Neutrino Masses at $v^{3/2}$

Nima Arkani-Hamed, Lawrence Hall, Hitoshi Murayama, David Smith and Neal Weiner
Department of Physics, University of California, Berkeley, CA 94720, USA;
Theory Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
(July 10, 2004)

Theories in which neutrino masses are generated by a conventional see-saw mechanism generically yield masses which are $O(v^2)$ in units where $M_{Pl} = 1$, which is naively too small to explain the results from SuperKamiokande. In supersymmetric theories with gravity mediated supersymmetry breaking, the fundamental small parameter is not v/M_{Pl} , but m_I/M_{Pl} , where m_I is the scale of supersymmetry breaking in the hidden sector. We note that m_I^3/M_{Pl}^2 is only slightly too large to explain SuperKamiokande, and present two models that achieve neutrino masses at this order in m_I , one of which has an additional suppression λ^2 , while the other has additional suppression arising from a loop factor. The latter model shares a great deal of phenomenology with a class of models previously explored, including the possibility of viable sneutrino dark matter.

I. INTRODUCTION

Neutrinos are exceedingly light compared to other fermionic elementary particles. For instance, the atmospheric neutrino data [2] suggest $m_\nu \simeq 0.06$ eV. To understand the smallness of neutrino masses, we usually invoke the presence of a scale M that is much larger than $v \approx 200$ GeV, the scale of electroweak symmetry breaking. This scale is conventionally taken to be roughly the reduced Planck mass, $M_{Pl} \approx 2 \times 10^{18}$ GeV, or the GUT scale, $M_{GUT} \approx 2 \times 10^{16}$ GeV. Setting $M = 1$, v becomes a small parameter of the theory. Charged fermion masses arise at order v , while neutrino masses arise at v^2 . Note that any particle of mass $\leq v^3$ is massless for all particle physics practicalities, although such masses could be interesting for cosmology.

The see-saw mechanism [1] is certainly the simplest explanation of why neutrino masses are so small - however, the result v^2/M gives neutrino masses which are $\sim 30 - 3000$ times too light to explain the atmospheric neutrino anomaly [2], for $M = M_{GUT}$ or M_{Pl} . Frequently this is fixed by instead taking $M \sim 10^{14} - 10^{15}$ GeV. In this paper we propose an alternative approach.

In supersymmetric theories, a standard framework has supersymmetry broken in a hidden sector at the intermediate scale $m_I \sim 10^{10} - 10^{11}$ GeV which allows supergravity mediation of supersymmetry breaking to the standard model to generate the weak scale $v \sim m_I^2/M_{Pl}$. We call this the intermediate-scale see-saw. An important question is at what order neutrino masses arise within this framework. If the relevant operator is $(LH)^2/M_{Pl}$, then $m_\nu \sim v^2/M_{Pl}$ as before. However, with $M = M_{Pl} = 1$, the small dimensionless parameter of the expansion is now m_I rather than v . There are now more possibilities, namely m_ν could occur at order $m_I^3, m_I^4, m_I^5 \dots$ corresponding to $v^{3/2}, v^2, v^{5/2} \dots$. We propose that scale for atmospheric neutrino oscillations arises at $O(m_I^3)$. The scale of solar oscillations can also arise at $O(m_I^3)$ with an extra suppression due to approximate flavor symmetries, or it can instead

occur at $O(m_I^4)$. Alternatively, the hierarchy between the atmospheric and solar scales can be traced to an approximately rank one loop integral, as discussed in section III.

II. NEUTRINO MASSES AT M_I^3

How might neutrino masses arise at $O(m_I^3)$? People have noted before that in a framework with large extra dimensions such that M_{Pl} is lowered to m_I , the ordinary see-saw result yields $m_\nu \sim v^2/m_I$ [3]. Such an approach is quite distinct from our perspective, where m_I is the small parameter. Other authors [4] have explored the possibility of generating neutrino masses from higher dimension operators involving a field with a vacuum expectation value (vev) at an intermediate scale, such as 10^{11} GeV. However, in these models, a supersymmetry-conserving vev generates the small couplings and masses, and the size of the vev is essentially a free parameter of the theory. In this paper, we will instead utilize the direct connection to supersymmetry breaking explored in [5]. Adopting this approach can have significant phenomenological ramifications, as we will find for the model of section III.

Right handed states, singlet under the standard model, might be light if they are protected by some global symmetry G , analogous to a symmetry used to prevent a Planck-scale $\mu H_u H_d$ term in the superpotential [8]. In [5], it was noted that if G is broken in the supersymmetry breaking sector, then it is quite natural to have light neutrinos. In the models presented in [5], the neutrino masses arose at $O(m_I^4)$.

We can employ the same framework to generate $O(m_I^3)$ masses instead. Consider the superpotential

$$[XNN + LNH]_F \quad (1)$$

where N is a standard model singlet and X is a field in the supersymmetry breaking sector that takes on

an A component vev $\langle X \rangle = m_I$ (for instance, the superpotential

$$\left[S(X\bar{X} - m_I^2) + \bar{X}^2 Y + X^2 \bar{Y} \right]_F \quad (2)$$

generates $F_S \sim F_Y = F_{\bar{Y}} \sim m_I^2$ and $A_X = A_{\bar{X}} \sim m_I$, but $F_X = F_{\bar{X}} = 0$ [6]). The Lagrangian then contains

$$[m_I NN]_F + [LNH]_F, \quad (3)$$

and when the Higgs takes on a vev, we have small Dirac masses for the neutrinos in addition to the Majorana mass for the right handed neutrino. Setting $M_{Pl} = 1$, the neutrino mass matrix is

$$m_{LR} = \begin{pmatrix} 0 & m_I^2 \\ m_I^2 & m_I \end{pmatrix}, \quad (4)$$

leading to a see-saw mass $m_\nu \approx m_I^3$ for the left handed neutrino. If we further assume that the Dirac masses are suppressed by λ_τ , for instance as might occur with flavor symmetries, the neutrino mass is $\lambda_\tau^2 m_I^3 \approx 0.1\text{eV}$. (A similar estimate $\lambda_\mu^2 m_I^3$ gives an approximately correct mass scale for the LOW solution to the solar neutrino problem if there are more than one N).

This model is exceedingly simple, but it illustrates the fact that once we take the small parameter of the theory to be m_I rather than v , there is no *a priori* reason why we should expect neutrino masses to occur at m_I^4 .

The phenomenology of this model is identical to that of ordinary see-saw neutrino physics, except that we do not expect the signals that might accompany broken GUT symmetries at the scale 10^{14}GeV . In contrast, the model that we consider next features additional weak-scale states and predicts a much richer phenomenology.

III. RADIATIVE NEUTRINO MASSES AT M_I^3

In [5], a model was proposed in which the right handed neutrinos get masses at $O(m_I^2)$, rather than at $O(m_I)$. If the Yukawa couplings are order one, then all neutrino masses are weak scale. As discussed in [5], this is remedied rather simply.

Suppose that a hidden sector field X acquires A and F component vevs of $O(m_I)$ and $O(m_I^2)$, respectively. Consider the operators

$$\frac{1}{M_{Pl}} ([XLNH_u]_F + [X^\dagger NN]_D), \quad (5)$$

which yield

$$\frac{m_I^2}{M_{Pl}} [NN]_F + \frac{m_I}{M_{Pl}} [LNH]_F + \frac{m_I^2}{M_{Pl}} [LNH]_A \quad (6)$$

when X acquires its vevs. We take there to be just one N superfield. The first term in (6) generates a weak scale

mass for N , while the second term generates Yukawas of order m_I/M_{Pl} . The neutrino mass matrix which we generate at tree level is (again setting $M_{Pl} = 1$)

$$m_{\text{tree}} = \begin{pmatrix} 0 & m_I^3 \\ m_I^3 & m_I^2 \end{pmatrix}. \quad (7)$$

Although this is not the canonical see-saw, as it involves a Dirac mass at $O(v^{3/2})$, it nevertheless yields the light neutrino mass of v^2/M_{Pl} as usual. The unusual feature is that the right handed neutrino is at the weak scale. The operators of (5) are easily justified, for instance by assuming ordinary R parity (under which N is odd), together with an R symmetry where N has R charge $2/3$, X has R charge $4/3$ and L and H_u have R charge 0 [7].

Once we have included the operator $[X^\dagger NN]_D$, it is impossible to forbid $[X^\dagger X X^\dagger NN]_D$ [9]. This operator induces a lepton number violating scalar mass of the form $\delta^2 \tilde{n} \tilde{n} + h.c.$, where $\delta \sim m_I^{5/2}/M_{Pl}^{3/2}$. The importance of this operator is that it allows a radiative contribution to the light neutrino mass at $O(m_I^3)$ from the diagram of figure 1.

FIG. 1. Diagram generating m_ν at order m_I^3 .

If we take $m_{\tilde{\nu}_L} \approx m_{\tilde{n}} \approx m_{X^0}$, and call them generically $\tilde{m}$, we generate a neutrino mass

$$m_\nu \approx \frac{g^2}{384\pi^2} \frac{A^2 v^2 \delta^2}{\tilde{m}^5}. \quad (8)$$

Taking $A \sim \tilde{m} \sim v$, this becomes

$$m_\nu \approx \frac{g^2}{384\pi^2} \left(\frac{m_I^3}{M_{Pl}^2} \right). \quad (9)$$

Note that this is *larger* than the lighter eigenvalue of (7) since it occurs at $O(m_I^3)$ rather than $O(m_I^4)$. A tree level neutrino mass at $O(m_I^3)$ is $O(\text{keV})$ - too heavy to be interesting. However, our mechanism automatically leads to a loop factor of order 10^{-4} giving masses of $O(.1\text{eV} - 1\text{eV})$ - within one order of magnitude of the scale necessary to explain the atmospheric neutrino anomaly!

Although the A terms couple $\tilde{n}$ to only a single linear combination of $\tilde{\nu}$'s, the loop diagram of figure 1 can generate more than one neutrino mass eigenvalue. For incoming ν_i and ν_j , the value of the loop integral, L_{ij} has nontrivial dependence on the corresponding sneutrino masses $\tilde{m}_i$ and $\tilde{m}_j$. The resulting mass matrix, $m_{ij} \propto$

$A_i A_j L_{ij}$ is not necessarily rank one, and we can expect a second and third eigenvalue. Although L_{ij} does depend on i and j , to a large extent it factors into $f_i f_j$, and is approximately rank one. Consequently, the second eigenvalue is suppressed greatly compared to first. We have investigated this numerically, and for a broad range of the parameters the second eigenvalue is typically a factor of 10^{-2} or smaller down from the first. Consequently, it may either be that the mass scale to explain the solar neutrino anomaly arises from this additional suppression, or that it arises at $O(m_I^4)$ from (7). Of course, one could alternatively use more than one N and a hierarchical A matrix to generate a hierarchical m_{ij} .

IV. NEUTRINO MASS ANARCHY

The possibility has been explored elsewhere that neutrino mass matrices have no ordering structure, such as a flavor symmetry [10]. Absence of flavor symmetry is even *more* reasonable in our framework - all suppressions arise naturally via loop factors or factorization, or occur at different orders in m_I .

If the parameters in the model display no apparent structure, that is, the A_i are all roughly equal, and likewise the $m_{\tilde{\nu}_i}$ are roughly - but not exactly - the same, then we have a natural justification for the large mixing observed between ν_μ and ν_τ . We would then expect the solution to the solar neutrino problem to similarly involve a large angle: either large angle MSW, vacuum oscillations or the LOW solution. The only small parameter required is $\theta_{13} < 0.16$, required by CHOOZ [11], but we can view this as an accident rather than a fine tuning.

Even if we have a flavor symmetry which explains the structure of the charged fermion masses, that would not necessarily preclude such a scenario [5]. If the structure of the A -terms were determined by a supersymmetry-preserving spurion λ_i , i.e.,

$$A_i \tilde{n} \tilde{\nu}^i h_u = \frac{[X]_F}{M_{Pl}} \lambda_i \tilde{n} \tilde{\nu}^i h_u, \quad (10)$$

then we expect a hierarchy in A_i related to that which we find in the charged leptons. However, if X carries a flavor index itself, i.e.,

$$A_i \tilde{n} \tilde{\nu}^i h_u = \frac{[X_i]_F}{M_{Pl}} \tilde{n} \tilde{\nu}^i h_u, \quad (11)$$

then the situation is quite different. Because the structure of X_i is determined in the supersymmetry breaking sector, it need not be related to the structure of the lepton masses. In this case, even with a flavor symmetry, we would expect large angles to arise in the neutrino sector.

V. PHENOMENOLOGY AND COSMOLOGY

The phenomenology of the model presented in section III is very interesting. It is essentially identical to that explored in [5] for the case of a single N superfield. The presence of a weak-scale $\tilde{n}$ that mixes through weak-scale A terms to the left-handed sneutrinos can profoundly affect the sneutrino spectrum. For instance, a sneutrino mass eigenstate is not subject to the Z -width constraint if it is mostly composed of $\tilde{n}$, and its mass can be different from that of $\tilde{l}_L$ by far more than just the D -term splitting. The A terms could potentially induce invisible Higgs decays into light sneutrino pairs, and $\tilde{\nu} \tilde{l}$ might be the dominant decay mode for the charged Higgs boson. Finally, cascade decays producing heavy sneutrinos that subsequently decay into a Higgs and light sneutrino could conceivably be the dominant source of Higgs production at the LHC. There are numerous other potential consequences, arising in particular scenarios, which we will not discuss here.

The presence of the additional $\tilde{n}$ state revives the possibility of sneutrino dark matter, as was explored in [5] in the context of both lepton number conserving and lepton number violating models. In the lepton number conserving case, direct detection experiments require $m_{\tilde{\nu}} < 3$ GeV [12]. In contrast, the lepton number violating scalar mass term central to the present model allows for easy evasion of this bound.

Direct detection experiments detect ordinary sneutrino dark matter through Z boson exchange. However, once a lepton number violating mass term is present, the CP-even state $\tilde{\nu}_+$ and CP-odd state $\tilde{\nu}_-$ are no longer degenerate in mass. Moreover, scalar couplings to the Z are off-diagonal, i.e., they couple $\tilde{\nu}_+$ to $\tilde{\nu}_-$, but not $\tilde{\nu}_-$ to $\tilde{\nu}_-$. The scattering of $\tilde{\nu}_-$ off of a nucleus through Z exchange is kinematically forbidden for $\Delta m > \beta_h^2 m_- m_A / 2(m_A + m_-)$, where $\Delta m = m_+ - m_-$ is the mass splitting between the CP-even and CP-odd states, m_A is the mass of the nucleus, and $\beta_h = 10^{-3}$ for virialized halo particles on average. Thus, even for sneutrino masses of $O(100$ GeV), direct detection limits are essentially harmless, stipulating only that the lepton number violating mass is adequately large. For example, taking $m_{\tilde{\nu}} = 100$ GeV and $m_A = 72$ GeV for a Ge target, we simply need $\Delta m > 20$ keV to prevent direct detection. Because $\Delta m = \delta^2 / m_{\tilde{\nu}}$, this corresponds to $\delta > 45$ MeV, which is of the order of what we expect from $m_I^{5/2} / M_{Pl}^{3/2}$.

Sneutrino dark matter will still scatter from the nuclei via Higgs exchange. The cross section per nucleon for this is small, however, given by [5]

$$\sigma \approx 1.8 \times 10^{-43} \sin^2 2\theta \quad (12)$$

$$\times \left(\frac{A}{100 \text{ GeV}} \right)^2 \left(\frac{130 \text{ GeV}}{m_h} \right)^4 \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 \text{ cm}^2, \quad (13)$$

where

$$\tilde{\nu}_{DM} = \tilde{\nu} \sin \theta + \tilde{n} \cos \theta. \quad (14)$$

The current upper bound on σ for a dark matter candidate with mass $\sim 100\text{GeV}$ is 10^{-41}cm^2 [12], and this bound is expected to be lowered by orders of magnitude in the near future [13]. Thus, this version of sneutrino dark matter may be detectable due to Higgs exchange in upcoming direct detection experiments.

Sneutrino dark matter with lepton number violation has been previously explored [14], precisely because it can evade direct detection limits. However a large mass splitting ($\Delta m_{\tilde{\nu}} \sim \text{GeV}$) was necessary to suppress $\tilde{\nu}_+ \tilde{\nu}_-$ coannihilation in the early universe to yield an appreciable relic density. Here, because the lightest sneutrino is an admixture of left- and right-handed states, the overall annihilation rate via MSSM processes is suppressed by an additional factor of $\sin^4 \theta$. As discussed in [5], acceptable relic abundances are obtained for a broad range of parameters. For example, for sneutrino masses less than M_W , the relic density is essentially determined by the annihilation rate via neutralino exchange, and one finds

$$\Omega h^2 \approx \left(\frac{M_{\tilde{W}}}{100\text{GeV}} \right)^2 \left(\frac{\sin \theta}{0.16} \right)^4, \quad (15)$$

where h is the reduced Hubble parameter and $M_{\tilde{W}}$ is the neutral wino mass. Such a simple approximate formula does not apply when the sneutrino mass is heavy enough so that production of W and Z pairs becomes relevant, but the abundance is still promising for reasonable parameter choices.

VI. CONCLUSIONS

While conventional see-saw models generate neutrino masses proportional to v^2 , in theories with gravity mediated supersymmetry breaking, it is also possible to generate neutrino masses proportional to $v^{3/2}$. Models of this type arise when flavor symmetries protect the masses of standard model singlet states, but are broken in the supersymmetry breaking sector of the theory.

Additional suppression to these masses can arise from Yukawa-type suppressions, or from loop factors, resulting in values for the neutrino mass in accordance with observations from Superkamiokande of an up/down neutrino asymmetry.

The model developed in section III, which features a right-handed sneutrino at the weak scale, is phenomenologically rich, with dramatic changes to collider signatures and the possibility of sneutrino dark matter. Although the sneutrinos in this model evade current detection limits on dark matter, the possibility exists for their detection at a future experiment.

Acknowledgements

We thank R. Rattazzi for pointing out an error in a previous version of this work. This work was supported in part by the U.S. Department of Energy under Contracts DE-AC03-76SF00098, in part by the National Science Foundation under grant PHY-95-14797.

-
- [1] M. Gell-Mann, P. Ramond, R. Slansky, In *Supergravity*, ed. by F. Van Nieuwenhuizen and D. Freedman (Amsterdam, North Holland, 1979) 315; T. Yanagida, in *Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe*, eds. O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979) 95.
 - [2] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), *Phys. Rev. Lett.* **81**, 1562 (1998) hep-ex/9807003.
 - [3] C.P. Burgess, L.E. Ibanez, F. Quevedo *Phys. Lett.* **B447** (1999) 257, hep-ph/9810535.
 - [4] P. Langacker, *Phys. Rev.* **D58** (1998) 093017, hep-ph/9805281; L.J. Hall, N. Weiner, *Phys. Rev.* **D60** (1999) 033005, hep-ph/9811299.
 - [5] N. Arkani-Hamed, et al., hep-ph/0006312.
 - [6] Assigning charges under $U(1)_R \times U(1)_L$ according to $S(2,0)$, $X(0,2)$, $\bar{X}(0,-2)$, $Y(2,4)$, $\bar{Y}(2,-4)$, $N(1,-1)$, $L(1,1)$, and $H(0,0)$ gives the superpotentials of equations (1) and (2).
 - [7] The other fields in the Supersymmetric Standard Model have R -charges $Q(0)$, $H_d(0)$, $E(+2)$, $U(+2)$, $D(+2)$.
 - [8] G.F. Giudice, A. Masiero, *Phys. Lett.* **B206** (1988) 480.
 - [9] In the original model of [5], $[X^\dagger N N]_D$ was not the only means of generating a weak-scale $\bar{N}N$ mass. Alternatively, a gauged $U(1)_{B-L}$, broken at the weak scale, can give the same mass without introducing significant radiative corrections to the neutrino mass.
 - [10] L. Hall, H. Murayama, N. Weiner, *Phys. Rev. Lett.* **84**, 2000, 2572.
 - [11] M. Apollonio, et al., *Phys. Lett.* **B466** (1999) 415, hep-ex/9907037.
 - [12] D.S. Akerib, et al., *Nucl. Phys. Proc. Suppl.* **70** (1999) 64, astro-ph/9712343; R. Bernabei et al., *Phys. Lett.* **B480** (2000) 23; L. Baudis, et al., *Phys. Rev.* **D59** (1999) 022001, hep-ex/9811045.
 - [13] L. Baudis et al., astro-ph/0005568; R.W. Schnee, et al., *Phys. Rep.* **307** (1998) 283.
 - [14] L.J. Hall, T. Moroi, H. Murayama, *Phys. Lett.* **B424** (1998) 305, hep-ph/9712515.