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105-KE BASIN ISOLATION BARRIER
LEAK RATE TEST
ANALYTICAL DEVELOPMENT

March 1995

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1.0 INTRODUCTION

This document provides an analytical development in support of the proposed leak rate test of the 105-KE Basin as described in WHC 1994a and 1994b.

The analytical basis upon which the K-basin leak test results will be used to determine the basin leakage rates is developed in this report. The leakage of the K-Basin isolation barriers under postulated accident conditions will be determined from the test results.

There are two fundamental flow regimes that may exist in the postulated K-Basin leakage: viscous laminar and turbulent flow. An analytical development is presented for each flow regime. The basic geometry and nomenclature of the postulated leak paths are denoted on Figure 1 and 2.

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Figure 1. K-Basin Proposed Leak Test Nomenclature.

Figure 2. 105-KE Basin Elevation View.

2.0 NOMENCLATURE

The basic symbols and nomenclature used for the discussion of fluid flow leakage are listed below. For consistency, the units for each symbol are given in terms of mass (M), length (L), and time (t).

Nomenclature

SYMBOL	DEFINITION	DIMENSIONS
A_L	Cross-sectional area of a leak path	L^2
A_B	Surface area of Storage Basin	L^2
A_{DC}	Surface area of Discharge Chute	L^2
A_t	Surface area of Storage Basin and Discharge Chute	L^2
A^*	Surface area parameter for Basin and Discharge Chute	L^2
b	Parameter in flow equation	$1/t$
D	Diameter of circular cross-section leak	L
E_B	Evaporation rate of the Storage Basin	L^3/t
E_{DC}	Evaporation rate of the Discharge Chute	L^3/t
E_{inst}	Evaporation rate of the VED Instrument	L^3/t
E^*	Evaporation rate parameter for the Storage Basin and Discharge Chute	L^3/t
f	Friction factor	Dimensionless
g_c	Conversion Constant ($32.174 \text{ Lb}_m\text{-ft/Lb}_f\text{-sec}^2$)	Dimensionless
g_0	Gravitational acceleration constant (32.2 ft/sec^2)	L/t^2
h_t	Height of a fluid column	L
h_2, h_b	Upstream static pressure head of a leak path	L
h_1, h_{dc}	Downstream static pressure head of a leak path	L
K_L	Leak rate correction factor	L
L	Length of leak path (between pressure zones P_1 and P_2)	L
n	Number of data pairs	Dimensionless
P_2	Upstream absolute pressure of a leak path	M/Lt^2

SYMBOL	DEFINITION	DIMENSIONS
P_1	Downstream absolute pressure of a leak path	M/Lt^2
Q_B, Q_L, Q_J, Q^*	Volumetric flow rate of Basin, Isolation Barrier Seal, Construction Joint or a General form	L^3/t
R_e	Reynolds number	Dimensionless
t	Time interval	t
V, V_B, V_{DC}	Total volume of fluid	L^3
V_s	Average fluid velocity	L/t
w_L, w_{Lj}	Constant in the laminar flow equation	L^2/t
w_T, w_{Tj}	Constant in the turbulent flow equation	L^2/t
w	Constant in the general flow equation	L^2/t
x_1, x_2, x_3	Differential height curve fit coefficients	
Z	Elevation	L
α	Parameter describing the flow regime	Dimensionless
ρ	Fluid density in a leak passage	M/L^3
σ	Standard deviation	
μ	Viscosity of fluid	M/Lt

3.0 CASE 1: DIFFERENTIAL WATER LEVEL ACROSS THE ISOLATION BARRIERS

3.1 VISCOUS LAMINAR FLOW

First assume that leakage from the basin is through the proposed isolation barrier to the discharge chute. In the following analysis, the surface evaporation rate and leakage through the expansion joint are neglected. The general parameters that define the viscous flow of a fluid through a straight leak passage of circular section are expressed in familiar form by the Hagen-Poiseuille equation (WHC 1992). The following equation is valid for Reynolds numbers, Re , (WHC 1992) less than 2000. This is derived by summing pressure, gravity and shearing forces of a fluid element in a cylindrical object for steady uniform flow,

$$P_2 - P_1 = \frac{32\mu V_s L}{D^2}.$$

and;

$$V_s = \frac{Q_L}{A_L}$$

therefore;

$$Q_L = \frac{D^2 A_L}{32\mu L} (P_2 - P_1)$$

To determine the volumetric flow rate in terms of static pressure due to a water column height the following relationship is used;

$$P = \frac{\rho g_0 h}{g_c}$$

therefore;

$$Q_L = \frac{D^2 A_L}{32\mu L} \rho \frac{g_0}{g_c} (h_2 - h_1)$$

From the above equation the leak flow rate is proportional to the difference in the static head of the water column above the leak path. The terms associated with the flow geometry and viscosity are unknown and are constant. Therefore equating these terms to a constant, w_L , results in the following equation.

$$Q_L = w_L (h_2 - h_1)$$

where:

$$w_L = \frac{D^2 A_L}{32 \mu L} \rho \frac{g_o}{g_c}$$

At any given point in time the above equation can be stated as;

$$Q_L(t) = w_L (h_2(t) - h_1(t))$$

In terms of the K-Basin nomenclature, ie., $h_2 = h_b$ and $h_1 = h_{dc}$;

$$Q_L(t) = w_L (h_b(t) - h_{dc}(t))$$

The flow constant, w_L , is in the form of a discharge coefficient. The unknown discharge coefficient can be determined from the K-Basin leak test results considering the following derivation. From the principle of conservation of mass, the leak rate can also be stated as;

$$Q_L(t) = A_B \frac{dh_b(t)}{dt} = -A_{dc} \frac{dh_{dc}(t)}{dt}$$

let;

and;

$$h_t = h_b - h_{dc}$$

$$\frac{dh(t)}{dt} = \frac{dh_b}{dt} - \frac{dh_{dc}}{dt}$$

noting;

$$\frac{dh_b}{dt} = -\frac{Q_L}{A_B} = -\frac{w_L}{A_B} h_t$$

$$\frac{dh_{dc}}{dt} = \frac{Q_L}{A_{DC}} = \frac{w_L}{A_{DC}} h_t$$

therefore;

$$\frac{dh_t}{dt} = -\frac{w_L}{A_B} h_t - \frac{w_L}{A_{DC}} h_t = -w_L \frac{A_B + A_{DC}}{A_B A_{DC}} h_t$$

Separating variables and integrating;

$$\int \frac{dh_t}{h_t} = -w_L \left(\frac{A_B + A_{DC}}{A_B A_{DC}} \right) \int dt$$

Integrating from initial height h_{ti} at time t_i to final height h_{tf} at time t_f results in;

$$\ln(h_{ti}/h_{tf}) = w_L \frac{A_B + A_{DC}}{A_B A_{DC}} (t_f - t_i)$$

or;

$$w_L = \frac{A_B A_{DC}}{A_B + A_{DC}} \frac{\ln(h_{ti}/h_{tf})}{(t_f - t_i)}$$

also;

$$w_L = \frac{A_B}{\left(\frac{A_B}{A_{DC}} + 1 \right)} \frac{\ln(h_{ti}/h_{tf})}{(t_f - t_i)}$$

This is the equivalent discharge coefficient for the isolation barriers based on measured test results assuming a leakage geometry and pressure differential which produces laminar flow.

To account for evaporation, the equation for w_L can be modified to the following form;

$$w_L = \frac{A_B}{\left(\frac{A_B}{A_{DC}} + 1\right)} \frac{\ln(h_{ti}/h_{tfe})}{(t_f - t_i)}$$

and;

$$h_{tfe} = h_{tf)_{ind}} + \Delta t \left(\frac{E_B}{A_B} - \frac{E_{DC}}{A_{DC}} \right)$$

where;

- h_{tfe} = water column height adjusted for evaporation effects, ft.,
- $h_{tf)_{ind}}$ = indicated water column height, ft.,
- E_B = measured evaporation rate of Storage Basin, cu ft/sec,
- E_{DC} = measured evaporation rate of Discharge Chute, cu ft/sec,
- Δt = Delta time, $t_f - t_i$, sec.

Note, the initial height is not adjusted for evaporation as evaporation is a time rate function and h_{ti} is at time zero. Also, the evaporation rate, cu. ft. per sec., is determined from an instrument with a known surface area. Therefore, the evaporation rate of the storage basin is determined by the following;

$$E_B = E_{inst} * \frac{A_B}{A_{inst}}$$

where;

- E_{inst} = measured evaporation rate of the VED instrument, cu ft/sec,
- A_{inst} = Surface area of VED instrument, sq ft.

To account for construction joint leakage and the evaporation rate of the discharge chute, (see Section 4.1), the equation above for h_{tfe} can be modified in the following form;

$$h_{tfe} = h_{tf)_{ind}} + \Delta t \left(\frac{E_B}{A_B} - \frac{E_{DC}}{A_{DC}} - \frac{Q_j}{A_{DC}} \right)$$

where;

- h_{tfe} = water column height adjusted for evaporation and construction joint leakage effects, ft.,
- $h_{tf)_{ind}}$ = indicated water column height, ft.,
- Q_j = Construction joint leakage, cu ft/sec.

3.2 TURBULENT FLOW

Although the majority of significant leaks in a fluid system may well be in the laminar flow regime at a low pressure, it is conceivable that with higher velocities, turbulent flow conditions will be attained in irregular leak passages. The region of transition between viscous flow and turbulent flow may be conservatively estimated to begin with a Reynolds number, Re , greater than 2000. Flow instabilities are usually present at the transition region between viscous and turbulent flow that make prediction of the type of prevailing flow uncertain. Transition effects as the fluids enter and leave the irregularly shaped leak passage further complicate the flow character in the region of turbulent flow. If the leak path in a fluid system is of a nature to permit the development of turbulent flow, the volumetric flow rate, Q , may be derived from the Darcy-Weisbach equation, assuming that the leak cross section approaches the geometry of a small-diameter tube. The equation commonly appears as (WHC 1992);

$$P_2 - P_1 = \frac{fL}{g_c D} \frac{\rho V_s^2}{2},$$

Where f is the friction factor, whose value is a function of the leak passage roughness and the Reynolds number for the flow conditions through the leak. The volumetric flow rate is the product of the flow area and the average flow velocity, ie.;

$$Q_L = A_L V_s$$

therefore, substituting for V_s ;

$$Q_L = A_L \sqrt{\frac{2Dg_c}{fL\rho} (P_2 - P_1)}$$

in terms of static pressure head;

$$Q_L = A_L \sqrt{\frac{2Dg_0}{fL} (h_2 - h_1)}$$

From the above equation the leak flow rate is proportional to the square root of the difference in the static head of the water column above the leak path. The terms associated with the flow geometry and fluid friction factor are unknown and or are constant. Therefore, equating these terms to a constant, w_T , results in the following equation.

$$Q_L = w_T \sqrt{(h_2 - h_1)}$$

At any given time the above equation can be stated as;

$$Q_L(t) = w_T \sqrt{h_2(t) - h_1(t)}$$

In terms of the K-Basin nomenclature, ie., $h_2 = h_b$ and $h_1 = h_{dc}$

$$Q_L(t) = w_T \sqrt{h_b(t) - h_{dc}(t)}$$

The flow constant, w_T , is in the form of a discharge coefficient. The unknown discharge coefficient can be determined from the K-Basin leak test results considering the following derivation. From the principal of conservation of mass, the leak rate can also be stated as;

$$Q_L(t) = A_B \frac{dh_b(t)}{dt} = -A_{dc} \frac{dh_{dc}(t)}{dt}$$

let;

and;

$$h_t = h_b - h_{dc}$$

$$\frac{dh(t)}{dt} = \frac{dh_b}{dt} - \frac{dh_{dc}}{dt}$$

noting;

$$\frac{dh_b}{dt} = -\frac{Q_L}{A_B} = -\frac{w_T}{A_B}\sqrt{h_t}$$

$$\frac{dh_{dc}}{dt} = \frac{Q_L}{A_{DC}} = \frac{w_T}{A_{DC}}\sqrt{h_t}$$

therefore;

$$\frac{dh_t}{dt} = -\frac{w_T}{A_B}\sqrt{h_t} - \frac{w_T}{A_{DC}}\sqrt{h_t} = -w_T \frac{A_B + A_{DC}}{A_B A_{DC}}\sqrt{h_t}$$

Separating variables and integrating;

$$\int \frac{dh_t}{\sqrt{h_t}} = -w_T \left(\frac{A_B + A_{DC}}{A_B A_{DC}} \right) \int dt$$

Integrating from initial height h_{ti} at time t_i , to final height h_{tf} at time t_f results in;

$$\sqrt{h_{ti}} - \sqrt{h_{tf}} = \frac{w_T}{2} \frac{A_B + A_{DC}}{A_B A_{DC}} (t_f - t_i)$$

or;

$$w_T = \frac{2A_B A_{DC}}{A_B + A_{DC}} \frac{\sqrt{h_{ti}} - \sqrt{h_{tf}}}{(t_f - t_i)}$$

also;

$$w_T = \frac{2A_B}{\left(\frac{A_B}{A_{DC}} + 1 \right)} \frac{\sqrt{h_{ti}} - \sqrt{h_{tf}}}{(t_f - t_i)}$$

This is the equivalent discharge coefficient for the isolation barriers based on measured test results assuming a leakage geometry and pressure differential which produces turbulent flow.

To account for evaporation, the equation for w_T can be modified to the following form;

$$w_T = \frac{2A_B}{\left(\frac{A_B}{A_{DC}} + 1\right)} \frac{\sqrt{h_{ti}} - \sqrt{h_{tfe}}}{(t_f - t_i)}$$

and;

$$h_{tfe} = h_{tf)_{ind}} + \Delta t \left(\frac{E_B}{A_B} - \frac{E_{DC}}{A_{DC}} \right)$$

where;

h_{tfe} = water column height adjusted for evaporation effects, ft.,
 $h_{tf)_{ind}}$ = indicated water column height, ft.,
 E_B = measured evaporation rate of Storage Basin, cu ft/sec,
 E_{DC} = measured evaporation rate of Discharge Chute, cu ft/sec,
 Δt = Delta time, $t_f - t_i$, sec.

Note, the initial height is not adjusted for evaporation as evaporation is a time rate function and h_{ti} is at time zero. Also, the evaporation rate, cu. ft. per sec., is determined from an instrument with a known surface area.

Therefore, the evaporation rate of the storage basin and discharge chute are determined by the following;

$$E_B = E_{inst} * \frac{A_B}{A_{inst}}$$

and;

$$E_{DC} = E_{inst} * \frac{A_{DC}}{A_{inst}}$$

where;

E_{inst} = measured evaporation rate of the VED instrument, cu ft/sec,
 A_{inst} = Surface area of VED instrument, sq ft.

The evaporation rate of the discharge chute is assumes that the evaporation from the process/reactor area is negligible.

To account for construction joint leakage (see section 4.1 below), the equation above for h_{tfe} can be modified in the following form;

$$h_{tfe} = h_{tf)_{ind}} + \Delta t \left(\frac{E_B}{A_B} - \frac{E_{DC}}{A_{DC}} - \frac{Q_j}{A_{DC}} \right)$$

where;

h_{tfe} = water column height adjusted for evaporation and construction joint leakage effects, ft.,

$h_{tf)_{ind}}$ = indicated water column height, ft.,

Q_j = Construction joint leakage, cu ft/sec.

3.3 LEAK TEST ACCEPTANCE CRITERIA FOR LAMINAR AND TURBULENT FLOW

In accordance with WHC 1994a, the K-Basin leak test acceptance criteria is specified at 750 gallons per hour at a 16 foot level differential. From section 3.1 the leak rate is calculated as (for laminar flow);

$$Q_L = w_L h_t$$

Substituting the maximum allowable leak rate, 750 gph, at the specified head differential, 16 ft, in the flow rate equation and solving for w_L , results in (note, the unit of time has been converted to days);

$$w_L = 150.4 \text{ sq.ft per day}$$

In order to determine the maximum allowable change in level differential as a function of time, the flow constant equation is repeated from section 3.1 as;

$$w_L = \frac{A_B A_{DC}}{A_B + A_{DC}} \frac{\ln(h_{ti}/h_{tf})}{(t_f - t_i)}$$

Rearranging, solving for the final differential height, h_{tf} , and noting that the initial time, t_i , is zero, results in;

$$h_{tf} = h_{ti} \cdot e^{-\left(w_L t \frac{A_B + A_{DC}}{A_B A_{DC}}\right)}$$

From WHC 1994a, the K-Basin surface areas are 8880 sq. ft and 940 sq. ft. respectively for the main storage basin and discharge chute. This results in the numerical form of the allowable differential level as a function of time;

$$h_{tf} = h_{ti} \cdot e^{-(0.1768 \cdot t)}$$

For the turbulent flow assumption, from section 3.2 the leak rate is calculated as;

$$Q_L = w_T \sqrt{h_t}$$

Substituting the maximum allowable leak rate, 750 gph, at the specified head differential, 16 ft, in the flow rate equation and solving for w_T , results in (note, the unit of time has been converted to days);

$$w_T = 601.6 \text{ ft}^{5/2} \text{ per day}$$

In order to determine the maximum allowable change in level differential as a function of time, the flow constant equation is repeated from section 3.2 as;

$$w_T = \frac{2A_B A_{DC}}{A_B + A_{DC}} \frac{\sqrt{h_{ti}} - \sqrt{h_{tf}}}{(t_f - t_i)}$$

Rearranging, solving for the final differential height, h_{tf} , and noting that the initial time, t_i , is zero, results in;

$$h_{tf} = \left(\sqrt{h_{ti}} - \left(w_T t \frac{A_B + A_{DC}}{2A_B A_{DC}} \right) \right)^2$$

Substitution of the K-Basin surface areas for the main storage basin and discharge chute as before results in the numerical form of the allowable differential level as a function of time;

$$h_{tf} = (\sqrt{h_{ti}} - 0.3535 t)^2$$

A plot of the maximum allowable laminar and turbulent flow rates and differential levels as a function of time is depicted in Figure 3. This figure covers the maximum 16 foot level differential for a period out to 24 days. Figure 4 is a subset of the full range, it depicts the maximum allowable laminar and turbulent flow rates and differential levels as a function of time at the test conditions of 11 inch level differential for a period of 24 hours.

During the actual K-Basin leak test, the measured differential levels as a function of time and the resulting volumetric flow rates should be plotted in a format similar to Figure 4. If the measured leak rates are above the maximum allowable laminar form, then the test shows an acceptable result.

Figure 3. Maximum Allowable Leak Rate and Differential Level at an Initial 16 foot Differential Level

Figure 4. Maximum Allowable Leak Rate and Differential Level
at an Initial 11 Inch Differential Level

3.4 LEAK TEST GENERAL FLOW MODEL

The following development was derived from WHC-1995. The general flow model relating the discharge flow, Q_L , to the differential height, h_t , is given as;

$$Q_L = wh_t^\alpha$$

Where the discharge coefficient, w , is similar to w_L or w_t given above. The parameter defines the flow model. (e.g., laminar flow is described by $\alpha=1$, and turbulent flow by $\alpha=1/2$). Let V denote volume, E denote evaporation rate, and Q denote leak rate. Then the fluid balance in the basin and in the discharge chute is described by;

$$\text{and; } \frac{dV_B}{dt} = -Q_L - E_B - Q_B$$

$$\text{respectively. } \frac{dV_{DC}}{dt} = Q_L - E_{DC} - Q_J$$

Together with $h_t = h_B - h_{DC}$, these equations can be combined with the flow model to give;

$$\text{where; } \frac{dh_t}{dt} = -(wh_t^\alpha + E^* + Q^*) / A^*$$

$$\frac{1}{A^*} = \frac{1}{A_B} + \frac{1}{A_{DC}}$$

$$\frac{E^*}{A^*} = \frac{E_B}{A_B} - \frac{E_{DC}}{A_{DC}}$$

and;

$$\frac{Q^*}{A^*} = \frac{Q_B}{A_B} - \frac{Q_J}{A_{DC}}$$

The leak rate, Q_B , is assumed to be zero and is included only for completeness.

Solution for Laminar Flow

The solution for laminar flow ($\alpha=1$) is readily found using an integrating factor $\text{Exp}((w/A^*)t)$ to give;

$$\text{where; } h_t(t) = h_{ti} e^{-\frac{w}{A^*}t} - K_L(t)$$

$$K_L(t) = \int_0^t e^{\frac{w}{A^*}(t'-t)} \left(\frac{E^* + Q^*}{A^*} \right) dt'$$

K_L is a correction for evaporation and leakage. If the exponential in the integral is assumed to equal unity, and the evaporation and leakage terms are assumed constant, then one obtains the results given in section 3.1. Neglecting the time-dependence of the exponential is equivalent to assuming that the correction, $K_L(t)$, is small enough that it only affects the measured height and not the flow itself.

General Case

If one assumes that the evaporation and leak terms are zero (or that the height measurements are pre-corrected for evaporation and construction joint leakage), then the case for arbitrary α becomes;

$$\text{which readily integrates to; } \frac{dh_t}{dt} = -\left(\frac{w}{A^*}\right)h_t^\alpha$$

$$h_t^{(1-\alpha)} = h_{ti}^{(1-\alpha)} - (1-\alpha)bt$$

where;

$$b = \frac{w}{A^*}$$

In order to fit $h_t(t)$ as a function of t , it is desirable to first expand $h_t(t)$ in a Taylor series. Expanding about $t=0$, to second order in t , one finds,

$$h_t(t) = h_{tj} - (h_{tj}^\alpha b) t + \frac{\alpha}{2} (b^2 h_{tj}^{2\alpha-1}) t^2 - \dots$$

Where all higher terms include a factor $(1-2\alpha)$. Thus this approximation is exact for turbulent flow ($\alpha=1/2$). For the case of laminar flow ($\alpha=1$), the series represents the exponential time-dependence found previously.

A form suitable for least-squares fitting is then obtained by writing;

where;

$$h(t) = x_1 + x_2 t + x_3 t^2$$

$$x_1 = h_{ti}$$

$$x_2 = -h_{ti}^\alpha b$$

$$x_3 = \frac{\alpha}{2} b^2 h_{ti}^{2\alpha-1}$$

The quantities of interest, h_{ti} , b , and α , are readily found by inverting these equations:

$$h_{ti} = x_1$$

$$\alpha = 2x_3 \left(\frac{x_1}{x_2^2} \right)$$

$$b = -\frac{x_2}{h_{ti}^\alpha} = -\frac{x_2}{x_1^\alpha}$$

From the differential height measurement, h_t , versus time, a 2'nd order polynomial regression curve can be fitted through the data. The flow rate at any given can then be calculated from the following (repeating the general flow equation);

$$Q_j = w(h_{t_j})^\alpha$$

substituting terms for w and h_t from the above development results in;

$$Q_j = \left(-\frac{x_2}{x_1^\alpha} \right) A^* (x_1 + x_2 t_j + x_3 t_j^2)^\alpha$$

Q_j is the instantaneous flow rate at any given time, t_j , less than a critical time, ie., the time at which the differential height is zero. The values for x_1 , x_2 and x_3 are determined from the regression curve fit. The flow parameter, α , is determined in terms of x_1 , x_2 and x_3 from the equation noted above and A^* , from the equation above is determined from the K-Basin areas.

3.5 STATISTICAL DATA ANALYSIS

Let $\{h_j\}$ denote a set of heights measured at a set of n discrete times $\{t_j\}$. Then the least-squares model to be fitted becomes;

This equation may be rewritten in matrix form as;

where H is the vector $\{h_1, h_2, \dots\}$ of measurements, x is the vector $\{x_0, x_1, x_2\}$ of parameters to be fitted, and B is the matrix;

$$B = \begin{pmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \vdots & \vdots & \vdots \\ 1 & t_n & t_n^2 \end{pmatrix}$$

The vector, $\hat{x}$, of estimated parameters is then readily computed from (See Hamilton 1964 e.g.)

where t denotes transpose. $\hat{x} = (B^t B)^{-1} B^t H$

The matrix B has one dimension equal to the number of data points, n . However, the matrix $B^t B$ is small (3×3) so that its inversion is easily done.

Uncertainties

Uncertainties are readily found by first computing the sum of squared residuals, σ_H^2 ,

$$\sigma_H^2 = \|H - B\hat{x}\|^2$$

(i.e., the square of the difference between the measured heights and the heights calculated from the estimated parameters) or;

$$\sigma_H^2 = \sum_{j=1}^n (h_j - \sum_{k=1}^3 b_{jk} \hat{x}_k)^2$$

The estimated variances for the fitted parameters are then given by the diagonal elements of the parameter covariance matrix given by

$$\text{Cov}(\hat{x}, \hat{x}) = \frac{1}{n-3} \sigma_H^2 (B^t B)^{-1}$$

Standard deviations are given by taking the square roots of the variances.

Uncertainties in the Estimated Height

While not of direct interest, the standard deviation in the height calculated from the estimated parameters can be obtained from the variance in $h(t)$ given by

$$\begin{aligned} \text{Var}[h(t)] &= \text{Cov}[h(t), h(t)] \\ &= T^t \text{Cov}(x, x) T \end{aligned}$$

where the vector T is given by;

$$T = \begin{pmatrix} 1 \\ t \\ t^2 \end{pmatrix}$$

Uncertainties in Parameters of Interest

As noted, the uncertainties in the fitted parameters, $\hat{x}$ are given by the covariance matrix $\text{Cov}(x, x)$. The parameter, x_1 , is directly equal to the height, h_{ti} , at time, $t=0$, so that the uncertainties in x_1 also represent those for h_{ti} . Likewise, since the uncertainty in the flow, Q_L , at time zero is found from the uncertainty in x_2 .

$$\begin{aligned} x_2 &= -(wh_{ti}^\alpha) / A^* \\ &= -Q_L / A^* \end{aligned}$$

Finally, the uncertainties in the flow model as described by α are expected to be dominated by the quadratic parameter, x_3 .

It should be emphasized that the uncertainty in the flow, Q_L , does not depend on α . While there may be some interest in the flow mechanism, the flow rate and its uncertainty can be found regardless. It may well happen, in fact, that the measurements are not capable of clearly distinguishing the type of the flow parameter, α .

These calculations may be performed by standard statistical packages. They could also be easily calculated in spreadsheets or with other mathematical software.

The calculated standard deviations give a good measure of the parameter uncertainties, and confidence limits with assigned probabilities are not recommended. This step could be taken, however, by assuming that the estimated parameters are normally distributed, a reasonable approximation for the large number of data points expected in the measurements. Then, in accordance with ANSI 56.8, the measured leak rates could be reported at the 95% Upper Confidence Level (UCL). What this means is that the measured leak rates shall be corrected to include the effects of data uncertainty. The 95% UCL is determined by adding to the mean value a factor of 1.65 times the standard deviation due to the data scatter (assuming a sufficiently large data set). The standard deviation of the data can be determined by the methods above (see also Appendix A).

4.0 CASE 2: EXPANSION JOINT LEAKAGE WITH OPEN ISOLATION BARRIER

The second case assumes leakage from the basin and discharge chute through the expansion joint. The objective is to determine the discharge coefficient for the expansion joint leakage from the K-Basin leak test results.

Assumptions:

$$h_t = 15.75 \text{ ft at time } 0,$$

4.1 VISCOUS LAMINAR FLOW

To determine the volumetric flow rate in terms of static pressure head the following relationship is used;

$$Q_J = \frac{D^2 A_L}{32 \mu L} \rho \frac{g_o}{g_c} (h_t - h_o) = w_{LJ} (h_t - h_o)$$

Where, in this case h_t is the total height of the water column in the basin and discharge basin and h_o is the hydrostatic pressure head under the K-Basin.

Assuming h_o is zero, then,

$$Q_J = \frac{D^2 A_L}{32 \mu L} \rho \frac{g_o}{g_c} h_t = w_{LJ} h_t$$

and;

$$Q_J = A_t \frac{dh_t}{dt} = w_{LJ} h_t$$

Where A_t is the total surface area of the basin and discharge chute.
Now separating variables and integrating;

$$\int \frac{dh_t}{h_t} = \frac{w_{Lj}}{A_t} \int dt$$

Integrating from initial height h_{ti} at time t_i , to final height h_{tf} at time t_f results in;

$$\ln(h_{tf}/h_{ti}) = \frac{w_{Lj}}{A_t} (t_f - t_i)$$

or;

$$w_{Lj} = \frac{A_t \ln(h_{tf}/h_{ti})}{(t_f - t_i)}$$

To account for evaporation, the equation above for w_{Lj} can be modified in the following form;

$$w_{Lj} = \frac{A_t \ln(h_{tfe}/h_{ti})}{(t_f - t_i)}$$

and;

$$h_{tfe} = h_{tf)_{ind}} + \Delta t \left(\frac{E_B}{A_B} + \frac{E_{DC}}{A_{DC}} \right)$$

where;

- h_{tfe} = water column height adjusted for evaporation effects, ft.,
- $h_{tf)_{ind}}$ = indicated water column height, ft.,
- E_B = measured evaporation rate of Storage Basin, cu ft/sec,
- E_{DC} = measured evaporation rate of Discharge Chute, cu ft/sec,
- Δt = Delta time, $t_f - t_i$, sec.

Note, the initial height is not adjusted for evaporation as evaporation is a time rate function and h_{ti} is at time zero. Also, the evaporation rate, cu. ft. per sec., is determined from an instrument with a known surface area. Therefore the evaporation rate of the storage basin and discharge chute are determined as before by the following;

$$E_B = E_{inst} * \frac{A_B}{A_{inst}}$$

and;

$$E_{DC} = E_{inst} * \frac{A_{DC}}{A_{inst}}$$

where;

E_{inst} = measured evaporation rate of VED, cu ft/sec,
 A_{inst} = Surface area of VED, sq ft.

4.2 TURBULENT FLOW

Similar calculations can be performed to determine the discharge coefficient for the expansion joint assuming turbulent flow. To determine the volumetric flow rate in terms of static pressure head the following relationship is used;

$$Q_J = A_L \sqrt{\frac{2Dg_0}{fL} (h_t - h_0)} = w_{Tj} \sqrt{h_t - h_0}$$

Where, in this case h_t is the total height of the water column in the discharge basin and h_0 is the hydrostatic pressure head under the K-Basin. Assuming h_0 is zero, then,

$$Q_J = A_L \sqrt{\frac{2Dg_0}{fL} h_t} = w_{Tj} \sqrt{h_t}$$

and;

$$Q_J = A_t \frac{dh_t}{dt} = w_{Tj} \sqrt{h_t}$$

Where A_t is the total surface area of the basin and discharge chute.
Now separating variables and integrating;

$$\int \frac{dh_t}{\sqrt{h_t}} = \frac{w_{Tj}}{A_t} \int dt$$

Integrating from initial height h_{ti} at time t_i to final height h_{tf} at time t_f results in;

$$\sqrt{h_{tf}} - \sqrt{h_{ti}} = \frac{w_{Tj}}{2A_t} (t_f - t_i)$$

or;

$$w_{Tj} = \frac{2A_t(\sqrt{h_{tf}} - \sqrt{h_{ti}})}{t_f - t_i}$$

To account for evaporation, the equation above for w_{Tj} can be modified in the following form;

$$w_{Tj} = \frac{2A_t(\sqrt{h_{tfe}} - \sqrt{h_{ti}})}{t_f - t_i}$$

and;

$$h_{tfe} = h_{tf})_{ind} + \Delta t \left(\frac{E_B}{A_B} + \frac{E_{DC}}{A_{DC}} \right)$$

where;

- h_{tfe} = water column height adjusted for evaporation effects, ft.,
- $h_{tf})_{ind}$ = indicated water column height, ft.,
- E_B = measured evaporation rate of Storage Basin, cu ft/sec,
- E_{DC} = measured evaporation rate of Discharge Chute, cu ft/sec,
- Δt = Delta time, $t_f - t_i$, sec.

Note, the initial height is not adjusted for evaporation as evaporation is a time rate function and h_{ti} is at time zero.

5.0 REFERENCES

- American Nuclear Society, 1979, *Containment System Leakage Testing Requirements*, ANSI/ANS 56.8
- Hamilton, W. C. 1964, *Statistics in Physical Science*, The Ronald Press Co., New York, NY
- WHC 1992, *A Survey of Fluid Flow Leakage*, WHC-SD-TP-RPT-003, Westinghouse Hanford Company, Richland, Washington.
- WHC 1994a, *Test Specification and Test Plan for 105-KE/KW Basin Isolation Barrier Leak Rate Test*, WHC-SD-SNF-TP-009, Westinghouse Hanford Company, Richland, Washington.
- WHC 1994b, *105-KE Basin Isolation Barrier Leak Rate Acceptance Test Procedure*, WHC-SD-SNF-ATP-005, Westinghouse Hanford Company, Richland, Washington.

Appendix A
Common Statistical Data Analysis Equations

The following equations were developed in ANS 1979 and are repeated here for information. Let Y be the dependent variable and t be the independent variable. Then the standard deviation of a data set is determined by;

$$\sigma = \left(\frac{1}{n-2} \sum_{j=1}^n [y_j - \bar{y}_j]^2 \right)^{1/2}$$

where;

y_i = Measured value at t_i ,

$\bar{y}_i$ = Estimated value at t_i e.g. from a regression curve fit,

Assume a linear least squares regression curve fit of the following form;

$$\bar{y}_j = mt_j + c$$

where;

m = slope of the line,

c = slope intercept,

t_j = dependent value.

Then the standard deviation of the slope is given by;

$$\sigma_m = kn^{1/2}$$

The standard deviation of the intercept is given by;

$$\sigma_c = k \left[\sum_{j=1}^n t_j^2 \right]^{1/2}$$

The standard deviation of the covariance of slope and intercept is given by;

$$\sigma_{mc} = k^2 \left[- \sum_{j=1}^n t_j \right]$$

where k is defined as;

$$k = \frac{\sigma}{\left(n \sum_{j=1}^n t_j^2 - \left(\sum_{j=1}^n t_j \right)^2 \right)^{1/2}}$$

For a leak rate defined by the ratio of slope over the slope intercept, i.e.;

$$Q_L = \frac{m}{C}$$

A leak rate at the 95% upper confidence level (UCL) can be defined by the following;

$$Q_{L.95} = \frac{m}{C} + t_{.95} \frac{\sigma_m}{C}$$

where the 95'th percentile of the t distribution, $t_{.95}$, is tabulated in Table A-10 for various degrees of freedom and is also defined by;

$$t_{.95} = \frac{1.6446 d_f + 3.5283 + 0.85602/d_f}{d_f + 1.2209 - 1.5163/d_f}$$

and the degree of freedom, d_f , is defined by;

$$d_f = n - 2$$

Table A-1. 95th Percentile of the t Distribution vs Degrees of Freedom

d_f	$t_{.95}$	d_f	$t_{.95}$	d_f	$t_{.95}$
1	6.314	12	1.782	23	1.714
2	2.920	13	1.771	23	1.711
3	2.353	14	1.761	25	1.708
4	2.132	15	1.753	26	1.706
5	2.015	16	1.746	27	1.703
6	1.943	17	1.740	28	1.701
7	1.895	18	1.734	29	1.699
8	1.860	19	1.729	30	1.697
9	1.833	20	1.725	40	1.684
10	1.812	21	1.721	60	1.671
11	1.796	22	1.717	120	1.658

note; at infinite degree of freedom, $t_{.95}$ is 1.645.