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FINAL TECHNICAL REPORT

March, 1995

**A COMPUTATIONAL MODEL FOR
COAL TRANSPORT AND COMBUSTION**

Grant Number: DE-FG22-91PC91297

Goodarz Ahmadi

Department of Mechanical and Aeronautical Engineering

Clarkson University

Submitted to

U.S. Department of Energy

Pittsburgh Energy Technology Center

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CONTENTS

	<u>Page</u>
I. ABSTRACT	1
II. PROJECT OVERVIEW	2
II.1 Project Objectives	2
II.2 Significance To Fossil Energy Program	3
II.3 Relationship To Other DOE Funded Projects	3
III. SUMMARY OF SCIENTIFIC AND TECHNICAL ACCOMPLISHMENTS	3
III.1 Theoretical Model Development	3
III.2 Computational Model Development	4
III.3 Experimental Study	4
IV. HIGHLIGHT OF ACCOMPLISHMENTS	5
IV.1 A Thermodynamically Consistent Rate-Dependent Model for Turbulent Two-Phase Flows	5
IV.2 A Thermodynamically Consistent Stress Transport Model for Rapid Granular Flows	6
IV.3 Analysis of Rapid Granular Flows Including Frictional Losses	7
IV.4 Granular Flows in Complex Domain	8
IV.5 Granular Flows with Bumpy Boundary	10
IV.6 Granular Gravity Flows with Bumpy Walls	11
IV.7 Two-Phase Flows	16
IV.8 Experimental Study	21
V. ACHIEVEMENTS OF THE PROJECT OBJECTIVES	25
VI. PROJECT PERSONNEL	29
VII. BIBLIOGRAPHY OF PUBLICATIONS EMANATING FROM THIS PROJECT	30
VII.1 Journal Articles and Reports	30
VII.2 Conference Presentations	31
VIII. TECHNICAL PAPERS AND REPORTS	33

A COMPUTATIONAL MODEL FOR COAL TRANSPORT AND COMBUSTION

Grant Number: DE-FG22-91PC91297

Project Period: September 1, 1991 to March 12, 1995

Contract Recipient: Clarkson University

Project Principal Investigator: Goodarz Ahmadi

DOE Project Officer: Dr. Mehrdad Massoudi

I. ABSTRACT

In this project, a comprehensive theoretical, computational and experimental study directed toward providing a fundamental understanding of particulate flows as applied to coal transport is performed. Thermodynamically admissible constitutive expressions for the phasic stress tensors, heat and fluctuation energy flux vectors for turbulent multiphase flows were derived. The material parameters of the model were evaluated from the limiting conditions of rapid flows of dry spherical granular particles, and single-phase turbulent fluid flows. The case of simple shear flows of glass beads-water mixtures was studied. The model was extended to cover chemically active gas-solid flows.

A thermodynamically consistent model for rapid flows of granular materials in a rotating frame of reference, along with a transport equation for the granular kinetic stress tensor was developed. The model parameters for the special case of spherical nearly elastic particles were evaluated. The results for the granular stresses and the normal stress differences were compared with the available simulation data and good agreement was observed.

Effects of frictional loss of energy on rapid granular shear flows were studied. The previously developed kinetic based model was used and the mean velocity, the fluctuation kinetic energy and the solid volume fraction profiles were evaluated under a variety of conditions and different friction coefficients.

A computational model for analyzing rapid granular in complex geometries was developed. The discrete element scheme was used and the granular flow down a chute was analyzed. The results were compared with the available experimental data, the model predictions, and the existing simulation results, and good agreements were observed. The model was used to analyze granular flows in a duct with an obstructing block. The effect of boundary condition was also included and the granular gravity flow was analyzed in details.

A computational model for analyzing turbulent two-phase flows with various loadings was developed. The special case of gas-solid flows in a vertical duct was analyzed and the model predictions were favorably compared with the available experimental data. Extension of the computational model to horizontal duct was also presented.

An experimental setup for generating simple shear flows of a mono-granular layer was designed and fabricated. A complete set of experimental data for mean velocity, fluctuation energy and solid volume fraction for shearing of 12 mm multi-color glass particles at various concentrations were obtained.

II. PROJECT OVERVIEW

In this section an overview of the project objectives, their significance to fossil energy program, and the relationship to other DOE projects are provided.

II.1 Project Objectives

The objective of this project was to develop an accurate model describing turbulent flows of two-phase coal mixtures, rapid flows of granular flows including bulk coal transport and turbulent coal combustion processes. The other main objective was to develop a computational model for predicting granular and turbulent two-phase flows in various regions of industrial interest. Experimental verification of the foundation of the model was also included in the study.

The specific objectives of the project were:

- i) *To develop a rate-dependent turbulence model (including normal stress effects) capable of predicting multidimensional, nonuniform and unsteady rapid flows of granular coal particles.*
- ii) *To experimentally verify the validity of the fundamental hypotheses of the model.*
- iii) *To establish the relationship between the material parameters and physical properties of the coal particles.*
- iv) *To develop an accurate rate-dependent model for analyzing reacting and non-reacting turbulent flows of coal slurries and multiphase mixtures.*
- v) *To apply the developed model and to solve a number of technologically important problems related to coal slurries and bulk transports. These include flows in bins, hoppers, chutes, stand pipes and coal combustors.*
- vi) *To verify the validity of the developed model by comparing the predicted results with the digital simulation and the available experimental data.*
- vii) *To develop a computer code incorporating the new model for predicting reacting*

and non-reacting turbulent flows of coal slurries, multiphase mixtures and bulk transport in complex geometries of industrial interest.

II.2 Significance To Fossil Energy Program

A completely satisfactory theory describing the bulk coal transport including the interstitial fluid effects does not exist. This is particularly the case for turbulent flows of dense coal particle-liquid mixtures and chemically active coal combustor flows. Coal-fluid mixtures and bulk transports, and operation of coal combustors account for a substantial portion of the cost of coal energy conversion systems. The major increase in cost arises from the need to over-design these facilities to guarantee reliability. Understanding the flow behavior of relatively dense coal slurries and bulk solids in various geometries including coal combustors, is indispensable to economical design of the needed equipment. In this project, a sound practical model for coal transport and a general frame work for chemically active gas-solid flows were developed. A series of experiments in mono-granular flow setup was performed. The resulting data provided a fundamental understanding of the dynamics of particulate flows. In addition, a computational predictive capability for analyzing rapid flows of granular coal particles, and turbulent flows of dense or dilute multiphase (gas-solid) coal mixtures was also developed. These findings provide the tool needed to improve the efficiency of coal energy systems.

II.3 Relationship To Other DOE Funded Projects

Most other DOE funded research projects were concerned with the chemistry of coal and coal processing. In this regard the present project was complementary to the other major efforts in that the important engineering (fluid mechanical) aspects of coal transport and reacting multiphase flows were addressed.

III. SUMMARY OF SCIENTIFIC AND TECHNICAL ACCOMPLISHMENTS

The major accomplished tasks of the projects are summarized in this section.

III.1 Theoretical Model Development

- i) An advanced thermodynamically consistent stress-transport model describing rapid granular flows was developed. This model contains the modern kinetic theories of granular materials as special cases.
- ii) An accurate thermodynamically consistent rate-dependent model describing turbulent flows of dilute and dense two-phase flows was developed. This new model accounts for the effects of phasic fluctuation energies, their transport and interactions.
- iii) An advanced thermodynamically consistent stress-transport model for turbulent flows was developed. This model contains the recent advanced turbulent models as special limiting cases.

- iv) An advanced thermodynamically consistent model for chemically active turbulent two-phase flows was developed. This model provides a general frame work for analyzing gas-particle flows with chemical reactions. The new model reduces to the advanced two-phase gas-particles turbulent flow models and chemically active mixtures theories in special limiting cases.

III.2 Computational Model Development

- i) A computational model for analyzing rapid granular parallel flows was developed, and several examples were solved. The previously developed kinetic model for granular flow which includes frictional losses was used in these analysis.
- ii) A computational model for analyzing rapid granular flows in complex geometry of industrial interest was developed. Examples of three-dimensional particulate flows in an obstructed duct, as well as gravity flow down an inclined chute were solved. The previously developed kinetic model for granular flow was used in these analysis.
- iii) A computational model for analyzing granular gravity flows over a bumpy inclined chute was developed. The earlier developed kinetic-based model which includes the effects of frictional energy losses was used in the computational model predictions. A new set of kinetic-based boundary conditions including the frictional energy losses was also developed. The model predictions were compared with the available experimental data and molecular dynamic results and good agreement was noticed.
- iv) A computational model for analyzing turbulent two-phase gas-particle flows was developed. The thermodynamically consistent model developed earlier was used in this analysis. The results for vertical and horizontal ducts were compared with the available experimental data and good agreement was observed.
- v) A computational model for analyzing liquid-solid laminar and turbulent two-phase flows on an inclined chute was developed. The previously developed thermodynamically consistent model was used and the model predictions were shown to be in agreement with the experimental data.

III.3 Experimental Study

- i) A shear flow device for experiment study of rapid shear flows of granular materials was designed and constructed. Experimental data for mean and phasic fluctuation granular velocities were obtained. It was shown that the granular fluctuations were anisotropic. The experimental data were also in agreement with the predictions of the proposed thermodynamically consistent model.

IV. HIGHLIGHT OF ACCOMPLISHMENTS

Brief descriptions of the highlight of accomplishments of the project are provided in this section. The details were given in the section on Technical Papers and Reports.

IV.1 A Thermodynamically Consistent Rate-Dependent Model for Turbulent Two-Phase Flows

A thermodynamically consistent theory for dense two-phase flow was developed. The new model includes the transport equations for the phasic fluctuation kinetic energies. Anisotropic and rate-dependent expressions for the phasic stress tensors were derived. For dense solid-liquid mixtures, the material parameters of the model were evaluated from the limiting conditions. The special case of simple shear flows of glass beads-water mixtures was studied. The coefficient of restitution for glass bead collision in water was experimentally measured and used in the model predictions.

Figure 1 shows the predicted shear and second normal stresses of the present model along with the data of Hanes and Inman for glass beads of 1.1mm diameter in water mixture. The dashed lines are the predicted particulate and fluid phasic stresses. It is observed that particulate phase contributions to the shear and normal stresses increase as solid volume fraction increases. Reasonable agreement is observed between the experimental data and the predictions of the present model.

Figure 1. Variations of total and phasic shear and normal stresses with solid volume fraction.

Figure 2 shows variations of shear stress with shear rate for two values of solid volume fraction along with the experimentally measured stresses of Hanes and Inman for 1.1mm glass beads in water. Although some deviations are observed, the predicted stresses are in good agreement with the experimental data.

Figure 2. Variations of shear stress with shear rate and the phasic fluctuation energy production with solid volume fraction for 1.1 mm glass beads in water.

Unlike the available models, the new rate-dependent formulation is capable of predicting the production and dissipation rates of phasic fluctuation energies. Figure 2 shows variations of nondimensional phasic fluctuation energy production with solid volume fraction for a mixture of 1.1 mm glass beads in water. This figure shows that for large solid volume fractions, the fluctuation energy is mainly produced by the particulate phase and the contribution of the fluid phase is negligible as $\hat{v}$ approaches the limiting maximum solid volume fraction of about 0.64. For smaller values of $\hat{v}$, the fluid phasic energy production become rather large and that of the solid phase decreases.

The details of the model were described in two technical reports (MAE-281, MAE-282, November 1993, also in press in the International Journal of Nonlinear Mechanics), copies of which are enclosed in the section on Technical Papers and Reports. In addition, a thermodynamical framework for analyzing chemically active gas-solid two-phase flows in a turbulent state of motion was also developed. The results were presented in a technical report (MAE-303, March 1995), a copy of which is provided in the section on Technical Papers and Reports.

IV.2 A Thermodynamically Consistent Stress Transport Model for Rapid Granular Flows

The basic conservation laws governing the flow of rapid granular materials in a rotating frame of reference along with a transport equation for the granular kinetic stress tensor and the corresponding Clausius-Duhem inequality were considered. The thermodynamics of the mean granular motion was described and based on the averaged entropy inequality, constitutive equations for the pressure-strain, collisional stress, and flux of granular stress tensors were formulated. The model parameters for the special case of

spherical nearly elastic particles were evaluated. The resulting stress transport model was used and the case of rapid simple shear flow of a collection of particles was analyzed. The preliminary results for the granular stresses and the normal stress differences were compared with the available simulation data and good agreement was observed.

Figure 3. Variations of nondimensional first and second normal stress differences with solid volume fraction.

Figure 3 shows variations of first and second normal stresses differences with solid volume fraction for various values of coefficient of restitution. The simulation results of Campbell is also shown in this figure for comparison. While some deviations are observed, the predicted normal stress differences are in good agreement with the simulation results. Additional details of the model were reported in a paper (International Journal of Engineering Science, Vol 30, 1483-1445, 1992), a copy of which is provided in the section on Technical Papers and Reports.

IV.3 Analysis of Rapid Granular Flows Including Frictional Losses

The kinetic model for particulate flows, including frictional energy losses, was used to analyze the rapid granular flows in a Couette geometry, gravity flows down an inclined chute and a vertical channel. The equations of motion were solved using a fourth-order Runge-Kutta numerical scheme. The corresponding mean velocity and solid volume fraction profiles for the flow in a vertical channel are shown in figure 4. The data of Savage (1979) is also shown in this figure for comparison. It is observed that the predicted roughly paraboloidal mean velocity profile is in good agreement with data of Savage. Figure 4 also shows that the fluctuation energy has a rather high value in the shearing region near the wall, and decreases to its minimum value near the centerline. The solid volume fraction has an opposite trend of variation. It has a somewhat low value near the wall, and increases to its peak value near the centerline. In addition, the cases of granular Couette and gravity flows, as well as granular motion over a vibrating surface

were also analyzed and the results were reported in a paper (Powder Technology, Vol. 77, 7-17, 1993), a copy of which is provided in the section on Technical Papers and Reports.

Figure 4. Variations of mean velocity, fluctuation energy, and solid volume fraction profiles for granular flows down a vertical channel.

IV.4 Granular Flows in Complex Domain

A computational model for analyzing granular flows in complex regions was developed. The special case of granular flow around a rectangular block in a channel with free surface was studied. The details of the computational model and the discretization scheme used were described in a published paper (Computer and Fluids, Vol. 22, 25-50, 1993), a copy of which is enclosed in the section on Technical Papers and Reports

Sample computer simulation results for the velocity conditions are shown in Figure 5. It is observed that the block significantly distorts the flow field. This figure shows that a source of granular materials seems to appear at the center line in front of the block which interacts with the incoming uniform velocity field. A reverse flow region forms in front of the block which penetrates the upstream flow field up to about one meter. On both sides of the block, the flow accelerates and a separation bubble is formed behind the block. Further downstream, the wake region can be seen. This figure also shows that the granular particles move upward in front of the block, while they move downward behind the block. These lead to the formation of source and sink type flows in front and behind the block as noticed at the surface layer.

The simulation results for the velocity and solid volume fraction field for the case of a submerged block are shown in Figure 6. It is observed that the block significantly distorts the flow field. In particular, on both sides of the block, the flow accelerates and behind the block a wake is formed. Figure 6 also shows the solid volume fraction contour plot in the plane of symmetry of the channel at time $t=60$ sec. It is observed that flow is

stratified and the solid volume fraction increases with the depth.

Figure 5. Velocity vector plot at the surface layer and the plane of symmetry at $t=10$ sec.

Figure 6. Velocity vector plots and solid volume fraction contours at $t=60$ sec.

IV.5 Granular Flows with Bumpy Boundary

The interactions of particle with the boundary is known to significantly affect flow and transport of granular materials. The kinetic model of granular materials including frictional losses was used in the analysis. The presence of bumpy boundary conditions leads to a strongly coupled system of governing equations which has to be solved numerically even for the simple case of Couette flow. A special discretization scheme was developed and the mean velocity, the fluctuation kinetic energy and the solid volume fraction profiles for granular flows between two parallel walls were evaluated. The results for different values of friction coefficients are plotted in figure 7. The molecular dynamic simulation results of Savage and Dai (1992) for frictionless particles are also shown in this figure for comparison. It is observed that the model predictions are in excellent agreement with the molecular dynamic simulation results.

Figure 7 shows that there is a significant amount of slip at the wall. The slip velocity decreases as the friction coefficient increases. It is also observed that the fluctuation velocity is rather large near the wall and approaches a constant value away from the wall. The solid volume fraction, on the other hand, is small near the wall and increases with distance from the wall. Figure 7 also shows that v is roughly constant in the core region. These results show that the commonly used assumptions of a linear mean velocity and constant values of kinetic energy and solid volume fraction for Couette flow of granular materials are not correct.

Figure 7. Variations of mean velocity, fluctuation energy and solid volume fraction profiles. Comparison with the molecular dynamic simulation of Savage and Dai (1992).

The details of the computational model and additional results were described in a report (MAE-274, July 1993) which is enclosed in the section on Technical Papers and Reports.

IV.6 Granular Gravity Flows with Bumpy Walls

Granular gravity flows over an inclined bumpy chute is an important flow and is often used as a bench-mark for test of various theories. The earlier developed kinetic-based theory of Abu-Zaid and Ahmadi (1990) for rapid flows of granular materials which includes the frictional losses of energy during particle-particle and/or particle-wall collisions is used in the analysis. The predicted mean velocity, fluctuation energy, and solid volume fraction profiles are evaluated and the results are compared with existing experimental data.

Figure 8. Sample prediction results for granular flow over a bumpy incline. Comparison with the experimental data of Drake and Shreve (1986).

Figure 8 compares the experimental data of Drake and Shreve (1986) with the predicted mean velocity, solid volume fraction and square-root (SR) of fluctuation kinetic energy profiles for a chute inclination angle of $\alpha = 42.75^\circ$. The experiment was performed in an inclined glass-walled channel (3.7 m long and 6.5 mm wide) using 6 mm diameter cellulose-acetate spherical particles. The bed was made of spheres like those in the flows, which were glued with random spacing to an aluminum bar. A granular layer height to particle diameter ratio of $H/d = 16$, a particle-particle restitution coefficient $r = 0.9$, particle-wall restitution coefficient $r_w = 0.5$, and $\mu = \mu_w = 0.3$ are used in the numerical simulation. It is observed that the predicted velocity and solid volume fraction profiles are

in good agreement with the experimental data. The predicted SR-fluctuation kinetic energy profile, however, overestimates the experimental data near the wall. As noted before, the experiment was performed for a mono-layer of granular particles in a two-dimensional channel, while the present model predictions are for fully three-dimensional particulate flows. Therefore, certain deviations should be expected.

Johnson et al. (1990) performed a series of experiments for gravity flow of 1 mm diameter glass beads down an inclined chute with a smooth aluminum plate. The chute was 1.4 m long and 6.35 cm wide, and was bounded by vertical glass walls 12.7 cm high. Their mean velocity data for an inclination angle of $\alpha=17^\circ$ and $H/d=20$ are reproduced in figure 9. The present model predictions for the same conditions are also plotted in this figure for comparison. Here, a coefficient of restitution of $r=0.91$ and a coefficient of friction of $\mu=0.45$ for glass beads as suggested by Johnson et al (1990) are used in numerical simulation. The experimental smooth wall surface condition is simulated by considering a wall with closely packed small bumps. Figure 9a shows that the calculated velocity profile is in good agreement with the experimental data. A significant amount of slip at the wall is also noticed in the experiment which is well predicted by the model.

Figure 9. Sample prediction results for granular flow over a bumpy incline. Comparison with the experimental data of Johnson et al. (1990).

From figure 9b, it is observed that the solid volume fraction remains roughly constant across the chute with a sharp gradient near the free surface. The dashed line in

this figure corresponds to the mean solid volume fraction of $v_0=0.65$ reported in the experiment of Johnson et al. (The solid volume fraction and fluctuation kinetic energy profiles were not measured by Johnson et al.). The predicted mean solid volume fraction here is 0.64 which is quite close to the experimental value.

Figure 9c shows the variation of SR-fluctuation kinetic energy (w) over the incline. It is observed that the SR-fluctuation energy has a large value near the wall and decreases rapidly to about 20% of its wall value at a short distance from the wall. It then decreases gradually toward the free surface. Unfortunately, there is no experimental data for the fluctuation kinetic energy for comparison in this case.

Figure 10. Sample prediction results for granular flow over a bumpy incline. Comparison with the experimental data of Johnson et al. (1990).

Figure 10 presents the model predictions for $H/d=6$ and a comparison with the corresponding experimental data of Johnson et al. (1990). The rest of parameters used in this simulation are the same as those of figure 9. Figure 10a shows that the calculated velocity profile is in reasonable agreement with the experimental data. The predicted slip at the wall, however, is slightly lower than that observed in the experiment. This could be due to the bumpy wall boundary model used in the simulation, while the experiment was performed on flat inclined chute. Figure 10b presents the calculated solid volume fraction profile. The dashed line in this figure corresponds to the experimental mean solid volume fraction. The solid volume fraction remains roughly a constant, except for its rapid

decrease in the saltation region, and a decreasing trend in the shearing zone near the wall. The calculated mean solid volume fraction is 0.543, while the experimentally reported value was about 0.61. Figure 10c shows the SR-fluctuation kinetic energy profile. It is observed that w has a large value near the wall and decreases with the distance from the chute surface.

Figure 11 shows variations of velocity, solid volume fraction and SR-fluctuation kinetic energy profiles for chute inclination angles of 13° , 17° and 21° . The other parameters are kept fixed at $r=0.9$, $\mu=0$, and $H/d=20$. The velocity profiles shown in figure 11a indicate that a large slip occurs at the bottom wall. As the inclination angle increases, the mean, as well as the slip velocities increase. Figure 11b shows that the solid volume fraction decreases with increasing α . The flow is also very sensitive to changes of the inclination angle, since a small change in α can have a dramatic effect on the flow properties. Saltation regions of very low solid volume fraction near the free surface are also clearly observed. The height of the saltation region increases as the inclination angle increases. Figure 11a also shows that the mean velocity is roughly constant in the saltation region.

Figure 11. Variation of mean velocity, solid volume fraction and SR-fluctuation energy profiles for granular flow over a bumpy chute at different inclination angles.

Figure 11b shows that the flow maintains a region of low density near the bottom

wall, despite the large overburden of material. This phenomenon has been observed experimentally in chute flows by Bailard (1978) and Johnson et al. (1990). This figure also shows that the solid volume fraction increases with distance from the wall to a maximum in the mid-section. It then decreases and becomes quite small in the saltation region.

Variations of the SR-fluctuation kinetic energy, w , are shown in figure 11c. It is observed that there is a region of large fluctuation kinetic energy near the wall. This is because one main source of the fluctuation energy production is the collisions between particles and wall and the occurrence of slip velocity. Thus, the highest fluctuation energy is found next to the wall. Away from the wall, the fluctuation kinetic energy generally decreases and reaches to its minimum value at the free surface. Figure 11c also shows that, as the inclination angle of the chute increases, w increases.

Figure 12. Variation of nondimensional normal and tangential stresses for granular flow over a bumpy chute at different inclination angles.

For the conditions of figure 11, the variations of non-dimensional normal stress, tangential stress, energy dissipation, energy production, and fluctuation energy flux are shown in figures 12 and 13. Figure 12 shows that the normal and tangential stresses have their highest values at the wall, and their magnitudes decrease with increasing distance from the wall. As the inclination angle increases, the non-dimensional normal and tangential stresses decrease. Figure 13 shows that the energy production, the energy dissipation and the fluctuation energy flux, generally, increase toward the wall. It is also observed that the energy dissipation and production rates decrease as α increases, while the fluctuation energy flux increases very near the wall and decreases at distances away from the wall. Figures 11 and 12 show the normal and tangential stresses, the energy production and dissipation rates, as well as the fluctuation energy flux are very small in the saltation region.

The details of the computational models and additional results were reported in technical report (MAE-275, September 1993) which is also under review for publication in the Journal of Fluid Mechanics. A copy of the report is provided in the section on Technical Papers and Reports.

Figure 13. Variations of nondimensional energy dissipation, energy production and fluctuation energy flux for granular flow over a bumpy chute at different inclination angles.

IV.7 Two-Phase Flows

A computational model for solving dense and dilute two-phase flows was developed. In this section, the computational model predictions for mean gas velocity, mean particle velocity, and phasic turbulence intensities for 0.5 and 1 mm particles are presented and compared with the experimental results of Tsuji et al. (1984). In addition, the variations of phasic shear and normal stresses, as well as the phasic fluctuation energy production and dissipation are also evaluated.

Using a Laser-Doppler Velocimeter (LDV), Tsuji et al. reported measurements of the phasic flow properties in a fully developed, two-phase, air-particle turbulent flow in a 30.5 mm vertical pipe. In their experiments, polystyrene spheres with density of $\rho_0 = 1020 \text{ kg/m}^3$ and diameters in the range 0.2 mm to 1 mm were suspended in air. Restitution coefficients of $r = 0.9$, for particle-particle collisions, and $r_w = 0.7$, for particle-wall collisions and a coefficient of dynamic friction $\mu = 0.2$ between a particle and the wall are used in the present study. These values are consistent with the observation of Govan et al. (1989),

who studied the trajectories of glass spheres transported in a small pipe under conditions similar to those of Tsuji et al. (1984).

The phasic mean velocity, turbulence intensity and solid volume fraction profiles for a mass loading of $m = 3.4$ of 0.5 mm polystyrene spheres are shown in figure 14. In this case, the flow Reynolds number is $Re = 2200$. Figure 14a shows an interesting feature that the air mean velocity profile becomes concave and the location of maximum velocity deviates from the channel centerline. This phenomena was observed in the experimental data of Tsuji et al. (1984), and was also reported by Vollheim (1965), who measured the mean air velocity in a vertical pipe in the presence of large particles at high loading ratios $m = 1-17$ by a Pitot tube. Figure 14a indicates that the change of mass loading does not have a significant effect on the mean particulate velocity profile, and the wall slip velocity of particles of the same size and density. Figure 14a also shows good agreement between the model predictions and the experimental data of Tsuji et al. (1984).

Figure 14. Sample prediction results for two-phase flows in a duct.
Comparison with the experimental data of Tsuji et al. (1984).

Variations of turbulence intensities of particulate and gas phases are shown in figure 14b. It is observed that the phasic turbulence intensities in the core region increase as the mass loading increases. The air turbulence intensity near the wall, however, is somewhat reduced as m increases. The experimental data of Tsuji et al. (1984) for the turbulence intensity are reproduced in this figure that are in reasonable agreement with the present model predictions. Figure 14c presents the solid volume fraction profile across the duct. It is observed that the particle concentration increases toward the channel centerline.

The influence of the mass loading ratio on the phasic turbulent shear stresses is shown in figure 15. Figure 15a shows that the gas turbulent shear stress increases toward the wall reaching to its maximum value at a distance of y/H of about 0.06 to 0.08, and then decreases sharply to zero at the wall. As the mass loading ratio increases, the gas turbulent shear stress decreases. Figure 15b presents the particulate phase shear stress profiles at different mass loading ratios. It is observed the shear stress of the particulate phase increases as m increases. This figure also shows that the phasic particulate stresses are comparable to those of the gas phase even at low solid volume fractions in the range of $v=0.002$ to 0.01 . The total shear stress of gas-particle mixture is shown in figure 15c. This figure indicates that the total shear stress increases as mass loading ratio increases.

Figure 15. Variation of turbulence shear stress profiles for 0.5 mm particles at different mass loadings.

Figure 16 shows the model predictions for a two-phase gas-particle flow at a mass loading of $m=0.6$ of 1 mm polystyrene spheres. The flow is upward and the Reynolds number in this case is 23,000. The maximum mean air velocity is 13.4 m/s which occurs at the channel centerline. Figure 16a shows the mean air and particle velocity profiles. The experimental data of Tsuji et al. (1984) for air velocity is also reproduced in this figure for comparison. It is observed that the model prediction for the air velocity is in good agreement with the experimental data. Compared with that for 0.5 mm particles, it is noticed that the mean particle velocity is reduced and the relative slip velocity between the particle and air is increased significantly.

The phasic turbulence intensities are shown in figure 16b. The air turbulence intensity has the general characteristics of clear air turbulence and the particle turbulence

intensity profile is roughly constant across the duct. Compared with the results for single-phase flows, it is observed that the air turbulence intensity generally increases due to the presence of 1 mm particles except for very near the wall where it decreases. Thus, the particles promote turbulence in the core region while suppress it near the wall. The experimental data of Tsuji et al. (1984) for air turbulence that are reproduced in figure 16b, are in qualitative with the model prediction. Figure 16c shows that the particle solid volume fraction profile is roughly a constant of about 0.0018 and tends to increase toward the channel wall.

Figure 16. Sample prediction results for two-phase flows in a duct.
Comparison with the experimental data of Tsuji et al. (1984).

For the flow condition described in figure 16, the phasic and total stresses are shown in figure 17. Figures 17a and 17b show that the phasic normal and shear stresses have profiles similar to those for 0.5 mm particles. However, the phasic fluid stresses increase significantly as the particle number density decreases (i.e., particle diameter increases at constant v). The total normal and shear stress profiles for the mixture are shown in figure 17c. The variations are similar to those observed for 0.5 mm particles, except that the total normal and shear stresses increase as much as two to three times as the particle diameter doubles.

The model predictions of the corresponding phasic momentum supply, particulate fluctuation energy supply, and the interaction energy dissipation rate are shown in figure 18. This figure shows that the overall behavior is quite similar to those observed for 0.5 mm particles. As the particle diameter increases, the phasic momentum supply terms decrease in direct proportion, while the particulate energy supply term, and the interaction energy

supply decrease slightly. The decrease of interaction momentum supply with increase of particle diameter is consistent with the variation of drag coefficient per unit mass with d .

Figure 17. Variation of phasic and total normal and tangential stresses for gas-solid turbulent flows.

Figure 18. Variation of momentum and energy supply terms and interaction energy dissipation for gas-solid turbulent flows.

The variations of corresponding phasic energy production and dissipation profiles are shown in figure 19. It is observed that the particulate energy dissipation is nearly constant across the channel. Figure 19 also shows that the phasic net energy productions exceed their energy dissipation rates. Comparing with the results for 0.5 mm particles, it is observed that the particulate energy production and dissipation rate decrease as the particle number density decreases (d increases), while the fluid energy production and dissipation rates increase.

Figure 19. Variations of energy production and dissipation profiles for gas-solid turbulent flows.

The computational procedure for analyzing turbulent, two-phase, gas particle flows and extensive comparisons with the experimental data were reported in a technical report (MAE-285, February 1994) which is also accepted for publication in the International Journal of Multiphase flows. A copy of the report is provided in the section on Technical Papers and Reports.

IV.8 Experimental Study

The main objective of the experimental effort is to provide a fundamental understanding of the mechanisms that control the features of granular particulate flows. The other main goal is to provide reliable data for velocity, concentration and fluctuation energy profiles for a simple shear flow of granular materials in order to verify the foundation of the thermodynamical formulation and the computational predictions.

A mono-granular simple shear flow setup was constructed and was used for the experimental study. A collection of multi-colored spherical glass balls which are 12 mm in diameter were used as the granular particles. A video camera is used to record the motions of particles. For different shear rates, the position of the balls in consecutive frames taken at 1/30th of a second apart were measured. Using this technique, the velocity vector of each particle was calculated. The shearing region was divided into 10-15 equivalent

horizontal segments. Averaging procedures are used to provide the experimental velocity and concentration profiles. Figure 20 shows the schematic of the device and the shearing cell. Figure 21 shows a picture of the experimental apparatus.

Figure 20. The schematic of the simple shear flow device.

Figure 21. Pictures of the simple shear flow apparatus.

Figure 22 shows sample axial and lateral mean velocity and concentration profiles. It is observed that a significant amount of slip exists. The slip is much larger at the inner wall when compared with that at the outer wall. It appears that the centrifugal force is important for this rotating Couette flow geometry. At the inner wall, the centrifugal force moves the particle away from the wall, reduces the normal force and, as a result, the slip velocity increases. At the outer wall, however, the particles are pushed toward the wall by the centrifugal force and the slip velocity decreases. Figure 22 also shows that the axial velocity is roughly linear in the central part of the shearing region. The mean lateral velocity fluctuates around the zero level.

Figure 22. Sample experimental data for the mean velocity and concentration profiles.

The experimental data for the nondimensional mean velocity variation across the shear cell are shown in figure 23. Figure 23a shows the variation for a mean solid volume fraction of 20% and different shear rates, while in figure 23b, the shear rate is kept approximately constant and the solid volume fraction is changed from 0.13 to 0.3. $U_{\max}$ in this figure corresponds to the case of maximum velocity of the glass particles in the cell adjacent to the outer wall of the shearing cell. It is observed that even in the normalized form the mean velocity exhibits a significant amount of slip near the inner wall. Figure 23 also shows that the mean velocity is roughly linear in the shearing region. From figure 23a it is observed that the amount of normalized slip velocity increases with the shearing rate, while the effect of solid volume fraction is not as distinct due to the scatter in the data.

The experimental data for the slip velocity variation at inner and outer walls of the shearing cell are shown in figure 24. It is observed that the slip at the outer wall is

generally larger than that at the inner wall. Figure 24 clearly shows that the slip velocity decreases with an increase of solid volume fraction, while it increases rapidly with an increase in shearing rate.

Figure 23. Experimental data for nondimensional mean velocities.

Figure 24. Variation of slip velocity with solid volume fraction and shearing rate.

The details of the experimental apparatus and procedure for taking data as well as extensive experimental data and comparisons with the theoretical models were reported in a technical report (MAE-304, March 1995). A copy of the report is enclosed in the section on Technical Papers and Reports.

V. ACHIEVEMENTS OF THE PROJECT OBJECTIVES

Achievements of the major objectives of the project are summarized in this section. Each specific objective is first stated and the relevant sections of the report and the resulting technical reports and papers are identified.

Objective i) *To develop a rate-dependent turbulence model (including normal stress effects) capable of predicting multidimensional, nonuniform and unsteady rapid flows of granular coal particles.*

Sections III.1, IV.2

S. Abu-Zaid and G. Ahmadi, "A Stress Transport Model for Granular Flows in a Rotating Frame," Int. J. Engng. Sci. 30, 1483-1495 (1992).

M. Massoudi and G. Ahmadi, "Rapid Flows of Granular Materials with Density and Fluctuation Energy Gradients," Int. J. Non-linear Mechanics 29, 487-492 (1994).

J. Cao, G. Ahmadi and M. Massoudi, "Gravity Granular Flows Down an Inclined Bumpy Chute with Friction," MAE Report-275, Clarkson University, Potsdam, NY, September (1993). Also submitted to J. Fluid Mechanics.

Objective ii) *To experimentally verify the validity of the fundamental hypotheses of the model.*

Sections III.3, IV.8

K. Elliott, G. Ahmadi and W. Kvasnak "Couette Flow of Granular Monolayer - An Experimental Study," MAE Report-304, Clarkson University, Potsdam, NY, March (1995).

Objective iii) *To establish the relationship between the material parameters and physical properties of the coal particles.*

Sections IV.2, IV.8

S. Abu-Zaid and G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulent Two-Phase Flows, Part I - Formulation," MAE Report-281, Clarkson University, Potsdam, NY, November (1993). Int. J. Non-linear Mechanics (In Press).

S. Abu-Zaid and G. Ahmadi, "A Stress Transport Model for Granular Flows in a Rotating Frame," Int. J. Engng. Sci. 30, 1483-1495 (1992).

Objective iv) *To develop an accurate rate-dependent model for analyzing reacting and non-reacting turbulent flows of coal slurries and multiphase mixtures.*

Sections III.1, IV.1

G. Ahmadi and J. Cao, "A Thermodynamical Formulation for Chemically Active Multiphase Turbulent Flows," MAE Report-303, Clarkson University, Potsdam, NY, March (1995).

S. Abu-Zaid and G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulent Two-Phase Flows, Part I - Formulation," MAE Report-281, Clarkson University, Potsdam, NY, November (1993). Int. J. Non-linear Mechanics (In Press).

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H. Ounis and G. Ahmadi, "Motions of Small Particles in a Turbulent Simple Shear Flow Field Under Microgravity Condition," Physics of Fluids A 3, 2559-2570 (1991).

G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulence, Part I - Formulation," Int. J. Non-Linear Mech. 26, 595-607 (1991).

Objective v) *To apply the developed model and to solve a number of technologically important problems related to coal slurries and bulk transports. These include flows in bins, hoppers, chutes, stand pipes and coal combustors.*

Sections III.2, IV.3, IV.5, IV.6, IV.7

J. Cao and G. Ahmadi, "Gas-Particle Two-Phase Turbulent Flow in a Vertical Duct," MAE Report-285, Clarkson University, Potsdam, NY, February (1994).
Int. J. Multiphase Flows (In Press).

J. Cao and G. Ahmadi, "Two-Phase Solid-Liquid Mixture Flows Down Inclined Chutes," MAE Report-298, Clarkson University, Potsdam, NY, December (1994).

S. Abu-Zaid and G. Ahmadi, "Analysis of Rapid Shear Flows of Granular Materials by a Kinetic Model Including Frictional Losses," Powder Technology 77, 7-17 (1993).

Objective vi) *To verify the validity of the developed model by comparing the predicted results with the digital simulation and the available experimental data.*

Sections IV.1, IV.2, IV.3, IV.5, IV.6, IV.7

J. Cao and G. Ahmadi, "Gas-Particle Two-Phase Turbulent Flow in a Vertical Duct," MAE Report-285, Clarkson University, Potsdam, NY, February (1994).
Int. J. Multiphase Flows (In Press).

J. Cao and G. Ahmadi, "Two-Phase Solid-Liquid Mixture Flows Down Inclined Chutes," MAE Report-298, Clarkson University, Potsdam, NY, December (1994).

J. Cao, and G. Ahmadi, "Numerical Simulation of Granular Couette Flows Between Two Rough Parallel Plates," MAE Report-274, Clarkson University, Potsdam, NY, July (1993). Also submitted to Particulate Science and Technology, An International Journal.

J. Cao, G. Ahmadi and M. Massoudi, "Gravity Granular Flows Down an Inclined Bumpy Chute with Friction," MAE Report-275, Clarkson University, Potsdam, NY, September (1993). Also submitted to J. Fluid Mechanics.

S. Abu-Zaid and G. Ahmadi, "Analysis of Rapid Shear Flows of Granular Materials by a Kinetic Model Including Frictional Losses," Powder Technology 77, 7-17 (1993).

Objective vii) *To develop a computer code incorporating the new model for predicting reacting and non-reacting turbulent flows of coal slurries, multiphase mixtures and bulk transport in complex geometries of industrial interest.*

Sections IV.3, IV.4, IV.6, IV.7

J. Cao and G. Ahmadi, "Gas-Particle Two-Phase Turbulent Flow in a Vertical Duct," MAE Report-285, Clarkson University, Potsdam, NY, February (1994). Also submitted to Int. J. Multiphase Flows.

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D. Ma, A.H. Eraslan and G. Ahmadi, "A Computer Code for Analyzing Transient Three-Dimensional Rapid Granular Flows in Complex Geometries," Computer Fluids 22, 25-50 (1993).

J. Cao, G. Ahmadi and M. Massoudi, "Gravity Granular Flows Down an Inclined Bumpy Chute with Friction," MAE Report-275, Clarkson University, Potsdam, NY, September (1993). Also submitted to J. Fluid Mechanics.

S. Abu-Zaid and G. Ahmadi, "Analysis of Rapid Shear Flows of Granular Materials by a Kinetic Model Including Frictional Losses," Powder Technology 77, 7-17 (1993).

VI. PROJECT PERSONNEL

Principal Investigator: Goodarz Ahmadi

Additional Project Personnel

A number graduate students were involved in various stages of this project, and their education benefited from this DOE funded research. A listing of the students and their current status and a brief description of their works are provided in this section.

Jianfa Cao (Ph.D Student), Computational Modeling of Granular and Two-Phase Flows.

Kenneth Elliott (Ex-M.S. Student), Experimental Study of Particulate Flows. (Plant Engineer, Nestle Corp., St. Louis, Missouri).

Willian Kvasnak (Ph.D Student), Experimental Study of Particulate Flows.

Amy Li (Ex-Ph.D Student), Computational Modeling of Dilute Two-Phase Flows. (Currently Consultant, Boston, MA).

Hadj Ounis (Ex-Research Associate) Particle Dispersion in Turbulent Flows. (Currently President of Ai7 Computer, New York).

Sameer Abu-Zaid (Ex-Ph.D Student), Theoretical Modeling of Granular and Two-Phase Flows. (Currently with Parsons Brinckerhoff, Energy System Group, New York, NY)

Danling Ma (Ex-Ph.D Student), Theoretical and Computational Modeling of Granular Flows. (Currently with Foft Company, Chicago, IL).

Showkat J. Chowdhury (Ex-Ph.D Student), Theoretical and Computational Modeling of Turbulent Flows (Currently Associate Professor, University of Dakka, Bangladesh).

VII. BIBLIOGRAPHY OF PUBLICATIONS EMANATING FROM THIS PROJECT

VII.1 Journal Articles and Reports

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K.A. Elrais, W. Eckerle, G. Ahmadi and A.H. Eraslan, "Simulation of Transient Three-Dimensional Natural Convection and Saturated Pool Boiling," *Int. J. Numerical Methods Heat Fluid Flow* 2, 139-154 (1992).

S.J. Chowdhury and G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulence, Part II - Numerical Results," *Int. J. Non-Linear Mech.* 27, 705-718 (1992).

S. Abu-Zaid and G. Ahmadi, "A Stress Transport Model for Granular Flows in a Rotating Frame," *Int. J. Engng. Sci.* 30, 1483-1495 (1992).

S.J. Chowdhury and G. Ahmadi, "Analysis of A Mixing Layer by a Rate-Dependent Turbulence Model," *ASCE, Journal of Engineering Mechanics* 119, 1700-1706 (1993).

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S. Abu-Zaid and G. Ahmadi, "Analysis of Rapid Shear Flows of Granular Materials by a Kinetic Model Including Frictional Losses," *Powder Technology* 77, 7-17 (1993).

S. Abu-Zaid and G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulent Two-Phase Flows, Part I - Formulation," *MAE Report-281*, Clarkson University, Potsdam, NY, November (1993). *Int. J. Non-linear Mechanics* (In Press).

S. Abu-Zaid and G. Ahmadi, "A Thermodynamically Consistent Rate-Dependent Model for Turbulent Two-Phase Flows, Part II - Simple Shear Flows," *MAE Report-282*, Clarkson University, Potsdam, NY, November (1993). Also submitted to *Int. J. Non-linear Mechanics*.

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VII.2 Conference Presentations

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G. Ahmadi and S. Abu-Zaid, "A Thermodynamically Consistent Model for Turbulent Two-Phase Flows," 28th Annual Technical Meeting of the Society of Engineering Science, Gainesville, FL, November 6-8, 1991.

G. Ahmadi and S. Abu-Zaid, "Analysis of Rapid Shear Flows of Granular Materials by a Stress Transport Model," 28th Annual Technical Meeting of the Society of Engineering Science, Gainesville, FL, November 6-8, 1991.

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G. Ahmadi and S. Abu-Zaid, "A Thermodynamically Consistent Stress Transport Model for Rapid Granular Flows," The 23rd Annual Meeting of the Fine Particle Society, Las Vegas, Nevada, July 13-17, 1992.

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J. Cao, G. Ahmadi and M. Massoudi, "Rapid Granular Flows Down an Inclined Bumpy Chute," 31th Annual Technical Meeting of the Society of Engineering Science, Texas A&M University, College Station, TX, October 10-12, 1994.

J. Cao and G. Ahmadi, "Turbulent Flow of Two-Phase Gas-Particle Mixture in a Vertical Duct," 31th Annual Technical Meeting of the Society of Engineering Science, Texas A&M University, College Station, TX, October 10-12, 1994.

VIII. TECHNICAL PAPERS AND REPORTS

*reprints and preprints
removed. (W)*

TWO-PHASE SOLID-LIQUID MIXTURE FLOWS DOWN INCLINED CHUTES

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Report No. MAE-298
December 1994

TWO-PHASE SOLID-LIQUID MIXTURE FLOWS DOWN INCLINED CHUTES

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ABSTRACT

Flows of several mixtures consisting of glass beads and different liquids down an inclined chute are studied. The upgraded thermodynamically consistent two-phase flow model which includes a low Reynolds number fluid phase turbulence closure is used in the analysis. The governing equations are solved with the aid of a semi-implicit numerical algorithm. The case of viscous flows of high concentration solid-liquid is first analyzed, and good agreement was obtained between the predicted mean velocity and solid volume fraction profiles and the available experimental data. The turbulent flow of glass bead-water mixture is then studied. Detailed distributions of phasic mean velocity, fluctuation kinetic energy and solid volume fraction are presented. Additional flow properties such as the phasic fluctuation energy production and dissipation rate, as well as the phasic and total normal and shear stress profiles are also presented and discussed.

INTRODUCTION

Turbulence modeling of multi-phase flows has been a subject of considerable interest in recent years. Currently, two main approaches have been proposed. One is based on an Eulerian treatment of the continuous phase and a Lagrangian stochastic representation of the particulate field. This Lagrangian method has been often applied at low particle concentrations where the mutual interaction between particles and the effect of particulate phase on the fluid phase can be neglected (Crowe et al. 1977; Sommerfeld et al. 1992; Li et al. 1994). The other main approach is based on entirely Eulerian method where the particulate phase is also treated as a continuous field. The continuum mechanical foundation of the so-called mixture theory was studied by Truesdell and Toupin (1960), Eringen and Ingram (1967), Bowen (1967), Nunziato and Walsh (1980), Ahmadi (1982, 1985), Massoudi (1986) and Johnson et al. (1991a,b), among others. In these earlier models, however, the effects of fluid turbulence, the particle fluctuation kinetic energy and the inter-particle collision were neglected. In another class of studies, turbulence modeling approach of single phase fluid was extended to low concentration turbulent flows. Earlier works in this direction were reported by Genchev and Karpuzov (1980), Ishii and Mashima (1984), Chen and Wood (1985) and Elghobashi and Abu-Arab (1983). Recently, Kashiwa (1987), Gidaspow et al. (1989), Louge et al. (1991), Sommerfeld et al. (1992), Ocone et al. (1993) and Sanjay et al. (1994) proposed more elaborated models that offered certain improvements. However, these models also did not consider the details of phasic fluctuation energy transfer and/or particle collisions effects for dense solid-liquid mixtures.

Earlier, Ahmadi and Ma (1990) used a phasic mass-weighted averaging technique to establish a thermomechanical formulation for turbulent multiphase flows. The corresponding system of field equations for determining phasic mean velocities, solid volume fraction and phasic fluctuation kinetic energies was derived. In particular, the formulation included distinct transport equations for the fluctuation kinetic energies of the particulate and fluid phases. Further more, this model also accounts for the particulate collisional stresses and phasic fluctuation energy interactions;

therefore, it is suitable for analyzing turbulent flows of relatively dense mixtures.

The inclined chute solid-liquid flow has many important applications in coal energy systems, clean coal technology, fluidized bed combustors, mining and food processing industries. In addition, solid-liquid flows also occur in nature as sediment transport in rivers and debris flows in sand storms. Earlier works on analyzing chute flows were mainly concerned with dry granular flows (Savage 1979; Drake and Shreve 1986; Johnson et al. 1990; Walton 1992), and a comprehensive theory describing dense solid-liquid mixture flows has not been available. Recently, Malekzadeh and Savage (1993) and Malekzadeh (1993) conducted a series of chute flow experiments for glass bead-liquid mixtures with the liquid being a blending composed of 20% Dibutyl Benzyl Phtalate (DBP) and 80% Alkyl Benzyl Phtalate (ABP). These experiments showed that the steady fully developed flows are possible for a range of chute inclination angles and loading ratios. Using a video-camera recorder, they measured the mean solid phase velocity and concentration profiles at different chute inclinations.

In this study, the upgraded thermodynamically consistent model of Ma and Ahmadi (1990) and Cao and Ahmadi (1994a,b) is used and the steady, fully-developed two-phase glass bead-liquid mixtures down inclined chutes is analyzed. On the chute surface a no-slip boundary condition for the liquid-phase and frictional slip-boundary condition for the particle phase are used in the analysis. The zero phasic shear stresses and fluctuation kinetic energy fluxes are prescribed on the free surface. The resulting governing equations were solved by using a semi-implicit finite difference method. The computational model is first applied to simulate a high concentration of 3 mm glass bead-liquid mixture flow down an inclined chute. It is shown that, while the liquid is in viscous flow regime, the particulate collisional stress is still significant. The model predictions for the phasic mean velocities and solid volume fractions are compared with the experimental data of Malekzadeh and Savage (1993) and Malekzadeh (1993), and reasonable agreement is observed. The model is then applied to turbulent flows of glass bead-water mixtures down an inclined chute at different conditions. Phasic velocity, solid volume fraction and normal and shear

stress profiles are presented. The phasic energy production and dissipation are also discussed.

GOVERNING EQUATIONS

The modified two-phase turbulence model of Ahmadi and Ma (1990) as described by Cao and Ahmadi (1994a,b) is used in this study. For a fully developed, two-phase, turbulent flow down an inclined chute with an inclination angle α , the flow variables are only functions of y and t . A schematic of the flow region is shown in figure 1. It is assumed that the fluid and particulate constituents are incompressible and the flow is isothermal. In this case, the continuity equations for particulate and fluid phase are identically satisfied. The momentum and fluctuation kinetic equations are then reduced to:

Momentum Balance

Particulate Phase

$$\rho_0 \nu \frac{\partial u}{\partial t} = \rho_0 \nu g \sin \alpha + \frac{\partial}{\partial y} [(\mu + \mu^T) \frac{\partial u}{\partial y}] + D_0(u^f - u) \quad (1)$$

$$-\nu \frac{dp^f}{dy} - \rho_0 \nu g \cos \alpha - \frac{\partial(\gamma \rho_0 \nu k)}{\partial y} = 0 \quad (2)$$

Fluid Phase

$$\rho_0^f \nu^f \frac{\partial u^f}{\partial t} = \rho_0^f \nu^f g \sin \alpha + \frac{\partial}{\partial y} [(\mu^f + \mu^{fT}) \frac{\partial u^f}{\partial y}] + D_0(u - u^f) \quad (3)$$

$$-\nu^f \frac{dp^f}{dy} - \rho_0^f \nu^f g \cos \alpha - \frac{2}{3} \frac{\partial(\rho_0^f \nu^f k^f)}{\partial y} = 0 \quad (4)$$

Fluctuation Kinetic Energy Balance

Particulate Phase

$$\rho_0 \nu \frac{\partial k}{\partial t} = \mu^T \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left(\frac{\mu^T}{\sigma^k} \frac{\partial k}{\partial y} \right) - \rho_0 \nu \epsilon + 2D_0(ck^f - k) \quad (5)$$

Fluid Phase

$$\rho_0^f \nu^f \frac{\partial k^f}{\partial t} = \mu^{fT} \left(\frac{\partial u^f}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left[\left(\mu^f + \frac{\mu^{fT}}{\sigma^k} \right) \frac{\partial k^f}{\partial y} \right] - \rho_0^f \nu^f \epsilon^f + 2D_0(k - k^f) \quad (6)$$

Global Continuity Equation

$$\nu^f + \nu = 1. \quad (7)$$

In equations (1)-(7), u is the mass-weighted average velocity, k is the fluctuation kinetic energy per unit mass, ϵ is the dissipation rate per unit mass, p^f is the mean pressure in the fluid phase, ν is the solid volume fraction, ρ_0 is the constituent density, μ is the coefficient of viscosity, μ^T is the coefficient of turbulence (eddy) viscosity, g is the acceleration of gravity, α is the angle of inclination, and σ^k is the turbulence Prandtl number for fluctuation kinetic energy. The superscript f refers to the fluid phase and a symbol without a superscript represents a particulate phase quantity.

Here the coefficients of turbulence viscosity and the dissipation rate for particulate phase are given as:

$$\mu^T = C^{*\mu} C^\mu \rho \nu d k^{1/2}, \quad \epsilon = a k^{3/2}, \quad (8)$$

where

$$\begin{aligned} C^\mu &= 0.0853[(\chi\nu)^{-1} + 3.2 + 12.1824\nu\chi], \\ C^{*\mu} &= \frac{1}{1 + \frac{T_L}{\tau}(1 - \frac{\nu}{\nu_m})^3}, \\ a &= \frac{3.9\nu\chi(1 - r^2)}{d}, \end{aligned} \quad (9)$$

where $\chi(\nu)$ is the radial distribution function. Throughout this paper, it is assumed that the particles are spherical and nearly elastic with a diameter d and a coefficient of restitution r . The parameters T_L and τ are the Lagrangian time macro-scale and the particle relaxation time, and are given as:

$$T_L = \frac{0.165k^f}{\epsilon^f}, \quad \tau = \frac{\rho_0\nu}{D_0}. \quad (10)$$

The drag coefficient D_0 is given as:

$$D_0 = \frac{18\mu_0^f\nu}{d^2} \frac{[1 + 0.1(Re_d)^{0.75}]}{(1 - \frac{\nu}{\nu_m})^{2.5\nu_m}}, \quad (11)$$

in which the particle Reynolds number is defined as:

$$Re_d = \frac{\rho^f d |u^f - u|}{\mu_0^f}. \quad (12)$$

The coefficient $C^{*\mu}$ is introduced in equation (8) to account for the reduction of collisional effect as particle relaxation time becomes small.

For spherical particles, Ma and Ahmadi (1986) found that:

$$\chi = \frac{1 + 2.5\nu + 4.5904(\nu)^2 + 4.515439(\nu)^3}{[1 - (\frac{\nu}{\nu_m})^3]^{0.678021}}, \quad (13)$$

with $\nu_m = 0.64356$. The increase in the particulate pressure is accounted for through the parameter γ which is given as:

$$\gamma = \frac{2}{3}(1 + 4\nu\chi) + \frac{1}{3}(1 - r^2). \quad (14)$$

In equation (5), the coefficient c is related to the ratio of the particle relaxation time to the Lagrangian time macro-scale of turbulence. i.e.,

$$c = \frac{1}{1 + \frac{\tau}{T_L}}. \quad (15)$$

For the phasic coefficients of mean viscosity, the following expressions as suggested by Abu-Zaid and Ahmadi (1989) are used:

$$\mu = \frac{\nu\mu_0^f}{(1 - \frac{\nu}{\nu_m})^{2.5\nu_m}}, \quad \mu^f = \frac{\nu^f\mu_0^f}{(1 - \frac{\nu}{\nu_m})^{2.5\nu_m}}. \quad (16)$$

These coefficients of mean viscosity are similar to those suggested by Ishii and Mishima (1984) and Sinclair and Jackson (1989), and are consistent with Einstein's equation for effective viscosity of dilute suspensions.

Note that the original expression for the phasic momentum interactions described by Ahmadi and Ma (1990) included the lift force. However, in equations (1) and (3), the effect of the lift force is neglected.

Eliminating $\frac{dv^f}{dy}$ between equations (2) and (4), it follows that

$$F_1 \frac{d\nu}{dy} = F_2, \quad (17)$$

where F_1 , F_2 , C_1 and C_2 are given as:

$$F_1 = C_2\nu k^f + (1 - \nu)\nu k \frac{d\gamma}{d\nu} + (1 - \nu)\gamma k, \quad (18)$$

$$F_2 = C_1(1 - \nu)\nu + C_2\nu(1 - \nu) \frac{dk^f}{dy} - (1 - \nu)\nu\gamma \frac{dk}{dy}, \quad (19)$$

$$C_1 = (\frac{\rho_0^f}{\rho_0} - 1)g \cos \alpha \quad (20)$$

$$C_2 = \frac{2}{3} \frac{\rho_0^f}{\rho_0}. \quad (21)$$

Cao and Ahmadi (1994a,b) described a suitable transport equation for the fluid phase energy dissipation rate which is valid up to the solid walls. Accordingly, the turbulence viscosity is given as:

$$\mu^{fT} = C^{\mu f} \rho_0^f \nu^f f^{\mu f} (k^f)^2 / \tilde{\epsilon}^f, \quad (22)$$

where the dissipation variable $\tilde{\epsilon}^f$ satisfies the following transport equation:

$$\rho_0^f \nu^f \frac{\partial \tilde{\epsilon}^f}{\partial t} = f_1 C^{\epsilon 1} \frac{\tilde{\epsilon}^f}{k^f} \mu^{fT} \left(\frac{\partial u^f}{\partial y} \right)^2 - f_2 C^{\epsilon 2} \frac{\tilde{\epsilon}^{f2}}{k^f} + E + \frac{\partial}{\partial y} [(\mu^f + \mu^{fT} / \sigma^{\epsilon f}) \frac{\partial \tilde{\epsilon}^f}{\partial y}]. \quad (23)$$

The fluid phase energy dissipation ϵ^f is given as:

$$\epsilon^f = \tilde{\epsilon}^f + \epsilon_0^f, \quad (24)$$

where ϵ_0^f is the extra dissipation. In equations (22) and (23), the damping functions $f^{\mu f}$, f_1 , f_2 , E and ϵ_0^f are given as:

$$f^{\mu f} = 1 - e^{-0.0115y^+}, \quad (25)$$

$$f_1 = 1, \quad (26)$$

$$f_2 = 1 - 0.22e^{-(Re_T/6)^2}, \quad (27)$$

$$E = -\frac{2\mu^f}{\rho_0^f \nu^f} \frac{\tilde{\epsilon}^f}{y^2} e^{-y^+/2}, \quad (28)$$

$$\epsilon_0^f = 2 \frac{\mu^f}{\rho_0^f \nu^f} \frac{k^f}{y^2}, \quad (29)$$

where Re_T and y^+ are defined as

$$Re_T = \frac{\rho_0^f \nu^f k^{f2}}{\tilde{\epsilon}^f \mu^f}, \quad y^+ = \frac{\rho_0^f \nu^f u^* y}{\mu^f}, \quad (30)$$

in which $u^* = \sqrt{\tau_w / \rho_0^f \nu^f}$ is the shear velocity, and τ_w is the gas shear stress at the wall.

The coefficients $C^{\mu f}$, $C^{\epsilon 1}$, $C^{\epsilon 2}$, and $\sigma^{\epsilon f}$ are given by:

$$C^{\mu f} = 0.09 C^{*\mu f}, \quad C^{\epsilon 1} = 1.35, \quad C^{\epsilon 2} = 1.80, \quad \sigma^{\epsilon f} = 1.3 \quad (31)$$

In equation (31), the coefficient $C^{*\mu^f}$ is also introduced to account for the effect of higher particulate solid volume fraction on damping the fluid turbulence, and is given as:

$$C^{*\mu^f} = \frac{1}{1 + \frac{\tau}{T_L}(\frac{\nu}{\nu_m})^3} \quad (32)$$

In the absence of particulate phase, i.e., $\nu = 0$, C^{μ^f} approaches to 0.09, and the model reduces to that of Chien (1982).

Boundary Conditions

The global conservation of mass implies that

$$m = \frac{\int_0^H \rho_0 \nu u dy}{\int_0^H \rho_0^f \nu^f u^f dy} = \text{const.} \quad (33)$$

where m is the ratio of particle-to-gas mass flow rates (loading), and H is the height of the mixture layer on the chute.

At the solid chute surface, no-slip boundary conditions for fluid phase are used. These are

$$u^f = 0, \quad k^f = 0, \quad \tilde{\epsilon}^f = 0, \quad \text{at } y = 0. \quad (34)$$

For the particulate phase, the frictional boundary conditions suggested by Jenkins (1992) are adopted. These boundary conditions were obtained based on the balance of tangential momentum and fluctuation kinetic energy of granular particles colliding with a solid wall. Accordingly,

$$S = -\hat{\mu}_w N, \quad (35)$$

and

$$K = -\frac{3}{8} N \sqrt{2k} \left[\frac{7}{2} (1 + r_w) \mu_w^2 - (1 - r_w) \right], \quad (36)$$

where $\hat{\mu}_w$ and r_w are the coefficients of friction and restitution for particle-wall collisions. Here, the particulate shear stress S , the normal stress N and the fluctuation kinetic energy flux K are given as:

$$N = \frac{\rho_0 \nu k}{3} [2(1 + 4\nu\chi) + (1 - r^2)], \quad (37)$$

$$S = (\mu + \mu^T) \frac{\partial u}{\partial y}, \quad (38)$$

$$K = -\kappa \sqrt{k} \frac{\partial k}{\partial y}, \quad (39)$$

where

$$\kappa = 0.0711 \rho_0 d (1 + r^2) (\chi^{-1} + 4.8\nu + 12.1184\nu^2\chi). \quad (40)$$

At the free surface, the total shear stress should be zero.

$$(\mu + \mu^T) \frac{\partial u}{\partial y} + (\mu^f + \mu^{fT}) \frac{\partial u^f}{\partial y} = 0. \quad (41)$$

Malekzadeh and Savage (1993a) and Malekzadeh (1993b) reported a series of experimental data for two-phase glass bead-liquid mixtures down an inclined chute. They found that the velocity profiles are usually convex, which implies that

$$\frac{\partial u}{\partial y} \geq 0, \quad \frac{\partial u^f}{\partial y} \geq 0. \quad (42)$$

Since the phasic viscosity is non-negative, equations (41) and (42) imply that

$$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial u^f}{\partial y} = 0, \quad \text{at } y = H. \quad (43)$$

In addition, we assume that the fluctuation energy fluxes for the particulate and fluid phase also vanish at the free surface. i.e.,

$$\frac{\partial k}{\partial y} = 0, \quad \frac{\partial k^f}{\partial y} = 0, \quad \text{at } y = H. \quad (44)$$

Furthermore, the dissipation rate satisfies

$$\frac{\partial \tilde{\epsilon}^f}{\partial y} = 0, \quad \text{at } y = H. \quad (45)$$

NUMERICAL PROCEDURES

Differential equations (1), (3), (5), (6) and (23) together with algebraic equations (7), (8) and (17) form a set of eight equations for determining the eight unknowns u , u^f , k , k^f , ν , ν^f , ϵ and $\tilde{\epsilon}^f$. The equations are discretized using the forward differencing in time and central differencing in space. Since the governing equations are parabolic in nature, they can be solved numerically by an iterative forward time

marching procedure. That is, results at the previous time step are used to evaluate the values at the next time step, and the iteration is continued until the final steady solution is reached. The details of the numerical procedure used may be found in the work of Cao and Ahmadi (1994a,b).

VISCOUS FLOW REGIME

The governing equations presented above are for general two-phase solid-liquid flows down an inclined chute in a state of turbulent motion. At low mean velocities, the turbulence fluctuation and turbulent stresses in the fluid phase become negligibly small, and the liquid resistance becomes limited to only viscous stresses. At high concentrations, however, the particulate collisional and fluctuation stresses may still remain significant. In this case, the fluctuation kinetic energy of fluid phase k^f and the fluid phase fluctuation stresses are neglected, and equation (6) for transport of k^f is ignored. The corresponding phasic velocity profiles, particulate solid volume fraction profile and particulate fluctuation kinetic energy profiles are then evaluated by solving equations (1)-(5) and (7)-(8).

Homogeneous Mixture Approximation

Homogeneous mixture assumption has often been used in the literature as a simplifying approximation for analyzing two-phase flows. Such an assumption is reasonable when the solid volume fraction is roughly constant and the mean phasic velocities are nearly identical. For sake of comparison, this limit case is described in this section.

Assuming a steady state condition and adding equations (1) and (3), for $u^f \approx u \approx \bar{u}$, the resulting equation becomes

$$\frac{\partial^2 \bar{u}}{\partial^2 y} = -\frac{\rho_e g \sin \alpha}{\mu_e}, \quad (46)$$

where ρ_e and μ_e are the constant effective density and viscosity of the mixture, and are given as

$$\rho_e = \rho\nu + \rho^f \nu^f, \quad (47)$$

$$\mu_e = \mu + \mu^f + \mu^T. \quad (48)$$

The exact mean velocity profile then is given by

$$\bar{u}(y) = -\frac{\rho_e g \sin \alpha}{2\mu_e}(2Hy - y^2). \quad (49)$$

Equation (46) and the corresponding solution given by (49) is for an inclined wide chute with a flow height H . For the flow down an inclined chute with a finite width, a , and a height H , the equation governing the mean velocity $u(y, z)$ is given as

$$\left(\frac{\partial^2}{\partial^2 y} + \frac{\partial^2}{\partial^2 z}\right)\bar{u} = -\frac{\rho_e g \sin \alpha}{\mu_e}. \quad (50)$$

The exact solution to equation (50) satisfying no slip boundary conditions at the side walls ($z = 0$ and $z = a$) and chute surface ($y = 0$), and zero slope at the free surface ($y = H$) is given by

$$\bar{u}(y, z) = -\frac{16\rho_e g \sin \alpha}{\pi^4 \mu_e} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{(2m-1)\pi z}{a} \sin \frac{(2n-1)\pi y}{2H}}{(2m-1)(2n-1)\left[\frac{(2n-1)^2}{4H^2} + \frac{(2m-1)^2}{a^2}\right]}. \quad (51)$$

At the chute mid-section $z = a/2$, $\bar{u}$ becomes

$$\bar{u}(y) = -\frac{16\rho_e g \sin \alpha}{\pi^4 \mu_e} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+1} \sin \frac{(2n-1)\pi y}{2H}}{(2m-1)(2n-1)\left[\frac{(2n-1)^2}{4H^2} + \frac{(2m-1)^2}{a^2}\right]}. \quad (52)$$

Equation (52) provides an analytical expression for the mean velocity profile at the centerline of a finite chute within the limit of homogeneous flow approximation.

RESULTS AND DISCUSSIONS

For several loading ratios (particle-to-liquid mass flow ratios), m , various particle diameters, d , different liquids and various chute angles, the particle-liquid mixture flows down an inclined chute are studied. The mean velocity, solid volume fraction and fluctuation kinetic energy profiles are evaluated and are compared with the available experimental data. The phasic normal and shear stress profiles and the fluctuation kinetic energy production and dissipation rates are also evaluated and discussed. The cases that the liquid phase in a viscous or a turbulent flow regime are treated separately.

Viscous Fluid Regime

The low speed high concentration solid-liquid flows down inclined chutes with the liquid phase being in the viscous flow regime for different inclination angles are first simulated. The simulation results are shown in figures 2-6 and are compared with the experimental data of Malekzadeh and Savage (1993a) and Malekzadeh (1993b). Malekzadeh's experiments were carried out for a chute with a length of 1.1 *m*, a width of 5.1 *cm* and a depth of 8 *cm*. Glass plates were used as the sidewalls of the flume and the bottom was a replaceable rough aluminum surface. The solid particles were 3-mm glass beads with a density of $\rho_0 = 2485 \text{ kg/m}^3$ and the liquid was a mixture of 80% Alkyl Benzyl Phtalate and 20% Dibutyl Benzyl Phtalate. This mixture had a density of $\rho_0^f = 1065 \text{ kg/m}^3$ and a viscosity of $\mu_0 = 0.0455 \text{ Ns/m}^2$. These values of experimental parameters were used in the computer simulation. In addition, a particle-particle restitution coefficient of $r = 0.5$, a particle-wall restitution coefficient of $r_w = 0.45$ and a particle-wall friction coefficient of $\hat{\mu}_w = 0.02$ were also assumed.

For the 3 mm glass beads-liquid mixture flowing down an inclined chute with an inclination of 13° , the simulated phasic velocity, solid phase fluctuation kinetic energy and solid volume fraction profiles are shown in figure 2. Here the flow height is $H = 14.5 \text{ mm}$ and the loading ratio is $m = 3.65$ (which corresponds to an average solid volume fraction of $\nu_a = 0.57$). Figure 2a shows that in the region close to the chute bed, the velocity gradient is rather large and the velocity profile becomes quite flat near the free surface. This figure also shows that there exists an slip velocity of about 0.02 m/s for the particulate phase at the wall, while the slip between the two phases is generally very small. A careful examination of figure 2a indicates that the particulate velocity is slightly larger than that of the liquid phase.

The experimental data of Malekzadeh and Savage (1993a) and Malekzadeh (1993b) for the chute mid-section (at $z=2.5 \text{ cm}$) for the particulate phase velocity profile are reproduced in figure 2a for comparison. It is observed that the predicted velocity is larger than the experimental data. This is mainly because the present simulation is for a wide chute, while the experimental data was for a narrow rectangular cross

section chute. The sidewalls of the experimental chute significantly retard the flow which is not accounted for in the present wide-chute simulation.

The analytical solutions given by equations (49) and (51) for the homogeneous mixture approximation are also shown in figure 2a for comparison. The corresponding effective density is, $\rho_e = 1873 \text{ kg/m}^3$, and the effective viscosity as calculated from equation (48) is $\mu_e = 2.74 \text{ Ns/m}^2$. Figure 2a clearly shows that the analytical solution for the wide-chute agrees with our numerical simulation quite well, while the solution for the rectangular cross-section chute is in reasonable agreement with the experimental data.

The variation of particulate fluctuation kinetic energy k is shown in figure 2b. It is observed that while the flow is in (low speed) viscous regime, the particulate fluctuation kinetic energy is relatively significant (particularly in the lower half of the channel). This figure also shows that the particulate fluctuation kinetic energy has a peak value near the wall, and then decreases rapidly toward the free surface. Figure 2c presents the comparison between the predicted solid volume fraction and the experimental data of Malekzadeh and Savage (1993a) and Malekzadeh (1993b). It is observed that the agreement between the numerical simulation results and the experimental data is quite good. This figure also shows that the variation of solid volume fraction is quite small and ν is nearly constant across the depth of the flow. This is perhaps the reason for the success of the homogeneous flow approximation in providing reasonably accurate results in this case.

Figure 3 presents the variations of the corresponding phasic mean viscosities μ and μ^f and the particulate phase collisional and fluctuation viscosity μ^T . It is observed that μ and μ^f are nearly constant, while μ^T first increases to its maximum value at a short distance from the wall, and then decreases toward the free surface. Comparing with μ and μ^f , however, μ^T is relatively small with its peak being about 5% of μ . This means that the effect of particle's collision is not very significant in such low speed but dense two-phase flows. The effective viscosity μ_e is shown in figure 3c, indicating that μ_e is roughly constant across the chute. This observation further justifies the use of an effective constant viscosity in equations (46) and (50).

Variations of the phasic normal and shear stresses, as well as the total normal and shear stresses are shown in figure 4. Note that the normal stresses shown here are the excess stresses beyond the hydrostatic pressure p^f , which is of the order of 270 N/m^2 near the wall. Since the liquid phase is essentially in the laminar flow regime with negligible fluctuation kinetic energy, the liquid phase excess normal stress is zero. Thus, the total normal stress is the same as the particulate normal stress as shown in figure 4a. The particulate (total) normal stress has a peak value near the bottom wall and then decreases toward the free surface. The phasic and total shear stress profiles are shown in figure 4b. The fluid phase shear stress has a large value at the solid wall and monotonically decreases toward the free surface, while the particulate phase shear stress first increases to a peak value near the wall and then decreases toward the free surface. As is expected, the total shear stress has a linear variation across the chute.

Using the present model, two other chute flows with the inclination angles of 15° and 17° are studied. The flow heights are, respectively, 11 mm and 10 mm in these cases. All other parameters are kept identical to those used earlier for figure 2. The corresponding phasic velocity, fluctuation kinetic energy and solid volume fraction profiles are shown in figures 5 and 6, respectively. From these figures, it is observed that the variations of phasic velocity, fluctuation kinetic energy and solid volume fraction profiles are quite similar to those shown in figure 2. The experimental data for the particulate velocity at the chute centerline as reported by Malekzadeh and Savage (1993a) and Malekzadeh (1993b) and the analytical solutions given by equations (49) and (51) for a wide and a rectangle cross-section chute are also presented in these figures for comparison. Values of $\rho_e = 1845 \text{ kg/m}^3$ and $\mu_e = 1.25 \text{ Ns/m}^2$ for the condition of figure 5 and $\rho_e = 1838 \text{ kg/m}^3$ and $\mu_e = 1.11 \text{ Ns/m}^2$ for the conditions of figure 6 were used in the analytical expression for the homogeneous mixture approximation. It is observed the agreement of model predictions with the experimental data is quite good. Unfortunately, no experimental data for fluctuation kinetic energy and solid volume fraction were reported for comparison.

Turbulence Regime

The results concerning turbulent flows of solid-liquid mixtures down an inclined chute under different conditions are reported in this section. The specific case of glass beads-water mixture flows is analyzed in details. A particle-particle restitution coefficient of $r = 0.55$, a particle-wall restitution coefficient of $r_w = 0.5$ and a particle-wall friction coefficient of $\hat{\mu}_w = 0.05$ are used in the analysis. The restitution coefficient used are in the range of values reported in the experimental measurements of Abu-Zaid and Ahmadi (1993) for glass beads in water.

For a mixture of 0.5 mm glass beads-water flowing down a 15° inclined chute, the predicted phasic velocity, fluctuation kinetic energy and solid volume fraction profiles are shown in figure 7. Here the flow height is 3 cm and loading ratio is $m=0.125$. The velocity profiles shown in figure 7a indicate that there exists considerable slip for the particulate phase at the wall and the fluid velocity profile has a very sharp gradient at the wall. The slip velocity between the particulate and liquid phases also increases in comparison with the viscous case described earlier. This figure also shows that the particulate phase slightly lags the liquid phase. This is in contrast to viscous case where the particles move somewhat faster than the liquid.

Figure 7b shows that the particulate fluctuation kinetic energy is less than that of water phase. Both the particulate and liquid phase fluctuation kinetic energy profiles have a peak value in the vicinity of the wall and then decrease with distance from the wall. While the fluid fluctuation kinetic energy exhibits a monotonic decrease toward the free surface, that of the particulate phase decreases to a minimum and then increases to a constant level near the free surface.

The solid volume fraction profile shown in figure 7c indicates that the particle concentration tends to increase with distance from the wall to a maximum, and then decrease very rapidly toward the free surface. In the upper half of the flow height, the solid volume fraction becomes very small because of the gravitational sedimentation effects.

Figure 8 presents the variations of the phasic coefficients of viscosity μ , μ^f , μ^T

and μ^{fT} across the chute. It is observed that the mean coefficient of viscosity μ and μ^f first increase with the distances from the wall, and then decreases toward the free surface. The combined collisional and kinetic particulate viscosity, μ^T , is nearly constant with a slightly increase near the wall. The fluid turbulence viscosity, μ^{fT} , however, monotonically increases with the distance from the wall. The collisional and turbulence viscosity μ^T and μ^{fT} are much larger than μ and μ^f , with μ^{fT} playing the dominant role.

The corresponding phasic normal and shear stress profiles, as well as the total normal and shear stresses are shown in figure 9. From figure 9a, it is observed that the phasic excess normal stresses are large at the wall, and decrease toward the free surface. The particulate normal stress is much smaller than that of the liquid phase. Figure 9b indicates that the particulate phase shear stress is very large at the chute surface, and decreases rapidly to a small value at short distance from the wall. The liquid phase shear stress has an increasing trend near wall, reaches to its maximum near the wall, and then decreases almost linearly to zero at the free surface. The total shear stress shown in figure 9c has a linear variation across the chute. The total excess normal stress also has a decreasing trend toward the free surface.

Figure 10 presents the phasic energy production and dissipation profiles. It is observed that both the particulate and the water phasic fluctuation kinetic energy productions and dissipations are quite large at the wall, and decrease rapidly with the distance from the wall. This figure also indicates that the energy production of each phase is nearly balanced by its energy dissipation rate and the amount of fluctuation energy transfer between the phases is relatively small.

For the 2 mm glass bead-water mixtures flowing down a 15° chute at the mass loading ratio of $m = 0.145$, the phasic mean velocity, fluctuation kinetic energy and solid volume fraction profiles are shown in figure 11. The general variations of phasic mean velocity, fluctuation kinetic energy and solid volume fraction are very similar to those shown in figure 7. As the particle diameter increases from 0.5 mm to 2 mm, however, the slip velocity between the two phases increases and the particulate velocity lags further behind liquid phase velocity. This is because the drag force

per unit mass exerted on the particles decreases as the particle diameter increases. Both particulate and water phase fluctuation kinetic energy increase as the particle size increases. Figure 11c shows that for relatively large particles, the solid volume fraction reaches to its maximum value at the chute bottom, and tends to decrease roughly monotonically toward the free surface. This is in contrast to the variation of solid volume fraction distribution near the chute surface for small particles.

The corresponding variations of phasic energy production and dissipation rates for the 2 mm glass bead-water mixtures are shown in figure 12. It is observed that both the particulate and liquid phase energy production and dissipation are very large at the wall, and decrease very rapidly toward the free surface. However, the particulate energy production is larger than its energy dissipation rate in the entire region. This indicates that the fluctuation kinetic energy is being transferred away from the particulate phase, and is being dissipated by the particle-fluid interaction and fluid turbulence. For fluid phase, the dissipation rate slightly exceeds the energy production rate.

Figure 13 presents the variation of the phasic velocity, fluctuation kinetic energy and solid volume fraction profiles for the 2 mm glass bead-water mixtures flowing down a 17° inclined chute. In this case, the loading ratio is $m = 0.167$. Variations of the phasic mean velocity, the fluctuation kinetic energy and the solid volume fraction profiles generally follow the same pattern shown in figure 11. It is also observed that as the inclination angle increases, both the particulate and liquid phase mean velocities, as well as the slip velocity of the particles at the wall increase. The phasic particulate and water fluctuation kinetic energies and the solid volume fraction also increase as the inclination angle increases.

CONCLUSIONS

The thermodynamically consistent two-phase flow model is used to simulate the flow of glass bead-fluid mixtures down an inclined chutes. The model predictions for flows in viscous regime are compared with the available experimental data. Based on the presented results, the following conclusions may be drawn:

1. The model predictions for the mean particulate velocities in the viscous regime are in good agreement with the experimental data of Malekzadeh and Savage (1993) and Malekzadeh (1993).
2. While for two-phase chute flows in the viscous regime, the viscous stresses dominate, the particulate collisional and kinetic stresses still plays a significant role.
3. For viscous two-phase flows, the concentration of particles across the chute is roughly constant. While the fluid phase fluctuation kinetic energy k^f is negligible, the particulate fluctuation kinetic energy is still significant.
4. In the viscous regime, there is a noticeable particle slip velocity at the chute wall. The particulate phase mean velocity is also slightly larger than that of the fluid phase.
5. For turbulent two-phase flow, the concentration of particles reaches its maximum at some distances away from the wall and then decreases to a very small value toward the free surface. The particulate and fluid fluctuation kinetic profiles also have peaks near the wall, and then decrease with the distance from the wall.
6. For turbulent regime, there is a significant slip velocity at the chute wall for the particulate phase. The slip between the particulate and the fluid phases is relatively large. The particulate mean velocity, however, is less than that of the fluid phase. As the particle size increases, the relative slip velocity increases and the particulate mean velocity lags further behind the fluid phase velocity.
7. For two-phase turbulent flows, as the inclination angle increases, both the particulate and fluid mean velocity increase. The particulate and fluid phase fluctuation kinetic energies and solid volume fraction also increase with the inclination angle.

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Figure 1. Flow Field Schematic

Figure 2: Variations of mean velocity, fluctuation kinetic energy and solid volume fraction profiles for 3 mm glass bead-liquid (20% DBP and 80% ABP) mixture down an inclined chute with 13° inclination. Comparison with the data of Malekzadeh (1993) and 1-D and 2-D analytical solutions.

$$\alpha = 13^\circ$$

$$m = 3.35, d = 3 \text{ mm}$$

Figure 3: Variations of phasic mean viscosity μ , μ^f , particulate eddy viscosity μ^T and effective viscosity μ_e for 3 mm glass bead-liquid (20% DBP and 80% ABP) mixture down an inclined chute with 13° inclination.

$$\alpha = 13^\circ$$

$$m = 3.35, d = 3 \text{ mm}$$

Figure 4: Variations of phasic normal and shear stress profiles and total normal and shear stress profiles for 3 mm glass bead-liquid (20% DBP and 80% ABP) mixture down an inclined chute with 13° inclination.

Figure 5: Variations of mean velocity, fluctuation kinetic energy and solid volume fraction profiles for 3 mm glass bead-liquid (20% DBP and 80% ABP) mixture down an inclined chute with 15° inclination. Comparison with the data of Malekzadeh (1993) and 1-D and 2-D analytical solutions.

Figure 6: Variations of mean velocity, fluctuation kinetic energy and solid volume fraction profiles for 3 mm glass bead-liquid (20% DBP and 80% ABP) mixture down an inclined chute with 17° inclination. Comparison with the data of Malekzadeh (1993) and 1-D and 2-D analytical solutions.

Figure 7: Variations of mean velocity, fluctuation kinetic energy and solid volume fraction profiles for 0.5 mm glass bead-water mixture down an inclined chute with 15° inclination.

$$\alpha=15^\circ$$

$$m=0.125, d=0.5 \text{ mm}$$

Figure 8: Variations of phasic mean viscosity μ , μ^f and phasic eddy viscosity μ^T , μ^{fT} for 0.5 mm glass bead-water mixture down an inclined chute with 15° inclination.

$$\alpha = 15^\circ$$

$$m = 0.125, d = 0.5 \text{ mm}$$

Figure 9: Variations of phasic normal and shear stress profiles and total normal and shear stress profiles for 0.5 mm glass bead-water mixture down an inclined chute with 15° inclination.

$$\alpha = 15^{\circ}$$

$$m = 0.125, d = 0.5 \text{ mm}$$

----- Energy dissipation

———— Energy production

Figure 10: Variations of energy production and dissipation profiles for 0.5 mm glass bead-water mixture down an inclined chute with 15° inclination.

Figure 11: Variations of mean velocity, fluctuation kinetic energy and solid volume fraction profiles for 2 mm glass bead-water mixture down an inclined chute with 15° inclination.

$$\alpha = 15^{\circ}$$

$$m = 0.145, d = 2 \text{ mm}$$

----- Energy dissipation

———— Energy production

Figure 12: Variations of energy production and dissipation profiles for 2 mm glass bead-water mixture down an inclined chute with 15° inclination.

Figure 13: Variations of mean velocity, fluctuation kinetic energy and solid volume fraction profiles for 2 mm glass bead-water mixture down an inclined chute with 17° inclination.