma_1) + a_4(s+\sigma_1) + a_5(s+\sigma_2) + a_6}{b_1(s+\sigma_1)^2 + b_2(s+\sigma_2)^2 + b_3(s+\sigma_1)(s+\sigma_1) + b_4(s+\sigma_1) + b_5(s+\sigma_2) + b_6}$$

$$= \frac{(a_1 + a_2 + a_3)s^2 + (2a_1\sigma_1 + 2a_2\sigma_2 + a_3\sigma_1 + a_3\sigma_2 + a_4 + a_5)s + (a_1\sigma_1^2 + a_2\sigma_2^2 + a_3\sigma_1\sigma_2 + a_4\sigma_1 + a_5\sigma_2 + a_6)}{(b_1 + b_2 + b_3)s^2 + (2b_1\sigma_1 + 2b_2\sigma_2 + b_3\sigma_1 + b_3\sigma_2 + b_4 + b_5)s + (b_1\sigma_1^2 + b_2\sigma_2^2 + b_3\sigma_1\sigma_2 + b_4\sigma_1 + b_5\sigma_2 + b_6)}$$

(4.7.2)

$$S_{21}(s) = \frac{P(s)}{E(s)}$$

$$= \frac{c_1}{b_1(s+\sigma_1)^2 + b_2(s+\sigma_2)^2 + b_3(s+\sigma_1)(s+\sigma_1) + b_4(s+\sigma_1) + b_5(s+\sigma_2) + b_6}$$

$$= \frac{c_1}{(b_1 + b_2 + b_3)s^2 + (2b_1\sigma_1 + 2b_2\sigma_2 + b_3\sigma_1 + b_3\sigma_2 + b_4 + b_5)s + (b_1\sigma_1^2 + b_2\sigma_2^2 + b_3\sigma_1\sigma_2 + b_4\sigma_1 + b_5\sigma_2 + b_6)}$$

(4.7.3)

Rearrange the parameters,

$$E(s) = B_1 s^2 + (B_2\sigma_1 + B_3\sigma_2 + B_4)s + (B_5\sigma_1^2 + B_6\sigma_2^2 + B_7\sigma_1\sigma_2 + B_8\sigma_1 + B_9\sigma_2 + B_{10})$$

$F(s) = A_1 s^2 + (A_2\sigma_1 + A_3\sigma_2 + A_4)s + (A_5\sigma_1^2 + A_6\sigma_2^2 + A_7\sigma_1\sigma_2 + A_8\sigma_1 + A_9\sigma_2 + A_{10})$

$P(s) = C_0$ (4.8)

Totally there are 23 parameters need to be approximated. The parameters have some unique properties:

1) parameters σ_1, σ_2 are positive real while the others could be complex;

2) parameter B_1 is unity;

3) deducting all the terms referring to σ_1, σ_2 in $E(s)$, $F(s)$ and $P(s)$, the system will be lossless;

4.3.2 Problem formulation

Then this problem can be formulated as an optimization problem:

Parameters vector (PV) to be approximated:

PV = $[A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, B_{10}, \sigma_1, \sigma_2, C_0]$

(4.9.1)

Cost function:

$$Fun = \sum_{i=1}^{m} \left|S_{21}^{ext}(s_{(i)}) - S_{21}^{mea}(s_{(i)})\right|^2 + \left|S_{11}^{ext}(s_{(i)}) - S_{11}^{mea}(s_{(i)})\right|^2 \quad (4.9.2)$$

$S_{21}^{mea}(s_{(i)})$ and $S_{11}^{mea}(s_{(i)})$ are known measured S_{21} and S_{11} values at m different sampling $s_{(i)}$; $s_{(i)} = \mathrm{j}\Omega_{(i)}$ is the complex domain; $\Omega_{(i)}$ is the normalized frequency for the low-pass prototype. The normalized frequency $\Omega_{(i)}$ has the relationship with the band-pass frequency $f_{(i)}$:

$$\Omega_{(i)} = \frac{f_0}{BW}\left(\frac{f_{(i)}}{f_0} - \frac{f_0}{f_{(i)}}\right) \quad (4.9.3)$$

where BW is the bandwidth of the band-pass filter; $f_0$is the resonant frequency.

$S_{21}^{ext}(s_{(i)})$ and $S_{11}^{ext}(s_{(i)})$ are the extracted transmission function and reflection function values at different $s_{(i)}$, where

$$S_{21}^{ext}(s_{(i)}) = \frac{P(s_{(i)})}{E(s_{(i)})}, \quad S_{11}^{ext}(s_{(i)}) = \frac{F(s_{(i)})}{E(s_{(i)})} \tag{4.9.4}$$

and

$$E(s_{(i)}) = B_1 {s_{(i)}}^2 + (B_2\sigma_1 + B_3\sigma_2 + B_4)s_{(i)} + (B_5\sigma_1^2 + B_6\sigma_2^2 + B_7\sigma_1\sigma_2 + B_8\sigma_1 + B_9\sigma_2 + B_{10}),$$

$$F(s_{(i)}) = A_1 {s_{(i)}}^2 + (A_2\sigma_1 + A_3\sigma_2 + A_4)s_{(i)} + (A_5\sigma_1^2 + A_6\sigma_2^2 + A_7\sigma_1\sigma_2 + A_8\sigma_1 + A_9\sigma_2 + A_{10}),$$

$$P(s_{(i)}) = C_0, \tag{4.9.5}$$

Constraints:

1) $B_1 = 1$;

2) σ_1, σ_2 are purely real, and $0 < \sigma_1, \sigma_2 < 1$;

3) Let $\hat{E}(s_{(i)}) = B_1 {s_{(i)}}^2 + B_4 s_{(i)} + B_{10}$, $\hat{F}(s_{(i)}) = A_1 {s_{(i)}}^2 + A_4 s_{(i)} + A_{10}$, and $\hat{P}(s_{(i)}) = C_0$, then $\hat{E}(s_{(i)})\hat{E}^*(-s_{(i)}) + \hat{F}(s_{(i)})\hat{F}^*(-s_{(i)}) = \hat{P}(s_{(i)})\hat{P}^*(-s_{(i)})$, here (*) means complex conjugation.

This optimization problem can be solved by applying Levenberg–Marquardt algorithm established in [14].

4.3.3 Numerical Example to test the method:

Here is an example to testify the method. For the 2-order BP prototype Chebyshev filter cited in Section 3.5.2, where $f_0 = 1GHz$, $BW = 100MHz$, $Q_1 = 701.447$, $Q_2 = 35.071$. The factors representing the losses are: $\sigma_1 = \frac{f_0}{BW}\frac{1}{Q_1} = \frac{1}{70.1447} = 0.014256$, $\sigma_2 = \frac{f_0}{BW}\frac{1}{Q_2} = \frac{1}{3.5071} =$ 0.28513.

From Equation (4.8), the theoretical characteristic polynomials are:

$$F(s) = s^2 + (\sigma_2 + \sigma_1)s + (\sigma_1\sigma_2 + 0.7128123\sigma_1 - 0.7128123\sigma_2 + 0.5000000)$$

$$E(s) = s^2 + (\sigma_2 + \sigma_1 + 1.4256246)s + (\sigma_1\sigma_2 + 0.7128123\sigma_1 + 0.7128123\sigma_2 + 1.5162022)$$

$$P(s) = 1.4313871 \tag{4.10.1}$$

Compare between Equation (4.10.1) and Equation (4.9.5), the theoretical parameters are:

$$[\,1,1,1, 1.4256246, 0,0,1, 0.7128123, -0.7128123, 0.5000000, 1, 1,1, 0,0, 0,1, 0.7128123, 0.7128123, 1.5162022, 0.014256, 0.28513, 1.4313871] \tag{4.10.2}$$

Plug the σ_1, σ_2 values into (4.10.1), the characteristic polynomials of the lossy filter will be:

$$F(s) = 1 \cdot s^2 + 0.2988 \cdot s + 0.3113$$

$$E(s) = 1 \cdot s^2 + 1.7244 \cdot s + 1.7331$$

$$P(s) = 1.4314 \tag{4.10.3}$$

Ignoring the factors related with σ_1, σ_2, the characteristic polynomials will be:

$$F(s) = 1 \cdot s^2 + 0.2993892 \cdot s + 0.3109808$$

$$E(s) = 1 \cdot s^2 + 1.7250138 \cdot s + 1.7336751$$

$$P(s) = 1.4313871 \tag{4.10.4}$$

These polynomials in Equation (4.10.5) are exactly the same as the ideal and lossless filter designed in Section 2.5, which indicates that the constraints are proper and effective.

Applying the Levenberg–Marquardt algorithm in Matlab, the parameters in Equation (4.9.1) are estimated as:

$$PV = [1.0000,\ \ 1.0525, 1.0526, 1.5587, 0.9243, 0.9246, 1.6674, 1.6677, 1.4529, 1.0000, 0.9619, 0.9620, 0.1474, 0.5114, 0.5118, 0.5119, 0.3321, 0.3316, 0.2490, 0.0780, 1.4314]. \quad (4.10.5)$$

Then the estimated polynomials are:

$$F(s) = 1 \cdot s^2 + 0.2998 \cdot s + 0.3113$$

$$E(s) = 1 \cdot s^2 + 1.7225 \cdot s + 1.7331$$

$$P(s) = 1.4314 \quad (4.10.6)$$

Comparing the Equation (4.10.6) and Equation (4.10.3), the estimation result is quite close to the original design.

Here below is the comparing between the original design and the extracted result:

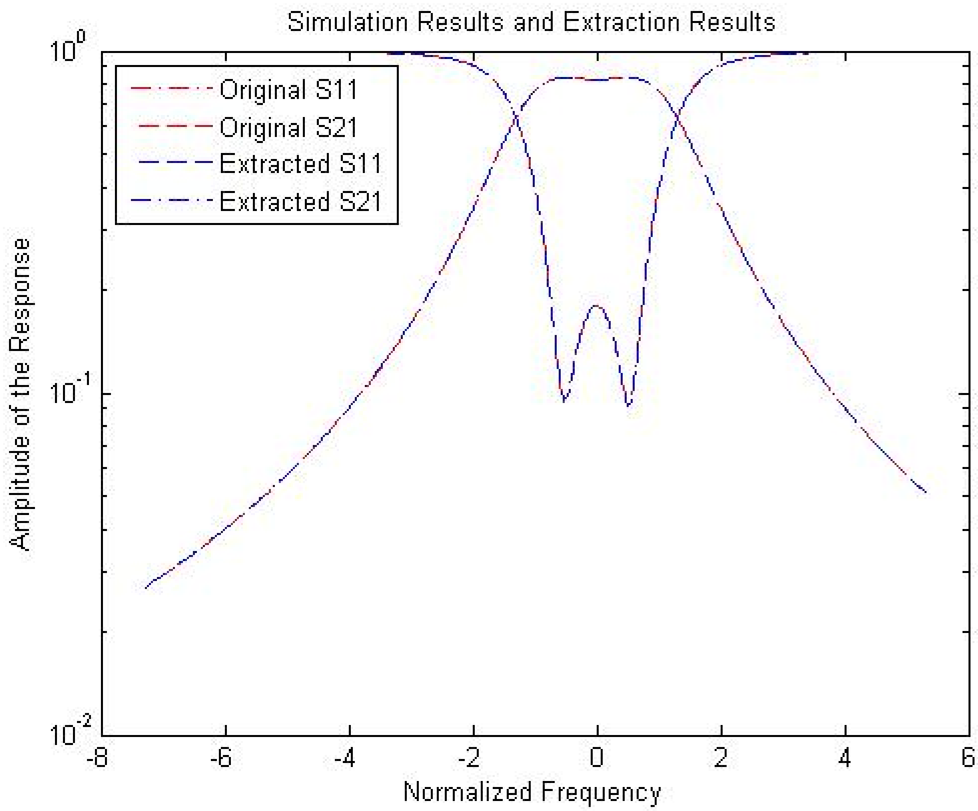


Fig. 4.3 Measured and Extracted Responses For the 2-Order Lossy Filter, Using the New Optimization Model

The red curves are the original data and the blue curves are the extracted responses. It can be seen that the matching of the curves is very good.

Similar to the method proposed in Section 4.2, the loss information has been accounted in the model, but the quality factors are estimated together with the other coefficients at one time without the recursive approaches. But due to the fact that the relationship between the coefficients are not revealed, the method does not extract the quality factors.

4.4 A new optimization model of the lossy filter

4.4.1 Analysis of the 2-order lossy filter

The model proposed in Section 4.3have 23 parameters for a 2-order filter. Is it possible to reduce the amount of the parameters? To help analysis, the LP prototype of the 2-order filter's schematic can be generated as below:

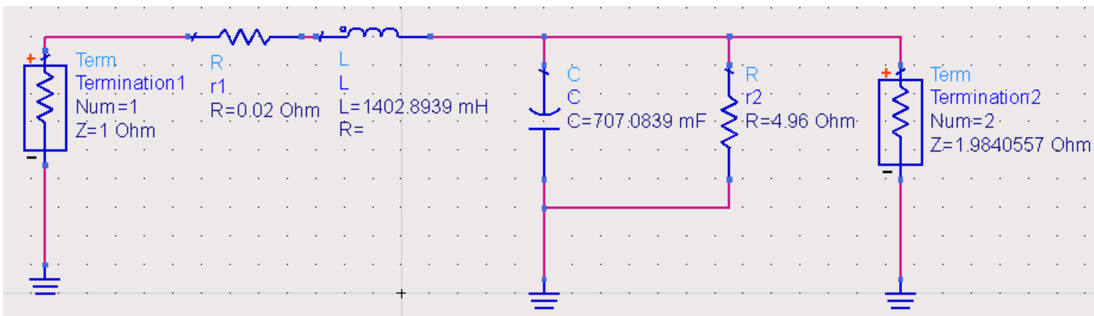


Figure.4.4 the 2-order LP prototype filter's schematic.

Then the S-parameters can be concluded as:

$$S_{11}(s) = \frac{R \parallel \left(\frac{1}{sC}\right) \parallel r_2 + sL + r_1 - 1}{R \parallel \left(\frac{1}{sC}\right) \parallel r_2 + sL + r_1 + 1}$$

$$= \frac{s^2 + \left(\frac{1}{C}\frac{1}{r_2} + \frac{1}{R}\frac{1}{C} + \frac{r_1}{L} - \frac{1}{L}\right)s + \left(\frac{1}{LC}\left(\frac{r_1}{r_2} + \frac{r_1}{R} - \frac{1}{r_2} - \frac{1}{R} + 1\right)\right)}{s^2 + \left(\frac{1}{C}\frac{1}{r_2} + \frac{1}{R}\frac{1}{C} + \frac{r_1}{L} + \frac{1}{L}\right)s + \left(\frac{1}{LC}\left(\frac{r_1}{r_2} + \frac{r_1}{R} + \frac{1}{r_2} + \frac{1}{R} + 1\right)\right)}$$

(4.11.1)

$$S_{21}(s) = \frac{1}{\sqrt{R}} \frac{2\left(R \parallel \left(\frac{1}{sC}\right) \parallel r_2\right)}{R \parallel \left(\frac{1}{sC}\right) \parallel r_2 + sL + r_1 + 1}$$

$$= \frac{\frac{2}{LC\sqrt{R}}}{s^2 + \left(\frac{1}{C}\frac{1}{r_2} + \frac{1}{R}\frac{1}{C} + \frac{r_1}{L} + \frac{1}{L}\right)s + \left(\frac{1}{LC}\left(\frac{r_1}{r_2} + \frac{r_1}{R} + \frac{1}{r_2} + \frac{1}{R} + 1\right)\right)}$$

(4.11.2)

where $s = j\Omega$ is the complex domain, Ω is the normalized frequency; the impedance of termination 1 on the left side is normalized as unity, 1; R is the impedance of termination 2 on the right side; L, C are the inductor and capacitor; r_1, r_2 are the resistors in the resonators, which represent the losses.

Let $t_1 = \frac{1}{L}$, $t_2 = \frac{1}{C}, t_3 = \frac{1}{R}, t_4 = r_1, t_5 = \frac{1}{r_2}$, then the S-parameters can be written as:

$$S_{11}(s) = \frac{F(s)}{E(s)} = \frac{F_2 \cdot s^2 + F_1 \cdot s + F_0}{E_2 \cdot s^2 + E_1 \cdot s + E_0}$$

$$= \frac{s^2 + (t_1t_4 + t_2t_5 + t_2t_3 - t_1)s + \big(t_1t_2(t_3t_4 + t_4t_5 - t_3 - t_5 + 1)\big)}{s^2 + (t_1t_4 + t_2t_5 + t_2t_3 + t_1)s + \big(t_1t_2(t_3t_4 + t_4t_5 + t_3 + t_5 + 1)\big)}$$

(4.11.3)

$$S_{21}(s) = \frac{P(s)}{E(s)} = \frac{P_0}{E_2 \cdot s^2 + E_1 \cdot s + E_0}$$

$$= \frac{2t_1t_2\sqrt{t_3}}{s^2 + (t_1t_4 + t_2t_5 + t_2t_3 + t_1)s + (t_1t_2(t_3t_4 + t_4t_5 + t_3 + t_5 + 1))}$$

(4.11.4)

where $F(s), E(s)$ and $P(s)$ are the characteristic polynomials. From Equations (4.11.3) and (4.11.4), the coefficients of these polynomials can be represented as:

$F_2 = 1;$

$F_1 = t_1t_4 + t_2t_5 + t_2t_3 - t_1;$

$F_0 = t_1t_2(t_3t_4 + t_4t_5 - t_3 - t_5 + 1);$

$E_2 = 1;$

$E_1 = t_1t_4 + t_2t_5 + t_2t_3 + t_1;$

$E_0 = t_1t_2(t_3t_4 + t_4t_5 + t_3 + t_5 + 1);$

$P_0 = 2t_1t_2\sqrt{t_3};$ (4.11.5)

Actually, F_2 and E_2 are all normalized as unity, $F_2 = E_2 = 1$, and they are fixed before being be approximated.

4.4.2 A new formulation of the 2-order lossy filter

Based on the analysis in Section 4.4.1, the new formulation of this problem will be:

Parameters vector to be approximated:

$$\mathrm{PV} = [t_1, t_2, t_3, t_4, t_5] \tag{4.11.6}$$

Cost function:

$$Fun = \sum_{k=1}^{m} |S_{21}^{ext}(s)_k - S_{21}^{mea}(s)_k|^2 + |S_{11}^{ext}(s)_k - S_{11}^{mea}(s)_k|^2 \tag{4.11.7}$$

$S_{21}^{mea}(s)_k$ and $S_{11}^{mea}(s)_k$ are known which are the measured S_{21} and S_{11} values at the kth sampling frequency, m is the total number of the samplings; $s = \mathrm{j}\Omega$ is the complex domain; Ω is the normalized frequency for the low-pass prototype filter. The normalized frequency Ω has the relationship with the band-pass frequency f:

$$\Omega = \frac{f_0}{BW}\left(\frac{f}{f_0} - \frac{f_0}{f}\right) \tag{4.11.8}$$

where BW is the bandwidth of the band-pass filter; $f_0$is the resonant frequency. Then

$$s = \mathrm{j}\Omega = \mathrm{j}\,\frac{f_0}{BW}\left(\frac{f}{f_0} - \frac{f_0}{f}\right) \tag{4.11.9}$$

$S_{21}^{ext}(s)_k$ and $S_{11}^{ext}(s)_k$ are the extracted transmission function and reflection function values at the kth sampling frequency, where

$$S_{21}^{ext}(s)_k = \frac{P(s)}{E(s)}_k, \quad S_{11}^{ext}(s)_k = \frac{F(s)}{E(s)}_k \tag{4.11.10}$$

and

$E(s) = s^2 + (t_1t_4 + t_2t_5 + t_2t_3 + t_1)s + \big(t_1t_2(t_3t_4 + t_4t_5 + t_3 + t_5 + 1)\big);$

$F(s) = s^2 + (t_1t_4 + t_2t_5 + t_2t_3 - t_1)s + (t_1t_2(t_3t_4 + t_4t_5 - t_3 - t_5 + 1));$

$P(s) = 2t_1t_2\sqrt{t_3}.$

(4.11.11)

Constraints:

1) t_1, t_2, t_3, t_4, t_5 are all purely real, and $t_1, t_2, t_3, t_4, t_5 > 0;$

2) $0 < t_1 \cdot t_4 < 1, 0 < t_2 \cdot t_5 < 1;$

3) Denote $\hat{E}(s_{(i)}) = s_{(i)}{}^2 + (t_2t_3 + t_1)s_{(i)} + t_1t_2(t_3 + 1)$, $\hat{F}(s_{(i)}) = s_{(i)}{}^2 + (t_2t_3 - t_1)s_{(i)} + t_1t_2(-t_3 + 1)$, and $\hat{P}(s_{(i)}) = 2t_1t_2\sqrt{t_3}$,

then

$$\hat{P}(s_{(i)})\hat{P}^*(-s_{(i)}) + \hat{F}(s_{(i)})\hat{F}^*(-s_{(i)}) = \hat{E}(s_{(i)})\hat{E}^*(-s_{(i)})$$

where (*) means complex conjugation.

$$\mathrm{P}(s_{(i)})P^*(-s_{(i)}) + F(s_{(i)})F^*(-s_{(i)}) < E(s_{(i)})E^*(-s_{(i)}) \quad (4.11.12)$$

where (*) means complex conjugation, $E(s), F(s)$ & $P(s)$ are defined in (4.11.11)

4.4.3 Analysis of the optimization problem

Here are some notes for the constraints:

For constraint 1, since $t_1 = \frac{1}{L},\ t_2 = \frac{1}{C}, t_3 = \frac{1}{R}, t_4 = r_1, t_5 = \frac{1}{r_2}$ are all related to physical components, these parameters must be positive and real numbers.

For constraint 2, there are $t_1 \cdot t_4 = \frac{1}{L} \cdot r_1 = \frac{f_0}{BW}\frac{1}{Q_1}$, and $t_2 \cdot t_5 = \frac{1}{C} \cdot \frac{1}{r_2} = \frac{f_0}{BW}\frac{1}{Q_2}$, where Q_1, Q_2 are the unloaded quality factors for each resonators; BW is the bandwidth of the band-pass filter; f_0 is the resonant frequency. Meanwhile, the quality factors are usually larger than several tens, even several thousands, and $\frac{BW}{f_0}$ is usually than 30%. Then $t_1 \cdot t_4$ and $t_2 \cdot t_5$ are smaller than 1, in the reality.

For constraint 3, $\hat{E}(s_{(i)})$, $\hat{F}(s_{(i)})$ and $\hat{P}(s_{(i)})$ are the characteristic polynomials without the parameters $t_4 = r_1$ & $t_5 = \frac{1}{r_2}$, which represent the losses. In other words, these are the lossless filter's polynomials. According to the law of energy conservation, the transfer function $\widehat{S_{21}}(s)$ and the reflection function $\widehat{S_{11}}(s)$ have the relationship as:

$$\left|\widehat{S_{11}}(s)\right|^2 + \left|\widehat{S_{21}}(s)\right|^2 = 1$$

then
$$\left|\frac{\hat{F}(s)}{\hat{E}(s)}\right|^2 + \left|\frac{\hat{P}(s)}{\hat{E}(s)}\right|^2 = 1$$

So that $\hat{E}(s_{(i)})$, $\hat{F}(s_{(i)})$ and $\hat{P}(s_{(i)})$ must fulfill the condition:

$$\left|\hat{F}(s_{(i)})\right|^2 + \left|\hat{P}(s_{(i)})\right|^2 = \left|\hat{E}(s_{(i)})\right|^2$$

For constraint 4, $E(s), F(s)$ & $P(s)$ are the characteristic polynomials of the lossy filter. For a lossy filter, the transfer function $S_{21}(s)$ and the reflection function $S_{11}(s)$ have the relationship as:

$$|S_{11}(s)|^2 + |S_{21}(s)|^2 < 1$$

then
$$\left|\frac{F(s)}{E(s)}\right|^2 + \left|\frac{P(s)}{E(s)}\right|^2 = 1$$

so that $E(s), F(s)$ & $P(s)$ must fulfill that:

$$\left|F\left(s_{(i)}\right)\right|^2 + \left|P\left(s_{(i)}\right)\right|^2 < \left|E\left(s_{(i)}\right)\right|^2$$

It can be seen that if the parameters to be approximated, t_1, t_2, t_3, t_4, t_5, are directly used in the cost function, the terms of the polynomials will be too complex and very hard to be operated. To simplify the analysis, notice that the coefficients of $F(s), E(s)$ and $P(s)$ shown in (4.11.5), $F_2, F_1, F_0, E_2, E_1, E_0, P_0$, can directly determine the cost function. Furthermore, if these coefficients are fixed, the parameters t_1, t_2, t_3, t_4, t_5 are also determined.

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4.4.4 Parameter Estimation Examples

From the problem formulation in Section 4.4.2, the estimation problem is a typical non-linear least square problem. It can be seen that the cost function in Equation (4.11.7) is related to division and it is not easy to apply the estimation algorithm. Then the cost function can be rearranged as:

$$Fun = \sum_{k=1}^{m} |P(s)_k - E(s) \cdot S_{21}^{mea}(s)_k|^2 + |F(s)_k - E(s) \cdot S_{11}^{mea}(s)_k|^2 \tag{4.12}$$

Then the estimation procedure will be simplified.

Here the Levenberg–Marquardt algorithm is applied in Matlab to estimate the parameters. A 2-order BP Chebyshev filter with the different quality factors $Q_1 = 35$, $Q_2 = 701$ is illustrated to test the method. Then the estimation parameter vector is:

PV = $[0.7130, 1.4142, 0.5039, 0.0200, 0.2016]$

Then the Q factors of this filter are: $Q_1 = \frac{f_0}{BW} \cdot \frac{1}{t_1 \cdot t_4} = 702.3$, and $Q_2 = \frac{f_0}{BW} \frac{1}{t_2 \cdot t_5} =$ 35.07. The extracted characteristic polynomials are:

$$F(s) = 1 \cdot s^2 + 0.2988 \cdot s + 0.3113$$

$$E(s) = 1 \cdot s^2 + 1.7244 \cdot s + 1.7331$$

$$P(s) = 1.4314 \qquad (4.13)$$

The extracted curves are shown in Fig.4.5:

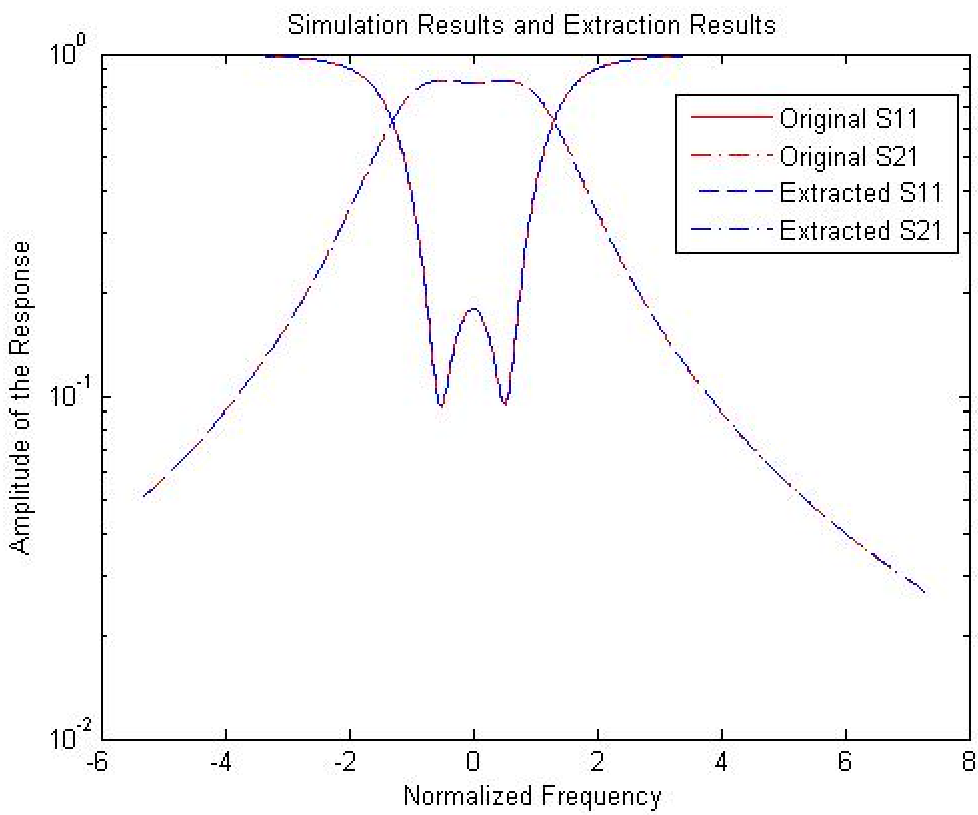

Fig 4.5 Simulated and extracted results of a 2-order BP lossy filter, using the new method

The red curves are the original S-parameters and the blue curves are the extracted S-parameters. The curves are matching very well.

The results show that this method can accurately extract the parameters of the lossy filter, not only the characteristic polynomials, but also the Q factors.

4.5 Conclusion

In this chapter, an enhanced Cauchy method and two optimization models are presented and discussed. All these three methods have good performance in the curves fitting. But the new Cauchy method proposed in Section 4.2 is not suitable for high order systems due to the large number of the coefficients to be estimated. And it can not quickly extract the quality factors. The optimization model proposed in Section 4.3 reduces the number of the coefficients, but it can not extract the Q factors. Another optimization model for the 2-order filter based on analyzing the prototype schematic is then proposed in Section 4.4. This model is able to extract the coefficients and the quality factors with good speed and accuracy. More studies are needed to understand how to generalize the results to higher order circuits.

CHAPTER 5

CONCLUSION AND FUTURE WORK

In this paper, the methods to extract the parameters of the filter have been discussed. Cauchy method and a two-stage optimization method have been introduced and tested using several different case studies. The results reveal the disadvantage of this this method that Cauchy method is not suitable for filters with un-even quality factors. To accurately extract coefficients and loss information of a filter, an enhanced Cauchy method and a model based on the prototype structure has been proposed. The enhanced Cauchy method has a good performance on producing the accurate extraction results but does not quickly extract the quality factors. It is also not suitable for high order filters. Then a new optimization model which can indicate the relationship of the parameters is posed. The example of the low order case shows the efficiency and accuracy of this method. However, more studies are needed to generalize the results to higher order filters.

In the future work, a more general formulation of filters is required. From the exploratory studies conducted in this thesis, further relationship of the parameters should be revealed and included in the new model. Meanwhile, more efficient and suitable optimization algorithms should be applied to obtain a high-performance extraction.

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