PRACTICAL ROBUST MIMO OFDM COMMUNICATION SYSTEM FOR
HIGH-SPEED MOBILE COMMUNICATION

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This thesis presents the design of a communication system (PRCS) which improves on all aspects of the current state of the art 4G communication system Long Term Evolution (LTE) including peak to average power ratio (PAPR), data reliability, spectral efficiency and complexity using the most recent state of the art research in the field combined with novel implementations. This research is relevant and important to the field of electrical and communication engineering because it provides benefits to consumers in the form of more reliable data with higher speeds as well as a reduced burden on hardware original equipment manufacturers (OEMs). The results presented herein show up to a 3 dB reduction in PAPR, less than $10^{-5}$ bit errors at 7.5 dB signal to noise ratio (SNR) using 4QAM, up to 3 times increased throughput in the uplink mode and 10 times reduced channel coding complexity.
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CHAPTER 1

INTRODUCTION

The event that motivated me to do this research was previous research I conducted on radio frequency power amplifier (RFPA) design during my undergraduate career. The research focus of the state of the art RFPA design at the time was in dynamic, power efficient designs that are highly linear with a variable peak efficiency depending on the behavior of the input signal. These requirements were to efficiently amplify the long term evolution (LTE) 4G signals from the base station which had very high peak to average power ratio (PAPR) greater than 11 dB. This causes regular linear amplifiers to either distort the signal at high power or be inefficient most of the time. I decided that research time would be better spent designing a practical communication system that avoids the problem of high PAPR, therefore saving original equipment manufacturers (OEMs) research and development (R&D) costs, reducing the cost of 4G deployment and related hardware. In addition to the PAPR concerns, LTE is susceptible to fast fading channels due to their pilot and resource allocation structure, causing high bit error rate (BER) and low throughput in certain situations. Lastly, LTE employs channel coding that provides good performance at the cost of high decoding complexity which, in a high data rate situation, can negatively impact peak speeds as well as increase design costs.

I started this research with the simple goals of designing a communication system that:

- Had better data reliability compared to 4G systems
- Had reduced PAPR compared to LTE
- Had reduced overall complexity

These goals seemed well within reach as the re-discovery of LDPC codes drove more research to low complexity coding implementations and the maturity of multiple input multiple output (MIMO) transmission schemes led to the development of robust detector methods as well as new spatial coding techniques. Lastly, the problem with high PAPR prompted research into
signal processing methods on the transmitter that reduced signal dynamic range without having to resort to single carrier transmission via frequency spreading.

The contributions of this research to the field of electrical and communication engineering consist of the novel system level implementation of state of the art signal processing techniques to increase the data reliability and reduce the dynamic range of the transmitted OFDM signal and the novel implementation of a soft output single tree search sphere decoder to provide ML decoding performance to an algebraic space time block code (STBC) with unequal antenna constellations using a look up table.

This thesis is organized as follows: in Chapter 2 the current state of the art in communication system design is discussed and analyzed, in Chapter 3 the problems with the current 4G communication systems are analysed in direct relation to the above research objectives, Chapter 4 details the experimental design and signal processing techniques used to solve the presented research problems with supporting theoretical and empirical evidence. Lastly, Chapter 5 concludes with a brief summary and conclusion of the results from Chapter 4 with the recommendations for future work on the topic.
2.1. Orthogonal Frequency Division Multiplexing (OFDM)

Orthogonal frequency division multiplexing (OFDM) is a multi-carrier spread spectrum technique currently used in all modern 4G communication systems [8] [10] because it provides protection against inter-symbol interference (ISI), allows for simple frequency domain equalization and provides scalable bandwidth based on the number of used subcarriers. These properties are desirable as it provides high data rate potential and resilience against multipath channels found in outdoor communication environments. However, because of the multi-carrier nature of OFDM it is sensitive frequency shift effects; specifically both Doppler shift from fast moving vehicles as well as oscillator mismatch between the receiver and transmitter. This means OFDM systems need precise synchronization to be used effectively.

The implementation of a general OFDM system can be seen in Figure 2.1 using multiple oscillators. In practice however, the discrete Fourier transform (DFT) and inverse DFT (IDFT) implemented using the fast Fourier transform (FFT) and inverse FFT (IFFT) algorithms allow for efficient creation of the orthogonal signals. In the DFT based OFDM system seen in Figure 2.2 an $N$-Point IFFT is taken from the transmitted symbols $\{X_l[k]\}_{n=0}^{N-1}$ to generate $\{x[n]\}_{n=0}^{N-1}$ the samples of the sum of N orthogonal subcarrier signals. Let $y[n]$ represent the received samples that correspond to $x[n]$ with additive noise $z[n]$ and channel $h[n]$ so that $y[n] = h[n] \ast x[n] + z[n]$. Taking the $N$-Point FFT of the received samples $\{y[n]\}_{n=0}^{N-1}$, the noisy version of the transmitted symbols $\{Y_l[k]\}_{n=0}^{N-1}$ is obtained at the receiver. The mathematical description of the OFDM transmitted samples is represented in [7] as

$$x[n] = \sum_{k=0}^{N-1} X[k] \exp(j2\pi nk/N)$$ (2.1)
the received samples with channel effects and noise as

\[ y[n] = \sum_{k=0}^{N-1} (H[k]X[k] + Z[k])\exp(j2\pi nk/N) \]  

(2.2)

and received symbols as

\[ Y[k] = \sum_{k=0}^{N-1} y[n]\exp(-j2\pi nk/N) \]  

(2.3)

Since all subcarriers of the OFDM signal are of finite duration \( T \), the spectrum of the signal is simply the sum of the frequency shifted sinc functions in the frequency domain as seen in Figure 2.3.

**Figure 2.1. General OFDM system**

**Figure 2.2. OFDM system using discrete fourier transform**

In the OFDM system, in order to protect against ISI caused by channel effects, techniques called cyclic prefix (CP) or zero padding (ZP) are used to extend the OFDM symbol time by either coping the last \( G \) samples of the time domain symbol to the front, or simply adding \( G \) zero samples, respectively. The channel effects can be understood by the
illustrative example in Figure 2.4 where two impulse responses with different delay lengths are shown along with their respective frequency response. Figure 2.5 shows the ISI effect over two consecutive OFDM symbols where the first symbol (dark line) is mixed up with the second symbol (light line) which incurs ISI. Therefore, the subcarriers are no longer orthogonal over the duration of the OFDM symbol. Now let $T_G$ denote the length of the CP in samples. Then, the extended OFDM symbols now have the durations of $T_{sym} = T_{sub} + T_G$. Figure 2.6 shows that if the length of the CP is set longer than or equal to the maximum delay of the channel the ISI effects (dotted line) on the next symbol are confined within the guard interval. Therefore, by taking the FFT for only duration $T_{sub}$ the received signal is unaffected by ISI and orthogonality is maintained.

Since each subcarrier component of an OFDM symbol has the spectrum of a shifted sinc function, it is clear the system will suffer large out-of-band power and incur adjacent channel interference (ACI) if not corrected. In order to avoid high complexity time domain

![Figure 2.3. Frequency spectrum of OFDM signal](image)
Figure 2.4. Top Left: Short channel impulse response, Top Right: Long channel impulse response Bottom Left: Flat frequency response, Bottom Right: Frequency selective response

Pulse shaping filters like the raised cosine (RC) filter, virtual carriers (VC) or null tones can be inserted at both ends of the transmission band before the IFFT operation. While these VC are effective at reducing ACI, they do reduce spectral efficiency by $\frac{N_{used}}{N}$.

Figure 2.5. ISI effects of a multipath channel on OFDM symbol
The performance advantage of the CP and ZP technique and the effects of ISI on bit error rate (BER) can be seen in Figure 2.7 compared to the theoretical bounds of both a frequency flat Rayleigh channel and a additive white Gaussian noise (AWGN) channel. The analytical BER expressions for any $M$-ary QAM signal for each respective channel are given as

\[ P_e = \frac{2(M - 1)}{M \log_2 M} Q\left(\sqrt{\frac{6E_b}{N_0} \cdot \log_2 M} \cdot \frac{\log_2 M}{M^2 - 1}\right) \]  

(2.4)

\[ P_e = \frac{M - 1}{M \log_2 M} \left(1 - \sqrt{\frac{3\gamma \log_2 M/(M^2 - 1)}{3\gamma \log_2 M/(M^2 - 1) + 1}}\right) \]  

(2.5)

2.2. Multiple Input Multiple Output (MIMO) Transmission

A multiple input multiple output (MIMO) transmission system is characterized by some $N$ and $M$ antennas at the transmitter and receiver with a complex scattering environment in between following a Rayleigh distribution. The illustrative example for an $N$ by $M$ system can be seen in Figure 2.8. The general MIMO system can be effectively modeled as

\[ y_j^{(t)} = \sqrt{\frac{E_x}{N_0 N_T}} \left[ h_{j1}^{(t)} h_{j2}^{(t)} \ldots h_{jN_T}^{(t)} \right] \left[ \begin{array}{c} x_1^{(t)} \\ x_2^{(t)} \\ \vdots \\ x_{N_T}^{(t)} \end{array} \right] + z_j^{(t)} \]  

(2.6)
Figure 2.7. BER performance for OFDM system with varied CP/ZF length (16QAM)

at the $j$th receive antenna during the $t$th symbol period where $t = 1, 2, \ldots, T$. $h_{ji}^{(t)}$ is the channel gain from the $i$th transmit antenna to the $j$th receive antenna over the $t$th symbol period where $i = 1, 2, \ldots, N_T$ and $j = 1, 2, \ldots, N_R$ and $z_j^{(t)}$ is complex Gaussian noise. $E_x$ is the average energy of each transmitted signal constrained by

$$\sum_{i=1}^{N_T} E \left\{ |x_i^{(t)}|^2 \right\} = 1, 2, \ldots, T$$

(2.7)

The linear MIMO system described in Equation 2.6 is useful in a communication system as it can be utilized to either increase data reliability or maximum data throughput either with space-time block codes (STBC) or the use of spatial multiplexing (SM).
2.2.1. Space-Time Coding

STBC are designed so that multiple copies of a data symbol are transmitted over different antennas at different time intervals so the collective set of received signals can be more easily distinguished. These codes can provide a reduction in BER through properties called diversity gain and coding gain. [16] Diversity gain is characterized by the increase in the slope of the error curve while coding gain is the amount of left shift in the curve.

One of the first and most well know STBC is the Alamouti code which is a specialized two transmit antenna complex orthogonal code described for two consecutive symbols $x_1$ and $x_2$ as

$$x_c = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (2.8)$$

The illustrative example in Figure 2.9 shows an Alamouti encoded signal transmitted over four symbol periods from two antennas. During the first period, $x_1$ and $x_2$ are transmitted simultaneously, while during the second these same symbols are transmitted again, except in the form $x_1^*$ and $-x_2^*$ and on different antennas. Periods 3 and 4 show the same procedure on symbols $x_3$ and $x_4$. 

\[ \text{Figure 2.8. Antenna configuration for a MIMO system} \]
The min rank criterion derived in [16] for orthogonal codes shows that for maximum likelihood (ML) based signal detection at the receiver, Alamouti codes are expected to provide a diversity gain of 2. This result also only holds true assuming the receiver has exact knowledge of the channel gains $h_{ji}^{(t)}$. After solving Equation 2.6 for two transmit and two receiver antennas over two symbol periods using the values from Equation 2.8 we can find the received symbols as

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \left( |h_1|^2 + |h_2|^2 \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{bmatrix} \quad (2.9)$$

In Equation 2.9 we can see that the other antenna symbol interference does not exist anymore, that is, the symbol $x_2$ dropped out of $y_1$ while $x_1$ also dropped out of $y_2$. This property is attributed to the orthogonality of the code in Equation 2.8. This unique feature allows for the simplified ML receiver structure

$$\hat{x}_{i,ML} = Q \left( \frac{\tilde{y}_i}{|h_1|^2 + |h_2|^2} \right), \quad i = 1, 2. \quad (2.10)$$

Where $Q(\cdot)$ denotes a slicing function that determines a transmitted symbol for a given constellation set. Since $x_1$ and $x_2$ can be guessed separately the overall complexity is reduced from $|C|^2$ to $2|C|$ where $C$ represents a constellation for symbols $x_i$. In addition, the scaling factor $|h_1|^2 + |h_2|^2$ warrants the second-order spatial diversity mentioned above.

Figure 2.10 shows a simulation of the error performance of the standard 2 by 2 case of the Alamouti code vs SISO transmission in a Rayleigh fading channel. As expected, the
BER curve slope is about twice that of the SISO transmission.

![BER curve vs E_{b}/N_{0}} for SISO and Alamouti 2x2 STBC with 16 QAM modulation](image)

**Figure 2.10.** Alamouti 2x2 STBC vs SISO Performance (16 QAM)

### 2.2.2. Spatial Multiplexing

A spatially multiplexed MIMO system (SM-MIMO) can transmit data at higher rates than MIMO systems using STBC or simple antenna diversity by transmitting different parallel data symbols at every time period $T$ instead of copies of the same symbols. As seen in Figure 2.11 however, effective decoding of the combined data streams needs to be done using a more complex signal detector. An effective detector used in current 4G communication systems is the MMSE detector which can be described using a weight matrix $W_{MMSE}$ to maximum signal-to-interference plus noise ration (SINR) as

$$W_{MMSE} = (H^H H + \sigma^2 I)^{-1} H^H$$

(2.11)
Using the MMSE weight above we obtain the following relationship

\[ \tilde{x}_{MMSE} = W_{MMSE} \cdot y \]  

(2.12)

**Figure 2.11.** Example of a SM-MIMO system using a 16 QAM constellation

In this section, however, the peak throughput capabilities of a SM-MIMO system will be introduced by looking at the maximum channel capacity of a MIMO system which can be increased by a factor of \( \min(N_T, N_R) \). The equation for the capacity of a deterministic channel is given as

\[ C = \max_{\text{Tr}(R_{xx}=N_T)} \log_2 \det \left( I_{N_R} + \frac{E_x}{N_T N_0} H R_{xx} H^H \right) \]  

(2.13)

Where \( R_{xx} = E\{x x^H\} \) or the autocorrelation of the transmitted signal vector, \( H \) is the \( N_R \times N_T \) deterministic channel matrix and \( I \) is the identity matrix. The capacity of a random ergodic process channel model is simply \( \bar{C} = E\{C(H)\} \) or the expected value. The theoretical capacities in a random channel of various antenna configurations vs SNR are given in Figure 2.12.
2.3. Turbo Codes

Turbo codes are capacity approaching codes constructed by concatenating 2 or more component recursive systematic convolutional codes (RSC) on different interleaved version of a data sequence. Capacity approaching means they approach the Shannon limit by providing up to $10^{-5}$ BER at an $E_b/N_0$ value of 0.7 dB in an AWGN channel given low code rates and large iterations. [12] Another key feature of turbo codes is the pair (or more) of soft-input soft-output serial decoders which pass decision information back and forth iteratively to produce more reliable error correction results.

A simple $1/2$ rate convolutional encoder with length $K$, memory $K - 1$, inputs $d_k$ and codeword bits $(u_k, v_k)$ is given as

$$u_k = \sum_{i=0}^{K-1} g_{1i}d_{k-i} \mod 2, \ g_{1i} = 0, 1$$  \hspace{1cm} (2.14)
\[ v_k = \sum_{i=0}^{K-1} g_{2i}d_{k-i} \mod 2, \ g_{2i} = 0, 1 \] (2.15)

Where \( G_1 = \{g_{1i}\} \) and \( G_2 = \{g_{2i}\} \) are code generators. An example using constraint length \( K = 3 \) gives generators as \( G_1 = \{1 1 1\} \) and \( G_2 = \{1 0 1\} \). To make a RSC encoder, the encoded information bits must be fed back into the encoder’s input and one of the code outputs is set equal to \( d_k \). This technique results in better error performance than simple nonsystematic codes. The new recursive encoder is described as

\[ a_k = d_k + \sum_{i=0}^{K-1} g_i' a_{k-i} \mod 2 \] (2.16)

To create a turbo encoder two RSC encoders are concatenated in parallel like in Figure 2.13 where a switch on \( v_k \) provides a puncturing sequence that makes the overall code rate 1/2, 3/4 or 4/5 in practice, instead of 1/3. The interleaver design is important to the overall code performance. [12]

![Turbo encoder block diagram of parallel RSC codes](image-url)

**Figure 2.13.** Turbo encoder block diagram of parallel RSC codes
The decoder is a set of serial fed decoders with feedback as seen in Figure 2.14. The input to the turbo decoder is the LLR per bit from a soft QAM demodulator of the data \( u_k = d_k \) and \( v_k \) at some time \( k \) expressed as \( L(x_k) \) and \( L(y_k) \) in the form

\[
L(x_k) = \log \left[ \frac{p(x_k|d_k = 1)}{p(x_k|d_k = 0)} \right] \quad \text{and} \quad L(y_k) = \log \left[ \frac{p(y_k|d_k = 1)}{p(y_k|d_k = 0)} \right]
\] (2.17)

The redundant information \( L(y_k) \) is de-multiplexed using a switch and sent to decoder 1 when \( v_k = v_{1k} \) from the punctured encoder or decoder 2 when \( v_k = v_{2k} \). The information processed by Decoder 1 is the non-interleaved output of Encoder 1 while the information processed by Decoder 2 is the output of Encoder 2 (which is simply the interleaved Encoder 1 data). Decoder 2 then makes use of Decoder 1 output by way of an inline interleaver so it is time ordered the same as the \( L(y_{2k}) \) input. Each decoder iteratively improves the LLRs over a set length of iterations before applying the maximum a posteriori (MAP) decision rule

\[
\hat{d}_k = 1 \quad \text{if} \quad L(\hat{d}_k) > 0 \] (2.18)

\[
\hat{d}_k = 0 \quad \text{if} \quad L(\hat{d}_k) < 0 \] (2.19)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2_14}
\caption{Turbo decoder block diagram of serial APP decoders with soft input}
\end{figure}

The performance of 5 state, 1048576 length codes of rate 1/2 in a AWGN channel are shown in Figure 2.15. The codes exhibit an error floor [3] which is a limit on the codes performance. This can be overcome with optimized puncturing structures.
2.4. Long Term Evolution (LTE)

Third Generation Partnership Project (3GPP) Long Term Evolution (LTE) is currently the most popular fourth generation (4G) wireless communication system [2] since it is built on 3GPP’s Universal Mobile Telecommunications System (UMTS) and High-speed Packet Access (HSPA) technology, the most popular 3G services before the roll out of 4G. Since LTE is backwards compatible with these WCDMA technologies, the carrier’s cost to upgrade is greatly reduced compared to other 4G systems. The LTE architecture eventually aims to provide an all IP service backbone for reduce cost per bit/call and better service provisioning and flexibility.

2.4.1. Physical Layer Structure

The LTE physical layer (PHY) is influenced by the requirements for high transmission rates, spectral efficiency and scalable channel bandwidths up to 20 MHz. It is uses state of
the art techniques like SM-MIMO and turbo codes to provide high throughput and error correction. The downlink mode uses the technique orthogonal frequency division multiple access (OFDMA) which is similar to OFDM scheme except multiple user data is spread across subsets of the total available subcarriers. To reduce PAPR the UL mode consists of single carrier OFDMA (SC-OFDMA) which is implemented as a discrete fourier transform spread over the subcarriers leading to a single carrier blocks within the OFDM spectrum. [10] This UL/DL structure can be seen in Figure 2.16.

![Figure 2.16. LTE Uplink and Downlink transmission schemes](image)

For channel estimation at the user equipment, pilot symbols are inserted into the OFDM time-frequency resource grid as seen in Figure 2.17 for the 2 by 2 antenna MIMO case. These 12 subcarrier 7 symbol blocks are the smallest assignable data elements in LTE. The pilots for the second antenna in the fourth and tenth subcarrier in the first symbol and the first and seventh in the fifth are left null on the first antenna’s transmission as to not interfere with the second antenna’s channel estimation and vice versa.
In the time domain, different time intervals within LTE are expressed as multiples of a basic time unit $T_s = 1/30720000$ or 32.55 ns which is the minimum sample time supported by WCDMA baseband chips. The radio frame has a length of 10 ms or $307200 \cdot T_s$. Each frame is divided into ten equally sized subframes of 1 ms or $30720 \cdot T_s$. Each subframe consists of two equally sized slots of 0.5 ms in length or $15360 \cdot T_s$. Finally, each slot consists of a number of OFDM symbols which can be either seven or six in length depending on CP length. Figure 2.18 shows the LTE frame structure in frequency division duplex (FDD) mode. The useful symbol time available for transmission is $66.7 \, \mu s$ or $2048 \cdot T_s$ in normal mode and the CP is $160 \cdot T_s$ or $5.2 \, \mu s$ for the first symbol and $144 \cdot T_s$ or $4.7 \, \mu s$ for the rest. This is done so that each slot of 0.5 ms is exactly divisible 15360.

In Table 2.1 we can see all the relevant parameters in LTE for the different channel bandwidth modes. Since the FFT size scales with bandwidth LTE can be considered a scalable OFDMA system (SOFDMA) on the downlink. LTE uses a large number of null guard subcarriers to protect against ACI.
Figure 2.18. LTE radio frame structure

<table>
<thead>
<tr>
<th>Channel Bandwidth (MHz)</th>
<th>1.25</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame Duration (ms)</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subframe Duration (ms)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subcarrier Duration (ms)</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sampling Frequency (MHz)</td>
<td>1.92</td>
<td>3.84</td>
<td>7.68</td>
<td>15.36</td>
<td>23.04</td>
<td>30.72</td>
</tr>
<tr>
<td>FFT Size</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>1536</td>
<td>2048</td>
</tr>
<tr>
<td>Occupied Subcarriers</td>
<td>76</td>
<td>151</td>
<td>301</td>
<td>601</td>
<td>901</td>
<td>1201</td>
</tr>
<tr>
<td>Guard Subcarriers</td>
<td>52</td>
<td>105</td>
<td>211</td>
<td>423</td>
<td>635</td>
<td>847</td>
</tr>
<tr>
<td>Number of Resource Blocks</td>
<td>6</td>
<td>15</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>Occupied Channel Bandwidth (MHz)</td>
<td>1.140</td>
<td>2.715</td>
<td>4.515</td>
<td>9.015</td>
<td>13.515</td>
<td>18.015</td>
</tr>
<tr>
<td>DL Bandwidth Efficiency</td>
<td>77.1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OFDM Symbols/Subframe</td>
<td>7/6 (short/long CP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP Length (Short CP) (µs)</td>
<td>5.2 (first) / 4.69 (rest)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP Length (Long CP) (µs)</td>
<td>16.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1. LTE downlink physical parameters

2.5. Mobile WiMAX (IEEE 802.16e)

The WiMAX IEEE 802.16 standard comes from the IEEE 802 family of network protocols and aims to extend the use of wireless access from local area networks (LAN) which are typically based on 802.11 to larger metropolitan area networks (MAN) and wide area networks (WAN). It uses SOFDMA physical layer structure with MIMO in both the uplink and downlink unlike LTE which uses SC-OFDMA in the uplink. Mobile WiMAX uses an all IP backbone from the first iteration allowing for greatly reduced price per bit/call compared
to third generation (3G) WCDMA systems. However, because it extends on 802 protocols the hardware is not backwards compatible with older 3G thus greatly increasing upgrade cost compared to LTE. [2] In addition, peak data rates are lower in WiMAX compared to LTE because of less sophisticated channel coding called Reed-Solomon Codes. [11]

2.5.1. Physical Layer Structure

In WiMAX the OFDM symbol has a variable CP of a certain duration Guard Time (GT) denoted as $T_G$. The ratio $T_G/T_d$ where $T_d$ is useful symbol time is denoted as $G$ in 802.16 documentation. The choice of $G$ is made relative to channel conditions and allows for WiMAX to optimize its temporal efficiency at the cost of increased complexity. $G$ can be one of 4 values: 1/4, 1/8, 1/16, 1/32. It is also up to the user equipment to search for the correct $G$ length and synchronize with the base station. The overall time structure of a WiMAX OFDM symbol is shown in Figure 2.19.

![Figure 2.19. WiMAX OFDM time domain symbol with CP](image)

The subcarrier allocation in WiMAX is divided into groups of subcarriers called subchannels. These subchannels are present in both the UL and DL and may or may not be adjacent in the OFDM spectrum. The subchannel size varies and is dependent on the channel bandwidth used. It can be seen in Figure 2.20 that subchannels have a time length of only one OFDM symbol, unlike LTE which uses a time-frequency grid of up to 7 symbol periods for its data allocation.

20
The pilot symbols, like the subchannel allocations, are done only in the frequency domain meaning that each OFDM symbol has their own pilot symbols. Unlike LTE where each pilot symbol has a fixed position in the resource block, WiMAX dynamically distributes pilots inside subchannels along with data based on certain permutation methods for better performance at the cost of increased complexity. Frequency domain pilot position for the WiMAX OFDM only mode can be seen in Figure 2.21.

In Table 2.2 we can see all the relevant parameters in WiMAX for the different channel
bandwidth modes. The 3 Mhz bandwidth is not used for OFDMA operation and is reserved for stationary OFDM operation only. The guard subcarrier ratio to FFT size and subcarrier spacing is smaller than LTE, meaning WiMAX has increased spectral efficiency at the cost of potentially more ACI in very noisy environments as well as increased sensitivity to Doppler frequency shift.

<table>
<thead>
<tr>
<th>Channel Bandwidth (MHz)</th>
<th>1.25</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Frequency (MHz)</td>
<td>1.4</td>
<td>2.8</td>
<td>5.6</td>
<td>11.2</td>
<td>22.4</td>
</tr>
<tr>
<td>FFT Size</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>2048</td>
</tr>
<tr>
<td>Number of used data subcarriers</td>
<td>72</td>
<td>200</td>
<td>360</td>
<td>720</td>
<td>1,440</td>
</tr>
<tr>
<td>Number of pilot subcarriers</td>
<td>12</td>
<td>8</td>
<td>60</td>
<td>120</td>
<td>240</td>
</tr>
<tr>
<td>Number of null guard band subcarriers</td>
<td>44</td>
<td>56</td>
<td>92</td>
<td>184</td>
<td>368</td>
</tr>
<tr>
<td>Number of Subchannels</td>
<td>2</td>
<td>n/a</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>Subcarrier frequency spacing (kHz)</td>
<td></td>
<td>10.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Useful symbol time (µs)</td>
<td></td>
<td>91.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Guard time (µs)</td>
<td></td>
<td>11.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OFDM symbol duration (µs)</td>
<td></td>
<td>102.9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2. Mobile WiMAX downlink OFDM/OFDMA physical parameters
CHAPTER 3

PROBLEM STATEMENT

3.1. Coding Complexity

As Discussed in Section 2.3 Turbo Codes provide exceptional performance, approaching within a few dB of the shannon limit in an AWGN channel. However, the complexity associated with decoding concatenated RSC codes is quite high. [4] The Turbo decoder structure requires decoding of each RSC code in series and then re-computation using apriori knowledge from the subsequent decoder. A similar example would be computing a maximum likelihood estimation over some \( N \) code bits forwards and then back again, obtaining ML metrics associated with states in both directions. [12] Using this example, it can be estimated that the lower bound on decoding time is at least two times that of a single component code using ML. In addition to the series decoding requirement, because the decoder needs to gain information on all states, the decoding procedure can not terminate early, making the decoding procedure take equally long in both high and low SNR situations. Figure 3.1 shows the simulation time in CPU seconds for Turbo Codes of block length 1944 in relation to SNR. It is clear from these results that Turbo Codes do not gain any speed benefit from a less noisy channel.

3.2. Peak to Average Power Ratio (PAPR)

High peak to average power ratio (PAPR) is an inherent problem problem of OFDM because the orthogonal multi-carrier nature leads to constructive and destructive interference in the time domain. This interference means that some time domain samples will be very large while some will be near 0. This problem is made worse in LTE by use of precoding channel estimation and compensation [16]. As seen in Figure 3.2 The precoded frequency domain symbols \( x \) are increased or reduced in power by weight matrix \( W \) such that the result at the receiver after channel effects is the original symbol value. The precoding technique can be represented as

\[
x' = Wx
\]  

(3.1)
where $W$, assuming MMSE based detection, is given as

$$W_{MMSE} = \beta \cdot \arg \min_W E \left\{ ||\beta^{-1}(HWx + z) - x||^2 \right\}$$  \hspace{1cm} (3.2a)$$

$$= \beta \cdot H^H \left( HH^H + \frac{\sigma^2}{\sigma_x^2} I \right)^{-1}$$  \hspace{1cm} (3.2b)$$

and $\beta$ is a constant to meet the total transmitted power constraint after precoding given as

$$\beta = \sqrt{\frac{N_T}{Tr(\Psi(\Psi)^H)}}$$  \hspace{1cm} (3.3)$$

where $\Psi$ is equal to the expression after $\beta$ in Equation 3.2b. This precoding leads to some symbols having even worse interference then without precoding. This problem with PAPR forced the designers of LTE to use SC-OFDMA on the uplink to save power on user equipment at the expense of UL data speed. Figure 3.3 shows the typical PAPR of an OFDM LTE system versus the worst precoded case of a highly frequency selective channel.

![Figure 3.1. Completion time of Turbo Codes versus SNR](image-url)
measurement is taken at the transmitter antenna using the formula

\[
PAPR = \frac{\max\{|x[n]|^2\}}{E\{|x[n]|^2\}}
\]  

(3.4)

where \(x[n]\) are precoded time domain symbols before channel effects, \(\max\{|x[n]|^2\}\) is the maximum symbol energy and \(E\{|x[n]|^2\}\) is the expected value of the energies of the set of symbols.

Figure 3.2. MIMO precoded equalization

Figure 3.3. Probability of LTE OFDM symbol PAPR above PAPR0
3.3. Sensitivity to Time Varying Channels

As mentioned earlier, LTE uses receiver-transmitter feedback precoding as its standard form of channel estimation. This estimation/compensation technique works by doing least squares (LS) based channel estimation at the receiver and sending it back to the transmitter to apply the corrections. This technique is better than receiver side LS estimation since it is immune to further noise enhancement. [16] To prove this we look at the standard receiver side estimation as a minimization of the cost function

\[ J(\hat{H}) = ||Y - X\hat{H}||^2 \]

\[ = (Y - X\hat{H})^HY - X\hat{H} \] \hspace{1cm} (3.5)

\[ = Y^HY - Y^HX\hat{H} - \hat{H}^HX^HY + \hat{H}^HX^HX\hat{H} \]

and by setting the derivative of the function with respect to \( \hat{H} \) to zero

\[ \frac{\partial J(\hat{H})}{\partial \hat{H}} = -2(X^HY)^* + 2(X^HX\hat{H})^* = 0 \] \hspace{1cm} (3.6)

and since \( X^HX\hat{H} = X^HY \) we get the solution to the LS estimation as

\[ \hat{H}_{LS} = (X^HX)^{-1}X^HY = X^{-1}Y \] \hspace{1cm} (3.7)

which is simply \( Y/X \) where \( Y \) and \( X \) are the received and transmitted pilot symbols respectively. Lastly, we can look at the mean square error (MSE) of the estimation as

\[ MSE_{LS} = E(H - \hat{H}_{LS})^H(H - H_{LS}) \]

\[ = E(H - X^{-1}Y)^H(H - X^{-1}Y) \]

\[ = E(X^{-1}Z)^H(X^{-1}Z) \] \hspace{1cm} (3.8)

\[ = EZ^H(XX^H)^{-1}Z \]

\[ = \frac{\sigma_z^2}{\sigma_x^2} \]
which is the inverse of the SNR. This implies that the estimation is subject to further noise enhancement if there is a sudden change in noise power at the receiver. Since the estimate is applied to the transmitter in the precoded case it can avoid any changing conditions in receiver SNR. However, if the channel gains in $H$ being sent to the transmitter for precoding get corrupted or lost, a large amount of errors can occur. In addition, LTE applies estimation on a resource block basis, that means only every 7 OFDM symbol periods. If the channel gains experience fast fading greater than $7 \cdot T_s$ even more errors can occur. Figure 3.4 shows LTE received symbols experiencing frequency selective Rayleigh fading, Figure 3.5 shows the received symbols after precoding at the transmitter. It’s clear that not only is the precoding estimation mediocre at correcting the channel effects, The two data streams have vastly different estimation quality leading to burst errors on the affected codeword.

**Figure 3.4.** LTE data symbols before channel compensation (4QAM 12 dB)
Figure 3.5. LTE received data symbols before demodulation (4QAM 12 dB)
4.1. Solution to Coding Complexity

4.1.1. Irregular Low Density Parity Check Codes (LDPC)

LDPC codes are capacity approaching error correcting codes invented by Gallager of M.I.T. in 1960. They were largely ignored until recently as the decoding procedure seemed impractical. Recent work and re-evaluation shows that they have similar performance to turbo codes with many implementation advantages.\cite{4} LDPC codes are a type of parity check block code with a collection of binary vectors of size $N$ where the symbols in the code satisfy $N_r$ parity check equations of the form

$$x_1 \oplus x_2 \oplus x_3 \oplus \ldots \oplus x_c = 0 \quad (4.1)$$

where $\oplus$ is the modulo 2 addition operation and $\{x_1, x_2, x_3, \ldots, x_c\}$ are a subset of code symbols from each equation. Each codeword of size $N$ contains $N - N_r = N_k$ information symbols. From this information, a parity check matrix can be formed where the rows represent the separate equations and the columns represent the digits in the code word. The parity check matrix for simple equations (hamming $(7,4)$) can be expressed as

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (4.2)$$

LDPC codes are unique in that the percentage of 1’s compared to 0’s in the parity check matrix is very low, every code digit is contained in the same number of equations and each equation contains the same number of code symbols. These properties allow for LDPC codes to be decoded quickly as there are very few parity checks that need to take place.

The decoding of LDPC codes is efficiently performed in a graph structure derived from the check matrix $H$ as seen in Figure 4.1 and Figure 4.2. Each row corresponds to
a check node (parity equation) and each 1 in the row represents a graph edge into a bit node. In addition, each column corresponds to a bit node (code symbol) and each 1 in the column represents a graph edge into a check node if that bit is involved in that specific parity equation.

**Figure 4.1.** Graph of check node to bit node message passing in an LDPC matrix

The decoding procedure is accomplished by passing messages along the graph lines from bit node to check node and vice versa. The messages are related to soft-bit probability values in the form of negative log-likelihood ratios per bit $i$ given as

$$\lambda_i = \log \left( \frac{P(x_i = 0)}{P(x_i = 1)} \right)$$  \hspace{1cm} (4.3)

where $x$ is the received vector of bits. The total vector parity LLR $\lambda_{\Phi(x)}$ can be computed using

$$\lambda_{\Phi(x)} = -2\tanh^{-1} \left( \prod_{i=1}^{n} \tanh \left( \frac{-\lambda_i}{2} \right) \right)$$  \hspace{1cm} (4.4)
The vector \( r \) of received symbols can be segmented into two parts: \( r_n \) which is the systematic part of the code word and \( r_{i \neq n} \) which is the symbols with parity bits of the code word. Therefore, we can express the a posteriori LLRs of the \( n \) bits as

\[
\lambda_n = \log \left( \frac{P(x_n = 0 | r_n, r_{i \neq n})}{P(x_n = 1 | r_n, r_{i \neq n})} \right) \tag{4.5}
\]

assuming an AWGN channel condition and priori knowledge of \( r \) we can write the intrinsic and extrinsic LLR components for bit \( n \) as

\[
\lambda_n = \frac{2}{\sigma^2} r_n + \log \left( \frac{P(x_n = 0 | r_{i \neq n})}{P(x_n = 1 | r_{i \neq n})} \right) \tag{4.6}
\]

If it is known that the parity of a vector \( x \) is even parity (0), the probability that a bit \( x_n \) is 1, given the received values of the rest of the vector information \( r_{i \neq n} \), is the same as the probability that the rest of the vector has odd parity. [4] Using this, we can express the bit \( n \) LLRs as

\[
\lambda_n = \frac{2}{\sigma^2} r_n - 2 \sum_{i=1}^{j} \tanh^{-1} \left( \prod_{i=2}^{k} \tanh \left( -\lambda_{i,l} - \frac{1}{2} \right) \right) \tag{4.7}
\]

which holds if the vectors \( x_1, x_2, \ldots, x_j \) are independent, where \( \lambda_{i,l} \) is the message from bit node \( i \) to check node \( l \) in terms of vector parity given as

\[
\lambda_{i,l} = \log \left( \frac{P(\Phi x_{i,l} = 0 | r_{i \neq n})}{P(\Phi x_{i,l} = 1 | r_{i \neq n})} \right) \tag{4.8}
\]

Finally, we can initialize the check node values and update procedure as

\[
u_{n_r,n}^0 = 0
\]

\[
u_{n_r,n}^l = -2 \tanh^{-1} \left( \prod_{i \in N \setminus n} \tanh \left( -\lambda_{i,l} - \frac{1}{2} \right) \right) \tag{4.9}
\]

and the bit node values and procedures as

\[
\lambda_{n_r,n}^0 = 0
\]

\[
\lambda_{n_r,n}^l = \frac{2}{\sigma^2} r_n + \sum_{n_r \in N_r} u_{n_r,n}^l \tag{4.10}
\]
Each bit node has a particular LLR associated with it computed from all the contributions from connected check nodes which are the extrinsic information explained above, and the received symbols $r_n$ which are intrinsic information. When a check node receives a message from a bit node it subtracts the value it passed on previously to reduce correlations with previous iterations.

Contrary to the properties of the LDPC code given previously, there exists some irregular LDPC codes which have a variable number of 1’s in the rows and columns. These codes distribute their 1’s using a technique called density evolution to allow certain parity equations in the matrix to provide more reliable information. [14] These irregular LDPC codes are known to provide better performance in both Rayleigh fading and AWGN channels. The performance can be further improved in Rayleigh channels by employing an interleaver or bit level scrambler over multiple codewords to prevent burst errors from frequency selectivity. This technique is employed in the system implemented in this thesis. The distribution of 1’s for the optimized irregular LDPC parity matrix used in this system can be seen in Figure 4.3.

![Distribution of 1’s in 802.11ae irregular LDPC code](image)

**Figure 4.3.** Plot of 1’s positions in 802.11ae Irregular LDPC parity matrix

As discussed previously, LDPC codes are capacity approaching codes similar to turbo codes but with implementation advantages in VLSI that make them more desirable for
practical, high data rate applications. These include the ability to process check and bit nodes concurrently as compared to the iterative requirements of turbo code RSC decoders. Also, at each update iteration the hard decision \( \tilde{x}_i \) on the negative LLR’s of all bits in vector \( x_i \) given as

\[
\tilde{x}_i = \begin{cases} 
1 & \text{if } -\lambda_i > 0 \\
0 & \text{if } -\lambda_i < 0 
\end{cases} 
\] (4.11)

can be checked with the parity equations in \( H \) by \( H \cdot \tilde{x}_i \mod 2 = 0 \). If said equation is satisfied then decoding ceases early. The computation benefits to these procedures can be seen Figure 4.4 for a small number of symbols. The performance benefits of irregular LDPC codes can be seen in Figure 4.5. Excluding the Error floor, irregular LDPC codes provide 2 to 3 dB BER improvement depending on code length.

![Figure 4.4. Completion time of Turbo Codes and LDPC Codes versus SNR](image-url)
4.1.2. Integer Based Linear Space-Time Block Codes

Thanks to the reduced complexity of LDPC codes, STBC can be used to further reduce errors without increasing complexity. Compared to the STBC used in the state of the art as described in Section 2.2 which have a rate of 1 (i.e. 1 symbol per 2 antennas) or spatial multiplexing technique which provides no coding gain in order to send 2 symbols per 2 antenna there exists full-rate, full-diversity algebraic STBC which can provide up to $n$ rate and $n^2$ diversity given an $n$ by $n$ square antenna configuration. [5] Recall the structure of standard rate 1 alamouti STBC as

$$X_A = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$  \hspace{1cm} (4.12)
where $x_1$ and $x_2$ are input symbols from a complex constellation which represent mapped input bits. Now we consider a general rate 2 code design given as

$$X_L = \begin{bmatrix} f_1(x_1, x_2) & f_2(x_3, x_4) \\ f_3(x_3, x_4) & f_4(x_1, x_2) \end{bmatrix}$$  \hspace{1cm} (4.13)$$

where $x_1, x_2, x_3, x_3$ are information symbols from a complex constellation and for all other currently known linear STBC $\{f_j(\cdot)\}$ are irrational linear combination of these symbols, with the most well known being the Golden code. [5] For every transmitted codeword, the encoding operation for the Golden code computes the linear combination of information symbols with irrational coefficients. When the Golden code is implemented on limited computation hardware there is a significant degradation of error performance due to finite precision operations which cause quantization error on the irrational numbers. Therefore, in order to provide reliable performance in limited or low power hardware environments, an integer based linear STBC is proposed which needs reduced number of bits for encoding operation as well as provides exact representation of coefficients. These codes can be obtained by the linear design

$$X_I = \begin{bmatrix} \langle \Phi_1, x_1 \rangle & \langle \Phi_1, x_2 \rangle & \langle \Phi_1, x_3 \rangle & \ldots & \langle \Phi_1, x_{n-1} \rangle & \langle \Phi_1, x_n \rangle \\ \gamma \langle \Phi_2, x_n \rangle & \langle \Phi_2, x_1 \rangle & \langle \Phi_2, x_2 \rangle & \ldots & \langle \Phi_2, x_{n-2} \rangle & \langle \Phi_2, x_{n-1} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma \langle \Phi_{n-1}, x_3 \rangle & \gamma \langle \Phi_{n-1}, x_4 \rangle & \gamma \langle \Phi_{n-1}, x_5 \rangle & \ldots & \langle \Phi_{n-1}, x_1 \rangle & \langle \Phi_{n-1}, x_2 \rangle \\ \gamma \langle \Phi_n, x_2 \rangle & \gamma \langle \Phi_n, x_3 \rangle & \gamma \langle \Phi_n, x_4 \rangle & \ldots & \gamma \langle \Phi_n, x_n \rangle & \langle \Phi_n, x_1 \rangle \end{bmatrix}$$  \hspace{1cm} (4.14)$$

where for $j, k \in \{1, 2, \ldots, n\}, \langle \Phi_k, x_j \rangle$ denotes the inner product of the $k$th row of the circulant matrix
and the vector $x_j = [x_{j,1}, x_{j,2}, x_{j,3} \ldots x_{j,n}]$ which are information symbols from some $M$-QAM square constellation. The coefficient values are $\gamma = i$ (where $i = \sqrt{-1}$) and $\alpha = 2^{m/2}$ where $m = \log_2 M$. Therefore, a code word where $x_j$ is a 4-QAM constellation the resulting values will be from a 16-QAM square constellation.

The performance of the integer based STBC design described above can be seen in Figure 4.6. It clearly shows moderate diversity gain using the hard-output ML decoding procedure.
4.2. Solution to High PAPR

4.2.1. Partial Null Subcarrier Switching

As previously discussed in Section 3.2 a major drawback of multi-carrier transmission standards like LTE is the occasionally high instantaneous PAPR when the orthogonal subcarriers are in-phase in the time domain. There exists a need to design a technique to reduce the PAPR of an OFDM / OFDMA multi-carrier communication system while still maintaining the orthogonal nature. The current standard methods for PAPR reduction include clipping, spreading, and null data carriers (NDC). clipping is a technique where any time domain signal larger than a certain value \( x_c \) is reduced at the transmitter before being sent to the receiver. This provides a controllable ceiling on the maximum PAPR but also reduces increase BER as the high power samples are now distorted. Spreading is the method used in LTE in the form of DFT spread SC-OFDMA where data is spread in the frequency domain over some subset of resource blocks. This technique does not distort the data but requires more advanced channel estimation to overcome channel effects like ISI, therefore increasing BER if a simple estimator is used. NDC is the technique used in WiMAX where a certain number of OFDM subcarriers are not transmitted on for that OFDM period. This technique maintains signal integrity so BER is unaffected but there is a data rate high of \( N_{NDC}/N_{OFDM} \).

The presented method called partial null subcarrier switching \([15]\) takes 1 null guardband subcarrier and switches it with an optimal data subcarrier in a region so that the PAPR is minimized for that symbol period. Let the OFDM symbol in the frequency domain be represented as a vector of length \( N_{OFDM} \) in the form \( x = \{x_1, x_2, x_3, \ldots, x_{N_{OFDM}}\} \). where the number of null guardband subcarriers is \( N_{null} \) and number of switchable data carriers is \( N_d \) have indexes \( I_d \) as some none consecutive subcarriers in \( x \). Let the indexes describing the position of null subcarriers in the OFDM symbol be \( I_n = \{1, 2, \ldots, N_n/2\} \cup \{N_{OFDM} - N_n/2, N_{OFDM} - N_n/2 + 1, \ldots, N_{OFDM}\} \) and the indexes describing the subset of data indexes to be searched over are given as \( I_p = \{I_i, I_i+1, I_i+2, \ldots, I_i+N_p\} \) where \( I_i \in I_d \) is the initial index in the OFDM symbol to search and \( N_p \) is the total size of the region to search.

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the shifting algorithm can be described as follows:

1. Initialize $I_p$ search subset, switching null $I_{sw} = N_{OFDM} - N_n/2$, initial PAPR $PAPR_i$ and counter to 0
2. Switch null in $I_{sw}$ index with $I_i$ + counter data
3. Compute IFFT and compare with $PAPR_i$
4. If new PAPR is less than $PAPR_i$, save current index value, else don’t
5. If $I_i$ + counter $\leq N_p$, increment counter +1, go to step 2. else stop

Once the above loop is finished the index in $I_p$ with the lowest PAPR is switched again and the final IFFT is computed to be sent out via the antenna to the receiver. A illustrative example of the shifting algorithm can be seen in Figure 4.7.

![Figure 4.7. Partial null subcarrier switching method example](image)

The advantages of the proposed method are that it has no data rate hit, complexity scales linearly with $N_p$, it is independent of OFDM symbol size $N_{OFDM}$, and computation is fast using the optimized IFFT operation. Also, no transmitter side information needs to be sent to the receiver assuming the receiver knows $I_{sw}$ (i.e. it must be a fixed value). All the receiver needs to do is detect the null power level in $I_d$ and switch with $I_{sw}$ since the average power of the frequency domain OFDM samples carrying data is $>> 0$. This
method is implemented in the system proposed in this thesis (PRCS) and its performance on $10^5$ symbols is shown in Figure 4.8 along with the LTE PAPR probabilities shown in the previous chapter.

![Figure 4.8. Probability of LTE and PRCS using null switching PAPR above PAPR0](image)

4.3. Solution to High Sensitivity to Time Varying Channels

4.3.1. Optimized Pilot Assisted Least Squares (LS) Estimation with DFT

Pilot assisted least squares (LS) estimation is used in many OFDM communication systems including the current 4G communication systems LTE and WiMAX [10] [9]. It is widely used because it provides a low complexity, efficient multipath channel estimation which can be used in various ways to remove channel effects. It has been proven [1] that evenly spaced constant power pilots provide the optimal estimation for an OFDM channel with all data carriers (i.e. $N_d = N_{OFDM}$). However, this doesn’t extend to OFDM systems
which use null subcarrier guard bands for controlling ACI and out-of-band power. Large improvements in BER have been shown to be possible by optimizing both the pilot position and energy profile using the cubic optimization technique.

First, we denote the set of indices in $x$, the frequency domain OFDM symbol $x = [x_1, x_2, \ldots, x_N]^T$, corresponding to the data carriers $\kappa_d$ and the pilot carriers $\kappa_p$. Therefore we have $x_p$ as pilot symbols and $[x]_k \neq x_p k \in \kappa_p$ as data symbols. We can express the MSE of channel on data carriers as

$$e = \text{diag} \left\{ \begin{bmatrix} \sigma_z^2 + \frac{\sigma_n^2}{\varepsilon_d} I_{|\kappa_d|} \end{bmatrix} \right\}$$  \hspace{1cm} (4.16)

and we want to minimize the $||e||_\infty$ norm in order to reduce BER which leads to the minimization problem

$$\arg \min_{E\{\|x\|,\varepsilon_p,\kappa_p\}} ||e||_\infty$$ \hspace{1cm} (4.17)

subject to $\varepsilon_p + \varepsilon_d = \varepsilon_s$, $x_n = 0_{|\kappa_n|}$

where $\varepsilon_p$ and $\varepsilon_d$ are energies in the pilot subcarriers and data subcarriers respectively and $x_n$ are the null subcarriers. However, solving the minimization equation above is a difficult nonlinear problem. Therefore by parameterizing the pilot positions with a cubic polynomial, Equation 4.17 can be simplified to a problem with continuous inputs. Further simplification by specifying the pilot positions in two continuous variables $\delta$ and $a_3$. For an arbitrary set of pilot indices $\{k_1, k_2, \ldots, k_{|\kappa_p|}\}$ it is possible to solve the optimization problem

$$\arg \min_{|x_p|^{-2}} \|z\|_\infty$$ \hspace{1cm} (4.18)

subject to $\|x_p\|_2^2 = \varepsilon_p |\kappa_p|$

$\{k_1, k_2, \ldots, k_{|\kappa_p|}\}$

using standard convex optimization solver libraries. The optimal energy value for for pilots in terms of signal energy is found by differentiating the MSE above to get

$$\varepsilon_p = \varepsilon_s \frac{\|z\|_\infty - \sqrt{\|z\|_\infty}}{\|z\|_\infty - 1}$$  \hspace{1cm} (4.19)
Solves the above equation and finds the optimized pilot location and energy profile as seen in Figure 4.9 with exact locations \( \kappa_p = \{29, 56, 85, 113, 143, 171, 200, 228\} \) and the pilot energy \( \varepsilon_p = 0.25 \cdot \varepsilon_d \). Also, the experimental MSE of an \( L = 4 \) channel can be seen in Figure 4.10.

![Comparison of FD Pilot Energy Profiles](image)

**Figure 4.9.** Energy profiles of default 802.16a OFDM mode pilots and convex optimized

While it is clear from the experimental results that the convex optimized pilot design increases the reliability of the LS channel estimation technique, it still suffers from possible noise enhancement at the receiver side. To help improve performance in receiver side estimation and correction the DFT-based estimation technique can be used. [16] This method eliminates the effect of noise in the time domain outside of the maximum channel delay \( L \). Let \( \hat{H}[k] \) denote the LS channel estimated gains at the \( k \)th subcarrier in the frequency domain. By taking the IDFT of the channel estimate \( \{\hat{H}[k]\}_{k=0}^{N-1} \) to get the time domain estimate as

\[
\text{IDFT}\{\hat{H}[k]\} = \hat{h}[n] = h[n] + z[n], \quad n = 0, 1, \ldots, N - 1
\]  

(4.20)
where $z[n]$ is the noise component in the time domain and $\{\hat{h}[n]\}$ is the total estimate including noise. We can ignore the coefficients in $\{\hat{h}[n]\}$ that contain only noise and define the new estimate coefficients as

$$
\hat{h}_{DFT}[n] = \begin{cases} 
  h[n] + z[n], & n = 0, 1, 2, \ldots, L - 1 \\
  0, & \text{otherwise}
\end{cases}
$$

(4.21)

and finally transforming the remaining L coefficients back to the frequency domain we get the improved estimate

$$
\hat{H}_{DFT}[k] = \text{IDFT}\left\{\hat{h}_{DFT}[n]\right\}
$$

(4.22)

It can be seen in Figure 4.11 that the LS-DFT method can provide exceptional resilience to channel noise, thus reducing MSE further, making it less susceptible to noise enhancement as most of the noise components are removed.
4.3.2. Single Tree Search Schnorr-Euchner Sphere Decoder (STS-SESD)

The main challenge in practical MIMO systems is the effective implementation of the detector which separates the spatially coded or multiplexed data streams. The most common of implementations is discussed in Chapter 2 and it is the hard decision linear MMSE detector which aims to maximize the SINR at the received antennas. The problem with linear detectors is that they have poor performance caused by uncorrected nonlinear interference and the inherent limitations of hard output demodulation. [16] The most powerful of detection scheme is the exact a posterior probability (APP) brute force maximum likelihood (ML) method which finds the exact per bit soft information over the constellation and all spatial dimensions. The problem with brute force ML is the exponential complexity based on spatial order and constellation size. Therefore, there exists a need to have a practical, efficient MIMO detector with tunable performance up to near ML. The Schnorr-Euchner sphere decoder (SESD) implemented in this thesis uses a single tree search, tree pruning method to efficiently find the max-log approximate soft information per bit over the ex-
panded integer based STBC constellation discussed previously in this chapter. In addition the max-log information is used to find the per bit ML solution of the STBC map using a lookup table before being sent to the channel decoder.

Consider a MIMO system with a $M_T = M_R$ square transmit and receive antenna configuration. The QAM modulated symbols created using the integer based STBC technique are mapped to $M_T$-dimensional vectors per time interval (each subcarrier). Each complex valued constellation point is from the square $2^{(r \text{bps})}$ constellation where bps is the bits per symbol on the transmit modulator and $r$ is the dimensionality of the STBC in the time dimension. We denote each symbol vector as $s$ where there is a bit level label associated with each denoted as $x$. Each entry of the bit level vectors is further denoted as $x_{j,b}$ where $j$ and $b$ refer to the $b$th bit in the symbol corresponding the the $j$th entry in the spatial $M_T$-dimensional vector $s = [s_1, s_2, \ldots, s_{M_T}]^T$. The received signal in the frequency domain per time interval is given as

$$y = Hs + n$$

where $H$ denotes the $M_T$ by $M_R$ channel matrix for 1 symbol period and $n$ is a complex Gaussian distributed noise vector of size $M_R$.

The soft-output MIMO detection requires the computation of LLRs for all bits in the bit level vectors $x$ which are computed using the low complexity max-log approximate method [13]

$$L(x_{j,b}) = \min_{s \in \chi^0_{j,b}} ||y - Hs||^2 - \min_{s \in \chi^1_{j,b}} ||y - Hs||^2$$

(4.24)

where $\chi^0_{j,b}$ and $\chi^1_{j,b}$ are sets of symbols vectors that have the $b$th bit in the label of the $j$th symbol (dimension) equal to 0 or 1 respectively. Checking these complementary bits is done using a simple look up table of the bit maps of the integer based STBC to reduce computation time. The lookup table is simply the bit assignments per symbol in $2^{(r \text{bps})}$ constellation per antenna, not an exhaustive code map. This is key to reducing memory requirements.

For each bit, one of the two minima is given by the metric $\lambda^{ML} = ||y - Hs||^2$ associated
with the ML solution of the detection problem

\[ s^{ML} = \arg \min_{s \in Q(2^{(\tau \text{bps})})} \| y - Hs \|^2 \]  

(4.25)

the other minimum can be written as a counter-hypothesis

\[ \overline{\lambda}^{ML}_{j,b} = \min_{s \in \chi_{j,b}^{ML}} \| y - Hs \|^2 \]  

(4.26)

where \( \overline{x}^{ML}_{j,b} \) denotes the binary complement of the \( b \)th bit in the bit level label \( x \) in the \( j \)th entry of \( s^{ML} \). Therefore, using the above metrics the max-log approximate LLRs can be written as

\[ L(x_{j,b}) = \begin{cases} 
\lambda^{ML} - \overline{\lambda}^{ML}_{j,b}, & x^{ML}_{j,b} = 0 \\
\overline{\lambda}^{ML}_{j,b} - \lambda^{ML}, & x^{ML}_{j,b} = 1 
\end{cases} \]  

(4.27)

From the Equation 4.27 we can conclude the max log MIMO detector efficiently identifies \( s^{ML}, \lambda^{ML}, \overline{\lambda}^{ML}_{j,b} \) for \( j = 1, 2, \ldots, M_T \) and \( b = 1, 2, \ldots, (\tau \text{bps}) \).

To perform a tree search on the ML solution problem using the sphere decoding algorithm, we first need to QR-decompose \( H \) according to \( H = QR \) where the matrix \( Q \) is unitary, and the upper triangle matrix \( R \) uses real valued positive entries on the main diagonal. We can left multiply Equation 4.23 by \( Q^H \) to get the modified expression

\[ \tilde{y} = Rs + Q^H n \quad \text{with} \quad \tilde{y} = Q^H y \]  

(4.28)

hence we can rewrite our hypothesis metrics \( \lambda^{ML} \) and \( \overline{\lambda}^{ML}_{j,b} \) as

\[ \lambda^{ML} = \arg \min_{s \in Q(2^{(\tau \text{bps})})} \| y - Rs \|^2 \]  

\[ \overline{\lambda}^{ML}_{j,b} = \min_{s \in \chi_{j,b}^{ML}} \| y - Rs \|^2 \]  

(4.29)

We next define the partial symbol vector structure (PSVs) for decoding a single time instance of the integer based STBC as \( s^{(i)} = [s_i, s_{i+1}, \ldots, s_{M_T}]^T \). All \( \tau \) time instances of the received spatial streams in Equation 4.28 are decoded separately to reduce decode complexity.
The PSVs are arranged in a tree that has its root at level $i = M_T$ and its leaves
at level $i = 1$ which correspond to the symbol vectors $s$. We can express the Euclidean
distances as $d(s) = ||\bar{y} - Rs||^2$ which can be computed recursively as $d(s) = d_1$ with partial
Euclidean distances (PEDs)

$$d_i = d_{i+1} + |e_i|^2, \quad i = M_T, M_T - 1, \ldots, 1$$  \hspace{1cm} (4.30)

initialization $d_{M_T} = 0$ and distance increments

$$|e_i|^2 = |\bar{y}_i - \sum_{j=i}^{M_T} R_{i,j}s_j|^2$$  \hspace{1cm} (4.31)

and since the dependence of the PED $d_i$ on the symbol vector $s$ is only through the PSV $s^{(i)}$,
we have transformed ML detection and the computation of max-log LLRs into a tree search
problem.

To traverse the tree structure in a STS fashion with minimal complexity, we ensure
that every node is visited at most once by searching for both the ML solution and all per bit
counter-hypotheses concurrently. The idea is to search a subtree originating from a given
node only if the result can lead to an update of at least one of the above metrics. The
algorithm is given as:

(1) Initialization: Initialize the metrics $\lambda^{ML} = \lambda^{ML}_{j,b} = \infty$ \((\forall j, b)\) and whenever a leaf
node corresponding to $x$ has been reached, the decoder distinguishes between two
cases:

(a) if a new ML hypothesis is found where $d(x) < \lambda^{ML}$, all $\lambda^{ML}_{j,b}$ for which $x_{j,b} = 
\bar{y}_{j,b}^{ML}$ are set to $\lambda^{ML}$ followed by the updates $\lambda^{ML} \leftarrow d(x)$ and $x^{ML} \leftarrow x$. This
ensures that at all times $\lambda^{ML}_{j,b}$ is the metric associated with a valid counter
hypothesis to the current ML hypothesis.

(b) In the case where $d(x) \geq \lambda^{ML}$ only the counter hypotheses have to be checked.

And $\forall j$ and $b$ such that $x_{j,b} = \bar{y}_{j,b}^{ML}$ and $d(x) < \lambda^{ML}_{j,b}$, the decoder updates
$\lambda^{ML}_{j,b} \leftarrow d(x)$.
(2) Pruning: Consider a given node \( s^{(i)} \) and its corresponding partial label \( x^{(i)} \) consisting of bits \( x_{j,b} \). Assume that the subtree originating from the node has not been expanded yet. The pruning criteria for \( s^{(i)} \) along with its subtree is compiled from two conditions. First, the bits in the partial label \( x^{(i)} \) are compared with the corresponding bits in the label of the current ML hypothesis \( x^{ML} \). All metrics \( \lambda^{ML}_{j,b} \) with \( x_{j,b} = x^{ML}_{j,b} \) found in the comparison may be affected when searching the subtree. Second, the metrics \( \lambda^{ML}_{j,b} \) corresponding to the counter hypotheses in the subtree of \( s^{(i)} \) may be affected as well. Therefore, the metrics which may be affected during the search in the subtree from node \( s^{(i)} \) are given by the set

\[
A\left(x^{(i)}\right) = \{a_l\} = \left\{ \lambda^{ML}_{j,b} | (j \geq i, b = 1, 2, \ldots, (\tau \text{bps})) \wedge (x_{j,b} = x^{ML}_{j,b}) \right\} \cup \left\{ \lambda^{ML}_{j,b} | (j < i, b = 1, 2, \ldots, (\tau \text{bps})) \right\} \tag{4.32}
\]

The node \( s^{(i)} \) along with all its subtree is pruned if its PED \( d\left(s^{(i)}\right) \)

\[
d\left(s^{(i)}\right) > \max_{a_l \in A(x^{(i)})} a_l \tag{4.33}
\]

After the tree has been fully traversed the matrix containing all the values of \( \lambda^{ML}_{j,b} \) is converted to the final metric \( L(x_{j,b}) \) as per Equation 4.27 using \( \lambda^{ML} \). Lastly to compute the ML solution per bit of the STBC \( x^{ML}_c \) we apply the equation

\[
x^{ML}_c (s, b) = \arg \max_{L(x_{j,b}) \in \gamma_s^{(r)}} L(x_{j,b}), \quad \forall \ b = 1, 2, \ldots, (\tau \text{bps}) \tag{4.34}
\]

where \( \gamma_s^{(r)} \) are \( s \) sets of \( j \) streams containing the spread information over \( \tau \) symbol periods.

Therefore, it is required that the detector stores \( \tau \) instances of the metric \( L(x_{j,b}) \) after computation. The resulting bits are the soft information of the data making up the STBC \( x_c \). An example using the software MATLAB is shown in Figure 4.12 where the most likely bits are chosen from the spread information. This decoding method provides the maximum possible performance provided by the max-log approximate LLR method. Performance measures are
shown in Figure 4.13. It can be seen that the STS-SESD STBC decoder in combination with a downstream LDPC belief propagation decoder allows for PRCS to come withing 3 dB of the practical capacity of a Rayleigh channel. [6]

![Figure 4.12. Max-log approximate LLRs over 2 symbol periods and most likely bits corresponding to the STBC](image)

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</tbody>
</table>

![Figure 4.13. Comparison of PRCS and LTE BER using various detector methods](image)

**Figure 4.13.** Comparison of PRCS and LTE BER using various detector methods

4.4. Channel and Noise Model

The single input single output (SISO) channel is modeled as a tapped delay line (TDL) or multiple weighted flat fading generators in the form

$$h[l] = \sum_{d=0}^{N_d-1} \sqrt{\frac{E_d}{2}} h_d[l]$$

(4.35)
where $N_D$ is the number of taps, $E_d$ is the power of the tap in dB scale and $l$ is the maximum delay of the channel. Each flat fading generator $h_d$ is a complex Gaussian random variable. Since the power delay profile (PDP) of a general channel model is based on measurements in the environment, the delay time may not coincide with an integer multiple of the sampling time $t_s$. Therefore, the delays must be adjusted for implementation in a discrete time simulation environment. This can be done simply by rounding, which shifts the tap to the closest sampling instance $N \cdot f_s$ where $f_s$ is the sample rate. The new tap delay is expressed as

$$t'_d = \left\lceil \frac{t_d}{t_s} \right\rceil$$

where $\left\lceil \ldots \right\rceil$ denotes the rounding operation.

The MIMO channel is computed by correlating some transmit and receiver correlation matrices $R_{TX}$ and $R_{RX}$ with $N_R$ by $N_T$ statistically independent TDL channels. Let the MIMO fading channel at some time instance $t$ be represented by an $N_R N_T$ by 1 vector $a_l = [a_1^{(l)}, a_2^{(l)}, \ldots, a_{N_R N_T}^{(l)}]^T$, which is a vector representation of $N_R$ by $N_R$ channel gains at some time $t$. The correlated MIMO channel can be found by multiplying an $N_R N_T$ by $N_R N_T$ matrix $C$, or correlation-shaping matrix, with $a_l$ as

$$\tilde{A}_l = \sqrt{P_l} C a_l$$

where $P_l$ is the average power of the current path. The total link spatial correlation matrix can be computed using $R_{TX}$ and $R_{RX}$ by

$$R = R_{TX} \otimes R_{RX}$$

where $\otimes$ denotes the Kronecker product. Using $R$ we can compute a root-power correlation matrix $\Gamma$ as

$$\Gamma = \sqrt{R}$$

$\Gamma$ is a non-singular matrix which can be decomposed into a symmetric correlation shaping
matrix using square-root decomposition as follows

$$\Gamma = CC^T$$  \hspace{1cm} (4.40)

Therefore, we can solve Equation 4.37 using the computed matrix $C$ from Equation 4.40 above. Lastly, the set of $N_R N_T$ by 1 vectors corresponding to correlated MIMO paths $\forall$ time instances $t$ need to be reshaped into a $N_R$ by $N_T$ by $l$ matrix. The standard channel model used for simulations in this thesis is the 3GPP model for deployment evaluation $\text{cost207RAx4}$ which is a weighted 4 tap channel with delay profile $h_d = [0, 0.2, 0.4, 0.6] \mu s$ and gain profile $h_g = [0, -2, -10, -20]$.

Once a three dimensional correlated channel matrix is computed the output data vectors $y_j$ needs to be computed according to the MIMO channel description in Figure 2.8. The output on the $j$th receive antenna as a function of the transmitted data vectors $x_i$ is described as

$$y_j[n] = \sum_{i=1}^{N_T} h_{ij}[l] * x_i[n]$$  \hspace{1cm} (4.41)

where $l$ is length of channel in samples and $n$ is the length of time domain OFDM symbol in samples. $*$ denotes the convolution operation as

$$\tilde{y}[n] = \sum_{i=0}^{N-1} x[n] h[n - m]$$  \hspace{1cm} (4.42)

After the transmitted data vectors $x_i$ have be subjected to correlated MIMO fading using the above description, AWGN needs to be properly computed based on transmit energy and added to all received data streams $y_j$. Assuming transmit energy in the frequency domain before IFFT operation is normalized to 1 watt as in $E\{Y_j\} = 1$ $\forall j$ the time domain average symbol energy per antenna will be $E\{y_j\} = 1/N_{OFDM} \forall j$ where $N_{OFDM}$ is the length of OFDM symbol including CP. The energy per modulated bit is given as

$$E_b = \frac{E_s}{\text{bps}}$$  \hspace{1cm} (4.43)

Where bps is modulated bits per symbol and $E_s$ is time domain average symbol energy per
antenna described above. The noise density per Hz is calculated as

\[ N_0 = \frac{E_b}{10^{\text{SNR}/10}} \]  

(4.44)

taking into account redundant symbols in the transmitted streams we have

\[ N_0' = N_0 \cdot \frac{1}{r_{\text{code}}} \frac{N_{\text{OFDM}}}{N_{\text{used}}} \]  

(4.45)

where SNR is the signal to noise ratio in dB scale, \( r_{\text{code}} \) is the code rate and \( N_{\text{used}} \) is the number of subcarriers per antenna which carry data. Lastly, the noise energy is calculated as

\[ \sigma = \sqrt{\frac{N_0'}{2}} \]  

(4.46)

and the \( j \) received noisy time domain signal vectors of length \( N_{\text{OFDM}} \) can be described as

\[ \hat{y}_j[n] = x_j[n] + \sigma \cdot z[n] \]  

(4.47)

where \( z \) is a complex Gaussian random variable with 0 mean and variance of 1 and \( x_j[n] \) is the combined faded time domain symbols symbols on the \( j \)th antenna.

4.5. Simulation and Physical Layer Structure

The full block diagram of the system simulation can be seen in Figure 4.14 where the ’MIMO Fading Channel’ and ’Additive White Gaussian Noise’ blocks are implemented using the previously described noise and channel methods in Section 4.4. The simulation runs over 100 LDPC codewords with full resource allocation for all transmission periods (OFDM mode) over a channel bandwidth of 3.35 MHz using 4-QAM and compares bit errors over all codewords. A block-by-block summary of the system starting with input data is detailed as follows:

1. The input data bits are uniformly distributed random integer in the range \([0, 1]\) for all 100 codewords.
2. The LDPC encoder encodes the data bits using an irregular LPDC generator matrix of length 1944 and scrambles the bits using a pseudorandom interleaver over all
codewords.
(3) The coded data bits are segmented into appropriately sized blocks based on code rate and modulated using 4-QAM for transmission over $N_{OFDM}$ orthogonal carriers.
(4) The QAM symbols are coded further using the integer based STBC of spatial and temporal size 2.
(5) The STBC data has pilot and null symbols inserted using the convex optimized design in Section 4.3.
(6) The OFDM operation is performed using IFFT after the partial null switching PAPR reduction technique; then TD samples are copied and added to the beginning in the form of a CP.
(7) The receiver removes the CP samples, reverts the OFDM operation using FFT and switches the null and data carriers back to original location.
(8) The FD channel response is estimated using LS-spline interpolation method over 1 OFDM symbol period and improved using the DFT technique.
(9) The null and pilot symbols are removed before being sending the user data to the decoders.
(10) The 'ML Block Code Decoder' block represents the STS-SESDD which efficiently computes the max-log LLR estimates over 2 spatial and symbol periods using FD channel estimates while also computing the per bit ML estimate of the integer STBC.
(11) The LDPC decoder de-scrambles the LDPC codewords and decodes each separately using the parity-check satisfied early termination condition for low complexity.
(12) The output bits are compared to the sent bits and errors are counted for performance analysis.

The physical layer structure of the system described in this thesis aims to take the best properties of both state of the art 4G systems LTE and WiMAX to provide the optimal balance of spectral efficiency, throughput, signaling overhead, channel estimation and backwards compatibility. This system uses the idea of resource blocks similar to LTE, however,
the resource blocks consist of 50 subcarriers in the frequency domain and 2 OFDM symbol periods in the time domain. This provides a balance between signaling overhead and granularity for OFDMA data allocation while still maintaining the frame/subframe structure of LTE. In addition, the resource blocks have pilot symbols inserted in every OFDM period similar to WiMAX for even better resilience against fast fading channels. The subframe structure with reference symbols for the 2 by 2 MIMO case is shown in Figure 4.15.

The sampling rate, subcarrier spacing, OFDM symbol duration, and CP length are the same as LTE to preserve the backwards compatibility with WCDMA and LTE chips. However, the number of guardband null carriers is less, increasing the occupied bandwidth. This is done to reduce the MSE at the channel edges caused by a large amount of null carriers. The consequence of this is the channel bandwidths are different than LTE and WiMAX. However, the bandwidth efficiency of 90% is maintained up to the maximum bandwidth of 20 MHz as seen in Figure 4.16. This increased bandwidth also means that it can scale up to 100 MHz bandwidth using a smaller FFT size and reduced complexity compared to LTE when it eventually gets deployed in 3GPP’s next release document. The relevant physical layer parameters can be seen in Table 4.1.

Figure 4.14. Block diagram of PRCS system simulation used in MATLAB
Lastly, performance parameters between LTE and PRCS are compared in Table 4.2. It can be seen that PRCS outperforms LTE in every category except for occupied bandwidth.
<table>
<thead>
<tr>
<th>Channel Bandwidth (MHz)</th>
<th>Frame Duration (ms)</th>
<th>Subframe Duration (ms)</th>
<th>Subcarrier Spacing (kHz)</th>
<th>Sampling Frequency (MHz)</th>
<th>FFT Size</th>
<th>Occupied Subcarriers</th>
<th>Guard Subcarriers</th>
<th>Number of Resource Blocks</th>
<th>Occupied Channel Bandwidth (MHz)</th>
<th>Bandwidth Efficiency</th>
<th>CP Length (Short CP) (µs)</th>
<th>CP Length (Long CP) (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.35</td>
<td>10</td>
<td>1</td>
<td>15</td>
<td>3.84</td>
<td>256</td>
<td>200</td>
<td>56</td>
<td>4</td>
<td>3</td>
<td>90%</td>
<td>5.2 (first) / 4.69 (rest)</td>
<td>16.67</td>
</tr>
<tr>
<td>6.7</td>
<td></td>
<td></td>
<td></td>
<td>7.68</td>
<td>512</td>
<td>400</td>
<td>112</td>
<td>8</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.3</td>
<td></td>
<td></td>
<td></td>
<td>15.36</td>
<td>1024</td>
<td>800</td>
<td>224</td>
<td>16</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td>23.04</td>
<td>1536</td>
<td>1200</td>
<td>336</td>
<td>32</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1. PRCS downlink/uplink physical parameters

<table>
<thead>
<tr>
<th>Bandwidth (MHz)</th>
<th>BW Efficiency (%)</th>
<th>PAPR (dB)</th>
<th>DL Rate (Mbps)</th>
<th>UL Rate (Mbps)</th>
<th>Code Rate</th>
<th>Spectral Efficiency (bps/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTE 3</td>
<td>90</td>
<td>11.503</td>
<td>4</td>
<td>1.69</td>
<td>0.5</td>
<td>1.34</td>
</tr>
<tr>
<td>PRCS 3.3</td>
<td>90</td>
<td>8.086</td>
<td>5.52</td>
<td>5.52</td>
<td>0.5</td>
<td>1.86</td>
</tr>
</tbody>
</table>

Table 4.2. PRCS vs LTE performance parameters
In conclusion, the research problems detailed in Chapter 3 have been solved using methods explained in Chapter 4 where a communication system capable of less than $10^{-5}$ bit errors at 7.5 db SNR using 4QAM, up to 10 times lower channel decoding complexity with early termination and up to 3 dB lower PAPR at the transmitter is designed. Both theoretical explanations and empirical simulation results are presented with discussion on physical layer structure for implementation as a 4G SOFDMA system.

The contributions to the state of the art in this thesis include the novel system level implementation of signal processing techniques to design a practical, low complexity communication system capable of capacity approaching performance in a frequency selective MIMO fading environment. Additionally, the novel implementation of a soft output single tree search sphere decoder to provide ML decoding performance to a full-rate algebraic STBC with unequal antenna constellations using a optimized look up table.

Further research on this topic should include custom computed convex optimized pilot structures with up to double the pilots per symbol period with pilots inserted in every second OFDM symbol. This change would allow for increased reliability in longer delay urban channel with reduced resilience to fast fading. The pilot structures could be changed depending on deployment needs with slight increase in signaling complexity. More exhaustive experimental results by testing cases up to 4 by 4 MIMO, 64QAM modulation and 100 Mhz channel bandwidth should be done since every aspect of the system is scalable with the only limiting factor being decoding and detection time constraints. It should also be noted that both performance and complexity of the integer based STBC will scale with dimensionality, therefore, advanced complexity reduction techniques from [13] should be implemented. The problem with obtaining these experimental results is there is much more simulation coding involved, including the custom convex solving problems as stated above as each SOFDMA mode needs their own carrier design, which was prohibitive in such a limited time frame.
Hardware prototyping using FPGAs and DSPs is another aspect that should be explored so that physical receiver limitations can be taken into account including phase noise and frequency mismatch at the receiver oscillators. Again, time constraints made hardware prototyping impossible.
BIBLIOGRAPHY


