Effects of Call Arrival Rate and Mobility on Network Throughput in Multi-Cell CDMA

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Outline

- **CDMA Capacity and Inter-cell Effects**
- **Feasible Region**
- **Mobility Model and Blocking**
- **Net Revenue**
- **Implied Costs**
- **Effects of Mobility**
- **Maximization of Subscribers**
- **Maximization of Revenue**
- **Shaping Feasible Region**

Probability of Outage

- **Single Cell**

$$P_{\text{out}}^i = \Pr \left(\sum_{l=2}^{n_i} v_l > \frac{(W/R)(1-\eta)}{E_b/I_o} \triangleq K_o \right)$$

$$\frac{I_o}{N_o} < \frac{1}{\eta} \quad \text{where } \eta < 1, \quad n_i : \text{users in cell } i,$$

$\Pr(v_l = 1) = \alpha$, the voice activity factor.

- **Multiple Cells**

$$P_{\text{out}}^i = \Pr \left(\sum_{l=1}^{n_i + \sum_{j=1}^M I_{ji}} v_l > \frac{(W/R)(1-\eta)}{E_b/I_o} \right)$$

M : total number of cells.

Probability of Outage

$$P_{\text{out}}^i = \Pr\left(\sum_{l=2}^{n_i} v_l > K_o\right) < \Pr\left(\sum_{l=1}^{n_i} v_l > K_o\right)$$

$$\text{Let } Z_i = \sum_{l=1}^{n_i} v_l$$

$$\text{where } \Pr(n_i = k) = \frac{(\lambda_i / \mu_i)^k}{k!} e^{-\lambda_i / \mu_i}, \quad k = 0, 1, 2, \dots$$

$$P_{\text{out}}^i < \Pr(Z_i > K_o) = e^{-\alpha \lambda_i / \mu_i} \sum_{k=\lceil K_o \rceil}^{\infty} (\alpha \lambda_i / \mu_i)^k / k!$$

Gaussian Approximation

We approximate the Poisson by a Gaussian variable with the same mean and variance:

$$P_{\text{out}}^i \approx Q\left(\frac{K_o - E(Z_i)}{\sqrt{\text{var}(Z_i)}}\right) = Q\left(\frac{K_o - \alpha \lambda_i / \mu_i}{\sqrt{\alpha \lambda_i / \mu_i}}\right)$$

$$Q(y) = \int_y^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

Probability of Outage for Multiple Cells

I_{ji} : Inter-cell interference from cell j to cell i .

$$P_{\text{out}}^i = \Pr \left(\sum_{l=1}^{n_i + \sum_{j=1}^M I_{ji}} v_l > \frac{(W/R)(1-\eta)}{E_b/I_o} \right)$$

M is the total number of cells.

Nine Cell Network

Inter-Cell Interference

$$I_{ji} = E \iint_{\text{cell } j} \frac{r_j^m(x,y) 10^{\varsigma_j/10}}{r_i^m(x,y)/X_i^2} \rho_j dA(x,y)$$

m is the path loss exponent.

ς_i is the decibel attenuation due to shadowing, and has zero mean and standard deviation σ_s .

$$E[X_i^2/\varsigma_i] = 10^{-\varsigma_i/10}$$

$$\rho_j = \frac{n_j}{\text{Area of cell } j}$$

Soft Handoff

User is permitted to be in soft handoff to its two nearest cells.

Soft Handoff

$$I_{ji} = \iint_{\text{region(a)}} \frac{r_j^m}{r_i^m} E \left[10^{\varsigma_j/10} X_i^2 / r_j^m 10^{\varsigma_j/10} < r_i^m 10^{\varsigma_i/10} \right] \rho dA(x,y)$$

$$I_{ki} = \iint_{\text{region(b)}} \frac{r_k^m}{r_i^m} E \left[10^{\varsigma_k/10} X_i^2 / r_k^m 10^{\varsigma_k/10} < r_i^m 10^{\varsigma_i/10} \right] \rho dA(x,y)$$

$$I_{ji} = \iint_{\text{region(c)}} \frac{r_j^m}{r_i^m} E \left[10^{\varsigma_j/10} X_i^2 / r_j^m 10^{\varsigma_j/10} < r_k^m 10^{\varsigma_k/10} \right] \rho dA(x,y)$$

$$I_{ki} = \iint_{\text{region(c)}} \frac{r_k^m}{r_i^m} E \left[10^{\varsigma_k/10} X_i^2 / r_k^m 10^{\varsigma_k/10} < r_j^m 10^{\varsigma_j/10} \right] \rho dA(x,y)$$

Inter-Cell Interference Factor

- K_{ji} : per user inter-cell interference factor from cell j to cell i
- n_j users in cell j produce an amount of interference in cell i equal to $n_j K_{ji}$

Feasible Region

$$\frac{E_b/N_o}{\alpha(E_b/N_o) \left(n_i - 1 + \sum_{j=1}^M n_j \kappa_{ji} \right) / (W/R) + 1} \geq \left(\frac{E_b}{I_o} \right)_{\text{req}}$$

for $i = 1, \dots, M$.

$$n_i + \sum_{j=1}^M n_j \kappa_{ji} \leq \frac{W/R}{\alpha} \left(\frac{1}{E_b/I_o} - \frac{1}{E_b/N_o} \right) + 1 \stackrel{\Delta}{=} c_{\text{eff}}$$

for $i = 1, \dots, M$.

Total Offered Traffic

- **New call arrival process to cell i is Poisson.**
- **Total offered traffic to cell i is:**

$$\rho_i = \lambda_i + \sum_{j \in A_i} v_{ji}$$

where λ_i is the mean of the Poisson Process,

v_{ji} is the handoff rate from cell j to cell i ,

A_i is the set of cells adjacent to cell i .

Handoff Rate

$$\begin{aligned} v_{ji} &= \lambda_j (1 - B_j) q_{ji} + (1 - B_j) q_{ji} \sum_{x \in A_j} v_{xj} \\ &= (1 - B_j) q_{ji} \rho_j \end{aligned}$$

where B_j is the Blocking probability for cell j ,
 q_{ji} is the probability that a call in progress
in cell j , after completing its dwell time,
goes to cell i .

Probability of State $\underline{n}$

$$p(\underline{n}) = \begin{cases} P_o \prod_{k=1}^M \frac{(\rho_k / \mu_k)^{n_k}}{n_k!} & \text{if } \underline{n} = (n_1, \dots, n_M) \in \text{Feasible Region} \\ 0 & \text{otherwise} \end{cases}$$

where M is the total number of cells,

ρ_k is the total offered traffic in cell k ,

$1/\mu_k$ is the mean dwell time in cell k ,

$$\mu_k = (1 - q_{kk})\mu,$$

P_o is a normalizing constant.

Fixed Point

- Given values of λ_i for $i = 1, \dots, M$
- Assume initial values for ρ_i for $i = 1, \dots, M$
- Calculate $p(\underline{\rho}, \underline{n})$
- Calculate B_i for $i = 1, \dots, M$
- Calculate v_{ij} for $i = 1, \dots, M$ and $j = 1, \dots, M$
- Calculate the new values for ρ_i for $i = 1, \dots, M$
and repeat

Net Revenue W

- **Revenue generated by accepting a new call**
- **Cost of a forced termination**

$$W = \sum_{j=1}^M \left\{ w_j \lambda_j (1 - B_j) - c_j (\rho_j - \lambda_j) B_j \right\}$$

Implied Costs

- Find derivative of W wrt new call arrival rate
- Capture knockon effect of increases in an arrival rate
- Maximize net revenue for a given blocking probability vector

Three Cell Network

	Low Mobility
q_{ii}	0.26
q_{ij}	0.02
	High Mobility
q_{ii}	0.00
q_{ij}	0.15

Blocking in Cell 3

Normalized Arrival Rate $\left(\frac{\rho_3}{(1-q_{33})\mu} \right)$

Net Revenue $W=1$ $C=2$

Net Revenue $W=1$ $C=10$

Optimization of Subscribers

- **Best Scenario:**

$$\begin{aligned} \max_{\underline{\lambda}} \quad & S = \sum_{i=1}^M \lambda_i, \\ \text{subject to} \quad & \underline{B} \leq \underline{\eta}, \\ & \underline{\lambda} \geq \underline{0}. \end{aligned}$$

$$\left(\text{Subscriber } s = \frac{S \text{ (erlangs)}}{0.025 \text{ (erlangs/u ser)}} \right)$$

Optimization of Subscribers

- **Worst Scenario:**

$$\begin{aligned} \max_{\underline{\lambda}} \quad & S = \sum_{i=1}^M \lambda_i, \\ \text{subject to} \quad & B(\alpha_1, \dots, \alpha_M) \leq \underline{\eta} \quad \forall \underline{\alpha} \ni \sum_{i=1}^M \alpha_i \leq S, \\ & \underline{\lambda} \geq \underline{0}. \end{aligned}$$

For all values of the arrival rates summing to S , the blocking probabilities do not exceed the threshold.

Maximized Subscribers

Optimization of Net Revenue

$$\begin{array}{ll} \max_{\underline{\lambda}} & W = \sum_{j=1}^M \{w_j \lambda_j (1 - B_j) - c_j (\rho_j - \lambda_j) B_j\} \\ \text{subject to} & \underline{B} \leq \underline{\eta}, \\ & \underline{\lambda} \geq \underline{0}. \end{array}$$

Maximized Net Revenue

$W = 1, C = 10$

Shaping Feasible Region

- **Increases capacity**
- **Fine tune the nominal power of the users through PCF**
- **PCF defined for each cell**
- **PCF is a design tool to maximize the capacity of the entire network**

Power Compensation Factor

- Interference is linear in PCF

$$I_{ji} = \iint_{\text{cell } j} \frac{\beta_j r_j^m 10^{\varsigma_j/10}}{r_i^m / X_i^2} \rho_j dA$$

$$n_i + \sum_{j=1}^M n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{eff} \quad \text{for } i = 1, \dots, M.$$

- Allows us to reshape the Feasible Region.

Equal Number of Users

For $n_1 = n_2 = \dots = n_M$:

$$n_1 = \min_i \left[\frac{c_{eff}}{1 + \frac{1}{\beta_i} \sum_{j=1}^M \beta_j \kappa_{ji}} \right] \text{ for } i = 1, \dots, M.$$

Let i^* be the index that minimizes the above equation.

The derivative $\frac{dn_1}{d\beta_k}$ can be calculated .

Implied Cost

$$\frac{dn_1}{d\beta_k} = \frac{-c_{eff} \beta_{i^*} \kappa_{ki^*}}{\left(\beta_{i^*} + \sum_{j=1}^M \beta_j \kappa_{ji^*} \right)^2} \quad \text{if } k \neq i^*$$

$$\frac{dn_1}{d\beta_k} = \frac{c_{eff} \sum_{j=1}^M \beta_j \kappa_{jk} - c_{eff} \beta_k \kappa_{kk}}{\left(\beta_k + \sum_{j=1}^M \beta_j \kappa_{jk} \right)^2} \quad \text{if } k = i^*$$

Shaping with PCF

$$\begin{aligned} & \max_{\underline{\beta}, n_1} && Mn_1, \\ \text{subject to} &&& 1 \leq \underline{\beta} \leq \underline{\beta}^{\max}, \\ &&& n_1 + \sum_{j=1}^M n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{eff}, \\ &&& \text{for } i = 1, \dots, M. \end{aligned}$$

Two Cell Network

Mobility

q_{ii}	0.04
q_{ij}	0.26

Feasible Region

Shaped Feasible Region

Maximized Subscribers

Maximized Net Revenue

$W = 1, C = 10$

Conclusions

- **For a given network topology we calculate:**
 - **Inter-cell and intra-cell interference and Feasible Region**
 - **Maximized net revenue**
 - **Maximized subscriber capacity**
 - **Effects of Mobility**
- **Shape feasible region to increase maximized net revenue and subscriber capacity**

Future Work

Design Problem: given a user demand distribution, what are the optimal location and coverage of Base Stations to:

- Maximize Net Revenue**
- Maximize Subscriber Capacity**