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Because of the potential application to power production, it is important to investigate a wide range of possible means to achieve nuclear fusion, even those that may appear initially to be infeasible. In antiproton catalyzed fusion, the negative antiproton shields the repulsion between the positively charged nuclei of hydrogen isotopes, thus allowing a much higher level of penetration through the repulsive Coulomb barrier, and thereby greatly enhancing the fusion cross section. Because of their more compact wave function, the more massive antiprotons offer considerably more shielding than do negative muons. The effects of the shielding on fusion cross sections are most predominate at low energies. If the antiproton could exist in the ground state with a nucleus for a sufficient time without annihilating, the fusion cross sections are so enhanced that at room temperature energies, values up to about 1000 barns (that for d+t) would be possible. Unfortunately, the cross section for antiproton annihilation with the incoming nucleus is even higher. A model that provides an upper bound for the fusion to annihilation cross section for all relevant energies indicates that each antiproton will catalyze no more than about one fusion. Because the energy required to make one antiproton greatly exceeds the fusion energy that is released, this level of catalysis is far from adequate for power production.

1. Introduction

The size, cost and complexity of projected thermonuclear fusion reactors are governed by the requirement to attain a minimum value of the temperature T of the thermalized Maxwellian plasma in the face of significant loss processes. Even for the fusion reaction with the highest reactivity, the d-t reaction ($^3\text{H}(d,n)^4\text{He}$) with $E_{\text{fus}}=17.6$ MeV, minimum fuel temperatures of about 10keV are required. For the so-called "advanced" fusion fuels, e.g., d- ^3He , d-d, p- ^{11}B , etc., the minimum fuel temperatures must be increased considerably to ones ranging from around 50keV to greater than 100 keV. These minimum temperatures are necessary so that energetic fuel ions in the tail of the Maxwellian have sufficient energy to begin to appreciably tunnel through the mutually repulsive Coulomb barrier to induce acceptable fusion energy reaction

rates. This suggests that one approach to achieving a step-change in the physics of fusion is to circumvent, at some level, this high temperature threshold set by the barrier penetration requirements of the conventional fusion cross-section. That is, to seek methods that realize higher fusion reactivities at lower temperatures relative to conventional thermonuclear plasmas.

One existing method of barrier reduction which does work at some level is muon-catalyzed fusion [1,2], which uses a negatively-charged muon to screen the repulsive Coulomb potential between fuel ions. In this paper, we examine an alternative method of Coulomb barrier amelioration: antiproton-catalyzed fusion. By analogy with muon-catalyzed fusion, this exploits the screening effect of a bound antiproton (negative charge, denoted $\bar{p}$ or p^- when its charge is stressed). The antiproton mass is about 9 times greater than the muon mass and, therefore, its ground state Bohr orbit is 9 times smaller. Given that the barrier transmission probability increases approximately exponentially with decreasing barrier thickness, we would predict a significant enhancement of the fusion cross section over even that of muon-catalyzed fusion. However, although the antiproton is stable in isolation (the muon has an intrinsic lifetime of only 2.2 μ s), we also would expect significant antiproton annihilation rates either from within the bound state with the fusion fuel nucleus or from direct annihilations with incident fuel nuclei.

In this paper, we report, for the first time, the energy-dependent cross sections for the d-d, d-t and p-d fusion reactions occurring between an incident fusion fuel nucleus and the bound state of an antiproton and a second fuel nucleus. Specifically, we consider the reactions: $(d^+p^-)+d^+$, $(d^+p^-)+t^+$, $(t^+p^-)+d^+$, and $(d^+p^-)+p^+$. These cross sections are obtained through a full calculation of the barrier transmission probability in the presence of antiproton screening. We also estimate the annihilation rate of the bound antiproton and, thereby, deduce an upper bound for the ratio of the fusion rate to annihilation rate in a typical fusion fuel medium.

We first proposed the consideration of $\bar{p}$ -catalyzed fusion in 1993 [3]. Earlier, Frolov and Thakkar [4] performed the first accurate calculation of the $p+d+t$ ground state, principally as part of a study of the binding of three charged particles. Korobov, Vinitsky, and Kuperin [5] extended such work to include $p+p+p$, $p+d+d$, $p+t+t$, and $p+d+t$ and provided values for the ratio of the fusion to annihilation rate for these bound systems. Here we seek values, although more approximate, for this same ratio in circumstances more general than bound states.

2. Model

We want a model that provides an estimate of the fusion to annihilation probability in a general mixture of the nuclei of hydrogen isotopes with antiprotons which is at least accurate enough to decide on the feasibility of employing antiproton catalyzed fusion for power production.

Our considerations in choosing such a model are as follows. It is likely, at least under most circumstances, that one $\bar{p}$ will catalyze, on the average, less than one fusion before it annihilates. There are two reasons for this to be true. First, the fundamental strength of annihilation is greater than that of fusion for most energies. Second, the coulomb repulsion of the fusing species at distances closer than shielding has an effect makes it less likely that they will be at nearly the same point in space than the annihilating species, for which there is a coulomb attraction. This proximity on a nuclear scale of the reacting particles is required for both fusion and annihilation to occur.

Our other consideration involves the requirements for energy breakeven in antiproton catalyzed fusion. Forward has estimated that it might be possible to lower the net energy required to make each stored antiproton from its current value to about 8 TeV [6], while a single fusion of hydrogen isotopes produces a maximum of 17.6 MeV (d+t). Thus, for energy breakeven, we would require the number of fusions catalyzed per antiproton to be about 4×10^5 . Since we think that number is less than one for most cases, we have a fair amount of leeway in choosing a model. Thus, we can seek an upper bound to the fusion to annihilation ratio in which we neglect some annihilation processes and make some simple assumptions in regard to shielding that maximize its effect.

In our model, every antiproton is presumed to shield a nucleus as if were in the ground state relative to that nucleus (maximum shielding) even though it is known that annihilation is likely before a $\bar{p}$ can enter this state. We neglect the appreciable annihilation probability it will have with the nucleons of that nucleus. We then calculate the fusion and annihilation cross sections of another nucleus incident upon the first nucleus and its bound $\bar{p}$ as a function of incident particle energy. The only lack of generality in this model is that the incident nucleus approaches both its potential fusion and annihilation partners with the same energy. Were such not true, the effect of shielding would be much less, if not absent, so we believe that this model does give a general upper bound to the fusion to annihilation ratio.

Since the fusion and annihilation probabilities and rates are proportional to the corresponding cross sections, the ratio of the fusion cross section to the annihilation cross section within the model is then our energy-dependent upper bound to

the number of fusions catalyzed per antiproton. We calculate this ratio and examine its energy dependence.

3. Calculation

We express the fusion cross section in the convenient form,

$$\sigma_f = T[E] S[E] / E, \quad (1)$$

where E is the energy of the incident nucleus in the center of mass system, the S -function, $S[E]$, incorporates the nuclear effects, and $T[E]$ is the barrier transmission factor, the ratio of the flux passing through the barrier to that incident upon it. Whereas $S[E]$ is independent of the approach of the target and incident nuclei, the transmission coefficient is profoundly affected by the presence of screening particles such as negative muons or antiprotons. We note that eq. (1) is completely general, providing the constituent terms, $T[E]$ and $S[E]$, are correctly determined individually. Bosch and Hale provide accurate values of the conventional fusion cross section, σ_f , for the d-d and d-t reactions based on experimental results and R-matrix theory. However they elect to use an approximation to $T[E]$ based on the Gamow formalism and then, for parameterization purposes, compute a resulting $S[E]$. Accordingly, we compute the exact Coulomb barrier transmission coefficient, $T_C[E]$, and then obtain the correct values of $S[E]$ from

$$S[E] = E \sigma_f[E] / T_C[E] \quad (2)$$

using the (accurate) values of σ_f from [7]. Our $S[E]$'s differ somewhat from those in [7] because we use the exact $T[E]$ ($T_C[E]$) rather than the approximate form. $S[E]$ for each pair is then used when we evaluate eq. (1) for σ_f in the presence of shielding by a $\bar{p}$ in the ground state around one of the nuclei. Calculation of $T_C[E]$ for unshielded nuclei becomes difficult for values of E around and below 1 keV due to the precision required in subtracting two numbers which are identical through 15 or more decimal digits. Fortunately, however, $S[E]$ has reached its limiting, constant value by this point, and we simply use that value for lower energies.

The Schroedinger equation used to obtain $T[E]$ when shielding is present is obtained as follows. We start with the full three body Schroedinger equation and assume that the wave function is the product of the center-of-mass wave function and two others. These are the unperturbed wave function of the antiproton and target nucleus and the wave function for the motion of the incident nucleus. The coordinates appearing in these wave functions are respectively those of the center of mass, the $\bar{p}$ - target nucleus relative coordinates, and the relative coordinates of the two nuclei. The center-of-mass motion is separated out, and the

resulting equation is multiplied on by the left by the wave function for the $\bar{p}$ - target relative motion and integrated over its coordinates. In the integration, a cross-derivative term that results from the non-orthogonality of the last two sets of coordinates drops out. That leaves an equation for the relative motion of the two nuclei which is, in electron atomic units,

$$(-1/2m_n) \nabla^2 + V[r] - E) \Psi = 0, \quad (3)$$

with $V[r] = (1/r + m_{Tp}) \exp[-2m_{Tp}r]$

and $E = E_L (m_T + m_p) / (m_T + m_I + m_p)$,

where ∇^2 is the kinetic energy operator for incident particle - target particle relative coordinates, m_n is the reduced mass of the two nuclei (the $\bar{p}$ mass does not appear here), r is the distance between the nuclei, E is the energy in the center-of-mass frame of the incident particle and the target nucleus plus the $\bar{p}$ together as a single particle, m_{Tp} is the target nucleus - $\bar{p}$ reduced mass, m_T is the target nucleus mass, m_p is the $\bar{p}$ mass, m_I is the incident nucleus mass, and E_L is the energy of the incident particle in the lab frame. $V[r]$ is the potential energy between the incident particle and the target particle as shielded by the $\bar{p}$ in an unperturbed 1S state. The addition of a $1/r^4$ long range potential with a proper cutoff would make V more accurate but might not have much effect. One might alternatively use the relative coordinates between incident particle and the center of mass of the target nucleus and $\bar{p}$ instead of those of eq. (3), but then the integration goes over the nuclear part of the potential energy and eq. (1) cannot be employed. Fusion occurs only for incoming S-waves, so, taking angular momentum to be zero, we solved eq. (3) to obtain $T[E]$ for a number of target/incident combinations involving p, d, and t.

4. Results

The values of $T[E]$ and σ_f resulting from the calculations for d-d, d-t, and t-d are given the figures. The σ_f values for p-d (not shown) were, as expected, some orders of magnitude below those shown, reflecting its much lower values of $S[E]$. The combination t-t was not done, but the results would be roughly the same as for d-d because their values of $S[E]$ are similar. The results given in fig. 1 and fig. 3 show that for all but the higher values of E , $T[E]$ is proportional to v , where $v = (2E_L/m_I)^{1/2}$ is the incident particle velocity. Since $S[E] \rightarrow \text{const.}$ as $v \rightarrow 0$, and E is proportional to v^2 , $\sigma_f \rightarrow \text{const.}/v$ as $v \rightarrow 0$, as inelastic cross sections must when the potential energy, V , is short range, as it is here due to the shielding. Note, in

particular, that at room temperature (0.025 eV), the d-t cross section has increased to about 1000 barns!

Fig. 1. Barrier transmission, $T[E]$, for d on antiproton-shielded d, for maximum shielding, compared to transmission with no shielding of the Coulomb repulsion.

We are not aware of prior recognition of the fact that shielded fusion cross sections increase indefinitely as $\text{const.}/v$ when $v \rightarrow 0$ (reaction rate $\rightarrow \text{const.}$). For this to be the case requires that at least one of the two nuclei undergoing fusion be surrounded by a negative charge equal in magnitude to its positive charge. When such is the case, the potential energy is short range (decreases faster than $1/r^2$ as $r \rightarrow \infty$), and the $\text{const.}/v$ limit for inelastic scattering applies irrespective of the form of the potential at any but large r . Thus, when shielding is provided by an electron, as in $D^0 + t^+$, the fusion cross section has $\text{const.}/v$ as its limit, although the value of the constant in this case is far less than when the shielding is provided by a negative muon or antiproton ($T[E]$ is very much less for electron shielding).

The results in fig. 2 and fig. 4 show that for E around and below 1 keV, the d-t and t-d fusion cross sections are about the same, with that for d on t being greater

by about 40% due to a more compact wave function in $\bar{p}t$ than in $\bar{p}d$ (hence better shielding). For the same energies, values of σ_f for d-t are greater than those for d-d by a factor of about 90, due to the much stronger tendency to fuse ($S(E)$ much larger). For values of E in this same energy domain, σ_f is fit very well by

$$\sigma_f = c (E / \text{keV})^{-1/2} \text{ barns}, \quad (4)$$

where $c = 0.105$ for d on shielded d
 $c = 6.69$ for t on shielded d
 $c = 9.26$ for d on shielded t.

Fig. 2. Fusion cross section for d on antiproton-shielded d, for maximum shielding, compared to the cross section with no shielding of the Coulomb repulsion.

For the annihilation cross section, we use the non-coulomb-enhanced $\bar{p}p$ annihilation cross section of Morgan and Hughes [8].

$$\sigma_a = 46 ((m_T + m_l + m_p) / 2m_p)^{1/2} (E / \text{keV})^{-1/2} \text{ barns}, \quad (5)$$

where the kinematic factor involving the masses makes the result appropriate to the present case. Including that factor, the constant in front of the energy factor is 72 for d on d and 80 for t on d or d on t. Equation (5) is valid for E from zero up to

a few tens of MeV. In reality there will be a slight enhancement in the values of σ_a due to the attraction between the antiproton and the target nucleus inside the shielded region, but we neglect this as well as the fact that the target nucleus might have one or two neutrons with which the antiproton may annihilate. These simplifications maintain the calculated fusion to annihilation ratio as an upper bound.

Fig. 3. Barrier transmission, $T[E]$, for d on antiproton-shielded t and t on antiproton-shielded d, for maximum shielding, compared to the common transmission for both cases with no shielding of the Coulomb repulsion.

In employing eq. (5) for the annihilation in this three body problem, we have, in effect, performed an approximate separation similar to that used for determining σ_f . In this case, however, we let the third factor in the product wave function depend on the relative coordinates between the incident nucleus and the $\bar{p}$ instead of the target nucleus. Here there is no need to solve the equation that results when integration is performed over the $\bar{p}$ - target nucleus relative coordinates, since we take the potential energy (shielded attractive) to be zero to obtain a lower bound for the annihilation and therefore know the solution. That solution leads to eq. (5) when the multiplicative factor accounting for the nuclear effects is

determined from experimental information (as done in [8]).

Taken together, eq. (4) and (5) indicate that for energies around and below 1 keV, the fusion to annihilation ratio in mixtures involving nuclei of hydrogen isotopes and antiprotons is less than 12%. This upper bound is consistent with the fusion and annihilation rates found by Korobov, Vinitsky, and Kuperin [5] for ground state exotic molecules involving a $\bar{p}$ and two of p, d, and t, where the maximum fusion to annihilation ratio they find is 7% ($\bar{p}dt$). In such molecules the particles have mean kinetic energies of about 0.1 to 0.4 keV.

The results shown in the figures and eq. (5) indicate that the upper bound to the ratio has a maximum value of very close to unity at 60 keV, occurring for both t-d and d-t at the peak of the resonance present in the ^5He compound nucleus. At still higher values of energy, the coulomb barrier is easily penetrated (or passed over), and shielding becomes irrelevant. Thus, our expectations in regard to annihilation dominating fusion, mentioned above, have been met.

Fig. 4. Fusion cross section for d on antiproton-shielded t and t on antiproton-shielded d, for maximum shielding, compared to the common cross section for both cases with no shielding of the Coulomb repulsion.

5. Conclusion

Our model provides an upper bound to the fusion to annihilation ratio that is valid with sufficient accuracy for all forms of a $\bar{p}$ - hydrogen nuclei mixture that we can envision. Hence, it appears that an antiproton can catalyze no more than about one fusion. This number of fusions is clearly energy inefficient, and is much less than that of which the less expensive negative muons are capable. Thus, consistent with our initial anticipation, it would appear that antiproton catalyzed fusion is not worth pursuing as a source of energy.

6. More Accurate Calculations

There are some questions of accuracy with the analysis presented here. First is the question of the accuracy of our approximate and forced separation of the three body problem. Second is the possibility that systematic errors are introduced into the fusion to annihilation ratio by, in effect, performing different separations for determining the two cross sections. The separations lead to physically reasonable pictures of the interactions in most respects, and that indicates the errors they introduce will not be great.

There is, however, a possible physical process that might not be adequately covered. That is the effect on fusion and annihilation of the motion of the $\bar{p}$ and target nucleus about their center of mass. The kinetic energy involved in this motion, 10 - 20 keV in the ground state (as we assume), exceeds the kinetic energy of the incident nucleus within most of the incident energy domain that we consider. One can argue that our choice of incident particle to target particle relative coordinates encompasses the effects of this motion. Further, the $1/v$ dependence of the cross sections for energies around and below 1 keV, and to at least some degree the dependence on v at higher energies, is due the kinematics of the motion at distances where the potential energy is inconsequential. The short range potential effects, both barrier and nuclear, that affect the constant multiplying $1/v$, involve energies well in excess of 10-20 keV, so they will not be substantially affected. Thus, we believe our results are reasonably accurate.

Nevertheless, it might be worthwhile to solve the three body problem by a method that separated out only the center of mass motion, thus directly including any effects of the relative motion of the $\bar{p}$ and target nucleus and permitting determination the fusion to annihilation ratio from the solution of the same equation (as was done in [5] for exotic $\bar{p}$ molecules).

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Figure Captions

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