THE CORRELATION OF COUPLED HEAT AND MASS TRANSFER EXPERIMENTAL DATA FOR VERTICAL FALLING FILM ABSORPTION

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ABSTRACT

Absorption chillers are gaining global acceptance as quality comfort cooling systems. These machines are the central chilling plants and the supply for comfort cooling for many large commercial buildings. Virtually all absorption chillers use lithium bromide (LiBr) and water as the absorption fluids. Water is the refrigerant. Research has shown LiBr to be one of the best absorption working fluids because it has a high affinity for water, releases water vapor at relatively low temperatures, and has a boiling point much higher than that of water. The heart of the chiller is the absorber, where a process of simultaneous heat and mass transfer occurs as the refrigerant water vapor is absorbed into a falling film of aqueous LiBr. The more water vapor absorbed into the falling film, the larger the chiller's capacity for supporting comfort cooling. Improving the performance of the absorber leads directly to efficiency gains for the chiller.

The design of an absorber is very empirical and requires experimental data. Yet design data and correlations are sparse in the open literature. The experimental data available to date have been derived at LiBr concentrations ranging from 0.30 to 0.60 mass fraction. No literature data are readily available for the design operating conditions of 0.62 and 0.64 mass fraction of LiBr and absorber pressures of 0.7 and 1.0 kPa.

Experiments were conducted on an internally cooled smooth tube 0.01905 m in outside diameter and 1.53 m in length. Tests were conducted with no heat and mass transfer additive. The data, for testing at 0.62 and 0.64 mass fraction of LiBr, were scaled and correlated into both Nusselt (Nu) and Sherwood (Sh) formulations. The average absolute error in the Nusselt correlation is about $\pm 3.5\%$ of the Nu number reduced from the experimental data. The Sherwood correlation is about $\pm 5\%$ of the reduced Sh data. Data from the open literature were reduced to the authors' Nu and Sh formulations and were within 5% of the correlations developed in the present study. Hence, this study provides correlations for the complex heat and mass transfer process that is validated against extensive experimental data. The study therefore contains useful information for the design of a vertical column absorber operating with no heat and mass transfer additive.

INTRODUCTION

The capacity of a chiller-or more specifically, the evaporator load-is directly proportional to the mass of refrigerant assimilated in the absorber. Improving the absorber's transport processes increases the chiller's capacity to support comfort cooling. The absorber is the largest of the machine's heat exchangers, and in the past its design was based on rules of thumb, resulting in heavy, costly, and oversized equipment. The design was created through a semi-empirical approach based on log-mean temperature difference. Experiments would be conducted and an empirical map would be derived for the overall heat transfer coefficient and the absorber load. The results were scaled to the desired machine capacity based on the experimental data as well as on the experience of the designer. Changing absorber configuration elements such as tube spacing, diameter,

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NOMENCLATURE

SYMBOL	DESCRIPTIVE	SI UNITS
A	Area	m ²
C _a	Concentration of absorbate	g-mole H ₂ O/cc soln
C _p	Specific heat	kJ/kg·K
D, d	Pipe diameter	m
D _{AB}	Diffusivity of a through b	m ² /s
g	Gravity	m ² /s
H _m	Overall liquid-side mass transfer coefficient	m/s
H	Overall heat transfer coefficient	J/ m ² · K · s
i _{fg}	Latent heat of water vapor	kJ/kg
k	Thermal conductivity	J/m · K · s
m	Mass flow rate	kg/s
ṁ	Absorbate mass flux	kg / m ²
q	Heat transfer rate	J/s
q''	Heat flux	J/ m ²
P _v	Absorber pressure	kPa
T	Temperature	°C
U	Average falling film velocity	m/s
x	Coordinate in direction of flow	m
y	Coordinate normal to direction of flow	m
A	Average film thickness	m
δ	Film thickness as function of t, x, and y	m
ρ	Density	g/cc
η'	Molecular weight of water	g/g-moles
ν	Kinematic viscosity	m ² /s
μ	Dynamic viscosity	kg/m · s
X	Mass fraction	kg LiBr / kg soln
σ	Surface tension	N/m
Subscripts		
CL	Coolant	
eq	Equilibrium	
ex	Exit	
f	Film	
in	Inlet	
	Maximum	
o	Outside diameter or outside radius	
v	Absorbate vapor	
wall	Tube wall	

Dimensionless Groups

ε	Aspect ratio	$\varepsilon = \delta / x$
Ja	Jacobs number	$\frac{C_p \Delta T}{i_{fg}}$
Ka	Kapitza number	$\frac{\sigma}{\rho (v_g^4)^{\frac{1}{3}}}$
Le	Lewis number	$\frac{\alpha}{D_{ab}} \equiv \frac{Sc}{Pr}$
Nu	Nusselt number	$\frac{H D_o}{k_f}$

Pe_H	Peclet heat
Pe_M	Peclet mass
Pr	Prandtl number
Re	Reynolds number
Sc	Schmidt number
Sh	Sherwood number

$$\frac{4\bar{U}\Delta}{\alpha} \equiv Re \cdot Pr$$

$$\frac{4\bar{U}\Delta}{D_{ab}} \equiv Re \cdot Sc$$

$$\frac{C_P\mu}{k} \equiv \frac{\nu}{\alpha}$$

$$\frac{4\bar{U}\Delta}{\nu} \equiv \frac{4\dot{m}}{\pi D\mu}$$

$$\frac{\nu}{D_{ab}}$$

$$\frac{H_m D_o}{\pi D\mu}$$

length, and material, or changing the operating conditions would invalidate the design; and experiments would be rerun to redevelop the empirical map.

Over the last three decades several researchers have analytically investigated simultaneous heat and mass transfer. Emmert and Pigford (1954) developed an approximate solution for isothermal mass transfer. Grigor'ycva and Nakoryakov (1977) and later Grossman (1983) analytically solved the energy and specie equations for the heat and mass transfer to a laminar falling film having a constant temperature or an adiabatic wall boundary condition. Wekken and Wassenaar (1988), Habib and Wood (1990), and Yang and Wood (1992) completed numerical studies that either complemented or furthered the earlier work on laminar falling film absorption. Morioka and Kiyota (1991), Yang and Jou (1993), Patnaik (1994), Kim et al. (1995), Conlisk (1995), and Patnaik and Perez-Blanco (1996) each completed analytical studies of the wavy laminar problem.

In all these studies the analysts assumed the liquid-vapor interface to be saturated at the pressure of the vapor in order to solve the problem. In some studies, transport coefficients were gleaned from such works as Bays and McAdams (1973) for heat transfer and Yih and Chen (1982) for mass transfer. The approach used by Patnaik and Perez-Blanco (1996) required local heat and mass transfer coefficients to characterize the transport associated with the local flow conditions.

The formulation for the liquid-vapor boundary conditions has received much discussion. At issue is the exchange of mass across the liquid-vapor interface, which involves the generation of heat. Water has a large heat of absorption—about 2700 kJ/kg—and heat effects associated with the mass exchange increase the temperature, which affects the pressure and composition and in turn affects the mass transfer. Aqueous LiBr is a homogenous, real mixture that requires empirical data to quantify its pressure, temperature, and composition. Hence, mixing rules for ideal mixtures are not applicable for calculating non-equilibrium conditions, further complicating the complex interface boundary condition. Without a priori knowledge of the local variations of

heat and mass transport, researchers assumed equilibrium to be the best approximation to the actual condition. The findings by Miller (1998) on the constant flux boundary simplify the solution because they imply that the local temperature gradient from the coolant to the interface remains approximately constant along the length of an absorber tube.

Miller (1998) also showed that the heat transfer affected the mass transfer coefficient, and that therefore a mass transfer coefficient derived from isothermal testing would not adequately describe simultaneous heat and mass transfer. The local mass transfer coefficient decreased while the heat transfer coefficient increased as the film transitioned into fully developed hydrodynamic flow. In fact, the use of independent heat and mass transfer coefficients actually decouples the transport. The more exothermic the heat release, the larger the temperature gradient through the liquid phase. Heat flows through the film and further influences the thermodynamic equilibrium, which in turn affects the mass transfer. The greater the heat effect achievable, the lower will be the concentration gradient available for mass transfer. Conlisk (1995) showed analytically that as the inlet film thins, the mass transfer coefficient decreases and the heat transfer coefficient increases. Therefore, approaching the problem with independently derived transport coefficients can be misleading in evaluating the load and the mass absorbed.

The problem was therefore approached from an overall rather than from a local perspective. A correlation was formulated by using the flows, temperatures, and concentrations entering and exiting the absorber to avoid the uncertainties in the local variations of the interface temperature and concentration. The correlation also used the tube diameter rather than the film thickness to avoid the confounding effect of the film's hydrodynamics. Hence, a correlation of simultaneous heat and mass transfer was formulated that is based on overall heat and mass transfer coefficients. The overall coefficients are derived from the absorber load and the mass absorbed. Given the findings of a constant flux boundary reported by Miller, the correlation can be scaled to the specific length and diameter of absorber tubes.

Validation of the correlation against extensive experimental data follows in the text of this report.

EXPERIMENTAL SETUP

A single absorber tube with a 0.01905-m outside diameter and a length of 1.524 m was tested to obtain baseline data for the purposes of formulating correlations of heat and mass transfer. Testing was conducted over a range of operating conditions that included near-field conditions for residential and commercial systems. In air-cooled applications the operating conditions would be about 62 wt% LiBr, with an absorber pressure of about 1.5 kPa and an inlet cooling water temperature of about 46°C. Testing was also conducted at a cooling water temperature of 35°C to simulate load rejection to a cooling tower. In commercial equipment, some of the solution leaving the absorber is circulated back and mixed with strong solution to improve wetting of the absorber tubes. Thus, the net concentration entering the absorber is about 60 to 62 wt%. Testing was therefore conducted at 60, 62, and 64 wt%, for absorber pressures of 1.5, 1.3, and 1.0 kPa, to also characterize operating conditions for commercial application.

UNCERTAINTY ANALYSIS

An uncertainty analysis was conducted to determine the maximum error for the load and the mass absorbed. Since the true value of a given measurement is not directly known, it is more accurate to speak of experimental uncertainty than of experimental error. The uncertainty analysis accounts for both the random error and the systematic error. Random error is due to the lack of repeatability and therefore describes the precision of measurement. Statistics are applied to the random error to establish measurement bands within which replication of measurement is assured to a given confidence interval. Systematic error is the bias from the true measurement and describes the inaccuracy of a calibration procedure. Miller (1998) gives the derivation and calculation of the uncertainty in absorber load and the mass absorbed.

The uncertainty in the absorber load measured on the coolant side was $\pm 7.0\%$ at 95% confidence. As expected, the major error in the load was due to the temperature gain of the coolant. The average uncertainty in the mass absorbed for all the reported experimental data is $\pm 25\%$ at 95% confidence. Note that the uncertainty analysis indicates the maximum possible error in the data. Measurements on the brine side and the coolant side showed the calculated heat balance within about 10% agreement.

ABSORPTION TEST RESULTS

We established the independent parameters of flow, temperature, concentration, and pressure to start testing. Flow rate was the easiest variable to change, and therefore the effects of the other variables were tested one parameter at a time for falling film mass flows ranging from 0.014 to 0.026 kg/s (Re numbers of 100–400). The load is defined as the heat removed from the absorber by the coolant; it is plotted in Fig. 1. The increase in load is

directly related to the mass of vapor absorbed into the falling film; it is plotted in Fig. 2. The more vapor absorbed, the greater the capacity of a chiller's evaporator. Absorber load, then, is a good indicator of system performance, although the mass absorbed is the important variable.

Stable wavy-laminar roll waves and, at the highest flows, second-transition turbulent laminar roll waves were observed during testing. The inception of the waves appeared to be

Fig. 1. Absorber load as a function of concentration, pressure, coolant temperature, and falling film mass flow rate.

Fig. 2. Mass absorbed as a function of concentration, pressure, coolant temperature, and falling film mass flow rate.

influenced by the solution flow rate. At the lowest flow of 0.014 kg/s, the inception of short waves of small amplitude were seen about 0.2 m from the inlet. The short waves transition into a longer roll-wave whose length and amplitude were at least twice those of the short wave. The results show that the mass absorbed and the load are not strongly affected by the falling film mass flow rate. Increasing the mass flow rate of the falling film shows an effect only at the lowest flow rates. These relatively flat data differed from the results of Grossman and Alsfeld (1996), who tested at a higher absorber pressure of 8.7 kPa, and introduced the solution into the absorber with about 15 to 20°C of subcooling. For the present study, the solution entered the absorber saturated at about the operating pressure for chiller equipment.

The data are separated and plotted in Fig. 3 to better indicate the effects of absorber pressure, concentration, and coolant temperature. Pressure and concentration both have strong effects on the mass absorbed. Testing at 64 wt% LiBr and 46°C coolant showed the mass absorbed to increase by almost 200% as the pressure was increased from 1 to 1.5 kPa [Fig. 3(a)]. Similar increases in mass absorbed were observed when the concentration was increased from 60 to 64 wt% LiBr for a coolant temperature of 46°C and pressure of 1.5 kPa [Fig. 3(h)]. For testing at 62 wt% and 1.3 kPa pressure, a decrease in coolant temperature from 46 to 35°C also caused the mass absorbed to nearly double [Fig. 3(c)].

In the absence of non-condensables, the resistance in the vapor phase is negligible, allowing the vapor pressure to be imposed on the interface. The driving force for absorbing the water vapor into the falling film is then the difference between the pressure of the absorber vapor on the interface and the partial pressure of the absorbate water in the liquid-vapor interface. Obviously, the driving force increases with an increase in the absorber pressure. The driving force would also increase as the concentration of LiBr increases. The higher the LiBr concentration, the lower the partial pressure of water in the brine, and therefore the greater the driving force for mass transfer. Lowering the cooling water temperature causes a steeper approach temperature for the falling film. The interface temperature drops, which lowers the partial pressure of water vapor within the interface. The driving potential then increases, in turn increasing the mass absorbed and the load. Hence, the plots shown in Fig. 3 are consistent with absorption theory. However, once the vapor has been absorbed, the driving force is liquid-side-controlled because of the high resistance to mass diffusion. After absorption, the mass transfer driving force is defined in terms of the difference in concentration between the interface and the bulk of the film. The higher the bulk concentration of LiBr, the larger this liquid-side driving force and the more mass absorbed into the bulk. The absorbate tends to lie on the interface, and continued mass transfer requires a cooling of the interface. The results show the coupling between mass transfer and heat transfer that is tied to the composition of the interface, the diffusion of absorbate from the

Fig. 3. The effect of the independent variables of pressure, concentration, and coolant temperature on mass absorbed.

interface, and most important, the transfer of heat from the interface.

Formulation of Nu and Sh Numbers

Grigor'yeva and Naroryakov (1977). Seban and Faghri (1978), and Grossman (1982) developed heat and mass transfer correlations by solving the energy and diffusion equations using the classical Gratz-Nusselt approach in which both diffusion and conduction are neglected in the direction of flow. Their correlations typically take the form

$$Nu = f\{Re, Pr, \epsilon\} \quad \text{and} \quad Sh = f\{Re, Sc, \epsilon\} \quad (1)$$

The Nu and Sh formulations account for the effects of viscosity, diffusivity, and inertia; but the coupled heat and mass transfer process has very strong pressure, concentration, and temperature effects, as seen in Fig. 3. Yang and Jou (1993)

accounted for the effect of pressure by including in Eq. 1 a pressure term normalized to the arbitrary vapor pressure of 1.133 kPa. The formulations in Eq. 1 do not explicitly account for the effect of concentration and pressure. Therefore, accurate correlation of simultaneous heat and mass transfer data requires a judicious selection of the temperature and concentration driving forces.

Heat Transfer

In conventional equipment, superheated brine is sprayed over an absorber tube bundle. As a spray, the brine flashes and is driven to equilibrium with the absorber pressure; it is saturated when it hits the first absorber tubes. In the mini-absorber used in the test stand, brine entered either slightly superheated or slightly subcooled. To simulate conventional operation, the dispenser was fabricated with vapor ports to allow the brine the opportunity to equilibrate before being introduced to coupled heat and mass transfer. Because the brine was initially saturated, the driving force was well defined at the top of the absorber tube. Miller (1998) describes an algorithm used to calculate the entering interface temperature and concentration based on saturation with the absorber pressure. The interface temperature at that instant is the maximum temperature observed on the interface of the falling film and is defined as $T_{f \max}$. The corresponding concentration is the lowest water concentration, which is defined as $C_{a \text{EQ}}$. Hence, both variables are useful for describing the overall driving forces for heat and mass transfer.

The value $T_{f \max}$ is dependent on the entering concentration, the temperature, and the pressure of the absorber, and would account for their independent effects. The entering coolant temperature is the lowest temperature that the film could ideally achieve. Therefore, an overall heat transfer coefficient was defined in terms of the load and this maximum achievable temperature gradient:

$$\frac{q}{A_o} = q''_{\text{wall}} \equiv H(T_{f \max} - T_{\text{CL-in}}) \quad (2)$$

The load can also be defined in terms of the temperature gradient along the wall as

$$q''_{\text{wall}} = -k \frac{dT}{dy} \bigg|_{y=0} \quad (3)$$

Using the scales of $\bar{T} = \frac{T}{(T_{f \max} - T_{\text{CL-in}})}$ and $\bar{y} = y/D$, and

then substituting for the overall heat transfer coefficient from Eq. 2 yields the following overall Nu number:

$$\text{Nu} \equiv \frac{H D_o}{k_f} = \frac{(q''_{\text{wall}}) D_o}{k_f (T_{f \max} - T_{\text{CL-in}})} \quad (4)$$

Inspection of Eq. 4 shows that the effects of coolant temperature, absorber pressure, and solution concentration are supported implicitly through the defined overall temperature gradient. If the entering concentration of vapor pressure increases, then $T_{f \max}$ will increase and account for the larger driving forces.

Mass Transfer

Care was taken to make sure the test section operated under vacuum pressure with only trace amounts of non-condensables. Therefore, the gas-film resistance was negligible, and the absorber pressure can be assumed to be imposed on the liquid-vapor interface. We do not, however, assume that the interface is saturated with the absorber pressure because the interface concentration is unknown. The absorption process is liquid-side-controlled, and the liquid-side coefficient from the interface to the bulk of the film expresses the driving force for mass transfer.

At the top of the absorber the concentration is $C_{a \text{EQ}}$. The mass diffusivity of aqueous LiBr is very low, on the order of $10^{-9} \text{ m}^2/\text{s}$, and any absorbed mass tends to stay on the interface unless it is mixed into the bulk of the film. The maximum water concentration would be observed at the absorber exit, tied to the film temperature. Therefore, we used an overall concentration driving force based on the continuity of absorbate from the inlet to the exit of the absorber. Use of the derived driving force avoids the uncertainty at the interface and takes advantage of the fact that the solute, LiBr, remains constant. Hence, we define an overall liquid-side mass transfer coefficient as

$$\frac{\dot{m}_v}{A_o} = \dot{m}_v'' \equiv H_m (C_{a \text{EX}} - C_{a \text{EQ}}) \quad (5)$$

The mass flux can also be defined in terms of the concentration gradient along the interface as

$$\dot{m}_v'' = -D_{ab} \eta' \frac{dC_a}{dy} \bigg|_{y=\Delta} \quad (6)$$

The scales of $\bar{C}_a = \frac{C_a}{(C_{a \text{EX}} - C_{a \text{EQ}})}$ and $\bar{y} = y/D$, are

substituted into Eq. 6 and the overall mass transfer coefficient from Eq. 5 is used to yield the following overall Sh number:

$$\text{Sh} = \frac{H_m D_o}{D_{ab}} = \frac{\dot{m}_v'' D_o}{\eta' (C_{a \text{EX}} - C_{a \text{EQ}}) D_{ab}} \quad (7)$$

We defined the overall liquid-side Sh number according to Eq. 7 to avoid use of the equilibrium condition along the interface. Neither the interface temperature nor the concentration are known, and they cannot be determined unless equilibrium is assumed with the absorber pressure. Reductions of liquid coefficients based on

Eq. 5 showed excellent trending of data, whereas an arithmetic or log-mean concentration difference yielded large scatter because they require interface measurements. Eq. 5 has two unknowns, the mass absorbed and the exit concentration. The concentration of water leaving the absorber can be related to the continuity of the solute and to the continuity of mass as follows:

$$C_{a_{ex}} = \frac{\rho}{\eta'} \left[\frac{\dot{m}_{eq}(1 - X_{ex}) + \dot{m}_v}{\dot{m}_{eq} + \dot{m}_v} \right] \quad (8)$$

Given the two equations for the mass absorbed and the exit concentration, both unknowns can be determined if a suitable correlation for the Sh number is formulated from the data. Diffusivity data is reported by Miller (1998) and is used in calculating the Sh number in Eq. 7.

HEAT AND MASS TRANSFER CORRELATIONS

The data shown in Fig. 1 for the load and in Fig. 2 for the mass absorbed were reduced to the Nu and Sh numbers, and the results are depicted in Fig. 4. All data collapse into a tight trend with the exception of Nu data for testing at 60 wt% and 35°C coolant. Sh data for 60 wt% testing fell within the bulk of the data; however, Nu data for 60 wt% testing appears as an anomaly in Fig. 4. The data were checked by repeating some testing at 60 wt%, and the results are shown in Fig. 4 by the + symbols. In fact, these data represent testing that was conducted one month later when testing with the thermographic phosphors (Miller and Keyhani 1999). Both 60 wt% data sets are similar in magnitude and prove that the earlier data are not spurious. All attempts using regression analysis to collapse Nu data for 60 wt% testing failed, and therefore, these data were excluded. Only the 62 and 64 wt% data were included in the regression analysis, and the 60 wt% data were then compared to the correlations of Nu and Sh numbers.

Regression analysis was conducted using statistical application software (SAS). The results for both the Nu and the Sh numbers are listed in Table 1. The Nu number was correlated as function of the Pr, Ka, and Ja numbers, while the Sh number was correlated as function of the SC and Ka numbers. The results of correlating the data are also depicted in Fig. 5. Here, the Nu and Sh data are scaled by the regression parameters listed in Table 1. The curve fit, $C_1 Re^{\alpha_1}$, is then superimposed onto the scaled data to compare the correlation to the reduced experimental data. Both correlation and experiment are in excellent agreement, as seen in Fig. 5. Analysis showed that the Ka number significantly improved the correlation, and its trends were consistent with the regression equation for the Nt number formulated by Miller (1998). Increasing the Ka number resulted in a decrease in the film thickness, which in turn caused an increase in both the heat and the mass transfer.

Miller (1998) found the Ka exponent in his Nt correlation to be negative (i.e., larger Ka yielding a thinner film). Regression yielded positive exponents for the Ka number in the formulations

(a) The overall Nu number was calculated using Eq. 4.

(b) The overall Sh number was calculated using Eq. 7.

Fig. 4. Overall Nu and Sh numbers as a function of concentration, coolant temperature, and falling film mass flow rate.

for the Nu and Sh correlations (i.e., larger Ka, thinner film, and increased load and mass absorbed). If the 60 wt% data were included in the regression, then the correlation parameter for the Ka number was too strong, and it made the Nu and Sh correlations somewhat biased by increasing the exponent of Ka to about 0.8. The SAS routines showed that the Nu correlation improved with inclusion of the Ja number; although its regression coefficient is small, it supports the Nu number by helping to account for film-to-coolant temperature effects. The negative coefficient for the Ja number reflects an increase in Nu number, which again is consistent with the defined heat transfer coefficient in Eq. 2 (i.e., $H = q''/\Delta T$, an increasing Ja number, implies an increasing ΔT , which in turn implies a tower Nu).

The Gratz-Nusselt approach would formulate the Nu and Sh correlations in terms of Pe_H and Pe_M numbers, which cancels the effect of viscosity. However, regression for the Nu number showed the coefficient for the Pr number to be significantly different from the coefficient for the Re number. For the Sh number, it was found that the regression yielded similar coefficients for the Re and SC numbers. Apparently, the effect of viscosity is more pronounced in the Nu formulation than in the Sh (Fig. 5). Also, the thermal boundary layer easily develops within 0.2 m of the inlet; while the concentration boundary layer requires about 1 m because of the strength of the Le number.

Table 1. Heat and mass transfer regression analysis

SAS regression analysis results		
Experimental conditions		
X	(wt% LiBr)	62-64
Pv	(kPa)	1.0-1.5
T _{CL}	(°C)	35-47
Range of regression variables		
Re number	100-400	100-400
Sc number	2500-3200	2500-3200
Pr number	21-25.5	21-25.5
Ka number	440-550	440-550
Ja number	0.005-0.018	0.005-0.018
Regression equation		
Coefficients		
Nu = C ₁ Re ^{a1} Pr ^{a2} Ka ^{a3} Ja ^{a4}		Sh = C ₁ Re ^{a1} Sc ^{a2} Ka ^{a3}
C ₁ = 0.8429		C ₁ = 0.0046
a1 = 0.145664		a1 = 0.771119
a2 = 0.048053		a2 = 0.771119
a3 = 0.330440		a3 = 0.593282
a4 = 4.071242		
Number of data points	86	86
Average absolute deviation	3.45%	4.28%
Maximum error

(a) The Nu correlation is compared to experiment.

(b) The Sh correlation is compared to experiment.

Fig. 5. Nu and Sh numbers correlated from 62 and 64 wt% LiBr data.

A point-by-point review is shown in Fig. 6 for both the Nu and Sh numbers for all experimental data. The ordinate in both 6(a) and 6(b) is the correlation, listed in Table 1, divided by the reduced experimental data. The results prove that the data is well described by the correlations for Nu and Sh numbers. The average overall deviation in Nu number is only 3.45% for the fitted data. The 60 wt% data for the Nu number is about 17% less than the experimental value. The Sh correlation has an average overall deviation of only 4.3% for the fitted data. Inclusion of 60 wt% Sh data causes the average error to increase to 6.7%, with the error for the 60 wt% SH data being about 15% low of the experimental value. The average error was calculated using the formula

$$\text{Averaged Error} = \frac{\sum_{j=1}^N \text{ABS} \left[\frac{1 - \text{Nu}_{\text{corr},j}}{\text{Nu}_{\text{exp},j}} \right]}{N} \quad (9)$$

Runs conducted at 60 wt% and 35°C coolant had the thinnest film as compared to all other data, yet the mass absorbed was comparable to testing at 62 wt% and 35°C coolant (see Fig. 1). It was therefore believed that viscosity and possibly surface tension effects caused the 60 wt% data to behave differently from the bulk of the data. To demonstrate the effect, the transport properties for testing at 60 wt% LiBr were evaluated at concentrations of 50, 55, and 56.5 wt% LiBr (i.e., we assumed the mass absorbed had some

(a) Nu correlation vs experiment.

(b) Sh correlation vs experiment.

Fig. 6. Point-by-point review of the **error** observed in the heat and **mass** transfer correlations.

variable property effect, and caused a local decrease in both viscosity and surface tension). If the transport properties of viscosity and surface tension were evaluated at about 55 wt% LiBr rather than at 60 wt%, then the data closely matched the Nu correlation curve [Fig. 7(a)]. The results affected both the Nu and Sh correlations; however, the effect was more pronounced for the Nu correlation [Fig. 7(a) and (b)].

Wasden and Dukler (1990) had shown that the convective motion of the waves accelerated fluid in the wave front into the mass of the slower substrate. For a hot absorbate, this would force a hotter fluid from the surface into the cooler substrate, thereby creating anomalous temperature gradients. Liquid viscosity, being a strong function of temperature, would drop at the wave front. The less viscous absorbate would be mixed into the substrate and would reduce the viscous shear effect on the wave. Shair (1971) modeled a variable viscosity laminar film and showed that the flow characteristics could be greatly affected by a change in the transport properties within the film. Further, the composition and diffusivity would also be affected by the heat inversion near the wave front. Yih and Seagrave (1980) analyzed gas absorption into liquid films with accompanying chemical reactions. They modeled a linear temperature drop across the film and varied the viscosity, diffusivity, kinetic rate, and gas solubility. They concluded that

(a) The Nu number for 60 wt% data correlates best using transport data evaluated at 55 wt% LiBr.

(b) The Sh number for 60 wt% data correlates best using transport data evaluated at 56.5 wt% LiBr.

Fig. 7. Simulations to observe the effect of viscosity and surface tension on the 60 wt% data.

heat transfer had a profound effect on mass transfer and found that an exponential representation was more accurate than an assumed linear profile. Their study is a useful guide and helps explain the anomalous behavior for testing at 60 wt%.

Observations of the wave structure during testing in the mini-absorber differed from those made during the hydrodynamic study of the falling film. Differences were due to the effects of absorption. During testing in the mini-absorber at 64 wt% LiBr, the solution flow was reduced to a point at which it would just wet the entire tube surface. A further decrease, to 0.013 kg/s, caused the film to break down, and the flow did not wet the entire tube. If the steam valve was closed, the entire surface would re-wet within 30 s. Once again, opening the steam valve caused dry-out to occur within 30 s. The process was successfully repeated several times. Each time the steam valve was opened, dry-out would occur, not instantaneously, but within about 30 s. The results show the strong influence of the absorbate on the hydrodynamics of the falling film. Unfortunately, we did not measure hydrodynamic data during simultaneous heat and mass transfer testing. However, visual observations indicated a larger wave structure during absorption than was observed during the hydrodynamic study. Coupling this observation with the results by Wasden and Dukler (1990) leads to the conclusion that the

viscosity of the falling film varies non-uniformly through the thickness of the film. The variable viscosity effect causes the waves to slide along a less viscous substrate because of the hot absorbate, which is forced from the surface of the wave front into the bulk of the film. The proposed dynamics would further accelerate the waves and yield peak-to-substrate ratios larger than those observed during hydrodynamic testing.

VALIDATION AGAINST INDEPENDENT DATA

Grossman and Alefeld (1996) introduced the brine to the test tube with about 10 to 20°C of subcooling. In commercial chillers the brine is sprayed into the absorber and adiabatically equilibrates with the absorbate vapor prior to hitting the absorber tubes. Therefore, in independent mini-absorber testing, the thermodynamic state of the brine when first exposed to the absorber tube was not as well defined as in the present study. Further, the film-side energy balance best matched the coolant side balance if the temperature and concentration of the entering brine were used in the film-side energy balance rather than the equilibrium state based on the absorber pressure. Hence, for the validations of Grossman and Alefeld the measured entering brine temperature was substituted for the $T_{f, \max}$ temperature.

Grossman's data were reduced using Eq. 4 for the Nu number and Eq. 7 for the Sh number. Data were scaled using the correlation regressions, listed in Table I, and plotted in Fig. 8 against the correlation for Nu and Sh. Grossman's data practically falls right on top of the Nu and Sh correlations and further substantiates the accuracy of the correlations formulated from the present study. The results are very promising. Kim et al. (1995) presented a mass transfer correlation based on an approach of gas absorption into a laminar falling film of aqueous LiBr (Sherwood et al. 1975). Kim's data are plotted on log-log scale and show variation over two decades of the ordinate. By comparison, the correlation presented herein is accurate to within an absolute error of 7% of extensive experimental data.

CONCLUSIONS

Experimental data for falling-film absorption are sparse. The dearth of experimental data strongly suggested the need for a combined experimental and analytical approach to the absorption phenomena. The data must be free of the confounding issue of non-condensables and must provide information in the operating ranges of present-day equipment. No literature data are readily available at the design operating conditions of 62 and 64 wt% LiBr and 0.7 through 1.5 kPa absorber pressure.

This study provides correlations for the complex heat and mass transfer process. The correlations are validated against extensive experimental data and contain useful information for guiding the design of a vertical column absorber with no heat and mass transfer additive. The state of the art in absorber design is still maturing because of the strong market influence incurred in the early 1970s. Rules of thumb coupled to empirical data have

(a) The Nu number predicts Grossman and Alefeld's data.

(b) The Sh correlation predicts Grossman and Alefeld's data

Fig. 8. Correlation for Nu and Sh numbers validated against independent data.

led to extrapolations, causing equipment to be overdesigned, heavy, and costly.

The correlation of Nu and Sh numbers provides a simple and direct calculation of the load and mass absorbed given the entering conditions for the absorber. The Sh correlation is coupled to the continuity of the solute, and the two independent equations yield the overall mass absorbed. Aqueous LiBr is a salt solution, and ions rather than molecules diffuse through the liquid. The theory of diffusion of salts at low concentrations is well developed; however, little information is known about the behavior of more concentrated solutions. The Sh correlation accounts for these intermolecular forces by using the diffusivity data measured by Kashiwagi (1985).

Simultaneous heat and mass transfer has strong pressure, concentration, and temperature effects. The Nu and Sh correlations support these effects through the defined heat and mass transport coefficients. Regression analysis revealed the Ka number as a strong regression parameter; its inclusion significantly improved the correlation of Nu and Sh numbers. Its formulation in Nu and Sh was consistent with the correlation for Nt number. An increasing Ka number resulted in a thinner film for wavy-laminar flow, which in turn would increase load and mass absorbed. Miller (1998) had shown the exponent for Ka number was negative in the Nt correlation; but in Nu and Sh correlations, the Ka power was positive and of the same magnitude as in the Nt

correlation. The average error, excluding 60 wt% data, is $\pm 3.5\%$ for the load and $\pm 4.3\%$ for the mass absorbed.

The 60 wt% data was an outlier as compared to data at 62 and 64 wt% LiBr because of the variable viscosity effects observed for the 60 wt% data having the thinnest falling films. Using viscosity and surface tension transport properties evaluated at lower concentrations of LiBr (about 55.5 wt% LiBr) caused the data to better match the Nu and Sh correlation curves. Evaluations at the lower concentration of 55 wt% LiBr caused the Re number for 60 wt% testing to increase to a maximum value of about 450.

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REFERENCES

- Bays, G., and McAdams, W. 1973. "Heat Transfer Coefficients in Falling Film Heaters: Streamline Flow." *Ind. Eng. Chemistry* 29: 1240-46.
- Conlisk, A. T. 1995. "Analytical Solutions for the Heat and Mass Transfer in a Falling Film Absorber." *Chem. Eng. Sci.* 50, no. 4: 651-60.
- Emmert, R. E., and Pigford, R. L. 1954. "A Study of Gas Absorption in Falling Liquid Films." *Chem. Eng. Prog.* 50, no. 2: 87-93.
- Grigor'yeva, N. I., and Nakoryakov, V. Ye. 1977. "Exact Solution of Combined Heat- and Mass-Transfer Problem during Film Absorption" (in Russian). *Inzh.-Fiz. Zh.* 33, no. 5: 893-98.
- Grossman, G. 1982. *Simultaneous Heat and Mass Transfer in Absorption of Gases in Laminar Liquid Films*. ORNL/TM-8366. Oak Ridge National Laboratory, Oak Ridge, Tenn.
- Grossman, G. 1983. "Simultaneous Heat and Mass Transfer in Film Absorption under Laminar Flow." *Int. J. Heat Mass Transfer* 26, no. 3: 357-70.
- Grossman, G., and Alefeld, G. 1996. *Investigation of Heat and Mass Transfer in Absorption Heat Pumps*. GIF Report, Contract no. 1-0187-100.10191, Technion-Israel Institute of Technology, Haifa, Israel.
- Habib, H. M., and Wood, B. D. 1990. "Simultaneous Heat and Mass Transfer for a Falling Film Absorber—The Two Phase Flow Problem." In *Solar Engineering*, Proceedings of the 12th Annual ASME Int. Solar Energy Conf., ASME, New York, pp. 61-67.
- Kashiwagi, T. 1985. "The Activity of Surfactant in High-Performance Absorber and Absorption Enhancement" (in Japanese). *Refrigeration (Reito)* 60, no. 687: 72-79.
- Kim, K. J.; Berman, N. S.; Chau, D.S.C. and Wood, B. D. 1995. "Absorption of Water Vapour into Falling Films of Aqueous Lithium Bromide." *Int. J. Refrig.* 18, no. 7: 486-94.
- Miller, W. A. 1998. "The Experimental Analysis of Aqueous Lithium Bromide Vertical Falling Film Absorption." Ph.D. diss., University of Tennessee, Knoxville, Tenn.
- Miller, W. A., and Keyhani, M. 1999. "The Experimental Analysis of Local Heat and Mass Transfer Data for Vertical Falling Film Absorption." In *Proc. ASME Advanced Energy Systems Division--1999*, vol. 39, sess. 7B, Nashville, Tenn.
- Morioka, I., and Kiyota, M. 1991. "Absorption of Water Vapor into a Wavy Film of an Aqueous Solution of LiBr." *JSME Int. J.*, Series II, 34, no. 2: 183-88.
- Patnaik, V. 1994. "Combined Heat and Mass Transfer in Wavy-Film Absorption." Ph.D. diss., Pennsylvania State Univ., University Park, Penn.
- Patnaik, V., and Perez-Blanco, H. 1996. "A Study of Absorption Enhancement by Wavy Film Flows." *Int. J. Heat and Fluid Flow* 17: 71-77.
- Seban, R. A., and Faghri, A. 1978. "Wave Effects on the Transport to Falling Laminar Liquid Films." *J. Heat Transfer* 100: 143-47.
- Shair, F. H. 1971. "Dispersion in Laminar Flowing Liquid Films Involving Heat Transfer and Interfacial Shear." *AIChE J.* 17, no. 4: 920-26.
- Sherwood, T. K.; Pigford, R. L.; and Wilke, C. R. 1975. *Mass Transfer*. McGraw-Hill, New York.
- Wasden, F. K., and Dukler, A. E. 1990. "A Numerical Study of Mass Transfer in Free Falling Wavy Films." *AIChE J.* 36, no. Y: 1379-90.
- Wekken, B. J. C. van der, and Wassenaar, R. H. 1988. "Simultaneous Heat and Mass Transfer Accompanying Absorption in Laminar Flow over a Cooled Wall." *Int. J. Refrig.* 11, no. 2: 70-77.
- Yang, R., and Jou, D. 1993. "Heat and Mass Transfer on the Wavy Film Absorption Process." *Can. J. Chem. Eng.* 71: 533-38.
- Yang, R., and Wood, B. D. 1992. "A Numerical Modeling of an Absorption Process on a Liquid Falling Film." *Solar Energy* 48, no. 3: 195-98.
- Yih, S. M., and Chen, K. Y. 1982. "Gas Absorption into Wavy and Turbulent Falling Liquid Films in a Wetted-Wall Column." *Chem. Eng. Comm.* 17: 123-36.
- Yih, S. M., and Seagrave, R. C. 1980. "Mass Transfer in Laminar Falling Liquid Films with Accompanying Heat Transfer and Interfacial Shear." *Int. J. Heat Mass Transfer* 23: 749-57.