Flexible Allocation of Capacity in Multi-Cell CDMA Networks

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Outline

- **Capacity and Probability of Outage**
- **Calculation of inter-cell interference**
- **Capacity region**
- **Power compensation factor**
- **Effects of different cell sizes**
- **Optimization of capacity**
- **Flexible allocation of capacity**
- **Results**

Nine Cell Network

Probability of Outage For Each Cell

Probability of Outage for Single Cell

$$P_{\text{out}}^i = \Pr\left(\sum_{l=2}^{n_i} v_l > \frac{(W / R)(1 - \eta) \triangleq K'_o}{E_b / I_o}\right)$$

$$\frac{I_o}{N_o} < \frac{1}{\eta} \quad \text{where } \eta < 1,$$

$\Pr(v_l = 1) = \alpha$, the voice activity factor,

n_i is the number of users in cell i .

Probability of Outage for Single Cell

$$P_{\text{out}}^i = \Pr\left(\sum_{l=2}^{n_i} v_l > K'_o\right) < \Pr\left(\sum_{l=1}^{n_i} v_l > K'_o\right)$$

$$\text{Let } Z_i = \sum_{l=1}^{n_i} v_l$$

$$\text{where } \Pr(n_i = k) = \frac{(\lambda_i / \mu_i)^k}{k!} e^{-\lambda_i / \mu_i}, \quad k = 0, 1, 2, \dots$$

$$P_{\text{out}}^i < \Pr(Z_i > K'_o) = e^{-\alpha \lambda_i / \mu_i} \sum_{k=\lceil K'_o \rceil}^{\infty} (\alpha \lambda_i \mu_i)^k / k!$$

Gaussian Approximation

- We approximate the Poisson by a Gaussian variable with the same mean and variance:

$$P_{\text{out}}^i \approx Q\left(\frac{K_o' - E(Z_i)}{\sqrt{\text{var}(Z_i)}}\right) = Q\left(\frac{K_o' - \alpha \lambda_i / \mu_i}{\sqrt{\alpha \lambda_i / \mu_i}}\right)$$

$$Q(y) = \int_y^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

Probability of Outage for Multiple Cells

- I_{ji} : Inter-cell interference from cell j to cell i .

$$P_{\text{out}}^i = \Pr \left(\sum_{l=1}^{n_i + \sum_{j=1}^M I_{ji}} v_l > \frac{(W/R)(1-\eta)}{E_b/I_o} \right)$$

M is the total number of cells.

Shadow and Rayleigh Fading

- **Assume power control overcomes both large scale path loss and shadow fading, but not Rayleigh fading.**
- **The average of the Rayleigh fading is the shadow fading on that path.**

Inter-Cell Interference

$$I_{ji} = E \iint_{\text{cell } j} \frac{r_j^m(x,y) 10^{\varsigma_j/10}}{r_i^m(x,y)/X_i^2} \rho_j dA(x,y)$$

m is the path loss exponent.

ς_i is the decibel attenuation due to shadowing, and has zero mean and standard deviation σ_s .

$$E[X_i^2/\varsigma_i] = 10^{-\varsigma_i/10}$$

$$\rho_j = \frac{n_j}{\text{Area of cell } j}$$

Soft Handoff

- User is permitted to be in soft handoff to its two nearest cells.

Soft Handoff

$$I_{ji} = \iint_{\text{region(a)}} \frac{r_j^m}{r_i^m} E \left[10^{\varsigma_j/10} X_i^2 / r_j^m 10^{\varsigma_j/10} < r_i^m 10^{\varsigma_i/10} \right] \rho dA(x,y)$$

$$I_{ki} = \iint_{\text{region(b)}} \frac{r_k^m}{r_i^m} E \left[10^{\varsigma_k/10} X_i^2 / r_k^m 10^{\varsigma_k/10} < r_i^m 10^{\varsigma_i/10} \right] \rho dA(x,y)$$

$$I_{ji} = \iint_{\text{region(c)}} \frac{r_j^m}{r_i^m} E \left[10^{\varsigma_j/10} X_i^2 / r_j^m 10^{\varsigma_j/10} < r_k^m 10^{\varsigma_k/10} \right] \rho dA(x,y)$$

$$I_{ki} = \iint_{\text{region(c)}} \frac{r_k^m}{r_i^m} E \left[10^{\varsigma_k/10} X_i^2 / r_k^m 10^{\varsigma_k/10} < r_j^m 10^{\varsigma_j/10} \right] \rho dA(x,y)$$

Inter-Cell Interference Factor

- K_{ji} : per user inter-cell interference factor from cell j to cell i
- n_j users in cell j produce an amount of interference in cell i equal to $n_j K_{ji}$
- K_{ji} are not necessarily zero because of soft handoff, users in cell i can cause inter-cell interference to cell i .

Capacity Region

$$\frac{E_b/N_o}{\alpha(E_b/N_o) \left(n_i - 1 + \sum_{j=1}^M n_j \kappa_{ji} \right) / (W/R) + 1} \geq \left(\frac{E_b}{I_o} \right)_{\text{req}}$$

for $i = 1, \dots, M$.

$$n_i + \sum_{j=1}^M n_j \kappa_{ji} \leq \frac{W/R}{\alpha} \left(\frac{1}{E_b/I_o} - \frac{1}{E_b/N_o} \right) + 1 \stackrel{\Delta}{=} c_{\text{eff}}$$

for $i = 1, \dots, M$.

Capacity Region

- **Capacity region: set of all feasible user configurations**
- **Uniform capacity: $n_i = c_1$ for all i**
- **Two level capacity: m_1 cells have capacity c_1 and m_2 cells have capacity c_2 where $m_1 + m_2 = M$**

Example: 2 Cell Network

K_{ij}	BS-1	BS-2
BS-1	0	4.0130
BS-2	0.0617	0

Capacity Region

Power Compensation Factor

- **Fine tune the nominal power of the users**
- **PCF defined for each cell**
- **PCF is a design tool to maximize the capacity of the entire network**

Power Compensation Factor

- Interference is linear in PCF

$$I_{ji} = \iint_{\text{cell } j} \frac{\beta_j r_j^m 10^{\varsigma_j/10}}{r_i^m / X_i^2} \rho_j dA$$

$$n_i + \sum_{j=1}^M n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{eff} \quad \text{for } i = 1, \dots, M.$$

Sensitivity Analysis

- **Derivative of capacity function with respect to the PCF**
- **Capture effect of increases in PCF in one cell on the capacity of whole network**
- **Tool to flexibly distribute capacity between cells**

Sensitivity Analysis

- **For uniform capacity case:**

$$c_1 = \min_i \left[\frac{c_{eff}}{1 + \frac{1}{\beta_i} \sum_{j=1}^M \beta_j \kappa_{ji}} \right] \text{ for } i = 1, \dots, M.$$

Let i^* be the index that minimizes the above equation.

The derivative $\frac{dc_1}{d\beta_k}$ can be calculated .

Sensitivity Analysis

$$\frac{dc_1}{d\beta_k} = \frac{-c_{eff} \beta_{i^*} \kappa_{ki^*}}{\left(\beta_{i^*} + \sum_{j=1}^M \beta_j \kappa_{ji^*} \right)^2} \quad \text{if } k \neq i^*$$

$$\frac{dc_1}{d\beta_k} = \frac{c_{eff} \sum_{j=1}^M \beta_j \kappa_{jk} - c_{eff} \beta_k \kappa_{kk}}{\left(\beta_k + \sum_{j=1}^M \beta_j \kappa_{jk} \right)^2} \quad \text{if } k = i^*$$

Optimization

- **Optimize the sum capacity: the sum of the capacities of the cells**
- **Constraint: PCF between a min and a max**
- **Use the derivatives in steepest descent algorithm**
- **New PCF is the factor that the nominal power needs to be increased by for every cell**
- **Each PCF is used by its Base Station in the Closed Loop Power Control**

Optimization of Uniform Capacity

$$\begin{aligned} & \max_{\bar{\beta}, c_1} && Mc_1, \quad (\text{sum capacity}) \\ \text{subject to} &&& 1 \leq \bar{\beta} \leq \bar{\beta}^{\max}, \\ &&& c_1 + \sum_{j=1}^M c_1 \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{\text{eff}}, \\ &&& \text{for } i = 1, \dots, M. \end{aligned}$$

Optimization of Two-Level Capacity

$$\begin{aligned} & \max_{\bar{\beta}, c_1, c_2} && m_1 c_1 + m_2 c_2, \quad (\text{sum capacity}) \\ \text{subject to} &&& 1 \leq \bar{\beta} \leq \bar{\beta}^{\max}, \\ &&& n_i + \sum_{j=1}^M n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{\text{eff}}, \quad \text{for } i = 1, \dots, M, \\ &&& n_i = c_1 \quad \text{for } i = 1, \dots, m_1, \\ &&& n_i = c_2 \quad \text{for } i = m_1 + 1, \dots, m_1 + m_2, \\ &&& m_1 + m_2 = M. \end{aligned}$$

Capacity Region

Hard Handoff vs. Soft Handoff

HH	BS-1	BS-2	BS-3	Cap	Opt	PCF
BS-1	0	0.330	3.573	4	17	1.00
BS-2	0.331	0	3.420	4	17	1.00
BS-3	0.076	0.072	0	4	17	4.53

SH	BS-1	BS-2	BS-3	Cap	Opt	PCF
BS-1	0.106	0.116	0.603	17	26	1.00
BS-2	0.117	0.099	0.591	17	26	1.00
BS-3	0.066	0.063	0.006	17	26	2.56

Seven Cell Network

Probability of Outage For Each Cell

Optimization

- **Uniform capacity**
 - **C1: 11 (PCF=1) to 20 (PCF Optimized)**
 - **Sum capacity: 77 to 140**
- **Two-level capacity**
 - **Small cells: 14 (PCF=1) to 22 (PCF Optimized)**
 - **Large cells: 7 (PCF=1) to 11 (PCF Optimized)**
 - **Sum capacity: 77 to 121**

Flexibility in Capacity Allocation

- **Case 1:**
$$\max_{\bar{\beta}, n_i} \sum_{i=1}^M n_i, \quad (\text{sum capacity})$$

subject to

$$1 \leq \bar{\beta} \leq \bar{\beta}^{\max},$$
$$n_i + \sum_{j=1}^M n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{\text{eff}},$$
$$n_i \geq 20,$$

for $i = 1, \dots, M$.

- **Result: sum capacity of 150.**

Flexibility in Capacity Allocation

- **Case 2:**
$$\max_{\bar{\beta}, n_i} \sum_{i=1}^M n_i, \quad (\text{sum capacity})$$

subject to

$$1 \leq \bar{\beta} \leq \bar{\beta}^{\max},$$
$$n_i + \sum_{j=1}^M n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{eff},$$

for $i = 1, \dots, M$.

- **Result: sum capacity of 152 but capacity of cell three drops from 20 to 13.**

Flexibility in Capacity Allocation

- **Case 3:**
$$\max_{\bar{\beta}, n_i} \sum_{i=1}^M n_i, \quad (\text{sum capacity})$$

subject to

$$1 \leq \bar{\beta} \leq \bar{\beta}^{\max},$$
$$n_i + \sum_{j=1}^M n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{\text{eff}},$$

for $i = 1, \dots, M,$

$$n_3 \geq 25.$$

- **Result: cell three has a capacity of 25 but sum capacity drops to 143.**

PCF of Cell i Capacity of Cell i

30
25
20
15
10
5
0
2
4
6

1

2

3

4

5

6

7

Cell i

- Case 1
- Case 2
- Case 3

Conclusions

- **Power Compensation Factor is a design tool to:**
 - **Flexibly distribute the capacity allocation between cells**
 - **Optimize the capacity of a network**
 - **Maximize the capacity of a single cell**
 - **Easy to implement in an existing network**
 - **Does not require the relocation or addition of Base Stations.**

Future Work

- **Pilot Power determines the cell geometry which determines the inter-cell and intra-cell interference**
- **Determine the sensitivity of capacity with respect to Pilot Power**
- **Determine the sensitivity of capacity with respect to Base Station location**
- **Complete Design: Location, forward power, reverse power.**