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STRESS-INTENSITY-FACTOR INFLUENCE COEFFICIENTS FOR SEMIELLIPTICAL INNER-SURFACE FLAWS IN CLAD PRESSURE VESSELS

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ABSTRACT: A problem of particular interest in pressure vessel technology is the calculation of accurate stress-intensity factors for semielliptical surface cracks in cylinders. Computing costs for direct solution techniques can be prohibitive when applied to three-dimensional (3-D) geometries with time-varying boundary conditions such as those associated with pressurized thermal shock. An alternative superposition technique requires the calculation of a set of influence coefficients for a given 3-D crack model that can be superimposed to obtain mode-I stress-intensity factors. This paper presents stress-intensity-factor influence coefficients (SIFICs) for axially and circumferentially oriented finite-length semielliptical inner-surface flaws with aspect ratios [total crack length ($2c$) to crack depth (a)] of 2, 6, and 10 for clad cylinders having an internal radius (R_i) to wall thickness (t) ratio of 10. SIFICs are computed for flaw depths in the range of $0.01 \leq a/t \leq 0.5$ and two cladding thicknesses. The incorporation of this SIFIC data base in fracture mechanics codes will facilitate the generation of fracture mechanics solutions for a wide range of flaw geometries as may be required in structural integrity assessments of pressurized-water and boiling-water reactors.

KEYWORDS: stress-intensity-factor influence coefficient, pressurized thermal shock, finite-length flaw, reactor pressure vessel, superposition

Several techniques have been employed by analysts to calculate accurate stress-intensity factors for semielliptical surface cracks in cylinders. Solution techniques that have been applied to this problem include direct three-dimensional (3-D) calculations using a

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finite-element method [1,2]. However, computing costs for this direct approach can be prohibitive when applied to 3-D geometries with time-varying boundary conditions such as those associated with pressurized thermal shock. Several investigators [3-5] have focused on an alternative superposition technique that requires calculation of a set of influence coefficients for a given 3-D crack model. In this technique, the crack surface of a specified geometry is subjected to four stress distributions in the radial direction: uniform, linear, quadratic, and cubic. Stress-intensity-factor influence coefficients (SIFICs) obtained from these solutions are then superimposed to obtain mode-I stress-intensity factors for other stress distributions, such as those due to internal pressure or thermal shock. Considerable economy can be achieved in this approach because the only direct 3-D solutions required are those used to calculate the influence coefficients.

The Heavy-Section Steel Technology Program at the Oak Ridge National Laboratory currently has the objective of developing a data base of validated SIFICs for a wide range of axially and circumferentially oriented finite-length semielliptical inner-surface flaw geometries for a range of clad-vessel geometries. The incorporation of this SIFIC data base into fracture mechanics codes such as FAVOR [6] will facilitate the generation of fracture mechanics solutions for a wide range of flaw geometries as may be required in structural integrity assessments of pressurized-water and boiling-water reactors.

In this paper, SIFICs are presented for axially and circumferentially oriented finite-length semielliptical inner-surface flaws with aspect ratios [total crack length ($2c$) to crack depth (a)] of 2, 6, and 10 for clad cylinders having an internal radius (R_i) to wall thickness (t) ratio of 10. The SIFICs are computed for flaw depths in the range of $0.01 \leq a/t \leq 0.5$, with particular emphasis on shallow flaws ($a/t \leq 0.1$) and cladding. SIFICs are computed for two cladding thicknesses ($t = 3.96$ and 6.35 mm). This paper first outlines the technique for computing the SIFICs. Next, results from the application of the superposition technique are compared with direct 3-D finite-element solutions. Finally, the calculated SIFICs are compared with results obtained by other investigators for $a/t \geq 0.2$ (SIFICs for shallow flaws from other sources were not available for comparison).

SUPERPOSITION TECHNIQUE

The stress-intensity factor (K_I) is calculated by a superposition technique proposed by Bueckner [7]. Instead of calculating K_I for the cracked structure using the actual loads, the calculation is performed with a distributed pressure applied to the crack surfaces only. This pressure is opposite in sign, but equal in magnitude and distribution, to the stresses along the crack line that are calculated for the uncracked structure with the actual loads applied. For an arbitrary stress distribution and the case of a 3-D semielliptical flaw, the truncated stress distribution is approximated with a third-order polynomial as indicated by Eq. (1):

$$\sigma(a') = C_0 + C_1(a'/a) + C_2(a'/a)^2 + C_3(a'/a)^3, \quad (1)$$

where $\sigma(a')$ is the stress normal to the crack plane at radial position a' , and a' and a are defined in Fig. 1. The K_I values are calculated for each of the individual terms (stress distributions) in Eq. (1) and are then added to obtain the total K_I value as indicated by Eq. (2):

$$K_I(a) = \sum_{j=0}^3 K_{Ij}(a) = \sum_{j=0}^3 C_j \sqrt{\pi a} K_j^*(a), \quad (2)$$

where

$$K_j^*(a) = K_{Ij}'(a) / (C_j' \sqrt{\pi a}) . \quad (3)$$

Values of $K_{Ij}'(a)/C_j'$ are calculated for each of the normalized stress distributions corresponding to each term in Eq. (1) (uniform, linear, quadratic, and cubic), using the 3-D finite-element analysis and an arbitrary value of C_j' , such as unity. The quantity $K_j^*(a)$ is referred to as the influence coefficient and, as indicated by Eq. (3), is dimensionless. Once the influence coefficients are obtained, they can be used with any values of C_j to obtain corresponding values of $K_I(a)$. For 3-D flaws, $K_j^*(a)$ values can be calculated for several points along the crack front, in which case $K_j^*(a)$ in Eq. (2) becomes $K_j^*(a, \phi)$, where ϕ is the elliptical angle denoting the point on the crack front.

FIG. 1--Axially oriented semielliptical flaw on inner surface of cylinder.

The presence of a thin layer of stainless steel cladding on the inner surface of reactor pressure vessels has a significant effect on the K_I values for inner-surface flaws because of very high thermal stresses generated in the cladding during a thermal transient. When using coefficients for 3-D flaws, it is necessary to represent the stress distribution in the uncracked cylinder with a third-order polynomial, and thus the discontinuity in the thermal stress at the clad-base material interface presents a problem. To accommodate the stress discontinuity associated with the cladding [5], influence coefficients were calculated for the cladding stresses alone; the corresponding K_I value can then be superimposed on the K_I value due to the stresses in the base material. This is accomplished by first calculating a K_I value for a continuous-function stress distribution obtained by a linear extrapolation of the stress distribution in the base material into the cladding. Then a K_I value is calculated for a stress distribution in the cladding which is obtained by subtracting the extrapolated distribution from the actual distribution in the cladding, which is also assumed to be linear. The total K_I value is simply the sum of the two. Because the stress distribution in the cladding is essentially linear, only a first-order polynomial is used for the cladding coefficients.

CALCULATION OF SIFICS (K_j^*)

For the present study, K_j^* values were computed using the ABAQUS [8] finite-element code. ABAQUS is a nuclear-quality-assurance certified (NQA-1) code that

employs a domain integral method for the computation of the J-integral. A 3-D finite-element model was generated for each crack depth and aspect ratio. The analysis matrix included a crack depth range from 2.16 mm to 108.0 mm (crack depth/vessel thickness ratio of 0.01 to 0.5) and three different aspect ratios ($2c/a$) of 2, 6, and 10. A total of 39 finite-element models and over 200 analyses were required to generate the K_j^* values.

The 3-D finite-element models were generated with the ORMGEN [9] mesh generating program. A typical finite-element mesh for an axial, inner-surface, semielliptical flaw is shown in Fig. 2 ($2c/a = 6$ and $a/t = 0.2$). The vessel in Fig. 2 has a $t = 216$ mm (including cladding), $R_i = 2184$ mm, and $\ell = 4699$ mm.

FIG. 2--Finite-element model for an axial inner-surface flaw having $R_i/t = 10$, $a/t = 0.2$, and $2c/a = 6$: (a) complete model, and (b) crack-tip region.

This model has 10792 nodes and 2255 twenty-noded isoparametric elements (reduced $2 \times 2 \times 2$ integration). The mesh refinement was increased (more nodes and elements) near the crack tip for models with shallower flaws. From symmetry conditions, only one-fourth of the vessel is included in the finite-element model. Figure 3 shows the finite-element mesh for a circumferential flaw ($2c/a = 6$ and $a/t = 0.3$).

FIG. 3--Finite-element model for a circumferential inner-surface flaw having $R_i/t = 10$, $a/t = 0.3$, and $2c/a = 6$: (a) complete model, and (b) crack-tip region.

In the process of calculating the SIFs, careful attention was paid to using adequately converged finite-element meshes and an appropriate cylinder length. The number of elements in the circumferential and axial directions and around the crack front was increased, one at a time, to the point where addition of one element changed the value of K_I by less than one percent. The resultant converged meshes for a 180°, half-length segment of the cylinder had ~30,000 degrees of freedom. The minimum length of the cylinder to negate end effects was estimated from the cylinder radial dimensions [10] to be ≈ 3353 mm. From previous studies [5], the appropriate length for a cylinder with a 6:1, inner-surface, semielliptical flaw ($a/t = 0.2$) was 1.4 times the estimated value, so a cylinder length of 4699 mm was used in these analyses. A second analysis was performed on a cylinder in which the length of the cylinder was doubled. The K_I values around the crack front changed less than one percent.

The adequacy of using SIFs was investigated by calculating K_I values by both the superposition (as implemented in the FAVOR code) and direct ABAQUS finite-element techniques. The first comparison was for the 6:1 axial semielliptical flaw with $a/t = 0.075$ and the thermal loading through the vessel wall as indicated in Fig. 4 (a). Representative material properties for these analyses (Table 1) were obtained from the NRC-EPRI cosponsored PTS Benchmarking Exercise [11]. The results of this comparison analysis indicate a maximum difference of $\approx 2\%$ along the crack front [Fig. 4 (b)]. The second comparison was for a 10:1 axial semi-elliptical flaw with $a/t = 0.5$ and the same loading. The results, given in Fig. 4 (b), show good agreement.

TABLE 1--Material properties for cylinder analyses.

Material Property	Base	Clad
Thermal Conductivity (k), W/m $\cdot$ K	41.54	17.31
Specific Heat (c_p), J/kg $\cdot$ K	502.4	502.4
Density (ρ), kg/m 3	7833	7833
Young's Modulus (E), GPa	193.05	186.16
Coeff. of Thermal Exp. (α), / $^{\circ}$ C	1.449E-05	1.638E-05
Poisson's Ratio (ν)	0.30	0.30
Fluid-film heat transfer coefficient (h) = 1816 W/m 2 $\cdot$ K

FIG. 4--(a) Thermal-stress profile in clad cylinder used to calculate K_I , and (b) comparison of K_I values calculated by superposition and direct finite-element techniques for axially oriented 6:1 and 10:1 flaws ($a/t = 0.075$ and 0.5 , respectively) for severe thermal transient.

Two analyses were performed on a clad cylinder having $R_i/t = 10$ and $a/t = 0.1$ with uniform and quadratic loading so a comparison of SIFs could be made for vessels with the same R_i/t and a/t ratios but different t . The SIFs around the crack front had less than 0.02 % difference between the analyses of the two cylinders ($t = 216$ mm and 108 mm).

At the outset of the present study, it was anticipated that SIFs for shallow circumferential flaws ($a/t \leq 0.1$) would be essentially identical to those computed for shallow axial flaws. This anticipated result was arrived at by examining SIFs computed and reported [12,13] previously for 2-D flaws. Figure 5 shows the maximum thermally induced value of K_I (with respect to time) as a function of crack depth for both long axial flaws and continuous circumferential flaws for a reference calculational model used in the NRC-EPRI cosponsored PTS Benchmarking Exercise [11]. It can be seen that the two

FIG. 5--Variation of maximum K_I (with respect to time) as a function of crack depth for inner-surface long axial flaws and continuous circumferential flaws for the stylized thermal transient reference calculational model.

flaw geometries give essentially identical results for $a/t \leq 0.1$, however, significant differences can be observed for deeper flaws. It could be expected that finite axial and circumferential flaws would show no differences for shallow flaws and less difference than that shown in Fig. 5 for deeper flaws due to the increased constraint provided by the 3-D geometry.

These expected results were indeed realized in the present study as can be observed in Fig. 6. Variations of the SIFs at the deepest point on the crack front are shown as a function of a/t ratios for $2c/a = 10$. It can be observed that there are essentially no differences between calculated SIFs for circumferential and axial semielliptical inner-surface flaws having an $a/t \leq 0.3$. Small differences ($\sim 5\%$) can be observed for flaw geometries having an $a/t = 0.5$ and $2c/a = 10$. This difference is less for $2c/a$ of 2 and 6. Calculations for shallow flaws indicated no differences between SIFs for finite axial and circumferential flaws.

Tables 2-10 give the results of this study in terms of non-dimensionalized K_j^* values for axially and circumferentially oriented, inner-surface flaws in a cylinder, with surface length-to-depth ratios of 2:1, 6:1, and 10:1, and with $t = 216$ mm (including the cladding), $R_i = 2184$ mm, and $\ell = 4699$ mm. Also, there are coefficients for two cladding thicknesses (t_{cl}), 6.35 mm and 3.96 mm. K_j^* values are obtained by computing K from J using a plane-strain relation, where K is the stress-intensity factor per unit stress applied to

the crack face and then dividing by $\sqrt{\pi a}$. While English units were utilized to compute the K_j^* values, the K_j values, which are nondimensionalized, in the tables are appropriate for any consistent set of units (English, SI, etc.).

FIG. 6--Influence coefficients for semielliptical inner-surface flaws ($2c/a = 10$) at $\phi = 90^\circ$: (a) four reference loads applied over crack face, and (b) uniform and linear load applied over portion of crack face within cladding.

TABLE 2--Influence coefficients for inside axial surface crack: $a/t = 0.01$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_{cl}=6.35$	K1 $t_{cl}=6.35$	K0 $t_{cl}=3.96$	K1 $t_{cl}=3.96$
2:1	0.00	0.764	0.153	0.061	0.034	0.764	0.153	0.764	0.153
	2.37	0.754	0.165	0.062	0.032	0.754	0.165	0.754	0.165
	16.6	0.690	0.192	0.079	0.040	0.690	0.192	0.690	0.192
	30.8	0.669	0.264	0.127	0.069	0.669	0.264	0.669	0.264
	45.0	0.660	0.335	0.196	0.124	0.660	0.335	0.660	0.335
	59.2	0.653	0.393	0.269	0.198	0.653	0.393	0.653	0.393
	73.4	0.651	0.434	0.329	0.268	0.651	0.434	0.651	0.434
	87.6	0.649	0.463	0.366	0.310	0.649	0.463	0.649	0.463
	90.0	0.649	0.468	0.372	0.317	0.649	0.468	0.649	0.468
6:1	0.00	0.670	0.134	0.048	0.024	0.670	0.134	0.670	0.134
	2.37	0.667	0.134	0.043	0.019	0.667	0.134	0.667	0.134
	16.6	0.654	0.170	0.055	0.009	0.654	0.170	0.654	0.170
	30.8	0.741	0.269	0.109	0.029	0.741	0.269	0.741	0.269
	45.0	0.827	0.381	0.199	0.100	0.827	0.381	0.827	0.381
	59.2	0.893	0.481	0.302	0.197	0.893	0.481	0.893	0.481
	73.4	0.938	0.559	0.389	0.290	0.938	0.559	0.938	0.559
	87.6	0.970	0.594	0.435	0.341	0.970	0.594	0.970	0.594
	90.0	0.975	0.601	0.443	0.350	0.975	0.601	0.975	0.601
10:1	0.00	0.515	0.090	0.020	0.006	0.515	0.090	0.515	0.090
	2.37	0.529	0.094	0.010	0.005	0.529	0.094	0.529	0.094
	16.6	0.610	0.146	0.033	0.005	0.610	0.146	0.610	0.146
	30.8	0.762	0.258	0.060	0.019	0.762	0.258	0.762	0.258
	45.0	0.889	0.389	0.171	0.066	0.889	0.389	0.889	0.389
	59.2	0.979	0.507	0.290	0.136	0.979	0.507	0.979	0.507
	73.4	1.033	0.593	0.389	0.249	1.033	0.593	1.033	0.593
	87.6	1.064	0.635	0.439	0.307	1.064	0.635	1.064	0.635
	90.0	1.069	0.642	0.447	0.316	1.069	0.642	1.069	0.642

TABLE 3--Influence coefficients for inside axial surface crack: $a/t = 0.0184$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_c/6.35$	K1 $t_c/6.35$	K0 $t_c/3.96$	K1 $t_c/3.96$
2:1	0.00	0.777	0.155	0.061	0.034	0.777	0.155	0.777	0.155
	2.37	0.767	0.167	0.062	0.032	0.767	0.167	0.767	0.167
	16.6	0.700	0.194	0.079	0.040	0.700	0.194	0.700	0.194
	30.8	0.677	0.266	0.127	0.069	0.677	0.266	0.677	0.266
	45.0	0.667	0.338	0.196	0.125	0.667	0.338	0.667	0.338
	59.2	0.660	0.397	0.270	0.198	0.660	0.397	0.660	0.397
	73.4	0.657	0.438	0.330	0.267	0.657	0.438	0.657	0.438
	87.6	0.654	0.467	0.366	0.310	0.654	0.467	0.654	0.467
	90.0	0.653	0.472	0.373	0.317	0.653	0.472	0.653	0.472
6:1	0.00	0.653	0.128	0.043	0.021	0.653	0.128	0.653	0.128
	2.37	0.654	0.128	0.038	0.016	0.654	0.128	0.654	0.128
	16.6	0.654	0.168	0.045	0.021	0.654	0.168	0.654	0.168
	30.8	0.758	0.271	0.099	0.026	0.758	0.271	0.758	0.271
	45.0	0.852	0.387	0.192	0.085	0.852	0.387	0.852	0.387
	59.2	0.920	0.492	0.298	0.187	0.920	0.492	0.920	0.492
	73.4	0.963	0.569	0.387	0.283	0.963	0.569	0.963	0.569
	87.6	0.994	0.609	0.434	0.335	0.994	0.609	0.994	0.609
	90.0	0.999	0.616	0.442	0.344	0.999	0.616	0.999	0.616
10:1	0.00	0.525	0.092	0.019	0.007	0.525	0.092	0.525	0.092
	2.37	0.538	0.096	0.009	0.005	0.538	0.096	0.538	0.096
	16.6	0.621	0.149	0.039	0.005	0.621	0.149	0.621	0.149
	30.8	0.777	0.262	0.050	0.022	0.777	0.262	0.777	0.262
	45.0	0.899	0.392	0.164	0.075	0.899	0.392	0.899	0.392
	59.2	0.982	0.509	0.283	0.127	0.982	0.509	0.982	0.509
	73.4	1.033	0.595	0.383	0.242	1.033	0.595	1.033	0.595
	87.6	1.063	0.637	0.433	0.300	1.063	0.637	1.063	0.637
	90.0	1.068	0.644	0.441	0.310	1.068	0.644	1.068	0.644

TABLE 4--Influence coefficients for inside axial surface crack: $a/t = 0.05$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_c/6.35$	K1 $t_c/6.35$	K0 $t_c/3.96$	K1 $t_c/3.96$
2:1	0.00	0.701	0.133	0.045	0.022	0.596	0.171	0.598	0.212
	31.0	0.681	0.276	0.131	0.070	0.615	0.278	0.238	0.119
	40.5	0.670	0.323	0.178	0.108	0.386	0.208	0.175	0.090
	50.0	0.663	0.367	0.231	0.158	0.268	0.147	0.146	0.075
	59.4	0.659	0.404	0.282	0.212	0.226	0.124	0.129	0.067
	68.9	0.654	0.432	0.325	0.263	0.206	0.112	0.119	0.062
	78.4	0.653	0.451	0.355	0.301	0.194	0.105	0.114	0.059
	87.9	0.651	0.472	0.379	0.327	0.186	0.101	0.108	0.056
	90.0	0.651	0.476	0.384	0.333	0.184	0.100	0.107	0.056
6:1	0.00	0.513	0.100	0.027	0.010	0.479	0.186	0.444	0.177
	31.0	0.764	0.275	0.100	0.022	0.700	0.346	0.336	0.166
	40.5	0.823	0.352	0.162	0.072	0.528	0.270	0.292	0.147
	50.0	0.875	0.429	0.235	0.137	0.445	0.232	0.271	0.137
	59.4	0.915	0.497	0.310	0.210	0.417	0.215	0.259	0.130
	68.9	0.944	0.551	0.375	0.279	0.400	0.205	0.251	0.126
	78.4	0.964	0.589	0.423	0.334	0.391	0.199	0.247	0.124
	87.9	0.984	0.611	0.449	0.363	0.384	0.195	0.243	0.122
	90.0	0.989	0.616	0.454	0.369	0.382	0.194	0.242	0.122
10:1	0.00	0.424	0.075	0.012	0.008	0.373	0.163	0.323	0.147
	31.0	0.781	0.261	0.044	0.016	0.713	0.345	0.372	0.182
	40.5	0.868	0.350	0.126	0.056	0.578	0.300	0.333	0.167
	50.0	0.932	0.437	0.211	0.082	0.501	0.258	0.309	0.155
	59.4	0.981	0.514	0.294	0.172	0.471	0.240	0.293	0.147
	68.9	1.017	0.575	0.367	0.251	0.450	0.229	0.283	0.142
	78.4	1.040	0.617	0.419	0.311	0.440	0.222	0.278	0.139
	87.9	1.057	0.639	0.446	0.341	0.431	0.217	0.272	0.136
	90.0	1.061	0.644	0.452	0.347	0.430	0.216	0.271	0.136

TABLE 5--Influence coefficients for inside axial surface crack: $a/t = 0.075$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_{cl}=6.35$	K1 $t_{cl}=6.35$	K0 $t_{cl}=3.96$	K1 $t_{cl}=3.96$
2:1	0.00	0.697	0.134	0.045	0.022	0.604	0.210	0.511	0.204
	32.3	0.683	0.284	0.137	0.075	0.250	0.127	0.138	0.063
	41.6	0.673	0.331	0.185	0.114	0.187	0.096	0.108	0.054
	50.9	0.666	0.373	0.236	0.163	0.157	0.081	0.092	0.047
	60.1	0.662	0.409	0.286	0.217	0.139	0.073	0.083	0.042
	69.4	0.657	0.436	0.327	0.266	0.129	0.067	0.076	0.039
	78.6	0.656	0.454	0.356	0.303	0.123	0.064	0.073	0.037
	87.9	0.654	0.474	0.379	0.328	0.118	0.061	0.070	0.036
	90.0	0.654	0.478	0.384	0.334	0.116	0.060	0.069	0.035
6:1	0.00	0.499	0.099	0.026	0.009	0.436	0.171	0.357	0.162
	32.3	0.772	0.285	0.102	0.018	0.356	0.177	0.208	0.099
	41.6	0.829	0.360	0.164	0.073	0.311	0.157	0.188	0.093
	50.9	0.878	0.434	0.237	0.138	0.289	0.146	0.177	0.089
	60.1	0.916	0.499	0.309	0.210	0.276	0.139	0.170	0.085
	69.4	0.943	0.551	0.372	0.278	0.268	0.135	0.166	0.083
	78.6	0.963	0.587	0.418	0.330	0.264	0.133	0.163	0.082
	87.9	0.982	0.609	0.443	0.358	0.266	0.134	0.165	0.082
	90.0	0.986	0.614	0.448	0.365	0.267	0.134	0.165	0.082
10:1	0.00	0.368	0.077	0.011	0.010	0.312	0.141	0.231	0.127
	32.3	0.796	0.276	0.049	0.025	0.394	0.195	0.234	0.112
	41.6	0.878	0.363	0.131	0.053	0.353	0.178	0.216	0.107
	50.9	0.939	0.447	0.215	0.089	0.328	0.165	0.203	0.101
	60.1	0.985	0.520	0.296	0.176	0.312	0.157	0.194	0.097
	69.4	1.018	0.578	0.365	0.252	0.302	0.152	0.188	0.094
	78.6	1.039	0.618	0.415	0.310	0.296	0.149	0.184	0.092
	87.9	1.056	0.639	0.441	0.339	0.291	0.146	0.181	0.090
	90.0	1.060	0.644	0.447	0.345	0.289	0.145	0.180	0.090

TABLE 6--Influence coefficients for inside axial surface crack: $a/t = 0.1$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_{cl}=6.35$	K1 $t_{cl}=6.35$	K0 $t_{cl}=3.96$	K1 $t_{cl}=3.96$
2:1	0.00	0.715	0.133	0.046	0.023	0.576	0.217	0.486	0.211
	27.2	0.687	0.256	0.116	0.060	0.206	0.102	0.116	0.052
	37.3	0.676	0.308	0.164	0.095	0.146	0.074	0.087	0.042
	47.4	0.669	0.357	0.220	0.145	0.119	0.061	0.072	0.036
	57.5	0.663	0.399	0.275	0.203	0.104	0.053	0.063	0.032
	67.7	0.659	0.431	0.323	0.258	0.095	0.049	0.057	0.029
	77.8	0.657	0.452	0.357	0.301	0.090	0.046	0.055	0.028
	87.9	0.655	0.474	0.383	0.329	0.086	0.044	0.052	0.026
	90.0	0.655	0.479	0.388	0.335	0.085	0.043	0.051	0.026
6:1	0.00	0.553	0.095	0.027	0.011	0.457	0.179	0.380	0.170
	27.2	0.741	0.242	0.084	0.011	0.278	0.137	0.164	0.079
	37.3	0.811	0.324	0.144	0.053	0.240	0.120	0.147	0.073
	47.4	0.869	0.407	0.220	0.118	0.221	0.111	0.136	0.068
	57.5	0.915	0.483	0.299	0.194	0.210	0.105	0.130	0.065
	67.7	0.948	0.544	0.371	0.270	0.203	0.102	0.126	0.063
	77.8	0.971	0.586	0.424	0.330	0.199	0.100	0.124	0.062
	87.9	0.991	0.610	0.453	0.362	0.196	0.098	0.122	0.061
	90.0	0.996	0.615	0.459	0.369	0.195	0.098	0.121	0.061
10:1	0.00	0.417	0.071	0.013	0.006	0.352	0.156	0.286	0.144
	27.2	0.752	0.228	0.041	0.017	0.310	0.151	0.186	0.089
	37.3	0.849	0.321	0.116	0.057	0.274	0.137	0.169	0.084
	47.4	0.922	0.415	0.202	0.067	0.252	0.126	0.157	0.078
	57.5	0.979	0.500	0.290	0.162	0.239	0.119	0.149	0.074
	67.7	1.020	0.569	0.370	0.247	0.230	0.115	0.143	0.072
	77.8	1.046	0.616	0.428	0.313	0.225	0.113	0.140	0.070
	87.9	1.065	0.640	0.457	0.346	0.220	0.110	0.137	0.069
	90.0	1.068	0.645	0.463	0.353	0.219	0.110	0.137	0.068

TABLE 7--Influence coefficients for inside axial surface crack: $a/t = 0.2$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_{cl}=6.35$	K1 $t_{cl}=6.35$	K0 $t_{cl}=3.96$	K1 $t_{cl}=3.96$
2:1	0.00	0.723	0.127	0.046	0.025	0.471	0.204	0.391	0.195
	19.8	0.695	0.214	0.089	0.044	0.130	0.063	0.071	0.031
	31.1	0.679	0.273	0.133	0.073	0.080	0.039	0.048	0.023
	42.5	0.671	0.332	0.192	0.120	0.061	0.031	0.038	0.019
	53.8	0.665	0.383	0.255	0.182	0.052	0.026	0.032	0.016
	65.2	0.660	0.423	0.312	0.245	0.047	0.023	0.029	0.014
	76.5	0.658	0.450	0.354	0.296	0.044	0.022	0.027	0.014
	87.9	0.656	0.475	0.384	0.329	0.041	0.021	0.025	0.013
	90.0	0.656	0.479	0.389	0.335	0.040	0.020	0.025	0.013
6:1	0.00	0.625	0.101	0.034	0.017	0.423	0.189	0.354	0.179
	19.8	0.699	0.194	0.066	0.019	0.157	0.074	0.093	0.043
	31.1	0.781	0.280	0.118	0.045	0.127	0.063	0.079	0.039
	42.5	0.856	0.375	0.195	0.101	0.116	0.058	0.072	0.036
	53.8	0.915	0.464	0.283	0.180	0.110	0.055	0.069	0.034
	65.2	0.958	0.538	0.366	0.265	0.106	0.053	0.066	0.033
	76.5	0.986	0.590	0.430	0.336	0.104	0.052	0.065	0.032
	87.9	1.010	0.619	0.464	0.373	0.102	0.051	0.064	0.032
	90.0	1.014	0.624	0.470	0.380	0.102	0.051	0.063	0.032
10:1	0.00	0.506	0.077	0.022	0.009	0.368	0.180	0.314	0.164
	19.8	0.694	0.183	0.050	0.025	0.174	0.078	0.106	0.050
	31.1	0.815	0.280	0.107	0.011	0.149	0.073	0.093	0.046
	42.5	0.915	0.387	0.190	0.083	0.137	0.068	0.085	0.043
	53.8	0.991	0.488	0.287	0.170	0.130	0.065	0.081	0.040
	65.2	1.045	0.572	0.379	0.263	0.125	0.062	0.078	0.039
	76.5	1.080	0.631	0.449	0.340	0.122	0.061	0.077	0.038
	87.9	1.103	0.660	0.483	0.378	0.120	0.060	0.075	0.037
	90.0	1.107	0.666	0.490	0.385	0.120	0.060	0.075	0.037

TABLE 8--Influence coefficients for inside axial surface crack: $a/t = 0.3$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_{cl}=6.35$	K1 $t_{cl}=6.35$	K0 $t_{cl}=3.96$	K1 $t_{cl}=3.96$
2:1	0.00	0.723	0.127	0.048	0.026	0.404	0.188	0.334	0.176
	17.4	0.708	0.203	0.083	0.042	0.102	0.049	0.056	0.025
	29.1	0.690	0.264	0.126	0.068	0.058	0.028	0.034	0.016
	40.9	0.680	0.326	0.185	0.114	0.043	0.021	0.026	0.013
	52.6	0.673	0.381	0.251	0.177	0.036	0.018	0.022	0.011
	64.4	0.668	0.423	0.310	0.242	0.032	0.016	0.020	0.010
	76.1	0.665	0.452	0.355	0.297	0.030	0.015	0.018	0.009
	87.9	0.662	0.478	0.385	0.331	0.028	0.014	0.017	0.009
	90.0	0.662	0.482	0.391	0.337	0.027	0.014	0.017	0.009
6:1	0.00	0.665	0.112	0.041	0.022	0.380	0.181	0.315	0.167
	17.4	0.715	0.190	0.068	0.027	0.117	0.054	0.069	0.032
	29.1	0.804	0.277	0.118	0.051	0.093	0.045	0.057	0.028
	40.9	0.886	0.376	0.194	0.104	0.085	0.042	0.053	0.026
	52.6	0.951	0.470	0.284	0.182	0.081	0.040	0.050	0.025
	64.4	0.998	0.549	0.372	0.270	0.078	0.039	0.049	0.024
	76.1	1.028	0.605	0.439	0.345	0.077	0.038	0.048	0.024
	87.9	1.053	0.635	0.475	0.384	0.075	0.038	0.047	0.024
	90.0	1.058	0.640	0.481	0.391	0.075	0.037	0.047	0.023
10:1	0.00	0.562	0.085	0.029	0.014	0.344	0.168	0.290	0.153
	17.4	0.707	0.176	0.052	0.016	0.128	0.059	0.078	0.037
	29.1	0.848	0.276	0.104	0.016	0.110	0.054	0.068	0.034
	40.9	0.962	0.389	0.188	0.082	0.102	0.051	0.064	0.032
	52.6	1.051	0.498	0.288	0.169	0.098	0.049	0.062	0.031
	64.4	1.115	0.590	0.385	0.265	0.096	0.048	0.060	0.030
	76.1	1.157	0.653	0.460	0.346	0.095	0.047	0.060	0.030
	87.9	1.183	0.685	0.496	0.387	0.094	0.047	0.059	0.029
	90.0	1.187	0.691	0.503	0.394	0.094	0.047	0.059	0.029

TABLE 9--Influence coefficients for inside axial surface crack: $a/t = 0.5$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_{cl}=6.35$	K1 $t_{cl}=6.35$	K0 $t_{cl}=3.96$	K1 $t_{cl}=3.96$
2:1	0.00	0.736	0.132	0.053	0.029	0.327	0.162	0.272	0.150
	15.4	0.746	0.203	0.083	0.043	0.079	0.037	0.045	0.020
	27.5	0.719	0.263	0.124	0.067	0.042	0.020	0.025	0.012
	39.6	0.704	0.327	0.183	0.112	0.029	0.014	0.018	0.009
	51.7	0.693	0.383	0.249	0.175	0.023	0.012	0.015	0.007
	63.7	0.685	0.426	0.311	0.242	0.021	0.010	0.013	0.006
	75.8	0.681	0.456	0.357	0.299	0.019	0.009	0.012	0.006
	87.9	0.676	0.483	0.389	0.334	0.018	0.009	0.011	0.006
6:1	90.0	0.676	0.488	0.395	0.340	0.017	0.009	0.011	0.005
	0.00	0.758	0.142	0.059	0.033	0.322	0.163	0.268	0.149
	15.4	0.814	0.213	0.083	0.040	0.091	0.041	0.054	0.025
	27.5	0.908	0.302	0.132	0.065	0.070	0.034	0.043	0.021
	39.6	0.998	0.405	0.208	0.116	0.065	0.032	0.040	0.020
	51.7	1.069	0.504	0.300	0.195	0.062	0.031	0.039	0.019
	63.7	1.120	0.588	0.392	0.285	0.061	0.030	0.038	0.019
	75.8	1.153	0.647	0.463	0.363	0.060	0.030	0.038	0.019
10:1	87.9	1.182	0.679	0.500	0.404	0.059	0.029	0.037	0.018
	90.0	1.187	0.685	0.506	0.411	0.059	0.029	0.037	0.018
	0.00	0.666	0.119	0.049	0.028	0.302	0.156	0.254	0.140
	15.4	0.822	0.208	0.077	0.033	0.097	0.044	0.060	0.028
	27.5	0.995	0.316	0.131	0.056	0.086	0.042	0.054	0.027
	39.6	1.138	0.440	0.216	0.112	0.083	0.041	0.052	0.026
	51.7	1.251	0.560	0.321	0.198	0.083	0.041	0.052	0.026
	63.7	1.335	0.662	0.425	0.298	0.083	0.041	0.052	0.026
	75.8	1.390	0.734	0.506	0.383	0.083	0.041	0.052	0.026
	87.9	1.423	0.770	0.546	0.427	0.083	0.041	0.052	0.026
	90.0	1.429	0.776	0.553	0.434	0.083	0.041	0.052	0.026

TABLE 10--Influence coefficients for inside circumferential surface crack: $a/t = 0.5$.

Aspect Ratio	Elliptic Angle	K0 Uniform	K1 Linear	K2 Quadratic	K3 Cubic	K0 $t_{cl}=6.35$	K1 $t_{cl}=6.35$	K0 $t_{cl}=3.96$	K1 $t_{cl}=3.96$
2:1	0.00	0.741	0.134	0.054	0.030	0.324	0.162	0.269	0.151
	15.4	0.750	0.205	0.084	0.044	0.079	0.038	0.045	0.020
	27.5	0.721	0.264	0.124	0.067	0.042	0.020	0.025	0.012
	39.6	0.706	0.328	0.183	0.112	0.029	0.014	0.018	0.009
	51.7	0.698	0.384	0.250	0.175	0.024	0.012	0.015	0.007
	63.7	0.692	0.430	0.312	0.243	0.021	0.010	0.013	0.007
	75.8	0.686	0.461	0.360	0.301	0.019	0.010	0.012	0.006
	87.9	0.682	0.488	0.392	0.336	0.020	0.010	0.012	0.006
6:1	90.0	0.682	0.493	0.398	0.343	0.020	0.009	0.013	0.006
	0.00	0.727	0.132	0.053	0.030	0.315	0.161	0.262	0.147
	15.4	0.786	0.205	0.079	0.037	0.087	0.039	0.052	0.024
	27.5	0.882	0.295	0.128	0.062	0.067	0.032	0.041	0.020
	39.6	0.974	0.398	0.205	0.114	0.062	0.031	0.038	0.019
	51.7	1.049	0.499	0.298	0.193	0.060	0.030	0.037	0.019
	63.7	1.103	0.584	0.390	0.284	0.058	0.029	0.036	0.018
	75.8	1.138	0.644	0.462	0.362	0.057	0.029	0.036	0.018
10:1	87.9	1.166	0.676	0.499	0.403	0.058	0.029	0.036	0.018
	90.0	1.171	0.682	0.506	0.410	0.058	0.029	0.036	0.018
	0.00	0.616	0.101	0.040	0.023	0.291	0.152	0.247	0.138
	15.4	0.770	0.195	0.071	0.028	0.090	0.039	0.055	0.026
	27.5	0.936	0.301	0.125	0.053	0.078	0.038	0.049	0.024
	39.6	1.076	0.424	0.211	0.109	0.075	0.037	0.047	0.024
	51.7	1.190	0.544	0.315	0.196	0.075	0.037	0.047	0.023
	63.7	1.275	0.647	0.420	0.295	0.075	0.037	0.047	0.023
	75.8	1.330	0.719	0.501	0.381	0.075	0.037	0.047	0.023
	87.9	1.363	0.755	0.542	0.425	0.075	0.037	0.047	0.024
	90.0	1.368	0.762	0.549	0.433	0.075	0.037	0.047	0.024

COMPARISON WITH OTHER INVESTIGATORS

A comparison of some of the 3-D influence coefficients generated in this study with similar coefficients obtained by other investigators [3,4] is shown in Figs. 7–9. To compare the coefficients with those obtained from Raju and Newman [3], the influence coefficients in Figs. 7 and 8 represent K_j^* values multiplied by $\sqrt{Q}$, where the shape factor (Q) is the square of the complete elliptic integral of the second kind. The shape factor for an elliptical crack is approximated by the following equation:

$$Q = 1 + 1.464 (a/c)^{1.65} \quad (4)$$

As indicated in Figs. 7 and 8, the coefficients being compared pertain to axially oriented, inner-surface flaws in a long cylinder with $a/t = 0.2$ and 0.5 and aspect ratios of 2:1 and 10:1.

FIG. 7--Influence coefficients for axially oriented semielliptical flaws ($a/t = 0.2$) on inner surface of cylinder: (a) 2:1 aspect ratio; (b) 10:1 aspect ratio

FIG. 8--Influence coefficients for axially oriented semielliptical flaws ($a/t = 0.5$) on inner surface of cylinder: (a) 2:1 aspect ratio; (b) 10:1 aspect ratio.

The coefficients in Fig. 9 (Ref. 4) are K_j^* values for an aspect ratio of 6:1 and $a/t = 0.5$. The results of all the comparisons indicate good agreement. The other investigators [3,4] did not publish SIFICs for the shallower flaw depths so a comparison could not be made for a/t ratios less than 0.2.

FIG. 9--Influence coefficients for an axially oriented semielliptical flaw with $a/t = 0.5$ and 6:1 aspect ratio.

SUMMARY AND CONCLUSIONS

The results of the present study show that there are essentially no differences in SIFICs for finite inner-surface axial and finite inner-surface circumferential flaws having flaw depths ($0.01 \leq a/t \leq 0.5$) and flaw aspect ratios ($2c/a = 2, 6, 10$). This important finding implies that SIFICs for axial flaws can be used for circumferential flaws as deep as halfway through the vessel wall with very little error provided that $2c/a \leq 10$. As might be expected, the greatest difference (~5%) between the two is obtained for flaw geometries with an $a/t = 0.5$ and $2c/a = 10$. For implementation into fracture mechanics codes, it is recommended that SIFICs calculated for axial flaws be used for circumferential flaws having an $a/t < 0.5$ and $2c/a \leq 10$. The SIFICs given in Table 10 should be utilized for circumferential flaws having an $a/t = 0.5$.

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