Solenoid Transport for Heavy Ion Fusion*

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Abstract

Solenoid transport of high current, heavy ion beams is considered for several stages of a heavy ion fusion driver. In general this option is more efficient than magnetic quadrupole transport at sufficiently low kinetic energy and/or large e/m , and for this reason it has been employed in electron induction linacs. Ideally an ion beam would be transported in a state of Brillouin flow, i.e. cold in the transverse plane and spinning at one half the cyclotron frequency. The design of appropriate solenoids and the equilibrium and stability of transported ion beams are discussed. An outline of application to a fusion driver is also presented.

1. Introduction

Solenoid magnets are now being considered for the transport and focusing of heavy ion beams in Heavy Ion Fusion (HIF) drivers and related experiments. This is at first sight surprising because the focusing strength of a solenoid lens is easily shown to be much smaller than that of a comparable magnetic quadrupole lens for the high energy, singly charged ions typically of HIF. The focal length of a short solenoid of effective length ℓ and peak field B is approximately

$$f = \frac{4[\hbar c]^2}{B^2 \ell}, \quad (1)$$

where $[\hbar c] = P/qe$ is the “magnetic rigidity” of an ion of momentum P and charge state q . By contrast, a quadrupole’s focal length scales linearly with $[\hbar c]$:

$$f = \frac{[\hbar c]}{B^2}, \quad (2)$$

where $B^2 = B/R$ is the quadrupole’s transverse field gradient and B is evaluated at the wire ($r=R$). For electron induction linacs, typically with $[\hbar c] \approx 0.01 \text{--} 0.1 \text{ T-m}$, copper wire solenoids with $B \approx 0.17 \text{ T}$ have been the obvious choice. However, the scaling of f with $[\hbar c]^2$ makes them appear unattractive compared with quadrupoles for $[\hbar c] = 2 \text{--} 200 \text{ T-m}$, that has been

characteristic of HIF driver designs. This is true even though a solenoid always focuses in both transverse directions, while at least two quadrupoles with opposed gradients are required for a net focusing effect. However, since the use of superconducting magnets with $B=4\text{--}10\text{ T}$ is anticipated for HIF drivers, it is reasonable to extend the application of solenoids up to $[4\text{--}10]\text{ T-m}$, e.g. 240 MeV Ne^+ or 36 MeV Cs^+ .

Several applications of solenoids to HIF are currently under consideration. These include beam transport in high current injectors [1], transport in induction linacs [2], transport in neutralized drift compression [3], and a simple final focus system for highly stripped ions. A common theme here is the recognition that highly neutralized transport (if successful) in the fusion target chamber, final focus, and drift compression, makes much lower ion masses and energies and much higher currents options for HIF. A representative example [2] is a 6.7 MJ driver comprised of 24 linacs, each accelerating $1.12 \times 10^{-3}\text{ Coulombs}$ of Ne^+ ions to $200\text{--}300\text{ MeV}$. Due to their very large space charge forces such beams cannot be focused in vacuum during and after final compression, but this appears to be possible for ion beams neutralized by electrons.

An induction linac driver for HIF has for the past 25 years been seen as an accelerator of many parallel, very rigid ion beams of multi-GeV energies, using complicated multiple-beam-channel quadrupoles for transport. This may in fact be an optimal approach for the economical generation of electricity once the required technologies are well developed. However, unlike laser-driven fusion, this approach suffers from a serious barrier to development at an

intermediate stage due to its lack of modularity. The use of parallel linacs of greatly reduced kinetic energy solves this problem. A single 200 MeV linac can establish much of the technical and economic basis for the entire driver and also be used for relevant experiments on targets.

The objective of this paper is to outline essential features of solenoids and their use for beam transport. It can serve as a basis for further work on fusion drivers.

2. Solenoid Magnets

For beam transport in vacuum we need a combination of high fields ($[4\text{--}10]\text{ T}$) and large beam radius ($a [0.05\text{--}0.15]\text{ m}$). This may be achieved with superconducting cable cooled to 4.2 K and wrapped in an azimuthally symmetric layer of $1\text{--}2\text{ cm}$ thickness to produce current density $J_\phi(r, z)$ at mean radius $R=1\text{--}2\text{ m}$. For the lower fields, copper-stabilized *NbTi* cable is suitable (gross averaged current density $J_c = 800\text{ A/mm}^2$ at 6 T), and for high fields stabilized *PbSn* could be used ($J_c = 500\text{ A/mm}^2$ at 10 T). Wire leads and windings are “paired” to cancel unwanted components of current that would create avoidable field aberrations.

The solenoidal windings of successive magnets are separated by gaps of length $g [10]\text{ cm}$ to accommodate acceleration, vacuum pumping, diagnostics, etc. Hence the field experienced by the transported ions cannot be uniform. Transport lattice design assumes a field that is periodic in longitudinal variable z or with the period (determined by gap locations) varying slowly with z . This is necessary in order to maintain a matched beam envelope,

i.e. beam radius that is also a periodic function of z , which minimizes radial oscillations and cancels the effects of some aberrations. The solenoids do not reverse polarity and the gaps are kept as short as practical in order to minimize return flux entering the induction cores that surround the cryostats. Also, transportable line charge density is proportional to the longitudinal mean of $\square_z^2$, so there is a significant penalty for using long gaps.

Cylindrical collars of permeable steel may surround the superconducting wire in order to further minimize flux into the cores and slightly increase the focal strength. At the transport system ends flux lines flare into the surroundings, similar to the hypothetical field of a monopole. However these end fields can be cancelled among parallel linacs with opposing fluxes, eliminating the need for an overall flux return path. Beams emerging from transport will be affected by the end fields of nearby linacs; the design of such a transition zone with compensating dipole and quadrupole magnets to eliminate un-symmetric fields acting on beams remains an outstanding problem.

Denote the current per meter in a wire layer of thickness $\square r$ by $K(z) = J_{\square} \square r$. Then the longitudinal magnetic field jumps across the wire:

$$\square \square_z = \square_0 K. \quad (3)$$

This provides a rough measure of the field produced by the wire layer inside a long solenoid:

$$\square_z \square \square_0 K \square (1.0T) \square \frac{K}{796 kA/m} \square. \quad (4)$$

For computing beam dynamics we need a much better expression for the magnetic field. If no iron is present, then the longitudinal component satisfies

$$\square^2 \square_z = \square \square_0 \frac{1}{r} \frac{\partial}{\partial r} r J_{\square}, \quad (5)$$

with Green function solution

$$\square_z = \frac{\square_0}{4\square} \square \int_{-\infty}^{\infty} dz' r' \frac{1}{|r - r'|} \frac{\partial}{\partial r'} r' J_{\square}(z'). \quad (6)$$

After an integration by parts, the on-axis field from a thin current layer at radius R is

$$\square_0(z) = \frac{\square_0 R^2}{2} \square \int_{-\infty}^{\infty} dz' \frac{K(z')}{\left[(z - z')^2 + R^2 \right]^{3/2}}. \quad (7)$$

This integral can be evaluated analytically for many relevant geometries. The external field ($r > R$) and field close to the wire are more difficult to evaluate, usually requiring numerical calculations (especially if iron is present). The field lines for a solenoid are given simply by the contours of constant $r A_{\square}(r, z)$, where $A_{\square}$ is the vector potential.

When the on-axis field $\square_0(z)$ is known, analytically or by an accurate measurement or computation, off-axis values for $r < R$ are readily obtained by a power series solution of $\square^2 \square_z = 0$:

$$\square_z(r, z) = \square_0 \square \frac{\square_0^2}{4} + \frac{\square_0^4}{64} \square \dots \quad (8)$$

From $\square \cdot \square = 0$ we also have

$$\square_r(r, z) = \square \frac{\square_0^2}{2} + \frac{\square_0^3}{16} \square \frac{\square_0^5}{384} + \dots \quad (9)$$

The leading terms of $\square_z$ and $\square_r$ are used in the computing the linearized dynamics of an ion. The higher order powers in r are fringe field aberrations, which one tries to minimize with short periodic gaps. The expansion of $\square_z$ in powers of r gives a good representation out to $r=R$ using 2-3 terms, but it fails

badly at the ends of the winding. Surprisingly, a thin wire layer has a logarithmically divergent field $\square_r$ at its ends. This is smoothed out with $\square_r/R > .1$, so that the peak field actually occurs at the wire edge near the middle of the magnet. The azimuthal symmetry of the solenoid winding automatically eliminates the higher order (unwanted) multipoles, which complicate the design and construction of dipoles, quadrupoles, etc.

For a semi-infinite wire layer ($K = 0$ for $z \geq 0$) equation (7) yields

$$\square_0(z) = \frac{\square_0 K}{2} \square \frac{z}{\sqrt{z^2 + R^2}} \square \quad (10)$$

Many useful formulas for design and beam dynamics can be obtained from equation (10) by adding or subtracting layers of different radii and end points. For a simple layer of length ℓ , centered at $z=0$, we get the single lens (textbook) formula

$$\square_0^{Lens}(z) = \frac{\square_0 K}{2} \square \frac{(z + \ell/2)}{\sqrt{(z + \ell/2)^2 + R^2}} \square - \frac{(z - \ell/2)}{\sqrt{(z - \ell/2)^2 + R^2}} \square \quad (11)$$

For a lattice with gaps of length g and period $P = \ell + g$, we obtain a formula suitable for beam envelope computations:

$$\square_0(z) = \square \square_0^{Lens}(z \square iP). \quad (12)$$

The radial component B_r , which is inevitably generated at gaps, is necessary for a consistent treatment of particle dynamics. It also defines a magnetic axis, which can be used for magnet alignment.

3. Beam Dynamics – Brillouin Flow

An ion is focused (i.e. deflected towards the system axis) by the interaction of its azimuthal velocity ($v_\square$) with $\square_z$. An ion moving straight ahead does not interact with $\square_z$, but it does receive an azimuthal velocity from its interaction with the $\square_r$ induced by changing $\square_z$. Therefore the focusing power of a short solenoid is second order in $\square_z$ (see equation (1)). For an axisymmetric system the transverse forces are:

$$F_r = qe(E_r + v_\square \square_z), \quad (13)$$

$$F_\square = qe(v_z \square_r - v_r \square_z). \quad (14)$$

A solenoid also produces a small longitudinal force ($\sim r^2$) that adds to the electric field from the gaps or space charge:

$$F_z = qe(E_z \square v_\square \square_r). \quad (15)$$

In general we wish to approximate Brillouin flow, where E_r from space charge and the centrifugal force ($M \square_\square^2/r$) are balanced by $v_\square \square_z$. This is the equilibrium of a cold beam with zero canonical angular momentum, i.e. emittance $\square = 0$ and

$$P_\square = r(Mv_\square + qeA_\square) = 0. \quad (16)$$

In a constant solenoidal field such a beam simply spins at the Larmor frequency ($\square \square_c/2$), and the radial electric field from space charge and centrifugal force are exactly balanced by the Lorentz force:

$$v_\square = \square \frac{\square_c r}{2} = \square \frac{qe \square_z}{M} \square \frac{r}{2}, \quad (17)$$

$$\square_r = \frac{\square}{2 \square \square} \frac{r}{a^2} = \square v_\square \square_z \square \frac{M \square_\square^2}{qe r}, \quad (18)$$

where $\square$ is the line charge density and a is the beam edge radius. In Brillouin flow the centrifugal force is one half the Lorentz force, with reverse sign. Solving for the line charge density we get

$$\begin{aligned} \Phi = \frac{\Phi_0}{2} \frac{qe}{\Phi} \Phi_z^2 a^2 \\ \Phi \left[66.5 \frac{\Phi C}{m} \frac{\Phi}{A/q} \frac{20}{\Phi} \right] \Phi_z^2 a^2. \end{aligned} \quad (19)$$

Note that the center-to-edge potential difference in Brillouin flow,

$$\Phi - \Phi_c = \frac{\Phi}{4\Phi_0} = \frac{\Phi}{2qe} \frac{\Phi_c a}{2} \quad (20)$$

is exactly the difference in kinetic energy attributed to the spin. In fact in Brillouin flow all ions with the same longitudinal velocity have the same total energy (kinetic plus potential).

Equation (19) suggests we can transport up to $66.5 \Phi C/m$ of Φe^+ (atomic mass $A=20$) with e.g. $\Phi_z = 10T$, $a = .1m$. This is much larger than the $.25 \Phi C/m$ readily achievable with high voltage electrostatic quadrupoles and also larger than the approximate $4.0 \Phi C/m$ achievable for Φe^+ with magnetic quadrupoles following a 2.0 MeV injector. Brillouin flow looks very good for large $(\Phi a) \Phi 1.0T\text{-}m$ and very poor for small $(\Phi a) \Phi .1T\text{-}m$. The beam potential $\Phi - \Phi_c$ has the very large value of 600kV when $\Phi = 66.5 \Phi C/m$, so a large potential for growth of transverse emittance from damping of space charge waves is apparent in this example. This is true even though analytic theory and simulations have shown that only a few percent of $\Phi - \Phi_c$ is actually available as free energy for such growth.

It is especially interesting that the longitudinal velocity (v_z) does not appear in the Brillouin flow formula for Φ . In fact a stationary cloud of charge can be confined in a solenoid if electrostatic end plugs are applied. However, this does not resemble either a typical magnetized plasma or most “non-neutral plasmas”, in which the gyro-

radius is small compared with a . In a statically confined cloud the beam radius varies inversely with Φ_z , so gaps would produce large bulges. For a typical transported beam the opposite behavior is expected, i.e. the matched radius is largest where Φ_z is largest. This requires finite longitudinal velocity and sufficiently short period lattice period P .

4. Beam Dynamics-Periodic Solenoids

For variable $\Phi_0(z)$ the beam radius satisfies an envelope equation, which may be derived from the axisymmetric forces, equations (13-15), with only linear terms included. The result is conveniently written in the form

$$\frac{d^2 a}{dz^2} = \Phi \frac{k_c^2}{4} a + \frac{Q}{a} + \frac{\Phi^2 + 4L^2}{a^3} \quad (21)$$

$\Phi \frac{1}{v} \frac{dv}{dz} \frac{da}{dz} \Phi \frac{1}{4v^2} \frac{d^2 v^2}{dz^2} a$
Here $a(z)$ is the beam edge radius, $v(z)$ is longitudinal velocity, $k_c(z) = \Phi_c/v$, Φ is un-normalized edge emittance, $L = \overline{P_\perp}/Mv$, and $Q(z)$ denotes the dimensionless perveance:

$$Q = \frac{\Phi}{4\Phi_0} \frac{2qe}{\Phi v^2}. \quad (22)$$

The envelope equation (21) describes a beam which in general is spinning, even when $\Phi_\perp = 0$. The effects of finite Φ_r and $\Phi_\perp$ and arbitrary $k_c(z)$ are included in its derivation. For a drifting beam, Φ and L are constant, but they decrease as $1/v$ during acceleration ($\overline{P_\perp}$ is constant). The final two terms in the envelope equation (21) represent the focusing effect of acceleration in the gaps and in the injector; they are essential in treating a solenoid-aided source and actually yield the Child-Langmuir source law for assumed constant a . Canonical angular momentum is clearly equivalent to

emittance and should therefore be minimized in order to allow the beam to be focused to a small radius on the fusion target. The solenoid field should therefore be made to vanish on a source's emitting surface.

The essential features of a matched envelope are apparent in the treatment of periodic quasi-Brillouin flow, i.e. a coasting cold beam with $\overline{P}_0 = 0$ and subject to periodic $\square_z$. Let

$$k_c(z) = k_0[1 + \square \cos(2\square z/P)], \quad (23)$$

where $\square \square .1 \square .5$ represents the effect of the gaps. Then by equation (21) the matched envelope radius satisfies

$$\frac{d^2 a}{dz^2} = \square \frac{k_c^2(z)}{4} a + \frac{Q}{a}, \quad (24)$$

with $a(z + P) = a(z)$.

For $\square = 0$ we have

$$a = a_0 = 2\sqrt{Q}/k_0. \quad (25)$$

For small $\square$,

$$a \square a_0 \square + \frac{(\square) \cos(2\square z/P)}{\square \frac{8\square^2}{P^2 k_0^2} \square 1} \square. \quad (26)$$

Note that this approximate solution actually has the impossible values $a < 0$ when $k_0 P \square \sqrt{8}\square$. This marks a transitional "zone of trouble" where the beam is between the two types of matched equilibria. When the envelope is treated numerically in this regime, either no matched solution is found or it has very large radial excursions. For $k_0 P \ll \sqrt{8}\square$ there is a normal matched (beam-like) solution with very low flutter $((a_{\max} \square a_0)/a_0 \ll \square)$, and for $k_0 P \gg \sqrt{8}\square$ the matched solution has the reverse (magnetized plasma-like) behavior with $(a_{\max} \square a_0)/a_0 \square \square$. Physically, the trouble zone corresponds

to a resonance between an envelope oscillation of period P and the periodic variation of B_z . The beam-like regime is preferred because of its very low flutter, its accessibility with typical HIF parameters, and also its stability. The low flutter of solenoid transport is expected to minimize halo formation and loss of ions to the pipe wall and also to minimize the effect of fringe-field aberrations.

5. Beam Dynamics - Stability

In examining the stability of the matched envelope, it is useful to first look at the motion of a single ion in the absence of space charge. In a frame rotating at the local Larmor frequency $(\square\square_c/2)$, a single ion's transverse position satisfies

$$\frac{d^2 r_\square}{dz^2} = \square \frac{k_c^2(z)}{4} r_\square, \quad (27)$$

$$x \square \cos \square \sqrt{k_c^2} z/2 \square. \quad (28)$$

So the phase advance per period P (undepressed tune) is

$$\square_0 \square \sqrt{k_c^2} P/2 \square k_0 P/2. \quad (29a)$$

The previously described "trouble zone" therefore corresponds to $\square_0 \square \sqrt{2}\square = 255^\circ$. However, our simple solution [equation (28)] for $x(z)$ is incorrect in an unstable band near $\square_0 = 180^\circ$, with relative bandwidth $\square \square$. The reason is that the period of variation of k_c with z comes into a half-integer resonance with the ion's oscillation period. Imagine a child standing on a park swing and pumping up and down at double the swing's frequency. There are also unstable single particle bands at $\square_0 = 360^\circ, 540^\circ$, etc., but with diminishing width.

The cold matched beam (including space charge) is readily shown to be unstable in a band around $\varphi_0 = 127^\circ$, with bandwidth depending on φ . To see this consider a warm beam (finite emittance), with matched radius $a_m(z)$ satisfying the envelope equation

$$a'' = -\frac{k_c^2(z)}{4}a_m + \frac{Q}{a_m} + \frac{\varphi^2}{a_m^3}. \quad (29b)$$

Small deviations $\delta a = a - a_m$ satisfy the linearized equation

$$\delta a'' = -\frac{k_c^2(z)}{4}\delta a - \frac{Q}{a_m^2}\delta a - \frac{3\varphi^2}{a_m^4}\delta a. \quad (30)$$

This equation involves z variations with period P from both k_c and a_m , but in analogy with the single particle equation we can write the approximate form

$$\delta a'' = -\frac{1}{P^2}[\varphi_0^2 + (\varphi_0^2 \mp \varphi^2) + 3\varphi^2]\delta a, \quad (31)$$

where the undepressed and depressed tunes (φ_0, φ) have eliminated k_c , φ and Q using the smooth limit relations:

$$\varphi_0 \approx k_0 P/2, \quad (32)$$

$$\varphi \approx \sqrt{P/a_0^2}, \quad (33)$$

$$QP^2/a_0^2 \approx \varphi_0^2 \mp \varphi^2. \quad (34)$$

We have

$$\delta a \approx \cos\left(\sqrt{2(\varphi_0^2 \mp \varphi^2)}z/P\right), \quad (35)$$

and therefore expect a half-integer instability in a band near

$$\varphi_0^2 \mp \varphi^2 = \frac{\varphi_0^2 \varphi^2}{\sqrt{2}} = (127^\circ)^2. \quad (36)$$

This prediction is well verified by numerical solutions of the envelope equation [4]. It is interesting that the prediction of instability occurs at the low value $\varphi_0 = 90^\circ$ when emittance dominates over perveance (so that $\varphi = \varphi_0$). The same smooth limit formulas also apply to quadrupole transport, except that the instability is not restricted to a narrow band, but is

predicted for a wide zone of (φ_0, φ) around and beyond the boundary equation (36). A second envelope instability for periodic solenoids [4], associated with a quadrupole distortion, appears in a band around

$$\varphi_0^2 + 3\varphi^2 \approx (180^\circ)^2, \quad (37)$$

e.g. at $\varphi_0 = 180^\circ$ for a cold beam.

At present we should be cautious about the application of these stability criteria since they also define stability boundaries for beams transported by quadrupoles. But the PIC simulations to date [5] suggest $\varphi_0 < 85^\circ$ is required for stable transport using quadrupoles, even when $\varphi \approx 0$. The reason for this discrepancy is unknown.

6. Applications in a Fusion Driver

Some parameters of a modular (24 linacs) driver were given in the Introduction. These essentially correspond to the largest line-charge density of Ne^+ ions, which can be transported in a linac, based on equation (19). However, the line density $\lambda = 67.5$ C/m must be produced for the linac and it must later be compressed and focused onto the fusion target. The first task clearly requires a large increase in the capability of injectors. One possibility is the accel-decel injector described by Kwan [1]. This may provide a factor of 4-10 increase in λ , which is temporarily accumulated in a high field solenoid following a triode source. Another large increase in λ may be available by applying magnetic insulation to the emitting anode, as has been done for light ions. In this approach very large extraction fields may be made possible by using a solenoid to suppress electron flow in the first gap [1]. A third route to high λ is through rapid compression following the injectors. This is plausible

for solenoid-transported beams due to their relative insensitivity to velocity variations within a pulse.

Once high γ has been reached a linac would operate in a “load and fire” mode, i.e. acceleration with constant pulse length in meters, so pulse duration would decrease inversely with velocity. The pulse would be held together longitudinally by shaping of the accelerating waveform rather than by applying special pulse-end “ear” fields. An overall parabolic pulse profile may be best suited for this mode.

When the desired energy of 200 MeV (prepulse) and 300 MeV (main pulse) is reached, the pulse durations are on the order of 200 ns. Final compression to 10-30 ns must be carried out in neutralized beam lines, due to the very large space charge fields that would otherwise be present in vacuum. Preliminary study [3] suggests the feasibility of this transport mode, but much work clearly remains to be done. A weak solenoid field ($B < 1.0\text{T}$) would be used in these beam lines to provide equilibrium for whatever emittance or angular momentum is present. Some bending and steering is also necessary, even though electrons are present.

Groups of beams are merged a short distance from the fusion target (10-

20 m). This must be carried out without disruption of the ion beam dynamics by un-symmetrical fields from nearby beam lines – an outstanding (but not impossible) design problem. Final focus of a beam bundle might be most simply done by a single large solenoid surrounding the target chamber. If the Neon ions have been stripped to charge state $q=+10$, their magnetic rigidity is reduced to $[B\rho] \approx 1.0\text{ T}\cdot\text{m}$, and the final large solenoid can be made from copper wire. The individual ions would approach the target with convergence angles of up to 100 mr, providing a spot radius of about 4.0 mm even for a very large edge emittance $\epsilon \approx 4 \times 10^{-4}\text{ m}\cdot\text{r}$. Several issues, which are obvious for this type of focus and now being investigated, are the affect of aberrations from the final lense and the need to make the focal system approximately achromatic. The latter issue is very serious since the velocity tilt required for compression ($\Delta v/v \approx .01-.1$) is not removable by the space charge force. Bends and pulsed focal magnets upstream from the final focus lens may be required for this.

References

- [1] J.W. Kwan, “Ion Sources and Injector Experiments at the HIF-VNL,” These Proceedings
- [2] W. Meier, “Systems Analysis for Modular Versus Multi-Beam HIF Drivers,” These Proceedings
- [3] D. Welch, “Simulations of Neutralized Final Focus,” These Proceedings
- [4] S. Lund and B. Bukh PRST-AB Vol. 7 (2004), 024801
- [5] I. Haber, et al. NIMPR-A 464 (2001), 342-350