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NUMERICAL CALCULATION OF TWO-PHASE TURBULENT JETS

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MASTER

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This work is dedicated to my grandparents.

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NOMENCLATURE

A_p	Particle's projected area.
B_d	Volume of dispersed phase particle.
C_D	Drag coefficient.
C_L	Lift coefficient.
C_p	Interfacial Pressure coefficient.
C_{VM}	Virtual Mass coefficient.
d_d	Dispersed phase diameter.
E	Energy spectrum.
f	Frequency of eddy fluctuations.
F	Interfacial Forces
G	Particle flux Distribution.
g	Acceleration due to gravity
j_g	Superficial gas velocity
j_l	Superficial liquid velocity
k, k_c	Turbulent kinetic energy of the continuous phase.
l_e	Turbulent length scale.
M	Interfacial Source terms for phase k.
N_{Re}	Particle Reynolds number.
p	Pressure
RT	Thermal energy of air molecules in atmosphere.
r_d	Dispersed phase radius.
S_k	Source terms for phase k in the code.

S_{ki}	Interfacial source of turbulence kinetic energy.
S_{ei}	Interfacial source of dissipation.
X_k	Phase indicator function.

Greek Symbols

α	Void fraction
ϵ	Energy dissipated per unit mass
ν	Kinematic viscosity
ν_c	Laminar kinematic viscosity of continuous phase.
ν_t	Turbulent kinematic viscosity
ρ	Density
ϕ	Any dependent variable solved in the code.
τ_c	Time constant for continuous phase.
τ_d	Time constant for dispersed phase.
Γ_k	Interfacial mass exchange for phase k.
ω	Angular frequency.
ω_c	Cut-off angular frequency.

Subscripts

b	Bubble
c	Continuous phase
cl	Centerline quantity.
d	Dispersed phase
i	Interfacial quantity
k	Any phase (c or d)
ki	Interfacial quantity for phase k.
p	Particles
r, R	Relative
t	Turbulent component.

Superscripts

'	Fluctuating component
---	-----------------------

-	Mean component
B	Basset Term
D	Drag
L	Lift
T	Turbulent component.
TD	Turbulent Diffusion
VM	Virtual Mass

ABSTRACT

Saif, Anis A. M.S., Purdue University, May 1995. Numerical Calculations for Two-Phase Turbulent Jets. Major Professor: Dr. Martin A. Lopez de Bertodano.

Two-phase turbulent round jets were numerically simulated using a multi-dimensional two-phase CFD code based on the two-fluid model. The turbulence phenomena were treated with the standard $k-\epsilon$ model. It was modified to take into account the additional dissipation of turbulent kinetic energy by the dispersed phase. Within the context of the two-fluid model it is more appropriate and physically justified to treat the diffusion by an interfacial force in the momentum equation. In this work, the diffusion force and the additional dissipation effect by the dispersed phase were modeled starting from the classical turbulent energy spectrum analysis. A cut-off frequency was proposed to decrease the dissipation effect by the dispersed phase when large size particles are introduced in the flow. The cut-off frequency combined with the bubble-induced turbulence effect allows for an increase in turbulence for large particles.

Additional care was taken in choosing the right kind of experimental data from the literature so that a good separate effect test was possible for our models. The models predicted the experimental data very closely and they were general enough to predict extreme limit cases: water-bubble and air-droplet jets.

1. INTRODUCTION

1.1 Purpose of Research

The subject of two-phase flows has an immense importance in a large variety of industrial and man-related activities. It obeys all the basic laws of fluid mechanics but the analysis involved is more complicated. Hence, starting from simplest one-dimensional techniques, this branch of fluid mechanics has ripened to a point where multi-dimensional treatment of some two-phase problems is possible. The system to be studied here is a two phase turbulent jet that has many varied applications ranging from design of nuclear reactors and chemical reactors to air-pollution studies. The two cases studied are a dispersed bubbly phase in a turbulent water jet and dispersed solid particles in a turbulent air jet. The relative importance of various forces, axial as well as radial, acting on the dispersed phase affects the jet structure. An attempt is made to predict these phenomena with a two-phase CFD model.

The two phase jet can be simulated mathematically in terms of two-fluid model. The dispersed phase is separated from the surrounding medium by a recognizable interface. Hence, the interfacial transfer terms represent a significant parameter. Unless these terms are accurately modeled, there is no advantage to the two-fluid model over the mixture models. It is therefore important to develop better constitutive relations to model the various interfacial forces like drag, lift, virtual mass and turbulent diffusion. Some of the constitutive relations have been successfully tested in the past while others require more development. Also, since the flow involved is turbulent, any attempt to model the system would not be complete unless a turbulence model, that considers production and dissipation of turbulence, is also included.

In the past, such simulations have been done but they either lack the particulate diffusion effect or the additional turbulence dissipation effect due to the presence of dispersed phase. Hence, the objective of the present study is to obtain a complete model of multi-dimensional two-phase turbulent jets which includes the effects mentioned above. The model is general enough to predict bubble-water and air-droplet jets.

1.2 Literature Review

1.2.1 Previous Work

An early theoretical investigation of the two-phase jets, consisting of a mixture of gas with solid particles or liquid droplets was done by Abramovich (1963) [1]. In his simplest analysis, he treated the jet as a single phase jet with pertinent corrections introduced in the expansion laws of jets to take into account the presence of particles. This method provided a useful insight into the complex problem but the results were not close to the experiments. Later two major approaches evolved for the analysis of multiphase systems: (1) locally homogeneous flow analysis and (2) separated flow analysis.

For the locally homogeneous flow analysis, it is assumed that interphase transport rates are much faster in comparison to the rate of development of the flow. The phases involved have the same velocity and they are in thermodynamic equilibrium. The two-phase system reduces to a flow of single-phase fluid that will have a different set of thermophysical properties due to changes in the concentrations of the two-phases as flow develops. Hence, the major advantage of this analysis is that the numerical solution of the governing equations is generally no more difficult than that for a single-phase flow. But since the central assumption of a local mechanical and thermodynamic equilibrium is not realized for most two-phase systems, the homogeneous flow analysis is not very useful.

In contrast, separated flow analysis allows for finite interface transfer rates and hence gives a more complete representation of multiphase processes. Since a complete

solution for this method is extremely complicated, an intelligent empiricism is needed to treat turbulence properties and interphase transfer phenomena. For the present study, a two-fluid model which is a separated flow analysis, is used. The model will be outlined in the next chapter.

Within the separated flow analysis for the two-phase turbulent jets two approaches have been tried so far: the Eulerian approach and the Lagrangian approach. The Eulerian approach uses a fixed frame of reference and treats both the continuous and the dispersed phases as though they are interpenetrating continua. In the Lagrangian approach, the motions of discrete particles are tracked through the flow field using a moving frame (Lagrangian) formulation while an Eulerian formulation is used to solve the conservation equations for the continuous phase. The effect of the particles on the gas phase is taken into account by introducing appropriate source terms in the gas phase conservation equations. Both methods have their merits and demerits: while the Eulerian method becomes almost impractical for polysized particulate suspensions, the Lagrangian method is not good when dealing with heavily laden suspensions where interactions between tracks become important, and the solution of a large number of tracks becomes computationally expensive. Furthermore, it is very difficult to introduce diffusion into the particulate phase with the Lagrangian approach (Moffat and Pericleous, 1990) [22]. In the literature, both approaches have been tried depending on the case being considered.

The Lagrangian approach has been applied in the past by Shuen et. al (1983), Neti and Mohamed (1990) and Mostafa and Mongia(1988) [26, 24, 23]. The Eulerian approach was used by Chen and Wood (1986) [6] . More recently, Bertodano et. al. (1994) applied the Eulerian method to bubbly two-phase flow in a pipe [4].

1.2.2 Important Aspects in Two-Phase Turbulent Jet Flows

The problem of the motion of a continuous jet flow containing a dispersed phase is exceedingly complex. The complexity is enhanced since most of these flows encountered in nature or in industry are turbulent. There are various phenomena due

to the presence of the dispersed phase, ranging from effects on basic flow characteristics to the structure of turbulence, which have been the subject of extensive study. Depending on density ratio between continuous phase and dispersed phase i.e. ρ_d/ρ_c ; we have air bubbles dispersed in a continuous water jet on one extreme to heavy particles dispersed in continuous air jet on the other extreme. These bubbles and particles modify the jet depending on their interactions with eddies in the continuous phase.

The presence of particles causes the jet to be more coherent i.e the centerline velocity decay is reduced with a corresponding reduction in the spreading rate. This happens because the turbulent mixing with the ambient fluid is reduced and hence the jet becomes narrower. The reduction in the turbulent mixing is caused by the damping of turbulence due to the presence of particles. Hence, as expected, when the loading ratio (mass of dispersed phase/ mass of continuous phase) is increased, the spreading of the jet is reduced. The same effect is also observed for a constant mass loading, when smaller particles are introduced in the flow. Now more particles can interfere with a greater number of eddies causing more reduction in turbulent motion (Fleckhaus, 1987) [11].

As mentioned previously, the study of two-phase jets would be incomplete without understanding the characteristics of the turbulent motion involved. The turbulent motion is assumed to consist of the superposition of eddies of various sizes with distinguishable upper and lower limits. The upper size limit of the eddies is determined by the size of the apparatus, whereas the lower limit is determined by viscosity effects. All these various sized eddies have a certain kinetic energy, determined by the intensity of velocity fluctuation of the corresponding frequency. Although in real turbulence a distinct frequency is not permanently present, it is possible on the average to allocate a certain amount of kinetic energy of turbulence to a distinct frequency. Such a distribution of energy between the frequencies is called an energy spectrum (Hinze, 1959) [14].

In a flow field, the largest eddies will cause fluctuations of low frequencies, whereas the smallest eddies will cause fluctuations of high frequencies. According to Kolmogorov's concept of spectral energy transfer, energy is extracted from the large-and slow-energy-containing eddies and cascaded down the spectrum to the fast small scale eddies which are responsible for the major part of turbulent energy dissipation. In the dissipative range of frequency, the energy spectrum has a decreasing trend with increasing frequency. When the dispersed phase is introduced in the flow, the energy spectrum starts decreasing more rapidly with increasing frequency. This happens because of the momentum transfer to the dispersed phase from the fast fluctuating eddies in the continuous phase. The dynamical equation for the energy spectrum is given as

$$\frac{\partial}{\partial t} E(k, t) = T(k, t) - 2\nu k^2 E(k, t) - 2 \frac{\rho_d}{\rho} \frac{1}{\tau_d} E(k, t) \quad [1.1]$$

Here k is the wave number. The left hand side of above equation describes how the energy of an eddy changes with time. The first term on the RHS represents the net energy increase in the eddy due to inertial interaction with all the other eddies. The second term refers to the viscous dissipation of the eddy. The third term on the RHS represents the effect due to the presence of the dispersed phase (Baw and Peskin, 1971) [2].

The response of dispersed-phase particles to turbulent fluctuations depends on the ratio between inertia and the viscous forces. If viscous forces dominate inertia, particles will follow the fluid's turbulent fluctuations whereas if inertia dominates over viscous forces, the particle trajectory is independent of the eddies. The deviation of particle motion from that of fluid, generates a fluctuating relative velocity resulting in additional dissipation of energy due to shearing action (Taweel and Landau, 1976) [28]. The change in the radial variation of mean velocities and turbulent intensities for an axisymmetric two-phase jet arising due to this additional dissipation will be dealt separately in the next chapter. At the same time, its interesting to note a reverse

phenomenon. When a dispersed phase is introduced, the fluctuations of the high-frequency energy-dissipating eddies are dampened and this results in a reduction of the rate of turbulent energy dissipation in the continuous phase (Taweel and Landau, 1976) [28].

Another important feature which should be considered in turbulent two-phase jet flow is the lateral phase distribution. A unique relationship exist between the lateral void distribution and the structure of turbulence in the continuous phase (Wang et.al.,1986) [31]. In their investigation on two-phase bubbly air/water flow in a pipe, Wang et. al. (1986) concluded that the observed wall peaking for up-flows and coring phenomenon in down flows could be predicted by considering the turbulence structure of the continuous phase and the lateral lift force acting on the bubbles. Turbulence induces a lateral pressure gradient which, because of differences in the inertia of bubbles and liquid, causes the bubbles to preferentially collect in the low-pressure zones. However, the effect of lift is more important.

1.2.3 Experimental Investigations

Two-phase flows, in general, have received considerable experimental attention. The data on two-phase jets is abundant in the literature. But there are very few good experimental data sets which presents the entire characteristics of two-phase turbulent jets including the effects of the dispersed phase on the carrier phase turbulence. Early experiments (e.g. Hetsroni and Sokolov, 1971, Serizawa et. al, 1975) used hot-wire anemometry [13, 25]. These data have been criticized on the ground that only a relatively small fraction of the droplets (or bubbles) which impinged on the hot wire were accounted for. Later measurements were all conducted with a Laser-Doppler anemometer which is a non-intrusive measurement. Though LDA yields good measurements of phase velocities, the void fraction measurements are speculative because LDA does not provide a point measurement (Sun and Faeth, 1985) [27].

The effect of the dispersed phase on turbulence properties of the carrier phase has received wide attention in recent years. Whether the addition of dispersed phase will

increase or decrease the turbulent intensity, depends on a critical parameter which is a ratio of dispersed particle diameter, d_d to the turbulent length scale, l_e . Some of the experimental data in the literature which shows either increase or decrease in turbulent intensities with this critical parameter is depicted in figure 1.1. (Gore and Crowe, 1989) [12]. They argued that the smaller particles which follow the fluctuations of turbulent eddies will reduce the turbulent intensity of the carrier phase since part of the turbulent energy of the eddies will transform into turbulent kinetic energy of the particles. On the other hand, the larger particles will tend to create turbulence (in their wake) near the scale of the most energetic eddies, thus increasing the turbulent intensity of the carrier phase.

According to Gore and Crowe's (1989) analysis, Hetsroni and Sokolov's (1971) data would prove ideal to test the dissipation model since they used smaller droplets of size $13 \mu\text{m}$ [13]. But its unlikely that the extremely low void fraction used in their experiment would show an adequate dissipation effect. Shuen et. al. (1983) though used higher void fractions but the particle size was higher [26]. Hence, we decided to use Bulzan's (1988) data which had sufficient small particles. (i.e.: a diameter of $39 \mu\text{m}$ and an inlet void of 10^{-5}) [5]. On the other extreme, for bubbly jets, we use Sun's (1985) detailed experimental data with bubbles of approximately 1 mm. to investigate turbulence in two-phase jets [27].

1.2.3.1 Sun's Data

One advantage of Sun's data is the relatively high velocity of the continuous (liquid) phase in their experiments. At high velocities ($v_c > 1.5\text{m/s}$), the turbulent diffusion force becomes more significant compared to the other lateral forces (i.e. lift) and hence it can be studied and correlated more accurately. To illustrate the above statement, the magnitude of some of these forces is sketched against the continuous phase velocity in Figure 1.2.

Sun (1985) studied the case of a non-condensable turbulent bubbly jet which involves interphase transfer of momentum only [27]. A bubbly jet was injected vertically

Figure 1.1 Change in turbulent intensity as function of length scale ratio.

Figure 1.2 Lateral Interfacial Forces vs. velocity

upward in a still water tank at atmospheric pressure and room temperature with identical injected and ambient liquids. The front and back surfaces of the windowed tank were constructed of plate glass to provide optical access to the flow. Water overflow, due to injection of water, was removed from the tank through a vertical standpipe. The flow was axisymmetric since the direction of injection and acceleration due to gravity were aligned. The large density difference between the phases implied a significant role of buoyancy on the relative motion (slip) between the phases. Measurements were limited to dilute bubbly jets having a nearly monodisperse bubble size distribution and a relatively low maximum void fraction less than 9 %.

This low void fraction flow regime justified the use of LDA for phase velocity measurements, since spurious scattering of the laser light beams was modest. The uncertainties in mean and fluctuating velocities were less than 5 %. Only axial and radial velocity fluctuations were measured. Therefore, the turbulence kinetic energy was estimated by assuming that the radial and tangential velocity fluctuations were equal which is generally true for jets. In addition to uncertainties introduced by this assumption, uncertainties in the kinetic energy of turbulence, k , are higher than velocity fluctuations since k is a quadratic of velocity fluctuations. Estimates indicated uncertainties in k of about 11 %.

Direct flash photography was used to measure the distribution of bubbles. It is a non-intrusive spatial averaging technique. In this technique, the number of bubbles in a region is measured during a short interval of time, so that contents of the region do not change during any signal observation. The photographs were viewed under a microscope for sizing and counting the bubbles. The data for bubble size and distribution were obtained by taking 20 pictures of the flow at each axial location and flow condition. The measurements were obtained from a line of sight observation through the flow which may be deconvoluted to get the local distribution of the bubble density. This measurement was estimated to have an uncertainty of about 12 %.

Four cases were studied: one single phase and three two-phase cases up to an inlet void fraction of 0.09. The measured properties included mean and fluctuating

velocities of both the phases and the distribution of void fraction in the flow field. The important parameters for the experiments of Sun (1985) are tabulated in Table 1.1 [27].

1.2.3.2 Bulzan's Data

Bulzan et. al (1988) conducted an extensive investigation of two-phase particle laden jets in which they introduced solid glass particles into an axially symmetrical round air jet [5]. The jet was injected vertically downward from a 19 mm. diameter tube with swirl numbers ranging from 0 to 0.33. The non-swirling case will be considered here to validate air model. The particle laden jet had a loading ratio of 0.2 with particles having an Sauter mean diameter of 39 μm . The air-phase velocities were measured using a single component phase-doppler anemometer. The air-phase velocity was measured by seeding the particle laden jet and the ambient surrounding with 1 μm nominal aluminium oxide particles. The velocity measured for these particles was taken as a representative for the velocity of the continuous phase. Measurements of mean and fluctuating axial velocities were reported.

The particle velocities were measured using the two-component Ar-Ion laser velocimetry (LV) system. The particle mass flux was measured using an isokinetic sampling probe. Samples were collected on filter paper for a timed interval and weighed. The probes had an inside diameter of 2 and 5 mm to ensure adequate resolution and reasonable sampling times in various regimes of flow. The measured particle mass fluxes when integrated across the jet were found to be within ± 10 percent of the calibrated particle flow rate. The important parameters for Bulzan's experiments are presented in Table 1.2 [5].

Table 1.1 Parameters for Sun's Experiments

Continuous Phase : Water
 Dispersed Phase : Air bubbles
 Diameter of Bubbles : 1 mm (approx.)
 Diameter of Jet (Inlet) : 5.08 mm.

Flow Parameters	Single Phase	Case I (2Φ)	Case II (2Φ)	Case III (2Φ)
Water Flow Rate (ml/s)	32.7	32.7	32.7	32.7
Air Flow Rate (ml/s)	0	0.82	1.64	3.27
Air Volume Flow Fraction (%)	0	2.4	4.8	9.1
Jet Inlet Velocity (m/s)	1.61	1.65	1.68	1.77
Jet Reynolds No.	8530	8740	8860	9380

Measurement Techniques :

Mean and Fluctuating Velocities : Laser Doppler Anemometry

Bubble Sizes and Distribution : Flash Photography

Table 1.2 Parameters for Bulzan's Experiments

Continuous Phase : Air
 Dispersed Phase : Glass Particles ($\rho = 2500 \text{ kg/m}^3$)
 Diameter of Particle : $39 \mu\text{m}$ (approx.)
 Diameter of Jet (Inlet) : 19 mm.

Flow Parameters	Single Phase	Case I (2Φ)
Air Inlet Velocity (m/s)	14.86	13.75
Particle Inlet Velocity (m/s)	0	13.8
Loading Ratio (kg_d/kg_c)	0	0.2
Jet Reynolds No.	18700	17300

Measurement Techniques :

Mean and Fluctuating Velocities : Laser Velocimetry.

Particle Mass Flux Distribution : Iso-kinetic sampling probe.

1.3 Objectives and Thesis Outline

The main objective of the present work is to develop a complete numerical simulation for solving two-phase turbulent jets. Models for the turbulent diffusion force and the dissipation of turbulent kinetic energy in the presence of particles are developed and validated against well established data from the literature.

Following the introduction, the mathematical model including the necessary constitutive equations is presented in Chapter 2. Chapter 3 includes the numerical model and some of the computational details. Also, the result of the numerical calculations and their comparison with the data are presented in Chapter 3. The thesis concludes with a summary and suggestions for future work in Chapter 4.

2. MATHEMATICAL MODEL

As mentioned previously, the mathematical model used here to simulate the two-phase turbulent jets is based on the two-fluid model which is a separated flow model that allows for finite interphase transfer rates. The two-fluid model was derived by Ishii(1975) using a rigorous time average technique of the single phase conservation equations [15]. Time averaging is the most practical averaging method because it is equivalent to the local measuring techniques used in two-phase flow (e.g.- the LDA and the hot wire anemometer). In a two-fluid model, the modeling of two-phase flow is based on an averaged Eulerian description for both phases. It is formulated by considering each phase separately, thus it is expressed by two sets of conservation equations of mass, momentum and energy with interaction terms appearing in the field equations. Constitutive relations are developed and used to represent the interfacial terms.

2.1 Field Equations

The equations governing the motion of each phase have the following general form:
[5]

Mass Conservation:

$$\frac{\partial}{\partial t} \alpha_k \rho_k + \nabla \cdot \alpha_k \rho_k \underline{v}_k = \Gamma_k \quad [2.1]$$

where α_k stands for the local void fraction of phase k, ρ_k is the density, $\underline{v}_k$ is the averaged velocity and Γ_k is the interfacial exchange of mass. In the present study for isothermal flows, Γ_k is zero.

Momentum conservation:

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_k \rho_k \bar{v}_k + \nabla \cdot \alpha_k \rho_k \bar{v}_k \bar{v}_k = & -\nabla \alpha_k p_k + \nabla \cdot \alpha_k (\underline{\tau}_k + \underline{\tau}_k^{Re}) \\ & + \alpha_k \rho_k \underline{g} + \underline{M}_{ki} + \bar{v}_{ki} \Gamma_k - \bar{\tau}_{ki} \cdot \nabla \alpha_k \end{aligned} \quad [2.2]$$

where $\underline{\tau}_k^{Re}$ represents turbulent Reynold stresses. $\underline{M}_{ki}$ is the interfacial momentum transfer (i.e., interfacial forces). Neglecting surface tension effects the jump condition is:

$$\underline{M}_{li} + \underline{M}_{gi} + \bar{v}_{li} \Gamma_l + \bar{v}_{gi} \Gamma_g = \bar{m}_i^{\sigma} \quad [2.3]$$

where $\bar{m}_i^{\sigma}$ is the surface tension source. The interfacial momentum transfer force comprises of force terms due to drag, virtual mass, Basset force, and lift.

$$\underline{M}_{ki} = \underline{M}_{ki}^D + \underline{M}_{ki}^{VM} + \underline{M}_{ki}^B + \underline{M}_{ki}^L \quad [2.4]$$

Various constitutive relations have been proposed for these interfacial terms. For the sake of completeness, energy conservation is listed though the present study is an isothermal case.

Energy conservation:

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_k \rho_k \bar{h}_k + \nabla \cdot \alpha_k \rho_k \bar{v}_k \bar{h}_k = & -\nabla \cdot \alpha_k (\bar{q}_k + \bar{q}_k^{Re}) \\ & + \alpha_k \frac{D_k}{Dt} p_k + a_i \bar{q}_{ki} + \bar{h}_{ki} \Gamma_k + \alpha_k (\underline{\tau}_k + \underline{\tau}_k^{Re}) : \nabla \bar{v}_k \end{aligned} \quad [2.5]$$

where h_k is the time averaged enthalpy, q_k is the heat flux and a_i is the interfacial area concentration. The interfacial jump condition is:

$$\sum_k \bar{h}_{ki} \Gamma_k + a_i \bar{q}_{ki} = \bar{e}_i^{\sigma} \quad [2.6]$$

where $\bar{e}_i^{\sigma}$ is the interfacial energy source. The interfacial transport of mass, momentum and energy are usually characterized as source terms per unit area multiplied by the

interfacial area. For example, the interfacial transport of heat for phase k in equation (2.5) is given by $a_i \bar{q}_{ki}$.

2.2 Constitutive Equations

The averaged source terms on the RHS of the conservation equations represents all the physics occurring at the microscopic level (i.e at the thin interface between the phases). These sources must be specified in terms of the averaged quantities on the LHS of the equations to solve the two-fluid model completely. Hence the constitutive relations have been based on analysis, experiments and judgement. The strategy chosen is to start with a well-established set of 1-D constitutive relations and expand and modify it to suit the multi-dimensional two-phase model.

Looking back at the momentum conservation equation, we see that the interfacial momentum transfer term can be represented as:

$$\underline{M}_{ki} = \underline{M}_{ki}^D + \underline{M}_{ki}^{VM} + \underline{M}_{ki}^B + \underline{M}_{ki}^L \quad [2.7]$$

The first three forces are widely used in 1-D analysis and have been well correlated by Ishii and Chawla (1979) [16]. In a numerical scheme, the virtual mass force has an important role to play: by increasing the inertia of the light phase the differential equation becomes less 'stiff' and numerical convergence is achieved faster. The Basset force accounts for the additional drag associated with the unsteady motion in the viscous fluid. The Basset time constant is usually much smaller than the acceleration time of the particles along their Lagrangian trajectories and hence is negligible in most cases including the case studied here.

The lift force is a lateral force which appear only in multi-dimensional models. The lift force has been modelled by Drew and Lahey (1987) [10]. Turbulent diffusion causes additional lateral displacement of the dispersed phase and have been originally included in the work of Lopez de Bertodano (1992) [3]. However, validated correlations for these forces over a wide range of conditions do not exist. The present

study focuses on modeling the force of turbulent diffusion and validating it with two types of two-phase turbulent jets namely, bubbly-water and air-droplet jets.

2.2.1 Drag Force

The drag source term on a dispersed phase(d) is given by:

$$\underline{M}_d^D = -\frac{1}{2} C_D \frac{\alpha_d}{B_d} A_p \rho_c \underline{\bar{v}}_R |\underline{\bar{v}}_R| \quad [2.8]$$

where $\underline{\bar{v}}_R = \underline{\bar{v}}_c - \underline{\bar{v}}_d$, is the relative velocity; A_p , is the projected area by dispersed phase; B_d , volume of each dispersed phase particle; α_d , dispersed phase void fraction; C_D , the drag coefficient.

Drag coefficient correlations for flows with multiple particles were developed by Ishii and Chawla (1979) [16]. The drag coefficients, C_D has been correlated for the whole spectrum of particle sizes and shapes. For the present calculations in the viscous-wake regime, the following correlation was used:

$$C_D = \frac{24}{N_{Re}} (1 + 0.1 N_{Re}^{0.75})$$

Here, N_{Re} is the bubble Reynolds number and is given as $N_{Re} = d_d \rho_c \underline{\bar{v}}_R / \mu_c$. In the present work, bubble breakup and coalescence, and distribution of bubble sizes have been ignored.

2.2.2 Virtual Mass

The virtual mass represents the force required to accelerate the apparent mass of the particle relative to the ambient fluid. Virtual mass terms have a stabilizing effect on the numerical calculation of two-phase flows by altering the eigenvalues of the system of equations. But, since these terms are important only at high frequencies and during rapid acceleration, they do not affect the final results in most cases (Drew et. al., 1979) [9]. Inclusion in a numerical scheme makes the differential equation less stiff, ultimately enhancing numerical convergence. For instance, Lahey et.al (1980) found that the inclusion of virtual mass terms reduced the computation time of a one-dimensional steady-state calculation by a factor of 40 [19].

The virtual mass source term given by Drew and Lahey (1987) for a single particle and extended to multiple particles according to Zuber (1964) is [10, 32]:

$$\underline{M}_d^{VM} = -C_{VM}\rho_c\alpha_d\frac{(1+2\alpha_d)}{(1-\alpha_d)}\left[\frac{D_d\bar{v}_d}{Dt}-\frac{D_c\bar{v}_c}{Dt}\right] \quad [2.9]$$

For an isolated, rigid spherical bubble, C_{VM} is equal to 0.5 corresponding to the limit of low void fraction (Drew, 1987) [10]. However, this value of C_{VM} ignores the effect of (nonspherical) bubble shape. It also ignores details of the interaction between bubbles. A virtual mass coefficient of 0.5 has been used in this study, since low void fraction cases were studied. For identical spherical bubbles, van Wijngaarden (1976), showed that $C_{VM} < \frac{1}{2}$ when one bubble travels in the wake of another, whereas $C_{VM} > \frac{1}{2}$ when two bubbles move side by side [30].

2.2.3 Lift Force

The lift force represents a force that arises from the relative motion of the phases in cross-stream velocity gradients. It is a lateral force which is shown to be important in unbounded shear flows and bounded shear flow as well. The coefficient of the lift force, like the drag force, depends on the particle size. It has been modelled by Drew and Lahey (1987) as [10]:

$$\underline{M}_d^L = -\alpha_d C_L \rho_c \bar{v}_R \times (\nabla \times \bar{v}_c) \quad [2.10]$$

where $C_L = C_{VM}$ for inviscid flows. Additionally, Lance (1991) performed some measurement for large bubbles and obtained $C_L = 0.25$ for bubbles between 1 mm. and 8 mm [20].

2.2.4 Basset Force

The Basset force is given by Ishii and Mishima(1984) as [17]:

$$\underline{M}_d^L = -\alpha_d \frac{9}{2} \frac{1}{r_d} \left[\frac{\rho_c \mu_c}{\pi}\right]^{1/2} \int_{-\infty}^t \frac{D_d}{D\zeta} (\bar{v}_c - \bar{v}_d) \frac{d\zeta}{\sqrt{(t-\zeta)}} \quad [2.11]$$

The Basset force is caused by the change in the boundary layer thickness during acceleration (i.e the boundary layer does not change instantaneously). The time constant for the buildup of the boundary layer is of the order:

$$t_B = \delta^2 / \nu_c$$

where δ is the thickness of the boundary layer. In general the time constant of the transients or the acceleration times of the particles along their Lagrangian trajectories are much greater than the Basset time constant and this force can be neglected.

2.2.5 Interfacial Pressure Source

The interfacial pressure appears in the last term in the momentum equation (2.2). At the interface

$$\bar{\underline{\underline{T}}}_{ki} \cdot \nabla \alpha_k \cong - \bar{p}_{ki} \nabla \alpha_k \quad [2.12]$$

neglecting laminar shear stresses. This term represents the averaged normal stresses over the interface and is different from the net variation of interfacial stresses which is handled by the previously described interfacial sources.

A simple case is the result for potential flow around a sphere:

$$\bar{p}_{ci} = \bar{p}_c - \alpha_d(1 - \alpha_d)C_p \rho_c |\bar{\underline{v}}_R|^2 \quad [2.13]$$

where $C_p = 1/4$. If the particle is not spherical or if it is moving in a wobbly path then $C_p > 1/4$. A value of $C_p = 0.25$ was used in this study.

2.2.6 Turbulent Diffusion Source

The interfacial momentum forces, which were presented previously, have been expressed as a function of mean gas and liquid velocities. The fluctuating components are introduced by modeling the turbulent diffusion term. The motivation to develop a turbulent diffusion term comes from the fact that the averaged two-phase continuity equations does not allow for a phasic diffusion term. That is, since the volume fraction of phase-k, α_k , is defined as an average, terms such as α'_k may not exist. In contrast, a diffusion term may occur in the momentum equation.

Following the analysis of Drew (1983) the averaged continuity equation of phase-k assuming incompressible flow with no phase source is [8]:

$$\frac{\partial}{\partial t} \overline{X}_k + \nabla \cdot \overline{X_k \underline{v}_k} = 0 \quad [2.14]$$

where $\overline{X}_k$ is the phasic indicator function and $\underline{v}_k$ is the instantaneous phasic velocity. Notice that this is a natural way to average because both these quantities are what a hot wire anemometer measures. Then by defining

$$\alpha_k \equiv \overline{X}_k \quad [2.15]$$

and

$$\underline{\bar{v}}_k \equiv \frac{\overline{X_k \underline{v}_k}}{\overline{X}_k} \quad [2.16]$$

the continuity equation becomes:

$$\frac{\partial}{\partial t} \alpha_k + \nabla \cdot \alpha_k \underline{\bar{v}}_k = 0 \quad [2.17]$$

Other investigators have started from this equation and averaged once more to obtain:

$$\frac{\partial}{\partial t} \bar{\alpha}_k + \nabla \cdot \bar{\alpha}_k \bar{\underline{v}}_k + \nabla \cdot \overline{\alpha'_k \underline{\bar{v}}_k} = 0 \quad [2.18]$$

where the last term on the LHS of equation (2.18) allowed them to have a turbulent diffusion term in the continuity equation. Unfortunately, this treatment is not rigorous. However, we may constitute the diffusion phenomena in the two-fluid model using turbulent Reynold stress term in the momentum equation.

Equation (2.2) can be rewritten in the form

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_k \rho_k \underline{\bar{v}}_k + \nabla \cdot \alpha_k \rho_k \underline{\bar{v}}_k \underline{\bar{v}}_k = & -\alpha_k \nabla p_k + \alpha_k \nabla \cdot (\underline{\tau}_k + \underline{\tau}_k^{Re}) \\ & + (\underline{\tau}_k + \underline{\tau}_k^{Re}) \cdot \nabla \alpha_k + \alpha_k \rho_k \underline{g} + \underline{M}_{ki} + \underline{\bar{v}}_{ki} \Gamma_k + (p_{ki} - p_k) \nabla \alpha_k \end{aligned} \quad [2.19]$$

Neglecting viscous stresses, $(\underline{\tau}_k + \underline{\tau}_k^{Re})$ may be separated into normal and shear stresses:

$$\rho_k \begin{pmatrix} \overline{u_k'^2} & \overline{u_k'v_k'} & \overline{u_k'w_k'} \\ \overline{u_k'v_k'} & \overline{v_k'^2} & \overline{v_k'w_k'} \\ \overline{u_k'w_k'} & \overline{v_k'w_k'} & \overline{w_k'^2} \end{pmatrix} = \rho_k \begin{pmatrix} \overline{u_k'^2} & 0 & 0 \\ 0 & \overline{v_k'^2} & 0 \\ 0 & 0 & \overline{w_k'^2} \end{pmatrix} + \rho_k \begin{pmatrix} 0 & \overline{u_k'v_k'} & \overline{u_k'w_k'} \\ \overline{u_k'v_k'} & 0 & \overline{v_k'w_k'} \\ \overline{u_k'w_k'} & \overline{v_k'w_k'} & 0 \end{pmatrix} \quad [2.20]$$

where the normal stresses account for the turbulent diffusion effect :

$$M_k^{TD} = \rho_k \begin{pmatrix} \overline{u_k'^2} & 0 & 0 \\ 0 & \overline{v_k'^2} & 0 \\ 0 & 0 & \overline{w_k'^2} \end{pmatrix} \begin{pmatrix} \partial \alpha_k / \partial x \\ \partial \alpha_k / \partial y \\ \partial \alpha_k / \partial z \end{pmatrix} \quad [2.21]$$

For the case of dispersed phase equation, we modify the density to $(\rho_d + C_{VM}\rho_c)$ to take into account the virtual mass effect. A factor of 3/2 is included to make the model compatible with data. The turbulent diffusion source in the dispersed phase becomes

$$\underline{M}_d^{TD} = - (3/2) (\rho_d + C_{VM}\rho_c) \overline{|\underline{v}_d'|^2} \nabla \alpha \quad [2.22]$$

An equivalent source on the continuous phase is

$$\underline{M}_c^{TD} = -\underline{M}_d^{TD} = (3/2) (\rho_d + C_{VM}\rho_c) \overline{|\underline{v}_d'|^2} \nabla \alpha \quad [2.23]$$

In terms of the energy spectrum, the fluctuating dispersed phase velocity is given by:

$$\overline{|\underline{v}_d'|^2} = \int_0^\infty E_d(\omega) d\omega \quad [2.24]$$

where ω = angular frequency and $E_d(\omega)$ = turbulent energy spectrum of the dispersed phase.

The dispersed phase spectrum is related to the energy spectrum of the continuous phase (Csanady, 1963) as [7]:

$$E_d(\omega) = \frac{E_c(\omega)}{1 + \tau_d^2 \omega^2} \quad [2.25]$$

where τ_d = time-constant for the dispersed phase and $E_c(\omega)$ = turbulent energy spectrum of the continuous phase.

The energy spectrum of the continuous phase is a function of its own time constant, τ_c (Hinze, 1959) and is given as [14]:

$$E_c(w) = \frac{2}{\pi} \frac{|\underline{v}'_c|^2}{1 + w^2 \tau_c^2} \quad [2.26]$$

From Equations (2.24), (2.25) and (2.26), we have

$$\overline{|\underline{v}'_d|^2} = \frac{2}{\pi} \frac{|\underline{v}'_c|^2}{\tau_d^2} \int_0^\infty \frac{1}{1 + w^2 \tau_d^2} \frac{\tau_c}{1 + w^2 \tau_c^2} dw \quad [2.27]$$

Performing the above integration yields:

$$\overline{|\underline{v}'_d|^2} = \frac{\tau_c}{\tau_c + \tau_d} \overline{|\underline{v}'_c|^2} \quad [2.28]$$

This says that the ratio of the dispersed phase to continuous fluid turbulent fluctuations is a function of their respective time constants. For high dispersed phase inertia leading to small τ_c/τ_d the dispersed phase will not tend to follow the fluid velocity fluctuations and hence $\overline{|\underline{v}'_d|^2}/\overline{|\underline{v}'_c|^2}$ is expected to be much less than unity.

The above integration is performed over the entire frequency spectrum assuming that the turbulent eddies of all sizes (or frequency) in the flow field are able to interact with the dispersed phase and displace them laterally. But this may not happen for large size particles (or bubbles) and hence a cut-off frequency should be used. In its simplest form, we can assume that only those turbulent eddies which are equal or bigger in size than the particles would be able to affect their lateral displacement.

According to the Kolmogorov's concept of energy transfer, the rate of dissipation of energy in small scale eddies, should be equal to the rate of energy supplied from the large scale eddies. Hence (Taylor, 1935) [29]

$$\epsilon \sim u^2/t \sim u^2/(l/u) \sim u^3/l$$

where 'u' and 'l' are velocity and length scale of turbulence respectively. The time scale of turbulence is also given in terms of velocity and length scales,

$$t \sim l/u \sim l/(\epsilon)^{1/3} \sim (l^2/\epsilon)^{1/3}$$

Hence, the angular frequency, ω becomes

$$\omega = 2\pi f \sim \frac{2\pi}{t} \sim 2\pi\left(\frac{\epsilon}{l^2}\right)^{1/3} \sim \frac{2\pi\epsilon^{1/3}}{l^{2/3}}$$

Taking the length scale of turbulence as the size of the smallest eddy that would induce a lateral motion in the dispersed phase which is also, at the limit, equal to the size of the particle, we have the angular cut-off frequency as,

$$\omega_c = \frac{2\pi\epsilon^{1/3}}{d_d^{2/3}} \quad [2.29]$$

Now, performing the integration in equation (2.27) from zero to ω_c , we get

$$\overline{|\underline{v}'_d|^2} = \frac{2 [(\tau_d/\tau_c) \tan^{-1}((\tau_d/\tau_c)\omega_c) - \tan^{-1}(\omega_c)]}{\pi(-1 + \tau_d^2/\tau_c^2)} \overline{|\underline{v}'_c|^2} \quad [2.30]$$

As a check we see that in the limit, $\omega_c \rightarrow \infty$, equation (2.30) reduces to equation (2.28). Inserting equation (2.30) into equation (2.22), we finally get

$$\begin{aligned} \underline{M}_d^{TD} &= - \left[\frac{2 [(\tau_d/\tau_c) \tan^{-1}((\tau_d/\tau_c)\omega_c) - \tan^{-1}(\omega_c)]}{\pi(-1 + \tau_d^2/\tau_c^2)} \right] \overline{|\underline{v}'_c|^2} \frac{3}{2} (\rho_d + C_{VM}\rho_c) \nabla\alpha \\ &= - \left[\frac{2 [(\tau_d/\tau_c) \tan^{-1}((\tau_d/\tau_c)\omega_c) - \tan^{-1}(\omega_c)]}{\pi(-1 + \tau_d^2/\tau_c^2)} \right] k_c (\rho_d + C_{VM}\rho_c) \nabla\alpha \quad [2.31] \end{aligned}$$

where k_c is the turbulent kinetic energy of the continuous phase. For isotropic turbulence, $k_c = 3/2 \overline{|\underline{v}'_c|^2}$. The turbulent diffusion force vanishes when $\nabla\alpha$ is zero, (i.e for uniform lateral distribution of the dispersed phase) which is expected. It is interesting to note that for higher dispersed phase inertia, the ratio of the time constants τ_c/τ_d is low and hence the lateral diffusion decreases.

2.3 Turbulence Model

Almost all phenomena related to multiphase flows are turbulent in nature and hence any study on two-phase flow would be incomplete without understanding the

structure of turbulence. As mentioned previously, a relationship between lateral void distribution and turbulent structure in pipe flow was established by Wang(1987) [31]. Also affected are the velocity profiles depending on whether turbulence is generated or dampened in the flow field due to the addition of the dispersed phase. For the present study, the k - ϵ model of turbulence is used. 'k' represents turbulent kinetic energy and ' ϵ ' refers to the rate of dissipation of turbulent kinetic energy. They determine momentum diffusivity of the turbulent motion. This turbulent diffusivity when added to its laminar component gives the effective kinematic viscosity of the turbulent flow.

2.3.1 Transport Equation of two-phase turbulence kinetic energy

Katoaka and Serizawa(1989) obtained an equation for the two-phase turbulent kinetic energy using two-phase time averaging method [18].

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_c k_c + \nabla \cdot \alpha_c \bar{v}_c k_c = & - \nabla \cdot \alpha_c \bar{v}_c' \left(k_c + \frac{p_c'}{\rho_c} \right) \\ & - \alpha_c \overline{v_c' v_c'} : \nabla \bar{v}_c - \alpha_c \nu_c \overline{\nabla v_c' : \nabla v_c'^T} + S_{ki} \end{aligned} \quad [2.32]$$

Here ' ν ' is the laminar kinematic viscosity and S_{ki} is the work done by the dispersed phase on the continuous phase. The other three terms on the RHS represent respectively: turbulent diffusion, shear-induced production of turbulence and viscous dissipation. The turbulent quantities are modeled based on a set of constitutive relations. The Reynolds stress tensor in the production term is modeled using a gradient hypothesis:

$$\overline{v_c' v_c'} = -\nu_t (\nabla \bar{v}_c + \nabla \bar{v}_c^T) \quad [2.33]$$

Similarly, the diffusion term is modeled as

$$\overline{v_c' \left(k_c + \frac{p_c'}{\rho_c} \right)} \cong -\nu_t \nabla k_c \quad [2.34]$$

where ν_t is the turbulent kinematic viscosity and is determined using the properties from the k- ϵ model, i.e.

$$\nu_t = 0.09 \frac{k_c^2}{\epsilon} \quad [2.35]$$

The third term is the turbulence dissipation rate

$$\epsilon \cong \nu \overline{\nabla \underline{v}'_c : \nabla \underline{v}'_c{}^T} \quad [2.36]$$

Thus the transport equation for the two-phase turbulence kinetic energy can be written as :

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_c k_c + \nabla \cdot \alpha_c \underline{\bar{v}}_c k_c &= \nabla \cdot [\alpha_c \nu_t \nabla k_c] \\ &+ \alpha_c \nu_t (\nabla \underline{\bar{v}}_c + \nabla \underline{\bar{v}}_c{}^T) - \alpha_c \epsilon + S_{ki} \end{aligned} \quad [2.37]$$

Similarly, the turbulence dissipation rate is modeled with another transport equation and is given by:

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_c \epsilon_c + \nabla \cdot \alpha_c \underline{\bar{v}}_c \epsilon_c &= \nabla \cdot \left[\alpha_c \frac{\nu_t}{\sigma_\epsilon} \nabla \epsilon_c \right] \\ &+ \frac{\alpha_c}{k_c/\epsilon} [C_{\epsilon 1} \nu_t (\nabla \underline{\bar{v}}_c + \nabla \underline{\bar{v}}_c{}^T)] - \frac{\alpha_c}{k_c/\epsilon} C_{\epsilon 2} \epsilon + S_{\epsilon i} \end{aligned} \quad [2.38]$$

where (Launder and Spalding, 1974) [21] $\sigma_\epsilon = 1.3$, $C_{\epsilon 1} = 1.44$ and $C_{\epsilon 2} = 1.92$.

The interfacial source of dissipation, $S_{\epsilon i} = (\epsilon_c/k_c) S_{ki}$. S_{ki} is modelled in the following section.

2.3.2 Particle Turbulence Dissipation Model

In the presence of a dispersed phase, the turbulent kinetic energy is also dissipated by the interaction between the particles and the eddies. This additional dissipation

occurs from inability of the dispersed-phase to completely follow the turbulent eddy fluctuations. The deviation of the dispersed-phase motion from that of the fluid, generates a fluctuating relative velocity resulting in dissipation of energy due to drag. The additional dissipated energy as a result of the fluctuating relative velocity between the phases is given in terms of the average rate of energy dissipated per particle as:

$$\begin{aligned}
 \epsilon_d &= \text{DragForce} * \text{RelativeVelocity} \\
 &= \frac{1}{2} A_p \rho_c C_d (\overline{|\underline{v}'_r| \underline{v}'_r \cdot \underline{v}'_r}) \\
 &= \frac{1}{2} \frac{\pi d_d^2}{4} \rho_c \frac{24(1 + 0.1 N_{Re}^{0.75})}{(d_d(|\underline{v}'_r|)/\nu_c)} (\overline{|\underline{v}'_r| \underline{v}'_r \cdot \underline{v}'_r}) \\
 &= 3\pi \nu_c \rho_c d_d (1 + 0.1 N_{Re}^{0.75}) (\overline{|\underline{v}'_r|^2})
 \end{aligned} \tag{2.39}$$

Now, fluctuating relative velocity is given by

$$\overline{|\underline{v}'_r|^2} = \int_0^\infty E_R(w) dw \tag{2.40}$$

The relative velocity spectrum is related to the energy spectrum of the continuous phase (Csanady, 1963) as [7]:

$$\frac{E_R(\omega)}{E_c(\omega)} = \frac{\tau_d^2 \omega^2}{1 + \tau_d^2 \omega^2} \tag{2.41}$$

The energy spectrum of the continuous phase is a function of its own time constant, τ_c (Hinze, 1959) and is given as [14]:

$$E_c(w) = \frac{2}{\pi} \overline{|\underline{v}'_c|^2} \frac{\tau_c}{1 + w^2 \tau_c^2} \tag{2.42}$$

Combining Equations (2.40), (2.41) and (2.42), we get

$$\overline{|\underline{v}'_r|^2} = \frac{\overline{|\underline{v}'_c|^2}}{2\pi} \int_0^\infty \frac{\tau_d^2 \omega^2}{1 + \tau_d^2 \omega^2} \frac{4\tau_c}{1 + w^2 \tau_c^2} dw \tag{2.43}$$

Performing the integration yields,

$$\overline{|\underline{v}'_r|^2} = \overline{|\underline{v}'_c|^2} \left(\frac{\tau_d}{\tau_c + \tau_d} \right) \tag{2.44}$$

Once more as in the case for turbulent diffusion, we note that above integration was performed over the entire frequency spectrum. The eddies that are smaller than the dispersed phase will not participate in the eddy-particle dissipation process. Hence invoking the concept of cut-off frequency from equation (2.29), and performing the integral (2.43) from zero to ω_c , we get

$$\overline{|\underline{v}'|^2} = \overline{|\underline{v}'|^2} \frac{2(\tau_d/\tau_c)[-(\tau_d/\tau_c)\tan^{-1}(\tau_c\omega_c) + \tan^{-1}(\tau_d\omega_c)]}{\pi(1 - \tau_d^2/\tau_c^2)} \quad [2.45]$$

In the limit, $\omega_c \rightarrow \infty$, equation (2.45) reduces to equation (2.44). Bertodano (1992) in his simulation of two-phase bubbly pipe flow had used large bubbles of size 5 mm (or small ω_c) and hence this dissipation effect was not significant in his study [3]. For particles of very small size ($\omega_c \rightarrow \infty$), inserting equation (2.44) into (2.39), we get the rate of dissipation as,

$$\epsilon_d = 3\pi\nu_c\rho_c d_d(1 + 0.1N_{Re}^{0.75}) (\overline{|\underline{v}'|^2}) \frac{\tau_d}{\tau_c + \tau_d} \quad [2.46]$$

The rate of dissipation of turbulent kinetic energy per unit volume is given by equation (2.46) multiplied by the number of particles per unit volume as :

$$\begin{aligned} \epsilon &= \epsilon_d \left(\frac{\alpha}{(\pi/6)d_d^3} \right) \\ &= 3\pi\nu_c\rho_c d_d(1 + 0.1N_{Re}^{0.75}) (\overline{|\underline{v}'|^2}) \frac{\tau_d}{\tau_c + \tau_d} \cdot \frac{6}{\pi} \frac{\alpha}{d_d^3} \\ &= \frac{18(1 + 0.1N_{Re}^{0.75})\mu_c}{\rho_d d_d^2} \rho_d \alpha (\overline{|\underline{v}'|^2}) \frac{\tau_d}{\tau_c + \tau_d} \end{aligned} \quad [2.47]$$

But $(\overline{|\underline{v}'|^2}) = 2k_c$

Hence, in terms of turbulent kinetic energy of the continuous phase, the additional dissipation due to the presence of dispersed particle is given by

$$\epsilon = \left[\frac{18(1 + 0.1N_{Re}^{0.75})\mu_c}{\rho_d d_d^2} \rho_d \alpha \right] k_c \frac{2\tau_d}{\tau_c + \tau_d} \quad [2.48]$$

The time scale of the particles can be obtained from their equation of motion and is given as,

$$\rightarrow \tau_d = \frac{\rho_d d_d^2}{18(1 + 0.1 N_{Re}^{0.75}) \mu_c} \quad [2.49]$$

Using equation (2.49), equation (2.48) reduces to :

$$\epsilon = \frac{2\rho_d \alpha k_c}{\tau_d} \frac{\tau_d}{\tau_c + \tau_d} \quad [2.50]$$

Chen and Wood (1986), suggested the following model for the dissipation of turbulent kinetic energy due to presence of heavy particles [6]:

$$\epsilon = \frac{2\rho_d \alpha k_c}{\tau_d} [1 - \exp(-0.5\tau_d/(k_c/\epsilon))] \quad [2.51]$$

⇒ For the case of homogeneous turbulence where $\tau_c = k_c/\epsilon$, Chen and Wood's model yields the same result as the present model when τ_d/τ_c is very large. At very small τ_d/τ_c , their model differs by a factor of 1/2. (cf.:Figure 2.1)

The models for turbulent diffusion force and turbulent dissipation effects due to the presence of the dispersed phase, which were developed in these sections, are used to solve the two-phase turbulent jets. The results from the numerical simulation are presented in the next chapter.

Figure 2.1 Dissipation Model : Comparison with Chen's model

3. NUMERICAL MODEL

The mathematical formulation of two-fluid model into PHOENICS has been introduced by Spalding (1985). In the present calculations, fluid 1 is defined as continuous phase and fluid 2 as dispersed one, respectively. The two fluids are assumed to share the same space at any instant. The code solves one Poisson pressure equation for both phases and an additional condition which states that their volume fractions α_c and α_d satisfy :

$$\alpha_c + \alpha_d = 1$$

at any location in the flow domain. The two fluids are assumed to have the same pressures. The numerical algorithm is Eulerian. The Eulerian approach uses a fixed frame of reference and continuum formulation of the conservation equations for both phases so that the continuous and the dispersed phases are treated as interpenetrating continua. The Eulerian method is called the IPSA algorithm. The Eulerian approach is favoured where the continuum approach can be applied best, e.g. in modeling immiscible fluids and when treating monosized particle suspensions.

Assumptions

- 1). Both the dispersed and the continuous phases behave macroscopically as continua.
- 2). The mean flow is steady, axisymmetric, incompressible and isothermal.
- 3). The flow field is represented by a set of elliptical pde's.
- 4). The dispersed phase is spherical in shape and monosized.
- 5). Continuous phase is Newtonian; both phases have constant physical properties.

3.1 Governing Equations

To determine the characteristics of two-phase turbulent jets, a set of partial differential equations is solved, together with the appropriate algebraic relations and boundary conditions. These differential equations are time-averaged mass-conservation and momentum equations. The k- ϵ turbulence model has been employed, and therefore additional conservation equations are solved for turbulence kinetic energy and for the dissipation rate of turbulence energy. These equations may all be expressed in the general form :

$$\frac{\partial(\alpha_k \rho_k \phi_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k v_k \phi_k - \alpha_k \rho_k \gamma_{\phi_k} \nabla \phi_k) = \alpha_k S_{\phi_k} \quad [3.1]$$

The terms and coefficients in this expression depend on the equation under consideration, and have to be specified individually. The dependent variable ϕ_k represents the conserved quantity, or unity for the mass-conservation equations. The first term represent the transient term which is zero for the present steady state case. The second term represent convective transport. The third term represents diffusive transport; γ_{ϕ_k} being the diffusive exchange coefficient. S_{ϕ_k} is the source term which for the momentum equation comprises the pressure-gradient term $-\nabla p$, inter-fluid friction and other interphase transfer terms. The IPSA algorithm is based on the SIMPLE algorithm proposed by Pathankar and Spalding (1972). In essence the mass and momentum equations are solved iteratively. The pressure is treated implicitly to overcome severe Courant constraints.

3.2 Computational Details

The governing equations have been solved numerically using the general purpose PHOENICS code. The equations are discretized on a non-uniform cylindrical polar grid using a finite volume procedure. For the cylindrical polar co-ordinate system, the y and z-coordinates represents radial and axial directions respectively. The symmetry condition at the jet axis, reduces the problem to 2-D by discarding the azimuthal variations. The variables solved are the pressure P1, the velocity components in the

y and z-directions, $V1$ and $V2$ and $W1$ and $W2$, the volume fractions $R1$ and $R2$ and the turbulent quantities KE and EP for the $k-\epsilon$ model. Specific subroutines were written to include the turbulence dissipation model and the source terms in the momentum equations : lift force, turbulent diffusion force, interfacial pressure term and bubble induced turbulence.

3.2.1 Boundary Conditions

The elliptical flow field implies that all the boundaries including the downstream one need to be specified. The boundary condition on the circumferential boundary is free stream condition. At the outlet, the pressure is at ambient atmosphere. The boundary conditions at the inlet are specified from the known inflow conditions. The normal gradients of the axial velocities, the kinetic energy of turbulence, k , and its rate of dissipation, ϵ , are zero on the axis of symmetry. While the radial velocity is zero at the centerline by virtue of symmetry.

3.2.2 Grid number and Layout

A grid of $40 * 100$ ($NZ * NY$) was employed in the majority of the runs. Convergence was tested by requiring the sum of the absolute residual values to be less than 10^{-6} . The residual value is calculated for each solved variable and is equal to the absolute difference between left-and right-hand sides of the difference equations solved at each node point multiplied by the total number of nodal points in the flow domain. The effect of grid number is also investigated. Figure (3.1) shows the numerical convergence for the case of bubbly two-phase jet as the grid is refined in the axial and radial direction.

3.2.3 Computer used

The calculations were carried out on a RISC 6000 cluster using Sun SPARCstation under Solaris 2.3/Unix 4.0.3 for output displays.

Figure 3.1 Numerical convergence test for a two phase jet.

3.2.4 PHOENICS Settings

PHOENICS consists of two main codes besides some auxillary programs which are post-processors and aid in visual display of outputs from the code. The two essential ones are a pre-processor called Satellite and a processor called Earth. The Satellite interpretes the problem specification provided by the user and from it creates a data file which Earth can understand and operate upon. The problem is specified by way of PHOENICS-input setting made in an input file called q1. An example of a typical q1-file is given in Appendix A.2. Earth contains the main flow-simulating coding sequence. It reads the data file provided by Satellite and executes the corresponding computations. Earth possesses Fortran subroutines which are accessible to the user. Grex is one of the exemplary subroutine which provides some fluid-property, boundary condition and source terms which will satisfy the most common occuring needs. In the other subroutine called Ground, the user can supply necessary boundary conditions and source terms which are not contained within Grex. A complete listing of the ground subroutine is presented in Appendix A.3. It contains coding related with initialization of variables and sources used in the momentum transfer equations which represent the interfacial transfer terms.

3.3 Results and Discussion

As a first step, numerical convergence of the code was tested by solving a standard single phase jet for which an analytical solution exists. The results are presented in Appendix A.1. For the two-phase jets, results from the numerical calculations were validated with the data of Sun (1985) for case of bubbly jet [27]. They used high continuous phase velocity where the force of turbulent diffusion becomes more significant compared to the other lateral forces, as previously discussed in section (1.2.3). Hence this data set is good to test the force of turbulent diffusion on bubbles. For air-particle jet, the data of Bulzan et.al.(1987) and Hetsroni and Sokolov (1971) were used [5, 13]. Bulzan's data shows significant dissipation of turbulent kinetic energy due to the presence of dispersed phase since they used small particles and high

dispersed phase loading. Therefore, it was a good test for the turbulent dissipation model. But their data was not good to test the model of turbulent diffusion since the time scale of the particles was comparable to the flow transient time. Hence, the particles may not get enough time to diffuse through the flow. Secondly, a convergence problem was encountered with the particle distribution near the centerline of the jet which prompted us to look for a better set of data to validate the turbulent diffusion model. The data of Hetsroni is good in that respect since the time scale of the particles is much smaller than the transient time of the flow. But due to a very low dispersed phase loading, Hetsroni's data was not used to test the dissipation effect due to the presence of dispersed phase since this dissipation was very small in their case.

3.3.1 Comparison with water-bubble data

Figures (3.2) to (3.8) show comparisons with Sun's (1985) data for $j_l = 1.61$ m/s and $j_g = 0.04$ m/s. The inlet void fraction for this case was approximately 2.5 %. Results are presented for axial locations of $z/d = 24$ and $z/d = 40$. The continuous phase velocity is predicted closely when the dissipation effect is included. While the viscous drag predicts the bubble phase velocity fairly well. Also predicted closely are the turbulent kinetic energy at both axial locations when the dissipation effect is included in the model. The results for the continuous phase velocity, with and without the dissipation effect, at the two axial locations of $z/d = 24$ and 40 are presented in figure (3.2) and figure (3.3) respectively. Figure (3.5) and figure (3.6) show similar results for the turbulent kinetic energy. It is quite obvious from these figures that the numerical results match better with the experimental data when the additional dissipation effect due to the dispersed phase is included in the model. The difference will become more pronounced for the higher void fraction case as discussed in the subsequent paragraphs.

The turbulent diffusion model presented earlier predicts the void fraction distribution fairly well. The data was compared with lift coefficients of $C_L = 0.0$ and 0.25.

Figure (3.7) and (3.8) at axial locations of $z/d = 24$ and 40 respectively, confirms the existence of lift on bubbles, and the lift coefficient of $C_L = 0.25$, recommended by Lance and Naciri (1991) gives an upper bound to the data. The uncertainty in the exact value of the lift coefficient is the major problem in this case.

Figure 3.2 Axial liquid phase velocity vs. radial distance.; Sun's data (Case 1)

Figure 3.3 Axial liquid phase velocity vs. radial distance.; Sun's data (Case 1)

Figure 3.4 Axial bubble phase velocity vs. radial distance.; Sun's data (Case 1)

Figure 3.5 Turbulent kinetic energy vs. radial distance.; Sun's data (Case 1)

Figure 3.6 Turbulent kinetic energy vs. radial distance.; Sun's data (Case 1)

Figure 3.7 Void fraction distribution vs. radial distance.; Sun's data (Case 1)

Figure 3.8 Void fraction distribution vs. radial distance.; Sun's data (Case 1)

Figures (3.9) to (3.15) show comparisons with Sun's (1985) data for $j_l = 1.61$ m/s and $j_g = .161$ m/s. The inlet void fraction for this case was 9.1 %. The results for axial bubble and axial liquid phase velocities are as good as in the previous case for low void fraction, but the turbulent kinetic energy at axial location of $z/d = 40$ is not well predicted. The bubble-induced turbulence, which is low for this case, was also included but it did not improve the solution significantly. The effect of additional dissipation of turbulent kinetic energy due to the presence of the dispersed phase is more significant for this case of higher inlet void fraction. As seen in figure (3.9) and figure (3.10) a better match with the data is obtained for the continuous phase velocity by including this dissipation effect. Similar conclusion can be drawn for the turbulent kinetic energy from figure (3.15). The plots for void distribution vs. radial distance at two axial locations shown in figure (3.13) and (3.14) show a similar trend to the previous case. The results confirm the existence of the lateral lift force on the bubbles but an exact value for the lift coefficient cannot be prescribed.

It was postulated earlier in section (2.3), that in the case of large bubbles dispersed in a turbulent flow field, the additional dissipation of turbulent kinetic energy by the dispersed phase would go down. Therefore, a cut-off frequency was introduced in the formulation of dissipation model. It was reasoned that as the size of the bubbles increases, it can interact with fewer eddies, (i.e. only with ones which are equal or bigger in size than the bubbles itself) and hence the additional dissipation due to eddy-particle interaction will decrease. This can be seen clearly from figures (3.16) and (3.17) where the turbulent kinetic energy increases with the increase in bubble size. It reaches the limit of no dissipation for very large bubbles. Also, since dissipation decreases, the turbulent mixing increases and hence the velocity profiles becomes flatter as seen in figures (3.18) and (3.19).

Figure 3.9 Axial liquid phase velocity vs. radial distance.; Sun's data (Case 3)

Figure 3.10 Axial liquid phase velocity vs. radial distance.; Sun's data (Case 3)

Figure 3.11 Axial bubble phase velocity vs. radial distance.; Sun's data (Case 3)

Figure 3.12 Turbulent kinetic energy vs. radial distance.; Sun's data (Case 3)
(PHOENICS calculations include dissipation)

Figure 3.13 Void fraction distribution vs. radial distance.; Sun's data (Case 3)

Figure 3.14 Void fraction distribution vs. radial distance.; Sun's data (Case 3)

Figure 3.15 Turbulent kinetic energy vs. radial distance.; Sun's data (Case 3)

Figure 3.16 Turbulent kinetic energy vs. radial distance.; With Cut-Off Frequency.

Figure 3.17 Turbulent kinetic energy vs. radial distance.; With Cut-Off Frequency.

Figure 3.18 Axial liquid phase velocity vs. radial distance.; With Cut-Off Frequency

Figure 3.19 Axial liquid phase velocity vs. radial distance.; With Cut-Off Frequency

3.3.2 Comparison with Air-particle data

For particle laden jets, the numerical calculations were validated against Bulzan's data [5]. Figure (3.20) compares the axial air phase velocity very well when the dissipation effect is included while figure (3.21) shows that the Stoke's drag law closely predicts the particle velocity. The turbulent kinetic energy is also predicted close to the experimental data when the dissipation is included as seen in figure (3.22). The plot of particle mass flux vs. radial distance matches fairly well with the data as seen in figure (3.23) though a problem was encountered with the convergence near the centerline of the jet. A more confident prediction was sought by comparing the particle flux distribution with the data of Hetsroni [13]. Figure (3.24) shows the comparison with Hetsroni's data. In the case of Hetsroni's data, the particle time-scale was small as compared to the transient time of the flow which allowed the particles to diffuse significantly in the lateral direction.

The effect of dissipation due to eddy-particle interaction is best seen in figures (3.20) and (3.22) for the axial air velocity and the turbulent kinetic energy respectively. The dissipation causes less turbulent mixing and hence the air velocity profile becomes narrower. Also, the turbulent kinetic energy decreases due to the high rate of dissipation.

Figure 3.20 Axial air phase velocity vs. radial distance.; Bulzan's Data

Figure 3.21 Axial particle phase velocity vs. radial distance.; Bulzan's Data

Figure 3.22 Turbulent kinetic energy vs. radial distance.; Bulzan's Data

Figure 3.23 Particle Flux Distribution vs. radial distance.; Bulzan's Data

Figure 3.24 Particle Flux Distribution vs. radial distance.; Hetsroni's Data

4. CONCLUSIONS AND RECOMMENDATIONS

This thesis dealt with numerical modeling of two-phase turbulent jets. The major objective of the work was to apply a complete two-fluid model to numerically simulate a two-phase jet. Also, the turbulent phenomena, which are important and inseparable with the two-phase jet analysis, were studied and an improved model was successfully applied.

4.1 Summary

The interfacial transfer terms in the two-fluid momentum equations are extremely important parameters. The forces of drag and virtual mass along with the lateral forces like lift and turbulent diffusion determine the characteristics of any two-phase turbulent flow. Also, the inclusion of the virtual mass force ensures numerical stability in practice. The lift force together with a turbulent diffusive force accounts for the observed spreading of the dispersed phase and the vertical variation in the centerline void fraction. In the case of turbulent jets, not many attempts have been made to include all these forces. In particular nobody has modelled the force of turbulent diffusion.

Since the averaged two-fluid continuity equation does not allow for the phasic diffusion term, in the past it was accounted by adding an additional diffusivity in the continuity equation. But this does not explain the physics involved and it is more appropriate to treat the diffusion as an interfacial force in the momentum equation. In the present work, the force of turbulent diffusion was developed making an analogy with thermal diffusion of air molecules in the atmosphere. It was applied successfully to predict the radial void distribution in bubbly jets and particle mass flux distribution

for a particle laden air jet. For the case of water-bubble jet, it was concluded that turbulent diffusion is more significant than the other lateral forces in determining the radial diffusion of bubbles. By not including the force of turbulent diffusion, the radial extent of the region containing bubbles is underestimated. The numerical results also confirmed the existence of the lift force on the bubbles though a single exact value of the lift coefficient could not be prescribed.

The profiles of the continuous phase velocity and the turbulent kinetic energy matches very well with various data. This demonstrates the successful application of our turbulence model for two-phase flows which included the additional dissipation effects due to the presence of the dispersed phase. The dissipation model was developed by giving a similar treatment as was done with the force of turbulent diffusion. Comparisons were also made without including dissipation effects and it was confirmed that the dissipation due to eddy-particle interactions is significant for small particles and should be included in a general two-phase $k-\epsilon$ model.

A cut-off theory for the dissipation was proposed and it was tested for the case of jets with 1 mm bubbles. The present dissipation model incorporates this theory very easily. It was reasoned that for the larger bubbles the dissipation of turbulent kinetic energy due to the presence of the dispersed phase decreases since they can interact with fewer eddies (only the ones which are equal or greater in size than the bubbles itself). This hypothesis merges well with the predictions of Bertodano (1992) for large bubbles, where the dissipation was not significant.

4.2 Recommendations for Future Work

In the present work, simple analytical models were developed making an analogy with actual physical phenomena occurring in nature. The models were powerful enough to accurately predict the two extreme kinds of jets without introducing any additional constants to match the data. The jets studied in this analysis were cases of dilute two-phase flows and the dispersed phase was monosized. As an extension of this work, it would be interesting to develop a two-fluid CFD model which could predict

two-phase flows with higher loading of dispersed phase and where the dispersed phase is polysized.

The experimental data chosen for the bubbly jet provided a good separate effect test for the turbulent diffusion force which was found to be the most significant lateral force present. But still some uncertainty exists in prescribing an exact value of the lift coefficient. An attempt should be made in this direction by working with different sets of data which can highlight different relative magnitudes of the two lateral forces i.e. the lift and the turbulent diffusion force.

The cut-off frequency proposed in the present work is a step in the right direction. It could explain the different amount of dissipation effects observed in the present work and the work of Bertodano (1992) due to the presence of the dispersed phase [4]. It is an interesting proposition which should induce further research. It should be tested against different sets of data pertaining to various bubble sizes and a more mechanistic cut-off limit should be established. Also, for a single data set containing polysized bubbles, the bubble size distribution will produce different dissipation rates radially. This effect should be included in the cut-off model and validated against the experimental data.

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APPENDICES

Appendix A: Numerical solution for single phase jet

The convergence test for the code was established by solving a standard, plane single-phase jet. The results were compared against the analytical solution. From a slit in the wall, a fluid issues out into infinite space filled with the same fluid. The mass discharge at the wall is $\rho b u$ where velocity 'u' can be considered uniform and 'b' is the width of the slit (c.f: Figure A.1).

The governing equation for the axial velocity is

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

with boundary conditions

$$u = 0 \quad \text{for } y = \pm \infty$$

$$u = 0 = v \quad \text{at } x = \infty$$

$$v = 0 = \partial u / \partial y \quad \text{at } y = 0$$

The analytical solution for this problem is obtained using the method of similarity. The velocity $u(x,y)$ is given as:

$$u = \left(\frac{3M^2}{32\nu\rho x} \right)^{1/3} \text{Sech}^2 \xi$$

where $M \equiv \text{Momentum Flux} = b \rho u^2$ and

$$\xi = \left(\frac{M}{48\rho\nu^2} \right)^{1/3} \frac{y}{x^{2/3}}$$

An elliptical solution procedure was used to solve the above single phase jet. For the laminar jet, a kinematic viscosity of $2 \times 10^{-4} \text{ m}^2/\text{s}$ was used since this is close to real turbulent jet data. Using a fixed value for kinematic viscosity instead of a turbulence model to predict the turbulent kinematic viscosity at each location eliminated any error in the solution due to the model itself and hence a separate effects convergence test was possible.

Three cases with 20, 40 and 80 nodes in the axial direction and with 20, 40 and 80 nodes respectively in transverse direction were tried. Figure A.2 shows the

Figure A.1 A single phase jet schematic.

comparison of the three runs with the analytical solution. It is seen clearly that the numerical results approach the analytical solution as the computational grid is refined.

Figure A.2 Numerical convergence test for a single phase jet.

Appendix B: q1-File: Input To Satellite

TALK=f;RUN(1, 1);VDU=X11-TERM

GROUP 1. Run title and other preliminaries
TEXT(SUN's DATA; Case1; NZ40,NY100; CL0.25; CTD1.0)

The locally-defined variables are as follows:

WJET	Jet velocity at the inlet	(m/s)
REYNO	Reynolds number	
DIAM	The diameter of the jet nozzle	(m)
EPSIN	The dissipation rate at the inlet	
GMIXL	The length scale at the inlet	
TKEIN	The turbulent kinematic energy at the inlet	

REAL(TKEIN, EPSIN, GMIXL, MAS1, MAS2, R1IN, R2IN)

REAL(WJET, REYNO, DIAM, AJET)

REAL(dbub, CL, CTD, CP, UR)

REYNO=8740; DIAM=508.E-5; AJET=(3.1416/4)*DIAM**2

MAS1=32700.E-6/AJET; MAS2=40750.E-8/AJET

RHO1=1000

RHO2=500

R1IN=MAS1*RHO2/(MAS2*RHO1+MAS1*RHO2); R2IN=1-R1IN

WJET=MAS1/(RHO1*R1IN)

TKEIN=WJET*WJET*0.0001

GMIXL=0.035*DIAM; EPSIN=TKEIN**1.5/GMIXL*.1643

Inlet dissipation rate=.1643*k**1.5/Lm

NZ=40

dbub=0.001

CL=0.25

CP=0.1

CTD=1.0

UR=50.0*dbub**0.875

rg(2)=UR

rg(5)=0.027661599

rg(5)=Time-scale of particle

GROUP 3. X-direction grid specification
 CARTES=F;XULAST=0.1

GROUP 4. Y-direction grid specification
 NREGY=2
 IREGY=1;GRDPWR(Y,3,DIAM/2,1)
 IREGY=2;GRDPWR(Y,93,DIAM*8,1)

GROUP 5. Z-direction grid specification
 GRDPWR(Z,NZ,DIAM*75,1.285)

GROUP 7. Variables stored, solved & named
 ONEPHS=F
 SOLVE(P1,V1,W1);SOLUTN(P1,P,P,Y,P,P,P)
 SOLVE(V2,W2,R1,R2)
 store(c1,c2,c3,c4)
 store(c5,c6,c7,c8,c9)
 store(c10,c11,c12,c13,c14,c15)
 store(c24,c25,c26,c27,c28,c29)
 store(c30,c31,c32,c33,c34,c35)
 STORE(TE,TP)

Te, Tp are characteristic time scales of turbulence
 and particles respectively

GROUP 8. Terms (in differential equations) & devices
 difcut=0.0

GROUP 9. Properties of the medium (or media)
 ENUL=99198.E-11
 TURMOD(KEMODL)
 STORE(ENUT,LEN1,GENK)

GROUP 10. Inter-phase-transfer processes and properties
 CFIPS=GRND7;CFIPA=WJET/10000;CFIPB=dbub
 STORE(CFIP,REYNO,CD,APRJ,VREL)

GROUP 11. Initialization of variable or porosity fields
 FIINIT(R1)=R1IN
 FIINIT(R2)=R2IN
 FIINIT(KE)=TKEIN;FIINIT(EP)=EPSIN
 FIINIT(W1)=WJET
 FIINIT(W2)=WJET
 restrt(all)

GROUP 13. Boundary conditions and special sources

Outer Boundary-- free stream

```
PATCH(HIGHY,NORTH,1,1,NY,NY,1,NZ,1,1)
COVAL(HIGHY,P1,1.OE1,0.0)
COVAL(HIGHY,W1,ONLYMS,0.0)
COVAL(HIGHY,V1,ONLYMS,0.0)
COVAL(HIGHY,KE,ONLYMS,1.E-10)
COVAL(HIGHY,EP,ONLYMS,1.E-6)
```

Lower Boundary--free stream

```
PATCH(LOWZ,LOW,1,1,#2,#2,1,1,1,1)
COVAL(LOWZ,P1,1.OEO,0.0)
COVAL(LOWZ,W1,ONLYMS,0.0)
COVAL(LOWZ,V1,ONLYMS,0.0)
COVAL(LOWZ,KE,ONLYMS,1.E-10)
COVAL(LOWZ,EP,ONLYMS,1.E-6)
```

Inlet Boundary-- uniform flow

```
INLET(UNIFORM,LOW,1,1,#1,#1,1,1,1,1)
VALUE(UNIFORM,P1,MAS1)
VALUE(UNIFORM,W1,WJET)
VALUE(UNIFORM,P2,MAS2)
VALUE(UNIFORM,W2,WJET)
VALUE(UNIFORM,KE,TKEIN)
VALUE(UNIFORM,EP,EPSIN)
```

Downstream Boundary

```
OUTLET(OUT,HIGH,1,NX,1,NY,NZ,NZ,1,1)
COVAL(OUT,P1,10,0)
COVAL(OUT,P2,10*RHO2/RHO1,0)
```

Turbulent Dissipation Source

```
PATCH(KEDISP,CELL,1,NX,1,NY,1,NZ,1,1)
COVAL(KEDISP,KE,GRND2,0)
COVAL(KEDISP,EP,GRND2,0)
```

Buoyancy source

```
PATCH(GRAV,VOLUME,1,NX,1,NY,1,NZ,1,1)
COVAL(GRAV,W2,FIXFLU,GRND)
```

Turbulent Diffusion

```
PATCH(GRAD,VOLUME,1,NX,1,NY,1,NZ,1,1)
COVAL(GRAD,V1,FIXFLU,GRND)
COVAL(GRAD,V2,FIXFLU,GRND)
rg(3)=CTD
```

Lift Force

```
PATCH(LIFTY,VOLUME,1,NX,1,NY,2,NZ,1,1)
COVAL(LIFTY,V1,FIXFLU,GRND)
COVAL(LIFTY,V2,FIXFLU,GRND)
rg(1)=CL
```

Interfacial Pressure Force

```
PATCH(PRESY,VOLUME,1,NX,1,NY,2,NZ,1,1)
COVAL(PRESY,V1,FIXFLU,GRND)
rg(4)=CP
```

GROUP 16. Termination of iterations

```
LSWEEP=800
```

GROUP 17. Under-relaxation devices

```
RELAX(P1,LINRLX,0.6)
RELAX(V1,FALSDT,ZWLAST/WJET)
RELAX(W1,FALSDT,ZWLAST/WJET)
RELAX(V2,FALSDT,ZWLAST/WJET)
RELAX(W2,FALSDT,ZWLAST/WJET)
RELAX(KE,FALSDT,ZWLAST/WJET/NZ)
RELAX(EP,FALSDT,ZWLAST/WJET/NZ)
```

GROUP 18. Limits on variables or increments to them

```
VARMIN(R1)=1E-10
VARMIN(R2)=1E-10
```

GROUP 20. Preliminary print-out

```
echo=t
```

GROUP 22. Monitor print-out

```
IZMON=NZ-2;IYMON=2;NPLT=1
```

GROUP 24. Dumps for restarts

```
nowipe=true
nsave=TEST
STOP
```

Appendix C: Ground Coding

```

SUBROUTINE GROUND
  INCLUDE 'lp2/d_includ/satear'
  INCLUDE 'lp2/d_includ/grdloc'
  INCLUDE 'lp2/d_includ/grdear'
  INCLUDE 'lp2/d_includ/grdbfc'

*****USER SECTION STARTS:*****

C    DATA NXDIM,NYDIM/10,10/
      data grhoc /1000.0/
      data grhod /1000.0/
      data ggrav /9.81/
C    grhod = rho_d + Cvm. rho_c

C*****

C--- GROUP 13. Boundary conditions and special sources
C                                Index for Coefficient - CO
C                                Index for Value      - VAL

13 CONTINUE
  GO TO (130,131,132,133,134,135,136,137,138,139,1310,
11311,1312,1313,1314,1315,1316,1317,1318,1319,1320,1321),ISC

C----- SECTION 12 ----- value = GRND

C    Buoyancy force

      if((npatch.eq.'GRAV').and.(indvar.eq.w2)) then
        call fn2(val, r2, 0.0, ggrav*(grhoc - 0.5*grhod))
      endif

C-----

C    Turbulent Diffusion Force

      if((npatch.eq.'GRAD').and.(indvar.eq.v1)) then
        call ggrad(ny,nz)
        call fn15(c31,ke,ep,0.0,0.165)
C    c31=time-scale(1)
        call fn28(c30,ep,0.165)
        call fn37(c30,ke,1.5)
C    c30=length scale
        call fn10(c35,w1,w2,0.0,-1.0,1.0)
        call fn40(c35)

```



```

C   c35=rel. vel
      call fn15(c32,c30,c35,0.0,0.5)
C   c32=time-scale(2)
      call fn67(c28,c31,c32,c31,1.0,1.0)
      call fn28(c33,c28,1.0)
      call fn21(c34,c32,c33,0.0,1.0)
C   c34=eff. time-scale
      call fn2(c26,c35,0.0,1000.0)
      call fn44(c29,c26,rg(5),0.1,0.75)
      call fn27(c29,c26)
C   1000.0 = ddrop/ENUL for SUN's case
C   c26= Nre (reynolds no.)
C   rg(5) = time-scale of particles from q1
C   c29= time-scale of particles(effective)

      call fn71(c3,ep,628.32,0.33333)
      call fn56(c4,c29,c3,c34,1.0)
      call fn6(c7,c4,0.9,1.0)
      call fn21(c8,c4,c7,0.0,1.0)
      call fn6(c9,c3,0.9,1.0)
      call fn21(c11,c3,c9,0.0,1.0)
      call fn15(c15,c29,c34,0.0,1.0)
      call fn3(c24,c15,-1.0,0.0,1.0)
      call fn56(c27,c15,c8,c24,1.0)
      call fn54(c27,c11,c24,-1.0)
      call fn25(c27,rg(3))

C   c27= Turbulent diffusion coefficient

      call fn2(val,c27,0.0,grhod)
      call fn26(val,ke)
      call fn26(val,c1)
      call fn27(val,dyg2d)
endif

      if((npatch.eq.'GRAD').and.(indvar.eq.v2)) then
        call ggrad(ny,nz)
        call fn2(val,c27,0.0,-grhod)
        call fn31(c12,c35,r2,0.25,2.0,1.0)
C   0.25 = C_v*0.5
        call fn10(c13,c12,ke,0.0,1.0,1.0)
        call fn26(val,ke)
        call fn26(val,c1)
        call fn27(val,dyg2d)
      endif

```

C LIFT FORCE IN MOMENTUM EQUATIONS

```

if((npatch.eq.'LIFTY').and.(indvar.eq.v1)) then
  call glift(ny,nz)
  call fn0(val,c5)
  call fn27(val,dyg2d)
  call fn0(c14,val)
  call fn21(val,c25,c14,0.0,-1.0)
endif

```

```

if((npatch.eq.'LIFTY').and.(indvar.eq.v2)) then
  call glift(ny,nz)
  call fn0(val,c5)
  call fn27(val,dyg2d)
  call fn0(c14,val)
  call fn21(val,c25,c14,0.0,1.0)
endif

```

C PRESSURE FORCE IN MOMENTUM EQUATIONS

```

if((npatch.eq.'PRESY').and.(indvar.eq.v1)) then
  call ggrad(ny,nz)
  call fn10(c35,w1,w2,0.0,-1.0,1.0)
  call fn40(c35)
  call fn3(val,c35,0.0,0.0,1.0)
  call fn25(val,rg(4)*grhoc)
  call fn26(val,c1)
  call fn27(val,dyg2d)
endif

```

RETURN

1312 CONTINUE

C--- GROUP 19. Special calls to GROUND from EARTH

19 GO TO (191,192,193,194,195,196,197,198,199,1910),ISC

194 CONTINUE

C * ----- SECTION 4 ---- Start of iterations over slab.

C LIFT SOURCE TERMS : CL * DENSL * VOID * REL .VEL. * VORTICITY

call fn10(c35,w1,w2,0.0,-1.0,1.0)

call fn21(c25,r2,c35,0.0,-rg(1)*grhoc)

RETURN

C*****

END

Subroutine ground in turn calls two other subroutines ggrad.f and glift.f which calculates "grad alpha" and "grad w1" respectively, to be used in interfacial transfer terms of turbulent diffusion and lift force.

```
*****
```

```

SUBROUTINE ggrad(ny,nz)
INCLUDE 'lp2/d_includ/grdloc'
common /namfn/ namfun, namsub
character*6 namfun, namsub
namsub = 'ggrad'

```

```
C----setup pointers to f-array
```

```

lr2 = 10f(r2)
if (iz.ne.1) lr2l = 10f(low(r2))
if (iz.ne.nz.and.nz.ne.1) lr2h = 10f(high(r2))
ldr2dy = 10f(c1)
ldr2dz = 10f(c2)

```

```
C----do loop over the whole slab
```

```

do 10 iy = 1,ny

    ic = iy
    icpy = ic + 1
    icmy = ic - 1

```

```
C----obtain void fractions in computational stencil
```

```

gr2 = f(lr2+ic)
gr2py = f(lr2+icpy)
gr2my = f(lr2+icmy)
if (iz.ne.1) gr2mz = f(lr2l+ic)
if (iz.ne.nz) gr2pz = f(lr2h+ic)

```

```
C----evaluate the void fraction differences
```

```

f(ldr2dy+ic) = gr2py - gr2
if (iy.eq.ny) f(ldr2dy+ic) = gr2 - gr2my

f(ldr2dz+ic) = gr2pz - gr2
if (iz.eq.nz) f(ldr2dz+ic) = gr2 - gr2mz

```

```

10    continue
    end

```

```
*****
```

```

SUBROUTINE glift(ny,nz)
INCLUDE 'lp2/d_includ/grdloc'
common /namfn/ namfun, namsub
character*6 namfun, namsub
namsub = 'glift'

```

C---setup pointers to f-array

```

lw1 = 10f(w1)
if (iz.ne.1)          lw1l = 10f(low(w1))
if (iz.ne.nz.and.nz.ne.1) lw1h = 10f(high(w1))
ldwldy = 10f(c5)
ldwidz = 10f(c6)

```

C---do loop over the whole slab

```

do 10 iy = 1,ny

    ic = iy
    icpy = ic + 1
    icmy = ic - 1

```

C---obtain axial liquid velocity in computational cross

```

gw1 = f(lw1+ic)
gw1py = f(lw1+icpy)
gw1my = f(lw1+icmy)
if (iz.ne.1) gw1mz = f(lw1l+ic)
if (iz.ne.nz) gw1pz = f(lw1h+ic)

```

C---evaluate the axial vel. differences

```

C      if (iy.eq.1) f(ldwldy+ic)=0.0
      if (iy.eq.ny) f(ldwldy+ic) = gw1 - gw1my
      f(ldwldy+ic) = gw1py - gw1

      if (iz.eq.1) f(ldwidz+ic)=0.0
      f(ldwidz+ic) = gw1 - gw1mz

```

```

10    continue
      end

```